Evolving discourse of practices for quality teaching in secondary school mathematics

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This paper presents initial findings from the project PRAQTAL: PRActices for Quality Teaching. Our project focuses on conceptualizing and identifying teaching practices for quality teaching of secondary school mathematics and physics. This work is driven by the understanding that to document, conceptualize, analyze and promote quality teaching, we need to constitute a discourse which articulates the diverse practices of teachers, in their multiple resolutions, and link them to educational theories on one hand and specific instruments on the other. For this purpose, we adopted Commognition as a conceptual framework. Here, we present our working definition of a teaching practice and our criteria for quality teaching practices. We discuss the procedures for identifying and documenting practices and their representation and illustrate our arguments with an example from the project's emerging database.

Keywords: Teaching practices, quality teaching, routines, secondary school mathematics.

Introduction

This paper presents initial findings from project PRAQTAL: PRActices for Quality Teaching. PRAQTAL focuses on conceptualizing and identifying practices for quality teaching of secondary school mathematics and physics. Why focus on teaching practices? Our team arrived at the need to explicate practice from two seemingly distinct paths: the first coming from an attempt to improve a teacher-education program from more "reflective" and "principles" based towards "practice" based. We noticed that the task of shifting the preservice teachers' traditional-teacher-centered approach towards more reform-student-centered approach (Gregg, 1995) calls for explicit discussion and explication of practices. The second path originated in a series of projects aimed at promoting innovative and effective use of technologies in mathematics education. One of the core observations of these projects was the gap between theories of teaching and learning, and the actions practitioners took in the classroom. Following the traditions of educational design research (Mor & Winters, 2007) and learning design (Mor, Craft & Hernández-Leo, 2013), these projects identified this gap with a need for articulating and sharing design knowledge: the knowledge of solving practical challenges, or affecting change, by translating abstract theory into action, and explaining action by reference to theory. The initial account of practitioner experience was expressed in the form of design narratives. The derived design knowledge was captured in design patterns (Warburton & Mor, 2015): statements of the form "in context X, you are likely to encounter the challenge Y, and can address it using method Z". The aim of the pedagogical design patterns community is to use this structure to articulate and share valuable elements of educational practice.

Teaching practices

The need to explicate practice, specifically for promoting teaching programs, resulted in the flourishing of many projects that aim at identifying and teaching teaching-practices (Grossman, 2018). Some projects were content specific, some focus on specific grade levels, others are general, but most grew from the needs of training programs for preparing teachers to practice. A prime example is the University of Michigan *Teaching Works* project (http://www.teachingworks.org/) that identified a set of high-leverage instructional practices to prepare beginning teachers "who are skillful at connecting with and helping their students develop." Several additional projects are discussed in Grossman's book *Teaching core practices in teacher education* (2018). McDonald et al. (2013) identify a major shift in teacher training: from a focus on the knowledge required for teaching to a greater attention to the core practices of teaching. This move towards core practices, argue McDonald et al, reflects an attempt to connect the development of knowledge of teaching with the capacity to enact this knowledge in the classroom. McDonald et al. review the shifts in pedagogical approaches in teacher training over the last half century, corresponding to the change in dominant perspective on teaching and learning. From an "acquisition" model of learning, to "participationist" model of learning. However, Grossman and McDonald (2008) warn us that the field of research on teaching lacks a structured method and vocabulary for describing, analysing, evaluating and improving practices of teaching.

Various definitions are offered for teaching-practices. One such definition is offered by Windschitl (2016), who defines teaching practices as the recurring professional work devoted to planning, enacting and reflecting on instruction. This definition emphasizes the role of teaching practices prior, during and after instruction in class. According to Windschitl, the purpose of teaching practices is to overcome the divide between teacher instruction and student learning. The CPC group (Core Practices Consortium) whose aim is to develop shared understanding and common language regarding what it takes to prepare teachers for practice, to improve learning opportunities available to all students. They have identified characteristics that most sets of core-practices adhere to: (1) high frequency in teaching; (2) practices that teachers can enact in classroom across different curricula or instructional approach; (3) practices that allow teachers to learn more about students and about teaching; (4) practices that preserve the integrity and complexity of teaching; and (5) practices that are research-based and have the potential to improve student achievement (Grossman, Hammerness and McDonald, 2009).

