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College Admissions with Entrance Exams: Centralized versus Decentralized*

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Abstract

We study a college admissions problem in which colleges accept students by ranking students' efforts in entrance exams. Students' ability levels affect the cost of their efforts. We solve and compare equilibria of "centralized college admissions" (CCA) where students apply to all colleges and "decentralized college admissions" (DCA) where students only apply to one college. We show that lower ability students prefer DCA whereas higher ability students prefer CCA. Many predictions of the theory are supported by a lab experiment designed to test the theory, yet we find a number of differences that render DCA less attractive than CCA compared to the equilibrium benchmark.

JEL Classification: *C78; D47; D78; I21*

Keywords: College admissions, incomplete information, student welfare, contests, all-pay auctions, experiment.

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1 Introduction

Throughout the world and every year, millions of prospective university students apply for admission to colleges or universities during their last year of high school. Admission mechanisms vary from country to country, yet in most countries there are government agencies or independent organizations that offer *standardized admission exams* to aid the *college admission* process. Students invest a lot of time and effort to prepare for these admission exams, and they differ in terms of their ability to do so.

In some countries, the application and admission process is centralized. For instance, in Turkey university assignment is solely determined by a national examination called YGS/LYS. After learning their scores, students can then apply to a number of colleges. Applications are almost costless as all students need only to submit their rank-order of colleges to the central authority.[1] On the other hand, Japan has a centralized "National Center test," too, but all public universities including the most prestigious universities require the candidate to take another, institution-specific secondary exam which takes place on the same day. This effectively prevents the students from applying to more than one public university.[2] The admissions mechanism in Japan is decentralized, in the sense that colleges decide on their admissions independent of each other. Institution-specific exams that prevent students from applying to all colleges have also been used and debated in the United Kingdom, notably between the University of Cambridge and the University of Oxford. Currently, students cannot apply to both the University of Cambridge and the University of Oxford.[3] Moreover, till 1994 the college admission exams in South Korea were only offered on two dates each year, and students were allowed to apply for only one college per exam date (see Avery et al., 2014, for more details). In the Soviet Union, everyone had to submit the original of the school certificate together with the application to a college, and colleges had institution-specific exams. Thus, college admissions were fully decentralized. Although most of the former Soviet republics and Russia have lately introduced centralized exams and a centralized college admissions process, some colleges, typically the best ones, still run their own entry exams and thus opt out of the centralized system. Moreover, China transitioned to a centralized college admissions system from a decentralized system in the 195's, mainly to avoid miscoordination in the assignment of students to colleges (see the online appendix of Chen and Kesten, 2017).

In the United States, students take both centralized exams like the Scholastic Aptitude Test

[1] Greece, China, South Korea, and Taiwan have similar national exams that are the main criterion for the centralized mechanism of college admissions. In Hungary, the centralized admission mechanism is based on a score that combines grades from school with an entrance exam (Biro, 2012).

[2] There are actually two stages where the structure of each stage corresponds to our description and modeling of the decentralized mechanism in section 4. The difference between the stages is that the capacities in the first stage are much greater than those in the second stage. Moreover, the Japanese high school admissions authorities have adopted similar mechanisms in local districts. Although the mechanism adopted varies across prefectures and is changing year by year, its basic structure is that each student chooses one among a specified set of public schools and then takes an entrance exam at his or her chosen school. The exams are held on the same day.

[3] We thank Aytek Erdil and Ken Binmore for discussions regarding the college admission system in UK.

(SAT), and also complete college-specific requirements such as college admission essays. Students can apply to more than one college, but since the application process is costly, students typically send only a few applications (the majority being between two to six applications, see Chade et al., 2014). Hence, the United States college admissions mechanism falls in-between the two extreme cases.

In this paper, we compare the institutional effects of different college admission mechanisms on the equilibrium efforts of students, student welfare, and sorting. To do this, we model college admissions with admission exams as contests (or all-pay auctions) in which the cost of effort represents the payment made by the students. We focus on two extreme cases: in the centralized model (as in the Turkish mechanism) students can freely apply to all colleges, whereas in the decentralized model (as in the Japanese mechanism for public colleges) students can only apply to one college. For tractability, in our main model we consider two colleges that differ in quality and assume that students have homogeneous preferences for attending these colleges.[4]

More specifically, each of the n students gets a utility of v_1 by attending college 1 (which can accommodate q_1 students) and gets a utility of v_2 by attending college 2 (which can accommodate q_2 students). College 2 is the better and college 1 is the worse of the two colleges. The students' utility from not being assigned to any college is normalized to 0. Hence, $0 < v_1 < v_2$. Following most of the literature on contests with incomplete information, we assume that an ability level in the interval $[0, 1]$, is drawn i.i.d. from the common distribution function, and the cost of exerting an effort e for a student with ability level a is given by $\frac{e}{a}$. Thus, given an effort level, the higher the ability the lower the cost of exerting effort. Note that for reasons of tractability, in this modeling specification we consider the effort exerted in preparation for the entrance exam as a pure cost. We thereby abstract from the fact that it can also lead to new knowledge, good habit formation, and improved abilities.

