THE NAVSTAR GLOBAL POSITIONING SYSTEM

Operation, Accuracy And Error Analysis.

by

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The United States Department of Defense, of course, should also be included in the "without whom" file, because they have provided a very interesting topic for study. Furthermore, thanks are also due to all those companies who kindly provided advertising brochures and/or scientific papers which dealt with the Global Positioning System. In this category, of particular note are GPS Survey Services, Measurement Devices Limited and Wimpol UK, (no "order of merit" implied or intended!) who were kind enough to discuss their experiences with GPS in detail.

ABSTRACT

The NAVSTAR Global Positioning System (GPS) is a United States Department of Defense satellite survey system which is being developed to replace the present TRANSIT system in the 1990's. When fully operational, the system will enable virtually instantaneous and continuous position to be determined throughout the world, 24 hours a day.

This dissertation aims to examine the operation of GPS, with particular interest in firstly the various sources of error within the system, and secondly methods of reducing or negating such errors. This leads to an examination of expected final accuracies for GPS.

The first three chapters discuss Global Positioning System design and operation in terms of three segments. The Space Segment covers the satellite orbit configuration and the structure of the electromagnetic signals which they transmit. The Control Segment describes how the GPS satellite orbits are obtained, and how predicted orbits are provided for each satellite. The User Segment is both wide and complex, comprising all those methods employed by receivers to obtain a position fix, using the signals transmitted by the GPS satellites. This chapter describes the four main GPS measurement modes (pseudo-range, carrier phase, interferometric delay and integrated Doppler), before introducing the other relevant variables in GPS receivers. Some of the more common GPS receivers are also examined in more detail at this point.

Following a discussion of height determination with respect to GPS, the main sources of error in the system are studied. Observation, ephemeris, clock, refraction, multipath and instrumental errors are all examined, along with the effect that satellite geometry has on the final accuracy.

Methods of improving the accuracy of GPS (for example, various forms of differencing, ionospheric and tropospheric refraction correction, cycle slip detection, antenna design) are then detailed, along with several other possible techniques.

The final chapter examines the variation in GPS accuracies, before comparing The system's performance with other survey systems. The peculiarities of GPS survey procedures are also dealt with, and the factors involved in drafting specifications for a GPS survey. A final section discusses GPS applications, and possible future developments within the system. Appendix A tabulates the major characteristics of 25 GPS receivers.

This dissertation was completed largely by library research, the information being collected from well over 100 articles, papers and brochures which discuss the Global Positioning System.

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List Of Abbreviations.

ABBREVIATION	
C/A	Coarse/Acquisition.
CEP	Circular Error Probability.
CIU	Control/Interface Unit.
cm	centimetres.
DA/R	Data Analyser/Recorder.
DMA	Defense Mapping Agency.
D.o.D.	Department of Defense.
drms	distance root mean square.
EDM	Electronic Distance Measurement.
FOC	Full Operational Capacity.
GDOP	Geometric Dilution Of Precision.
GOES	Geosynchronous Orbiting Earth Satellite.
GPS	Global Positioning System.
HDOP	Horizontal Dilution Of Precision.
INS	Inertial Navigation System.
IOC	Interim Operational Capacity.
ISS	Inertial Survey System.
ISTAC	International SERIES Technology Applications Corporation.
km	kilometres.
maser	Microwave Amplification by StimulatedEmission of Radiation.
MCS	Master Control Station.
MITES	Miniature Interferometric Terminals for Earth Surveying.
mm	millimetres.
NAD	North American Datum.
NATO	North American Treaty Organisation.
NAVSTAR	NAVigation Satellite Time And Range.
nsecs	nanoseconds.
NSWC	Naval Surface Weapons Centre.
P	Precise.
PDOP	Position Dilution Of Precision.
ppm	parts per million.
PPS	Precise Positioning Service.
rms	root mean square.
SERIES	Satellite Emission Radio Interferometric Earth Surveying.
SLR	Satellite Laser Ranging.
SPS	Standard Positioning Service.
TACAN	TActical Air Navigation.
TDOP	Time Dilution Of Precision.
TEC	Total Electron Content.
USGS	United States Geological Survey.
UTC	Coordinated Universal Time.
UTM	Universal Transverse Mercator.

VDOP	Vertical Dilution Of Precision.
VHF	Very High Frequency.
VLBI	Very Long Baseline Interferometry.
VOR/DME	VHF Omnidirectional Range/Distance Measurement Equipment.
WGS	World Geodetic System
WVR	Water Vapour Radiometer.

CHAPTER ONE - INTRODUCTION

1.1. Project Background And Aims.

Triangulation and spirit levelling, coupled with observations to the sun or stars, for long was the only means of determining position accurately on the Earth's surface, with respect to a defined origin and coordinate system. The development of quick and accurate electromagnetic distance measurement (EDM) systems subsequently allowed trilateration and traversing to supercede triangulation as the most effective means of accurate horizontal position coordination. Over the last 20 years the survey profession has seen the intoduction of the next generation of control, and detail, positioning; that of satellite observation.

Absolute or relative position on (or near) the surface of the Earth can be procured by taking observations to satellites which orbit the Earth. The first satellite theory was derived using the Earth's natural satellite, the moon. Observations included timing eclipses of the moon, and recording the occultation of stars behind the moon. The latter method was employed in the 1960's to determine the position of the Hawaiian Islands, with respect to the United States' coast. The development of artificial satellites quickly rendered observations of the moon obsolete for two main reasons. Firstly a satellite's point image facilitated more accurate pointing. Secondly, the greater number of satellites allowed more observations than were possible to the moon during any similar time period. Further developments introduced electronic "observations" , using electromagnetic wave theory, to artificial satellites. Much present day satellite theory however, is still based on the early observations to the moon.

Position fixing using electronic observations to artificial satellites has become increasingly common for several reasons:

a) Unlike the theodolite/EDM techniques, intervisibility is not required between the survey stations, and so points much further apart can be coordinated, or longer distances measured, than is possible by the more "traditional" methods.
b) This form of positioning has an all weather, continuous operation capability, which cannot be said for optical survey systems, including field astronomic techniques (which of course do not suffer the intervisibility problem).
c) Whereas a theodolite based system requires a stable platform upon which to perform the observations, satellite positioning is possible at sea, albeit the results are not as reliable as those which would be obtained with the same system operating in a stable environment.
d) Position determined by satellite observation is three-dimensional. This is also the case in inertial-type systems, however the traditional positioning systems deal with height separately from horizontal position. The main advantage of a three-dimensional approach is the continuity in terms of

measurement and computation; position is treated as a whole, and not as two parts.

The first and last of the above advantages have revolutionised survey practice, in that they allow the control points to be positioned at the points of interest. This cuts out the need to transfer position from control points, which are often few and far between, or non-existent, in many parts of the world, including the oceans.

Artificial satellites have a history of 31 years, during which a multitude of satellites have been developed for a variety of purposes. The so called TRANSIT satellite system was the first to have a major effect on survey practice. This system is now more than 20 years old, and a new system, the Global Positioning System (GPS), is presently being developed to replace it. Many people in the survey and navigation industry are predicting that the latter system will have an enormous impact on their profession. Indeed, the System is already having an effect in some areas. Thus, because satellite surveying is possibly becoming the major technique for control survey and navigation, and because the GPS is the most recent satellite survey system "off the draughting board", the author feels that the System is worthy of study.

The period of study for the dissertation was from August 1986 to August 1987. The stucture and content of the project was determined after consulting both the Project Supervisor, Professor Petrie, and the Project Sponsors, Shell International Petroleum Company.

Literally hundreds of papers and articles have been published which deal exclusively with some aspect of the GPS. Almost without exception, however, these articles examine only one small area within the whole subject of GPS, often dealing with the chosen topic in great detail. This is because the subject as a whole is too complex to be encompassed by a brief paper in a scientific journal. Those few articles which do discuss GPS as a whole are necessarily shallow in their study. These papers generally date from early in the history of the system, and are intended to give only a skeletal, introductory outline to GPS.

By 1987, Shell International Petroleum Company had begun to take an interest in GPS, and were keen to learn more of its potential. The Company were particularly interested in the sources of error within the system, and how these could be reduced or negated. Naturally, they also required information concerning the final accuracy obtainable usingGPS.

Thus Shell's interest in a detailed, but overall analysis of GPS errors and accuracy, combined with the lack of such a study by August 1986, largely determined the aims of the project. Initially some form of analysis of a GPS data set (the data being provided by the sponsors) was to be included. Unfortunately this part of the project had to be abandoned when it became obvious that there would be difficulties in obtaining the data in sufficient time for satisfactory statistical analysis. Instead, the methods of increasing GPS accuracy were examined in more detail.

The information contained within this dissertation has been extracted, distilled, compared and contrasted from a large number of papers which discuss some aspect of GPS. Generally speaking, there seem to be three main sources for such articles. Firstly there are papers published in scientific journals. In some cases these articles are easily obtained, however many of the relevant articles have been published in journals printed in the United States of America, and are thus difficult to obtain, unless they have an outlet in Britain. Secondly, there is a large source from companies which manufacture GPS receivers. Again, many of these companies are in the USA, and it can prove difficult to obtain the relevant articles. Thirdly there have been a number of conferences concerning the Global Positioning System, (directly or indirectly) and the proceedings of such are a valuable source of information. Unfortunately, again, these can be difficult to procure.

This dissertation, therefore, would be of particular use to those who have a need to learn about the Global Positioning System, but who do not have the time or wherewithal to obtain and read a sufficient number of relevant papers which would provide the information required. Such people, it is envisaged, might include those in the oil industry, survey industry, and oil service industry (offshore/hydrographic survey). This dissertation might be helpful to those who wish to make an informed decision as to whether or not GPS would be of use to their company.

As previously mentioned, GPS, as a whole, is relatively complex, and so after a brief introduction to the system, some time is spent examining the various aspects of positioning, using GPS. Following a short chapter which deals with height determination, with reference to GPS, the various inherent errors in the system are introduced and discussed. Various means of accuracy enhancement are then examined in detail, before looking at the expected final accuracies of the system. Finally the peculiarities of GPS survey procedures are detailed, along with a discussion of GPS applications, and possible future trends in the system.

Before studying the Global Positioning System, however, it is worthwhile to briefly consider its forerunner, the TRANSIT system.

1.2. TRANSIT.

The TRANSIT system, otherwise known as NAVSAT, or merely "satellite Doppler", was developed at the Applied Physics Laboratory of the John Hopkins University, in the United States of America. TRANSIT became operational in January 1964 and as a military system was designed to update submarine inertial positioning systems. In July 1967, the system was released for commercial exploitation, where it flourished; presently non-military users outnumber their military counterparts by approximately 10 to 1. TRANSIT consists of several satellites orbiting the Earth, a tracking network which observes, computes and predicts each satellite's orbital path, and receiver/processor units located on Earth. At present there are seven Transit satellites, each in polar orbits, circling the Earth at an altitude of 1,075 kilometres. This altitude results in an orbital period of about 107 minutes. For the period that a TRANSIT satellite is above the observer's horizon, the position of the receiving antenna can be obtained. The period between the set of one satellite and the rise of the next is dependent on the observer's latitude. This period is about 30 minutes at 70° N and 70 minutes at the Equator. Thus, for each user the system operates continually, but not continuously, 24 hours a day, in all weather conditions. A single satellite pass (usually lasting about 16 minutes) is sufficient to fix position when navigating, however at least 30 to 40 passes are necessary if an accuracy of one metre is required.

In essence, each satellite is a navigation beacon, which transmits two very stable signals (149.988 MHz and 399.968 MHz), and a navigation message. The latter is a series of 6103 binary digits (bits) which describe the satellite's position, in Keplerian elements, as a function of time. The navigation message is modulated on to the two transmission frequencies, is two minutes long and is transmitted at two minute intervals. Each message begins (and thus also ends) with a timing mark. The last 25 bits of each message form a synchronisation word that identifies the time mark and the start of the next two minute message. Once the receiver has recognised the synchronisation word, it can start to "translate" the remainder of the message. This, in turn, allows the position of the satellite to be determined at each instant of the observation.

To obtain a position fix, the location of the receiver must now be related to the known satellite orbit. This is accomplished by measuring the Doppler shift of the transmitted signals. As the satellite moves closer to the receiver, a greater number of cycles per second are received than were transmitted (the Doppler shift), accounting for the decreasing number of wavelengths between the transmitter and the receiver. In fact, for each wavelength that the satellite moves closer, one extra cycle will be received. This relationship indicates that the Doppler frequency count is a direct measure of the change in distance between the receiver and the satellite, during the Doppler count interval. For a detailed account of the principles of Doppler positioning see Cross (in preparation) or Haugh, Moffitt and Anderson, 1980.

In practice, the TRANSIT system begins with an estimated receiver position, obtained, for example, by dead reckoning. The receiver/processor then calculates the shift required to match the slant ranges predicted by the estimated position, to the slant ranges obtained via the Doppler counts. Receiver motion must also be accounted for by the processor.

TRANSIT is operated by the Navy Astronautics Group at Point Mugu, California. Tracking stations, whose coordinates are accurately known, are located at Prospect Harbour, (Maine), Rosemount, (Minnesota), and Wahiawa (Hawaii). These stations receive the signals as the satellites pass, measure the Doppler frequency shift, and record the Doppler frequency as a function of time. This information is then sent to Point Mugu, where it is employed to determine each satellite's orbit, and to predict each orbit many hours into the future. This information, in turn, is uploaded to each satellite, where it becomes the navigation message. Uploading occurs every 12 hours, and the predicted position of each satellite is accurate to about 20 metres.

A larger, worldwide network of approximately 25 tracking stations provide satellite positional information to the United States Defense Mapping Agency. This information is used to produce a "precise ephemeris" for each satellite, which is only available after the event, but which is accurate to about one metre.

TRANSIT transmits at two frequencies so that a correction for ionospheric refraction can be made. This is discussed, with reference to GPS in sections 5.4 and 6.2.

1.3. Global Positioning System - Historical Background.

Although TRANSIT accuracies have improved from 100 metres initially to one metre by 1972 (Anderle, 1985), the system does have several drawbacks:

a) Accuracy is limited by oscillator uncertainties.

b) The system is not entirely continuous, a position fix being possible only while a TRANSIT satellite is above the observer's horizon.

c) To obtain a position fix of even low order accuracy, at least 16 minutes of observation is required.

d) Higher order ionospheric effects are not accounted for, reducing the final accuracy.

e) At least two satellite passes, typically separated by 90 minutes, are required to remove a near singularity in computing a three-dimensional position, and several days on site are required to reduce the errors to give solutions accurate to one metre.

Despite the supplementary coverage provided by a number of terrestrial radio-navigation aids such as Loran C, Omega, TACAN (TACtical Air Navigation), Ontrac and Decca, the increasing navigation and survey accuracies required by the U.S. Military initiated the development of a second generation satellite survey/navigation system. The U.S. Air Force's 621 B, project, begun in the 1960's, developed into the Global Positioning System program, which is a joint services project involving the U.S. Army, Navy, Marine Corps, Defense Mapping Agency (DMA), Department of Transportation, NATO and Australia. More recently the National Oceanic and Atmospheric Administration and the U.S. Geological Survey (USGS) have also been involved. The system is sometimes also called "NAVSTAR", an acronym for NAVigation Satellite Time And Range.

Three agencies were initially responsible for the development of hardware.

a) The Naval Surface Weapons Center (NSWC) demonstrated the expected measurement precision using a system developed by Standard Telecommunications Incorporated, and subsequently work commenced on the "GEOSTAR TI4100" receiver for a group of agencies comprising the DMA, the National Geodetic Survey and the USGS.

b) The Massachusetts Institute of Technology developed the single frequency MACROMETER, and also a dual frequency receiver.

Three of the dual frequency sets are now on site at three of the POLARIS VLBI (Very Long Baseline Interferometry) sites in Texas, Florida and Massachusetts, and can be used to refine the satellite orbits and connect them to the positions of the VLBI sites.

c) A Jet Propulsion Laboratory initiative led to experimental models of two instruments, the SERIES and SERIES X receivers.

The receivers developed by each of these groups were tested in 1984. The results indicated that centimetre accuracy was possible using each of the models.

1.4. Global Positioning System Operation - An Introduction.

The Global Positioning System satellites transmit two signals, which are generally called the L1 and the L2 signals. The L1 signal is modulated by two codes, the (Coarse/Acquisition code) and the P code (Precise code). These codes are streams of pseudo-random binary digits (see 2.1.2.). The L2 signal carries only the P code. The two codes are further modulated by a formatted navigation message, which contains Keplerian elements of each satellite's predicted orbit. In this respect, GPS is similar to TRANSIT. The navigation message also provides information for relating the GPS time system (which is kept by highly accurate atomic clocks) to Universal Time. This signal and code structure allows position to be determined by four main methods. Depending on which method is employed, the configuration of the receiver/processor will differ, and the achievable final accuracy will also vary.

1.Pseudo-range Measurement.

This is the most common technique of position fixing using GPS, and indeed it is the method for which the system was designed. The other methods are subsequent developments.

Essentially each satellite transmits a signal at a known time, and the time at which the signal reaches the receiver is recorded. The distance between the satellite and the receiver can then easily be calculated. Position is determined by resection, distances being calculated to three or more satellites, usually. One or both codes are used to identify the times of transmission and reception of the signal.

The generation of each code is synchronised to the satellite clock time reference. The ground receiver/processor maintains a time reference which is used to generate a replica of the code transmitted by the satellite; the size of time shift that the receiver/processor system requires to align the replica code with the code received from the satellite is a measure of the signal propagation time from satellite to receiver. The distance between satellite and receiver may then be found by applying the speed of light to the time of propagation. This form of range measurement is termed "pseudo-ranging" and will be in error by the difference between the satellite clock and the receiver clock. Figure 1 illustrates the resection for a two-dimensional position fix (including a small receiver clock error, or "bias"). Here pseudo-

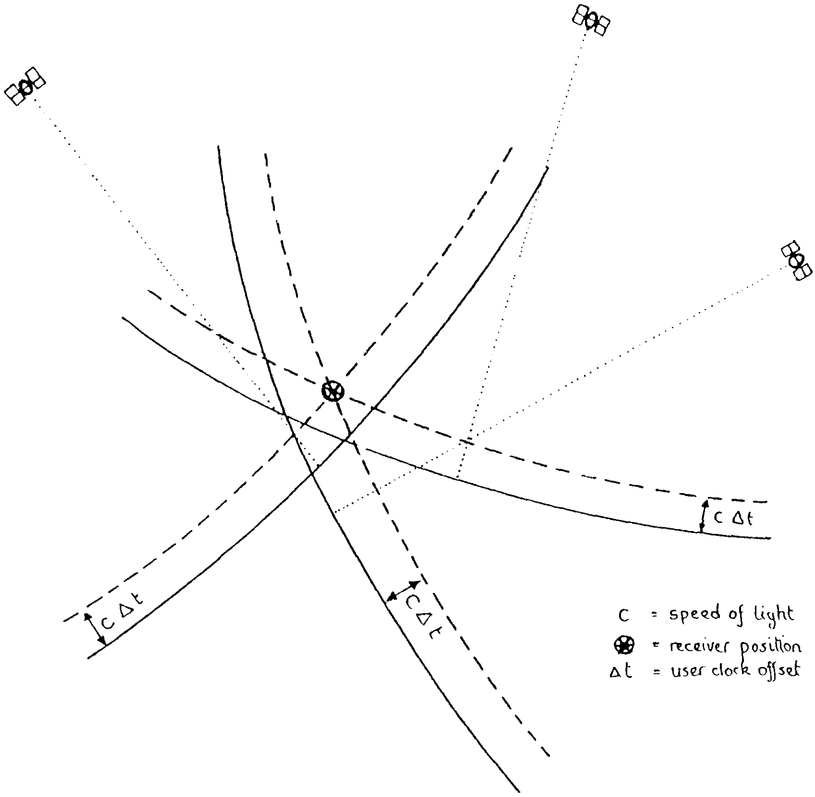


FIGURE 1: TWO-DIMENSIONAL RESECTION USING GPS.

range measurements are made to three satellites. If the receiver's clock had been precisely synchronised with GPS time (and no other errors were present) the intersection of two ranging circles would have provided the receiver's position. There will certainly be at least a small receiver clock bias, however, and so a third pseudo-range is required to solve for this fixed range error. In order to determine three-dimensional position, a pseudo-range to a fourth satellite is necessary. In this case, the initial uncertainty of receiver clock bias results in an error volume, formed by the intersection of four position spheres. Again the fixed range error is determined and added, or subtracted, to the pseudo-ranges to produce a point intersection.

For each known variable (northing, easting, height and receiver time) the user will require one fewer pseudo-range. Obviously satellite position is also required (these being the control "points" at the instant of observation) and so the parameters for this time-dependent variable are included in the satellite transmission (as with TRANSIT). The C/A code, has a frequency 10 times lower than that of the P code, and is designed either to provide accuracy sufficient for navigation purposes, or is employed to acquire the second, more precise, code which gives better resolution.

2.Carrier Phase Measurement.

Receivers have been developed which can measure the phase of the carrier signals. As with EDM systems, this fractional phase value, coupled with a knowledge the number of complete cycles (wavelengths) between the satellite and receiver, can be employed to calculate a distance. In order to measure the carrier phase, the codes first must be removed. If the binary sequence of the codes is known, then the original, unmodulated, carrier can be reconstructed and carrier phase measurement is then possible. Conversely, if the codes are not known, a so called "codeless receiver" can reconstruct the unmodulated carrier signal, removing the codes by squaring the incoming signal (see section 3.1.3.). If the latter technique is employed, then the navigation message is lost along with the codes and so ephemeris information must be obtained from elsewhere. Carrier phase measurement is capable of higher resolution than pseudo-ranging, but it has its own set of problems which must be dealt with if this technique's inherent accuracy is to become available. It is perhaps misleading to suggest that carrier phase observation is "better" than pseudo-ranging, because the two are used for different purposes. The former is a relative positioning technique, while the latter is largely employed as a means of navigation, to find absolute position.

3.Interferometric Delay Measurement.

In this approach the total GPS signal (carriers and codes from all visible satellites) is recorded as a noise signal at both stations on a baseline. The difference between the two signals is then extracted. This can be done using "traditional" interferometric techniques, or by what is called "codeless spectral compression", in which the phase of the codes themselves are employed.

4.Integrated Doppler Measurement.

This is essentially the same measurement as that employed by TRANSIT receivers. In this technique, changes in the satellite-receiver range over the observing period are determined by recording the changes in frequency of the received GPS signal, whose transmitted frequency is accurately known. The measuring process here is the same as that for carrier phase measurement. In this case, however, rates of change of phase are utilised, as opposed to instantaneous phase recordings. This type of observation can be used to find position, however it is more commonly employed as an aid to one of the other techniques.

Each measurement type is examined in much more detail in Chapter Three.

1.5.Global Positioning System - Error Sources.

The source and magnitude of errors will depend on which type of measurement being is made, as well as on other factors, including the procedures implemented before and during the survey, and the methods adopted for processing the data received. Errors will occur at the satellite, during the propagation of the signal, and at the receiver. Chapter Five discusses these problems in detail.

a)Errors Occurring at the Satellite.

The main errors in this category are ephemeris errors and satellite clock bias. As noted above, the track of the satellite is required to compute the receiver's position; it is impossible to obtain the exact orbital path (or "ephemeris") of the satellites, and errors in the ephemeris become apparent in the position fix. The satellite clock will also contain a bias due to frequency offset and frequency drift. This error is also translated into the position fix.

b)Errors Occurring During the Propagation of the Signal.

The transmitted electromagnetic signal is affected by the atmosphere through which it travels. The effect of this phenomenon isvarying velocity between the satellite and the receiver. Such atmospheric effects are usually divided between those which occur in the ionosphere and those occurring in the troposphere. Changes in velocity result in erroneous measurements; the magnitude of the error depends on the conditions prevalent at the time of observation. Electromagnetic waves are also capable of reflecting off surfaces, which causes multiple signals, that can be picked up by the receiver. This "multipath" condition results in further problems in determining position accurately.

c) Errors Occurring at the Receiver.

Further errors occur in the receiver. Not only is there receiver clock bias, as illustrated in Figure 1 (facing page 7), but there are also various other instrumental effects, such as noise and channel bias.

Finally the geometry of the observed satellite constellation will also have an effect on the final accuracy obtained.

It is convenient to discuss the operation of GPS in three segments. These are:

a) The Space Segment.
b) The Control Segment.
c) The User Segment

Chapter Two deals with the Space and Control Segments, while the rather extensive User Segment is discussed in Chapter Three.

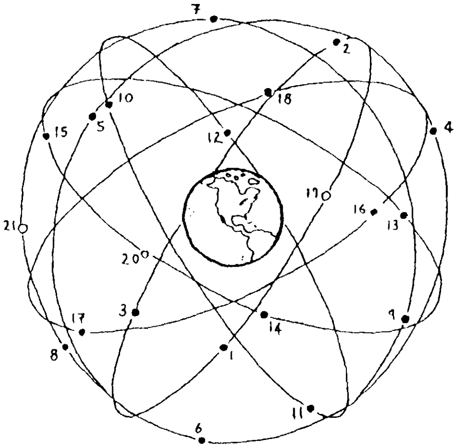
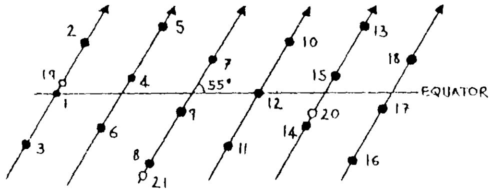


FIGURE 2 : GPS CONSTELLATION.



● NOMINAL CONSTELLATION.
○ ACTIVE SPARES.

FIGURE 3 : LINEAR ILLUSTRATION OF ORBITS.

CHAPTER TWO - THE SPACE AND CONTROL SEGMENTS

2.1. The Space Segment.

1. Constellation.

The Fully Operational Capacity (FOC) of the GPS is to consist of 18 satellites; three each in six approximately circular orbits, plus three active spares, giving a total of 21 satellites. The satellites will orbit at 20,183 kilometres altitude, with the orbital plane inclined 55° to the equatorial plane. The orbital planes will be 60° apart in longitude and the three satellites in each plane will be 120° apart. The orbit plane phases have a 40° separation, so that a satellite in one plane has a satellite 40° ahead (North) of it in an adjacent plane to the East. Figures 2 and 3 depict this FOC configuration.

Figure 4 (facing page 11) illustrates the United States Department of Defense's original program schedule. Phase II has been disrupted by the Space Shuttle "Challenger" disaster in January 1986. There are presently seven research and development Block I satellites transmitting reliable data (spring 1987; see figure 5, facing page 11). These satellites have an equatorial inclination of 63° and orbit in two planes, 120° between each satellite. The U.S. Department of Defense (according to Stansell, 1986) places high priority on obtaining a 12 satellite Interim Operational Capacity (IOC) as quickly as possible. This constellation would provide continuous two-dimensional coverage. The Department of Defense, however, is not so concerned with speedy provision of the 18 satellite FOC. Stansell hopes that the actual implementation of the system will closely follow the "optimistic curve" (figure 6, facing page 12) until IOC, and predicts that thereafter deployment may drift towards the "pessimistic curve".

2. Signal Transmission.

The relationship between carrier and code signals is shown in figure 7, facing page13. Each satellite broadcasts radio-navigation signals on two frequencies in order to allow the measurement and correction of ionospheric refraction (see section 6.2.). These frequencies are known as L1 (1575.42 MHz, wavelength 190 mm) and L2 (1227.60 MHz, wavelength 240 mm). The L1 carrier is modulated by two codes (the C/A code and the P code), while the L2 signal carries only one code (the P code). The modulation consists of phase changes of either 0° (to represent a zero) or 180° (to represent a binary "one"); this binary biphase modulation can be represented by:

$$y = A(t)\cos(\omega t - \phi) \qquad (1)$$

where the amplitude function A(t) is a sequence of +1 and -1 values. So the codes are streams of binary digits. They are, in effect "pseudo-random noise", in other words they have noise-like

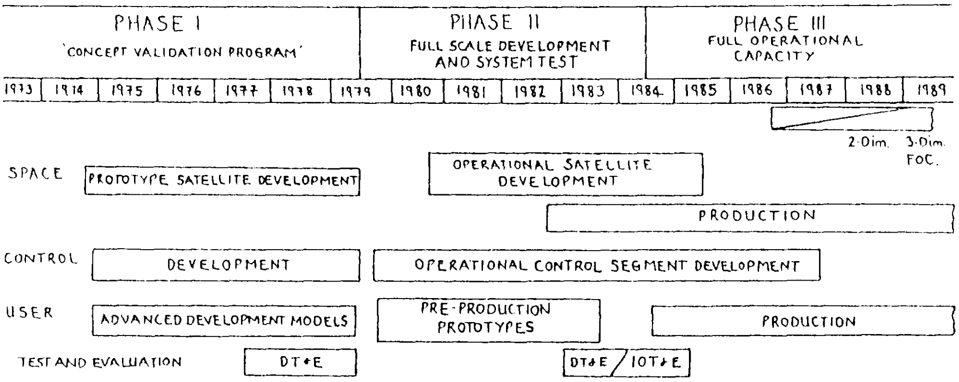


FIGURE 4 : GPS PROGRAMME SCHEDULE (PROPOSED).

