Does it make sense to talk about $N\Delta$ phase shifts?

H. Garcilazo
Institut für Theoretische Physik, Universität Hannover, D-3000 Hannover 1, Federal Republic of Germany

T. Mizutani*
Paul Scherrer Institut, CH-5232 Villigen, Switzerland

M. T. Peña
Centro de Física Nuclear da Universidade de Lisboa, Instituto Nacional de Investigação Científica, P-1699 Lisboa Codex, Portugal

P. U. Sauer
Institut für Theoretische Physik, Universität Hannover, D-3000 Hannover 1, Federal Republic of Germany

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The question of whether one can consistently define and extract nucleon-delta scattering parameters, phase shifts, and inelasticities from the partial-wave $NN \to N\Delta$ amplitudes is discussed. We have studied the unitarity relation and identified the conditions under which the extraction of such quantities is meaningful. Then these conditions were tested in several coupled-channel models of the NN-$N\Delta$ system.

I. INTRODUCTION

The reaction $pp \to np\pi^+$ was analyzed by Wicklund *et al.*[1] and by Shypit *et al.*[2,3] in the kinematic domain in which the proton and pion form a delta resonance in the final state. The analysis determined the major NN-$N\Delta$ partial-wave amplitudes, and from them $N\Delta$-$N\Delta$ phase shifts were extracted whose energy dependence turned out to be smooth. That fact is taken as proof against the existence of broad dibaryon resonances.

Since the Δ is not a stable particle but a rather wide πN resonance ($\Gamma_\Delta \sim 114$ MeV at the resonance position), the $N\Delta$ phase shift does not appear to be well defined. In fact, by looking at the formal structure of the $NN \to N\Delta$ amplitude in his coupled-channels model rewritten in a distorted-wave form, Lee[4] has concluded that due to the width of the delta, the quantities extracted in Refs. 1–3 may not be identified as the $N\Delta$ phases, but something more intricate, the physical meaning of which could only be disentangled with the help of models.

Although the phase shifts are not the observables, they provide us with quite valuable information concerning the nature of the underlying interaction in a given scattering process. For example, if they could be defined properly, the phase shifts (and inelasticities) allow for a natural representation of the NN-$N\Delta$ amplitude linked by unitarity to the NN-NN amplitude, which then enables us to determine the $N\Delta$-$N\Delta$ amplitude which is otherwise not accessible experimentally.

As an attempt to clarify this situation, we have decided to study the scattering problem of a nucleon and an unstable delta within a (meson-exchange) potential model. This may allow us to establish under which conditions the $N\Delta$ phase shift can be consistently defined. We then have studied how well these conditions may be satisfied by exploiting several realistic models for the coupled NN-$N\Delta$ system.

II. TWO-CHANNEL CASE WITH A STABLE Δ PARTICLE

In order to facilitate our later discussion, let us first consider a simple NN-$N\Delta$ coupled-channel problem with a stable Δ and with real interaction potentials. For simplicity, let us further assume that one NN partial wave couples to only one $N\Delta$ partial wave. We denote the NN system as channel 1 and the $N\Delta$ system as channel 2. Then the off-shell coupled-channel equations in the overall c.m. system, upon suppressing the partial-wave indices, read

$$T_{ij}(q_i,q_j) = V_{ij}(q_i,q_j) + \sum_{k=1}^{2} \int_0^\infty q_k^2 dq_k \, V_{ik}(q_i,q_k) \times G_k(E,q_k) T_{kj}(q_k,q_j) \ , \qquad (1)$$

where q_1 and q_2 are the NN and $N\Delta$ off-shell relative momenta, respectively. The Green's functions are

$$G_1(E,q_1) = \frac{1}{E - q_1^2/2\mu_1 + i\epsilon} \ , \qquad (2)$$

$$G_2(E,q_2) = \frac{1}{E - (m_\Delta - m_N) - q_2^2/2\mu_2 + i\epsilon} \ , \qquad (3)$$