In the centralized college admissions problem (CCA), all students rank college 2 over college 1. Hence, the students with the highest q_2 efforts get into college 2, students with the next highest q_1 efforts get into college 1, and students with the lowest $n - q_1 - q_2$ efforts are not assigned to any college. In the decentralized college admissions problem (DCA), students need to simultaneously choose which college to apply to and how much effort to exert. Then, for each college $i \in \{1, 2\}$, students with the highest q_i efforts *among the applicants to college* i get into college i.

It turns out that the equilibrium of CCA can be solved by standard techniques, such as in Moldovanu et al. (2012). In this monotone equilibrium, higher ability students exert higher efforts, and therefore the students with the highest q_2 ability levels get admitted to the good college 2, and students with ability rankings between $q_2 + 1$ and $q_1 + q_2$ get admitted to the bad college 1 (Proposition 1).

Finding the equilibrium of DCA is not straightforward. In equilibrium, there is a cutoff ability level that we denote by c. All higher ability students (with abilities in $(c, 1]$) apply to the good

[4]In section 6, we discuss the case with three or more colleges.

college, whereas lower ability students (with ability levels in $[0, c]$) use a *mixed strategy* when choosing between the good and the bad college. Students' effort functions are continuous and monotone in ability levels (Theorem 1). We also establish that the equilibrium we have found is the unique symmetric and monotone equilibrium.

Our paper therefore contributes to the all-pay contests literature. To the best of our knowledge, ours is the first paper to model and solve a game of competing contests with multiple prizes where the players have private information regarding their abilities and sort themselves into different contests.[5]

After solving for the equilibrium of CCA and DCA and proving their uniqueness, we compare the equilibria in terms of students' interim expected utilities. We show that students with lower abilities prefer DCA to CCA when the number of seats is smaller than the number of students (Proposition 2). The main intuition for this result is that students with very low abilities have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero due to the possibility of fewer applications than the capacity of a college. Moreover, we show that students with higher abilities prefer CCA to DCA (Proposition 3).[6] The main intuition for this result is that high-ability students (i) can only get a seat in the good college in DCA, whereas they can get seats in both the good and the bad college in CCA, and (ii) their equilibrium probability of getting a seat in the good college is the same across the two mechanisms. Moreover, DCA is dominated by CCA with respect to the sorting of students to colleges. This follows directly from the fact that miscoordination can occur in equilibrium in DCA but not in CCA. Such miscoordination can lead to the imperfect sorting of students and unfilled seats. Nevertheless, DCA can be ex ante preferable for the students, since it can entail lower equilibrium effort levels than CCA. This is intuitive given that a student in DCA only competes with the subset of students that have applied for the same college.

We test the theory with the help of lab experiments. Based on the findings from previous experimental studies, overexertion of effort can be expected for CCA. Whether similar deviations from the theoretical predictions are observed for DCA is an open question. Thus, the experiments allow us to test whether competing for the two colleges separately in DCA mitigates overexertion or not. We implement five markets for the college admissions game that are designed to capture different levels of competition (in terms of the supply of seats, the demand ratio, and the quality difference between the two colleges). We compare the two college admission mechanisms. The findings regarding students' ex-ante expected utilities, their effort levels, and the students' preferences regarding the two mechanisms given their ability are well organized by the theory. However, the experimental subjects in both treatments exert a higher effort than predicted. The overexertion of

[5]There is a large literature on competing auctions and other competing mechanisms. Competing contests with unit prizes and incomplete information are analyzed by DiPalantino and Vojnovic (2009). We discuss this literature in the next subsection.

[6]More specifically, we obtain a single crossing condition: if a student who applies to college 2 in DCA prefers CCA to DCA, then all higher ability students also have the same preference ranking.

effort is particularly pronounced in DCA, which makes it relatively less attractive for the applicants compared to CCA. We also find less sorting of students by abilities in DCA than in CCA, which is in part due to out-of-equilibrium choices of the subjects. Overall, the results resonate with the fact that in Japan, our prime example for DCA, every year a large group of students do not succeed in the entrance exam and then have to study for at least another year at private institutions to take the exam again. These students are called ronin (translated as masterless samurai).[7]

The rest of the paper is organized as follows. The introduction (section 1) ends with a discussion of the related literature. Section 2 introduces the model and preliminary notation. In sections 3 and 4 we solve the model for the Bayesian Nash equilibria of the centralized and decentralized college admission mechanisms, respectively. Section 5 offers comparisons of the equilibria of the two mechanisms while section 6 provides extensions. Section 7 presents our experimental results. Finally, section 8 concludes. Omitted proofs and additional figures are given in the Appendix.

