Baryons in a soft-wall AdS-Schwarzschild approach at low temperature

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Recently we derived a soft-wall anti–de Sitter-Schwarzschild approach at small temperatures for the description of hadrons with integer spin and adjustable number of constituents (mesons, tetraquarks, dibaryons, etc.). In the present paper we extend our formalism to states with half-integer spin (baryons, pentaquarks, etc.), presenting analytical results for the temperature dependence of their masses and form factors.

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I. INTRODUCTION

The study of the temperature dependence of hadron properties gives an opportunity for getting a deeper understanding of several physical phenomena, such as the evolution of the early Universe, and the formation and phase transitions in both hadronic and nuclear matter. One of the powerful methods to get insights into the thermal properties of hadrons is holographic QCD. For the recent progress achieved by holographic QCD in this direction see Refs. [1–14], and Ref. [14] for a short overview. In Ref. [14] we proposed a modification of the soft-wall model at finite temperature T in order to have consistency with QCD properties. In particular, we argue that in order to reproduce a temperature behavior of the quark condensate one should include a T-dependence of the dilaton field (which is the parameter of spontaneous breaking of chiral symmetry related to the pseudoscalar meson decay constant) and the warping of the anti–de Sitter (AdS) metric due to temperature. In particular, we proposed that the dilaton field has a specific T-dependence, which is dictated by the temperature behavior of the chiral quark condensate in QCD [15–17], derived using chiral perturbation theory (ChPT) [18]. In this way we postulated the temperature dependence of the dilaton field, using its relation to the chiral quark condensate at zero temperature. A thermal behavior of the dilaton has been previously proposed in Ref. [10]. In the present case we aim for consistency with QCD, which makes the study important in order to improve the understanding of hadronic properties at finite temperature. In Ref. [14] we were interested in the low temperature limit and in the derivation of analytical formulas for the mass spectrum of mesons and their form factors within a soft-wall AdS/QCD model [19–24]. In particular, we considered two possible sources of temperature dependence: (1) the warping of the AdS metric due to temperature, (2) the temperature dependence of the dilaton-background field. This field produces confinement and is responsible for the breaking of conformal invariance and the spontaneous breaking of chiral symmetry in holographic QCD.

In the present paper we extend the ideas and formalism of Ref. [14] to the case of hadrons with half-integer spin, and it is structured as follows. In Sec. II we present the details for the construction of an effective action for AdS fields with half-integer spin at small temperatures, and apply it to the calculation of the mass spectrum and form factors. In Sec. III we discuss a derivation of a new quantity—the hadron light-front wave function at finite temperature. It is obtained using the matching of form factors obtained in our approach and the Drell-Yan-West (DYW) formula [25] for the hadronic form factors in light-front QCD. Finally, in Sec. IV, we summarize the results of the paper.

II. FRAMEWORK

A. Effective action and hadron masses at low temperatures

In this section we start with the derivation of the five dimensional action for the fermion bulk field $B_{M_1 \ldots M_J}$, with arbitrary total half-integer spin J at low temperature T. Our formalism is based on the analogous action at zero temperature [22], and includes the issues proposed in Ref. [14]. The AdS-Schwarzschild metric is specified by

$$ds^2 = e^{2A(z)}\left[f_T(z)dt^2 - (d\vec{x})^2 - \frac{dz^2}{f_T(z)}\right] \qquad (1)$$

where $x = (t, \vec{x})$ is the set of Minkowski coordinates, z is the holographic coordinate, R is the AdS radius and $A(z) = \log(R/z)$. Here $f_T(z) = 1 - z^4/z_H^4$, where z_H is the position of the event horizon, which is related to the black-hole Hawking temperature by $T = 1/(\pi z_H)$. As in the case of boson AdS fields, we also introduce the exponential prefactor $\exp[-\varphi(z)]$ in the effective action. It contains the background (dilaton) field $\varphi(z) = \kappa^2 z^2$ where κ is a scale parameter of the order of a few hundred MeV. This dilaton field breaks conformal invariance, produces confinement and is responsible for the spontaneous breaking of chiral symmetry in holographic QCD. In addition to the dilaton, we introduce in the action the thermal prefactor

