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In didactic literature there are many practical materials to use experiments in mathematics classroom. Even though, experiments seem to be beneficial in mathematics education, little is known about what teachers think concerning this method. In this explorative study we want to examine the teachers’ beliefs about experiments and which relations they see between experiments and mathematics. We asked teachers from various school types in Germany, with an open-ended questionnaire. Our results indicate that most teachers see experiments either as controlled and planful procedure or as “trial and error”. Furthermore, teachers state mainly two relations between experiments and mathematics.

Keywords: Experiments, teachers’ beliefs, grounded theory

Introduction

Despite the fact, that teachers’ beliefs have been of increasing interest in educational research, the lack of a common shared definition or understanding of beliefs is obvious (Pajares, 1992; Philipp, 2007). Most authors locate beliefs within the affective domain but attest beliefs to be more cognitive and stable than attitudes and emotions (McLeod, 1992; Philipp, 2007; Hannula, 2012). We refer to beliefs as “Psychologically held understandings, premises, or propositions about the world that are thought to be true” (Philipp, 2007, p. 259). It has been proven that beliefs influence teachers’ activity (Grigutsch, 1996). Thus, beliefs are the basis for later actions and are part of the teacher's professional knowledge (Kuntze & Reiss, 2005).

However, during the last years the focus of interest has changed from rather global beliefs to domain specific beliefs or beliefs concerning single methods (e.g. Eichler & Schmitz, 2018). One method, that has not been taken into account so far, are experiments or hands-on laboratory activities. Since experiments play an important role in both scientific mathematics and mathematics education, we want to shed light on teachers’ beliefs concerning the nature of experiments and the relations between experiments and mathematics.

Experiments

Experiments play a crucial role in the epistemology of nature science. The experimental testing of hypotheses, which have been derived from theories, can be seen as the core of scientific work in subjects like physics and chemistry. It is, therefore, not surprising that experiments have an outstanding position in science education as well. For example, in physics lessons experiments and laboratory work fill about 30% of the time spent in class (Tesch & Duit, 2004). Before illustrating the role of experiments in mathematics we will give a brief overview of the discussion about the question what exactly an experiment is.

Characteristics of Experiments

It is rather surprising that there is no common shared definition of experiments. Borba and Villarreal (2005) summarized definitions from several dictionaries concluding that "in an experiment, conditions or factors are manipulated and facts are observed in order to prove or disprove a given hypothesis" (p. 66). They added that besides the testing of hypotheses, experiments can be used to "discover something unknown […] or to provide examples of a known truth" (p.66). Ganter and Barzel (2012) stress that experiments form a cyclical process, starting with a concrete hypothesis, the planning of an experiment and followed by the performing of the experiment itself and analysing the results – which themselves can lead to a following cycle. It should not remain unmentioned that in every day sense – and even in some dictionaries – every hands-on activity or laboratory work is considered to be an experiment, even if the process is not controlled and follows rather the principles of "trial and error" (Borba & Villarreal, 2005).

Experiments and (the Learning of) Mathematics

Even if it is not often mentioned or recognized: experiments and empirical observations play an important role in mathematics and the learning of the same. Polya (1945) described that:

> "mathematics has two faces; it is the rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself" (p. vii).

Experiments are part of the proving process in the sense that they help to identify structures and they give ideas how to proof theorems (Kortenkamp, 2014). A (young) subdomain of mathematics that uses (mainly) computer-supported experiments and simulations for this purpose is called experimental mathematics (Borwein et al., 2004). Besides the role experiments play in the proving process, mathematics and experiments meet each other in the experimental processes as it is performed in nature science, too. Quantitative experiments cannot be analyzed without mathematics – the data derived from an experiment undergoes a mathematical modeling process. Henning (2013) postulates that one of the most important functions of mathematics lies in supporting the understanding of our world.