Lampert (2010) addresses the difference between teaching practices and the practice of teaching. The practice of teaching involves adopting the identity of a teacher, doing what teachers do and believing what teachers believe in. This raises the question whether a practice can be learned in isolation or whether it requires participation in collective activity.

For the purpose of our project we need a working definition of teaching-practice and quality teaching-practice. To this end, we adopt the socio-cultural commognitive conceptual framework that conceptualizes mathematics (and any other discipline) as a special type of communication or discourse, including unique ways of saying and doing (Sfard, 2008), and learning mathematics as changes in the learner's mathematics discourse. Adopting the commognitive conceptual framework, we identify teaching-practice with a teaching-routine. Let us explain: according to our initial understanding, teaching practice is *a recognizable pattern of actions used by teachers in a given context to achieve specified aims*. This definition includes three constituents: pattern of action, context and aims. By the term “pattern of actions” we refer to actions that are repeated in situations that the performer would consider as similar. The assignment of a practice to a given context is crucial. Practices are inherently situated in material, social and intentional environments, and only make sense within these environments. Finally, practices are not arbitrary, they serve a goal. Identifying practices with routines will address those three requirements. Following Lavie, Steiner and Sfard (2019), we define a teaching-practice similar to their definition of routines: *the task the teacher saw herself performing together with the procedure she executed to perform the task* (p. 161). Such task could be *having to address a student's idea that she had not thought of earlier*, or *eliciting students’ thinking*.

Thus, our goals in project PRAQTAL are to identify typical, interesting or challenging tasks that the teachers set themselves before, during and after a lesson, and possible related teaching-practices. That is, for each task we identify the various procedures that teachers chose to execute to perform the task. The plurality is used here to denote that our point of departure is that for each task, different teachers may consider different procedures to perform.

The advantage of this definition is that a task and therefore also a routine are recursive structures: a task could be parted to sub-tasks and a routine to sub-routines. This recursive structure is apparent in Table 1. This conceptualization of teaching-practices allows us to view teaching from different “zoom-ins” or different granularities: the highest level (left-most column in Table 1) provides us with the rational for everything that takes place in class. This is apparent with the highest meta-level of "providing students opportunity to become explorative participants in the mathematics discourse". At the other end, the *procedure* and *empirical example* (two right-most columns of Table 1), provide us detailed steps to follow and perform, with different levels of sub-tasks in between. This relates to one of the issues with which we were concerned early on - the question of the “granularity” of practices: is “pausing for 5 seconds after asking a question” a teaching practice? Is “leading a classroom discussion” one? The first seems too miniscule, the second too expansive. The answer we found is that practices are networked in a fractal manner (as alluded to in the project name): they are spread across a continuous space with granularity on one axis and generality on the other. Complex practices are composed of other smaller-size practices. This granularity is apparent by Table 1. So “pausing for 5 seconds…” could be written in the procedure column of Table 1 as a part of procedure for performing a “larger size” task, such as “eliciting students’ thinking”.

Quality teaching and quality teaching-practices

Now we are left with the task of defining what counts as a quality teaching-practice (QTP). Defining mathematics as a specific type of discourse with unique ways of saying and doing, and learning mathematics as becoming more central participants in the mathematics discourse (Sfard, 2008), makes itself evident that QTP will be those that are most likely to prompt and support learners in participating in this discourse. Specifically, in the lessons we observed in our study and in literature, we identified three meta-tasks (or principles) that seem to underlay today's thinking about mathematics teaching and seem to be shared by all mathematics teachers: (1) Provide students opportunities to become central participants in mathematics discourse; (2) Help students to develop a positive identity as mathematics learners; and (3) Encourage students to participate in equitable, egalitarian mathematical discourse. Therefore, our definition for quality-teaching-practice is *a teaching-practice that a teacher would perform that is aligned with the above three meta-tasks*. By the words "aligned with" we mean that the QTP supports at least two of those tasks, and does not violate any of those tasks.

Sourcing, Documenting and Representing Practices

Having considered the definition of a QTP, we set out to identify and articulate such objects. In this section we explain how we identify teaching-practices. Our identification of a teaching-practice includes identifying typical, challenging or interesting tasks that the teacher saw herself as having to perform, and then identifying various procedures that could be enacted to perform the task. Our two primary sources of data are video recordings of secondary-school mathematics lessons taught by expert-teachers, and literature. Each video-taped lesson is fully transcribed, and subtitles are added to the video. We also have video-taped discussions with the teachers about the lesson before and after the lesson, written documents about the lesson (such as the teacher's lesson plan) and access to the teacher for any questions that we have during our analysis of the lesson. This is highly required as our analysis, which is based on identifying the tasks that the teacher considered herself facing when choosing to perform certain actions, is interpretative in nature. Having the teachers react to our findings allow us to learn more about teaching practices that often remain implicit.