1.1 Related literature

College admissions have been studied extensively in the economics literature. Following the seminal paper by Gale and Shapley (1962), the theory literature on two-sided matching mainly considers centralized college admissions and investigates stability, incentives, and the efficiency properties of various mechanisms, notably the deferred-acceptance and the top trading cycles. The student placement and school choice literature is motivated by the centralized mechanisms of public school admissions, rather than by the decentralized college admissions mechanism in the US. This literature was pioneered by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003). We refer the reader to Sönmez and Ünver (2011) for a recent comprehensive survey regarding centralized college admission models in the two-sided matching literature. Recent work regarding centralized college admissions with entrance exams include Abizada and Chen (2015) and Tung (2009). Abizada and Chen (2015) model the entrance (eligibility) criterion in college admissions problems and extend models of Perach et al. (2007) and Perach and Rothblum (2010) by allowing the students to have the same scores from the central exam. On the other hand, by allowing students to submit their preferences after they receive the test results, Tung (2009) adjusts the multi-category serial dictatorship (MSD) analyzed by Balinski and Sönmez (1999) in order to make students better off.

One crucial difference between the modeling in our paper and the literature should be emphasized: in our paper student preferences affect college rankings over students through contests among students, while student preferences and college rankings are typically independent in the

[7]Each year, around 110,000 students become ronins. Most of the prestigious universities such as the University of Tokyo and Kyoto University are public universities that rely on their own entrance exams. Among their incoming students in 2016 were 34.1% and 41.2% ronins, respectively (`http://www.u-tokyo.ac.jp/content/400043116.pdf` and `http://www.kyoto-u.ac.jp/ja/admissions/undergrad/documents/h28shotoukei_1.pdf`. These webpages were accessed on December 24, 2016).

two-sided matching models and school-choice models.

The analysis of decentralized college admissions in the literature is more recent. Chade et al. (2014) consider a model where two colleges receive noisy signals about the caliber of applicants. Students need to decide which colleges to apply to and application is costly. The two colleges choose admissions standards that act like market-clearing prices. The authors show that in equilibrium, college-student sorting may fail, and they also analyze the effects of affirmative action policies. In our model, the colleges are not strategic players as in Chade et al. (2014). Another important difference is that in our model the students not only have to decide which colleges to apply to, but also how much effort to exert in order to do well in the entrance exams. Che and Koh (2016) study a model in which two colleges make admission decisions subject to aggregate uncertainty about student preferences and linear costs for any enrollment exceeding the capacity. They find that colleges' admission decisions become a tool for strategic yield management, and in equilibrium, colleges try to reduce their enrollment uncertainty by strategically targeting students. In their model, as in Chade et al. (2014), students' exam scores are costlessly obtained and given exogenously. Avery and Levin (2010), on the other hand, analyze a model of early admission at selective colleges where early admission programs give students an opportunity to signal their enthusiasm to the college they would like to attend. More recently, motivated by the South Korean college admission system that went through a policy change in 1994, Avery et al. (2014) compare the two (with and without early admissions) regimes and conclude that lower-ranked colleges may gain in competition with higher-ranked colleges by limiting the number of possible applications.

In another related paper, Bodoh-Creed and Hickman (2016) also model college admissions as a Bayesian game where heterogeneous students compete for seats at colleges. They present a model in which there is a centralized allocation mechanism mapping each student's score into a seat at a college. Bodoh-Creed and Hickman (2016) are mostly interested in the effects of affirmative action policies and the solution concept used is "approximate equilibrium" in which the number of students is assumed to be large so that students approximately know their rankings within the realized sample of private costs. Similarly, Olszewski and Siegel (2014) consider contests with many players and prizes and show that the equilibrium outcomes of such contests are approximated by the outcomes of an appropriately defined set of mechanisms. In contrast to Bodoh-Creed and Hickman (2016) and Olszewski and Siegel (2014), our results are also applicable when the number of agents is not large. In a related paper, Morgan et al. (2012) study competition for promotion in a continuum economy. They show that a more meritocratic profession always succeeds in attracting the highest ability types, whereas a profession with superior promotion benefits attracts high types only under some assumptions.

In the extensions section 6.3, we also consider a case where we approximate a large number of students with a continuum in the presence of two colleges.[8] Due to the law of large numbers, we obtain a three-level step function under both DCA and CCA: low types "stay out," medium types

[8]We would like to thank an anonymous referee for suggesting that we analyze this case.

go to college 1, and high types go to college 2. Moreover, all agents who apply to the same college exert the same effort. This kind of equilibrium is not obtained in the aforementioned three papers that consider continuum economies, due to differences in the modeling assumptions. Hence, the extension of our model to a continuum of agents is not a special case of these three papers.

In another recent paper by Salgado-Torres (2013), students and colleges participate in a decentralized matching mechanism called costly signaling mechanism (CSM) in which students first choose a costly observable score to signal their abilities, then each college makes an offer to a student, and finally each student chooses one of the available offers. Salgado-Torres (2013) characterizes a symmetric equilibrium of CSM which is proven to be assertive and also performs some comparative statics analysis. CSM is decentralized just like the decentralized college admissions model developed in this paper. However, CSM cannot be used to model college admission mechanisms (such as the ones used in Japan) that require students to apply to *only one* college.