with the reduced masses

$$\mu_1=\frac{m_N}{2}\ , \tag{4}$$

$$\mu_2=\frac{m_N m_\Delta}{m_N+m_\Delta}\ . \tag{5}$$

The redefinitions

$$F_{ij}(q_i,q_j)=-\sqrt{\pi\mu_i q_i}\,T_{ij}(q_i,q_j)\sqrt{\pi\mu_j q_j}\ , \tag{6}$$

$$v_{ij}(q_i,q_j)=-\sqrt{\pi\mu_i q_i}\,V_{ij}(q_i,q_j)\sqrt{\pi\mu_j q_j}\ , \tag{7}$$

$$g_k(E,q_k)=-\frac{q_k}{\pi\mu_k}G_k(E,q_k)\ , \tag{8}$$

make our subsequent discussion somewhat simpler, and lead to the equations for the new amplitudes F_{ij}:

$$F_{ij}(q_i,q_j)=v_{ij}(q_i,q_j)+\sum_{k=1}^{2}\int_0^\infty dq_k v_{ik}(q_i,q_k)\times g_k(E,q_k)F_{kj}(q_k,q_j)\ , \tag{9}$$

while the new propagators (8) obey

$$\mathrm{Im}g_k(E,q_k)=\delta(q_k-k_k)\ , \tag{10}$$

so that

$$\int_0^\infty dq_k \mathrm{Im}g_k(E,q_k)=1\ . \tag{11}$$

In Eq. (10) k_1 and k_2 are the on-shell momenta for the NN and $N\Delta$ channels, respectively, determined by the scattering energy E. Since we have assumed real potentials, the solutions of Eq. (9) satisfy the off-shell unitarity relations

$$\mathrm{Im}F_{ij}(q_i,q_j)=\sum_{k=1}^{2}\int_0^\infty dq_k F_{ik}(q_i,q_k)\times \mathrm{Im}g_k(E,q_k)F^*_{kj}(q_k,q_j)\ . \tag{12}$$

Then, as a result of Eq. (10), the on-shell unitarity relations are simply

$$\mathrm{Im}F_{ij}(k_i,k_j)=\sum_{k=1}^{2}F_{ik}(k_i,k_k)F^*_{kj}(k_k,k_j)\ . \tag{13}$$

From Eq. (13) it follows that the amplitudes $F_{ij}(k_i,k_j)$ can be represented as

$$F_{11}(k_1,k_1)=\frac{1}{2i}(\eta e^{2i\delta_N}-1)\ , \tag{14a}$$

$$F_{22}(k_2,k_2)=\frac{1}{2i}(\eta e^{2i\delta_\Delta}-1)\ , \tag{14b}$$

$$F_{12}(k_1,k_2)=\tfrac{1}{2}(1-\eta^2)^{1/2}e^{i(\delta_N+\delta_\Delta)}\ , \tag{14c}$$

where the three real parameters η, δ_N, and δ_Δ are the inelasticity, and the NN and $N\Delta$ phase shifts, respectively, which are functions of the scattering energy. From Eqs. (14a), (14b), and (14c), one may obtain F_{22} from the knowledge of the amplitudes F_{11} and F_{12}.

III. TWO-CHANNEL CASE WITH UNSTABLE Δ PARTICLE

Let us now consider the case where the Δ is not a stable particle so that the Δ propagator of Eq. (3) (in the presence of a spectator nucleon) is to be replaced by

$$G_2(E,q_2)=\frac{1}{S_\Delta(E,q_2)+(i/2)\Gamma_\Delta(E,q_2)}=-\frac{2}{\Gamma_\Delta(E,q_2)}\sin\delta_{33}e^{i\delta_{33}}\ , \tag{15}$$

where $\Gamma_\Delta(E,q_2)$ is the width of the Δ and $S_\Delta(E,q_2)$ can be obtained from a microscopic model of the pion-nucleon P_{33} resonance or by fitting it directly to the P_{33} phase shift at pion-nucleon energies determined by E and q_2; the simplest form of $S_\Delta(E,q_2)$ may be identified as the inverse of $G_2(E,q_2)$ of Eq. (3). The reduced mass μ_2 in Eqs. (6)–(8) will then be replaced by