LAUNCH SEQUENCE NUMBER	ORBITAL POSITION NUMBER	ASSIGNED VEHICLE PRN CODE	NASA CATALOGUE NUMBER	INTERNATIONAL DESIGNATION	LAUNCH DATE (DD MM YY)	STATUS
1	0	4	10684	1978-020A	22·02·78	quartz clock
2	4	7	10893	1978-047A	13·05·78	not operating
3	6	6	11054	1978-093A	07·10·78	operating
4	3	8	11141	1978-112A	11·12·78	operating
5	1	5	11690	1980-011A	09·02·80	not operating
6	5	9	11783	1980-032A	26·04·80	operating
7					18·12·81	launch failed
8	2	11	14189	1983-072A	14·07·83	operating
9	1	13	15039	1984-059A	13·06·84	operating
10	4	12	15271	1984-097A	08·09·84	operating
11		3	16129	1985-093A	09·10·85	operating
12					?·08·88	launch plan
13-19					J.A,J.J.S,O,D·89	launch plan

FIGURE 5 : GPS SATELLITE IDENTIFICATION AND LAUNCH SCHEDULE.

properties, the most important of which is that the stream has maximum autocorrelation at zero lag. The transmitted signal is also "spread spectrum", which means that it is transmitted over a much wider bandwidth than is necessary in order to transmit the information being sent. This enhances its random appearance. The codes are necessary for the pseudo-ranging process (section 3.1.1.). Figure 8, (facing page 14) illustrates the structure of the two codes.

The P-code ("Precise" code, or Precise Positioning Service, "PPS") has a modulation rate of 10.23 MHz (the rate at which it changes from a one to a zero or vice versa), a rather long period of 38 weeks and is modulated on both the L1 and the L2 carrier. TheC/A code ("Coarse/Acquisition" codeor Standard Positioning Service, "SPS") is modulated on to the L1 carrier, has a rate of 1.023 MHz and a period of 1 millisecond. Each satellite transmits a different C/A code and a different section (one week in length) of the P code. The two codes are in phase quadrature on the L1 carrier; that is they are 90° apart in phase so that they can be readily separated. The two "chipping" rates of 10.23 MHz and 1.023 MHz translate into code chip sizes of 29.3 metres (P code) and 293 metres (C/A code) respectively (figure 8). Although the actual measurement accuracy depends on many factors, the ultimate accuracy ratio between the P and C/A codes is dictated by this measurement resolution ratio of 10; the P code being ten times more accurate than the C/A code, in theory. The purpose of the C/A code is to allow coarse range acquisition (to be followed by precise positioning using the P code) and low accuracy (perhaps 100 metre) navigation. At present, both codes are generated in synchronisation with a rubidium vapour atomic clock in each satellite. The Production satellites (Phase II) however, will employ more accurate caesium clocks, which will be kept in synchronisation by the control segment (see section 2.2) to an accuracy of approximately 20 nanoseconds.

Both the P and C/A codes are further binary biphase modulated at 50 bits per second by the data message. This is transmitted in five subframes, each 300 bits long (and therefore lasting 30 seconds, whereupon it is repeated). The beginning of each subframe contains GPS time (the "Z" count), a "hand-over-word" (HOW), to gain rapid access to the P code, and the following information:

Data Block 1 (subframe 1)	: satellite clock correction parameters, age of data and various flags.
Data Block 2 (subframes 2 and 3)	: satellite ephemeris parameters.
Message Block (subframe 4)	: ionospheric delay model and other user information.
Data Block 3 (subframe 5)	: alphanumeric information, clock correction parameters and health status of all the GPS satellites (one satellite per subframe). This almanac is required to find the other satellites.

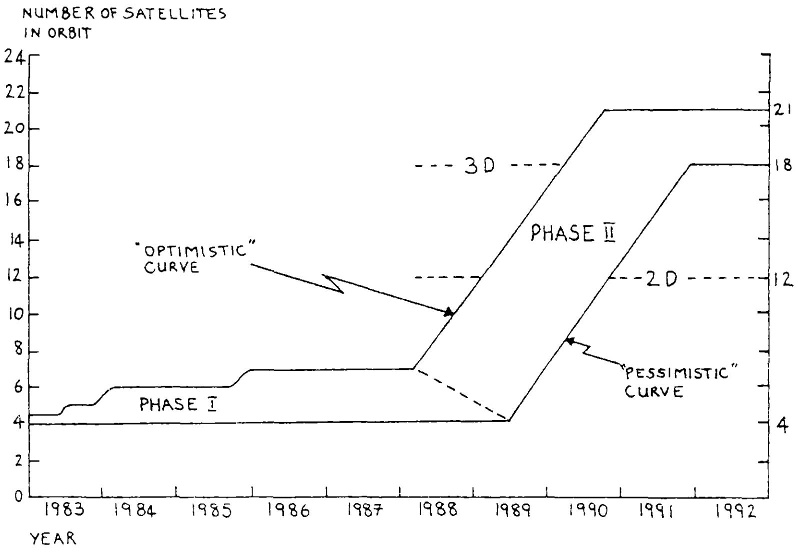


FIGURE 6 : GPS SATELLITE AVAILABILITY; PAST AND PREDICTED.

Data Block 3 rotates through 25 subframes of data. Every hour (nominally) Data Blocks 1 and 2 are updated; Data Block 3 and the Message Block are changed at upload time (see section 2.2).

2.2. The Control Segment.

In order for the GPS to provide position, the Control Segment comprises four Monitor Stations, an Upload Station and a Master Control Station (MCS), which together will:

a) estimate satellite ephemerides and frequency standards.
b) predict future satellite orbital paths and frequency drifts
c) format and upload this information to the satellites for subsequent retransmission to the user (in the data modulation).

The MCS at Vandenberg Air Force Base, California, oversees the operation of the Control Segment. Firstly, a reference orbit is is calculated for each satellite for a period of several weeks, using past tracking data provided by the Naval Surface Weapons Center (Dahlgreen), or taken from the integrator in the MCS (see figure 9, opposite page 15). Monitor stations in Guam, Alaska, Hawaii and California collect the following information from each satellite in view (nominally every six seconds):

L1 pseudo-range measurement.
L1-L2 pseudo-range difference measurement.
Delta pseudo-range on L1 (integrated Doppler, or change in pseudo-range with time).
Meteorological data (temperature, pressure, relative humidity).
Satellite navigation data.

This information is sent to the MCS for processing. Figure 9 illustrates the following procedure, which occurs at the MCS. The preprocessor (PREP) "unpacks" and scales the measurement data, edits bad data and corrects for:

a) the measurement time tag;
b) ionospheric delay;
c) tropospheric delay;
d) relativistic effects;
e) antenna phase centre offsets; and
f) Earth rotation during signal transit.

The smoother edits all wild points (those calculated satellite positions which are spurious) and processes all measurements contained within the 15 minute Kalman filter cycle for each Monitor Station pair. The remaining (good) pseudo-range and delta pseudo-range measurements (see section 3.1) are fitted to a polynomial by a least squares adjustment. From here, one smoothed pseudo-range residual and one smoothed delta pseudo-range (along with the appropriate statistics; standard deviations, variances, and so on) are sent to the Kalman filter. The filter processes the smoothed measurements to estimate:

a) satellite position and velocity residuals;

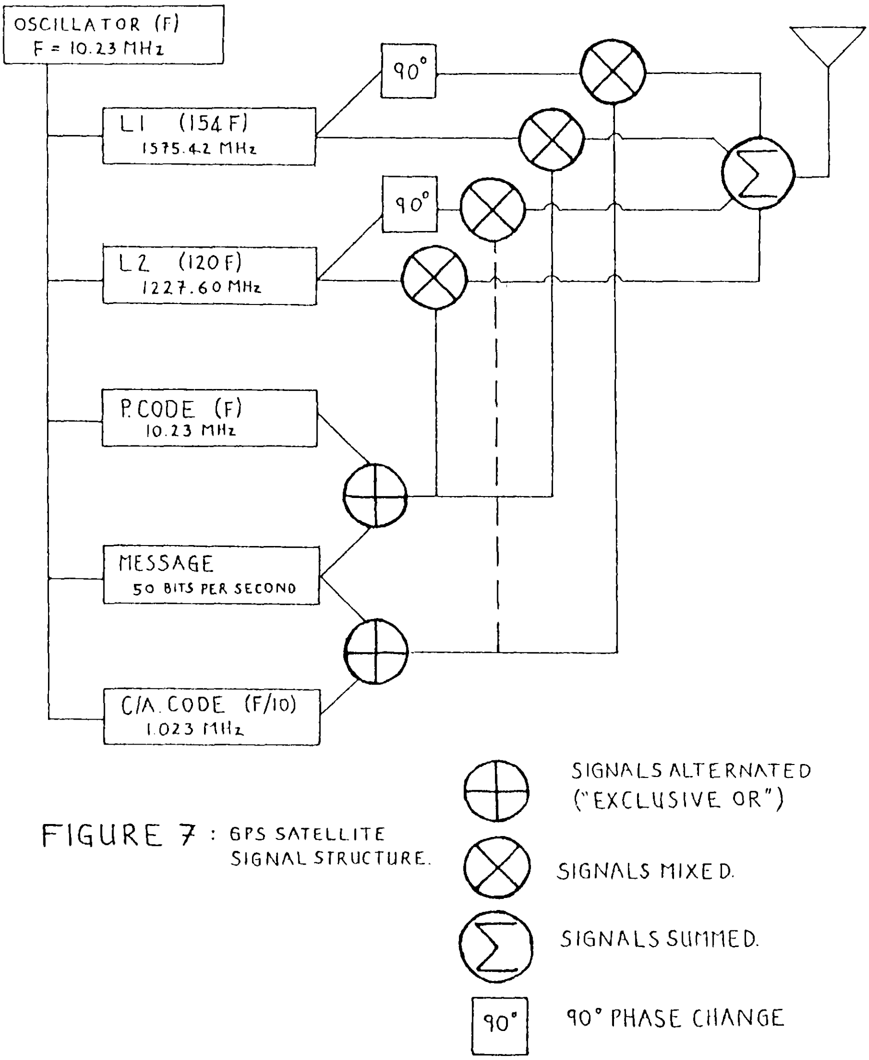


FIGURE 7 : GPS SATELLITE SIGNAL STRUCTURE.

b)solar radiation pressure model scaling parameter residuals;
c)satellite clock offset states;
d)Monitor station clock offset states;
e)tropospheric delay residual states; and
f)polar wander residual states.

The tracking data acquired by the Monitor Stations each day are used to update that day's portion of the reference orbit. The extractor obtains the relevant sections of the reference trajectory required by the preprocessor, filter and predictor. The predictor employs the most recent Kalman estimates of the residuals to predict the earth-fixed cartesian coordinates of the satellite's position. Each set of coordinates is then uploaded to the appropriate satellite (along with clock bias parameters) to become part of the navigation message; uploading occurs every eight hours.

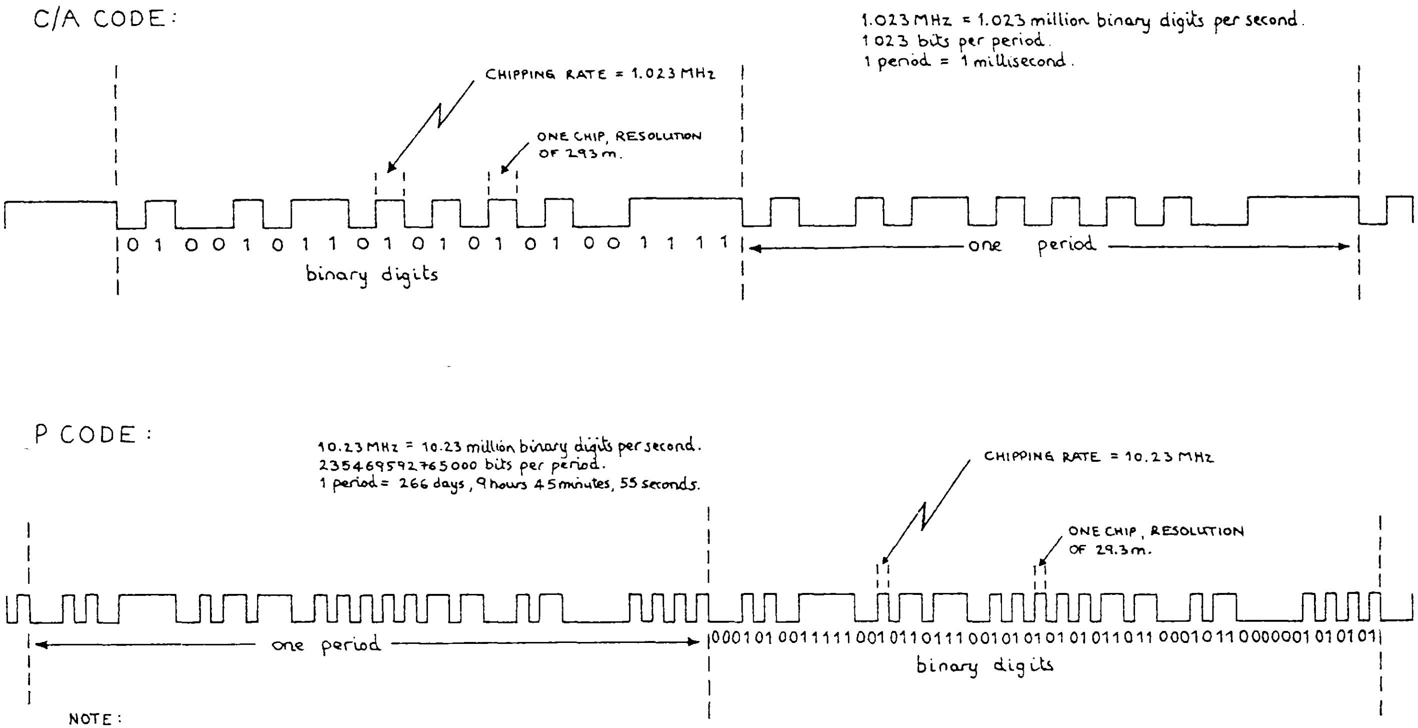
Gravitational and non-gravitational forces (solar radiation, albedo, air drag and such like) perturb GPS satellite motion from a Keplerian ellipse. So in order to describe the orbits adequately during the time interval for which the ephemeris information is transmitted (at least one hour), a representation based on Keplerian elements plus perturbations, is employed. The parameters used are listed in figure 10 (opposite page16). The parameters are in terms of ephemeris reference time, t_{oe} , which is nominally the centre of the transmission period, in GPS time. The t_{oe} is measured in seconds from the beginning of the GPS week (Sunday midnight at present). The parameters are obtained from a curve fit to the predicted satellite ephemeris over an interval of four to six hours. They only describe the ephemeris over the period of applicability, equal to the transmission period plus an interval of 1.5 to 5 or more hours.

2.3. United States Department of Defense Policy.

The United States Department of Defense has stated an intention to incorporate methods of accuracy denial into the transmitted signal structure of the Production Satellites. This is for reasons of national security. The Policy states that:

"...D.o.D. intends that the SPS signal will be broadcast in the clear and will be available for use by any properly equipped user. There will be no annual or other direct fee associated with the use of this signal. The SPS will be made available to civil, commercial and other users on an international basis at the highest level of accuracy consistent with the U.S. national security interests. It should be noted that at the direction of the Defense Subcommittee of the Senate Appropriations Committee, the GPS has been designed and engineered in a manner to protect the user fee option should it be appropriate in the future. If Congress does direct user fee implementation in the future, an appropriate time would be allowed to transition user equipment into a user fee configuration."

"The PPS signal will be encrypted and will be made available initially to U.S. and selected allied military users. Limited civil use of the PPS may be authorised if it can be demonstrated



NOTE:

chipping rate = the rate at which the signal changes from a one to a zero, or vice-versa.
one chip = a one or a zero.
the waveforms depicted are for illustration only; they are not segments of the actual codes.

FIGURE 8 : CODE STRUCTURE.

that such use is in the national interest, adequate security protection can be provided, and comparable accuracy cannot be obtained from another source. Indirect costs such as non-recurring cost recoupment and purchase of decryption and anti-spoof devices may be applicable to the use of PPS."

(Quoted from "Comprehensive Global Positioning System (GPS) User Policy.". Revised in: "A Program for Providing GPS PPS Capability and Services to Selected Non-U.S. Government Civil Users").

The Program states that a fee structure will be established so that the cost of providing the PPS (P code) can be recovered. Classified equipment and information will be controlled by an on-scene Government Field Technician. It is proposed that the fee will cover application processing costs, a daily hire charge for the use of government equipment and services, travel expenses of the technician and a share of the cost of hiring and training technicians. This sounds rather complex, however if current development plans are successful, receivers that operate with a security module may be unclassified, even when loaded with a classified key (the section which decodes the encrypted message). The program states that under these conditions, a technician may not be necessary. It is not clear how the D.o.D. would judge whether or not a prospective civil GPS PPS user would be acting in the U.S. national interests.

The accuracy denial could be achieved by:

a) Altering the phase and frequency of the transmitted signal ("dither").
b) Applying random errors or transmitting a lower accuracy broadcast ephemeris.
c) Including incorrect satellite clock correction parameters.
d) Altering the C/A code modulation intermittently.
e) Combining these and other interference techniques.

At present the accuracy of the C/A code is similar to that planned for the P code for the FOC. The intention is to degrade the C/A to 100 metres (horizontal) and 156 metres (vertical) for point positioning. This suggests the equivalent of a degradation of the broadcast ephemeris from 2 - 5 ppm (present best) to a maximum of 20 ppm. Ashkenazi (1987) also states that it is not clear whether the adopted denial procedure will prevent access to the L1 carrier frequency by using cross-correlation with the C/A code, and the L2 carrier frequency by using the signal squaring technique. Furthermore it is not clear if the frequencies of the two code modulations are to be affected by accuracy denial procedure.

It is predicted by many that the C/A code degradation policy will not be implemented. Whether or not it is, the biases introduced by degradation are likely to be eliminated by differential techniques (see section 6.1) over baselines of up to a few 100 km. It is important to note that codeless receivers would be unaffected by D.o.D. accuracy denial, however independently produced epemerides would be required in these cases, as the broadcast ephemeris would not be available (see

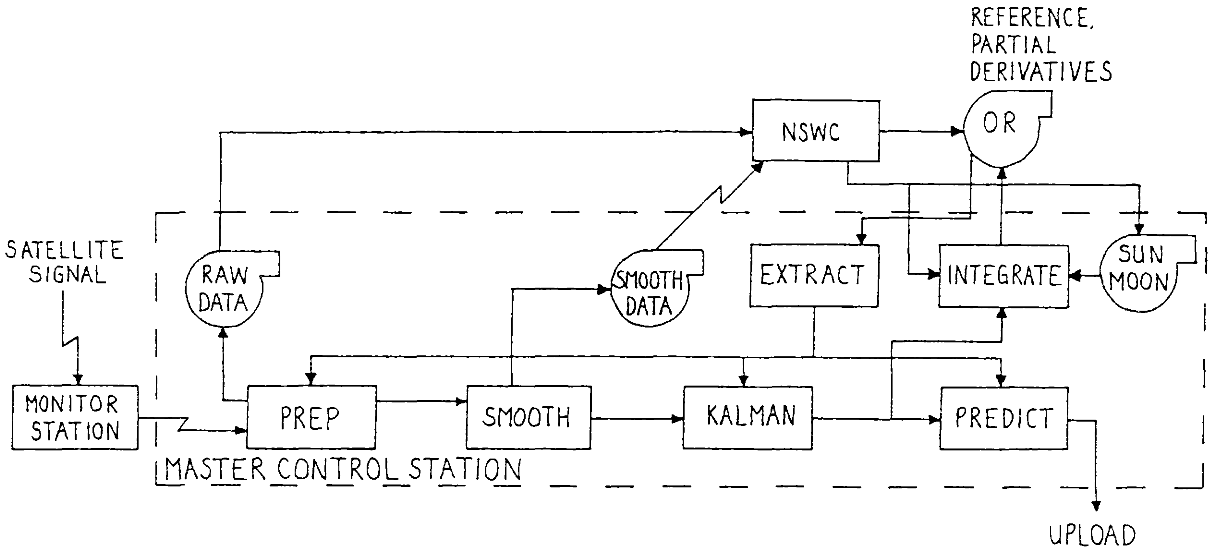


FIGURE 9 : MCS EPHEMERIS DETERMINATION PROCESSING.

5.2). Davidson et al (1983) suggest that the denial effects will depend on the relationship between the correlation time of the degradation and the time span of the differential observations. If the correlation time is longer than the observation time, then the degradation can be treated as a set of biases, along with station and clock parameters (5.3). A shorter degradation correlation time, and the degradation time may be treated as noise and reduced by averaging.

i - dot	Increment in inclination angle
M_0	Mean anomaly at reference time.
Δn	Mean motion difference from computed value.
e	Eccentricity.
$\sqrt{A}$	Square root of the semi-major axis.
Ω_0	Right ascension at reference time.
i_0	Inclination angle at reference time.
ω	Argument of perigee.
$\dot{\Omega}$	Rate of right ascension.
C_{us}	Amplitude of the sine harmonic correction term to the argument of latitude.
C_{uc}	Amplitude of the cosine harmonic correction term to the argument of latitude.
C_{rs}	Amplitude of the sine harmonic correction term to the orbit radius.
C_{rc}	Amplitude of the cosine harmonic correction term to the orbit radius.
C_{is}	Amplitude of the sine harmonic correction term to the angle of inclination.
C_{ic}	Amplitude of the cosine harmonic correction term to the angle of inclination.
t_{oe}	Ephemeris reference time.
AODE	Age Of Data (Ephemeris).

FIGURE 10: EPHEMERIS REPRESENTATION PARAMETERS.

CHAPTER THREE - THE USER SEGMENT

3.1. Measurement Modes.

The User Segment varies widely, depending largely on survey requirements and the time and capital available for the survey. The GPS may be used for highly accurate work, such as geodetic surveys or deformation studies, or for lower accuracy navigation. The configuration of the specific receiver sets will vary depending on the application for which they are intended. Receivers may be land based, operate at sea, in the air, or in near Earth space.

The range between a satellite and a receiver is the basic quantity observed in the NAVSTAR system. Three techniques have evolved to determine this range and a fourth measures changes in range. The most distinguishing feature of any GPS receiver is the type of measurement made. Some instruments employ just one form of measurement, whilst others combine more than one to produce a position fix. The four types of measurement are described below.

1. Pseudo-range.

The range is obtained by observing the time delay between the transmission and reception of the GPS signal. This range is contaminated by the receiver's clock offset with respect to the GPS time framework, and also by other error sources; thus the observation is termed the "pseudo-range". Pseudo-ranging can be carried out using the C/A code or the P code. Dual frequency C/A code pseudo-ranging is not possible because the C/A code is only modulated on to the L1 frequency. Thus if dual frequency measurement is required from pseudo-ranging, the P code must be available.

The time delay is obtained via a code correlation process. The following procedure occurs as GPS signals reach the receiver (adapted from Ashkenazi, 1983).

a) The receiver generates a replica C/A code which is on GPS time plus (or minus) the unknown receiver clock bias.

b) This replica code is then correlated (aligned in time) with the incoming (received) GPS signal. Once the two codes have been correlated, their product gives only +1 values for the resulting amplitude function (equation 1, page 10). Figure 11 (from D.E. Wells, 1985, facing page 17) illustrates how the incoming signal is reduced by differencing it (see section 3.1.2 below) with a local carrier (at A). If the two codes are aligned, then the signal resulting from the product of the reduced frequency received signal and the replica code (point B) will have the code removed because the 180° phase changes cancel out. The low frequency data message is not cancelled, and can be decoded (D in figure 11). The correlation peak detector checks for the presence of the code and corrects the delay of the replica code, at C, to maintain alignment (correlation). This completes what is known as the delay lock loop; the time delay is then scaled into distance by the speed of light, to form the pseudo-

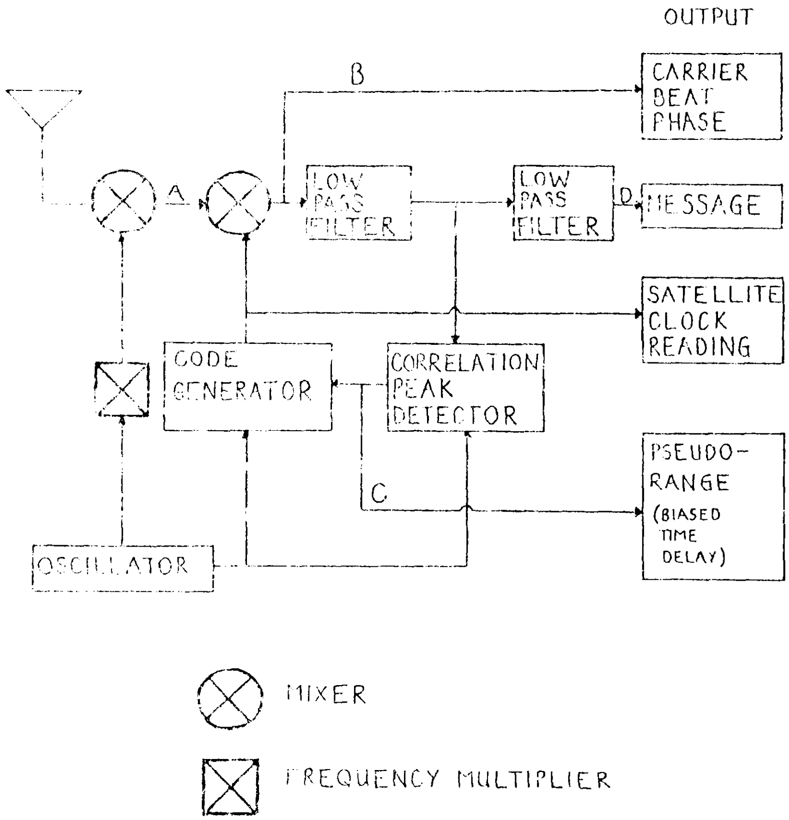


FIGURE 11 : CODE CORRELATING CHANNEL.

range. There will normally be several correlation peaks; some are due to pseudo-ranges of multipath signals (section 5.5). The system will select (not always correctly) the pseudo-range with the strongest signal.

There are many techniques for implementing the correlation process. One method employs at least two delay lock loop "channels". One channel correlates the received signal with a replica code which is timed to be slightly in advance of the actual received signal (the early channel), while a second channel correlates a replica which is slightly delayed (the late channel). Comparison of the received satellite signal power in each channel is a measure of the correction to be made to the receiver's estimate of the pseudo-range. When the received power in the two channels is equal, the pseudo-range is correct. A third channel may be employed to correlate the received signal with correctly timed replicas of the satellite code (A.R. Pratt, 1987).

This is C/A code pseudo-ranging. The algorithm outlined in Figure 12 (facing page 18) is employed in conjunction with the broadcast orbital information (figure 10) and the observed pseudo-ranges to derive Earth-fixed, Earth-centred coordinates. If the P code is used, then the following additional processes occur.

c) The C/A code is cleared out of the signal, and the "hand-over-word" instructs the receiver to generate the particular segment of the 38 week P-code which corresponds to the specific transmitting satellite at the given GPS time (already obtained by the C/A code measurement).

d) The receiver generated P code is shifted approximately with respect to the received P code by using the C/A code derived pseudo-range.

e) Further cross-correlation between the two P codes results in a more precise transit time, leading to a more accurate pseudo-range.

f) Steps (d) and (e) are performed using the L2 carrier frequency to obtain a second pseudo-range and the resulting two ranges are combined in pre-determined proportions to correct for first order ionospheric effects (see section 6.2).

g) The algorithm outlined in Figure 12 is employed, again in conjunction with the broadcast orbital information and the ranges produced in (f) above, to derive the coordinates of the receiver.

