Veto Players and Equilibrium Uniqueness in the Baron-Ferejohn Model*

Levent Celik† Bilgehan Karabay‡

October 25, 2015

Abstract

In political economy, the seminal contribution of the Baron-Ferejohn bargaining model constitutes an important milestone for the study of legislative policy-making. In this paper, we analyze a particular equilibrium characteristic of this model, equilibrium uniqueness. The Baron-Ferejohn model yields a class of payoff-unique stationary subgame perfect equilibria (SSPE) in which players' equilibrium strategies are not uniquely determined. We first provide a formal proof of the multiplicity of equilibrium strategies. This also enables us to establish some important properties of SSPE. We then introduce veto players into the original Baron-Ferejohn model. We state the conditions under which the new model has a unique SSPE not only in terms of payoffs but also in terms of players' equilibrium strategies.

Keywords: Multilateral bargaining; equilibrium uniqueness; veto players.
JEL classification codes: C72, C78.

*We would like to thank the editor, three anonymous referees, Sahin Avcioglu, Hulya Eraslan, John Hillas, John McLaren, David Primo, Matthew Ryan and Avner Shaked for many helpful comments. We are also grateful to the University of Virginia for their hospitality during authors' visits. We acknowledge the research support provided by the Czech Science Foundation (project ref. # GAČR P402/12/0666). All errors are our own.

†National Research University Higher School of Economics, Myasnitskaya 20, 101000, Moscow, Russia; and CERGE-EI (a joint workplace of Charles University and the Economics Institute of the Academy of Sciences of the Czech Republic), Prague, Czech Republic. E-mail: lcelik@hse.ru. URL: https://sites.google.com/site/celiklev.

‡School of Economics, Finance and Marketing, RMIT University, 445 Swanston Street, Bld. 80, Level 11, Melbourne, VIC 3000, Australia. E-mail: bilgehan.karabay@gmail.com. URL: http://bilgehan.karabay.googlepages.com.

1 Introduction

Institutional procedures play a vital role in legislative decision-making. In this regard, the Baron-Ferejohn (1989) closed-rule divide-the-dollar game is one of the most widely used legislative bargaining models to study distributive politics and government policy-making.[1] It has some methodological advantages over the models used in social choice theory. For instance, since it utilizes non-cooperative game theory, equilibrium existence is guaranteed even if the core is empty. In fact, in the game we analyze here, where a fixed surplus is divided under majority rule, the core is indeed empty. This feature of the Baron-Ferejohn model makes it very convenient to study various institutional aspects of political economy.

We believe that it is important to know as much as possible about a workhorse model like Baron and Ferejohn's. In this study, we focus on a particular feature of this model, namely the multiplicity of equilibrium strategies. Baron and Ferejohn (1989) show that any outcome (meaning any division of the dollar) can be supported as a subgame perfect equilibrium (SPE) using infinitely nested punishment strategies as long as there are at least five players and the discount factor is sufficiently high. They then restrict attention to stationary subgame perfect equilibria (SSPE) in which the continuation payoffs for all structurally equivalent subgames are identical. This restriction allows them to obtain a unique payoff-equivalent equilibrium.[2] However, even with SSPE, *equilibrium strategies cannot be uniquely determined*.

The key elements of this multiplicity problem can be highlighted with the following example. Consider a country with a parliamentary system that needs to select its new government with majority rule. None of the political parties can achieve a majority on its own, and therefore, they need to form a coalition government.[3] Now suppose the head of state appoints a random party the right to form a coalition.[4] If we adapt the Baron-

[1] See Snyder, Ting and Ansolabehere (2005) and the references therein for further examples (especially footnote 6 on page 5). In addition, see Bowen (2014) and Celik, Karabay and McLaren (2013) for its recent use in trade policy.

[2] Eraslan (2002) proves uniqueness in SSPE payoffs.

[3] Countries generally differ in procedures they follow in designing inter-party bargaining over a new government (see Diermeier and van Roozendaal, 1998).

[4] The selection of a formateur also differs from one country to another. In some countries it is the head of states (a monarch or an elected president) who appoints the formateur, in others it is an informateur (a

Ferejohn model to this example, it predicts that the proposer will form a minimum winning coalition, that the first proposal will be accepted, and that the set of equilibrium payoffs for the parties involved can be uniquely calculated. However, it does not tell us the composition of the winning coalition. In fact, there are multiple mixed strategy equilibria each of which involves a different set of parties as coalition partners selected with different probabilities.[5] The multiplicity of SSPE arises from the flexibility to choose the randomization probabilities with which proposers select coalition members. It is possible to show the multiplicity of equilibrium strategies via examples. One case in hand is provided in footnote 16 of Baron and Ferejohn (1989). However, *no formal proof has been provided.* In that paper as well as the ones that follow, the main emphasis is on equilibrium payoffs. Our aim in providing a formal proof is to shift the focus to equilibrium strategies.

In this paper, *we first provide a formal proof of the multiplicity of equilibrium strategies for a general n-player symmetric Baron-Ferejohn game* (Proposition 1).[6] This proof, while useful on its own, also enables us to establish three important properties that SSPE must satisfy (Lemmas 1-3). We believe these are very useful results for any applied theorist using the Baron-Ferejohn model. We also provide, for expositional purposes, an example with three players that highlights some of the properties and restrictions SSPE have to satisfy.

One of the reasons the original Baron-Ferejohn game allows a very large set of mixed strategy SSPE is the *ex ante* symmetry of the players. Important asymmetries may create incentives for players to choose some of the coalition members with pure strategies. One example is when some of the players have veto power. Veto players are decision-makers whose agreement is required to adopt a new policy. Since every elected proposer must make each veto player a member of the winning coalition, the set of equilibrium strategies substantially shrinks when there are veto players. In Section 3 of our paper, we investigate the effects of introducing veto players on equilibrium multiplicity in the Baron-Ferejohn model. We provide the conditions under which the game has a unique SSPE in terms of not

senior, experienced, 'elder statesman').

[5]The formation of coalition governments can be quite uncertain with respect to which party will be included in the coalition (see Müller and Strøm, 2000; and Laver and Schofield, 1998).

[6]A similar exercise for the open-rule version of the Baron-Ferejohn game is done by Primo (2007).

only payoffs but also players' equilibrium strategies. Agents with veto power exist and play an important role in decision-making processes. In this respect, we believe that analyzing the effect of veto players on equilibrium characteristics of the Baron-Ferejohn model is valuable. Throughout the analysis, we use the coalition government example we introduced above to highlight our results. All formal proofs for the general n-player game are relegated to the appendix.

2 The Baron-Ferejohn game

A q-quota rule symmetric Baron-Ferejohn game is an infinite-horizon sequential multilateral bargaining game with the following structure. Let $N = \{1, 2, ..., n\}$ denote the set of players ($n \geqslant 3$ and odd). At the beginning of the game, one of the n players is randomly selected (equivalently, recognized) with equal probability $\frac{1}{n}$ to make a proposal for the division of \$1. Let $X = \{x \in \mathbb{R}^n_+ : x_i \geqslant 0 \text{ and } \sum_{i=1}^{n} x_i \leq 1\}$ denote the set of feasible allocations, where x_i is the share player i receives. In addition, denote u_i as the utility of player i and assume that utility is linear in money such that $u_i = x_i$, $\forall i$.