For each video-taped lesson, we focus on the teaching-actions performed by the teacher and ask: *what is the task that the teacher may achieve by those actions?* We return to the teachers and discuss our suggestions with them. We then ask whether the teaching-practice, that is, the task and the related procedure, supports at least two of the three meta-tasks defined earlier and does not violate any of them. If we find that the teaching-practice is repeated, either in a specific lesson or across different lessons, or if we find similar teaching-actions reported upon in the literature, we designate it as a candidate for quality-teaching-practice. We use Table 1 as our primary working-tool. In Table 1, the leftmost "meta-task" column is fixed, and includes the three meat-tasks described earlier (in Table 1 we only present the first of the three meta-tasks in the sake of brevity). The other columns become more and more fixed as we continue with our analysis. Rubrics in the two right-most columns keep adding as we analyze lessons. We document the performed action(s) found in the analyzed lessons in the "empirical example" column of Table 1 and list the various tasks and sub-tasks that a teacher performing these actions could be facing.

To clarify our methodology, an example from a 90-minute lesson on complex-numbers is demonstrated next. The lesson was taught in an advanced 12th grade mathematics classroom in an Israeli high-school. This lesson was the first on this topic. Before focusing on the specific example, we describe the lesson's thematic structure: In this lesson the teacher faced the challenge of teaching a new mathematical object, a set of numbers that the students are not yet familiar with – the complex numbers. Her choice was to follow the idea that ontogenetic processes often follow phylogenetic ones, and walk her students through the historical development of those numbers, in four steps: (1) realize that as long as you only have real numbers, there is a no such thing as square root from negative number, (2) conclude that a 3rd power polynomial has at least one root. (3) introduce the students to the suggestion of Girolamo Cardano which led to mathematics results including square roots of negative numbers, and then Rafael Bombeli's suggestion of determining such entities as mathematics objects – numbers. (4) Show that the new entity "complex number" is a number by showing that it has similar attributes as the corresponding operations on rational and real numbers. Our example focuses solely on this fourth section, during which the following discussion took place [timestamp 57:50]:

Teacher	… so I see that this addition has different characteristics that are… the existence of a neutral element in addition
Student	but why do we need this?
Teacher	To know the structure. I want to convince you that this structure of the complex-numbers justifies the name "numbers". Ok, why do I call these numbers? In what sense are those numbers? I want to convince you that this behaves similarly to real numbers regarding operations.

In this short excerpt, we find a student's interesting question "why do we need this?" This question is interesting as it seemed from the video that the student was not having a difficulty with the mathematics procedures that the students and teacher were performing neither was she trying to say that "she is not interested", or that "she does not want to study". It seemed that she was trying to make sense of the reasons for which the teacher was showing that the set of complex numbers have different specific characteristics (thus – proving that it is a field). The teacher's answer addresses the three meta-tasks: (1) *Provide students opportunities to become participants in mathematics discourse:* in mathematical discourse, when an object is introduced by expanding a familiar class (category), it is imperative to check its properties against the definition of the class to which it is supposed to belong, or by considering its characteristics (the object of *number* is not well defined in mathematics). More generally, when a new object is introduced or when an object is identified, this identification should adhere to the conditions of a mathematical definition. As was found in various studies, this is not a simple endeavor, and therefore requires explicit teaching:

> The discursive activity of defining seems to be pushed aside by our strong tendency to use words in a direct, unmediated manner, without accounting for this use and without monitoring its appropriateness. This inclination for unmediated, spontaneous use of words is the basic characteristic of human communication. …. In schools, one's spontaneous uses of words are supposed to be translated into scientific. For this modification to happen, the students will have to learn to suspend their spontaneous discursive decisions for the sake of reflective, meta-discursively mediated choices of words. This, as was already observed, is a difficult thing to learn (Nachlieli & Sfard, 2003. pp. 3-355–3-356).

(2) *Enable students to develop a positive identity as mathematics learners*: the teacher's answer explicates her encouragement of students' questions, of seeing the students as capable of participating in a challenging mathematics discussion.