Our paper is also related to the all-pay auction and contests literature. Notably, Baye et al. (1996) and Siegel (2009) solve for all-pay auctions and contests with complete information. We refer the reader to the survey by Konrad (2009) about the vast literature on contests. Related to our decentralized mechanism, Amegashie and Wu (2006) and Konrad and Kovenock (2012) both model "competing contests" in a *complete information* setting. Amegashie and Wu (2006) study a model where one contest has a higher prize than the other. They show that sorting may fail in the sense that the top contestant may choose to participate in the contest with a lower prize. In contrast, Konrad and Kovenock (2012) study all-pay contests that are run simultaneously with multiple identical prizes. They characterize a set of pure strategy equilibria and a symmetric equilibrium that involves mixed strategies. In our decentralized college admissions model, the corresponding contest model is also a model of competing contests. The main difference in our model is that we consider incomplete information as students do not know each others' ability levels.

A series of papers by Moldovanu and Sela (and Shi) studies contests with incomplete information, but they do not consider competing contests in which the participation in contests is endogenously determined. In Moldovanu and Sela (2001), the contest designer's objective is to maximize expected effort. They show that when cost functions are linear or concave in effort, it is optimal to allocate the entire prize sum to a single first prize. Moldovanu and Sela (2006) compare the performance of dynamic sub-contests whose winners compete against each other with static contests. They show that with linear costs of effort, the expected total effort is maximized with a static contest, whereas the highest expected effort can be higher with contests with two divisions. Moldovanu et al. (2012) study optimal contest design where both awards and punishments can be used. Under some conditions, they show that punishing the bottom is more effective than rewarding the top.

There is a large literature on competing auctions and mechanisms; notable examples are Ellison et al. (2004), Biais et al. (2000), McAfee (1993), and more recently, Moldovanu et al. (2008), Virág

(2010), and Ovadia (2014). Two papers that are most related to our papers are DiPalantino and Vojnovic (2009) and Büyükboyacı (2016). DiPalantino and Vojnovic (2009) consider multiple contests where each contest gives a single prize and show the existence of a symmetric monotone equilibrium using the revenue equivalence theorem. They are mostly interested in the participation rates among different contests and establish that in the large system limit (i.e., as the population gets large) the number of players that participate in a given contest class is a Poisson random variable. Büyükboyacı (2016), on the other hand, theoretically and experimentally compares the performance of one contest with a single prize and two parallel contests each with a single prize. In her model agents can be either a high-ability or a low-ability type. Her main finding is that the designer's profit is higher in the parallel tournaments when the contestants' low- and high-ability levels are sufficiently differentiated. In a companion paper to this piece, Hakimov (2016) presents the results of a field experiment with a microfinance institution to compare a system with parallel contests to a standard contest when monetary prizes are awarded to the employees of the firm. Among other things, he finds that the effort levels are on average higher in the case of parallel (decentralized) contests.

This paper also contributes to the experimental literature on contests and all-pay auctions, summarized in a recent survey article by Dechenaux et al. (2014). Our setup in the centralized mechanism with heterogeneous agents, two non-identical prizes, and incomplete information is closely related to a number of existing studies by Barut et al. (2002), Noussair and Silver (2006), and Müller and Schotter (2010). These studies observe that agents overbid on average compared to the Nash prediction. Moreover, they find an interesting bifurcation, a term introduced by Müller and Schotter (2010), in that low types underbid and high types overbid. Regarding the optimal prize structure, it turns out that if players are heterogeneous, multiple prizes can be optimal to avoid the discouragement of weak players, see Müller and Schotter (2010). Higher effort with multiple prizes rather than with a single prize was also found in a setting with homogeneous players by Harbring and Irlenbusch (2003).

We are not aware of any previous experimental work related to our decentralized admissions mechanism where agents simultaneously choose an effort level and decide whether to compete for the high or the low prize.

The paper also belongs to the experimental literature on two-sided matching mechanisms and school choice starting with Kagel and Roth (2000) and Chen and Sönmez (2006).[9] These studies as well as many follow-up papers in this strand of the literature focus on the rank-order lists submitted by students in the preference-revelation games, but do not study effort choice. Thus, the rankings of students by the schools are exogenously given in these studies unlike in our setup where the colleges' rankings are endogenous.

[9] A recent example of theory combined with experiments in the school choice literature is Chen and Kesten (2016; 2017).

2 The Model

The college admissions problem with entrance exams, or simply the problem, is denoted by $(S, \mathcal{C}, (q_1, q_2), (v_1, v_2), F)$. There are two colleges – college 1 and college 2. We denote colleges by C. Each college $C \in \mathcal{C} := \{1, 2\}$ has a capacity q_C which represents the maximum number of students that can be admitted to college C, where $q_C \geq 1$.