Once a proposal is made, each player simultaneously votes and if the proposal $x \in X$ receives q votes, $1 < q < n$, it is accepted and the game ends.[7] Otherwise, the game proceeds to the next period in which another player (possibly the same) is randomly selected to make a proposal. This process continues until an agreement is reached. If no agreement is ever reached, each player receives a zero payoff. When voting on a proposal, players compare their current payoff with the alternative of continuing to the next period.[8] All players discount the future at a common rate of $\delta \leq 1$.

We are now ready to describe the solution concept. Let H_t be the history of the game that contains identity of the proposers, proposals that have been put forward and actions taken up to period t. A feasible action $a^i_t(H_t)$ for player i in period t can be described as follows

[7] When the voting rule is unanimous, i.e., when $q = n$, the Baron-Ferejohn game has a unique SSPE. Since this is obvious, we assume $q < n$ for the rest of the analysis.

[8] To eliminate unreasonable equilibria, weakly dominated strategies are ruled out.

$$a_t^i\left(H_t\right) \in \begin{cases} X \text{ and } \{accept,\ reject\} & \text{if player } i \text{ is the proposer,} \\ \{accept,\ reject\} & \text{otherwise.} \end{cases}$$

A strategy s^i for player i is a sequence of actions $\{a_t^i\left(H_t\right)\}_{t=1}^{\infty}$, and a strategy profile s is an n-tuple of strategies $(s^1, s^2, ..., s^n)$, one for each player. We restrict our attention to stationary subgame perfect equilibria (SSPE). A strategy profile s is subgame perfect if and only if unilateral deviations from s at a single stage are not beneficial (it satisfies one-stage deviation condition, see p.110 in Fudenberg and Tirole) and is stationary if it is time and history independent. A strategy profile is stationary subgame perfect if it is stationary and subgame perfect. Intuitively, in a stationary equilibrium, a player who is recognized to make a proposal in any two different periods behaves the same way in both sessions (in the case of a mixed-strategy equilibrium, this generally means choosing the same probability distribution over offers).[9]

In the following theorem, we rephrase Theorem 1 of Eraslan (2002), which characterizes the set of SSPE.

Theorem 1 *Let player $i \in N$ denote the proposer and x_j^i represent the share she allocates to the j^{th} player. Then, the set of SSPE can be described as follows:*

$$a^i = \left(x_i^i, x_j^i\right) \in X \textit{ with probability } g^i(\psi^i),$$
$$a^j = accept \textit{ iff } x_j^i \geqslant \delta V_j \textit{ for all } j \neq i,$$

where

$$x_i^i = 1 - \sum_{j \neq i}^{n} \delta V_j \psi^{ij},$$
$$x_j^i = \delta V_j \psi^{ij}, \textit{ for all } j \neq i,$$

where V_j represents the equilibrium continuation payoff of player j and is given by

$$V_j = \frac{1}{n}\left(1 - \sum_{k \neq j}^{n} \delta V_k \sum_{\psi^j \in C^j} \psi^{jk} g^j(\psi^j) + \sum_{k \neq j}^{n} \delta V_j \sum_{\psi^k \in C^k} \psi^{kj} g^k(\psi^k)\right). \tag{1}$$

[9] Baron and Kalai (1993) argue that stationarity is an attractive restriction since it is the "simplest" equilibrium such that it requires the fewest computations by agents.

In the above expressions, $\psi^i = (\psi^{i1}, ..., \psi^{ii-1}, \psi^{ii+1}, ..., \psi^{in})' \in C^i$ *is an* $(n-1)$*-dimensional vector (excluding* ψ^{ii}*) of ones and zeros,* $g^i(\psi^i)$ *is the probability distribution of coalitions that player* i *can form and* C^i *is the set of* $(n-1)$*-dimensional vectors that solve the following program:*

$$\min_{(\theta^{ij})_{j\neq i}} \sum_{j\neq i}^{n} \theta^{ij}\delta V_j, \text{ subject to}$$

$$\sum_{j\neq i}^{n} \theta^{ij} = q-1 \text{ and}$$

$$\theta^{ij} \in \{0,1\}.$$

Proof. The proof is provided in Eraslan (2002) and thus omitted.

Notice that in an SSPE, a proposer makes an allocation of $1 such that it will be accepted by exactly $q-1$ other players besides herself since she wants to maximize her own share of the dollar. In other words, she offers a positive share to only $q-1$ other players, thereby forming a minimum winning coalition. As a result, we can interpret those players who receive a positive share as coalition partners. To formalize this idea, consider the following. For any $i \neq j$, let p^{ij} represent the probability that i^{th} player, as a proposer, gives j^{th} player its discounted continuation payoff

$$p^{ij} = \sum_{\psi^i \in C^i} \psi^{ij} g^i(\psi^i). \tag{2}$$

In other words, p^{ij} represents the probability that player i includes player j in the winning coalition.[10] Given the randomization probabilities p^{ij}, it is possible rewrite equation (1) as

$$V_j = \frac{1}{n}\left(1 - \sum_{k\neq j}^{n} p^{jk}\delta V_k + \sum_{k\neq j}^{n} p^{kj}\delta V_j\right) \tag{3}$$

[10] An example may be helpful. Consider a 5-player game with $q = 3$, and assume that player 1 is the proposer. There are 6 possible coalitions that player 1 may form: $\psi_1^1 = (1,1,0,0)$, $\psi_2^1 = (1,0,1,0)$, $\psi_3^1 = (1,0,0,1)$, $\psi_4^1 = (0,1,1,0)$, $\psi_5^1 = (0,1,0,1)$ and $\psi_6^1 = (0,0,1,1)$ with corresponding probabilities g_i^1 for $i = 1, ..., 6$ and $\sum_{i=1}^{6} g_i^1 = 1$. Hence, we have: $p^{12} = g_1^1 + g_2^1 + g_3^1$, $p^{13} = g_1^1 + g_4^1 + g_5^1$, $p^{14} = g_2^1 + g_4^1 + g_6^1$ and $p^{15} = g_3^1 + g_5^1 + g_6^1$.

Thus, δV_j denotes the payoff the j^{th} player expects to get if she votes no to the current proposal, and the bargaining is carried over to the next period. We establish a useful property of continuation payoffs in our first lemma.

Lemma 1 $\sum_{j=1}^{n} V_j = 1$, *where* V_j *is as given in equation (1).*

Proof. See the Appendix.

In other words, the equilibrium continuation payoffs of the players must add up to the total size of the surplus to be shared. As a result, Lemma 1 makes clear that there is no waste. Moreover, given the symmetry of the players, we can state our second lemma.

Lemma 2 *Given that all players are symmetric, their equilibrium continuation payoffs must be equal, i.e.,* $V_1 = V_2 = ... = V_n = \frac{1}{n}$.

Proof. See the Appendix.

Therefore, as long as there is symmetry among players, each player's expected share is equal. In the next lemma, we show that the equilibrium strategies are balanced (see Baron and Kalai, 1993); i.e., all players have an equal probability of being included in minimum winning coalitions when added up over all proposing players.