The role of experiments in the mathematics classroom is closely connected to the relations between experiments and mathematics in scientific settings. In the existing literature we identified three learning situations related to experiments in mathematics:

- *Proving processes:* Regardless that proving is less present in school than in scientific mathematics, experimental work plays the same role in the proving process in school. Experiments can help to find a rule that could be proved and the ideas for the proof itself (Kortenkamp, 2014). "Experimental proofs" are part of the proving process of secondary students at school (Brunner, 2014).
- *Modeling:* Modeling is a key competence in mathematics lessons (Blum & Niss, 1991). But since modeling activities can be performed with "real" data derived from experiments as well as with data found in textbooks, the question arises whether the implementation of

experiments can enrich the modeling process. Following Carreira and Baioa (2018; 2011), experiments can enhance the credibility of modeling tasks and foster students' understanding of the real situation that is modeled and the involved mathematics. The use of "real" data from experiments can also enrich the validation of models and opens teachers' and students' view for the question what characterizes a good model and how one should act upon measurement errors which are inherent of experiments (Carrejo & Marshall, 2007).

- *Learning of new concepts:* Furthermore, experiments can support students' learning of new concepts. Ganter and Barzel (2012) showed that experiments can create a fruitful start for the learning of functional relations and, in particular, the manipulation of a variable during an experiment can help students to distinguish between dependent and independent variable.

Furthermore, experiments can increase students' interest in mathematics and their motivation (Beumann, 2016). As conclusion one might say that experiments can enrich mathematics lessons in various ways and learning situations. However, experiments are neither explicitly mentioned in the German mathematics curriculum, nor do the play a crucial role in teacher education. Thus, teachers' beliefs concerning experiments and the teachers' use of experiments in the classroom is widely unexplored. This contribution is a first step to investigate teachers' beliefs concerning experiments and their connection to mathematics.

Beliefs and Experiments

Some recent studies addressed the beliefs of teachers in nature sciences concerning the role of experiments in the classroom and the goals obtained by using experiments. Science teachers' reasons to use experiments in their teaching practice include that experiments involve hands-on-activities, increase students' motivation and their conceptual and procedural understanding of nature sciences (Lavonen, 2012). In the special case of mathematics, teachers value the use of hands-on activities in classroom and believe that mathematics lessons should be student-centered (Wang & Cai, 2007) – experiments fall within both: hands-on activities and student-centered methods.

However, to the best of our knowledge there is currently no study (neither in science education nor mathematics education) focusing on teachers' beliefs concerning the nature of experiments and how experiments are connected to mathematics and mathematics learning. For a profitably use of experiments in class, teachers need to have appropriate beliefs about this method itself as well as about the relation to mathematics.

Research Questions

Since on the one hand experiments can be beneficial for the learning of mathematics, and on the other hand teachers' beliefs affect their classroom practices (Grigutsch, 1996), we want to explore beliefs that teachers hold concerning experiments. This leads to the following questions:

1) Which beliefs are held by mathematics teachers concerning the notion experiment?
2) Which relations between experiments and mathematics do mathematics teachers see?

Methods

Since the answers in liker-scale surveys are not easy to interpret when dealing with beliefs (Phillips, 2007) we decided to use open-ended questions within a paper and pencil questionnaire. The

questions where formulated as follows: 1) What do you understand under the notion "Experiment"? Give a brief explanation and typical characteristics. 2) In which way do experiments and mathematics fit together? Which relations between mathematics and experiments do you see?

The questionnaire was handed to 50 in-service teachers who worked at different types of schools in North Rhine Westphalia, Germany (primary school, lower and upper secondary schools). All of them were mathematics teacher and 24 taught another STEM-subject (like chemistry or physics) as well. All teachers participated voluntarily. We choose that variety of participants in order to gain a more global (yet not necessarily generalizable) picture of the teachers' beliefs. However, not all teachers answered every question in the questionnaire.