There are n students. We denote the set of all students by S. Since we suppose homogeneous preferences of students, we assume that each student has the cardinal utility v_C from college $C \in \{1, 2\}$, where $v_2 > v_1 > 0$. Thus, we sometimes call college 2 the good college and college 1 the bad college. Each student's utility from not being assigned to any college is normalized to be 0. We assume that $q_1 + q_2 \leq n$.[10]

Each student $s \in S$ makes an effort $e_s \geq 0$ and is assigned to one college or receives no seat in any college. The mechanism takes into account the efforts by the students when determining the admissions.[11] The students are heterogeneous in terms of their abilities, and the abilities are their private information. More specifically, for each $s \in S$, $a_s \in [0, 1]$ denotes student s's ability. Abilities are drawn identically and independently from the interval $[0, 1]$ according to a continuous distribution function F that is common knowledge. We assume that F has a continuous density $f = dF > 0$. For a student s with ability a_s, putting in an effort of e_s results in a disutility of $\frac{e_s}{a_s}$. Hence, the total utility of a student with ability a from making effort e is $v_C - e/a$ if she is assigned to college C, and $-e/a$ otherwise.[12]

Before we move on to the analysis of the equilibrium of centralized and decentralized college admission mechanisms, we introduce some necessary notation.

2.1 Preliminary notation

First, for any continuous distribution F with density f, for $1 \leq k \leq m$, let $F_{k,m}$ denote the distribution of the k^{th}−(lowest) order statistics out of m independent random variables that are identically distributed according to F. That is,

$$F_{k,m}(a) \quad := \quad \sum_{j=k}^{m} \binom{m}{j} F(a)^j (1 - F(a))^{m-j}. \tag{1}$$

Moreover, let $f_{k,m}(\cdot)$ denote $F_{k,m}(\cdot)$'s density:

$$f_{k,m}(a) \quad := \quad \frac{d}{da} F_{k,m}(a) = \frac{m!}{(k-1)!\,(m-k)!} F(a)^{k-1} (1 - F(a))^{m-k} f(a). \tag{2}$$

[10]Many college admissions, including ones in Turkey and Japan, are competitive in the sense that the total number of seats in colleges is smaller than the number of students who take the exams.

[11]In reality the performance in the entrance exams is only a noisy function of efforts. For tractability, we assume that efforts completely determine the performance in the tests.

[12]Note that when $e = a = 0$, $-e/a$ is not well-defined. We assume $-e/a = 0$ for this case.

For convenience, we let $F_{0,m}$ be a distribution with $F_{0,m}(a) = 1$ for all a, and $f_{0,m} \equiv dF_{0,m}/da = 0$.

Next, define the function $p_{j,k} : [0,1] \to [0,1]$ as follows: for all $j, k \in \{0, 1, \ldots, n\}$ and $x \in [0,1]$,

$$p_{j,k}(x) := \binom{j+k}{j} x^j (1-x)^k. \tag{3}$$

The function $p_{j,k}(x)$ is interpreted as the probability that when there are $(j+k)$ students, j students are selected for one event with probability x and k students are selected for another event with probability $(1-x)$. Suppose that $p_{0,0}(x) = 1$ for all x. Note that with this definition, we can write

$$F_{k,m}(a) \;\; = \;\; \sum_{j=k}^{m} p_{j,m-j}(F(a)). \tag{4}$$

3 The Centralized College Admissions Mechanism (CCA)

In the centralized college admissions game, each student $s \in S$ simultaneously makes an effort e_s. Students with the top q_2 efforts are assigned to college 2 and students with the efforts from the top $(q_2 + 1)$ to $(q_1 + q_2)$ are assigned to college 1. The rest of the students are not assigned to any college.[13] We now solve for the symmetric Bayesian Nash equilibrium of this game.[14] The following proposition is a special case of the all-pay auction equilibrium which has been studied by Moldovanu and Sela (2001) and Moldovanu et al. (2012).

Proposition 1. *In CCA, there is a unique symmetric equilibrium β^C such that for each $a \in [0,1]$, each student with ability a chooses an effort $\beta^C(a)$ according to*

$$\beta^C(a) = \int_0^a x \, \{ f_{n-q_2,n-1}(x)\, v_2 + (f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x))\, v_1 \} \, dx.$$

where $f_{k,m}(\cdot)$ for $k \geq 1$ is defined in Equation (2) and $f_{0,m}(x)$ is defined to be 0 for all x.

Proof. Suppose that β^C is a symmetric equilibrium effort function that is strictly increasing and differentiable. Consider a student with ability a who chooses an effort as if her ability is a'. Her expected utility is

[13] The game reflects how the how the Turkish college admission mechanism works if student preferences are homogeneous. Since all students would put college 2 as their top choice and college 1 as their second choice in their submitted preferences, the resulting assignment would be the same as the assignment described above. In a school choice context, this can be described as the following two-stage game. In the first stage, there is one contest where each student s simultaneously makes an effort e_s. The resulting effort profile $(e_s)_{s \in S}$ is used to construct a single priority profile $\succ$ such that a student with a higher effort has a higher priority. In the second stage, students participate in the centralized deferred acceptance mechanism where colleges use the common priority $\succ$.