Lemma 3 *In every SSPE, the probability that player* j *is included in a winning coalition is given by* $\frac{1}{n}(1 + \sum_{i \neq j}^{n} p^{ij}) = \frac{q}{n}$. *In other words, SSPE strategies are balanced.*

Proof. See the Appendix.

Given Lemmas 1-3, we can state our first main result.

Proposition 1 *The set of randomization probabilities* $\{p^{ij}\}$ *in a* q*-quota rule symmetric Baron-Ferejohn game is not singleton.*

Proof. See the Appendix.

One solution to the symmetric Baron-Ferejohn game, for instance, has all players choosing each possible minimum winning coalition with an equal probability (this in turn implies that $p^{ij} = \frac{q-1}{n-1}$). Another solution involves, if we imagine players placed around a circle, each player choosing the $q-1$ players on her right with pure strategy.

To make the exposition clear and highlight some of the above results, focus on a 3-player game with $q = 2$ (i.e., three-player simple majority rule game). Recall the example we considered in the introduction about a country with a parliamentary system. Assume now that there are three political parties (players) with equal number of seats in the parliament (thus no party has majority control). A coalition government needs to be formed and assume that in accordance with the number of seats they hold, each party has an equal chance to be the formateur.[11] To form a coalition government, the formateur party needs one other party's support and assume that in accordance with Riker's size principle (Riker, 1962), only minimum winning coalitions are formed, i.e., all-party coalitions do not occur.

Using equation (3), the continuation payoff of each party can be written as

$$\begin{aligned} V_1 &= \tfrac{1}{3}\left[\left(1 - p^{12}\delta V_2 - p^{13}\delta V_3\right) + \left(p^{21} + p^{31}\right)\delta V_1\right], \\ V_2 &= \tfrac{1}{3}\left[\left(1 - p^{21}\delta V_1 - p^{23}\delta V_3\right) + \left(p^{12} + p^{32}\right)\delta V_2\right], \\ V_3 &= \tfrac{1}{3}\left[\left(1 - p^{31}\delta V_1 - p^{32}\delta V_2\right) + \left(p^{13} + p^{23}\right)\delta V_3\right]. \end{aligned} \tag{4}$$

In light of Lemma 2, we have $V_1 = V_2 = V_3 = \frac{1}{3}$. Therefore, equations in (4) become

$$\begin{aligned} p^{12} + p^{13} &= p^{21} + p^{31}, \\ p^{21} + p^{23} &= p^{12} + p^{32}, \\ p^{31} + p^{32} &= p^{13} + p^{23}. \end{aligned} \tag{5}$$

[11] One key uncertainty about coalition government formation can be the designation of a formateur. Diermeier and Merlo (2004) analyze formateur selection process for 11 parliamentary democracies over the period 1945-1997. They conclude that the data supports the proportional selection, where formateurs are selected randomly proportional to the distribution of seat shares in the parliament as suggested by Baron-Ferejohn model.

Furthermore, given that each proposer needs one other vote to obtain majority support, we must have

$$p^{12} + p^{13} = p^{21} + p^{23} = p^{31} + p^{32} = 1. \tag{6}$$

Equations (5) and (6) imply that $p^{12} = p^{23} = p^{31}$.[12] As a result, randomization probabilities are not uniquely determined. Put another way, the probabilities that a given party will be chosen as a coalition partner are not unique. However, this is not to say that any configuration of randomization probabilities is consistent with SSPE behavior. In particular, in this 3-player example, if the 1^{st} player elects the 2^{nd} player $k\%$ of the time in a given SSPE, then the 2^{nd} player must be electing the 3^{rd} player, and the 3^{rd} player must be electing the 1^{st} player $k\%$ of the time as well (where the choice of k is unrestricted, $k \in [0, 100]$).

3 Baron-Ferejohn game with veto players

There is a large literature on veto players in political science (for examples, see Tsebelis, 2002). Most of the work in this literature analyzes the relationship between policy stability and veto players, i.e., how policy stability is affected by the number and ideological differences of veto players. According to Tsebelis (2002, p.34), all political institutions including parliaments, party systems, regime types can be translated into veto player framework. In this sense, veto players play an important role in policy-making. In general, we can speak of two types of veto players: individual (the U.S. president, permanent members of the U.N. security council) or collective (the House and the Senate in the U.S.). Individual veto players can block the adoption of a new policy unilaterally whereas collective veto players can block the new policy if all of them agree to do so. For example, each permanent member of the U.N. Security Council can unilaterally prevent adoption of a proposal, even if it has received the required number of votes. Over the history of the U.N., the veto has been used many times. In fact, not only its use but also the possibility of its use can affect U.N. actions (for example, the veto threat by France for the second U.N. Security Council resolution on Iraq).

[12] This can also be seen from combining equation (5) with Lemma 3.

In the context of Baron-Ferejohn framework, there are other papers that employ veto power. Winter (1996) examines the change in veto players' power (payoffs) with respect to a change in (i) the negotiation length (deadline) and (ii) the number of non-veto players. Primo (2006) studies spending limits and executive veto authority, and shows that while imposing a cap on spending is welfare improving, the effect of a veto on spending depends on the presence of a cap as well as the ideology of the executive. Nunnari (2012) analyzes the effect of veto power in a game with an endogenous *status quo* policy. In his model, unlike the original Baron-Ferejohn model, when a proposal is accepted the game does not end, rather a new committee member is randomly recognized to propose a new division of the dollar. He shows the existence of Markow equilibrium of this game and finds that the policy eventually converges to the one where the veto player obtains the whole dollar. Celik, Karabay and McLaren (2015) examine the effect of veto power in the context of trade-policy determination. However, none of these papers focus on the relationship between the existence of veto players and multiplicity of equilibrium strategies.

Our paper is also close in spirit to McCarty (2000a) and (2000b). Those papers, as in ours, also indicate that the existence of veto power ties down the set of equilibrium strategies. However, there are a few differences between our paper and his papers. First and foremost, unlike ours, his focus is not on the multiplicity of equilibrium strategies. McCarty (2000a) analyzes the effect of executive veto on legislative spending, whereas McCarty (2000b) distinguishes between veto and proposal power. Second, we only consider absolute veto power, meaning that it is not possible to override a veto. As a result, the equilibrium coalition size is not different from the no-veto case as long as the number of veto players does not exceed the quota rule. In contrast, in McCarty (2000a) and (2000b), the existence of veto players often causes larger coalitions to form in equilibrium and the size of the coalition depends on the override rule. Our treatment of veto players can be considered as a special case of his papers with absolute veto power. We have followed this approach since our focus is specifically on the equilibrium multiplicity rather than the general effects of veto players, such as coalition size, payoffs, etc.

There are applications of veto players in other venues, too. Consider, for example, an indivisible asset to be traded. Assume that this asset is jointly owned by r individuals (sellers) and that there are $n-r$ potential buyers, where $1 \leq r < n$. Assume also that the sale of this asset will create surplus for both sides but how this surplus will be shared depends on the relative position of the sellers and buyers. In this example, the quota rule is $r+1$ and there are r veto players since the asset cannot be sold unless all of the sellers agree to it. As can be seen from these examples, veto power and veto players are quite relevant and vital part of any decision-making.