Neglecting errors other than the receiver clock bias, the observation equation for the pseudo-range is:

$$R = \left\{(X^i - X_j)^2 + (Y^i - Y_j)^2 + (Z^i - Z_j)^2\right\}^{1/2} + c\Delta t \qquad (2)$$

where R = The pseudo-range.
c = The speed of light.
(X^i, Y^i, Z^i) = The position of the satellite (in an earth-fixed reference frame) at the instant of observation.
(X_j, Y_j, Z_j) = The position of the receiver.
Δt = The synchronisation error between the satellite and receiver clocks (that is, the receiver clock bias).

It is important to note that for the system to generate the

$A = (\sqrt{A})^2$ Semi-major axis.

$n_0 = \sqrt{\frac{\mu}{A^3}}$ Computed mean motion.

$t_k = t - t_{oe}$ Time from epoch.

$n = n_0 + \Delta n$ Corrected mean motion.

$M_k = M_0 + n t_k$ Mean anomaly

$M_k = E_k - e \sin E_k$ Keppler's equation for eccentric anomaly.

$\cos v_k = (\cos E_k - e)/(1 - e \cos E_k)$

$\sin v_k = \sqrt{1-e^2} \sin E_k / (1 - e \cos E_k)$

True anomaly.

$\phi_k = v_k + \omega$ Argument of latitude.

$\delta u_k = C_{us} \sin 2\phi_k + C_{uc} \cos 2\phi_k$ Argument of latitude correction.

$\delta r_k = C_{rc} \cos 2\phi_k + C_{rs} \sin 2\phi_k$ Radius correction.

$\delta i_k = C_{ic} \cos 2\phi_k + C_{is} \sin 2\phi_k$ Correction to inclination.

2nd harmonic perturbations

$u_k = \phi_k + \delta u_k$ Corrected argument of latitude.

$r_k = A(1 - e \cos E_k) + \delta r_k$ Corrected radius.

$i_k = i_0 + \delta i_k$ Corrected inclination.

$X_k' = r_k \cos u_k$

$Y_k' = r_k \sin u$

Positions in orbital plane.

$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) t_k - \dot{\Omega}_e t_{oe}$ Corrected longitude of ascending node.

$X_k = X_k' \cos \Omega_k - Y_k' \cos i \sin \Omega_k$

$Y_k = X_k' \sin \Omega_k + Y_k' \cos i \cos \Omega_k$

$Z_k = Y_k' \sin i_k$

Earth fixed coordinates.

μ = WGS 84 value of the earth's universal gravitational parameter.

$\dot{\Omega}_e$ = WGS 84 value of the earth's rotation rate.

t = GPS time at time of transmission.

FIGURE 12 : BROADCAST EPHEMERIS COMPUTATIONAL PROCEDURE.

replica codes, an a priori knowledge of the random GPS codes is required.

2. Carrier Beat Phase.

The pseudo-range is the most commonly utilised "observable" in the GPS system, however the code chip sizes of 293 m (C/A) and 29.3 m (P) limit the accuracy attainable from this type of measurement. The two carriers, however, have wavelengths of 190 mm (L1) and 240 mm (L2) and so determination of the phase of these frequencies should allow much higher measurement resolution (fractions of the circa 200 mm wavelengths).

When signals of two frequencies (A and B for example) are mixed, two additional frequencies are obtained which are equal to the sum or difference of the two original frequencies (A+B and A-B, respectively). The carrier beat phase (or "reconstructed carrier phase" or just "carrier phase") is the phase of the signal which remains when the incoming (Doppler shifted) satellite is beat with the reference frequency generated in the receiver. This signal is produced by the code-correlating ("reconstructing") procedure described in part (2) of the pseudo-ranging process (see figure 11). The TI 4100 operates in this way.

As noted in 1.4., the original carrier phase can also be measured, without knowledge of the codes, by squaring the incoming signal. This form of measurement is dealt with in 3.1.3., below.

The phase of the carrier beat changes from 0° through to 360° as the distance between the satellite and the receiver increases by one wavelength of the carrier frequency. The phase also depends on differences in the initial phase and frequency between the receiver and satellite clocks. The signals are recorded at epochs determined by the clock in the receiver. The range from satellite (i) to receiver (j) can be expressed:

$$R = (N_j^i * \lambda_{L1}) + \lambda_{L1}/2\ \phi_j + c\Delta t_{L1} \qquad (3)$$

where R = The range between satellite and receiver.
N_j^i = The unknown integer number of wavelengths.
λ_{L1} = The wavelength of the reconstructed L1.
ϕ_j = The phase observed at the receiver.
Δt = The clock bias (satellite - receiver).
c = The speed of light.

In terms of the recorded phase, the carrier beat phase at epoch t can be represented as follows:

$$\phi_j^i(t) = \phi^i(t) - \phi_j(t) - (f/c)\ p_j^i + N_j^i \qquad (4)$$

where $\phi_j^i(t)$ = The carrier beat phase between satellite i and receiver j.
$\phi^i(t)$ = The phase at satellite i.
f = The constant frequency of the oscillators.
c = The speed of light.
p_j^i = The distance between the satellite andthe

receiver.

$\phi_j(t)$ = The phase at receiver j.

N_j^i = The whole number of cycles between thesatellite and the receiver.

The carrier beat phase does not provide the whole number of cycles that exist between the satellite and receiver at the initial measurement epoch, represented by N_j^i in equation 4, page 18. This is the inherent two pi ambiguity as found in EDM systems. In a navigation mode, the carrier beat phase may be "complete instantaneous", if the receiver automatically records the number of wavelengths which are added or subtracted as the satellite to receiver distance alters. Otherwise the measurement is simply a number between zero and one cycle and is termed the "fractional instantaneous phase measurement". The ambiguity consists of three components:

$$\alpha_j + \beta^i + N_j^i \qquad (5)$$

where α_j = The fractional initial phase in the receiver.

β^i = The fractional initial phase in the satellite.

N = The integer bias in the initial measurement.

The first two terms in equation 5 are due to various contributions to phase bias, such as unknown clock phase, circuit delays, atmospheric effects and so on. If the receiver uses a code correlation technique to obtain the carrier beat phase, then the P and C/A codes must be known and so pseudo-ranges can be observed and used to resolve the ambiguity. If the codes are not known and a signal squaring technique has been employed, or if more accurate results are required, then the ambiguity may be resolved in a software oriented manner. Section 6.4 deals with this problem.

3. Interferometric Phase Delay.

In Very Long Baseline Interferometry (VLBI), two or more receiving stations are set up on known points. As a satellite travels across the antenna pattern the phase difference between the same signal arriving at the stations will vary according to a sine wave pattern, with periods of maximum (constructive) interference occurring when the waves are in phase and destructive interference when the waves are 180° out of phase. The track of the satellite may be determined from the recorded phase changes. The system does not depend on the signals having a regular or pseudo-regular structure. In the Interferometric Phase Delay technique, this process of satellite tracking is "turned on its head"; the satellite track is known and if one of the receiver locations is known, then the positions of other receivers may be found. For observations of a particular satellite at a given instant, the interferometric phase delay, "t" can be expressed as:

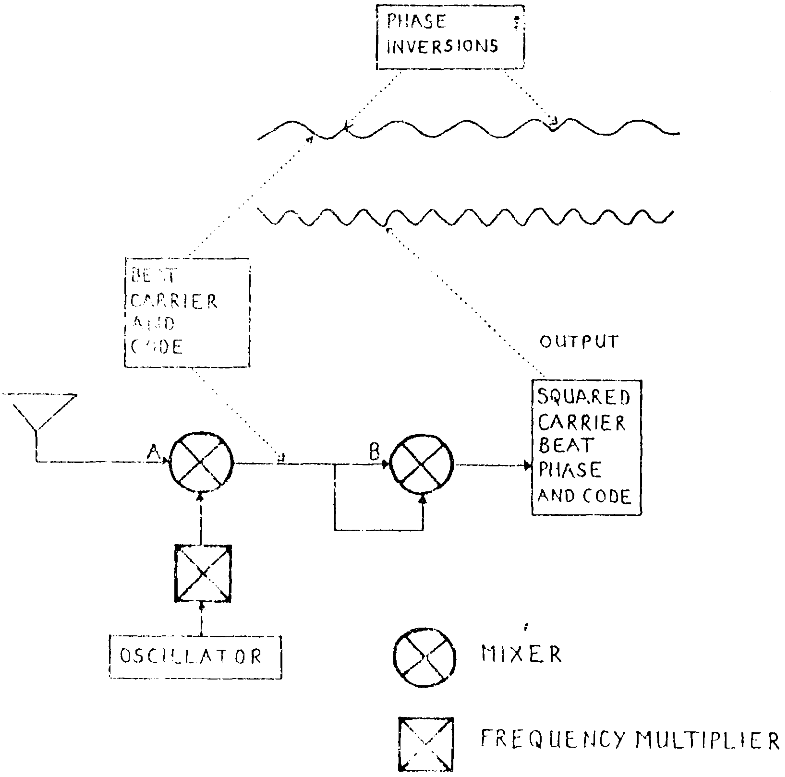


FIGURE 13 : SQUARING CHANNEL.

$$t_{1,2} = 1/c \left\{ |\bar{s} - \bar{r}_j| - |\bar{s} - \bar{r}_k| \right\} + \bar{I}^{cl} \tag{6}$$

where $t_{1,2}$ = The interferometric delay.
c = The speed of light.
s = The vector from the earth's centre to the satellite.
$\bar{r}_{j,k}$ = The vector from the earth's centre to receiver j or k.
$\bar{I}^{cl}$ = The difference between the epoch settings of the clocks at the two receivers (that is, their departure from synchronisation).

In practice, the effects of parallax and satellite motion are also taken into account when modelling the interferometric phase.

At any instant, it can be seen that the observed delay contains information concerning both the projection of the baseline vector along the direction of the satellite, and the clock error. As with pseudo-ranging and carrier beat phase measurement, in order to determine all three components of the baseline vectors simultaneously with the clock error, the signals from at least four satellites must be observed, and the resultant four linear equations solved simultaneously for the four unknowns. Again the time dependent positions of the satellites are obviously also required for the solution. There are currently two interferometric techniques used in GPS surveying, and a third method has been discussed.

a) Standard Interferometry.

This technique is exemplified by the MACROMETER II. The instrument employs the standard interferometric method, as discussed above, using a squaring channel to recover the demodulated carrier signal, as follows:

If the received signal (equation 1, page) is multiplied by itself (B in Figure 13, from D.E. Wells, 1985), a second harmonic of the carrier is obtained which does not contain the code modulation:

$$y^2 = A^2 \cos^2 (\omega t + \phi) = A^2 \left\{ 1 + \cos (2\omega t + 2\phi) \right\} / 2 \tag{7}$$

Since $A(t)$ is equal to ± 1, A^2 is always equal to +1. Thus the resulting signal y^2 is pure carrier, but at two times the original frequency. Unfortunately all noise on the signal will also be squared. Figure 13 indicates that, in reality, the incoming signal is differenced with a local reference frequency (point A) to produce the carrier beat phase, at a much lower frequency than that of the original carrier; this improves the signal to noise ratio. P code "spreading" results in a GPS signal bandwidth (see section 3.2.3) of approximately 20 MHz; often this can be reduced to a few Hertz by compression.

The squaring process produces only the carrier beat phase; no pseudo-ranges are obtained and neither are the data messages,nor the timing pulses inherent to the pseudo-ranges. So although it requires no knowledge of the GPS codes, satellite ephemerides have to be obtained independently for the observation period, and accurate time must be kept.

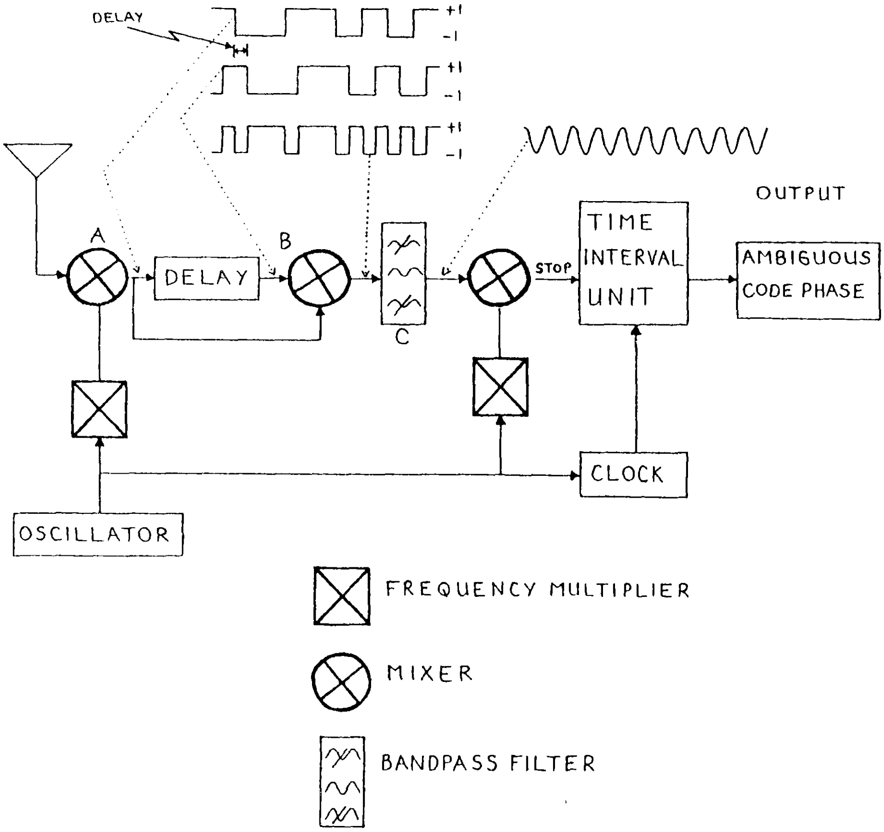


FIGURE 14: CODE PHASE CHANNEL

b) Spectral Compression.

The second technique ("codeless spectral compression") does not square the signal. The GPS transmissions are instead treated as undifferentiated "white noise" and recorded as such. This technique has been successfully implemented by ISTAC (International SERIES Technology Applications Corporation) using the SERIES (Satellite Emmission Radio Interferometric Earth Surveying) concept. The ISTAC 1991 and 2002 receivers track the phases of the C/A and P code, also called the "sub-carriers", using directive antennae, and without knowledge or recovery of the actual codes or use of the carriers. In order to do this only knowledge of the code frequencies, or chipping rate, of the codes is required. A "code phase channel" (figure 14) provides a measurement of the ambiguous phase of either the C/A or the P code (from Wells et al, 1986).

a)The received signal is mixed to a lower audio or sub-audio frequency (at point A in fig. 14): this is spectral compression.

b)The signal is then multiplied by a delayed version of itself. The delay is equal to half the chipping rate; 487 nanosecs for C/A or 49 nanosecs for the P code (point B).

c)The resulting signal is bandpass filtered (at C), allowing the code modulation rate to be recovered.

d)The positive going zero crossings of this sine wave are used to stop a counter, which was started by a one pulse per second signal from the receiver clock (point D).

e)Position is found using a combination of Doppler positioning and phase ranging using the phase differences. The measurement time interval is the satellite-receiver range, modulo 978 nanosecs (293m) if employing the C/A code or 97.8 nanosecs (29.3m) if using the P code.

There is virtually no two pi ambiguity to resolve because the code wavelengths are so long. This low ambiguity advantage is offset by the lower measurement resolution, for example approximately 29 m for P code, compared with 190 mm for the L1 carrier phase. As with the squaring technique, this measurement requires no knowledge of the GPS codes and so could operate successfully under conditions of degraded accuracy. As noted above, one caveat is the lack of real-time positioning, because the broadcast ephemeris is not recorded and pseudo-ranges are not observed. Again, accurate time is also required.

c) Miniature Interferometric Terminals For Earth Surveying.

A third interferometric method has been discussed (MITES-Miniature Interferometric Terminals for Earth Surveying) which involves simple phase measurements of sinusoidal wave carriers. In order to determine the two pi ambiguity, and to eliminate ionospheric errors however, it requires additional signals to be transmitted from the GPS satellites. As with SERIES, phase delays would be obtained simultaneously at two terminals, but in this case they are obtained for a set of different radio frequencies ("tones") covering a wide band. Measurement of the frequencies of the received signals would be made concurrently to enable conversion of the phase differences to corresponding delay differences. These delay differences could be interpreted in the standard manner to produce the baseline. Approximately ten tones would be required, and thus the present two frequency transmission is inadequate for this technique. This system could be developed in the future.

4. Integrated Doppler.

The integrated Doppler technique makes use of the carrier beat phase of both the L1 and the L2 frequencies. The phase changes over given time intervals, and this continuous wave carrier, subject to Doppler shift, is analysed. The continuously integrated Doppler is the same quantity as the complete instantaneous phase measurement, except that the integrated Doppler measurement assumes the two pi ambiguity to be zero. The standard Doppler technique may be applied to this signal; the accumulated Doppler shift over an interval of time is obtained by differencing the reconstructed carrier with a frequency generated by the receiver. Zero crossings of the resulting signal are then counted. The observation equation for this technique is :

$$\begin{aligned}\Delta R &= |\bar{p}^{i}(t_2) - \bar{p}_j| - |\bar{p}^{i}(t_1) - \bar{p}_j| \\ &= c/f_o \left\{N_{2,1} - (f_o - f_g)(t_2 - t_1)\right\} \qquad (8)\end{aligned}$$

where R = The change in range over the time interval $(t_2 - t_1)$.
$\bar{p}^{i}$ = The position vector of the satellite.
$\bar{p}_j$ = The position vector of the receiver.
c = The speed of light.
f_o = The observed frequency.
$N_{2,1}$ = The accumulated Doppler count during $(t_2 - t_1)$.
f_g = The generated frequency.

Although the GPS will contain a greater number of satellites than the present TRANSIT (Doppler) system, they orbit at a much higher altitude. The result of this is a reduced Doppler shift and correspondingly smaller range-rate measurements than those associated with the TRANSIT system. The measurement is contaminated by oscillator frequency variations and atmospheric refraction.

The integrated Doppler phase measurements give only the changes in range over the observing period, thus the absolute range differences at the initial period must be determined. The initial range can be found by performing range measurements using the L1 signal, and thus "time tagging" the Doppler measurements.

3.2. Other Receiver Characteristics.

1. Channels.

The types of measurement that a receiver makes is its most distinguishing feature, electronically. This will affect the type of channel incorporated into the receiver. The number of channels that a receiver contains and the way that they are employed is

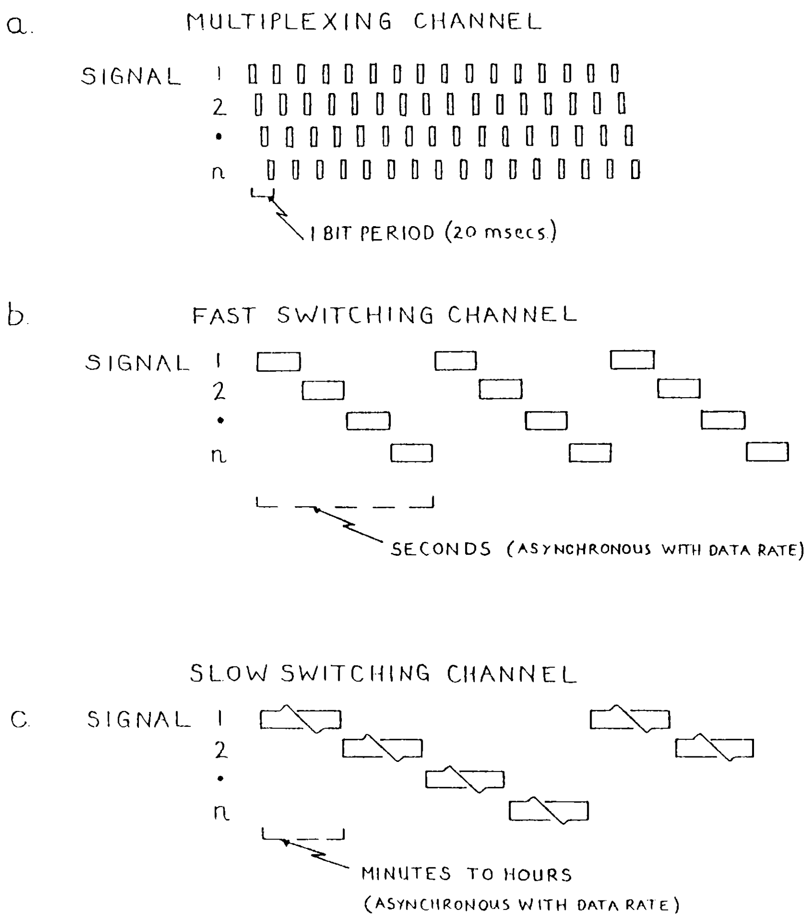


FIGURE 15: MULTIPLEXING AND SWITCHING.

also important. A channel consists of the hardware and software required to track the signal from one satellite. Each channel may be a code-correlating, squaring, or code phase channel, as described in the previous section. There are two types of code-correlating channel, the difference lying in the method of accumulating the satellite message.

a) Multiplexing channel.

This type of channel will sequence through a number of satellite signals, each from a specific satellite at a specific frequency, at a rate which is synchronous with the satellite message bit rate (20 millisecs per bit). One cycle, observing each of the satellites in the designated network once, will therefore require a multiple of 20 milliseconds to complete. This type of channel builds up a picture of the satellite message obtaining one bit from each satellite, per sequence cycle. A multiplexing channel is illustrated in figure 15a (from Wells, 1985).

b) Switching channel.

Here the satellite signals (again each a specific satellite and at a specific frequency) are sequenced at a rate which is slower than, and asynchronous to the message data rate. During each signal dwell time, the channel collects information, which it uses to build up a picture of the message from each satellite. Due to the dwell time asynchronicity with the message data rate, eventually all parts of each satellite message are sampled, and the picture is complete. There are two kinds of switching channel:

i) Fast-switching: in this case the sequencing cycle is short enough to enable the channel to recover the integer part of the carrier beat phase through software prediction. One cycle will last several seconds at most.

ii) Slow-switching: describes a channel whose sequencing time takes minutes to hours.

Both types of switching channel are illustrated in Figure 15b,c.

The way that these different types of channel are configured in a GPS receiver is wide and varied. A single channel receiver obviously has just one channel which may correlate (by multiplexing or sequencing) or square the carrier signal received or it may employ a code phase channel. Many types of receiver have more than one channel ("multi-channel" receivers); these channels may all be code correlating, squaring or both types of channel may be incorporated in the one receiver (a so called "hybrid" receiver) for specific reasons. Possible configurations for a GPS receiver include:

a) A single multiplexing channel tracking several satellite signals.

b) A single switching channel tracking several satellite signals.

c) A single code phase channel tracking all "visible" satellites.

d) A multi-channel facility, tracking the same signal continuously; when receiver motion over even a fast-switching sequencing period is significant this may be the only way of retaining signal lock.

e) A multi-channel fast-switching through all "visible" satellites.

f) A multi-channel, with some channels tracking one signal each continuously, while other channels switch through all available satellites, possibly to collect ephemeris data from all visible satellites.

GPS receiver technology varies in many ways other than the particular signals observed and the type and number of channels contained within the receiver. The most important variables are introduced below; their effects are discussed in Chapters Five and Six.

2. Single/Dual Frequency.

If C/A code pseudo-ranges are utilised, then only single frequency operation is possible. With all other measurement modes, however, there is the option to operate on two frequencies. Dual frequency operation is more complex, and therefore expensive, than single frequency mode, but a more accurate picture of the magnitude of atmospheric errors is obtained by this method, and so better results become available.

3. Receiver Bandwidth.

The bandwidth of a receiver is the frequency range over which it receives signals. This characteristic has an effect on the final accuracy of the position fix.

4. Antenna Design.

The design and construction of the receiving antenna can vary markedly. Some are more accurate than others, however consideration must be given to other factors such as expense, portability, and ruggedness.

5. Integration Of External Information.

Some receivers are capable of assimilating extra information concerning the receiver's position, provided by other sensors. This may be in terms of height, time, velocity, inclination from the vertical (in marine applications) and so on. The more additional accurate information available, then the better will be the position obtained. The system is no longer solely a "GPS receiver" as such, but this point is not important, as long as the cost and availability of each external sensor is considered in tandem with its effect on the final accuracy of the system.

3.3. Receiver Utilisation.

1. Static Positioning.

This is the simplest way of employing the NAVSTAR system. In this case, the absolute positions of points are determined using one or more GPS receivers, independently. In many receivers, if an unobstructed view of the sky can be retained while moving from station to station, then new integer ambiguities need not be introduced (the receiver remains locked in), and position fixing can proceed much more quickly than would have been possible otherwise.

2. Kinematic Positioning.

This refers to applications in which the motion of a receiver's host vehicle is sought. This is often also called "dynamic positioning".

3. Relative Positioning.

The relative location of two (or more) points can be obtained by tracking the same satellite signals simultaneously with at least two receivers. Relative positioning is "static" if the survey involves obtaining the differences in coordinates between a pair of points in a baseline or pairs of points in a network. Conversely in "kinematic relative positioning", one receiver will remain stationary on a coordinated control point, while another receiver is placed on the vehicle whose trajectory is required.

Positioning is described as being "real-time" if the coordinates of the receiver, whether it be static or mobile, are determined within a few seconds of the time of observation and not "after the event". In kinematic relative applications this must involve a real-time data link between the control station and the mobile receiver. Instruments which either square the signal or employ a code phase channel and, in neither case, incorporate a code-correlating channel cannot position in real-time because they do not decode the navigation message and so cannot obtain the broadcast ephemeris required for "immediate" position computation.

In relative positioning many of the errors inherent to the system are correlated among the measurements made by the receiver. These errors can be modelled and accounted for by estimating biases and weighting the observations accordingly. Signal differencing is a more common method of reducing some of the errors in GPS observations. This involves subtracting one observation from another; such a process reduces or negates the effect of errors common to the measurements which are differenced. There are several ways in which observations can be differenced; measurements may be subtracted between receivers, satellites, time epochs and between the L1 and L2 frequencies. The resulting difference (a single difference) can be further differenced with another single difference to produce a double difference, and in so doing, errors are further reduced. The process of signal differencing is discussed in detail in section 6.1.

3.4. Common Receivers.

The main characteristics of a large number of the GPS receivers which are on the market are listed in Appendix A. It is often difficult to make valid comparisons between different GPS receivers because the quantities being measured and the terms of reference vary considerably for both physical and electronic characteristics. One obvious example is the power supply. In some instances this is internal and therefore it will be included in the dimensions and weight statistics. Other instruments employ separate power sources whose properties are not quantified, apart from the power they contribute. Furthermore the prices quoted are approximate only. For these reasons, comparisons between specific receivers should be treated with caution.

The statistics quoted in Appendix A give only a skeletal description of the GPS receivers, and so the following section describes some of the more commonly used instruments in more detail. Accuracies quoted are taken from publicity brochures and/or papers which describe the receiver in question. As far as possible, the consistency of the figures quoted were checked by cross correlation among the various articles. Unfortunately the term "accuracy" is rather vague, but in many cases there was no indication of what form of accuracy was being quoted. Thus again the terms of reference are variable here; for example between root mean square errors and circular or spherical error probabilities. The figures quoted, however, do indicate which category a receiver falls into (low accuracy navigation, second/third order survey, high precision geodetic survey and so on).