$$\mu_2=-\frac{1}{\pi}\int_0^\infty q_2 dq_2 \mathrm{Im}G_2(E,q_2)\ , \tag{16}$$

where μ_2 is now a function of the energy E which is determined by the nucleon-nucleon on-shell momentum k_1. Thus Eqs. (6)–(8) remain unchanged. The only change is that Eq. (10) no longer holds for the $N\Delta$ ($k=2$) channel, but Eq. (11) still is valid:

$$\int_0^\infty dq_2 \mathrm{Im}g_2(E,q_2)=1\ . \tag{17}$$

We note that when E is below the πNN threshold, $\mathrm{Im}G_2(E,q_2)$ is zero; thus Eq. (16) and the simplification as exercised in Eqs. (6)–(8) do lose their meaning. Since the $N\Delta$ channel is closed in this case, the redefinition of the amplitudes involving this channel is not needed anyway. Since our present interest is concerned with the $N\Delta$ phases, we may safely assume that we are always above the pion production threshold; thus no modification of the formulas is required.

If we continue to assume real potentials, the unitarity relations (12) now become [some discussion on the effect of retarded (complex) potentials which are introduced by the requirement of three-body unitarity will be given in Sec. IV]

$$\mathrm{Im}F_{11}(k_1,k_1)=|F_{11}(k_1,k_1)|^2+\int_0^\infty dq_2|F_{12}(k_1,q_2)|^2\mathrm{Im}g_2(E,q_2)\ , \tag{18a}$$

$$\mathrm{Im}F_{22}(q_2,q_2')=F_{21}(q_2,k_1)F^*_{12}(k_1,q_2')+\int_0^\infty dq_2''F_{22}(q_2,q_2'')\times \mathrm{Im}g_2(E,q_2'')F^*_{22}(q_2'',q_2')\ , \tag{18b}$$

$$\mathrm{Im}F_{12}(k_1,q_2')=F_{11}(k_1,k_1)F^*_{12}(k_1,q_2')+\int_0^\infty dq_2''F_{12}(k_1,q_2'')\times \mathrm{Im}g_2(E,q_2'')F^*_{22}(q_2'',q_2')\ . \tag{18c}$$

We stress where that $\mathrm{Im}g_2(E,q_2)$ being not any more a δ function is a manifestation that for a given energy E a unique on-shell momentum cannot be assigned to the $N\Delta$ system due to the finite Δ decay width. It is thus appropriate to keep the off-shell momenta q_2's in the above expressions.

To proceed further we note that $\mathrm{Im}g_2(E,q_2)$ is normalized to unity, and that from Eqs. (8) and (15) $\mathrm{Im}g_2(E,q_2)\propto\Gamma_\Delta(E,q_2)$; thus this quantity may be regarded as a measure of the "on-shell" momentum distribution of the $N\Delta$ system within the resonance width. So the quantity

$$\langle A(E)\rangle=\int_0^\infty dq_2\,A(E,q_2)\mathrm{Im}g_2(E,q_2) \tag{19}$$

has the meaning of the expectation value of A over the available width of the Δ resonance (or, equivalently, average over the Δ mass distribution, in a covariant language). Thus, in Eqs. (18), we are constructing products of amplitudes and averaging them over the Δ width. We can now carry out this averaging procedure a little bit further: We multiply Eq. (18b) by $\mathrm{Im}g_2(E,q_2)\mathrm{Im}g_2(E,q_2')$ and integrate over q_2 and q_2', and similarly multiply Eq. (18c) by $\mathrm{Im}g_2(E,q_2')$ and integrate over q_2'. We thus find

$$\mathrm{Im}F_{11}(k_1,k_1)=|F_{11}(k_1,k_1)|^2+\langle|F_{12}(k_1)|^2\rangle\ , \tag{20a}$$

$$\mathrm{Im}\langle F_{22}\rangle=|\langle F_{12}(k_1)\rangle|^2+\langle F_{22}F_{22}^*\rangle\ , \tag{20b}$$

$$\mathrm{Im}\langle F_{12}(k_1)\rangle=F_{11}(k_1,k_1)\langle F_{12}^*(k_1)\rangle+\langle F_{12}(k_1)F_{22}^*\rangle\ , \tag{20c}$$