Our focus is the effect of veto players on a specific equilibrium characteristic of the Baron-Ferejohn game, equilibrium uniqueness. Let us describe the new game in more detail. Consider an n-player, q-quota rule symmetric Baron-Ferejohn game with r veto players such that $1 \leq r < q < n$.[13] The set of SSPE for this modified game can be described as in Theorem 1. The only difference is that in the minimization problem, there is an extra constraint such that $\theta^{ij} = 1$ if j is a veto player. In other words, any proposer must include all veto players in the winning coalition, so $p^{ij} = 1$ for any j who is a veto player. Now, consider the following lemma which is a modified version of Lemma 2.

Lemma 4 *The equilibrium continuation payoff of all veto players is the same,* $V_i^{veto} = V_j^{veto} = ... = V^{veto}$. *Moreover, the equilibrium continuation payoff of all non-veto players is the same and strictly smaller than the equilibrium continuation payoff of veto players,* $V_i = V_j = ... = V < V^{veto}$.

Proof. See the Appendix.

We can categorize players into two groups, veto and non-veto. Lemma 4 establishes that the continuation payoffs are identical within each group, and that the continuation payoff of veto players is strictly larger than that of non-veto players. The next lemma provides the necessary condition for equilibrium uniqueness.

[13] If $r \geqslant q$, then only veto players are included in any winning coalition and the problem becomes trivial since there is no need to choose any coalition partners. Of course, there is no multiplicity of SSPE in this case. Hence, to make the problem interesting, we assume $r < q$.

Lemma 5 *In a q-quota rule Baron-Ferejohn game with n players, all of whom have equal recognition probabilities and r of whom have veto power, a necessary condition to obtain a unique SSPE (not only in terms of payoffs but also in terms of players' equilibrium strategies) is to have $r = q - 1$.*

Proof. See the Appendix.

We are now ready to state our second main result.

Proposition 2 *A q-quota rule Baron-Ferejohn game with n players, all of whom have equal recognition probabilities and r of whom have veto power, has a unique SSPE (not only in terms of payoffs but also in terms of players' equilibrium strategies) if and only if $q = 2$ and $r = 1$.*

Proof. See the Appendix.

Thus, when an agreement requires only two players' consent ($q = 2$), the existence of a single veto player gives us a unique solution not only in terms of payoffs but also in terms of strategies. This result is valid for any $n \geqslant 3$. In the context of the asset example mentioned earlier, if the asset is owned by a single seller who faces many potential buyers, then the resulting SSPE will be unique. This is true since the sale of the asset requires the consent of the seller and one of the potential buyers.

To highlight this result, we continue to use the coalition government example we considered before but this time we assume that one of the three parties has veto power, say party 1. One way to motivate this is the presence of strong ideological differences between parties. Assume that there are three parties with equal seats and also assume that two of these parties would not form a coalition with each other due to opposing ideological views.[14] This

[14]For example, after the recent elections of June 2015 in Turkey, Nationalist Movement Party (Turkish: Milliyetci Hareket Partisi (MHP)) announced that it will not be involved in any coalition government that includes People's Democratic Party (Turkish: Halkların Demokratik Partisi (HDP)); see http://www.hurriyet.com.tr/gundem/29306673.asp.

makes the remaining third party the veto player, which can form a coalition with others.[15] In addition, we continue to assume that only minimum winning coalitions occur.

Using equation (3) and noting that $p^{21} = p^{31} = 1$, the continuation payoff of each party can be written as

$$V_1^{veto} = \frac{1}{3}\left(1 - p^{12}\delta V_2 - p^{13}\delta V_3\right) + \frac{2}{3}\delta V_1^{veto}, \tag{7}$$

$$V_2 = \frac{1}{3}\left(1 - \delta V_1^{veto}\right) + \frac{1}{3}p^{12}\delta V_2, \tag{8}$$

$$V_3 = \frac{1}{3}\left(1 - \delta V_1^{veto}\right) + \frac{1}{3}p^{13}\delta V_3. \tag{9}$$

Using Lemma 4, we have $V_1^{veto} > V_2 = V_3$. Given that $V_2 = V_3$, equations (8) and (9) necessarily imply

$$p^{12} = p^{13}. \tag{10}$$

Equations (10) and (6) imply $p^{12} = p^{13} = \frac{1}{2}$. Moreover, solving equations (7), (8) and (9), we obtain

$$V_1^{veto} = \frac{2-\delta}{6-5\delta} \text{ and } V_2 = V_3 = \frac{2(1-\delta)}{6-5\delta}.$$

A straightforward comparison establishes that $V_1^{veto} > V_2 = V_3$.

Recall that the standard Baron-Ferejohn game does not generate a unique SSPE due to the multiplicity of equilibrium strategies, i.e., multiplicity of the randomization probabilities p^{ij}.[16] In this new game with one of parties having veto power, we reach a unique SSPE also in terms of equilibrium strategies since both p^{12} and p^{13} are uniquely determined.

[15] Other examples outside the coalition government context prevail as well. One such example is the amendment of Canadian Constitution, which is provided in Winter (1996). The British Parliament had the veto authority to overturn any proposal for the amendment of Canadian Constitution between the years 1867 and 1982. This veto power was changed in 1982 with another rule which required that the proposal for amendment must be supported at least two-thirds of the provinces in Canada and also that the supporting provinces must have 50% of the population. At that time, Ontario and Quebec together had more than 50% of the population. That means they together had a veto power without constituting a winning coalition. Another example is from finance, called "golden share". Golden share grants minority shareholders veto rights on certain issues in shareholders' meetings.

[16] Note that in the standard Baron-Ferejohn game, continuation values are uniquely determined.

When $q = 2$ and $r = 1$, each non-veto player is bound to choose the only veto player as a coalition partner with pure strategy. Moreover, since we need that non-veto players have equal continuation payoffs in any SSPE, the veto player, as a proposer, must equally randomize between the non-veto players. From the results obtained in the proof of Proposition 2, we can state the following.

Remark *When $q > 2$, the minimum number of veto players required to obtain a unique SSPE (not only in terms of payoffs but also in terms of players' equilibrium strategies) is $n - 1$, which is not viable given that $r < q < n$.*

Thus, when $q > 2$ there are many SSPE that differ in terms of randomization strategies. The underlying reason is similar to the one in the original Baron-Ferejohn model. We know from Lemma 5 that a necessary condition to obtain equilibrium uniqueness is $r = q - 1$. This automatically implies that for $q > 2$, we must have $r > 1$. But with two or more veto players, there is a flexibility with the choice of the randomization probabilities with which veto players select non-veto players as coalition partners. To see this, consider the following example. Suppose $n = 4$, $q = 3$ and $r = 2$, and order players such that $i = 1, 2$ are veto players and $i = 3, 4$ are non-veto players. Then, two possible SSPE (among others) are as follows: (1) when proposer, player 1 chooses players 2 and 3 with pure strategy, player 2 chooses players 1 and 4 with pure strategy, player 3 as well as player 4 chooses players 1 and 2 with pure strategy; (2) when proposer, player 1 chooses player 2 with pure strategy and randomizes equally between players 3 and 4, player 2 chooses player 1 with pure strategy and randomizes equally between players 3 and 4, player 3 as well as player 4 chooses players 1 and 2 with pure strategy. Both of these are legitimate SSPE because each non-veto player appears in a winning coalition with the same probability (i.e., strategies are balanced in both).