[14] Focusing on symmetric equilibria (where monotonicity and differentiability is assumed) is standard in the literature on contests. Analyzing the possibility of asymmetric equilibria is beyond the scope of this paper.

$$v_2\, F_{n-q_2,n-1}(a') + v_1\; (F_{n-q_1-q_2,n-1}(a') - F_{n-q_2,n-1}(a')) - \frac{\beta^C(a')}{a}.$$

The first-order condition at $a' = a$ is

$$v_2\, f_{n-q_2,n-1}(a) + v_1\,(f_{n-q_1-q_2,n-1}(a) - f_{n-q_2,n-1}(a)) - \frac{[\beta^C(a)]'}{a} = 0.$$

Thus, by integration and as the boundary condition is $\beta^C(0) = 0$, we have

$$\beta^C(a) = \int_0^a x\, \{f_{n-q_2,n-1}(x)\, v_2 + (f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x))\, v_1\}\, dx.$$

The above strategy is the unique symmetric equilibrium candidate obtained via the "first-order approach" by requiring no benefit from local deviations. Standard arguments show that this is indeed an equilibrium by making sure that global deviations are not profitable (for instance, see section 2.3 of Krishna, 2002). □

4 The Decentralized College Admissions Mechanism (DCA)

In the decentralized college admissions game, each student s chooses one college C_s and an effort e_s simultaneously. Given the college choices of students $(C_s)_{s\in S}$ and efforts $(e_s)_{s\in S}$, each college C admits students with the top q_C effort levels among its set of applicants ($\{s \in S \,|\, C_s = C\}$).[15]

For this game, we focus on "symmetric and monotone" Bayesian Nash equilibrium. More specifically, we consider the case in which (i) the students' strategies only depend on their ability levels and not their names, and (ii) when we consider the effort levels of students who are applying to a particular college, higher ability students choose higher efforts.

A natural equilibrium candidate is to have a cutoff $c \in (0, 1)$ such that students with abilities in $[0, c]$ to apply to college 1, and students with abilities in $(c, 1]$ to apply to college 2. It turns out that there is no equilibrium of this kind. In such an equilibrium, (i) type c has to be indifferent between applying to college 1 or college 2, (ii) type c's effort is strictly positive in case of applying to college 1, and 0 when applying to college 2. Hence, there is a discontinuity in the effort function. These two conditions together imply that a type $c + \epsilon$ student would benefit from mimicking type c. We show this in Proposition 4 in Appendix B.1. Hence, we obtain that there is no symmetric and monotone pure strategy Nash equilibrium.

[15] In a setup with homogeneous student preferences, this game reflects how the Japanese college admissions mechanism works: all public colleges hold their own tests and accept the top performers among the students *who take their tests*. In the school choice context, this can be described as the following two-stage game. In the first stage, students simultaneously choose which college to apply to, and without knowing how many other students have applied, they also choose their effort level. For each college $C \in \{1, 2\}$, the resulting effort profile $(e_s)_{\{s\in S \,|\, C_s = C\}}$ is used to construct one priority profile $\succ_C$ such that a student with a higher effort has a higher priority. In the second stage, students participate in two separate deferred acceptance mechanisms where each college C uses the priority $\succ_C$.

Therefore, some students have to use mixed strategies when choosing which college to apply to. Next, as we formally show in Proposition 5 in Appendix B.1, when the students use mixed strategies in a symmetric and monotone equilibrium, they choose the same effort level when they apply to either of the colleges. This is surprising at first sight, yet it follows from a "revelation principle" argument: when students mix, they have to be indifferent between applying to either colleges, but since both games are Bayesian incentive-compatible, expected utilities being the same implies expected payments or efforts being the same.[16] In this equilibrium, lower ability students choose the same effort level independent of whether they are applying to college 1 or 2. Note that this is an equilibrium property, not a restriction on effort functions. In other words, students are allowed to choose different effort levels when they are applying to different colleges, yet they choose the same effort level in equilibrium.

Before we move on to the derivation of a mixed-strategy Nash equilibrium, a brief recap of our earlier arguments and an explanation of the equilibrium properties are in order.[17] As the equilibrium of the kind where students with abilities in $[0, c]$ apply to college 1, and students with abilities in $(c, 1]$ apply to college 2 does not exist, some students have to use mixed strategies when choosing which college to apply to. The students who mix are the lower ability students. For high-ability students the probability of getting a seat in the good college is high and they cannot be made indifferent by getting a seat in the bad college 1 for sure. Thus, the student at the cutoff ability is indifferent between getting a seat at the bad college 1 for sure or getting a seat in the good college 2 with some probability. All students below the cutoff use mixed strategies when choosing which college to apply to. Due to the revelation principle, a student who uses a mixed strategy when choosing which college needs to exert the same effort independent of the choice of the college. Therefore, in a symmetric and monotone equilibrium, (i) we have to have mixing in college choices, and (ii) mixing entails identical effort. Hence, we are led to consider the case where all types below a certain cutoff mix when deciding which college to apply to and choose the identical effort in either case.

In what follows, we consider a symmetric and monotone equilibrium in which low-ability students use mixed strategies while high-ability students are certain to apply to the better college. More specifically, $(\gamma(\cdot), \beta^D(\cdot); c)$ where $c \in (0, 1)$ is a cutoff, $\gamma : [0, c] \to (0, 1)$ is the mixed strategy that represents the probability of lower ability students applying to college 1, and $\beta^D : [0, 1] \to \mathbb{R}$ is the continuous and strictly increasing effort function that is differentiable in $[0, c)$ and $(c, 1]$. Each student with type $a \in [0, c]$ chooses college 1 with probability $\gamma(a)$ (hence chooses college 2 with probability $1 - \gamma(a)$), and makes effort $\beta^D(a)$. Each student with type $a \in (c, 1]$ chooses college 2 for sure, and makes effort $\beta^D(a)$.