The data analysis of this study is based on the "Grounded Theory" by Glaser and Strauss (1998). We constantly compared the data with each other by using open codes. Thus, we identified various beliefs concerning experiments. These beliefs have been discussed and grouped in categories. Given the limited space, we only give an overview of the main categories in this contribution.

Results

Category	**Description**	**Example**	**Quantity**
Procedure *(characteristics of the experimental process)*			
Experiments as planful and controlled procedure	Experiments are characterized by planful and controlled actions to verify a given hypothesis	"An experiment is an action, that is conducted under fixed circumstances […] to check a hypothesis or generate one."	17
Experiments as "trial and error"	Experiments are not planful or controlled. They do not need a hypothesis and are characterized by trying out and attempting different things.	"Try things out – involves that sometimes it goes wrong."	22
Experiments as autonomous hands-on activity	Every kind of hands-on activity that provides autonomous working is viewed as experiment – regardless if it is planful or just trying.	"To solve a task with hands-on activities and practical solutions."	10
Aim *(aims connected to experiments)*			
Experiments aiming on (better) understanding	The aim of experiments is to (better) understand previously known things.	"Better understand relations."	13
Experiments aiming at (new) knowledge	The aim of experiments is to provide new (at least from the viewpoint of the experimenting person) insights and knowledge.	"Experiments are a method to gain new insights."	16

Table 1: Beliefs concerning the nature of experiments

Regarding the nature of experiments, we distinguish beliefs concerning the procedure of experimenting and the aims which are connected to experiments. Most teachers stated that they see experiments as either a controlled and planful action to check a given hypothesis or as uncontrolled trying without guiding hypothesis and plan. However, some teachers wrote that they see every hands-on activity as an experiment as far as it involves autonomous working. Besides these characteristics of the conduction of an experiment, most teachers stated that experiments aim at a certain goal: 16 teachers believe that experiments should lead to new insights and new knowledge – at least this knowledge should be subjectively new for the person that obtains the experiments. Other teachers focus more on the aim to better understand previously gained knowledge (Table 1).

Concerning the relation between mathematics and experiments we could identify two main beliefs (Table 2): 13 teachers stated that experiments are part of the process to generate new mathematical knowledge. Experiments are the inductive starting point of this process. 9 teachers see mathematics as part of the experimental process in other sciences – in this view mathematics is used to describe and model experiments. Two teachers stated no direct connection between mathematics and experiments but listed characteristics that both have in common. 3 teachers wrote that they see no relation between mathematics and experiments. However, despite the fact that we explicitly asked for a connection between experiments and mathematics, many teachers only wrote about the connection of experiments and classroom practices.

Category	**Description**	**Example**	**Quantity**
Experiments as a method in the mathematical process	Experiments are used in the mathematical process to generate new hypothesis that can be proven later.	"Mathematical statements are the result of a long process, obtained with the help of experiments"	13
Mathematics as toolbox in the experimental process	Mathematics is used to describe and evaluate experiments in other sciences and everyday life.	"Experiments and their results can usually be described with mathematics."	9
Common characteristics of Mathematics and Experiments	Mathematics is not part of the experimental process or vice versa but both have common characteristics.	"Mathematics as well as experiments is based on plans to solve certain problems, the systematic conduction of the plan and the reflection upon the results."	2

Table 2: Beliefs concerning the relation between mathematics and experiments

Like beliefs concerning the nature of mathematics, beliefs about experiments are not disjoint in the sense that teachers can either believe that experiments are planful and controlled or trial and error. The different beliefs can be held in various combinations. We illustrate this fact by describing the beliefs of three teachers more precisely:

Mr. A teaches mathematics and biology at an upper secondary school and wrote that he used experiments in his mathematics lessons before. For him a "clear mandatory setting and procedure" characterizes experiments. But he is aware of the fact that in every day life "experimenting" can be

viewed as less planful and less controlled: “Colloquial, experimenting can be aimless without mandatory setting and determined procedure.” He sees the connection between mathematics and experiments in modeling activities: “Insights from experiments can be mathematically modeled. The mathematical models themselves can be used for further predictions concerning the phenomenon. These predictions can be checked by new experiments. Mathematics and experiments can support each other in this sense.”