$$e^{-\lambda_T(z)}, \qquad \lambda_T(z) = \alpha\frac{z^2}{z_H^2} + \gamma\frac{z^4}{z_H^4} + \xi\frac{\kappa^2 z^6}{z_H^4}, \qquad (2)$$

where the dimensionless parameters α, γ, and ξ parametrize the z^2, z^4, and z^6 thermal corrections. The parameters γ and ξ have been fixed in Ref. [14] to guarantee gauge invariance and massless ground-state pseudoscalar mesons (π, K, η) in the chiral limit, and to suppress the six power of the radial dependence in the holographic potential:

$$\gamma = \frac{J(J-3)+3}{5}, \qquad \xi = \frac{2}{5}. \qquad (3)$$

The parameter α encodes the contribution of gravity to the restoration of chiral symmetry at a critical temperature T_c, and the term $\alpha z^2/z_H^2$ was considered to be a small perturbative correction to the quadratic dilaton $\varphi(z)$. Then, for convenience, we related the holographic coordinate z to the Regge-Wheeler (RW) tortoise coordinate r via the substitution [26,27]:

$$r = \int\frac{dz}{f_T(z)} = \frac{z_H}{2}\left[-\arctan\frac{z}{z_H} + \frac{1}{2}\log\frac{1 - z/z_H}{1 + z/z_H}\right]. \qquad (4)$$

Here we use the plus sign in the right-hand side (r.h.s.) of Eq. (4). As in Ref. [14], we restrict ourselves to the leading-order (LO) and next-to-leading-order (NLO) terms in the expansion of z in powers of r:

$$z = r\left[1 - \frac{t_r^4}{5} + \mathcal{O}(t_r^8)\right], \qquad t_r = r/z_H. \qquad (5)$$

Using (5) the generalized exponential prefactor in the case of the boson AdS bulk fields reads

$$\mathcal{P} = \exp\left[-\varphi_T(r) - \gamma\frac{r^4}{z_H^4}\right]. \qquad (6)$$

In the following we construct the action for fermion AdS fields $B_{M_1\ldots M_J}$ dual to hadrons with total half-integer spin J and number of the constituents N. As was shown in Ref. [22] in the case of baryons the exponential prefactor in the action can be absorbed by a redefinition of AdS fermion fields. Here we extend this idea to the prefactor $\mathcal{P}$, performing redefinition the AdS fermion field $\mathcal{B}_{M_1\ldots M_J}$ as

$$\mathcal{B}_{M_1\ldots M_J}(x,r,T) = \mathcal{P}^{-1/2}B_{M_1\ldots M_J}(x,r,T). \qquad (7)$$

Now the action for fermion bulk fields $B_{N_1\ldots N_J}(x,r,T)$ reads:

$$S_B = \int d^4x dr\sqrt{g}\bar{B}_{N_1\ldots N_J}(x,r,T)\hat{\mathcal{D}}_\pm(r)B^{N_1\cdots N_J}(x,r,T),$$
$$\hat{\mathcal{D}}_\pm(r) = \frac{i}{2}\Gamma^M\left[\overleftrightarrow{\partial}_M - \frac{1}{4}\omega_M^{ab}[\Gamma_a,\Gamma_b]\right] \mp [\mu(r,T) + U_F(r,T)]. \qquad (8)$$

Here $\mu(r,T) = \mu/f_T^{3/10}(r)$ is the temperature dependent five-dimensional mass of the AdS fermion with half-integer spin $\mu = N + L - 3/2$, where N and L are the number of partons and orbital angular momentum, respectively. $U_F(r,T) = \varphi_T(r)/f_T^{3/10}(r)$ is the dilaton temperature dependent potential, in which $\varphi_T(r)$ is the T-dependent dilaton field derived in Ref. [14]:

$$\varphi_T(r) = K_T^2 r^2 = (1+\rho_T)\kappa^2 r^2,$$
$$\rho_T = \left(\frac{9\alpha\pi^2}{16} + \delta_{T_1}\right)\frac{T^2}{12F^2} + \delta_{T_2}\left(\frac{T^2}{12F^2}\right)^2 + \mathcal{O}(T^6). \qquad (9)$$