Having veto players introduces an *ad hoc* constraint in the bargaining game that changes the composition of randomization probabilities. Therefore, a one-to-one comparison between the non-veto and the veto games in terms of equilibrium multiplicity is difficult. One way to examine the effect of introducing veto players is to compare the number of equations with the number of unknowns. Without any veto players, there are $(n-1)n$ unknowns ($n-1$ possible randomizations for each of the n players) and $2n-1$ linearly independent equations ($n-1$ equations implied by the construction of continuation payoffs given in equation (3) and n equations implied by the property of balanced strategies – please see the proofs of Lemmas 2 and 3, and Proposition 1 for more details). With veto players, the number of unknowns decreases to $(n-1)(n-r)$ when $r < q-1$ ($n-r-1$ possible randomizations for each of the $n-r$ non-veto players, plus $n-r$ possible randomizations for each of the r veto players) and to $r(n-r)$ when $r = q-1$ ($n-r$ possible randomizations for each of the r veto players). Similarly, the number of equations decreases to $2n-1-r$ when $r < q-1$ and to $n-1$ when $r = q-1$ (again follows from the construction of continuation payoffs and the property of balanced strategies – please see the proofs of Lemmas 4 and 5, and Proposition 2 for more details).[17]

Another way to examine the effect of introducing veto players is to see how it narrows down the number of possible coalition formations. In a q-quota game with r veto players, each veto player's problem is to offer an acceptable payoff to each one of the remaining $r-1$ veto players, and to an additional $q-r$ players from the pool of $n-r$ non-veto players. Hence, the total number of possible coalitions that a veto player can form is $\frac{(n-r)!}{(q-r)!(n-q)!}$. Note that this is the same number of coalitions that could be formed by each player in a $(q-r+1)$-quota game with $n-r+1$ players and zero veto players. In contrast, if there were no veto players, then the total number of possible coalitions for each player would be $\frac{(n-1)!}{(q-1)!(n-q)!}$. Hence, introduction of veto power substantially lowers the number of possible coalitions each veto player can form (except for when $r = 1$). Similarly, for each non-veto player as the proposer, the main task is to decide which $q-r-1$ of the remaining non-veto players shall be included in the winning coalition, besides all veto players. This would generate $\frac{(n-r-1)!}{(q-r-1)!(n-q)!}$ possible coalitions, which is the same number of coalitions as in a $(q-r)$-quota game with $n-r$ players and no veto players. And once again, compared with $\frac{(n-1)!}{(q-1)!(n-q)!}$, each non-veto

[17] The discontinuity at $r = q-1$ is due to the fact that when $r = q-1$, non-veto players have no choice but form the winning coalition with veto players alone besides themselves.

player has a substantially smaller number of possible winning coalitions to form.

4 Conclusion

This paper first presents a formal proof of the multiplicity of equilibrium strategies in the original (symmetric) closed-rule Baron-Ferejohn game. In doing so, we also establish important properties that stationary subgame perfect equilibria must satisfy. We then analyze a new version of the game by introducing veto players. Agents with veto power exist and play an important role in decision-making processes. We show that when the quota rule is 2, the existence of a single veto player provides us with a unique equilibrium not only in terms of payoffs but also in terms of strategies. We highlight our results using a coalition government example, where two of the three political parties needs to form an alliance to establish the new government. We believe that these results will be of great interest to applied theorists using the Baron-Ferejohn model.

Appendix

Proof of Lemma 1. This can be seen analytically by summing equation (1) over j

$$\sum_{j=1}^{n} V_j = \sum_{j=1}^{n} \frac{1}{n} \left(1 - \sum_{k \neq j}^{n} p^{jk} \delta V_k + \sum_{k \neq j}^{n} p^{kj} \delta V_j \right),$$

Next, using equation (2), we obtain

$$\sum_{j=1}^{n} V_j = \sum_{j=1}^{n} \frac{1}{n} \left(1 - \sum_{k \neq j}^{n} \delta V_k \sum_{\psi^j \in C^j} \psi^{jk} g^j(\psi^j) + \sum_{k \neq j}^{n} \delta V_j \sum_{\psi^k \in C^k} \psi^{kj} g^k(\psi^k) \right), \text{ or}$$

$$\sum_{j=1}^{n} V_j = \sum_{j=1}^{n} \frac{1}{n} - \frac{1}{n} \left(\sum_{j=1}^{n} \sum_{k \neq j}^{n} \delta V_k \sum_{\psi^j \in C^j} \psi^{jk} g^j(\psi^j) - \sum_{j=1}^{n} \sum_{k \neq j}^{n} \delta V_j \sum_{\psi^k \in C^k} \psi^{kj} g^k(\psi^k) \right)$$

Note that

$$\sum_{j=1}^{n} \sum_{k \neq j}^{n} \delta V_k \sum_{\psi^j \in C^j} \psi^{jk} g^j(\psi^j) = \sum_{j=1}^{n} \sum_{k \neq j}^{n} \delta V_j \sum_{\psi^k \in C^k} \psi^{kj} g^k(\psi^k),$$

so we have

$$\sum_{j=1}^{n} V_j = \sum_{j=1}^{n} \frac{1}{n} = 1. \blacksquare$$

Proof of Lemma 2. Consider an n-player game with q-quota rule, where $1 < q < n$. In this game, in any winning coalition, there will be q players including the proposer. Without loss of generality, order the continuation values such that $V_1 \leq V_2 \leq ... \leq V_n$. First, suppose that $V_{n-1} < V_n$. By Lemma 1, this implies that $V_n > \frac{1}{n}$. Since player n has the highest continuation value, she is in a winning coalition only when she is the proposer. This means that $V_n = \frac{1}{n}(1 - \sum_{i=1}^{n-1} p^{ni}\delta V_i) \leq \frac{1}{n}$, since $V_i \geqslant 0$ for all i, a contradiction. Hence, $V_{n-1} = V_n$. We can continue in the same fashion until we reach player q (in other words, the last one we analyze is $V_q < V_{q+1} = ... = V_n$). This establishes that

$$V_q = V_{q+1} = ... = V_n. \tag{11}$$

After that, we continue as follows. Assume that $V_{q-1} < V_q$. This implies that $V_1 < \frac{1}{n}$. In this case, players 1, 2,... , $q-1$ are always in the winning coalition and they will be offered a payoff of δV_1, $\delta V_2, ...$ δV_{q-1} as a coalition partner. Thus, using equation (11), the first player's continuation payoff can be written as

$$V_1 = \frac{n-1}{n}\delta V_1 + \frac{1}{n}\left(1 - \sum_{j=2}^{q-1}\delta V_j - \delta V_q\right). \tag{12}$$