where the average quantities are defined as

$$\langle F_{12}(k_1)\rangle=\int_0^\infty dq_2 F_{12}(k_1,q_2)\mathrm{Im}g_2(E,q_2)\ , \tag{21a}$$

$$\langle F_{22}\rangle=\int_0^\infty dq_2\int_0^\infty dq_2'\mathrm{Im}g_2(E,q_2)F_{22}(q_2,q_2')\mathrm{Im}g_2(E,q_2')\ , \tag{21b}$$

$$\langle|F_{12}(k_1)|^2\rangle=\int_0^\infty dq_2|F_{12}(k_1,q_2)|^2\mathrm{Im}g_2(E,q_2)\ , \tag{21c}$$

$$\langle F_{22}F_{22}^*\rangle=\int_0^\infty dq_2\int_0^\infty dq_2'\int_0^\infty dq_2''\mathrm{Im}g_2(E,q_2)F_{22}(q_2,q_2'')\mathrm{Im}g_2(E,q_2'')F_{22}^*(q_2'',q_2')\mathrm{Im}g_2(E,q_2')\ , \tag{21d}$$

$$\langle F_{12}(k_1)F_{22}^*\rangle=\int_0^\infty dq_2''\int_0^\infty dq_2'\mathrm{Im}g_2(E,q_2'')F_{12}(k_1,q_2'')F_{22}^*(q_2'',q_2')\mathrm{Im}g_2(E,q_2')\ . \tag{21e}$$

Then, as in Eqs. (14), one is tempted to parametrize F_{11}, $\langle F_{22}\rangle$, and $\langle F_{12}\rangle$ as

$$F_{11}(k_1,k_1)=\frac{1}{2i}(\eta e^{2i\delta_N}-1)\ , \tag{22a}$$

$$\langle F_{22}\rangle=\frac{1}{2i}(\eta' e^{2i\delta_\Delta}-1)\ , \tag{22b}$$

$$\langle F_{12}(k_1)\rangle=\tfrac{1}{2}(1-\eta''^2)^{1/2}e^{i(\delta_N+\delta_\Delta')}\ , \tag{22c}$$

where the three inelasticity parameters η, η', and η'' are all independent as well as the two phases δ_Δ and δ_Δ'. All scattering parameters are functions of the scattering energy E. However, this representation only makes sense physically, if all $N\Delta$ phase-shift parameters and all inelasticity parameters become approximately equal, i.e., $\delta_\Delta\simeq\delta_\Delta'$, and $\eta\simeq\eta'\simeq\eta''$. In fact, they become exactly equal:

$$\eta=\eta'=\eta''\ , \tag{23a}$$

$$\delta_\Delta=\delta_\Delta'\ , \tag{23b}$$

if the coupled unitarity relation Eq. (13) holds among F_{11}, $\langle F_{22}\rangle$, and $\langle F_{12}(k_1)\rangle$, i.e., the following conditions are met:

$$\langle|F_{12}(k_1)|^2\rangle=|\langle F_{12}(k_1)\rangle|^2\ , \tag{24a}$$

$$\langle F_{22}F_{22}^*\rangle=|\langle F_{22}\rangle|^2\ , \tag{24b}$$

$$\langle F_{12}(k_1)F_{22}^*\rangle=\langle F_{12}(k_1)\rangle\langle F_{22}\rangle^*\ . \tag{24c}$$

It is worth remarking here that the conditions Eqs. (24) are equivalent to the following requirement: Physical observables identified as averages over the Δ width should be obtained from the average amplitudes $\langle F_{12}\rangle$ and $\langle F_{22}\rangle$. This point will be touched upon later.

IV. TESTING THE CONDITIONS

In the previous section we have identified the conditions Eqs. (24) under which the $N\Delta$ phase parameters may be defined consistently. So our task now is to find out to what extent these conditions may be satisfied in reality. We shall carry this out here in the context of meson-exchange potential models of the coupled NN-$N\Delta$ system. In particular, we consider two dominant partial waves, $J^P=2^+$: ${}^1D_2(NN)$-${}^5S_2(N\Delta)$, and $J^P=3^-$: ${}^3F_3(NN)$-${}^5P_3(N\Delta)$. Coupling to higher $N\Delta$ orbital angular momentum states are disregarded since the effects are quite small. Four different force models are employed.