1. Texas Instruments TI 4100.

The Texas Instruments TI 4100 is designed to provide data simultaneously for both absolute and relative positioning and can be used for both low accuracy navigation and first order geodetic surveying. The system comprises a receiver/processor, an antenna (plus preamplifier) and a data recorder. The receiver employs a single code-correlating multiplexing channel to track up to four satellites simultaneously. Eight primary measurements are made on each of the two GPS carrier frequencies (that is, four L1 pseudo-ranges, four L2 pseudo-ranges, four L1 integrated Doppler counts, and four L2 integrated Doppler counts) and navigation messages from the four observed satellites are also obtained. The simultaneous measurement of the continuously counted integrated Doppler with P code pseudo-ranging allows both the ambiguity to be resolved and the offset between the receiver clock and GPS time to be estimated. The dual-frequency approach allows accurate determination of the ionospheric delay (see 5.4.1). The instrument requires two large batteries as a power source and a cassette recorder or field computer to record the data. When navigating, the display unit provides several measures of the course taken, for example, velocity, altitude, cross track error, range and bearing to the next waypoint and the speed and course made good since the previous waypoint.

The U.S. Department of Defense's proposed encryption of the P code throws some doubt as to the long term utility of the TI 4100, however the instrument's 1-2 ppm accuracy over baselines from 10 km to over 1000 km encouraged sales while the instrument was being manufactured. The TI 4100 is, however, no longer in production.

2. MACROMETER V-1000.

The V-1000 was the first GPS receiver designed and built for the civil surveying market. It consists of a receiver/recorder (and power supply), an omni-directional antenna and the P-1000 processor.

Six squaring channels provide carrier beat phase measurements of the GPS L1 signal; thus no knowledge of the P or C/A codes is required and so V-1000 performance should be unaffected by encryption policy. This is an interferometric system and so at least two receivers are required in order to carry out a survey. The P-1000 is used firstly to determine the optimum schedule for satellite observations and from this station occupation can be calculated. The P-1000 is then used to create almanac files (A-files) that are site and time specific. These files define the observation schedule (which satellites at which times). Observation consists of recording the superposition of all phase signals. The file contains sixty discrete observation times and the corresponding expected satellite signal frequencies of each of the six satellites to be received. Only these six preselected satellites can be observed and thus there is little flexibility once the initial observing schedule has been determined. In order that these signals can be separated, the relative observation times of the two (or more) receivers must be known accurately. This is achieved by either synchronising all units before the survey using either clock setting hardware and a time interval counter, or external time standard, such as a hydrogen maser. Time measurements are usually taken after the survey to provide a complete history of relative clock drift. Atmospheric conditions are also required to allow a tropospheric correction. A 350 watt petrol generator, automotive battery, time receiver, time interval counter, cassette recorder, battery charger, and 12 volt DC to 110 AC invertor are also required to operate the V-1000 system. A telephone modem is also necessary if information is to be transmitted between office and field.

Two to three hours of observation will provide accuracies of 1 ppm over baselines less than 100 km with this system; so the accuracy of the V-1000 is comparable to that of the TI 4100.

3. MACROMETER II.

The V-1000 is no longer produced by the manufacturers Aero

Service, and has been replaced by a similar but dual-frequency model, the Macrometer II, which allows a correction for ionospheric refracton. This comes as two units (receiver/display and clock/battery) and the antenna. The MACROMETER II system provides similar accuracy to that possible using the V-1000, however such results are obtained in 15-30 minutes.

4. Mini-Mac LGS 200

This is the third generation of Aero Service's MACROMETER technology. The highly compact unit contains an L1 C/A code receiver which is capable of tracking eight satellites simultaneously. There is a dual band option through the addition of an L2 codeless card. The system tracks the C/A code pseudo-ranges as well as the carrier phase, and so both precise time and the broadcast ephemeris data are available, allowing near real-time positioning.

5. Wild-Magnavox WM-101.

The WM Satellite Survey Company is a joint venture of Wild Heerbrugg Survey Corporation and Magnavox Survey Systems Inc. The WM-101 will track up to 9 satellites simultaneously, using four C/A code-correlating channels. Three channels track three satellites sequentially with a two second dwell time, while the fourth is used to update ephemeris data and check interchannel biases. The L1 carrier beat phase is also recorded and obviously the satellite data message is also decoded. This information, along with phase rate, cycle count, time signal strength and interchannel calibration readings, is recorded on tape and post processed position or position differences are obtained by means of software used in a micro computer. Real-time navigation is also possible using the 101's internal processor.

The WM-101 provides a static positional accuracy of about plus or minus 5 metres with 4 to 5 minutes of observation; in a dynamic mode, position is determined to an accuracy of 20 metres. Post-processed differential accuracy reaches approximately 10 mm plus 2 ppm of the separation.

There are plans to upgrade the WM 101 so that the L2 signal can be accessed using either a squaring channel, or code-correlation with the P code, allowing ionospheric refraction calibration. Automatic recording of atmospheric parameters is also being proposed; at present atmospheric readings can be entered manually to allow the receiver to correct for tropospheric effects.

6. GPS Land Surveyor 1991.

The 1991 is based on the ISTAC SERIES technology, but it is far more efficient, compact and much cheaper than SERIES. Because it uses the interferometric approach the system measures relative positions, and so at least two receivers are required. The receiver/antenna is tripod mounted and, coupled with a data recording unit/clock, measures biased range differences on the L1 frequency. An externally produced ephemeris is necessary, as the system is "codeless". A code-correlating receiver, such as the Trimble 4000A may be used to provide the broadcast ephemeris, and in conjunction with a portable computer allows computations on site. Once the start and stop times of the observation session are keyed into the recording unit, the procedure is automatic, data being recorded on a blank tape every 15 seconds.

Reported accuracies are around plus or minus 60 mm for 30 minutes of observations and 250 mm in just 5 minutes.

7. ISTAC Model 2002.

The 1991 was replaced in mid-1985 by the Model 2002, which is based on the same principles. In the new model a battery-powered field computer replaces the recorder. The 2002 consists of an antenna/receiver; a clock interface unit (CIU) and a data analyser/recorder (DA/R). The DA/R provides satellite identification in the field using a set of almanac data, which will be valid for one month, as long as no major orbital changes have occurred. The best results are obtained if the CIUs are synchronised to one another before each day of observation, and are within 60 milliseconds of UTC. If synchronisation is impossible, associated software is capable of solving for clock offset. The 2002 utilises all visible satellites simultaneously with up to 35 software assignable channels available.

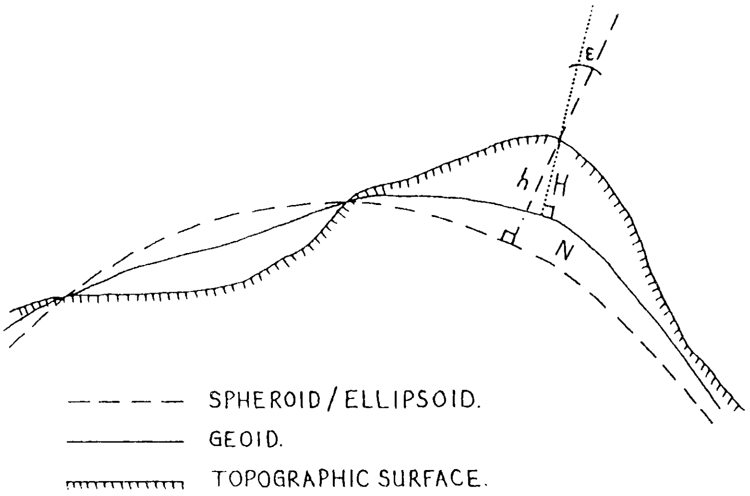
Results indicate accuracies of 300 mm with 3 minutes observation and 50 mm after one hour. Longer observation times will give results accurate to 2 ppm of the baseline measured. There are plans to upgrade the 2002 to a two frequency model.

8. Trimble 4000S.

The 4000S automatically acquires and simultaneously tracks up to four satellites by C/A code-correlation. Carrier phase and integrated Doppler measurement is also carried out in order to smooth the C/A code pseudo-ranges. The usual pros and cons apply to this system. No external frequency or ephemeris is required, and neither is synchronisation of receivers before deployment, however the system will be affected by future C/A code degradation. The 4000S may be used for navigation, providing better than 15 metre level accuracy; in a geodetic (static relative) mode, it will give "centimetre level differential measurements in three dimensions" (from a publicity brochure), measuring the carrier phase to better than two degrees of the phase angle.

9. Trimble 4000SD, 4000SDL.

Trimble have recently introduced a 10 channel, dual frequency model (the 4000SD). This model is capable of tracking up to 10 satellites on L1, or 5 satellites on L1 and 5 satellites on L2. The 4000SDL is a lightweight model (15 kgs), and is designed to act as a mobile, while the 4000SD remains stationary on a reference point.



ε = DEVIATION OF THE VERTICAL.

H = ORTHOMETRIC HEIGHT.

h = ELLIPSOIDAL HEIGHT.

N = GEOIDAL HEIGHT (GEOID-ELLIPSOID SEPARATION)

FIGURE 16 : HEIGHTS.

CHAPTER THREE - HEIGHT DETERMINATION USING GPS

4.1. Introduction.

The Earth-centred, Earth-fixed coordinates which the NAVSTAR system provides are normally converted to a desired frame of reference such as UTM or latitude/longitude/height in a specific reference spheroid (for example WGS72 or NAD27). Most surveyors are concerned with orthometric heights, as measured above some datum by levelling. The most common datum is the geoid, a geopotential surface which coincides with mean sea level, approximately. Heightings produced by the GPS, however, are ellipsoidal heights, referenced to the specified ellipsoid or spheroid. Figure 16 illustrates the difference between these two heights. It can be seen that in order to derive the orthometric height from GPS measurements, the geoid-ellipsoid separation is required. This is defined by:

$$N = h - H \qquad (9)$$

where N = The geoid height (geoid-spheroid separation).
h = The ellipsoidal height (from GPS).
H = The orthometric height (from levelling).

Whereas the ellipsoid is a regular surface which can be described mathematically, the geoid surface is irregular due to variations in the topographic surface (Figure 16) and variations in density within the Earth. So in a GPS survey where the orthometric heights of points are required, a model of the geoid surface must be obtained for the area of interest. Such models may be produced by astrogeodetic, geopotential, gravimetric or geometric means. The astrogeodetic method is of little relevance here since it is a costly, time-consuming process, and quicker, more accurate methods are available.

Absolute geoid determination is possible using these methods (especially by the geometric technique), but more accurate results are obtained by relative geoid measurements, using a differential technique:

$$(N_B - N_A) = (h_B - h_A) - (H_B - H_A) \qquad (10)$$

where h_A = ellipsoidal height at A, N_A = geoid height at A
h_B = ellipsoidal height at B, N_B = geoid height at B
H_A = orthometric height at A, H_B = orthometric height at B

The major errors in geoid determination techniques are present at both ends of a baseline, and so they will tend to cancel when $(N_B - N_A)$ is calculated.Thus, as with horizontal positioning, differential procedures will result in more accurate results.

4.2. Geopotential Models.

The geoid height N, at a point P with spherical coordinates (ϕ_P, λ_P, R_P) can be calculated from the truncated spherical harmonic series:

$$N_P = GM/R_P \, \gamma_P \, (a/R_P) \sum_{m=0}^{n} P_{nm} (\sin \phi_P)(c_{nm} \cos m + s_{nm} \sin m\lambda_P) \qquad (11)$$

where a = The mean radius of the earth.
γ_P = Normal gravity at P.
GM = The gravitational constant times the mass of the Earth.
$P_{nm}(\sin \phi_P)$ = Legendre's polynomial of degree n and order m.
c_{nm}, s_{nm} = Dimensionless spherical harmonic coefficients.

Artificial satellite orbits depart from an ellipse largely due to the irregularities of the Earth's gravity field. Therefore by measuring perturbations of satellite orbits, c_{nm} and s_{nm} may be obtained, and thus the shape of the geoid is determined. This is possible only to a certain level of accuracy because other factors such as solar wind, the Earth's electrostatic force, and the attraction of other celestial bodies also have an effect on satellite orbital paths and these other factors are difficult to quantify accurately. Furthermore the term $(a/R)^n$ rapidly diminishes for large values of n, and so higher degree harmonics are harder to obtain. The result of this is that geoid estimation using only satellite observation produces a "smoothed" representation of the geoid surface. Geoid surface resolution can be improved by incorporating surface gravity data and marine geoid heights, which are derived from altimetry. The resulting models can be used in conjuction with GPS heights.

Production of ΔN ($N_B - N_A$) from geopotential models is not highly accurate; even high degree models can only resolve geoid features with extents of about 100 km. However, the method is computationally straightforward because such a model is geocentric in nature and it comprises a set of coefficients which can be plugged in for each set of coordinates. Thus in areas where the geoid is smooth, or where accuracy requirements are not stringent, and in areas where vertical benchmarks are few and far between, this procedure may be implemented and incorporated into GPS processing software.

4.3. Gravimetric Models.

Stokes formula gives a value for the geoid height, using surface gravity measurements:

$$N'_P = R/4\pi G \int_{\alpha=0}^{2\pi} \int_{\psi=0}^{\pi} \Delta g f(\psi) \delta \sigma \qquad (12)$$

where N'_P = The geoid height at point P.
R = The mean radius of the earth (6371 km).
G= The mean value of gravity over the earth(979.0 gals).
Δg = The gravity anomaly (the difference between gravity on the geoid and gravity on the ellipsoid at the equivalent point).
$f(\psi)$ = Stokes function k for a closed surface with thickness zero, where ψ is the angle subtended by geocentric radii to P and the element of surface area. As far as P is concerned, it represents a weighting factor which

weights the anomalies according to their angular distance from P.

$\delta\delta$ = An element of area containing a gravity anomaly Δg and located at an angular distance ψ from the computation point P.

Stokes's formula represents the summation of gravity anomalies for the whole Earth and therefore the accurate determination of N depends on knowing Δg at sufficiently close intervals over the Earth's surface. However the contribution of N'_P decreases rapidly with distance from P. Spherical harmonic models of the geopotential can be used to calculate contributions to the geoid height from areas away from the point of interest. In practice, formula (12) can be converted into one which is a function of ϕ and λ ; then if the Earth's surface is divided into blocks of size ($\delta\phi$, $\delta\lambda$), and the mean anomaly Δg and f (ψ) is determined for each block, a value for N'_P may be found.

Relative geoid heights accurate to decimetre level are possible using this technique, provided surface gravity data coverage is sufficient. Sub-decimetre accuracy over continental areas has been obtained by, for example, Engelis et al (1984), by employing accurate local gravity data and a high degree geopotential model. The final accuracy will depend on the precise scheme implemented for equation 12 and on the magnitude of errors in gravity anomaly values and errors in the gravity field model.

4.4. Geometric Models.

By employing orthometric heights obtained directly from vertical benchmarks or indirectly from geodetic levelling, in conjunction with ellipsoidal heights from GPS (or TRANSIT), geoidal heights may be obtained using equation 9 (page 30). Two approaches are possible.

a) Ellipsoidal heights are obtained at vertical benchmarks using GPS receivers. The geoidal height is then calculated at these points and a geoid contour map constructed by interpolation. Geoid height can then be estimated at points of interest, where the GPS observations have been made, and orthometric heights deduced.

b) If the ellipsoidal surface, determined from GPS measurements, and the geoid surface (determined from benchmarks) are made to coincide at one benchmark, then the other benchmarks will have two height values. For a point (x,y) this can be modelled as:

$$N = Ax + By + C \qquad (13)$$

where A = The east-west tilt of the surface.
B = The north-south tilt of the surface.
c = The separation of the surfaces.

Three well distributed benchmarks, whose ellipsoidal heights are known (by GPS) are the required minimum to define each surface. Extra points with known ellipsoidal and orthometric heights, however, will provide redundancy, allowing a more accurate result using a least squares adjustment. A more complex, curved surface will provide still better results.

These latter two methods are the most direct methods of providing orthometric height using the GPS. The geometric procedures are easier to comprehend, compile and carry out than geopotential or gravimetric techniques. However, in areas where benchmarks are scarce and the geoid surface is complex, the gravimetric technique will often be more accurate and easier to implement.

The accuracy to which ellipsoidal height is obtained (that is the vertical accuracy of the GPS measurements) will depend on the same factors as those affecting horizontal accuracy. Satellites, however, can only be observed when they are above the horizon, and so the balance of satellites (an important method of reducing systematic errors, refraction in particular) in the vertical plane will be inherently poorer than that horizontally, where satellite observations can be balanced north/south and east/west. This factor, along with generally poorer resolution of geoidal height results in orthometric height accuracy being poorer than horizontal positioning accuracy using GPS.

CHAPTER 5 - ERROR SOURCES.

5.1. Observation Errors.

The observed time delays or phase changes will be erroneous due to the limitations of the receiver's electronics. This error is random, and generally proportional to the wavelength of the signal. Wells et al (1986) indicate that the observation resolution is about one per cent of the signal wavelength, and so the corresponding observation errors for the various signals will be approximately:

a) C/A code; 3 metres.
b) P code; 30 centimetres.
c) carrier; 2 millimetres.

5.2. Ephemeris Errors.

The accuracy of a conventional survey is partly dependent on the accuracy to which the locations of the control points are known. Similarly the final accuracy of a GPS survey is affected by the accuracy of the GPS satellite ephemerides employed.

Ephemeris errors are the result of unintentional or intentional errors (U.S. D.o.D. accuracy denial) in the tracking, computation, or broadcasting of the satellite orbital paths. For point positioning, errors in the GPS ephemerides introduce errors in position of roughly the same magnitude as the ephemeris errors (but see section 6.8.4.d). An ephemeris may be either "Broadcast" or "Precise".

1. Broadcast Ephemeris.

The Broadcast ephemeris is the predicted GPS satellite orbital path which is transmitted in the Navigation Message (section 2.1) and allows real-time positioning, using pseudo-ranges. Although the official literature quotes accuracies of less than 20 metres, accuracies of 40 to 100 metres are perhaps more commonly achieved (King et al, 1985). As was noted in section 2.1, each day's tracking data are used in tandem with the reference orbit to predict the following day's satellite paths. King et al (1985) suggest that accuracy will decrease as a function of time from the initial upload at the start of the day, and possibly as a function of the age of the reference orbits (calculated from the start of the week - Sunday midnight). The magnitude of the ephemeris error may be reduced as the Control Segment refines the orbit prediction models and/or introduces more frequent uploading of orbital data to the satellites.

2. Precise Ephemerides.

A Precise or "post-processed" ephemeris may be determined after the event, using observations made to a satellite. This is generally a least squares estimate of the satellite's track during the observation period. Precise ephemerides are presently made available by the U.S. National Geodetic Survey, from orbit determination at the NSWC, using seven days of pseudo-range data from eight worldwide tracking stations. The final computation and processing scheme to be implemented in the operational phase has not yet been fully decided and at present the ephemerides are provided without restriction to civil users.

Accuracy is affected by the quality and coverage of the tracking data, both in space and in time, and by the geometric and dynamic models applied during the orbit computation. Orbital accuracies of about 2.0 metres are required to obtain an accuracy of one centimetre for baselines approximately 100 kilometres long (0.1 ppm, Davidson et al, 1985). The NSWC precise ephemerides are given in the WGS-72 framework and have accuracies of 10-20 metres, which is sufficient for surveying at the 0.5 to 1 ppm level.

Post-processed ephemerides may be obtained from other U.S agencies, private firms and research institutes which possess their own tracking network

a) because the NGS ephemerides are not accurate enough;
b) because the NGS ephemerides are not delivered quickly enough (they are available about two weeks after the event); or
c) for research purposes.

There is evidence to suggest that it is possible to produce ephemerides accurate to better than 0.1 ppm (about one metre errors in the orbit) using regional dedicated tracking networks and special processing techniques (Beutler et al, 1986). These ephemerides would still be accurate to about 0.5 ppm in areas outside the region. Accuracies of a few metres are already being produced using observations from stations equipped with hydrogen maser frequency standards (King et al, 1985).

The Broadcast ephemeris should satisfy most survey requirements and independently produced orbits should only be required if the C/A code is degraded according to Department of Defense specifications. Codeless users will always require independently produced orbits, of course.

5.3. Clock Errors.

The measurement process is subject to oscillator frequency variations in both the satellite and the receiver clocks. The systematic components of these drifts can be modelled, but there will always remain random errors which cannot be modelled and errors in the modelling itself. If these go uncorrected positioning by GPS is limited to several metres.

1. Satellite Clock Instability.

The transmitting frequency ($f_i(t)$) can be modelled (King et al, 1985):

$$f_i(t) = f_o + a_i + b_i(t-t^\circ) \qquad (14)$$

where f_o = The nominal transmitting frequency (1575.42 MHz for L1).

t = The epoch of observation.
t° = The reference epoch chosen; for example near the centre of the span of the observation.
a_i = The frequency offset.
b_i = The frequency drift coefficient.

a_i/f_0 is a measure of the fractional frequency accuracy.
b_i/f_0 is a measure of the fractional frequency drift.

For caesium receivers:
a_i/f_0 is better than 10^{-12} over a six hour period.
b_i/f_0 is better than 10^{-15} s^{-1} over a six hour period.

For rubidium oscillators:
a_i/f_0 is about 10^{-11} over a six hour period.
b_i/f_0 is about 10^{-14} s^{-1} over a six hour period.

Single differences between receivers (section 6.1.3) will largely eliminate satellite clock errors.

2. Receiver Clock Instability.

Receiver clock fractional stabilities are generally no better than about one part in 10^{10} ; a stability of one in 10^{12} will produce a change of one metre in the signal pathlength over one hour. Receiver clock instabilities, however, may be modelled in a similar manner to satellite clock instabilities. Single differencing between satellites (section 6.1.2) will greatly reduce, if not negate, receiver clock biases.

In double differencing (see section 6.1.4), there are three important types of receiver clock error (King et al 1985):

a) If there is an offset from UTC present in both receivers, then the ephemeris will be interpreted for the wrong time. In this case, the magnitude of the error introduced into the baseline measurement is the product of the timing error and the satellite angular velocity (about $1.5 * 10^{-4}$ rads s^{-1}). Thus for 1 ppm accuracy, the receivers must be within 7 milliseconds of UTC. This form of error could also affect point positioning results.
b) There may be an offset between the two receivers (that is lack of synchronisation); in this case, the size of the error will be the product of the offset and the radial satellite velocity (1 kms). Receiver clock offset should be under 3 microseconds to reduce this error to less than 1 centimetre.
c) A rate difference between the two receivers is also possible but will not cause a problem unless the oscillators are badly out of adjustment or have not been adequately warmed up. A rate difference of 1 part in 10^{9} (that is, 1.57 Hz at L1) will produce an epoch difference of 4 microseconds after 1 hour.

5.4. Refraction.

Until now, constant velocity has been assumed for the signals

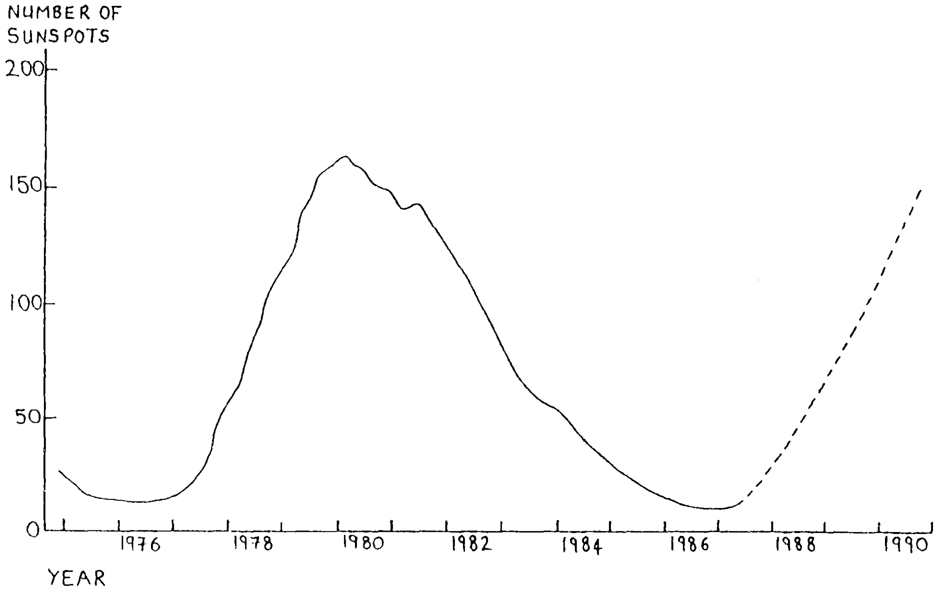


FIGURE 17: SOLAR ACTIVITY; SUNSPOTS.

travelling from the satellite to the receiver. Unfortunately this is not the case because GPS signals, like all electromagnetic waves, are refracted by the atmosphere through which they pass. The atmosphere may be conveniently divided into two areas in order to describe and analyse its effect on wave propagation.

1. Ionospheric Refraction.

The ionosphere is that part of the atmosphere from 50 km to 1,000 km where ionisation of atmospheric gases occurs. The ionospheric contribution to pathlength variation is proportional to the highly variable Total Election Content (TEC) of the ionosphere and inversely proportional to the square of the signal frequency. The ionosphere is a dispersive medium and the phase delay (phase divided by frequency) is equal, but opposite in sign, to the effect on group delay (which is the derivative of phase with respect to frequency). Thus carrier phase measurements are subject to ionospheric phase advance (that is, its phase is increased negatively: see equation 15), while pseudo-range measurements using the codes are affected by ionospheric group delay, which is added to the expected delay, of course. Pseudo-ranges can alter by 5 to 50 metres due to ionospheric effects.

For phase observations, the ionospheric effect is equal to:

$$\partial\phi = -40.28/fc \int N\ e\ \delta s \qquad (15)$$

where $\delta\phi$ = The ionospheric effect (in cycles of phase).
c = The speed of light (ms).
f = The transmitted carrier frequency.
$\int N\ e\ \delta s$ = The TEC.

Ne is a function of the elevation angle of the satellite and the electron density. The electron density in turn is dependent on latitude, the time of day, season of the year, and the current point within the eleven year sunspot cycle. Figure 17 illustrates the sunspot cycle; it can be seen that sunspot activity is at a minimum at present, and so ionospheric refraction effects are currently having their least effect. A two to four times increase in ionospheric delay and irregularity is expected over the next few years. The layer of the ionosphere which induces the greatest refraction disappears at night, and mid-latitudes tend to have a more homogeneous ionosphere than equatorial or high latitude areas. Errors due to ionospheric refraction should therefore be smaller in mid-latitudes, and from observations taken at night.

Short term variations in the TEC may also cause amplitude and phase scintillation. These effects are especially severe in high and low latitudes, and are more difficult to account for than the "regular" advance or delay of GPS signals.

Figure 18 (opposite page 38) graphs the ionospheric variation over a thirty minute period. The variation is illustrated in terms of the ionospheric delay at the L1 frequency. The data was produced using TI4100 integrated dual frequency carrier phase advance measurements on GPS SV8. (Lachapelle et al, 1987). The ionospheric variation indicated is considered average for high

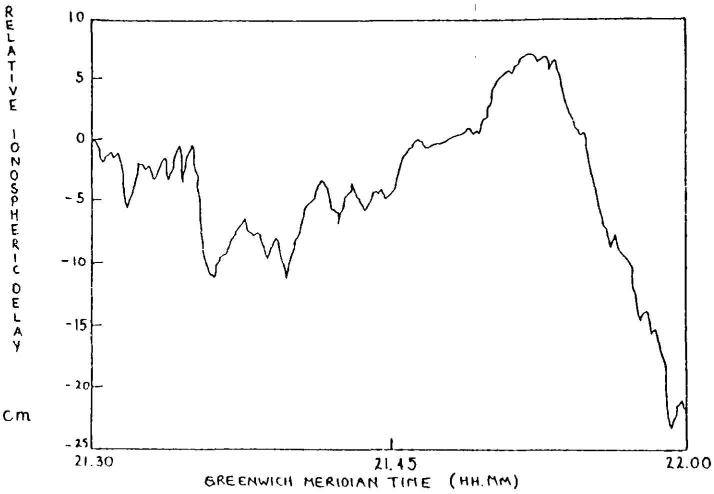


FIGURE 18: IONOSPHERIC DELAY AT L1 FREQUENCY, 34° ELEVATION, AT CAMBRIDGE BAY, DAY 225 1985.