In addition, using Lemma 1 and equation (11), we obtain

$$\sum_{j=2}^{q-1} V_j = 1 - V_1 - (n-q+1)V_q. \tag{13}$$

Next, substituting equation (13) into equation (12), we get

$$V_1 = \frac{n-1}{n}\delta V_1 + \frac{1}{n}\left(1 - \delta\left(1 - V_1 - (n-q+1)V_q\right) - \delta V_q\right), \text{ or}$$

$$V_1 = \frac{n-1}{n}\delta V_1 + \frac{1}{n}\delta V_1 + \frac{1-\delta}{n} + \frac{n-q}{n}\delta V_q,$$

Simplifying the above equation gives us

$$V_1 = \frac{1}{n} + \frac{n-q}{n}\frac{\delta}{1-\delta}V_q.$$

Since $V_q \geqslant 0$, this is a contradiction to the claim that $V_1 < \frac{1}{n}$. Therefore, it must be the case that $V_{q-1} = V_q$. We can continue in the same fashion until we reach player 1 (meaning that the last one to check is $V_1 < V_2 = ...V_q$). This establishes the result. ∎

Proof of Lemma 3. In a q-quota symmetric Baron-Ferejohn game, the probability that player j is included in a wining coalition is given by $\frac{1}{n}(1+\sum_{i\neq j}^{n} p^{ij})$. In what follows, we will determine the value of $\sum_{i\neq j}^{n} p^{ij}$. Lemma 2 implies that each player is always offered the same share whenever she is in a winning coalition (except for when she is the proposer). Thus, using Lemma 1, Lemma 2 and equation (3), we obtain

$$V_j = \frac{1}{n}\left(1 - \sum_{i\neq j}^{n} p^{ji}\delta V_i + \sum_{i\neq j}^{n} p^{ij}\delta V_j\right), \text{ for all } j = 1, 2, ..., n, \text{ or}$$

$$\frac{1}{n} = \frac{1}{n}\left(1 - \delta\frac{1}{n}\sum_{i\neq j}^{n} p^{ji} + \delta\frac{1}{n}\sum_{i\neq j}^{n} p^{ij}\right),$$

which implies that

$$\sum_{i\neq j}^{n} p^{ij} = \sum_{i\neq j}^{n} p^{ji}, \text{ for all } i,\ j \text{ and } i \neq j. \tag{14}$$

Notice also that

$$\sum_{i\neq j}^{n} p^{ji} = \sum_{i\neq j}^{n}\sum_{\psi^j\in C^j}\psi^{ji}g^j(\psi^j) = \sum_{\psi^j\in C^j}\sum_{i\neq j}^{n}\psi^{ji}g^j(\psi^j) = \sum_{\psi^j\in C^j} g^j(\psi^j)\sum_{i\neq j}^{n}\psi^{ji}. \tag{15}$$

There is a total of $\frac{(n-1)!}{(q-1)!(n-q)!}$ possible coalitions the i^{th} player may form when she is the proposer. All of these possibilities occur with certain probabilities which add up to 1, i.e., $\sum_{\psi^j\in C^j} g^j(\psi^j) = 1$. In addition, we know by definition that $\sum_{i\neq j}^{n}\psi^{ji} = q-1$ (see Theorem 1). Hence, equation (15) becomes

$$\sum_{i\neq j}^{n} p^{ji} = q-1, \text{ for } j = 1, ..., n.^{18} \tag{16}$$

Using equation (14),

$$\sum_{i\neq j}^{n} p^{ij} = \sum_{i\neq j}^{n} p^{ji} = q-1, \text{ for } j = 1, ..., n.$$

[18] Consider our previous example given in footnote 10, where $n = 5$, $q = 3$, and player 1 is the proposer. Recall that: $p^{12} = g_1^1 + g_2^1 + g_3^1$, $p^{13} = g_1^1 + g_4^1 + g_5^1$, $p^{14} = g_2^1 + g_4^1 + g_6^1$ and $p^{15} = g_3^1 + g_5^1 + g_6^1$. This implies that $\sum_{j=2}^{5} p^{1j} = 2(g_1 + g_2 + g_3 + g_4 + g_5 + g_6) = 2$.

Hence,

$$\frac{1}{n}(1+\sum_{i\neq j}^{n}p^{ij})=\frac{1}{n}(1+q-1)=\frac{q}{n}. \blacksquare$$

Proof of Proposition 1. This result directly follows from Lemmas 1, 2 and 3. We already know from Lemma 2 that $V_i=\frac{1}{n}$, $\forall i$. Thus, what remains to be determined are $n(n-1)$ randomization probabilities ($n-1$ possible randomizations for each of the n players). We have $n-1$ linearly independent equations given by equation (14), and n equations given by equation (16). Since $n(n-1)>2n-1$ for any $n\geqslant 3$, the solution to randomization probabilities is not unique. $\blacksquare$

Proof of Lemma 4. First, note that Lemma 1 is still valid. Consider an n-player game with q-quota rule, where $q<n$. There are also r veto players, with $r<q$ for quota rule to be effective. In this game, in any winning coalition, there will be q players including the proposer. Without loss of generality, assume that players $1,2,...,r$ are veto players and players $r+1,r+2,...n$ are non-veto players. In addition, order the continuation values of non-veto players such that $V_{r+1}\leq V_{r+2}\leq ...\leq V_q\leq ...\leq V_n$. First, suppose that $V_n>V_{n-1}$. Since player n has the highest continuation value among non-veto players, she is in a winning coalition only when she is the proposer. This means that $V_n=\frac{1}{n}(1-\sum_{k=1}^{n-1}p^{nk}\delta V_k)$. On the other hand, $V_{n-1}=\frac{1}{n}\left(1-\sum_{k=1}^{n-2}p^{(n-1)k}\delta V_k\right)+\frac{1}{n}\left(\sum_{k\neq n-1}^{n}p^{k(n-1)}\delta V_{(n-1)}\right)$. Note that the second term in this expression may be positive, for instance, when $q=n-1$, or when q is smaller but $V_n>V_{n-1}=...=V_1$. Given our ranking of continuation payoffs, it must be true that $1-\sum_{k=1}^{n-1}p^{nk}\delta V_k=1-\sum_{k=1}^{n-2}p^{(n-1)k}\delta V_k$, since otherwise the minimization problem defined in Theorem 1 is violated. In simple terms, both proposers $n-1$ and n choose the same least-costly way of obtaining support for their proposal. But this implies that $V_n\leq V_{n-1}$, a contradiction. Hence, $V_{n-1}=V_n$. We can continue in the same fashion until we reach player q (in other words, the last one we analyze is $V_q<V_{q+1}$). This establishes that

$$V_q=V_{q+1}=...=V_{n-1}=V_n. \tag{17}$$

After that, we continue as follows. Assume that $V_{q-1}<V_q$. This implies that $V_{r+1}<$