[16] More specifically, types up to c obtain the same utility regardless of the college they apply to, and by the envelope theorem the utility is the integral of the probability of winning times the value of winning. This integral equation implies that the probability of winning times the value of winning is the same in both colleges, so the equal utility implies that the effort is the same as well.

[17] We would like to thank an anonymous referee for suggesting this discussion.

We now move on to the derivation of symmetric and monotone Bayesian Nash equilibrium. Let a symmetric strategy profile $(\gamma(\cdot), \beta(\cdot); c)$ be given. For this strategy profile, the ex-ante probability that a student applies to college 1 is $\int_0^c \gamma(x) f(x)dx$, while the probability that a student applies to college 2 is $1 - \int_0^c \gamma(x) f(x)dx$. Let us define a function $\pi : [0, c] \to [0, 1]$ that represents the ex-ante probability that a student has a type less than a *and* she applies to college 1:

$$\pi(a) := \int_0^a \gamma(x) f(x)dx. \tag{5}$$

With this definition, the ex-ante probability that a student applies to college 1 is $\pi(c)$, while the probability that a student applies to college 2 is $1 - \pi(c)$. Moreover, $p_{m,k}(\pi(c))$ is the probability that m students apply to college 1 and k students apply to college 2 where $p_{m,k}(\cdot)$ is given in Equation (3) and $\pi(\cdot)$ is given in Equation (5).

Next, we define $G(\cdot) : [0, c] \to [0, 1]$, where $G(a)$ is the probability that a type is less than or equal to a, conditional on the event that she applies to college 1. That is,

$$G(a) := \frac{\pi(a)}{\pi(c)}.$$

Moreover let $g(\cdot)$ denote $G(\cdot)$'s density. $G_{k,m}$ is the distribution of the $k^{th}-$order statistics out of m independent random variables that are identically distributed according to G as in equations (1) and (4). Also, $g_{k,m}(\cdot)$ denotes $G_{k,m}(\cdot)$'s density.

Similarly, let us define $H(\cdot) : [0, 1] \to [0, 1]$, where $H(a)$ is the probability that a type is less than or equal to a, conditional on the event that she applies to college 2. That is, for $a \in [0, 1]$,

$$H(a) = \begin{cases} \frac{F(a)-\pi(a)}{1-\pi(c)} & \text{if } a \in [0, c], \\ \frac{F(a)-\pi(c)}{1-\pi(c)} & \text{if } a \in [c, 1]. \end{cases}$$

Moreover, let $h(\cdot)$ denote $H(\cdot)$'s density. Note that h is continuous but is not differentiable at c. Let $H_{k,m}$ be the distribution of the $k^{th}-$order statistics out of m independent random variables distributed according to H as in equations (1) and (4). Also, $h_{k,m}(\cdot)$ denotes $H_{k,m}(\cdot)$'s density.

We are now ready to state the main result of this section, which characterizes the *unique* symmetric and monotone Bayesian Nash equilibrium of the decentralized college admissions mechanism. The sketch of the proof follows the theorem, whereas the more technical part of the proof is relegated to Appendix B.

Theorem 1. *In DCA, there is a* unique *symmetric and monotone equilibrium* $(\gamma, \beta^D; c)$ *where a student with type* $a \in [0, c]$ *chooses college 1 with probability* $\gamma(a)$ *and makes effort* $\beta^D(a)$*; and a* *student with type $a \in [c,1]$ chooses college 2 for sure and makes effort $\beta^D(a)$. Specifically,*

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx.$$

The equilibrium cutoff c and the mixed strategies $\gamma(\cdot)$ are determined by the following four requirements:

(i) $\pi(c)$ uniquely solves the following equation for x

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x).$$

(ii) Given $\pi(c)$, c uniquely solves the following equation for x

$$v_1 = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(x)-\pi(c)}{1-\pi(c)}\right).$$

(iii) Given $\pi(c)$ and c, for each $a \in [0,c)$, $\pi(a)$ uniquely solves the following equation for $x(a)$

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)-x(a)}{1-\pi(c)}\Big) = v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{x(a)}{\pi(c)}\Big).$$

(iv) Finally, for each $a \in [0,c]$, $\gamma(a)$ is given by

$$\gamma(a) = \frac{\pi(c)B(a)}{(1-\pi(c))A(a) + \pi(c)B(a)} \in (0,1),$$

where

$$\begin{aligned} A(a) &:= v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big), \\ B(a) &:= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big). \end{aligned}$$