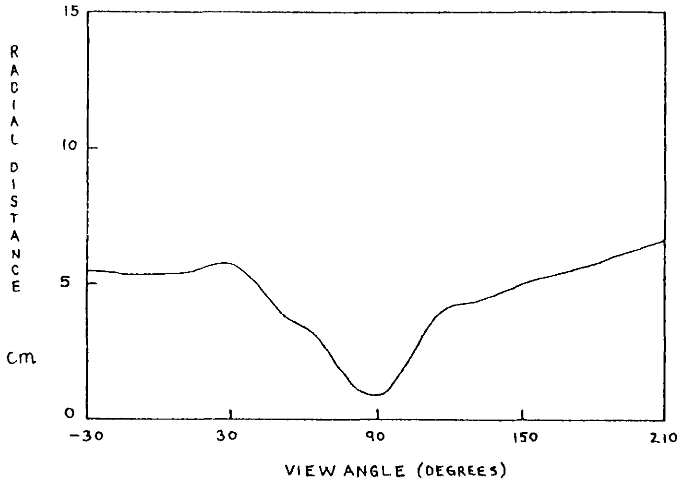


FIGURE 19: PHASE CENTRE VARIATION OF A COMMERCIAL GPS ANTENNA.

latitude regions.

For single frequency observations made in a normal, day-time, mid-latitude environment, the ionospheric effect on a baseline will be approximately one to two parts per million of the baseline for a two hour observation period (King et al, 1985). Daily and short term variations may, however be much more significant. Lachapelle et al (1987) quote a test in Hawaii (20° N) where a daily change of 20 ppm was observed over baselines up to 400 km long. Davidson et al (1983) suggest that for a (maximum) TEC of $2.8 * 10$ el/m^{-1} the delays for the L1 and L2 frequencies are 45 metres and 75 metres respectively.

2. Tropospheric Refraction.

The effect of the troposhere (that region of the atmosphere immediately above the ground to about 50 km altitude) on propagation delays is a function of satellite elevation at the time of observation and various tropospheric variables such as temperature, atmospheric pressure due to dry gases, and partial pressure of water vapour. The effect increases from a few metres delay for a satellite observed at 90° elevation to about 25 metres when observing 5° above the horizon. The effect is exponential below 15° elevation angles and has been known to reach 50 metres during horizontal observation. There are two components to the tropospheric effect. The "dry" part constitutes approximately 90% of the total zenith range error but can be estimated to about 0.02% (that is, about 0.5 cm) from surface pressure measurements. Horizontal gradients in the temperature structure versus height, however, must be considered.

The "wet" component of the tropospheric error is not as easy to model due to its dependence on water vapour content and distribution, which may vary considerably even over short distances and over brief time intervals. Thus conditions along the path are often not closely correlated with surface conditions. Several models (Hopfield, Black, Yionoulis, all from Saastamoinen, 1973), employing surface pressure, relative humidity and temperature, have been developed to try to account for this delay (see section 6.3), which can reach several parts per million of the baseline length.

The atmosphere also induces a small bending effect, however this is on a millimetre scale at elevation angles greater than 15 degrees.

5.5. Multipath And Imaging.

For an accurate measurement between the satellite and receiver, the direct line between the two points is required. Unfortunately electromagnetic waves are capable of reflecting off surfaces, and therefore both direct and reflected signals can be obtained by the receiver, causing erroneous measurements . If a single strong reflection at the receiver is considered, then the signal accepted by the antenna is the vector addition of both the direct and the reflected signal. The resultant of these signals will usually have a different phase angle from that of the direct wave alone, and so an error results. The magnitude of the error depends on the strength of the reflected signal and its phase angle with respect to the direct ray. Such a problem is termed "multipath"; it is prudent to assume that it may be present in all measurements and so suitable precautions should be taken. Sites often have their own multipath "signature" because multipath varies from day to day in a cyclic manner. The effect is not dependent on baseline length but will vary according to three factors:

a)The character of the environment surrounding the receiver. The strength of the reflected signal depends largely on the nature of the reflecting surface. Coefficients of reflection as high as 60° can be obtained from some surfaces (Burnside, 1982). Many different types of surface can be responsible for multipath, including the ground, trees, oil platform superstructure and certain sea states. The actual phase difference produced depends on the difference in path length (between direct and reflected signal), the angle of incidence and the permittivity and conductivity of the reflecting surface. There may also be a phase change at the point of reflection. Multipath can also occur at the satellite, however differential techniques should reduce this error to less than 10 cm (Lachapelle et al, 1987).

b)The frequency of the carrier and modulation will alsoaffect the strength of reflection and the phase difference between the direct and reflected wave. The code modulation of the GPS signals provides an inherent rejection of the signals which do not occur within one code chip size of the direct pseudo-range. Maximum errors using pseudo-ranges are therefore 29.3 metres and 293 metres for the P and C/A codes respectively. The carrier wavelengths are approximately 20 cm and so the effect of multipath on carrier phase measurement is two orders of magnitude smaller than that on pseudo-range measurement. This is an important distinction between code and carrier measurement techniques.

c)The type of antenna employed will also affect the severity of multipath experienced. There is presently much debate as to which type of antenna will give the most favourable results (see section 6.8.3).

Imaging is another effect similar to that of multipath. A nearby object can act as a mirror and thus produce a "mirror image" of the antenna. This "second antenna" produces a coupling effect which affects the phase characteristics of the incoming signal, which in turn produces an erroneous measurement of the range.

4.5. Instrumental Effects.

Receiver clock instability is the most significant instrumental effect in terms of error analysis, and has been discussed in conjunction with satellite clock bias. Other receiver/processor hardware and software, however, may be the source of smaller, but nonetheless significant, errors.

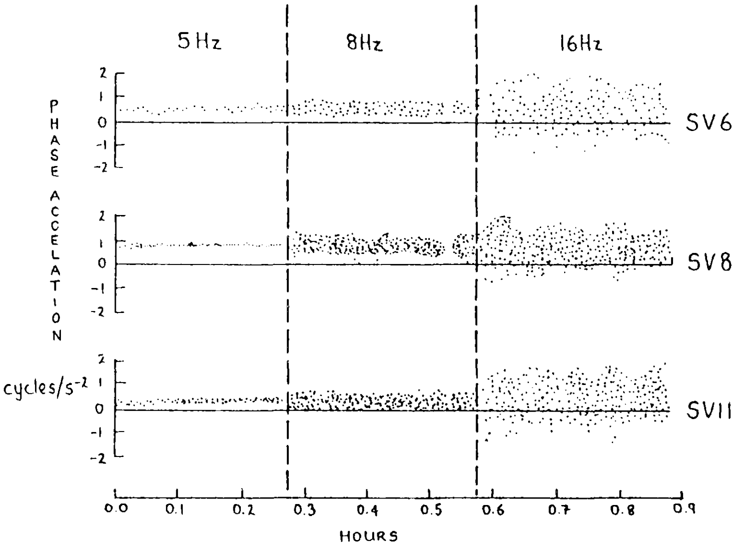


FIGURE 20: EFFECT OF TRACKING BANDWIDTH ON MEASUREMENT NOISE.

1. Antenna Phase Centre Variation.

The electrical phase centre of the receiving antenna varies from the antenna's geometric centre , depending on the configuration and individual properties of the receiving elements, on the phase of the incoming signal, and on surrounding environmental factors. Lachapelle et al (1987) employed phase response patterns to measure phase centre variations of three GPS antennae. Variations of up to about 60 mm were observed (Fig. 19, facing page 38). Two antennae of the same design (quadrifilar helix) showed different phase variation patterns, thus indicating that not only the particular type but also the quality of construction of the antenna is important. Phase centre variations differed considerably (80 mm) between two cases; once where there was no multipath effects present and another where strong multipath was induced. This demonstrates that the local environment can also affect the position of the phase centre.

2. Receiver Channels.

The number of receiver channels incorporated in an instrument is an important factor in error analysis. A single channel receiver must track each satellite signal sequentially and so measurements must be made at different points in time. This will produce small errors. These errors can be reduced by employing a multiplexing receiver which will switch very rapidly from one signal to the next, sampling four or five signals every few tens of milliseconds. This increases the number of fixes obtained in any period of time, leading to higher accuracy, but the decreased time spent at each satellite results in a loss of signal energy which translates into a lower measurement precision. A multichannel receiver, tracking several satellites simultaneously and continuously, will not suffer these problems. Furthermore, positional update is limited only by computational throughput. However the receiver delay along the electrical path (filtering, decoding and so on) between reception and recording will vary from channel to channel. Theoretically these interchannel biases can be eliminated by locking all channels to the same satellite and determining the biases in a calibration procedure, but for this to be effective, the characteristics of the channels must not change throughout the observation period. This is unlikely due to changes in temperature (both environmental and internal) and pressure, and so calibration must be performed either periodically by locking all channels to one receiver or by providing one channel to continuously compare and record interchannel biases.

3. Tracking Bandwidth.

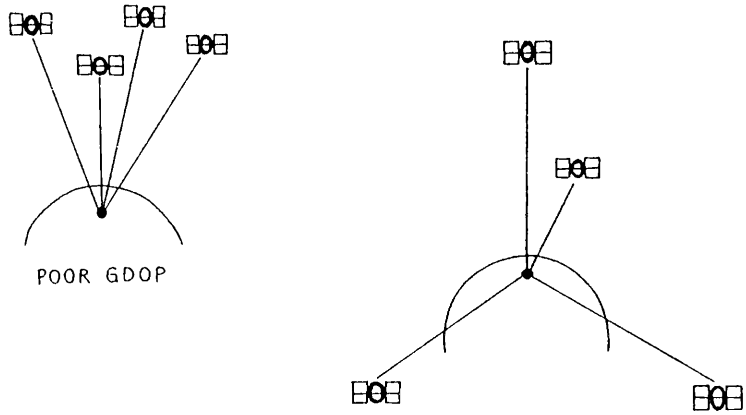
This is the frequency range over which the receiver "observes". A narrow tracking bandwidth (for example 0.7 Hz) will result in a more accurate measurement than that produced from a system employing a wider bandwidth (say 16 Hz) because measurement noise is cut out to a greater degree by the narrower observing window. Figure 20 illustrates carrier phase measurement noise for three tracking bandwidths, measured in terms of phase acceleration (from Lachapelle et al, 1987). Phase lock (that is, signal retention) must be maintained to avoid cycle slips (see section 5.6.4.), but this is more difficult using a narrow tracking bandwidth. Thus there is a trade off between the possibility of cycle slippage and increased measurement accuracy. The system could be likened to catching falling tennis balls into a bag, during a snowstorm. A large bag (wide bandwidth) will ease the ball catching (phase lock or signal acquisition), but will allow a lot of snow to fall into the bag as well (snow being analogous to signal noise). A smaller bag, representing a narrower bandwidth, will make catching the tennis balls more difficult (a dropped ball equalling cycle slippage or phase loss), but will reduce the amount of snow (noise) also being caught.

The main dynamic characteristics of the received L1 GPS signal at a fixed location on the Earth's surface is a Doppler shift of up to +/- 3800 Hz and a maximum rate of change of Doppler shift of approximately 0.6 Hzs (Pratt, 1987). For static surveying with the GPS, these characteristics are all that need be considered when establishing the optimum tracking bandwidth. In GPS navigation equipment, however, the host vehicle's movements must also be considered. Highly dynamic vehicles such as fighter aircraft will produce maximum shifts similar to those of the satellites themselves, while other vehicles will induce much smaller frequency offsets. Thus tracking bandwidth is application dependent. Ideally it should be variable, in which case the bandwidth employed could be that which maximises measurement accuracy, whilst discouraging cycle slips. It may even be possible to utilise a wide bandwidth while navigating, when accuracy requirements are not stringent, and then close down the bandwidth as the target is approached. This would allow more accurate position determination, and cycle slips would be less likely due to the lower velocity. Upon reaching the target, the minimum bandwidth can be employed to provide the best position fix while the vehicle is stationary. The bandwidth would subsequently be opened up when the target is left in order to find the next point of interest.

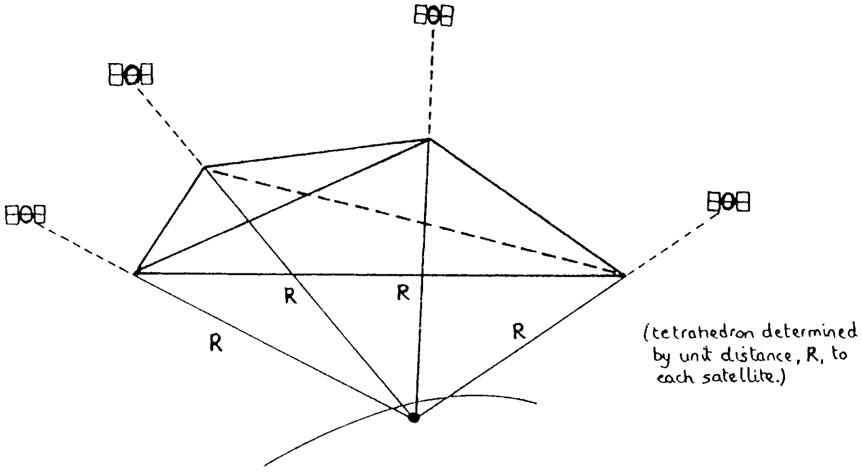
4. Cycle Slips.

A cycle slip is a sudden gain or loss of some whole number of cycles, one cycle equaling one code or carrier wavelength. Tracking stops for a moment due, for example, to occultation of a satellite, signal interference, atmospheric irregularities or excessive change in velocity or direction of the receiver. After reacquisition, the fractional phase measurement is the same as if tracking had been maintained, but the integer number of cycles is incorrect. If the slip is not corrected in the processing, by the addition or subtraction of the appropriate number of wavelengths, then a gross error results. The ability to detect such errors depends on the magnitude of one cycle relative to the magnitude of noise also present in the observation. For short baselines (about 10 km), the root mean square (rms) noise level in a double-difference observation is typically about 0.1 cycle (Bock et al 1985) and therefore a slip of just one cycle is

FIGURE 21: SATELLITE GEOMETRY.


POOR GDOP

GOOD GDOP



(tetrahedron determined by unit distance, R, to each satellite.)

FIGURE 22: GDOP DETERMINATION.

conspicuous in a single baseline, single pair of satellites observation network. This is not the case in a multiple-station, multiple-satellite network, where one cycle slip may be distributed into fractional slips by automatic adjustment procedures. Oscillator errors usually dominate in one-way phase measurements, which means that only obvious (large) slips are detectable.

The magnitude of the error varies proportionally with the number of cycles which have been missed. As noted above, a receiver employing a narrower tracking bandwidth will be more susceptible to cycle slippage. Short periods of slippage may be acceptable for navigation receivers, but will normally not allow high accuracy surveying.

5. Data Measurement Rate.

The rate at which measurements are observed, and the rate at which these measurements (or derivatives, such as averages) are made available to the user can affect the final accuracy of the system. For navigation systems, especially those of a highly dynamic nature, the measurements must be made at very short intervals (approximately one second) in order to record vehicle movement successfully. As discussed in section 6.5, velocity linearity is assumed in many cycle slip detection and correction methods; this assumption is all the more valid for receivers using short measurement intervals.

Most receivers output a filtered position fix. This is adequate for real-time positioning, however the raw data is necessary to produce the highest accuracy results. The filtering process results in a delay which will reduce accuracy under high kinematic navigation. Lachapelle et al (1987) demonstrate a three second time lag in a TI 4100 , using a rapid vehicle stop test.

5.7. Geometry.

In conventional resection the geometry of the control framework, with respect to the resected point, affects the final accuracy of the position fix. Similarly, the particular geometry of the satellites being observed will also affect the accuracy of positioning. For a given set of measurements to four satellites, the accuracy of the computed position is a number of times worse than the measurement precision, which is equal to the sum of ephemeris, receiver and propagation errors. The single measurement ranging error is multiplied by the quantified spatial relationship of the satellites. This spatial parameter, called the geometric dilution of precision, (GDOP) generally ranges from around one or two to about four for "good" to"average"satellite geometries. Poor satellite geometries will have a GDOP greater than five (and can go as high as 50). Thus if the measurement precision was 1.5 metres and the GDOP was 3, then the final position accuracy would be 4.5 metres. Simple illustrations of "good" and "poor" GDOP are given in Figure 21. The optimum four satellite geometry consists of one satellite at the observer's zenith, and the other three situated around the horizon, at 120

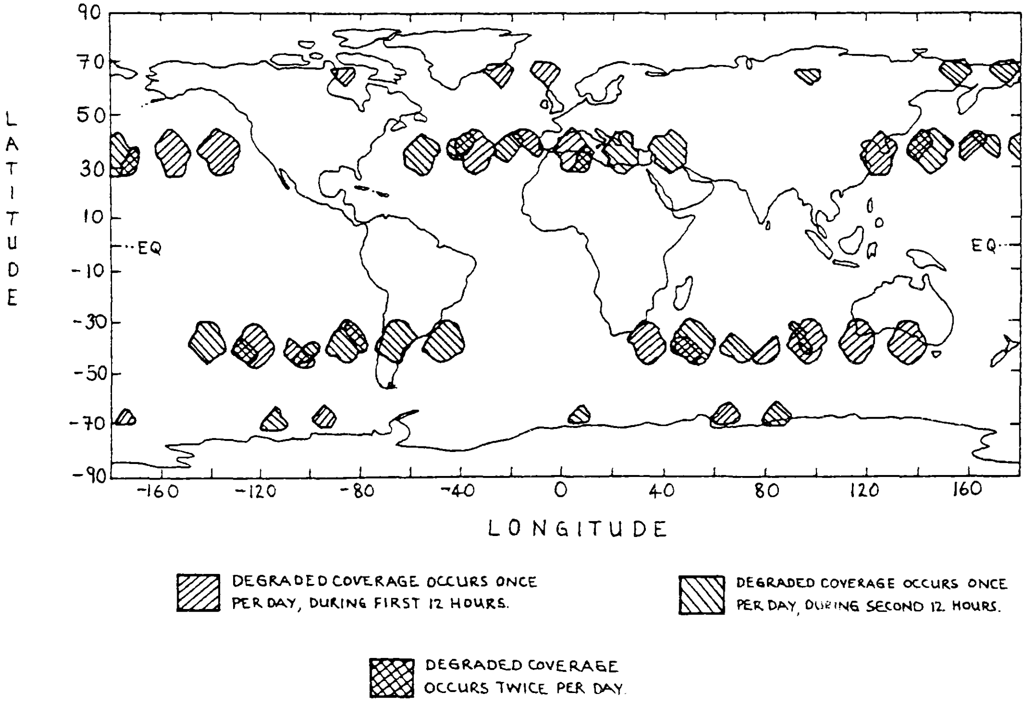


FIGURE 23 : OUTAGE AREAS (PDOP>6).

intervals.

GDOP may be divided as follows:

GDOP : Geometric Dilution (Northings, Eastings, Height, Time).
PDOP : Position Dilution (Northings, Eastings,Height).
HDOP : Horizontal Dilution (Northings, Eastings).
VDOP : Vertical Dilution (Height).
TDOP : Time Dilution (Time).

In general, satellite geometries will have better HDOPs than VDOPs (see Chapter 4 for explanation).

Several schemes have been developed to select the best GDOP from the available satellites. The obvious scheme is to combine measurements from all the satellites in view, above a selected mask angle. However, since there may be up to seven satellites visible, user equipment would have difficulty acquiring, tracking ranging and computing a real-time continuous solution, especially those operating on two frequencies. The ISTAC model 2002, however, does implement this technique, although not in a real-time mode, because it is "codeless". One of the more common approaches relates the PDOP value to the volume of a tetrahedron (see Figure 22, facing page 42). The tetrahedron is formed by the unit vectors from the receiver to each of four satellites in view, and has volume V. PDOP is then inversely proportional to V and thus maximising V will produce the optimum geometry. Note that the best satellite configuration in terms of GDOP may not coincide with the best configuration in terms of measurement precision.

The proposed GPS constellation does contain several "outages", which are time intervals in specific areas during which the dilution of precision is greater than a specified number (perhaps about six). The three active spare satellites are to be configured to reduce the number of outages to a minimum. The specific areas are depicted in Figure 23; the duration and size of the areas affected by these outages will depend on the particular mask angle employed by the user.

In addition to these predictable outages, other periods of poor geometry will undoubtably occur due to satellite malfunctions (equipment failure, jamming, shading etc). Obviously the exact location and extent of these unpredictable outages will depend on the number of simultaneous failures and the location of the failures. As far as satellite failure is concerned, however, the general areas would still be along the 65°N, 35°N, 40°S, 65°S parallels of latitudes as shown in Figure 23.

For the areas of degraded coverage, the PDOP value often becomes very large for a period of a few minutes. Where there are two outages in one 24 hour period, one will generally have a PDOP value of between 15 and 20, while the other will exceed 20. For the remainder of each 24 hour period, PDOP will be between one and five. At any time there can only be one to three outages occurring anywhere in the world, and no others can appear until a multiple of 40 minutes later. During these periods there is no degraded coverage.

CHAPTER SIX - ACCURACY ENHANCEMENT

6.1. Signal Differencing.

A single carrier or code phase observation to a single satellite (a one way phase measurement, that is) will contain most, if not all, the errors described in the previous chapter. If, however, pairs of single phase or pseudo-range measurements are made, then errors common to both observations (ephemeris, propagation, and clock errors) can be eliminated or at least reduced by signal differencing. This technique has been applied in surveying as a means of negating errors common to two measurements. Equation 10 (page 30) demonstrates the effect in relative geoid determination. Differences can be taken between satellites, receivers, epochs and frequencies. All but "between-epochs" are simultaneous measurements, in other words, the observations are referred to time frame epochs which are either exactly equal, or else they are extremely closely spaced in time. In the latter case, the misalignment can be allowed for using correction terms in the observation equations used in the adjustment, rather than by parameter estimation.

By employing the differencing technique, the position of a mobile receiver can be determined relative to a fixed master station. This scheme could be developed to involve permanent GPS stations. Lachapelle et al (1987) suggest that if a baseline of 1000 km is the subject of differential positioning, then the broadcast orbital error will be about one metre. Furthermore, unmodelled atmospheric effects will also be reduced and the receiver located at the base station can easily detect unpredictable errors caused by satellite clock or ephemeris problems (for example, those cited in section 6.8.4.d). Biases introduced by C/A code degradation should be largely eliminated by differential observation, over baselines up to a few hundred kilometres.

1. Between-Satellites.

The difference between two carrier phase beat frequency readings (equation 4, page 18) obtained in one receiver (j) from two satellites (h and i), at time (t), is expressed as:

$$\phi_j^{hi}(t) = \phi_j^h(t) - \phi_j^i(t)$$

$$= \phi^i(t) - \phi^h(t) - f/c\,(p_j^h - p_j^i) + N_j^i - N_j^h \qquad (16)$$

where $\phi_j^{hi}(t)$ = The phase observation between satellites h and i and receiver j.
$\phi_j^h(t)$ = The beat frequency of receiver j and satellite h.
$\phi_j^i(t)$ = The beat frequency of receiver j and satellite i.
$\phi^h(t)$ = The phase value of satellite h.
$\phi^i(t)$ = The phase value of satellite i.
f/c = The inverse of the wavelength of the carrier.
p_j^h = The range between receiver j and satellite h.

p_j^i = The range between receiver j and satellite i.
N_j^h = The integer unknown between receiver j and satellite i.
N_j^h = The integer unknown between receiver j and satellite h.

This type of observation largely eliminates the effects of instabilities in the receiver clock (α_j in equation 5, page 19). As the satellites carry atomic oscillators, pseudo-range or phase observations from several satellites enable relatively accurate positioning of a receiver which contains only a crystal clock. This form of differencing is limited, therefore, by the satellite clock accuracy (as well as ephemeris accuracy, atmosphere and such like); usually to several metres.

2. Between-Receivers.

The difference between two signals received at two stations, but from the same satellite, is generally called a "between-receivers" difference. For an observation at epoch (t), this takes the form:

$$\phi_{jk}^i\ (t) = \phi_j^i\ (t) - \phi_k^i\ (t)$$

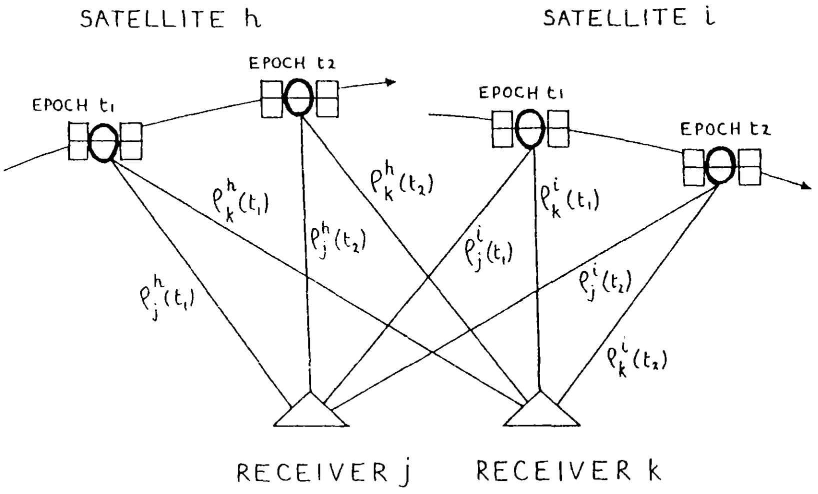
$$= \phi_k\ (t) - \phi_j\ (t) - f/c\ (p_j^i - p_k^i) + N_k^i - N_j^i \qquad (17)$$

where ϕ_{jk}^i (t) = The phase observation between receivers j and k and satellite i.
ϕ_k^i (t) = The beat frequency of receiver k and satellite i.
ϕ_j (t) = The phase value of receiver j.
ϕ_k (t) = The phase value of receiver k.
p_k^i = The range between receiver k and satellite i.
N_k^j = The integer unknown between receiver k and satellite i.

(other symbols are as with equation 16 notation)

This type of observation eliminates satellite clock irregularities (β^i in equation 5, page 19), and is generally not often used in the field because most receivers contain crystal clocks whose biases will dominate the measurement whether or not satellite clock instability has been cancelled. GPS tracking stations, however, will employ between-receiver differences, since they are equipped with atomic frequency standards.

The above differences are termed "single differences" for obvious reasons. Two other forms of single difference are possible. A "between-epochs" difference is taken between two complete carrier beat phase measurements made by the same receiver, on the same satellite signal. This is the same as integrated Doppler measurements and cancels all three terms in equation 5 (both clock phase biases and the integer ambiguity), because all are common to both measurements. Clock errors however, will remain. King et al (1985) indicate that such an observation results in poorer relative position estimates than other forms of differencing since some of the information is



BETWEEN-RECEIVER	BETWEEN-SATELLITE	BETWEEN-EPOCH
$[\rho_j^h(t_1)-\rho_k^h(t_1)]$	$[\rho_j^h(t_1)-\rho_j^i(t_1)]$	$[\rho_j^h(t_2)-\rho_j^h(t_1)]$
$[\rho_j^i(t_1)-\rho_k^i(t_1)]$	$[\rho_j^h(t_2)-\rho_j^i(t_2)]$	$[\rho_k^h(t_2)-\rho_k^h(t_1)]$
$[\rho_j^h(t_2)-\rho_k^h(t_2)]$	$[\rho_k^h(t_1)-\rho_k^i(t_1)]$	$[\rho_j^i(t_2)-\rho_j^i(t_1)]$
$[\rho_j^i(t_2)-\rho_k^i(t_2)]$	$[\rho_k^h(t_2)-\rho_k^i(t_2)]$	$[\rho_k^i(t_2)-\rho_k^i(t_1)]$

FIGURE 24: SINGLE DIFFERENCES.

removed during the differencing (the integer ambiguity is not produced). A single erroneous phase measurement, however, affects only two consecutive epoch differences, and so between-epoch observation can be employed in cycle slip editing.

These three types of differencing are illustrated in Figure 24. A difference may be taken between frequencies, producing a linear combination of the L1 and L2 signals which has a coarser or finer wavelength.

3. Double Differencing.

Two single differences can be further differenced to form a double difference measurement. The most common form is a receiver-satellite double difference, but other types may be formed. The observation equation for a receiver-satellite double difference takes the form:

$$\phi^{hi}_{jk}\ (t) = \phi^{h}_{jk}\ (t) - \phi^{i}_{jk}\ (t)$$

$$= -\ f/c\ (\rho^{h}_{k} - \rho^{h}_{j} - p^{i}_{k} + p^{i}_{j}) + N^{i}_{j} - N^{i}_{k} - N^{h}_{j} + N^{h}_{k} \qquad (18)$$

where ϕ^{hi}_{jk} = The double difference observation among satellites h and i and receivers j and k.