$\frac{1-\sum_{i=1}^{r} V_i^{veto}}{n-r}$. In this case, in addition to veto players, players $r+1$, $r+2$,..., $q-1$ are always in the winning coalition and they will be offered a payoff of δV_{r+1}, δV_{r+2}, ... δV_{q-1} as a coalition partner. Thus, using equation (17), player $r+1$'s continuation payoff can be written as

$$V_{r+1} = \frac{n-1}{n}\delta V_{r+1} + \frac{1}{n}\left(1 - \sum_{j=1}^{r}\delta V_j^{veto} - \sum_{j=r+2}^{q-1}\delta V_j - \delta V_q\right). \tag{18}$$

In addition, using Lemma 1 and equation (17), we obtain

$$\sum_{j=1}^{r} V_j^{veto} + \sum_{j=r+2}^{q-1} V_j = 1 - V_{r+1} - (n-q+1)V_q. \tag{19}$$

Next, substituting equation (19) into equation (18), we get

$$V_{r+1} = \frac{n-1}{n}\delta V_{r+1} + \frac{1}{n}\left(1 - \delta\left(1 - V_{r+1} - (n-q+1)V_q\right) - \delta V_q\right), \text{ or}$$

$$V_{r+1} = \frac{n-1}{n}\delta V_{r+1} + \frac{1}{n}\delta V_{r+1} + \frac{1-\delta}{n} + \frac{n-q}{n}\delta V_q,$$

Simplifying the above equation gives us

$$V_{r+1} = \frac{1}{n} + \frac{n-q}{n}\frac{\delta}{1-\delta}V_q. \tag{20}$$

Similarly, we can write veto player i's continuation payoff (where $i \in \{1, 2, ..., r\}$) as

$$V_i^{veto} = \frac{n-1}{n}\delta V_i^{veto} + \frac{1}{n}\left(1 - \sum_{j\neq i}^{r}\delta V_j^{veto} - \sum_{j=r+1}^{q-1}\delta V_j - \delta V_q\right). \tag{21}$$

Furthermore, using Lemma 1 and equation (17), we obtain

$$\sum_{j\neq i}^{r} V_j^{veto} + \sum_{j=r+1}^{q-1} V_j = 1 - V_i^{veto} - (n-q+1)V_q. \tag{22}$$

Next, substituting equation (22) into equation (21), we get

$$V_i^{veto} = \frac{n-1}{n}\delta V_i^{veto} + \frac{1}{n}\left(1 - \delta\left(1 - V_i^{veto} - (n-q+1)V_q\right) - \delta V_q\right), \text{ or}$$

$$V_i^{veto} = \frac{n-1}{n}\delta V_i^{veto} + \frac{1}{n}\delta V_i^{veto} + \frac{1-\delta}{n} + \frac{n-q}{n}\delta V_q,$$

Simplifying the above equation gives us

$$V_i^{veto} = \frac{1}{n} + \frac{n-q}{n}\frac{\delta}{1-\delta}V_q. \tag{23}$$

If we compare equations (20) and (23), we see that $V_{r+1} = V_i^{veto}$ for any $i \in \{1, 2, ..., r\}$. Therefore, the initial requirement that $V_{r+1} < \frac{1-\sum_{i=1}^{r} V_i^{veto}}{n-r}$ becomes

$$V_{r+1} < \frac{1 - rV_{r+1}}{n-r}, \text{ or}$$

$$V_{r+1} < \frac{1}{n}.$$

If we look at equation (20), we see that $V_{r+1} \geqslant \frac{1}{n}$, since $V_q \geqslant 0$, a contradiction. Thus, we must have $V_{q-1} = V_q$. We can continue in the same fashion until we reach player $r+1$ (in other words, the last one we analyze is $V_{r+1} < V_{r+2}$). So far, we have

$$V_{r+1} = ... = V_n = V. \tag{24}$$

Given equation (24), we can write veto player i's continuation payoff (where $i \in \{1, 2, ..., r\}$) as

$$V_i^{veto} = \frac{n-1}{n}\delta V_i + \frac{1}{n}\left(1 - \sum_{j \neq i}^{r} \delta V_j^{veto} - \delta(q-r)V_q\right). \tag{25}$$

In addition, using Lemma 1 and equation (24), we obtain

$$\sum_{j \neq i}^{r} V_j^{veto} = 1 - V_i^{veto} - (n-r)V_q. \tag{26}$$

Next, substituting equation (26) into equation (25), we get

$$V_i^{veto} = \frac{n-1}{n}\delta V_i^{veto} + \frac{1}{n}\left(1 - \delta\left(1 - V_i^{veto} - (n-r)V_q\right) - \delta(q-r)V_q\right), \text{ or}$$

$$V_i^{veto} = \frac{n-1}{n}\delta V_i^{veto} + \frac{1}{n}\delta V_i^{veto} + \frac{1-\delta}{n} + \frac{n-q}{n}\delta V_q,$$

Simplifying the above equation gives us

$$V_i^{veto} = \frac{1}{n} + \frac{n-q}{n}\frac{\delta}{1-\delta}V_q. \tag{27}$$

Since equation (27) is true for any veto player, we have

$$V_1^{veto} = V_2^{veto} = ... = V_r^{veto} = V^{veto}.$$

The final step of the proof is comparing V with V^{veto}. Assume that $V^{veto} \leq V$. This implies that $V^{veto} \leq \frac{1}{n}$, which is a contradiction, since as can be observed from equation (27), $V_i^{veto} > \frac{1}{n}$ for any $\delta > 0$. ■

Proof of Lemma 5. Using Lemma 1 and equation (3), we obtain

$$V_i^{veto} = \frac{1}{n}\left(1 - \delta(r-1)V_i^{veto} - \delta V_j \sum_{k=r+1}^{n} p^{ik}\right) + \frac{n-1}{n}\delta V_i^{veto}, \text{ for } i = 1, ..., r,$$

$$V_j = \frac{1}{n}\left(1 - r\delta V^{veto} - \delta V_j \sum_{\substack{k \neq j \\ k=r+1}}^{n} p^{jk}\right) + \frac{1}{n}\left(\delta V_j \sum_{k \neq j}^{n} p^{kj}\right), \text{ for } j = r+1, ..., n. \tag{28}$$

Notice also that for a veto player i and a non-veto player j, it is true that

$$\sum_{k=r+1}^{n} p^{ik} = \sum_{k=r+1}^{n} \sum_{\psi^i \in C^i} \psi^{ik} g^i(\psi^i) = \sum_{\psi^i \in C^i} g^i(\psi^i) \sum_{k=r+1}^{n} \psi^{ik}, \text{ for } i = 1, ..., r, \tag{29}$$

$$\sum_{\substack{k \neq j \\ k=r+1}}^{n} p^{jk} = \sum_{\substack{k \neq j \\ k=r+1}}^{n} \sum_{\psi^j \in C^j} \psi^{jk} g^j(\psi^j) = \sum_{\psi^j \in C^j} g^j(\psi^j) \sum_{\substack{k \neq j \\ k=r+1}}^{n} \psi^{jk}, \text{ for } j = r+1, ..., n. \tag{30}$$