(A) Realistic potentials which we have adopted are the following: For the diagonal NN potential, we took

$$V_{11}=V_{\mathrm{Paris}}-V_{12}(G_2+G_2t_{22}G_2)V_{21}|_{E=0}\ , \tag{25}$$

where

$$t_{22}=V_{22}+V_{22}G_2t_{22}\ . \tag{26}$$

This guarantees that the solution of the coupled-channel equations is identical to that of the Paris potential at $E=0$. For the transition potentials V_{12}, we used the static potential model with the π and ρ exchanges developed by Hajduk *et al.*[5] In this model all the meson-nucleon-nucleon and meson-nucleon-delta vertices are regularized by a square-root-monopole form factor with a cutoff mass $\Lambda=1200$ MeV/c.

For the $N\Delta$-$N\Delta$ potential, we adopted a standard

TABLE I. Phase parameters of the $NN\ {}^1D_2$ and $N\Delta\ {}^5S_2$ channels for models (A)–(D) at three different nucleon-nucleon laboratory energies (in MeV). The experimental $N\Delta$ phases extracted in Ref. 3 at 643 and 796 MeV are, respectively, $\delta'_\Delta=31°\pm3°$ and $\delta'_\Delta=13°\pm2°$.

Model	$T_N^{\rm LAB}$	δ_N (deg)	η	η'	η''	δ_Δ (deg)	δ'_Δ (deg)
	600	17.4	0.72	0.72	0.72	−1.5	−1.3
(A)	800	−2.4	0.54	0.55	0.56	2.9	2.9
	1000	−16.5	0.69	0.71	0.71	2.4	2.9
	600	16.7	0.72	0.72	0.73	−0.4	−0.3
(B)	800	−1.7	0.57	0.58	0.59	0.5	0.6
	1000	−15.2	0.70	0.72	0.72	−1.3	−0.9
	600	12.8	0.85	0.85	0.85	2.4	2.8
(C)	800	1.5	0.74	0.74	0.75	11.8	11.4
	1000	−6.2	0.85	0.86	0.86	13.7	13.2
	600	12.0	0.89	0.89	0.89	−1.5	−1.5
(D)	800	4.2	0.78	0.79	0.79	−10.0	−10.1
	1000	−3.5	0.81	0.82	0.82	−18.5	−18.5

three-body picture: the u-channel one-pion-exchange (OPE) process which contains all the three-body singularities, to which we added $\frac{1}{2}$ of the static OPE potential.[6] The $\pi N\Delta$ form factor was taken consistently with the NN-$N\Delta$ transition potential, viz., a square-root monopole with $\Lambda=1200$ MeV/c. Relativistic kinematics was used for the pion and nucleons, and the Δ propagator (15) was constructed directly from the P_{33} phase shift and was extrapolated to the subthreshold region as

$$G_2(E,q_2)=\frac{\alpha}{s_2(q_2)+\beta},\quad s_2(q_2)<(m_N+m_\pi)^2\ , \quad (27)$$

where $s_2(q_2)$ is the pion-nucleon invariant mass squared. The constants α and β were determined such that the propagator and its first derivative are continuous at $s_2(q_2)=(m_N+m_\pi)^2$. The Δ propagator and the $N\Delta$-$N\Delta$ potential have been constructed consistently with the requirements of three-body unitarity.[7]

(B) From the model (A) described above, we constructed a second model by adding a residual $N\Delta$-$N\Delta$ interaction.[8] This interaction was assumed to be a static potential generated by the exchange of a ρ meson in the u channel and by the t-channel exchange of π, ρ, ω, and σ mesons, with the $\rho N\Delta$ and meson-delta-delta coupling constants taken from the naive SU(3) quark model, and a square-root-monopole form factor with $\Lambda=1200$ MeV/c was used at every vertex.[8]

(C) We have also considered the coupled-channel model developed recently by Bulla[9] which is quite similar to the model of Lee,[10] in which the transition potentials have a monopole cutoff form factor with $\Lambda=650$ MeV/c. In this model the delta propagator is obtained from an energy-dependent separable potential using a monopole form factor with $\Lambda=288$ MeV/c and the $N\Delta$-$N\Delta$ potential is constructed consistently with the delta propagator so as to satisfy three-body unitarity.