(3) *Encourage students to participate in egalitarian mathematical discourse:* In the classroom, when a teacher introduces an object, students often accept it as belonging to that class/category. It is as if they accept the teacher as the primary authority. In this example the teacher encourages her students to break away from the ritual participation of accepting a "truth" because it was stated by the teacher, and leads them to an empowered, explorative participation, where they identify objects by reference to a definition. This is apparent through the words "I want to convince you" – the teacher stresses that she needs to do the work of convincing the students, that she does not expect them to just accept whatever she tells them. She does not place herself as the primary authority.

The QTP that we elicited from this excerpt, based also on literature (e.g. Sfard & Nachlieli, 2003), includes the following task and practice:

Task. When the teacher faces the task of introducing new objects, or of expanding a familiar class (category), the new object needs to be identified by checking its properties against the definition of the class, or by considering its characteristics.

Procedure.

1. The teacher raises the question, or invites the students to raise the question, or addresses the question if arises: is this object compliant with the definition of the base class?
2. The teacher guides the students in independent verification of the new object against the conditions of the definition.
3. The teacher ascertains that the students continue to work with the object as compliant with the definition.

Meta-task	Task	Sub-task	Procedure	Empirical example
1. The task of providing students opportunity to become explorative participants in the mathematics discourse	1.1 modeling explorative mathematics discourse 1.2 inviting students to participate in explorative math discourse	1.1.1 modeling objectified mathematics discourse	When introducing a new object that belongs to a familiar class, the teacher explicates the need to check whether this object compliant with the definition of the base class, see steps 1-3 in the procedure as described above.	"I want to convince you that this structure of the complex-numbers justifies the name "numbers"." Following, the teacher and students prove that the set of complex numbers are a field.

Table 1: Recursive structure of a teaching-practice

The design of a practice

The mission of the PRAQTAL project is not just to identify and document quality teaching practices, but also to make those available to pre- and in-service teachers. For this purpose, we develop an accessible presentation of practices which is communicative and appealing to practitioners.

On our website, we present practices as tri-partite structure, described in Diagram 1.

(1) A short 30 second video that explains the task that the teaching-practice focuses on, elaborating briefly on a conceptual question relating to a math related topic and teaching/learning challenges	⇨	(2) A structured text listing the aims, context and pattern of actions, and video examples. These are followed by an "in depth" section, offering theoretical justifications, limitations and risks, and links to other related practices. To formulate this part, we use a "practice template" (see template and example)	⇨	(3) Short clip of a real classroom video that demonstrates and exemplifies the teaching practice discussed. The video is taken from a real secondary class after acquiring all ethics approval.

Diagram 1: The tri-partite structure of a teaching practice

Discussion

In this paper we presented the initial work of project PRAQTAL. We began this account with our personal paths into the domain of teaching practices, and noted how these resonated with a general trend in educational research and teacher training. Most of the work in the field focuses on deriving practice recommendations from theoretical frameworks and developing teacher training programs based on these. Our approach was to first move beyond the intuitive treatment of the core concepts, and systematically define the constructs of *teaching-practice* and *quality-teaching-practice*, based on the commognitive conceptual framework. We adopt commognition for three main purposes: (1) for the conceptualization of our basic terms, such as: "practice" and "quality teaching practice", (2) to develop a methodology for discourse analysis of transcripts of mathematics lessons that helps us identify teaching practices, and (3) findings from commognitive studies serve as main literature from which we identify possible teaching practices that we then look for empirically (an example is teaching practices that promote meta-level learning, a commognitive idea that we continue to develop in the field of teaching practices. This is beyond the scope of this paper). On these foundations, we proceeded to formulate a methodology for sourcing, articulating and communicating practices. Our methodology blends discourse analysis and design based traditions in mathematics education research, and the standard of quality stems from a commognitive framework. We illustrate this methodology through an example. At this stage, we have the building blocks for a combined scientific and pragmatic inquiry into the teaching practices of mathematics. The next steps are to build an extensive language of practices, validated empirically and theoretically, and map the connections between them. In parallel, we will develop our framework for practices-oriented teacher training. True to our constructivism roots, we reject the urge to "deliver knowledge of practices" to teachers and teachers in training, and instead aim to base our offering on co-construction and critical discussion of representations of practices. We invite the community to join us in these endeavours.

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