The quantity ρ_T encodes the T-dependence of the dilaton, in the form of an $T^2/(12F^2)$ expansion dictated by QCD [15] and F is the pseudoscalar coupling constant in the chiral limit; $\omega_M^{ab} = (\delta_M^a\delta_r^b - \delta_M^b\delta_r^a)/(rf_T^{1/5}(r))$ is the spin connection term; $\sigma^{MN} = [\Gamma^M, \Gamma^N]$ is the commutator of the Dirac matrices in AdS space, which are defined as $\Gamma^M = \epsilon_a^M\Gamma^a$ and $\Gamma^a = (\gamma^\mu, -i\gamma^5)$. The term with partial derivative is defined as

$$\Gamma^M\overleftrightarrow{\partial}_M = \Gamma^M(\overleftarrow{\partial}_M - \vec{\partial}_M) = g^{MN}\epsilon_N^a\Gamma_a(\overleftarrow{\partial}_M - \vec{\partial}_M),$$
$$\epsilon_N^a = \frac{R}{z}\delta_N^a. \qquad (10)$$

Using the axial gauge $B_z(x,r,T) = 0$ we expand the fermion AdS field into left- and right-chirality components:

$$B_{N_1\ldots N_J}(x,r,T) = B^R_{\mu_1\ldots\mu_J}(x,r,T) + B^L_{\mu_1\ldots\mu_J}(x,r,T),$$
$$B^{L/R} = \frac{1\mp\gamma^5}{2}B, \qquad \gamma^5 B^{L/R} = \mp B^{L/R}. \tag{11}$$

Then we perform a Kaluza-Klein expansion for the four-dimensional transverse components of the AdS fields

$$B^{L/R}_{\mu_1\ldots\mu_J}(x,r,T) = \sum_n B^{L/R}_{n,\mu_1\ldots\mu_J}(x)\Phi^{L/R}_{nJ}(r,T), \tag{12}$$

where n is the radial quantum number and $B^n_{\mu_1\ldots\mu_J,n}(x)$ is the tower of the Kaluza-Klein (KK) modes, dual to hadrons with half-integer spin J. $\Phi^{L/R}_{nJ}(r,T)$ are their extradimensional profiles (wave functions) depending on the temperature. After substituting $\Phi^{L/R}_{nJ}(r,T) = e^{A(r)(J-2)}\phi^{L/R}_{nJ}(r,T)$ in the rest frame of the AdS field with $\vec{p}=0$, one can derive the Schrödinger-type equations of motion [22] for $\phi^{L/R}_{nJ}(r,T)$

$$[-\partial_z^2 + U_{L/R}(r,T)]\phi^{L/R}_{nJ}(r,T) = M_n^2(T)\phi^{L/R}_{nJ}(r,T). \tag{13}$$

$U_{L/R}(r,T)$ is the effective potential at finite temperature for the left/right bulk profile $\phi^{L/R}_{nJ}(r,T)$. It can be decomposed into a zero temperature term $U_{L/R}(r)\equiv U_{L/R}(r,0)$ and a temperature dependent term $\Delta U_{L/R}(r,T)$

$$U_{L/R}(r,T) = U_{L/R}(r) + \Delta U_{L/R}(r,T),$$
$$U_{L/R}(r) = \kappa^4 r^2 + 2\kappa^2\left(m\mp\frac{1}{2}\right) + \frac{m(m\pm1)}{r^2},$$
$$\Delta U_{L/R}(r,T) = 2\rho_T\kappa^2\left(\kappa^2r^2 + m\mp\frac{1}{2}\right), \tag{14}$$

where $m = N+L-3/2 = \tau - 3/2$.

Note that Eq. (13) is solved using the boundary conditions for the modes $\phi^{L/R}_{mJ}(r,T)$ in the ultraviolet (UV) and infrared (IR) limits:

$$\phi^{L/R}_{nJ}(r,T) \sim r^{N+L-1\pm1/2} \text{ at small } r,$$
$$\phi^{L/R}_{nJ}(r,T) \to 0 \text{ at large } r. \tag{15}$$

Also the normalizable modes $\Phi_{nJ}(r,T)$ and $\phi_{nJ}(r,T)$ obey the following normalization conditions:

$$\int_0^\infty dr\, e^{2A(r)(2-J)}\Phi^{L/R}_{mJ}(r,T)\Phi^{L/R}_{nJ}(r,T)$$
$$= \int_0^\infty dr\phi^{L/R}_{mJ}(r,T)\phi^{L/R}_{nJ}(r,T) = \delta_{mn}. \tag{16}$$