Mr. B teaches mathematics and physics at an upper secondary school and used experiments in his mathematics lessons before. For him, experiments follow a very clear and determined procedure: “Experiments consist of the observation of results, the investigation of dependences from single parameter, a defined procedure, and the reflection on the objects and used methods.” In contrast to Mr. A he sees no direct connection between mathematics and experiments but he stresses that the experimental process and the process of creating new mathematical knowledge have some characteristics in common: “Similar to experiments, an appropriate question and the development of ideas, hypothesis and conceptions belong to mathematics. On this basis theories can be developed and later be proven.”

Mrs. C works at an upper secondary school, too, but teaches no nature science and has not used experiments in her lessons so far. Mrs. C’s beliefs concerning the nature of experiments are less sophisticated. For her experimenting means to “try things out.” However, in her opinion experiments are an integral part of mathematical discoveries: “Mathematics is based on experimental work, like searching and finding schemas – that’s how discoveries are made!”

The exemplary comparison of Mr. A and Mr. B reveals, that the teachers’ beliefs concerning the nature of experiments as well as their background (both are teaching a second STEM-subject and used experiments in their lessons before) do not directly determine which connections to mathematics they see. Both teachers stress that experiments follow a defined procedure. However, Mr. A sees a clear connection between experiments and mathematics, while Mr. B does not see this connection. This result was observed amongst many teachers in our sample. Furthermore, we found no effect of STEM-teaching on the teachers’ beliefs concerning the nature or experiments.

Discussion and Outlook

The teachers’ beliefs concerning the nature of experiments mainly reflect the discussion on a scientific definition of experiments (Borba & Villarreal, 2005) with the two (extreme) poles “experiments as trial and error” and “experiments as controlled procedure”. Given the fact, that modeling plays an important role in modern mathematics education, it is rather surprising that not more teachers saw this connection between mathematics and experiments. Especially since many teachers in our sample taught a nature science as well and, therefore, should be familiar with experiments. In our understanding, every analysis of a quantitative experiment in nature sciences needs mathematical modeling to some degree. However, 25% of the teachers reported that they see experiments as part of the process to gain new mathematical knowledge which is in line with Brunner’s (2014) model of proving processes in school. Furthermore, (computer-supported) mathematical experiments are of growing importance in scientific mathematics which should be reflected in mathematics education as well, according to Kortenkamp et al. (2014).

One limitation for our analysis is that we have no information about the teachers' beliefs concerning the nature of mathematics (mathematical worldview). These beliefs might affect which connections teachers see between experiments and mathematics.

With this pilot study we made a first step to understand teachers' beliefs concerning experiments in mathematics. Our ongoing research will now focus on two facets: 1) The reasons that lie behind teachers' beliefs concerning experiments (e.g. teachers' background and connections between teachers' beliefs concerning the nature of mathematics and their beliefs concerning experiments) and 2) the question whether (and how) mathematics teachers implement experiments in their mathematics lessons. In order to gain deeper insights into the connection of the teachers' beliefs and their use of experiments in mathematics lessons, semi-structured interviews as well as classroom observations will be used.

References

Beumann, S. (2016). *Versuch´s doch mal – Eine empirische Untersuchung zur Förderung von Motivation und Interesse durch mathematische Schülerexperimente*. (Doctoral dissertation) Ruhr-Universität-Bochum. Retrieved from https://hss-opus.ub.ruhr-uni-bochum.de/opus4/frontdoor/deliver/index/docId/5122/file/diss.pdf.

Blum, W. & Niss, M. (1991). Applied mathematical problem solving, modelling, applications and links to other subjects – State, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37–68.