ϕ^{h}_{jk} = The single difference taken between receivers j and k and satellite h.

p^{h}_{k} = The range between receiver k and satellite h.

N^{h}_{i} = The integer unknown between receiver i and satellite h.

(other symbols are as in the notation employed in equations 16 and 17)

Whereas in the single differences, epoch-to-epoch clock differences which are not eliminated must be modelled or estimated, with this type of measurement all the satellite and receiver instabilities are removed. Thus here phase bias is an integer if noise is ignored. For this reason, the double difference type of observation is very common in GPS surveying. The integer nature of the biases will deteriorate as baseline length increases, because non-cancelling effects, such as ionospheric and tropospheric refraction, and ephemeris errors, contaminate the solution.

4. Triple Differencing.

A pair of double differences may be further differenced, usually between epochs, to produce triple differences:

$$\phi^{hi}_{jk}\ (t_{1,2}) = \phi^{hi}_{jk}\ (t_2) - \phi^{hi}_{jk}\ (t_1)$$

$$= -\ f/c\ \left\{\rho^{hi}_{jk}\ (t_2) - p^{hi}_{jk}\ (t_1)\right\} \qquad (19)$$

where $\phi^{hi}_{jk}\ (t_{1,2})$ = The triple difference observation among receivers j and k, satellites h and i and epochs t_1 and t_2.

$\phi_{jk}^{hi}(t_2)$ = The double difference at epoch t_2.
$\phi_{jk}^{hi}(t_1)$ = The double difference at epoch t_1.
$p_{jk}^{hi}(t_1)$ = The range differences between receivers j and k and satellites h and i at time t_1.
$p_{jk}^{hi}(t_2)$ = The range differences between receivers j and k and satellites h and i at time t_2.

(other symbols are as in the notation employed in equations 16, 17 and 18)

In this observation, the cycle ambiguity has been eliminated, and the only remaining unknowns are the coordinate differences between the receivers (j and k). Unfortunately, this means that the ambiguity cannot be restrained to an integer (see 6.4.), which reduces errors due to refraction. This is why double differences generally provide better results than triple differences. Loss of lock, however, will cause a spike (or "outlier") in the triple difference data, and this characteristic can be used in cycle slip editing.

5. Correlation.

There has been considerable discussion concerning the various forms of differencing (for example Goad, 1985, Ashkenazi & Yau, 1986). Although double differencing eliminates satellite and receiver clock biases, single phase measurements appear more than once, even for a single baseline, and thus the baseline data sets are correlated. Only one way phase data and single difference data can be considered to be geometrically independent. In order to produce accurate results, correlations in double and triple differences should be modelled. This can be done by either including an appropriate weight matrix in the least squares adjustment, or by decorrelating double difference data, at an epoch, using a transformation procedure to generate a linear combination of the double difference observations. Goad (1985) notes that for both of these methods some means of identifying duplicative observations is necessary. Thus, using Goad's example, duplicate measurement $\phi_{3,2}^{4,9}$ should not be included when

$$\phi_{3,2}^{4,9} = \phi_{3,1}^{4,9} - \phi_{2,1}^{4,9} \qquad (20)$$

and the right hand side measurements have been considered.

Goad (1985) outlines a technique which allows the data to remain uncorrelated (no matter how many stations are observing simultaneously) while also managing to incorporate the integer nature of the biases. This "base-station, base-satellite" concept involves the following steps:

a) Designate a base-station and a base-satellite arbitrarily.
b) For each phase measurement, if either the satellite or station involved in the phase measurement is "base", then the normal formula for the observation applies.
c) If neither the base-station nor the base-satellite are involved, then employ the "K formulation" for the observation:

$$K_2^9 = \left\{\bar{N}_2^9(t) - \bar{N}_2^4(t)\right\} - \left\{\bar{N}_1^9(t) - \bar{N}_1^4(t)\right\} \qquad (21)$$

where station 1 and satellite 4 are the bases and:

$$\bar{N}_j^i = N_j^i + \phi^i - \phi_j \qquad (22)$$

so that, in this case:

$$\phi_2^q(t) = - f/c\ p_2^q + \bar{K}_2^q + \bar{N}_2^4(t) + \bar{N}_1^q(t) - \bar{N}_1^4(t) \qquad (23)$$

Under optimal conditions, the values of K should be nearly integer.

A study by Ashkenazi (1986) indicated that pure phase and single differences are indeed uncorrelated, and similar results should be obtained from either type of data. Double differenced data should also produce similar results if the correlations are modelled correctly. It is important to note, however, that significantly different results can be obtained if the correlations are ignored. Ashkenazi does not recommend triple difference processing, except as a means of pre-processing the data.

6. Alternatives to Differencing.

One way phase and single difference observations can be employed successfully by treating the clock errors as nuisance parameters. There are two common methods.

a) The oscillator errors at every epoch can be eliminated by implementing a partitioning scheme with the least squares normal equations (see King et al 1985, Goad 1985, Hatch & Larson 1985). By doing this the so called "epoch parameters" (those parameters which change from epoch to epoch; typically the oscillator errors) are taken out as each set of observations for each epoch is processed. This method allows unequal measurement precision from different receivers to be modelled withrelative ease. Partitioning also overcomes the correlation problems encountered by double differencing.

b) A second technique (Bock et al, 1985) involves orthogonalisation of the data to obtain nuisance-free observations containing no clock terms. If the observations at each epoch have equal variances, then the orthogonalised observations will also be uncorrelated. So this method allows the correlation problem to be dealt with. On the other hand unmodelled systematic errors, such as cycle slips, prove difficult to detect in orthogonalised observations. Such errors, however, can often be isolated by examining differences between the non-orthogonalised observations.

6.2. Ionospheric Refraction Correction.

The optimum way to account for ionospheric refraction is by dual-frequency operation. The effect of the ionosphere is inversely proportional to to the square of the signal frequency, and so by observing at both the L1 and the L2 frequency, the effect can be modelled. The carrier phase advance error can be found using the following equation:

$$\phi_c = \phi_{L1} - 1.984(\phi_{L2} - 0.779) \qquad (24)$$

where ϕ_c = The ionospheric refraction-free phase observation.
ϕ_{L1} = The observed L1 phase.
ϕ_{L2} = The observed L2 phase.

Pseudo-ranging is subject to ionospheric group delay, and the correction to the L1 pseudo-range is expressed as follows (from Henson and Collier, 1985):

$$R_{c1} = R_{L1} + 1.546(R_{L1} - R_{L2}) \qquad (25)$$

where R_{c1} = The ionospheric refraction-free L1 range.
R_{L1} = The observed L1 pseudo-range.
R_{L2} = The observed L2 pseudo-range.

Relative positioning tends to cancel ionospheric effects. For short baselines, both sites record the satellite signals through virtually the same part of the ionosphere, and so the differential delay is often negligibly small. King et al (1985) suggest that for short baselines (up to a few tens of kilometres) 0 is noisier than the corresponding single frequency measurement because errors due to multipath are magnified when the two frequencies are combined and become the dominant errors over small distances. Thus single frequency measurements can be preferable for short baselines. Single frequency observation is also advantageous when the ionospheric delay is constant with time (again the multipath error will dominate in dual frequency operation). Ionospheric correlation deteriorates approximately linearly with site separation, and so, over longer baselines, ionospheric differences are usually significant and therefore dual-frequency operation pays dividends (see Lachapelle and Cannon, 1986). By this method, the effect can be reduced to about one part per million.

Single frequency users must either ignore the ionospheric correction or employ a simple mathematical model to allow for the effect. A pseudo-range delay model (Klobuchar, 1983) is broadcast by the GPS satellites, and its parameters are continually updated. However the model's overall root mean square error in mid-latitude regions is quoted at 50% of the actual correction (Lachapelle et al, 1987), and at least one study (Lachapelle & Wade, 1982) has suggested that applying this correction produces poorer results than the original, uncorrected measurements. Single frequency users can resort to night-time observation, which will reduce ionospheric effects by a factor of five or greater. A "mask angle" is always employed so that satellite signals are not recorded once the satellite has sunk below a specified elevation angle. This prevents the use of signals which have travelled through large parts of the ionosphere and troposphere; the equivalent in conventional surveying is the elimination of glancing rays. The mask angle is usually between 5 and 20 degrees. A higher mask angle will reduce the atmospheric component in the final error, but will increase the GDOP value.

6.3. Tropospheric Refraction Correction.

The dry component is quantitatively the more important part of the tropospheric error and can be modelled to an accuracy of about 0.02% using surface pressure measurements. In the zenith direction it can be approximated by (Wells et al, 1986):

$$\Delta R_D = 2.27 * 10^{-3} P \tag{26}$$

where ΔR_D = The dry component range correction (metres).
P = The surface pressure (millibars).

Ashjaee (undated, and 1985) introduces another model for the dry component, based on the elevation of the satellite observed:

$$\Delta R_D = 2.4255/(\sin E + 0.025) \tag{27}$$

where E = The elevation angle of the observed satellite.

Saastamionen's (1973) model deals with both wet and dry components and can be used for observations greaterthan 20° above the horizon:

$$t_{atm} = 7.595 * 10^{-12} \sec z \left\{ P + ((1255/T) + 0.05)e - \tan^2 z \right\} \tag{28}$$

where t_{atm} = The atmospheric delay (seconds).
z = The zenith angle of the satellite.
P = The pressure (millibars).
T = The temperature in Kelvin.
e = The partial pressure of water vapour (millibars).

The partial pressure of water vapour can be obtained (as a fraction of one) from the following equations:

$$e = 6.108 \; RH \exp \left\{ (17.15\, T - 4684)/(T - 38.45) \right\} \tag{29}$$

where RH = The relative humidity.
and the pressure is given by:

$$P = P_S \left\{ (T - 4.5h)/T_S \right\} \tag{30}$$

where P = The required pressure.
P_S = The surface pressure.
T_S = The surface temperature.
h = The height above sea-level (kilometres).

Obviously to create this model, measurements of surface temperature, pressure and relative humidity are required at the receiver locations. Ware et al (1983) note that this model cannot correct for any azimuthal variation in the index of refraction. Ashkenazi et al (1982) found that various tropospheric correction models for TRANSIT produced coordinate component differences, especially in height, of the order of one to two metres, so obviously modelling the troposphere causes some problems. Refractive index is independent of frequency up to about 15 GHz, and so this point is applicable to GPS.

For surveys of points separated by short distances, between-stations or double differencing will allow cancellation of the tropospheric effect to a large degree. The variability of the water vapour content, however, precludes removing the tropospheric effect over distances greater than a few tens of kilometres using differential observation techniques. Lachapelle et al (1987) note that this is particularly true in marine applications, where the GPS receiver control station is typically on land while the mobile receiver is at sea, usually a few hundred kilometres away.

Dual channel water vapour radiometry at 22 and 31 GHz (MacDoran, 1979) has demonstrated the ability to measure water vapour path delay to an accuracy of 12 to 15 mm (less at elevations higher than 20°), and thus allow better determination of the wet component than would otherwise be possible. Ware et al (1983) provide a model for making water vapour radiometer (WVR) corrections to the wet component:

$$\phi_W = V * 6.5/\lambda_{L1} \tag{31}$$

where ϕ_W = The tropospheric phase delay due to the wet component.
V = The integrated water vapour (centimetres) measured by the WVR.
λ_{L1} = Half the L1 carrier frequency (centimetres).

The model appears to have a repeatability of 1 ppm during disturbed weather conditions, which is consistent with calm weather, non-WVR corrected data. Unfortunately WVRs are expensive and bulky, and so at present they will only be employed in the highest accuracy work. Prices may drop in the future, and advancing technology will undoubtably enable the production of more portable WVRs.

6.4. Phase Ambiguity Resolution.

Equation 5, page 19, demonstrates that in the undifferenced carrier beat phase and single-difference phase observations (equations 16 and 17, pages 44 and 45), the integer cycle ambiguity is indistinguishable from other contributions to phase bias, particularly clock errors. Clock biases are eliminated in double-differenced observations, however (section 6.1.3), and so phase ambiguity may be resolved using this type of measurement.

One way of estimating the integer values is to employ pseudo-range measurements, with sufficient averaging to reduce the uncertaintity to less than half the carrier wavelength. In this way, the ambiguity can be resolved independently for the two L band frequencies. Multipath effects can disrupt this type of solution; frequencies within about 10 MHz of the carrier are used to determine the pseudo-range, and so even a small amplitude reflected wave could shift the apparent time delay by half a wavelength of the carrier, knocking the integer up or down by one. Bossler, Goad & Bender (1980) suggest that by employing a measurement rate of six seconds the ambiguity could be resolved in less than an hour by this technique.

A second technique, discussed by MacDoran (1979) and Ashkenazi, (1983) indicates that over a two hour period, the satellites will traverse about 60°, causing a linear phase versus frequency shift, if a baseline error is present. Thus the ambiguity can be resolved using software which applies the fact that while the baselines remain fixed, the satellite geometry changes with time. This technique is slower than the one previously discussed, however the multipath problem is an order of magnitude less serious because the separation between the frequencies here (1575 and 1227 MHz) is much larger than the 10.23 MHz P code pseudo-range switching frequency.

Departures from integer can be attributed to measurement noise, ephemeris and atmospheric errors, circuit delays and such like. Ideally values for the bias parameters obtained from a preliminary solution will be close to integers and uncertain by less than one cycle. These estimates are rounded to the nearest integer and the least squares solution is repeated with the integers held fixed at these values. Statistical tests then indicate whether or not the appropriate integers were selected. For baselines less than 20 to 30 kilometres in length, the precision of the estimated biases is usually high enough to allow the integer value to be resolved (Bock et al, 1985). This is the "biases-fixed " solution. If the biases selected are not within the set confidence limits then the baseline is solved with the biases completely "free", that is, not constrained to integer value. Whether or not the biases can be fixed depends on the magnitude of errors introduced by residual ephemeris and propagation medium effects noted above, which generally increase with baseline length . The geometric strength of the observations and receiver biases will also have an effect. If these terms are included, then equation 24 (page 49) becomes:

$$\phi_c = \phi_{L1} + N_1 - 1.984 \left\{ \phi_{L2} + N_2 - 0.779 (\phi_{L2} + N_1) \right\} + \phi_e \qquad (32)$$

where ϕ_c = The combined phase value.
ϕ_{L1} = The fractional phase value at L1.
ϕ_{L2} = The fractional phase value at L2.
ϕ_e = Phase errors resulting from effects other than those due to ionospheric refraction.
N_{L1} = The integer phase value at L1.
N_{L2} = The integer phase value at L2.

This equation applies to double-differenced phases as well as undifferenced carrier phase observations. King et al (1985) suggest that, in order to resolve the ambiguity in dual frequency operation, the total contribution from the troposphere, ionosphere and ephemeris in the combined phase observation (ϕ_c), should be less than 10 cm with the code correlating receivers or

5 cm with codeless instruments.

6.5. Cycle Slip Correction.

As with all other scientific procedures, prevention is better than cure. Cycle slips can largely be prevented by employing a wide tracking bandwidth and, with respect to kinematic operations, by keeping sudden changes in velocity or direction of the host vehicle to a minimum. Cycle slips, however, will occur in both stationary and kinematic applications and so some means of detecting such slips must be adopted in most GPS software.

There are several approaches to isolating cycle slips. One method is to calculate residuals, holding the best known stations fixed (suitable station positions may be obtained by processing between-epoch differenced phases, which are largely free of the effects of cycle slippage). Slips are then manually identified as discontinuities in the double difference residuals. The times at which slips occurred are noted and the observations corrected and reprocessed. This is a time consuming process, and various other more sophisticated methods have been discussed:

a)Beutler et al (1984) fit a polynomial function through the gaps which appear in the data. Cycle slips are discovered if the zero order terms of the polynomials alter over a session of observations. The holes in the data are again located manually by visual identification, as above, and as such, this procedure is fairly time consuming. This method is not sensitive to small cycle slips.

b)Lachapelle et al (1986) discuss a phase velocity trend technique of cycle slip detection. Here the observed phase difference measured over a short interval is compared with the phase difference estimated using cycle slip free phase velocities obtained at two successive code measurement epochs. A difference between the two indicates a slippage. This method assumes phase velocity linearity between the two code phase measurement epochs, so a smaller measurement interval will give more reliable results. This is an important point when considering the rapid phase velocity changes which occur in high kinematic applications. A receiver such as the TI 4100 which can measure phase every 160 milliseconds, maximises the linearity assumption and can therefore reliably detect cycle slips even under harsh kinematic conditions.

c)In Remondi's (1985) technique, preliminary stations are determined by employing double-differenced phase observations which have been further processed between epochs to produce triple-differences. Large discontinuities in the double-differenced phases are then automatically isolated by searching the triple-difference residuals for outliers (see section 6.1.5). These outliers are assumed to be cycle slips, and are rounded to the nearest integer value.Each cycle slip is then removed from all subsequent observations taken by the culprit channel, where the cycle slip occurred. Triple-differences are convenient to use here because they are not affected by satellite or receiver clock errors. This procedure is automatic.

d)Goad (1985) describes a "dual frequency phase ratio" method of detecting cycle slips. The author's (1985) base-station, base-satellite concept is employed; the cycle slips pertaining to the base-station or base-satellite are transferred to the subsequent data coming from all other sites and satellites. This transfer of slippage cancels out when the data is double-differenced or when the station-satellite clock offsets at each epoch are estimated. Having edited the cycle slips, the station coordinates can be recomputed using the edited data. Lachapelle et al (1987) note that while this method does not assume phase velocity linearity (allowing it to operate in a highly dynamic environment) it is not as reliable as method (b) above; they suggest that a combination of the two might be satisfactory.

6.6. Averaging.

As with conventional surveying, GPS errors can be classified under three headings; gross, systematic and random. Gross errors, although often contributing the largest proportion to the total error in an observation, are usually the easiest type of error to detect and correct. Examples of gross errors in GPS include cycle slippage, multipath effects (this may also have a systematic element) and incorrectly measured antenna height. This type of error is largely a result of carelessness or failure of equipment. Careful checking and personnel alive to the possibility of such errors, coupled with automatic procedures to negate those gross errors which cannot be avoided (such as cycle slips) should be sufficient to eliminate these errors.

Systematic errors (those which can be mathematically modelled) abound in GPS; clock bias, atmospheric effects and interchannel differences, to name a few. These types of errors can often be eliminated during the processing stage by modelling or by differencing, because the errors are similar at each end of a baseline.

Random errors are those variations remaining in the observations after gross and known systematic errors have been removed, for example instrument noise generated by the tracking loops (section 5.1.) or residual errors after systematic modelling. This type of error can have any value, but the magnitude of the error will generally vary according to the laws of probability. Thus most observations will fall around a mean, which is very close to the true value of the observation. Simple averaging is an excellent way of dealing with random errors, and can also reduce some of the effects of systematic and gross errors. The number of measurements can be increased by observing for longer, or by employing a multichannel receiver. Unfortunately averaging is only practical in static applications, although the larger data measurement rate provided by multichannel receivers can be used to enable some form of averaging in low kinematic conditions. Because of its nature, multipath tends to repeat from day to day. The phenomenon is often indicated by the repetition of a systematic pattern in the post-fit residuals. Longer or repeated observation will thus normally reduce the problem by averaging multipath variation. Observing a greater number of satellites will help to average out ephemeris errors and Lachapelle et al (1986) suggest that a moving average technique could be employed to negate cycle slip effects in quasi-real time. Obviously the level of stability of the data and results recorded over longer time spans, or to a greater number of satellites will also indicate the accuracy and reliability of the solution. Ashjaee (1985) suggests that although position fixes can be averaged, smoothing provides better results because in this case the noise is reduced before the effect of GDOP multiplies the error.

6.7. Combining Measurement Modes.

The advantages of code, phase and Doppler techniques have been combined in many instruments. The integrated Doppler and carrier phase measurements give only the changes in range over the observing period; the absolute range difference at lock-on must still be determined. In code-correlating instruments, this initial range is often found by performing pseudo-range measurements using the L1 signal. This is a simple and effective way of solving the two pi ambiguity. The pseudo-ranges can also be used as a gross error check.

Conversely, receivers which primarily employ the code phase or pseudo-ranges to obtain position may measure the Doppler shift on L1, for example, in order to smooth the range measurements. Carrier phase observations can be used for the same purpose; in both cases, the change in frequency or phase over a specified time is used to derive a change in range over that time. As the measurement accuracy of continuous phase data is about two orders of magnitude better than that of P code ranging (three orders for C/A code), this provides a fine grid for improved accuracy ranging, and better consistency for point to point positioning in kinematic applications. In the latter cases, the velocity of the host vehicle is also more readily determined. The carrier phase is much less susceptible to multipath than pseudo-ranging, due to its much shorter wavelength, and so the addition of carrier tracking can act to reduce multipath. Ashjaee (1985) suggests that Doppler aiding of C/A code phase, real-time relative observation can provide sub-metre accuracy. One reservation here is that high quality Doppler integration is required; cycle slips cannot be tolerated. Phase smoothed pseudo-ranges are accurate to about one metre, as opposed to ten metres for unaided C/A code (Lachapelle et al, 1987). Approximately doubled accuracy is quoted for phase smoothed P code measurements (Keel and Oland 1986, Lachapelle et al, 1987).

Integrated Doppler measurements can also be used to compute the whole number of carrier cycles between carrier phase measurements.

6.8. Other Methods.

1. Integration of Other Sensors.

GPS generally determines four unknowns, the three components of either position or a baseline, and time. If one of these unknowns is provided by an additional sensor, then either fewer satellites need be observed, or greater accuracy can be obtained. The former is particularly important in the present situation of

FIGURE 25: PDOP PLOT, 5° MASK ANGLE, NORTHERN HUDSON BAY.

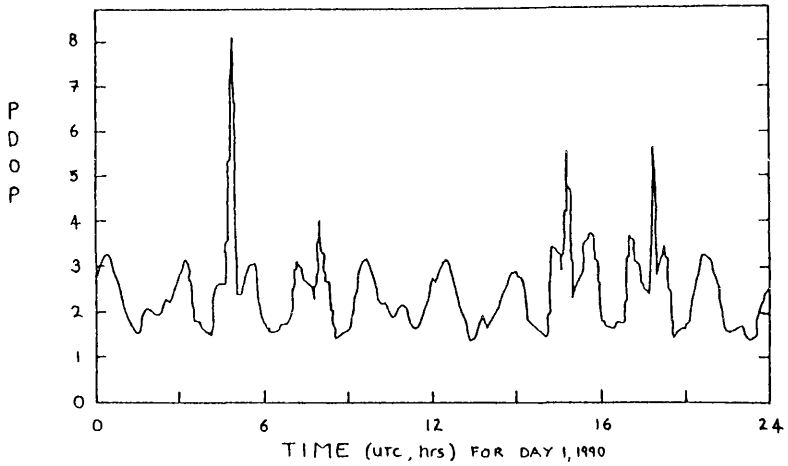


FIGURE 26: HDOP PLOT, 5° MASK ANGLE, NORTHERN HUDSON BAY.

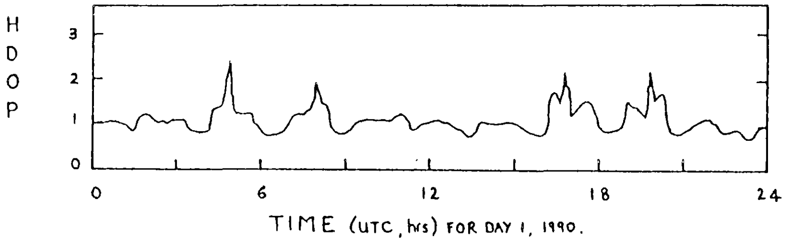
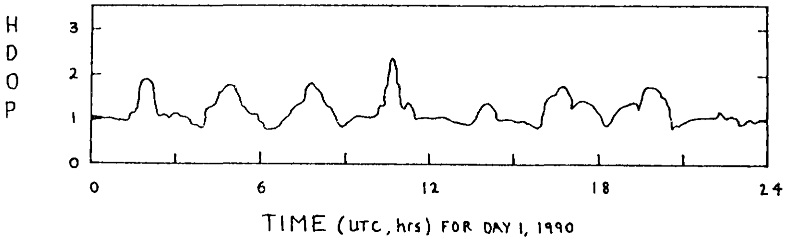


FIGURE 27: HDOP PLOT, 10° MASK ANGLE, 5M HEIGHT CONSTRAINT, NORTHERN HUDSON BAY.



limited satellite coverage.

Height and time are the most readily available information which can be used to constrain the solution. In marine operations, two-dimensional position is usually the requirement, and so HDOP is the measure of accuracy. Figure 25 shows the PDOP for northern Hudson Bay, while that of Figure 26 shows the same area's HDOP. Comparison indicates the considerable improvement which may be obtained if positioning is reduced to two dimensions. Furthermore outages can be significantly reduced by employing a constraint, as exemplified by Figure 27 (Figures 25, 26, and 27 all Lachapelle et al, 1987). Height is easily determined in marine operations, although the sea state and geoid heights perhaps limit accuracy to a few metres. Time information can be obtained by using a caesium frequency standard. Reliability and cost effectiveness favour height constraining in shipborne applications (caesium clocks are expensive). If accurate time can be kept in addition, however, navigation can proceed under only two satellite coverage, which is a tremendous advantage at present. Such a combination could also increase reliability and accuracy during outages. As GPS receiver price drops, however, the attraction of employing a costly caesium clock may wane.

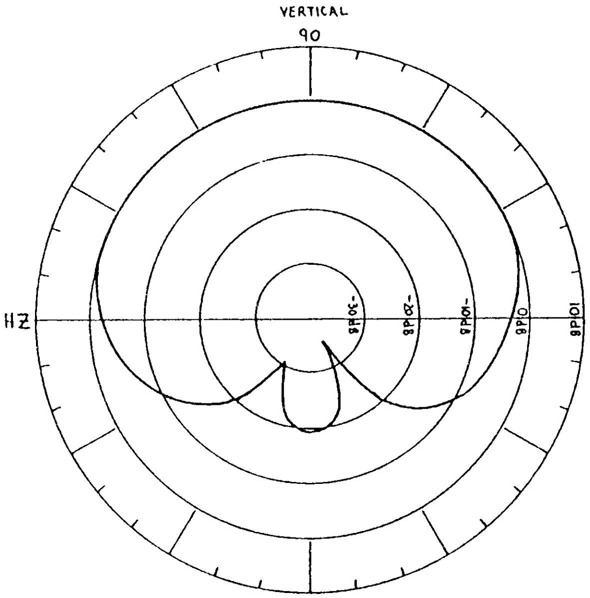
Numerous investigations have been concerned with systems which combine GPS and Inertial Navigation (or Survey) Systems (INS or ISS). Many studies suggest that an integrated GP-IN System would provide attitude and positional parameters sufficiently accurate and reliable for most marine and airborne applications. The short term accuracy and rapid response of the INS aids navigation between GPS updates, while the high accuracy GPS range and range-rate data can be used to calibrate the INS error parameters. Eller (1985) suggests that the resulting system improves on an unaided GPS receiver by:

a) providing high rate/accuracy position and velocity estimates during high kinematics.
b) reducing position and velocity error growth during GPS signal outage.
c) improving jamming resistance.
d) making vehicle attitude available.

Antenna movement is a major problem in marine applications. Inexpensive inclination monitors, however, could provide pitch and roll information, to accuracies of about 0.1° (Lachapelle et al, 1987), which would be employed to remove the motion of the antenna. Furthermore similar information from the ship's log, gyros and so on could be used to fill the gaps which result from phase loss or satellite switching.

2. Observing More Satellites.

By taking measurements to a greater number of satellites, redundancy is provided and ephemeris errors are averaged (as noted in section 6.6.). Four satellite observation is the minimum, provided that no external information is available. Five, six, seven and even nine channel receivers are available to provide the capacity for observing more than four signals. Other b.



b. 'TI 4100' at L1 (conic spiral antenna).

a. 'WM101' at L1 (quarter wavelength volute, helix).

a.