At both zero temperature $T=0$ and finite temperature the Schrödinger-type equations of motion (EOMs) have analytical solutions. At $T=0$ the wave function

$$\phi^{L/R}_{nJ}(r,0) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m_{L/R}+1)}}\kappa^{m_{L/R}+1}r^{m_{L/R}+1/2}$$
$$\times\, e^{-\kappa^2r^2/2}L_n^{m_{L/R}}(\kappa^2r^2),$$
$$m_{L/R} = m\pm\frac{1}{2} \tag{17}$$

corresponds to the mass spectrum

$$M^2_{nJ}(0) = 4\kappa^2\left(n+m+\frac{1}{2}\right). \tag{18}$$

Note that $m_L = \tau - 1$ and $m_R = \tau - 2$.

At finite T the solution reads

$$\phi^{L/R}_{nJ}(r,T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m_{L/R}+1)}}K_T^{m_{L/R}+1}r^{m_{L/R}+1/2}$$
$$\times\, e^{-K_T^2r^2/2}L_n^{m_{L/R}}(K_T^2r^2), \tag{19}$$

which corresponds to the mass spectrum

$$M^2_{nJ}(T) = 4K_T^2\left(n+m+\frac{1}{2}\right) = 4\kappa^2(1+\rho_T)\left(n+m+\frac{1}{2}\right). \tag{20}$$

Here

$$L_n^m(x) = \frac{x^{-m}e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^{m+n}) \tag{21}$$

are the generalized Laguerre polynomials. The above formulas are degenerate for any value of half-integer J and are valid for fermionic hadrons composed of N constituents, angular orbital momentum L, and radial quantum number n.

B. Form factors of fermionic hadrons at low temperatures

In this section, following our study in Refs. [14,22], we proceed to results for the form factors of hadrons with half-integer spin (baryons, pentaquarks, etc.) at low temperature. The corresponding form factors are induced by the coupling of AdS fields dual to hadrons with external vector AdS fields dual to the electromagnetic field. First, we calculate the vector bulk-to-boundary propagator at low temperatures, using the universal action derived in Eq. (8). The corresponding EOM for the Fourier transform of the bulk-to-boundary propagator $V(Q,r,T)$, in Euclidean metric $Q^2=-q^2$, reads:

$$\partial_r\left(\frac{e^{-\varphi_T(r)}}{r}\partial_r V(Q,r,T)\right) - Q^2\frac{e^{-\varphi_T(r)}}{r}V(Q,r,T) = 0. \tag{22}$$

This EOM is solved using the boundary conditions for the mode $V(Q,r,T)$ in the ultraviolet (UV) and infrared (IR) limits:

$$V(Q,r,T)=1 \quad \text{at}\ r=0, \qquad V(Q,r,T)\to 0 \ \text{at large}\ r. \tag{23}$$

EOM (22) is similar to the EOM for the case of zero temperature, and the only difference is that the temperature dependence is absorbed in the T-dependence of the dilaton parameter. Therefore, the solution for the bulk-to-boundary propagator at small temperature is straightforward [28]:

$$\begin{aligned} V(Q,r,T) &= \Gamma(1+a_T)U(a_T,0,K_T^2 r^2) \\ &= K_T^2 r^2 \int_0^1 \frac{dx}{(1-x)^2} x^{a_T} e^{-K_T^2 r^2 \frac{x}{1-x}}, \qquad a_T=\frac{Q^2}{4K_T^2}, \end{aligned} \tag{24}$$

where $\Gamma(n)$ and $U(x,y,z)$ are the gamma and Tricomi functions, respectively. Now we can calculate the form factor $F_{nJ}(Q^2,T)$, depending on the Euclidean momentum squared Q^2, for fermionic hadrons with quantum numbers (n, J, L) and number of constituents N at low temperature. The master formula is [22]

$$F_{nJ}^{L/R}(Q^2,T) = \int_0^\infty dr (\phi_{nJ}^{L/R}(r,T))^2 V(Q,r,T). \tag{25}$$