(D) This is identical to model (C) except that the residual $N\Delta$-$N\Delta$ interaction[8] is added.

With the above interactions we have solved a coupled Lippmann-Schwinger–type equation for F_{ij}'s, and extracted the phase parameters according to Eqs. (22). The result is presented in Tables I and II. Clearly, we find

TABLE II. Phase parameters of the $NN\ {}^3F_3$ and $N\Delta\ {}^5P_3$ channels for models (A)–(D) at three different nucleon-nucleon laboratory energies (in MeV). The experimental $N\Delta$ phases extracted in Ref. 3 at 643 and 796 MeV are, respectively, $\delta'_\Delta=6°\pm3°$ and $\delta'_\Delta=5°\pm2°$.

Model	$T_N^{\rm lab}$	δ_N (deg)	η	η'	η''	δ_Δ (deg)	δ'_Δ (deg)
	600	0.1	0.90	0.92	0.92	3.2	3.6
(A)	800	−5.4	0.66	0.68	0.69	11.2	12.1
	1000	−17.5	0.67	0.67	0.68	17.4	18.1
	600	0.3	0.90	0.91	0.92	3.3	3.8
(B)	800	−5.5	0.66	0.68	0.68	11.5	12.4
	1000	−17.3	0.67	0.67	0.68	15.4	15.9
	600	−3.2	0.96	0.97	0.97	1.3	1.5
(C)	800	−5.5	0.83	0.84	0.85	1.9	1.7
	1000	−10.9	0.80	0.81	0.81	1.0	1.1
	600	−3.1	0.96	0.96	0.97	1.4	1.5
(D)	800	−5.6	0.83	0.84	0.84	2.4	2.3
	1000	−10.9	0.80	0.81	0.81	1.1	1.2

that within a given model the three inelasticity parameters η, η', and η'', all stay very close to each other. This is true also with the two $N\Delta$ phase shifts δ_Δ and δ'_Δ. Incidentally, we remark that since our u-channel $N\Delta$-$N\Delta$ potentials are nonstatic, there is an additional contribution to the unitarity relation Eqs. (18) due to three-body unitarity.[7] However, the fact that Eqs. (23) are well satisfied (particularly the near equality of the inelasticity parameters) means that this nonstatic effect is quite small. So, in fact, conditions in Eqs. (24) are quite well satisfied by the averaged amplitudes, which means that from the knowledge of the NN-NN amplitude and the average NN-$N\Delta$ amplitude $\langle F_{12}(k_1)\rangle$ one may indeed deduce the average $N\Delta$-$N\Delta$ amplitude $\langle F_{22}\rangle$. It is important to stress here that Eqs. (24) are satisfied quite irrespective of the model interactions employed here. This, we think, is a rather important consequence of the present study.

It should be useful to plot some Argand diagrams of the amplitudes we have obtained above. They are shown in Figs. 1 and 2 for the NN-NN and $N\Delta$-$N\Delta$ channels in the $J=2(^1D_2, ^5S_2)$, and $J=3(^3F_3, ^5P_3)$ partial waves, respectively. In the figures numbers signify the nucleon-nucleon laboratory energies in the units of 100 MeV. At 400 MeV we are near the πNN threshold and consequently the $N\Delta$-$N\Delta$ amplitude lies almost at the origin. In some cases the $N\Delta$ amplitudes rotate clockwise and in other cases they rotate counterclockwise, whereas the NN amplitudes always move counterclockwise. This should be due to the fact that while the adopted interactions are constrained (not very strongly, though) by the NN phase parameters from the existing phase-shift analyses, no such constraint has been adopted from the available data in the $NN \to N\Delta$ channel,[1] although those data are quite scarce. Therefore, the Argand amplitudes of Figs. 1 and 2 should be taken as illustration for the potential of the method in extracting $N\Delta$-$N\Delta$ scattering amplitudes. In this respect, we should also remind the reader that we have not included the coupling to the πd channel, which is known to be very important and which will affect the inelasticity in the $^1D_2$-$^5S_2$ channel:[11] Its inclusion leads to a three-channel problem adding the $\pi d \to \pi d$, $\pi d \to NN$, and $\pi d \to N\Delta$ amplitudes. In this case the unitary parametrization of the T matrix is more complicated than Eqs. (14) since it involves six real parameters; however, one can still determine the $N\Delta$-$N\Delta$ amplitude from the knowledge and the nondiagonal amplitudes once the extended conditions of Eqs. (24) are still well satisfied.