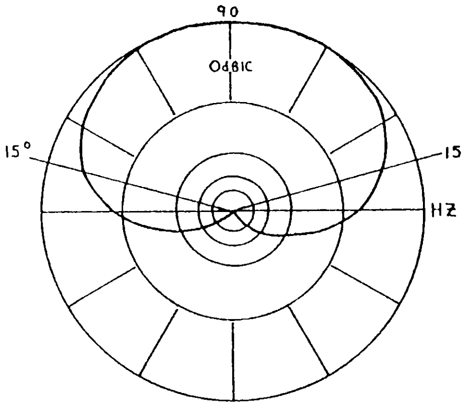


FIGURE 28: ANTENNA PATTERNS.

advantages accrue from recording the signals from more than the minimum number of satellites. If five, rather than four are observed, then (from Pratt, 1987):

a) When one of the observed satellites passes below the mask angle, there is automatically a suitable replacement.

b) Similarly, if the satellite signal fades or fails, there is another already being tracked which can take its place.

c) The extra satellite can be used to provide a measure of integrity of the GPS signals.

3. Antenna Design.

At least four GPS antenna types are used at present; these are the quadrifilar helix, conic spiral, dipole and microstrip antennae (see Tranquilla, 1986 and IRE Transactions on Antennas and Propagation in particular, for a description of the characteristics of each type). The most important features of the antenna design, apart from their ability to receive the signal, of course, is their phase centre stability and their susceptibility to the effects of multipath. The dipole antenna presently employed by MACROMETER is probably the most accurate, with phase centre variation at the millimetre level. Unfortunately it is rather bulky. The microstrip antenna is still at the experimental stage, but it may be able to produce highly accurate results, having a phase centre variation of less than one centimetre, while being quite compact and light. The angle cut-off pattern requirements are dictated by the particular application. Highest accuracy results can be obtained by employing an antenna whose amplitude response pattern (the amplitude sensitivity of the antenna as a function of vertical observing angle) attenuates signals received from below the cut-off angle, which is usually about 15° (see figure 28a). The MACROMETER does this with a one metre square ground plane, hence the antenna's bulk. An absorbing collar similarly enhances a microstrip's ability, with fewer losses to portability (Collins, 1986). Another method of achieving the same result is to utilise the polarisation characteristics of the received signal. The GPS signals are right-hand circularly polarised, but reflected signals are left-hand circularly polarised, so that an antenna which maintains its right-hand circular polarisation over the entire sphere is an elegant way of reducing multipath effects. The Wild-Magnavox 101 deals with multipath in this manner. Most of the antennae employed are omnidirectional, however some studies suggest that a directive (high gain) antenna (for example beam steering or adaptive null steering) which has low response in the direction of reflected waves, and is pointed in the direction of the observed satellite, should reduce multipath effectively. Others (Counselman & Gourevitch, 1981, for example) argue that, if observing and data analysis techniques are properly conducted, a high gain antenna is not necessary for high accuracy work. Furthermore such an antenna would result in decreased phase stability, loss of lock occurring more readily. Directive and ground plane features are not plausible for marine operations, where omnidirectional characteristics and adequate response below the horizon is necessary to allow for ship pitch and roll. The antenna characteristics depicted in figure 28b are ideal for marine operations, because this type of antenna has sufficient response below the horizon.

In differential work, if the antenna phase centres are assumed to move in a common manner (as they will if the antennae are as near identical in constuction as possible) then aligning the antennae in the same azimuth will reduce phase centre variation to a minimum, because phase centre motion will be common and thus cancel.

4. Miscellaneous.

a)Continuous observation of the health status of the satellite constellation will reduce the possibility of inadvertent use of unhealthy satellites. Durrant (1986) lists some of the error messages which could be encountered:

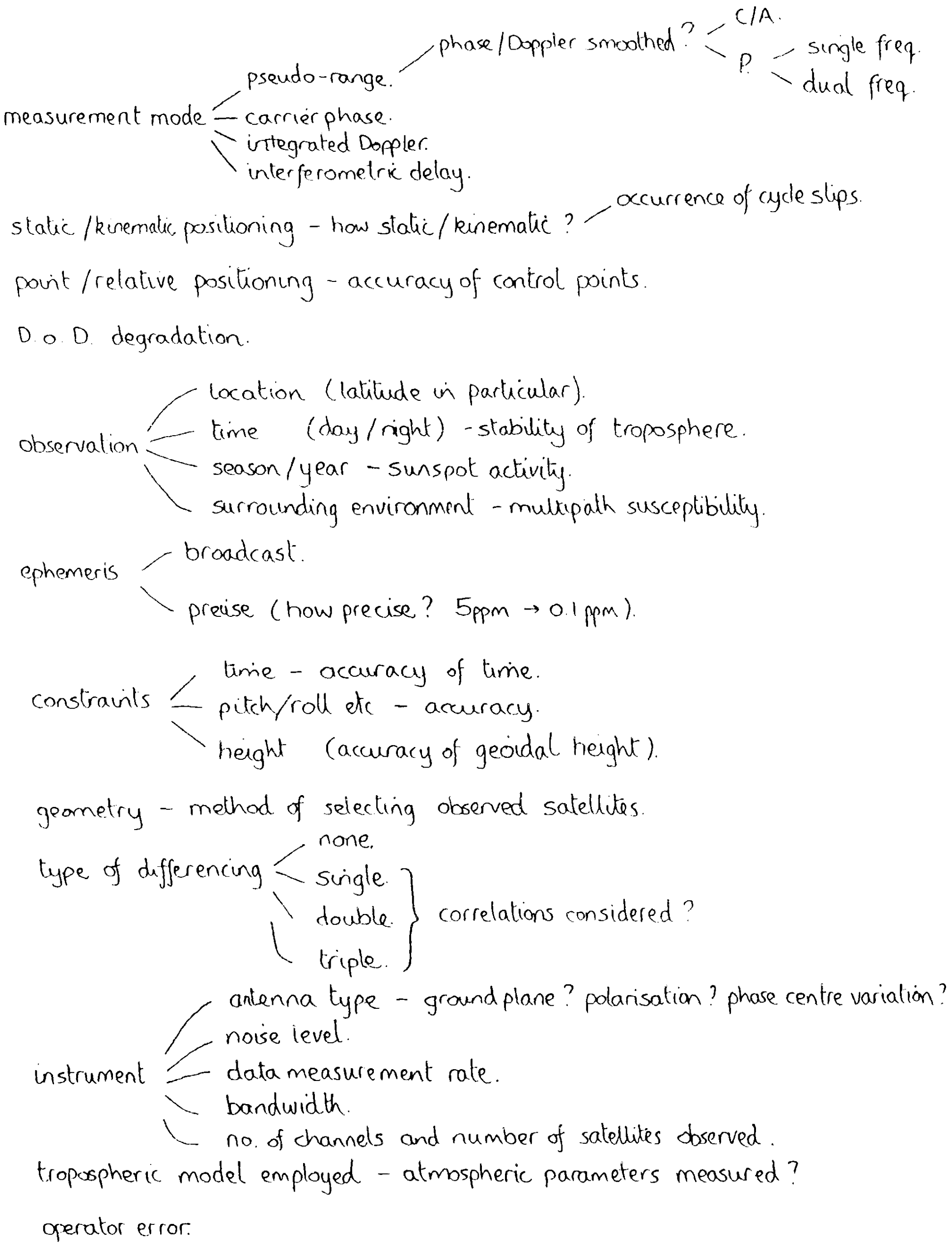
i) SV06. Experienced clock difficulties resulting in 2 to 4 times the errors normally expected. 2 Jul 0000 UT to 3 Jul 2142 UT.
ii) SV03. Bit hit in synchronisation table generated parity errors insubframe 5, all pages vice page 11, all words 1 Jun 0705 to 1505 UT.
iii) SV13. Received bit hits in subframes 1 through 5 due to problems with its navigation synchronisation table; first detected 24 May 0110 UT; cleared by 0350 UT.

b)Multichannel tracking reduces errors because:

i) simultaneous measurements from each channel causes complete cancellation of errors due to clock instability. Thus a multichannel receiver may perform better than a single channel receiver which is supported by an atomic clock.
ii) continuous tracking of satellites eliminates the need for reacquisition. This enables receivers to track using a narrower bandwidth, which in turn reduces signal noise.

c)While real-time filtered and smoothed position fixes are advantageous for many applications, the raw data must be available for post processing in work where the highest accuracy is required.

d)Ephemeris errors are reduced by the ratio of the baseline distance to the satellite's altitude; this is one of the reasons why interferometry and differenced observation is common in GPS surveying. Note also, however, that ephemeris errors are related to the length of the baseline, errors being about one part per million of the baseline.



- measurement mode
  - pseudo-range.
    - phase/Doppler smoothed?
      - C/A.
      - P.
        - single freq.
        - dual freq.
  - carrier phase.
  - integrated Doppler.
  - interferometric delay.
- static/kinematic positioning - how static/kinematic?
  - occurrence of cycle slips.
- point/relative positioning - accuracy of control points.
- D.o.D. degradation.
- observation
  - location (latitude in particular).
  - time (day/night) - stability of troposphere.
  - season/year - sunspot activity.
  - surrounding environment - multipath susceptibility.
- ephemeris
  - broadcast.
  - precise (how precise? 5ppm → 0.1 ppm).
- constraints
  - time - accuracy of time.
  - pitch/roll etc - accuracy.
  - height (accuracy of geoidal height).
- geometry - method of selecting observed satellites.
- type of differencing
  - none.
  - single. } correlations considered?
  - double.
  - triple.
- instrument
  - antenna type - ground plane? polarisation? phase centre variation?
  - noise level.
  - data measurement rate.
  - bandwidth.
  - no. of channels and number of satellites observed.
- tropospheric model employed - atmospheric parameters measured?
- operator error.

FIGURE 29: FACTORS AFFECTING FINAL GPS ACCURACY.

CHAPTER SEVEN - GPS ACCURACIES AND APPLICATIONS

7.1. Global Positioning System Accuracy.

Figure 29 displays a large number of the factors which will affect the final accuracy of a position fix using GPS.

The Department of Defense quotes the absolute accuracy of the Global Positioning System as follows:

a) PPS (P code) : within 10 m, 50 % of the time (CEP).
within 25 m, 95 % of the time (CEP).

b) SPS (C/A code) : within 25 m, 50 % of the time (CEP).
within 55 m, 95 % of the time (CEP).

where CEP = Circular Error Probability.

These figures are for the horizontal accuracy; the vertical accuracy will be slightly poorer. It must not be forgotten that the D. o. D. proposes to degrade the C/A code accuracy to 100 metres (2 distance root mean square), and that there may well only be selected access to the P code (section 2.3).

It should be clear by now, however, that these figures are not the final word on GPS accuracy. Higher resolution is obtained using the carrier frequencies, or by operating in an interferometric mode. Many studies have quoted accuracies or repeatabilities obtained using the various methods. The final results obviously depend on the factors listed in Figure 29; so many factors are involved that the same technique can produce different results. Figure 30, facing page 60, is a guideline to the expected system accuracy for the various measurement techniques, under good GDOP conditions, average atmospheric conditions and low/nil multipath.

7.2. The Impact of GPS On The Survey/Navigation Industry.

The advantages that GPS has over other common survey and/or navigation methods should be apparent by now. Figure 31 (opposite page 61) compares the accuracy performance of various survey systems.

From Figure 31 it is easy to see why GPS is already creating an impact in the survey and navigation industry, despite the fact that the system is experimental, and only about one third of the satellites are in orbit. The extent and form of the final effects procedures are not fully apparent at present. Several points, however, can be made concerning these impacts, which range from those which will certainly occur to those which can only be predicted. The extent of the impact that GPS will have depends partly on the eventual cost of GPS receivers, and also on Department of Defense and Department of Transportation policy concerning the Federal Radionavigation Plan. The main areas of predicted impact are discussed below. Some of the effects can already be seen in some areas.

MEASUREMENT MODE	STATIC POINT POSITIONING	STATIC RELATIVE POSITIONING	KINEMATIC POINT POSITIONING	KINEMATIC RELATIVE POSITIONING
C/A CODE pseudo-range	20-40m	5-10m	20-40m	5-10m (marine)
P CODE pseudo-range	10-20m	3-10m	10-20m	0.5-1m (land) 3 -10m (marine)
CARRIER BEAT PHASE	0.2-0.5m	0.5 -4 ppm	——	——
INTERFERO-METRIC PHASE DELAY	——	0.5 -4 ppm	——	3 -10m

FIGURE 30 : FINAL GPS ACCURACIES.
(under present constellation (1987) ,and good GDOP)

1. Control Networks.

GPS will replace the TRANSIT system, which is presently employed for establishing and maintaining high order control. This is because GPS surveys are more accurate (1 ppm over distances greater than about 30 km, compared with at least one metre over a few hundred kilometres for TRANSIT) and can be completed in hours as opposed to days. The GPS will also largely replace conventional means of setting up control. Triangulation, trilateration and traversing all suffer from poor error propagation and furthermore the GPS has other advantages over these traditional methods:

a) Station intervisibility is not necessary.
b) Sites can be selected in terms of survey requirements rather than by network configuration considerations, with the proviso that their sky view must be unobstructed.
c) Observation towers are unnecessary.
d) Relatively low skill is required to operate a GPS receiver (compared with that required for a geodetic theodolite).
e) Unattended measurement is possible, and large area, high accuracy controlcan be completed in hours or days instead of weeks or months.

It may be that GPS will also be an efficient and economic means of establishing low order control. King et al (1985) suggest that there will probably be a threshold station-separation distance above which GPS could compete effectively with conventional theodolite/EDM systems. Ten kilometres is suggested as a possible critical distance.

The internal consistency and accuracy of most GPS network surveys are higher than those of present survey control networks; in the future the entire geodetic network may well be reconstructed and redefined using GPS. It may be that the GPS will result in no national or international control networks as such. Re-establishing first order control points with GPS will be less expensive than monumenting such a position, and Wells et al (1986) suggest that continuously tracking GPS monitor stations, evenly distributed around the globe, could satisfy most monumentation requirements. Advantages accruing from such a system include :

a) Access to the control points by the surveyor is not necessary.
b) No station descriptions are required.
c) Continual update of control point coordinates, with automatic access to such information.
d) A single datum, worldwide.
e) Automatic integration of information into a national map data base.

Development and testing of this type of system is already proceeding.

2. Coordinate Systems.

To elaborate on points (d) and (e) immediately above, if the GPS does become widely adopted as the primary positioning system,

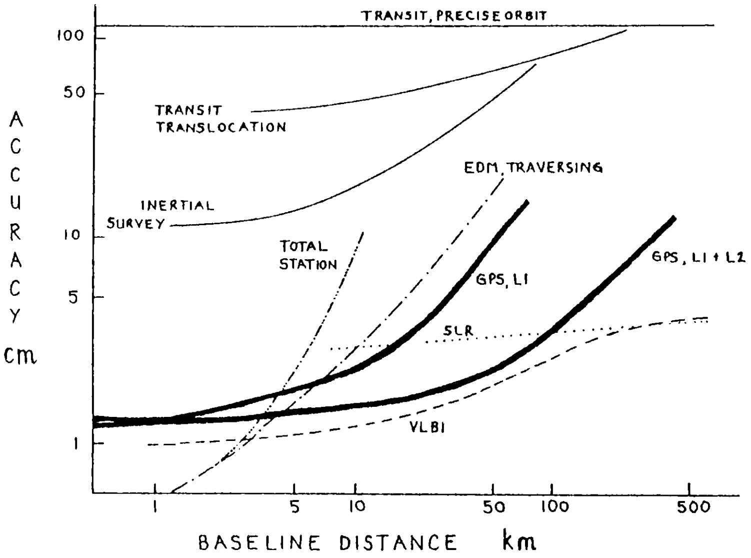


FIGURE 31: ACCURACY COMPARISON FOR VARIOUS SURVEY SYSTEMS.

then GPS and local geodetic coordinates could be simply related. A unified worldwide coordinate system should develop from this, which in turn could form the skeleton of many local or national information systems.

3. Navigation Systems.

The United States Federal Radionavigation Plan will largely have the final word on the eventual impact of GPS as a navigational aid. At present it seems as though the U. S. military are destined to become heavily dependent on GPS, the older navigation systems being phased out gradually. Military use of TRANSIT was scheduled for replacement by GPS in 1994, but this date may be revised due to the delays in the launch programme. Other proposed phase out-dates include Loran-C and Omega by 1992, VOR/DME (VHF Omnidirectional Range/Distance Measuring Equipment), TACAN and radio beacons by 1997. These are only military replacement dates, however; VOR/DME, Omega and Loran-C will operate until at least 2000. On the other hand, government operation of TRANSIT is expected to cease in 1994. GPS would also expect to have an impact on commercial navigation systems for short range work. Syledis, Trisponder, Pulse 8, Miniranger and other such systems will presumably be under threat if GPS receiver prices continue to drop. The short range systems are at a disadvantage to GPS in that they:

a) require constant upkeep of shore-based transmitting stations.
b) only operate in specific areas; and
c) are susceptible to loss of lock or jamming, particularly around metal structures such as oil rigs or platforms.

A single system such as GPS, which caters for almost all requirements, is desirable because it would vastly reduce the congestion of radio airwaves, which at present can be a problem. Of course, consideration must be made for a back-up navigation system.

4. Other Effects.

These are wide and far ranging. Some are already apparent, others can only be surmised and presumably still others are unforeseen. The following is a list of a few of those which fall into the first two categories.

a) Crustal deformation studies. Presently such work is executed using VLBI and SLR techniques which are expensive and whose equipment is bulky. GPS is inexpensive compared with VLBI and SLR and so crustal motion studies should become much more common once GPS is fully operational.
b) GPS heighting is likely to replace geodetic levelling over long distances and in mountainous terrain because it is cheaper, more accurate and quicker, despite the fact that dual frequency and WVR measurements will be required.
c) Aerial Triangulation will be possible with little or no ground control if GPS is used in conjunction with an Inertial Navigation System.
d) A much higher degree of accuracy will be possible in marine

and aerial gravity measurements, using GPS integrated withan INS. This will allow geoid determination on a much larger scale.

Most of the applications mentioned above are dealt with in more detail in section 7.5, along with other areas in which GPS may flourish.

7.3. GPS Survey Procedures.

It is difficult to discuss GPS survey procedures because standards and specifications for such a survey have not yet been documented formally, and procedure varies markedly, depending largely on the type of survey, and on which type of receiver, or set of receivers, is employed. Broad guidelines, however, can be constructed.

1. Preliminary Analysis.

Once the initial decision has been made to employ the GPS, the next stage in the process is to perform a preliminary cost analysis in order to determine the most appropriate method of completing the survey within the stated accuracies. Provided that GPS receivers have not already been purchased, this will involve a decision as to which type, and subsequently which particular model, to employ. The process can be categorised as follows (adapted from Hannah, 1985):

a) Define the survey tasks to be executed, including the specified positional accuracy, real-time requirements and so on.

b) Decide on P code eligibility. At present, all users are able to employ the Precise code, however this may not be the case once FOC is realised. If access to the P code is available, then all measurement modes can be considered; otherwise codeless or hybrid technology must be employed.

c) A decision based on accuracy and real-time requirements must be made to determine whether the Broadcast ephemeris is adequate, or whether the appropriate precise ephemerides need to be purchased. If the latter is the case, then the source of the Precise orbits must also be considered.

d) With respect to (a) and (b) above, a decision should be made between single or dual frequency operation, and about the necessity, or lack of it, for WVRs to reduce tropospheric effects.

e) Once the above decisions have been made, highlighting the type of receiver required, the appropriate available instruments can be considered in terms of cost, reliability, accuracy differences, portability, servicing and so on.

f) If orthometric heights are required from the survey, then the method of determining geoid heights should be outlined and the appropriate equipment should be obtained or available at the required time.

g) Finally the form of acquisition can be considered; purchase, lease or perhaps joint venture are the main possibilities.

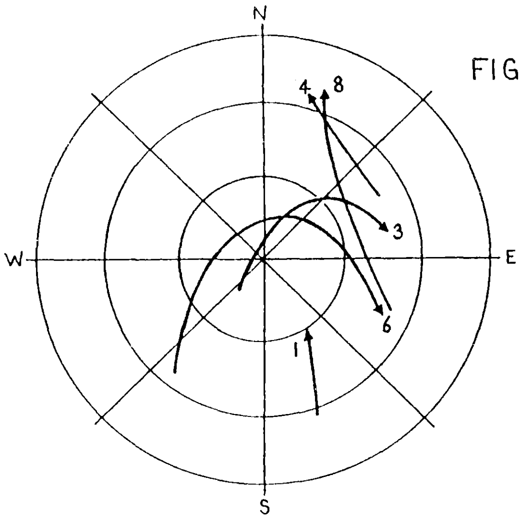


FIGURE 32: TYPICAL SKY PLOT.

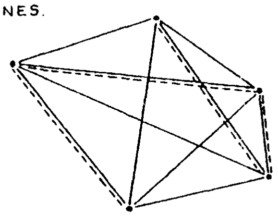
FIGURE 33: INDEPENDENT LINES.

- Station

——— Baselines

===== Independent lines (one example)

n = 5
n(n-1)/2 = 10 lines
n-1 = 4 independent



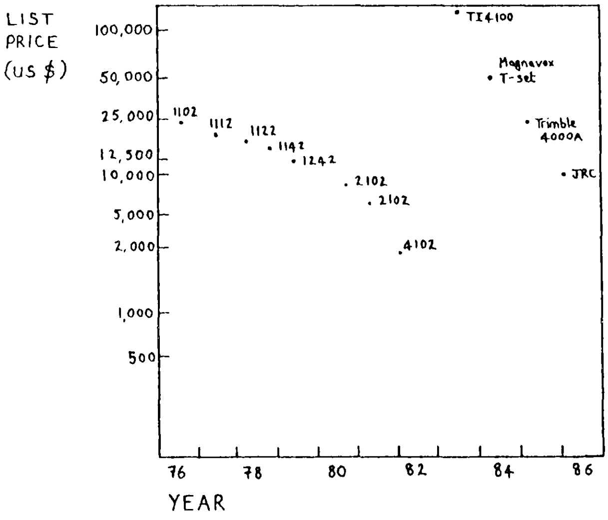


FIGURE 34: TRANSIT AND GPS PRICES.

. 1122 MAGNAVOX MODEL NUMBER

note: price scale is logarithmic.

Having determined the best receiver for the job, consideration must be made of logistical aspects, such as size and type of power source, cable length required and availability the degree of "ruggedisation" of the particular receiver, in order that the appropriate preparations and precautions can be made and taken. Some of these aspects may have played a part in the original determination of the appropriate receiver.

2. Planning and Reconnaissance.

Comprehensive planning is important in any survey, from the simplest low accuracy chain survey to the most complex national contol network survey employing, for example, theodolite and EDM or an Inertial Survey System. Planning in surveys employing the GPS is particularly important, if not crucial, at present because GPS receivers are expensive to purchase or hire, and the observation window is brief (four to six hours per day maximum) due to the incomplete satellite constellation. One poorly executed session could substantially affect the profit margin of a GPS survey.

Before even conducting field reconnaissance, some aspects of the survey must be considered (from King et al, 1985):

a) The points for which coordinates are required should be plotted.

b) The horizontal and vertical control points in the vicinity of the survey area should also be plotted.

c) This allows computation of approximate coordinates of survey points and therefore calculation of approximate baseline lengths; these in turn may be used to check for gross errors, and in the calculation of weight matrices when the time comes.

d) A "sky plot" (Figure 32) should be prepared for each site, showing the tracks of all satellites which may be utilised. Normally this will mean the available GPS satellites, but the ISTAC Model 2002, for example, is not restricted to these satellites and may be able to employ others, such as the GLONASS satellites. The sky plot is necessary in order to design the observing schedule, and, if appropriate, to determine the subset of satellites which will be observed to give the best available geometry. This in turn allows all receivers to be programmed to track the same set of satellites, and aids the reconnaissance for suitable sites.

e) The sites' approximate coordinates, and the selected satellites can be employed in a computer simulation whichwill calculate the observation period required to obtainthe stated accuracy levels. Obviously the accuracy characteristics ofthe receivers being employed are a necessary input to the simulation. Travel times between the observing stations should also be estimated.

f) The product of the pre-survey planning is a provisional observation itinerary, indicating which sites are to be occupied by which receivers, and all details concerning vehicle and personnel movements.

Field Reconnaissance.

As with conventional surveys, the survey procedure should be reconnoitered so that:

a) The specific survey points are found or selected and monumented (if necessary). Points should be selected with due consideration for the sky plot and the satellite set to be observed. Obstructions should not cut out the signals during the observing session and note should be taken of instances where a tower or a tall tripod is necessary to clear the receiver's view.

b) Terrain and station accessibility can be inspected with respect to vehicle access, required cable lengths and such like. This will verify or alter the planned vehicle and personnel movements. Detailed descriptions of station access should be completed so that receivers can be located on the correct site quickly and accurately. The time taken to reach the station from the main road (if helicopters are not being used) should be noted and verified with the itinerary.

3. Pre-observation.

The chores to be completed immediately before the actual observation will depend on the receivers utilised. Codeless receivers will need to be synchronised to Coordinated Universal Time (UTC). In order to do this, all the receivers must be brought together. One receiver is then synchronised to UTC, within a few milliseconds, by employing one of the following systems:

a) The domestic television network.
b) Communication satellites (for example GOES).
c) Short wave radio time-tick transmissions.
d) An atomic clock.
e) Another GPS receiver which acquires time information from the GPS satellites.

The other receivers are then synchronised to this receiver, to within one microsecond. All the receivers should then be compared with one another, using a time interval counter, to ensure synchronisation is complete. These timing accuracies are those required for one part per million accuracy or better. The code correlating receivers do not need to be "manually" synchronised because they are capable of decoding the GPS time mark, and so are automatically synchronised to GPS time during the observations.

Batteries should be charged, generators and vehicles fuelled, and the appropriate initialisation procedures completed for data storage media. The receivers themselves can often be warmed up en route to the survey point, by switching them to standby. Specific pre-observation procedures, peculiar to each receiver model will obviously be documented in the instrument manual.

4. Field Operation.

Once at the site, the antenna is tripod mounted over the station, centred and the height of the antenna's phase centre above the survey point is measured. The antenna cable may then be connected to the receiver and tracking commences after initialisation of the receiver. The form of initialisation varies from receiver to receiver, and might include, for example, the insertion of the A files for the V-1000, the start and stop times of the observing session, the approximate coordinates of a single satellite, and so on. Once tracking has started, the operator can check that data acquisition is running smoothly, and that the data is being recorded correctly. Receiver performance should then be monitored continually and if necessary, atmospheric conditions should also be recorded at the appropriate times. Gravity measurements may be required to obtain geoid separations. These may be taken during the observing session, or during the reconnaissance, or may form a separate survey. A log book should be maintained, recording a description of the site and the progress of the data collection, including receiver malfunctions and operator errors. If an azimuth is required, this can be observed by conventional means, during the observing session.

At the end of the observing session, the data tape can be removed, labelled and copied if necessary; calibration checks and antenna height measurements should be repeated. Often when moving to a new site, the approximate coordinates of one satellite will have to be re-entered. This is not the case with other receivers such as the TI 4100, which can maintain lock even while in transit. For this to be possible, however, the antenna must retain an unobstructed "view"of the satellite while it is being moved.

At the end of the day's observing, codeless receivers should be brought together again and their synchronisation checked. This is done by using a time interval counter as before, and allows a complete history of clock drift to be determined and used in the position computation.

5. Processing.

Verification of data quality and sufficient quantity is the first task in the processing stage. Often this is possible on site using a field computer, so that if further observation is required, it can be done there and then. Once this is done, the data can be edited and smoothed as necessary, cycle slips corrected or flagged, and the appropriate differences calculated. If the Broadcast ephemeris was received, it must be decoded. Often the first step involving the ephemeris will be transforming the ephemeris elements into the coordinate system in which the station coordinates are expressed. From here (X,Y,Z) values are computed for each observation time, normally using an orbit generating program or polynomial interpolation (King et al, 1985). Codeless receivers will employ either a Precise ephemeris (which rules out real-time position fixing), or an inexpensive receiver which does decode the Broadcast message.