Borba, M. C., & Villarreal, M. E. (2005). *Humans with Media and the Reorganization of Mathematical Thinking. Information and Communication Technologies, Modeling, Visualization and Experimentation.* Springer.

Borwein, J., Bailey, D., Girgensohn, R. (2004). *Experimentation in Mathematics. Computational Paths to Discovery*. New York: A K Peters/CRC Press.

Brunner, E. (2014). *Mathematisches Argumentieren, Begründen und Beweisen. Grundlagen, Befunde und Konzepte*. Heidelberg: Springer.

Carreira, S., & Baioa, A. M. (2011). Students' Modelling Routes in the Context of Object Manipulation and Experimentation in Mathematics. In G. Kaiser et al. (Eds.). *Trends in Teaching and Learning of Mathematical Modelling* (pp. 211–220). Springer:

Carreira, S., & Baioa, A. M. (2018). Mathematical modelling with hands-on experimental tasks: On the student's sense of credibility. *ZDM - Mathematics Education, 50*(1–2), 201–215.

Carrejo, D. J., & Marshall, J. (2007). What is mathematical modelling? Exploring prospective teachers' use of experiments to connect mathematics to the study of motion. *Mathematics Education Research Journal, 19*(1), 45–76.

Eichler, A., & Schmitz, A. (2018). Domain Specificity of Mathematics Teachers' Beliefs and Goals. In B. Rott, G. Törner, J. Peters-Dasdemir, A. Möller, & Safrudiannur (Eds.), *Views and Beliefs in Mathematics Education. The Role of Beliefs in the Classroom.* (pp. 137–146). Berlin: Springer.

Ganter, S. & Barzel, B. (2012). Experimentell zum Funktionalen Denken: Eine empirische Untersuchung zur Wirkung von Schülerexperimenten als Ausgangspunkt mathematischer Begriffsbildung - Teilprojekt 7. In W. Rieß, M. Wirtz, B. Barzel & A. Schulz (Eds.), *Experimentieren im mathematisch-naturwissenschaftlichen Unterricht. Schüler lernen wissenschaftlich denken und arbeiten*. (pp. 265–284). Münster: Waxmann.

Glaser, B. G., & Strauss, A. A. L. (1998). *Grounded theory: Strategien qualitativer Forschung.* Bern: Huber.

Grigutsch, S. (1996). *Mathematische Weltbilder von Schülern. Struktur, Entwicklung, Einflußfaktoren*. (Doctoral dissertation) Universität Duisburg.

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137–161.

Henning, H. (Ed.) (2013). *Modellieren in den MINT-Fächern.* Münster: Verlag für wissenschaftliche Texte und Medien.

Kortenkamp, U. (2004). Experimental Mathematics and Proofs in the Classroom. *ZDM - International Journal on Mathematics Education, 36*(2), 61–66.

Kuntze, S. & Reiss, K. (2005). Situation-specific and generalized components of professional knowledge of mathematics teachers – Research on a video-based in-service teacher learning program. In H. L. Chick & J. L. Vincent (Hrsg.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education (PME), Vol. 3* (pp. 225–232). Melbourne: University.

Lavonen, J., Jauhiainen, J., Koponen, I. T., & Suonio, K. K. (2012). Effect of a long-term in-service training program on teachers' beliefs about the role of experiments in physics education. *International Journal of Science Education, 26*(3), 309–328.

McLeod, D. B. (1992). Research on Affect in Mathematics Education: A Reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 575–596). New York: Macmillan.

Pajares, M. F. (1992). Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct. *Review of Educational Research, 62*(3), 307–332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte: Information Age.

Tesch, M., & Duit, R. (2004). Experimentieren im Physikunterricht - Ergebnisse einer Videostudie. *Zeitschrift Für Didaktik Der Naturwissenschaften, 10*, 51–69.

Wang, T., & Cai, J. (2007). United States teachers' views of effective mathematics teaching and learning. *ZDM - International Journal on Mathematics Education, 39*(4), 315–327.