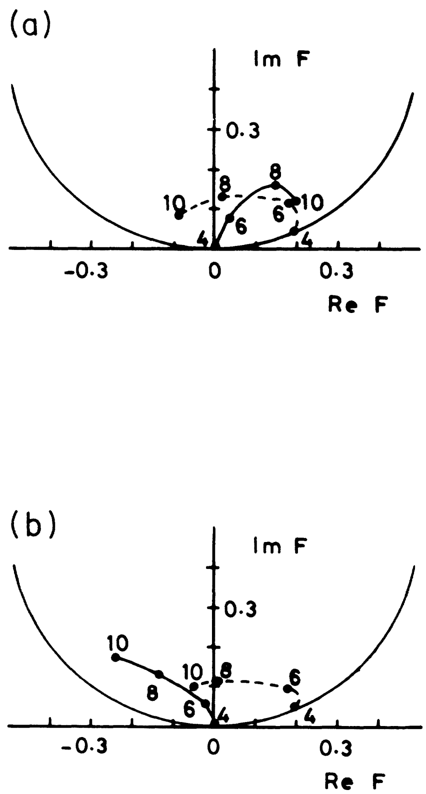

FIG. 1. Argand diagrams for the NN $^1D_2$ (dashed lines) and $N\Delta$ $^5S_2$ (solid lines) channels obtained from (a) model (C) and (b) model (D). The numbers indicate the nucleon-nucleon laboratory energies in units of hundreds of MeV.

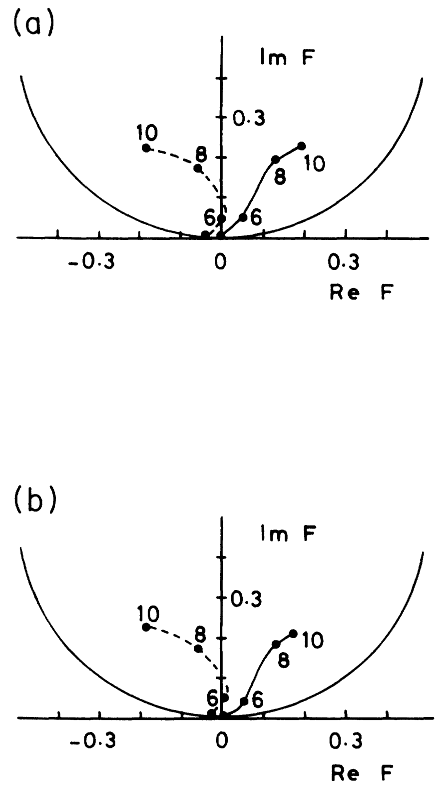

FIG. 2. Argand diagrams for the NN $^3F_3$ (dashed lines) and $N\Delta$ $^5P_3$ (solid lines) channels obtained from (a) model (A) and (b) model (B). The numbers indicate the nucleon-nucleon laboratory energies in units of hundreds of MeV.