Using the ephemerides, the epochs of the observations and the approximate values of site coordinates, theoretical values of the observations can be calculated. Comparison between the observed and computed values and a least squares adjustment will provide more accurate coordinates for the occupied sites. The processing time will depend on the accuracy required, the software employed and the amount of data lost due to cycle slips, equipment malfunction, loss of lock, and also on the amount of data collected. Moderate accuracy is often obtainable in less than 30 minutes. The solution may be computed in a baseline by baseline approach, or a more rigorous network adjustment may be performed. The former procedure has been more common up until now; data for each pair of stations of n(n-1)/2 baselines is reduced (see Figure 33, facing page 63) and then partial results are combined into an "occupation" adjustment, enforcing the closure relations between baselines forming closed loops (McArthur et al, 1983). In this approach the correlations between the various combinations of vectors are not known, since instead of producing (n-1) independent vectors, it yields n(n-1)/2 vectors. McArthur et al (1983) examine three processing methods:

a) (n-1) vectors per session are selected from the n(n-1)/2 total, the other trivial vectors being rejected. This appeals because it reflects the correct number of observations, but obviously some data is ignored using this method.

b) Combine all n(n-1)/2 vectors observed during one session in one adjustment and then combine these session adjustment results ((n-1) sets of coordinate results) in an overall network adjustment. This method seems to be the most rigorous, because each session adjustment reflects the geometric contributions of all observations through the resulting covariance matrix for the set of coordinate differences. It also produces the correct number of degrees of freedom. However, data from each site are used more than once, resulting in strong correlations which are not represented. This could be overcome by processing all the observations in a network simultaneously.

c) Employ all the observed vectors from the entire survey, whether independent or not, in a unified network adjustment. As with (b) above, weighting is a problem because all the trivial vectors are included.

McArthur et al note that the results using each method are statistically equivalent, but due to the neglect of interstation correlations and the arbitrary vector selection procedures in some network adjustments, the simultaneous adjustment of the whole network is the type of approach advocated.

Finally results should be prepared for presentation, and data, text, log books and results archived.

7.4. GPS Survey Specifications.

Wells et al (1986) note that presently the only sure way of evaluating GPS receiver accuracy and performance is by testing the receiver on an accurately known baseline or network. Hotham & Williams (1985) discuss the development of GPS survey specifications. The more important items which should be included, further to those relevant from conventional techniques, are listed below.

a) Number of horizontal control points to be occupied.
b) Number of vertical control points to be occupied.

c) Maximum and minimum station spacing.
d) Distribution of existing network control.
e) Azimuth reference: distance from station not less than X metres.
f) Minimum number of satellites observed simultaneously.
g) Number of receivers observing simultaneously.
h) Period of observation not less than X hours.
i) Number of repeated baselines (to check repeatability).
j) Minimum independent occupations per site (to check receiver biases, antenna centring, height offset and so on).
k) Minimum number of satellites observed during each session.
l) Single or dual frequency operation.
m) Presence or absence of WVR measurements.
n) Data measurement rate.
o) Frequency standard warm up time.
p) Maximum cut off angle, above the horizon.
q) Antenna set up measurement specifications.
r) Minimum number of antenna phase centre measurements per session.
s) Quantity and Quality of meteorological observations.

Factors which should be included in the data processing specifications (Well et al 1986) are as follows:

a) Maximum cut off angle.
b) Ephemeris source and age.
c) Measurement data quality and quantity.
d) Measurement data rejection criteria.
e) Measurement residual maxima.
f) Baseline or session adjustment specifications.

A minimally constrained three-dimensional adjustment would reveal the survey's overall precision and consistency. Having done this, a constrained adjustment using existing control coordinates, accounting for scale , orientation and undulation relationships between the GPS satellite and the control network datums, would allow the GPS survey to be fixed to the local control.

7.5. Applications.

One of the most important features of the GPS is its worldwide, 24 hour, all weather capability.This type of survey system, as exemplified by the TRANSIT system, has the potential to be widely used by a number of different operators over a large field ranging from the military through to commercial and recreational civil applications. The GPS can be employed on land, at sea,in the air or in near earth space. The following section discusses most of the areas in which GPS could be employed. In some cases it already is; others require the FOC before a switch to the new system is justified in terms of accuracy or ease of operation. Some of the applications discussed depend on new technology currently being researched or developed, while others await the development of a large market, which will promote commercial competition and large scale production, resulting in low cost receivers.

1. Land Applications.

a)Geodetic Control.

The required accuracy for first order geodetic networks is of the order of $1 * 10^{-6}$ to $5 * 10^{-6}$ for distances between 20 km and 100 km. GPS can already give accuracies of 1 ppm over these distances, using post-processed relative carrier phase observations. The GPS is much faster than, and more accurate than the TRANSIT system, which is presently one of the commonest methods for establishing such control. Furthermore the GPS does not suffer from the error propagation which degrades conventional surveying technique accuracies over long distances.

GPS stations are not constrained by considerations such as intervisibility and network conditioning, and so the control stations can be positioned in easily accessible places and where they are needed, unlike most triangulation, trilateration or traverse control stations. This eliminates the need for tower construction and line cutting, except where small areas need to be cleared to allow the antenna an unobstructed view of the sky.

b)Surveying

The GPS is applicable in topographic, cadastral and geophysical surveying, where accuracies of $1 * 10^{-4}$ to $1 * 10^{-5}$ are generally required. In many instances, two GPS receivers provide a better service than a conventional total station, there being no distance limitation, no intervisibility or reference azimuth requirement and of course 24 hour operation is possible. GPS allows direct positioning of geographical features, or the seismic energy source and geophones in geophysical surveys, thus, for these applications, control frameworks are unnecessary. Wells et al (1986) also note that all coordinates produced by the GPS are in the same unique system. This eases the creation of land information data bases and facilitates the use of position information in different applications without complex transformation procedures.

c)Geoid, Gravity, and Deflection of the Vertical.

If the ellipsoid height determined by GPS is supplemented by orthometric heights determined by accurate geodetic levelling, then a direct and accurate measure of the geoid can be obtained from equation 9 , page 30. GPS can determine geoid undulations of the order of 5 to 10 cm, while gravimetric geoid determination is only accurate to about 10 cm. Conversely, the geoid heights obtained by the GPS or by gravimetric techniques can be used to determine more accurate global geoid shape, allowing orthometric height to be found using GPS, and accurate to perhaps between 10 and 20 cm. Precise navigation is possible using relative GPS measurements, and if the vehicle's changing position is input to an INS, the inertial sensors can detect changes in the gravity field and provide profiles of gravity and deflection of the vertical over the traverse.

d) Deformation Monitoring.

Global crustal motion, in the form of plate boundary movements, seismic and post-seismic adjustments, large-scale deformation due to impending volcanic activity or isostatic recovery, is difficult to monitor accurately because therelative accuracy required is of the order of $1 * 10^{-7}$ to $1 * 10^{-8}$ over intercontinental distances. To achieve the 0.1 ppm baseline accuracy required, precise models of satellite orbit (to about two metres absolute accuracy), atmospheric propagation delay and clock biases are necessary.

Smaller scale local deformation studies do not require such demanding precision (but still approximately one part per million), and thus the GPS can also be successfully employed to study changes in tower or dam structures, for example, or subsidence due to mining activities.

e) Navigation and Monitoring Vehicle Movement.

Low cost GPS receivers will replace the navigation of civil vehicles by visual comparison between the surroundings and a conventional map. Accuracies of about 20 metres (absolute) are sufficient for such purposes, which suggests that real-time navigation using GPS integrated with an electronically stored map will be a reality in the near future. This type of system would be employed by official fleets such as ambulance, fire engines and police. Monitoring systems would also be possible if the vehicles are capable of automatic transmission of position. This facility should be attractive to commercial users such as taxi, haulage, security and delivery firms.

f) Recreational Navigation.

This application awaits the advent of low cost, simple and lightweight GPS instruments. Accuracies of a few metres to several hundred metres are adequate for mountaineers, hikers, safari expeditions and a host of other non- or semi-professional activities.

2. Marine Applications.

a) Marine Navigation.

GPS will be an excellent primary navigation system once the constellation reaches 12 satellites (around 1990), allowing worldwide two-dimensional navigation. While ocean navigational accuracy requirements are relaxed, at perhaps several hundred metres, harbour exit and approach requires much higher accuracies (25 to 50 metres). Thus all stages of marine navigation can be dealt with by GPS, and there are significant advantages to be gained by employing a single navigation system (see section 7.3), although a back-up system will be mandatory. River navigation and high accuracy search and rescue missions will be facilitated by carrier phase GPS measurements.

b) Positioning Requirements.

A significant advantage over the present TRANSIT system is GPS's ability to provide an accurate position fix every few seconds. Rig and buoy positioning, anchor handling, and three-dimensional seismic surveys require real-time accuracies of two to five metres. To achieve this level of accuracy, phase measurements are necessary. This, however, presents problems. The antenna must be mounted high above the centre of gravity of the ship, in order that the GPS signal be received without interference from prohibitively intense multipath and imaging. This results in accentuation of the ship's pitch, roll and yaw movements, which in turn will encourage loss of lock and cycle slips. Accurate solutions are thus required for cycle slippage. Antenna movement also decreases the accuracy with which the system can determine the path of the ship. As noted in section 6.8.1, this can be reduced if antenna movement is monitored by additional equipment. However multipath can still be a problem when the antenna is mounted high on the ship's main mast, because reflected signals can be received from the sea, and from other parts of the ship.

c) Hydrographic Surveying/Mapping.

GPS can be used in relative pseudo-range mode to provide accuracies of two to five metres for harbour, wharf, shoal and other similar hydrographic activities.

d) Marine Geodesy and Oceanography.

Seabed topography, gravity measurements, ocean current monitoring and other types of marine studies all require moderate to high accuracy standards. Often the ship's velocity is also a necessary input (accurate to 10 cms for gravity surveys): GPS carrier phase and integrated Doppler measurements can provide the necessary accuracy in this case.

e) Pleasure Boat Navigation.

As with civil land vehicle navigation, low cost, easy to use, compact GPS receivers will find a ready market among small craft owners, both recreational (yacht and power boats) and commercial (fishing boats).

3. Aerial Applications.

a) Navigation.

The planned GPS constellation results in small pockets of decreased accuracy, which occur in specific areas, at specific times (see section 5.7). Unless provision is made to clear these outages, by launching extra satellites, the GPS could not provide continuous, worldwide en route navigation. The system could however, be employed in conjunction to conventional systems, and would be advantageous because GPS accuracy in "clear" areas is superior to all other radio positioning systems.

b) Sensor Positioning.

GPS can provide the positional accuracy necessary for various surveying and mapping procedures in which the sensors are airborne. New methods of surveying and data capture are becoming possible due to GPS, especially at the high end of the accuracy scale. Photogrammetry requires variable accuracy (0.5 to 30 metres, depending on the photographic scale), all of which can be provided by GPS. Using this method, ground control points are redundant, allowing instantaneous positioning, and freedom of flight path choice and avoiding access problems, such as surveying in control points , and targetting. Tests are proceeding to determine whether the GPS can provide the control alone, or whether integration with inertial systems is necessary (for example Goldfarb and Schwartz 1985).

Digital elevation modelling of the Earth's surface is possible now by laser profiling; airborne positional accuracy of 0.5 to 1 metre vertically and a few metres horizontally is required: again GPS fits the bill. Lasers are also being used in a dual frequency mode for airborne shallow water bathymetry. Location of the depth profile requires horizontal positional accuracy of 15 metres; relative pseudo-ranging can provide this level. Attempts have been made to take gravity readings from the air, but positional information needs to be accurate to two metres, velocity accurate to 10 centimetres per second and some method of distinguishing between gravitational and inertial accelerations is required to solve cross-coupling problems. Relative phase GPS, integrated with INS measurements should be able to provide the required accuracy. The major advantage of aerial gravity surveys would be rapid acquisition, especially in remote areas, and large area coverage in short periods.

For these fixed wing applications, the antenna is usually installed on the upper section of the fuselage, which minimises shading and multipath. Helicopter positioning is another possible application for GPS. In this case, however, the antenna location is more complex. In large helicopters, the antenna can be fixed above the main rotor, but on smaller designs, the top of the tail fin may be a better location. GPS aided helicopter applications might include buoy positioning, laser profiling and control point positioning.

c) Non-precision Approach.

Relative GPS can provide approach navigation for airports, provided that the airport is outwith the outage areas (figure 27, opposite page 56).

4. Space And Military Applications.

a)Space Applications.

Space vehicles require navigation and positioning. Again GPS can provide such information. Orbit computations can be improved, or single discrete positions provided for satellites such as those employed for remote sensing.

b)Military Applications

One user which encompasses all the foregoing media is the military. Their applications are often similar to those above; navigation and positioning for infantry, land vehicles, ships, aircraft, missiles and possibly even space vehicles in the future. GPS signals are also employed for target acquisition, accurate time, missile guidance and tracking, range tracking and command control. The GPS satellites also carry sensors to detect and monitor nuclear detonations (Wells et al, 1986).

7.6. Future Developments.

Electronic equipment prices continue to drop, even for mature durables such as digital watches, televisions and microcomputers. Stansell (1984) suggests that this trend is even more apparent for more complex, innovative instruments such as satellite navigation equipment. Figure 34, facing page 63, graphs the falling price trend both for a series of TRANSIT navigation equipment marketed by Magnavox, and also various GPS receivers. There is every reason to assume that present and future GPS products will continue along a similar path. The system's advantages of accuracy, reliability, coverage, simplicity and availability will result in large scale, wide variety usage, which will further reduce prices by way of competition and mass production.

Very Large Scale Integrated (VLSI) circuits are being manufactured which heavily reduce the receiver parts count, resulting in decreased cost, size and power consumption. Gate array structures of increasing size and speed on a single integrated circuit are becoming possible, while similar technological advances will soon enable the entire signal down-conversion process and synthesisor to be contained on a few integrated circuits. The digital signal processor and all the computational power required for a five channel GPS receiver can now be held on a single chip (Pratt, 1987). Thus GPS receiver size and weight will continue to decrease until eventually a receiver will be available which can be strapped to the user's wrist. These may appear on the market before the end of this century. Supportive evidence of such advances is given by the United States Department of Defense, which has recently contracted the development of a hand-held GPS receiver for military use.

As far as actual receiver design is concerned, it is already becoming standard to measure the carrier beat phase and either incorporate the results as a smoothed pseudo-range fix, or to output the data for post-processing. Dual-frequency receivers which do not depend on access to the codes should become very common, as will receivers which can track more than the minimum number of satellites. This may well include observations to satellites other than those in the GPS constellation. The Soviet Union has commenced the implementation of a satellite navigation system which is similar to the GPS format; the GLONASS global positioning system. Receivers capable of utilising both the superpowers' satellite systems could become a reality, increasing the accuracy and reliability of satellite navigation/surveying as a whole.

The company "Aero Service Division" have an orbit improvement project which involves setting up their own tracking network of base stations, which should improve the accuracy obtained from GPS to about 0.1 ppm or better. Permanent control stations, which control errors such as those from the ephemeris, and keep an accurate tag on ionospheric effects and system status, will surely become commonplace in the future. Developments in real-time communication links to these stations are already emerging and other improvements in technology, which will undoubtably come, should result in extremely high accuracy real-time navigation and surveying.

APPENDIX A - GPS RECEIVER CHARACTERISTICS

GPS RECEIVER NAME	MACROMETER V-1000	MACROMETER II	TEXAS INSTRUMENTS TI 4100	GPS LAND SURVEYOR MODEL 1991	GPS LAND SURVEYOR MODEL 2002
MEASURE-MENT MODE/S	Interferometric carrier phase	interferometric carrier phase	pseudo-range, carrier phase	P code phase (SERIES)	P code phase (SERIES)
FREQUENCY	L1	L1 L2	L1 L2	L1	L1
CODE	none	none	C/A P	none	none
NUMBER OF SATELLITES TRACKED	6	6	4	ALL	ALL
POSITIONAL ACCURACY	25cm in 1hr 1cm in 4hrs	1-2 ppm using ephemeris provided.	25cm in 1hr 1cm in 4hrs pseudo-ranging: 30mins 14m SEP P 47m SEP C/A	25cm in 5mins 6cm in 30mins	5cm SEP in 1hr 30cm SEP in 3 mins 2ppm over long lines
VELOCITY ACCURACY	—	—	0.15 m/s	—	—
MAXIMUM ACCELERATION	—	—	—	—	—
MAXIMUM VELOCITY	—	—	—	—	—
DIMENSIONS AND WEIGHT	Receiver 69×53×64cm 73kg Antenna 91×91×15cm 18kg	2 Receivers 27kg combined Antenna as V-1000	Receiver 46×37×20cm 25kg Antenna 28×18cm 2kg	Main Unit 23kg 66×33×23cm Clock/Battery 14kg 46×36×15cm Antenna 5kg 28×28×8cm	Main Unit/Antenna 7kg 23×42 cm Clock/Battery 18kg 42×28×26cm Data Recorder 7kg
POWER CONSUMPTION	115 V AC 350 W 12 V DC	12 V DC 180 W	22-32 V DC 110W	24 V DC 12 W	internal battery
OPERATING TEMPERATURE (AND HUMIDITY)	Antenna -50°C to +70°C	as V-1000	-20°C to +50°C	?	?
FIX INTERVAL	—	—	—	—	—
TIME TO FIRST FIX	—	—	—	—	—
ANTENNA CHARACTER-ISTICS	omnidirectional dipole 3mm phase centre stability with ground plane	as V-1000	conical spiral	?	flat microstrip
OTHER INFORMATION	GOES sat. receiver and time interval counter also required. 2 receiver operation ephemeris available through contract	squaring channels ephemeris provided 3-5 days after the event.	1 multiplexing channel = 8 "pseudo" channels 4L1, 4L2 no longer in production	2002's "ancestor"	not restricted to GPS satellite observation
COST (American Dollars unless stated)	no longer for sale hire still possible	$ 240,000 for 2 units + GOES rx and time interval counter	$ 120,000	?	$ 60.000 incl. software

GPS RECEIVER NAME	TRIMBLE 4000S	TRIMBLE 4000A	MAGNAVOX/WILD WM 101	COLLINS NAVCORE	GEO/HYDRO SYSTEM III
MEASURE-MENT MODE/S	pseudo-range, carrier phase, code phase, Doppler	pseudo-range	pseudo-range, carrier phase	pseudo-range, Doppler (?)	carrier phase, code phase, integrated Doppler
FREQUENCY	L1	L1	L1	L1	L1
CODE	C/A	C/A	C/A	C/A	C/A
NUMBER OF SATELLITES TRACKED	4 or 5	4 or 5	9	4	5
POSITIONAL ACCURACY	pseudo-range 2m carrier phase 1cm + 2ppm over short lines in 30mins	15m SEP GDOP < 6 7m CEP HDOP < 3 Altitude acc. to 1m	5m in less than 1hr 10mm + 2ppm in relative positioning	25m SEP 10m SEP relative	2m in 20mins 1-2ppm in relative positioning with 'geodetic' antenna.
VELOCITY ACCURACY	2 cm/sec rms in low kinematic environment	0.1m/s under constant velocity	—	0.5 m/sec	?
MAXIMUM ACCELERATION	0.5g	$5 m/s^2$	—	1g	?
MAXIMUM VELOCITY	250m/s	250m/s	—	600 knots	?
DIMENSIONS AND WEIGHT	Receiver 23kg 22 × 45 × 48cm Antenna & Ground Plane 15 × 44cm 1.4kg.	Receiver 20.5kg 18 × 46 × 48 cm Antenna 1.4kg 15 × 18cm	Receiver 14kg 17 × 51 × 39cm Antenna 1.5kg 18 × 21 cm	Receiver 3.6kg 23 × 21 × 12cm Antenna 0.6kg 17 × 18 cm	Receiver 9.9kg 18 × 46 × 48 cm Antenna & Shield 28 × 28 × 6cm 60 × 60cm 4 kg
POWER CONSUMPTION	115/230 V AC 12 V DC 60 W	115/230 V AC 100W 20-35V DC 50W	10-15V DC 20W	10 - 40 V DC 30W	?
OPERATING TEMPERATURE (AND HUMIDITY)	Receiver 0°C to +50°C Antenna -20°C to +75°C	Receiver 0°C to +50°C Antenna -30°C to +75°C	Receiver -25°C to +55°C Antenna -40°C to +70°C	-20°C to +55°C 95% non-condensing humidity	?
FIX INTERVAL	pseudo-range 1sec carrier phase 15secs computer params. 10secs	1sec	—	1sec	2secs
TIME TO FIRST FIX	5mins warm 30mins cold	5mins warm 30mins cold	—	< 5mins warm < 20mins cold	?
ANTENNA CHARACTER-ISTICS	incorporates ground plane helix ?	helix	quadrifilar helix volute; right hand polarisation.	right hand polarised	?
OTHER INFORMATION	4 channels	4 channels	4 channels dual-frequency model proposed	1 channel fast switching	5 chanels based on the 4000S.
COST US DOLLARS	\$ 44,000 incl. software	\$ 25,000	\$ 90,000 incl. software	\$ 18,000	?

GPS RECEIVER NAME	MAGNAVOX 10X navigator	MAGNAVOX 400A	MAGNAVOX MX 1100 - GPS navigator	MAGNAVOX MX 4400	MAGNAVOX MX T-SET
MEASUREMENT MODE	pseudo-range Loran-C	pseudo-range	pseudo-range	pseudo-range	pseudo-range
FREQUENCY	L1	L1	L1	L1	L1
CODE	C/A	C/A	C/A	C/A	C/A
NUMBER OF SATELLITES TRACKED	?	?	4	?	?
POSITIONAL ACCURACY	25m rms	25m hz 35m vt	35m using 3 sats 90% probab.	15m rms 5.1m 30mins averaging	35m 2D 90% probab 5m relative
VELOCITY ACCURACY	0.1 km/hr	0.1 m/s	?	0.05 m/s	?
MAXIMUM ACCELERATION	?	2g	6 m/s²	6 m/s²	6 m/s²
MAXIMUM VELOCITY	?	600 knots	400 m/s	400 m/s	400 m/s
DIMENSIONS AND WEIGHT	Receiver 4kg 26 × 29 × 11cm (incl. antennas) CDU 1.4kg 11 × 8 × 26 cm	Receiver 2.7kg 18 × 30 × 6 cm Antenna 2.5 lbs 2.6 × 20 × 13 cm	?	Receiver 7.2 kg 35 × 32 × 10 cm Antenna < 5kg 33 × 20 cm	Receiver 16kg 37 × 45 × 36 cm Keyboard 2kg 9 × 46 × 20 cm Antenna : variable
POWER CONSUMPTION	9 - 28 V 30 W	18 - 36 V DC < 15W	?	10 - 28 V DC 20 W	90 - 128 V 130 W
OPERATING TEMPERATURE (AND HUMIDITY)	-10°C to +60°C	Receiver 0°C to +55°C Antenna -55°C to +70°C 95% noncondensing humidity	?	Receiver -20°C to +50°C Antenna -40°C to +70°C 100% relative hum	Receiver +5°C to +40°C Antenna -40°C to +70°C 90% relative hum
FIX INTERVAL	1 sec	1 sec	1.6 secs	1.2 secs	?
TIME TO FIRST FIX	< 4 mins	2 mins warm 4 mins cold	?	3 mins max warm	
ANTENNA CHARACTERISTICS	?	?	circular polarisation	circular polarisation	circular polarisation
OTHER INFORMATION	includes Loran sensor 2 multiplex channels	2 sequencing channels	includes TRANSIT sensor 2 channels	2 channels	5 channels training set receiver requires controlled environment
COST (US DOLLARS UNLESS STATED)	$16 → 30,000 depending on configuration	?	$ 45,000	$ CANADIAN 25,000	$ 50,000

GPS RECEIVER NAME	POLYTECHNIC XR1	POLYTECHNIC XR3(M)	ALLEN OSBOURNE GPS TTR-5A	JLR JLR-4000	MOTOROLA EAGLE MINIRANGER
MEASURE-MENT MODE	pseudo-range	?	pseudo-range	pseudo-range	pseudo-range, carrier phase
FREQUENCY	L1	L1	L1	L1	L1
CODE	C/A	C/A	C/A	C/A	C/A
NUMBER OF SATELLITES TRACKED	?	5	?	?	?
POSITIONAL ACCURACY	30m	30m and "much better with relative positioning"	5m 1σ < 50cm relative	30m rms	< 25m SEP GDOP < 4 < 1m relative
VELOCITY ACCURACY	?	?	?	0.1 m/s rms	?
MAXIMUM ACCELERATION	$4 m/s^2$	$40 m/s^2$	?	1g	?
MAXIMUM VELOCITY	25m/s	m/s^2	?	600knots	?
DIMENSIONS AND WEIGHT	47 × 13 × 41 cm 12.3kg	100^3 inches 15 lbs	Antenna 3lbs 20 × 20 × 20cm Receiver/Processor 43 × 40 × 43cm 28lbs	Receiver/Processor 27 × 14 × 30cm 5kg Antenna 0.7kg 43 × 6 cm	Receiver/Processor 30 × 18 × 6cm 2kg Antenna 1.4kg 11 × 11 × 5 cm.
POWER CONSUMPTION	110-120 220-240 V AC 24 V DC 40W	110/220 V AC 19-32 V DC 15W	110/230 V AC	100/115/220/230 V AC 10-40 V DC	18-32 V DC 10-17 V DC 30 W
OPERATING TEMPERATURE (AND HUMIDITY)	Receiver +5°C to +45°C benign env. required Antenna -25°C to +85°C	-40°C to +70°C 100% humidity	Receiver -10°C to +50°C Antenna -20°C to +65°C	Receiver 0°C to 50°C Antenna -25°C to +70°C	Receiver -20°C to +55°C Antenna 40 to +85°C 90% noncondensing.
FIX INTERVAL	5-10 secs	1 sec	?	?	?
TIME TO FIRST FIX	?	5 mins warm 30 mins cold	< 90 secs with internal almanac	< 1 min warm 15 mins cold	?
ANTENNA CHARACTER-ISTICS	?	optional null steering anti-jam (otherwise "standard")	omnidirectional	quadrifilar helix omnidirectional	?
OTHER INFORMATION	Sequential	1 multiplex channel expandable to 5 channels	code correlating	1 sequential channel	4 channels
COST (US DOLLARS UNLESS STATED)	?	?	?	$ CANADIAN 32,000	$19,000

GPS RECEIVER NAME	SERCEL TR5S	SPERRY GPS CORE MODULE	JMR Geotrak	JMR SatTrak	Norstar 1000
MEASUREMENT MODE	pseudo-range, carrier phase	pseudo-range	pseudo-range, carrier phase, integrated Doppler	pseudo-range	pseudo-range, carrier phase
FREQUENCY	L1	L1	L1	L1	L1
CODE	C/A	C/A	C/A	C/A	C/A
NUMBER OF SATELLITES TRACKED	5	?	7	4	?
POSITIONAL ACCURACY	15m instantaneous	30M 2drms 95% probab.	?	50m SEP nav. 30m CEP 30m SEP surv. 20m CEP 5m relative	10m 1σ 2m 1σ relative
VELOCITY ACCURACY	?	0.2 KT rms	?	?	?
MAXIMUM ACCELERATION	0.5g	?	?	?	?
MAXIMUM VELOCITY	100 km/hr	?	?	?	?
DIMENSIONS AND WEIGHT	Receiver 27 kg 36 × 53 × 45 cm Antenna 5 kg	Receiver 5 kg 14 × 26 × 25 cm Antenna 2 kg 25 × 12 cm	Receiver 58 × 37 × 28 cm 25 kg	Receiver/Processor 30 × 40 × 17 cm Antenna 6 × 35 cm 10 kg	18 × 45 × 55 cm 15 kg
POWER CONSUMPTION	24 V AC	100/110/220 V AC	10-18 V DC	11-15 V DC (1 hr internal battery)	11 - 32 V DC 100-250 V AC
OPERATING TEMPERATURE (AND HUMIDITY)	Receiver +09°C to 40°C Antenna -20°C to +50°C	Receiver -10°C to +55°C 0 to 95% humidity Antenna -20°C to +70°C	-20°C to +50°C	?	-40°C to +60°C 90% noncondensing humidity
FIX INTERVAL	0.6 sec	2-3 secs	1 sec	1 sec	1 sec
TIME TO FIRST FIX	1 min 90% of the time	approx 5 mins	10 mins	?	5 mins
ANTENNA CHARACTERISTICS	ground plane for precise applications	?	?	?	?
OTHER INFORMATION	5 channels		multiplex	1 sequencing channel	5 or 7 channels
COST (US DOLLARS UNLESS STATED)	$67,500	?	?	$ 35,000	$ 55,000 10,000 extra for 7 chans.

APPENDIX B : REFERENCES AND BIBLIOGRAPHY.

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