Note that at finite temperature the form factor $F_{nJ}(Q^2,T)$ is properly normalized with $F_{nJ}(0,T)=1$, because of $V(0,r,T)=1$ and $\int_0^\infty dr(\phi_{nJ}^{L/R}(r,T))^2=1$. The form factor $F_{nJ}^{L/R}(Q^2,T)$ has the correct power scaling at large Q^2 consistent with quark counting rules: it is independent of the quantum numbers n and J, but depends on the number of constituents N and the orbital angular momentum L:

$$F_{nJ}^{L/R}(Q^2) \sim \frac{1}{(Q^2)^{m_{L/R}+1}} \tag{26}$$

or

$$F_{nJ}^{R}(Q^2) \sim \frac{1}{(Q^2)^{\tau-1}}, \qquad F_{nJ}^{L}(Q^2) \sim \frac{1}{(Q^2)^{\tau}}. \tag{27}$$

Using Eqs. (17) and (24) we get for the ground state $n=0$ fermion

$$F_{0J}^{L/R}(Q^2,T) = \frac{\Gamma(a_T+1)\Gamma(m_{L/R}+2)}{\Gamma(a_T+m_{L/R}+2)}. \tag{28}$$

Results for radial excitations with any value for n are readily obtained. For example, for the first two radial excitations $n=1$ and $n=2$ the form factors are

$$\begin{aligned} F_{1J}(Q^2,T) &= \frac{\Gamma(a_T+1)\Gamma(m_{L/R}+4)}{\Gamma(a_T+m_{L/R}+4)} \\ &\quad + a_T(m_{L/R}+1)\frac{\Gamma(a_T+2)\Gamma(m_{L/R}+2)}{\Gamma(a_T+m_{L/R}+4)}, \end{aligned} \tag{29}$$

$$\begin{aligned} F_{2J}(Q^2,T) &= \frac{\Gamma(a_T+1)\Gamma(m_{L/R}+6)}{\Gamma(a_T+m_{L/R}+6)} \\ &\quad + a_T \frac{\Gamma(a_T+2)\Gamma(m_{L/R}+3)}{\Gamma(a_T+m_{L/R}+6)} \\ &\quad \times \bigg[(m_{L/R}+5)(2m_{L/R}+3) \\ &\quad + \frac{1}{2}(m_{L/R}+1)a_T(a_T+5)\bigg]. \end{aligned} \tag{30}$$

Next we can perform a small T-expansion of the form factors. For the ground state form factor we get

$$\begin{aligned} F_{0J}(Q^2,T) &= F_{0J}(Q^2,0) + \Delta F_{0J}(Q^2,T), \\ F_{0J}(Q^2,0) &= \frac{\Gamma(a+1)\Gamma(m_{L/R}+2)}{\Gamma(a+m_{L/R}+2)}, \\ \Delta F_{0J}(Q^2,T) &= \rho_T a \frac{\Gamma(a+1)\Gamma(m_{L/R}+2)}{\Gamma(a+m_{L/R}+2)} \\ &\quad \times [\psi(a+m_{L/R}+2)-\psi(a+1)], \end{aligned} \tag{31}$$

where $a=Q^2/(4\kappa^2)$ and $\psi(n)=\Gamma'(n)/\Gamma(n)$ is the polygamma function. Note that all results for the fermionic hadron form factors follow from the results for mesonic hadrons [14] with the substitution $m \to m_{L/R}$.

Note that the analytical formulas for masses and form factors of hadronic states with half-integer spin and adjustable quantum numbers n and L at finite temperature are derived in present manuscript for the first time in literature. Analysis of these quantities at zero temperature started in Ref. [31], where originally the soft-wall AdS/QCD action for the nucleon was proposed. It included a term describing the nucleon confining dynamics and the electromagnetic field, and their minimal and nonminimal couplings. Later, in Ref. [32], this action was used for the calculation of generalized parton distributions of the nucleon. In Ref. [33] it was extended to take into account higher Fock states in the nucleon and additional couplings with the electromagnetic field in consistency with QCD constituent counting rules [34] for the power scaling of hadronic form factors at large values of the momentum transfer squared in the Euclidean region. In Ref. [22] soft-wall AdS/QCD was developed for the description of baryons with adjustable quantum numbers n, J, L, and S. In another development, in Refs. [29,35,36], the nucleon properties have been analyzed using a Hamiltonian formalism. However, their calculation of the nucleon electromagnetic properties ignored the contribution of the non-minimal coupling to the Dirac form factors, and therefore, the analysis done in Refs. [29–36], is in our opinion not fully consistent. In Ref. [35] the ideas of Ref. [29] have been extended by the inclusion of higher Fock states in the nucleon, in order to calculate nucleon electromagnetic form factors in light-front holographic QCD. In this paper the Pauli form factor is again introduced by hand, using the overlap of the $L = 0$ and $L = 1$ nucleon wave function. Additionally, the expression for the neutron Dirac form factor has been multiplied by hand by a free parameter r. Recently, in Ref. [37] a version of the soft-wall AdS/QCD approach with the presence of a modified warp factor in the metric tensor, was proposed. Notice that in Ref. [22] we proved that any modification of the warp factor in the metric tensor can be compensated by an appropriate choice of the holographic potential. The form of such potentials for AdS field with different spins were also analytically derived in [22].