V. FINAL REMARKS

Our present study was triggered in part by Lee's formal observation[4] discussed in the Introduction. To the extent of what we have found, his concern may be safely put away: The phase of a partial wave $NN \to N\Delta$ amplitude can indeed be identified as the sum of the NN and $N\Delta$ phase shifts. As a matter of fact, it is our feeling that the distorted-wave representation of the $NN \to N\Delta$ amplitude devised in his discussion may not be particularly suited for invoking a useful insight. As an example, let us use our simple model of Sec. II with the stable Δ and repeat the derivation of the NN-$N\Delta$ amplitude following the steps of Lee.[4] Then one finds that the amplitude takes the following form:

$$F'_{12}(k_1,k_2)=e^{i(\hat{\delta}_N+\bar{\delta}_\Delta)}f(k_1,k_2,\hat{\delta}_N,\bar{\delta}_\Delta) \ , \quad (28)$$

where $\hat{\delta}_N$ is a complex phase, the real part of which is equal to δ_N of Eqs. (14) while $\bar{\delta}_\Delta$ is *real* in this simplified model and is the $N\Delta$ phase shift in the *absence* of the coupling to the NN channel. The fact that $\hat{\delta}_N$ is *complex* makes the function f also complex. Now the amplitude Eq. (28) must be identical to the one in Eq. (14c). So the sum of the phase of the function f in Eq. (28) and $\bar{\delta}_\Delta$ must give δ_Δ, which cannot be inferred easily from this distorted-wave formula, however.

Within the models we have adopted for the NN and $N\Delta$ interactions, it is fair to conclude that the $N\Delta$ phase shifts (and the corresponding inelasticities) may be consistently defined from the relevant partial-wave amplitudes averaged over the Δ width. That this holds quite well irrespective of the different behavior of the $N\Delta$ phase shifts for different models supports that this conclusion may be rather model independent and thus general. One of the possible reasons behind this fact might be that the Δ could effectively be regarded as a *narrow* resonance, although even at the πN threshold it cannot be neglected.

Last, we need to clarify the following point: How can one relate the phase parameters (or amplitudes) we have defined above, viz., $\langle F_{12}(k_1)\rangle$, etc., with those extracted from data? Let us first discuss the case with the $NN \to N\Delta$ process.[1-3] The total theoretical cross section for the process $NN \to N\Delta$ reads

$$\sigma(NN\to\Delta N)=\frac{2\pi}{k_1^2}\sum(2J+1)\langle |F_{12}(k_1)|^2\rangle \ , \quad (29)$$

and in fact requires the averaged $N\Delta$ amplitude as of Eq. (21c). In contrast, the analysis of the experiment[1-3] uses

$$\sigma(NN\to\Delta N)=\frac{2\pi}{k_1^2}\sum(2J+1)|\langle F_{12}(k_1)\rangle|^2 \ , \quad (30)$$

for a comparison with data and extracts the averaged amplitudes $\langle F_{12}\rangle$ and phase shifts corresponding to $\langle F_{12}\rangle$ from Eq. (30). When the conditions in Eqs. (24) are met, as in the numerical study of this paper, Eqs. (29) and (30) become the same (recall the remark towards the end of Sec. III). This last expression is just identical to Eq. (30) of Ref. 1 once W_p, the amplitude extracted from the data, is identified with $\langle F_{12}(k_1)\rangle$. We argue that this identification is quite natural since W_p apparently depends only on the total energy, which means that this is certainly some averaged quantity (over the Δ width). In this respect we should stress here again that the essential conditions are Eqs. (24), which give meaning to the extracted amplitudes W_p, thus the phase parameters.

The relation to the phase parameters extracted from other channels[12-16] is less clear: In Refs. 12–14 what is called the coupled-channel K-matrix approach was employed to reproduce the phase parameters in the NN channel, which at the same time provided the corresponding quantities in the $N\Delta$ channel. This method relies principally on the multichannel unitarity relation, and the finite-width effect is incorporated only into the Chew-Mandelstam function, which is essentially the $N\Delta$ propagator like our $g_2(E,q_2)$. References 15 and 16 attempted to extract the $N\Delta$ phases from the discrepancy between the data and the model results in the elastic πd channel. Apparently, the $N\Delta \to N\Delta$ amplitudes were taken as if the Δ has a zero width, and the effect of the finite width was simulated by introducing a complex on-shell momentum in relating the amplitude and the corresponding t matrix [the relation like Eq. (6)]. In both cases, the relation to our averaged amplitudes (or phase parameters) needs to be further investigated. Of course, if it turns out that the Δ may be treated as a narrow resonance, then all the different treatment of the finite-width effect should give the same result.

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*Permanent address: Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

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