For each veto player, there is a total of $\frac{(n-r)!}{(q-r)!(n-q)!}$ possible coalitions she may form when she is the proposer. All of these possibilities occur with certain probabilities which add up to 1, i.e., $\sum_{\psi^i \in C^i} g^i(\psi^i) = 1$. In addition, we know that $\sum_{k \neq j}^{n} \psi^{ik} = q - 1 = (r-1) + \sum_{k=r+1}^{n} \psi^{ik}$. Then, we have $\sum_{k=r+1}^{n} \psi^{ik} = q - r$. Hence, equation (29) becomes

$$\sum_{k=r+1}^{n} p^{ik} = q - r, \text{ for } i = 1, ..., r. \tag{31}$$

Similarly, for each non-veto player, there is a total of $\frac{(n-r-1)!}{(q-r-1)!(n-q)!}$ possible coalitions she may form when she is the proposer. All of these possibilities occur with certain probabilities which add up to 1, i.e., $\sum_{\psi^j \in C^j} g^j(\psi^j) = 1$. In addition, we know that $\sum_{k\neq j}^{n} \psi^{jk} = q - 1 = r + \sum_{\substack{k\neq j \\ k=r+1}}^{n} \psi^{jk}$. Then, we have $\sum_{\substack{k\neq j \\ k=r+1}}^{n} \psi^{jk} = q - r - 1$. Hence, equation (30) becomes

$$\sum_{\substack{k\neq j \\ k=r+1}}^{n} p^{jk} = q - r - 1, \text{ for } j = r+1, ..., n. \tag{32}$$

Furthermore, since for all non-veto players continuation value is the same as indicated in Lemma 4, and the first term on the right hand side of equation (28) is also the same due to equation (32), the second term must be the same as well, implying

$$\sum_{k\neq i}^{n} p^{ki} = \sum_{k\neq j}^{n} p^{kj} \text{ for all } i,\, j = r+1, ..., n \text{ and } i \neq j. \tag{33}$$

Note that using Lemma 1 and equation (23), we can uniquely determine V^{veto} and V. For the randomization probabilities, the necessary condition to obtain uniqueness is that $r = q - 1$. To see this, assume in contrast that $r < q - 1$. In this case, we need to solve for $(n - r - 1)$ randomization probabilities for each one of the $(n - r)$ non-veto players. In addition, we have $(n-r)$ randomization probabilities for each one of the r veto players, thus a total of $r(n - r) + (n - r)(n - r - 1)$ unknowns. On the other hand, we have r equations implied by equation (31), $(n-r)$ equations implied by equation (32), and $(n-r-1)$ equations implied by equation (33), thus a total of $n + (n - r - 1)$ equations. In order to have the number of unknowns smaller than or equal to the number of equations, we must have

$$r(n - r) + (n - r)(n - r - 1) \leq n + (n - r - 1), \text{ or}$$
$$r \geqslant n - 1 - \frac{1}{n-2}.$$

Given that $r < q - 1$ and that $r < q < n$, we need to consider $n > 3$ only. However, in that case, the above requirement becomes $r \geqslant n - 1$, which is not feasible since $r < q < n$. ■

Proof of Proposition 2. In light of Lemma 5, we can restrict our attention to the case where $r = q - 1$. The right-hand side of equation (32) is zero for all for $j = r + 1, ..., n$. Hence, $p^{jk} = 0$ for all $k \neq j$ and $j = r + 1, ..., n$. This leaves us with $(n - r)$ randomization probabilities for each one of the r veto players, thus a total of $r(n-r)$ unknowns. On the other hand, we have r equations implied by equation (31), and $(n-r-1)$ equations implied by equation (33).[19] In order to have the number of unknowns smaller than or equal to the number of equations, we must have

$$\begin{aligned} r(n-r) &\leq r + (n-r-1) \\ &\Rightarrow r \leq 1 \text{ or } r \geqslant n-1. \end{aligned}$$

Given that $r < q < n$ and $r = q - 1$, the only possible way to obtain uniqueness is when $r = 1$, and $q = 2$. ■

References

[1] Baron, D.P. and J. A. Ferejohn, 1989. Bargaining in legislatures. *American Political Science Review* 83, pp. 1181-1206.

[2] Baron, D.P. and E. Kalai, 1993. The simplest equilibrium of a majority-rule division game. *Journal of Economic Theory* 61, pp. 290-301.

[3] Bowen, T.R., 2014. Legislated protection and the WTO. *International Economic Review* (forthcoming).

[4] Celik, L., B. Karabay, and J. McLaren, 2013. Trade policy making in a model of legislative bargaining. *Journal of International Economics* 91, pp.179-190.

[5] Celik, L., B. Karabay and J. McLaren, 2015. When is it optimal to delegate? The theory of fast-track authority. *American Economic Journal: Microeconomics* 7, pp. 347-389.

[6] Diermeier, D. and A. Merlo, 2004. An empirical investigation of coalitional bargaining procedures. *Journal of Public Economics* 88, pp. 783-797.

[19] As an example, consider the 3-player game we analyze in the main text. Since player-1 is assumed to be a veto player, it is true that $p^{21} = p^{31} = 1$ and $p^{23} = p^{32} = 0$. Therefore, the unknown randomization probabilities are p^{12} and p^{13}. On the other hand, the linearly independent equations are: $p^{12} + p^{13} = 1$, implied by equation (31), and $p^{12} = p^{13}$, implied by equation (33).

[7] Diermeier, D. and P. van Roozendaal, 1998. The duration of cabinet formation processes in Western multi-party democracies. *British Journal of Political Science* 28, pp. 609-626.

[8] Eraslan, H., 2002. Uniqueness of stationary equilibrium payoffs in the Baron-Ferejohn model. *Journal of Economic Theory* 103, pp. 11-30.

[9] Fudenberg, D. and J. Tirole, 1991. *Game Theory*. Cambridge, MA: MIT Press.

[10] Laver, M. and N. Schofield, 1998. *Multiparty Government: The Politics of Coalition in Europe*. Ann Arbor: University of Michigan Press.

[11] McCarty, N.M., 2000a. Presidential pork: executive veto power and distributive politics. *American Political Science Review* 94, pp. 117-129.

[12] McCarty, N.M., 2000b. Proposal rights, veto rights, and political bargaining. *American Journal of Political Science* 44, pp. 506-522.

[13] Müller, W. and K. Strøm, 2000. *Coalition Governments in Western Europe*. Oxford: Oxford University Press.

[14] Nunnari, S., 2012. Dynamic legislative bargaining with veto power. Working paper, University of California, San Diego, October.

[15] Primo, D.M., 2006. Stop us before we spend again: institutional constraints on government spending. *Economics and Politics* 18, pp. 269-312.

[16] Primo, D.M., 2007. A comment on Baron and Ferejohn (1989): The open rule equilibrium and coalition formation. *Public Choice* 130, pp.129-135.

[17] Riker, W.H., 1962. *The Theory of Political Coalitions*. New Haven: Yale University Press.

[18] Snyder, J., M. Ting and S. Ansolabehere, 2005. Legislative bargaining under weighted voting. *American Economic Review* 95, pp. 981-1004.

[19] Tsebelis, G., 2002. *Veto Players: How Political Institutions Work*. Cambridge, MA: MIT Press.

[20] Winter, E., 1996. Voting and vetoing. *American Political Science Review* 90, pp. 813-823.