III. HADRONIC LIGHT-FRONT WAVE FUNCTION AT FINITE TEMPERATURE

Here we discuss the derivation of a new quantity—$\psi_M(x, \mathbf{k}_\perp, T) \equiv \psi_M(x, \mathbf{k}_\perp; \mu_0)$, the hadron light-front wave function at finite temperature and initial scale μ_0, based on the matching of form factors obtained in our approach $F_M(Q^2, T)$ and the DYW formula [25] for the hadronic form factors in light-front QCD:

$$F_M(Q^2, T) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi^\dagger_M(x, \mathbf{k}'_\perp, T)\psi_M(x, \mathbf{k}_\perp, T), \tag{32}$$

where $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1 - x)\mathbf{q}_\perp$ and $Q^2 = \mathbf{q}^2_\perp$. Here M is the quantum number related to twist τ (number of partons N and orbital angular momentum L) as

$$M = m = N + L - 2 = \tau - 2 \tag{33}$$

for bosonic hadrons, and

$$\begin{aligned} M &= m_L = m + 1/2 = N + L - 1 = \tau - 1, \\ M &= m_R = m - 1/2 = N + L - 2 = \tau - 2 \end{aligned} \tag{34}$$

for left- and right-handed fermionic hadrons, respectively.

For simplicity we consider the case of ground state mesons and baryons with $n = 0$. As a result of the matching we derive the following unified expression for the LFWF for bosonic and fermionic hadrons at the initial scale μ_0

$$\begin{aligned} \psi_M(x, \mathbf{k}_\perp, T) = N_M \frac{4\pi}{K_T} \sqrt{\log(1/x)}(1 - x)^{\frac{M-2}{2}} \\ \times \exp\left[-\frac{\mathbf{k}^2_\perp}{2K^2_T} \frac{\log(1/x)}{(1 - x)^2}\right] \end{aligned} \tag{35}$$

where

$$N_M = \sqrt{M + 1}. \tag{36}$$

Note that the dilaton K_T and function $\psi_M(x, \mathbf{k}_\perp, T)$ vanish at the critical temperature T_c [14]:

$$\frac{T^2_c}{12F^2} = N_f \left[\sqrt{\frac{N^2_f + 1}{N^2_f - 1} - 2\beta + \beta^2} - 1 + \beta\right], \tag{37}$$

where

$$\beta = \frac{9\alpha\pi^2}{16} \frac{N_f}{N^2_f - 1} \tag{38}$$

and N_f is the number of quark flavors.

Note the hadron light-front wave function is the basic block for the definition of matrix elements and hadronic properties in the light-front QCD. Using T-dependent light-front wave function one can calculate different parton distributions in hadrons, form factors, and structure functions (see for details, e.g., Refs. [20,29,30].

IV. SUMMARY

We have extended a soft-wall AdS/QCD model at small temperatures proposed in Ref. [14] for the description of bosonic hadrons, to fermionic hadrons (baryons, pentaquarks, etc.). The approach implements important features of QCD at zero and low temperatures: (1) the dilaton field, responsible for spontaneous breaking of chiral and conformal symmetry, which plays an important role in the temperature dependence of hadronic properties, (2) the T-dependence, which coincides with the quark condensate dependence in QCD. We present analytical results for the temperature dependence of both the mass spectrum and form factors of fermionic hadrons.

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