Proof. Suppose that each student with type $a \in [0,1]$ follows a strictly increasing effort function β^D that is differentiable in $[0,c)$ and $(c,1]$, and a type $a \in [0,c]$ chooses college 1 with probability $\gamma(a) \in (0,1)$, and a type in $(c,1]$ chooses college 2 for sure. A necessary condition for this to be an equilibrium is that each type $a \in [0,c]$ has to be indifferent between applying to college 1 or 2. Thus, for all $a \in [0,c]$,

$$
\begin{aligned}
&v_1\left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))G_{m-q_1+1,m}(a)\right) \\
&= v_2\left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))H_{m-q_2+1,m}(a)\right). \qquad (6)
\end{aligned}
$$

Note that in the above equality we skip the cost of efforts. This is because we have already established that students choose the same effort when applying to either college. The left-hand side is the expected utility of applying to college 1, while the right-hand side is the expected utility of applying to college 2. To see this, note that $\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c))$ and $\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c))$ are the probabilities that there are no more than $(q_1 - 1)$ and $(q_2 - 1)$ applicants in colleges 1 and 2, respectively. For $m \geq q_1$, $p_{m,n-m-1}(\pi(c))G_{m-q_1+1,m}(a)$ is the probability of getting a seat in college 1 with effort a when there are m other applicants in college 1. Similarly, for $m \geq q_2$, $p_{n-m-1,m}(\pi(c))H_{m-q_2+1,m}(a)$ is the probability of getting a seat in college 2 with effort a, when there are m other applicants in college 2.

The indifference condition (6) at $a = 0$ and $a = c$ will lead to those in (i) and (ii), which imply (iii) and (iv) together with the indifference condition (6), see Appendix B.2.

Finally, we derive the unique symmetric effort function β^D by taking a "first-order approach" in terms of $G(\cdot)$ and $H(\cdot)$ which are determined by the equilibrium cutoff c and the mixed strategy function γ. Consider a student with type $a \in [0, c]$. A necessary condition for the strategy to be an equilibrium is that she does not want to mimic any other type a' in $[0, c]$. Her utility maximization problem is given by

$$
\max_{a' \in [0,c]} v_2\left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))H_{m-q_2+1,m}(a')\right) - \frac{\beta^D(a')}{a}.
$$

where the indifference condition (6) is used to calculate the expected utility.[18] The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

[18] Equivalently, we can write the maximization problem as

$$
\max_{a' \in [0,c]} v_1\left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))G_{m-q_1+1,m}(a)\right) - \frac{\beta^D(a')}{a},
$$

With the same procedure, this gives the equivalent solution as

$$
\beta^D(a) = v_1 \int_0^a x \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))g_{m-q_1+1,m}(x)dx
$$

for each $a \in [0, c]$.

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition (which is $\beta^D(0) = 0$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$$

for all $a \in [0, c]$

Next, consider a student with type $a \in [c, 1]$. A necessary condition is that she does not want to mimic any other type a' in $[c, 1]$. Her utility maximization problem is then

$$\max_{a' \in [c,1]} v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a') \right) - \frac{\beta^D(a')}{a}.$$

Note that although the objective function is the same for types in $[0, c]$ and $[c, 1]$, it is not differentiable at the cutoff c. The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition of continuity (which is $\beta^D(c) = v_2 \int_0^c x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$$

for each $a \in [c, 1]$.

To complete the proof, we need to show that not only local deviations, but also global deviations cannot be profitable. In Appendix B.3, we check this and hence show that the uniquely derived symmetric strategy $(\gamma, \beta^D; c)$ is indeed an equilibrium. □

Before we move to interim utility comparisons between CCA and DDA, two remarks about the equilibrium in DCA are in order. First the equilibrium effort function $\beta^D(a)$ is a continuous and monotone increasing function and it is differentiable everywhere except at the kink point $a = c$. Second, the equilibrium mixing probability function $\gamma(a)$ is continuous in $[0, c]$, yet it may be increasing or decreasing and we do not necessarily have $\gamma(c) = 1$.

5 Comparisons

As illustrated in sections 3 and 4, the two mechanisms result in different equilibria. It is therefore natural to ask how the two equilibria compare in terms of interim student welfare. We denote by $EU^C(a)$ and $EU^D(a)$ the expected utility of a student with ability a under CCA and DCA, respectively.

Our first result concerns the preference of low-ability students.

Proposition 2. *Low-ability students prefer DCA to CCA if and only if $n > q_1 + q_2$.*

Proof. First, let us consider the case of $n > q_1 + q_2$. For this case it is not difficult to see that $EU^C(0) = 0$ (because the probability of being assigned to any college is zero), and $EU^D(0) > 0$ (because with a positive probability, type 0 will be assigned to a college). Since the utility functions are continuous, it follows that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^D(x) > EU^C(x)$.

Next, let us consider the case of $n = q_1 + q_2$. For this case, we have $EU^C(0) = v_1$. This is because with probability 1, type 0 will be assigned to college 1 by exerting 0 effort. Moreover, we have $EU^D(0) < v_1$. This is because type 0 should be indifferent between applying to college 1 and college 2, and in the case of applying to college 1, the probability of getting assigned to college 1 is strictly smaller than 1. Since the utility functions are continuous, it follows that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^C(x) > EU^D(x)$. □

Intuitively, when the seats are over-demanded (i.e., when $n > q_1 + q_2$), very low-ability students have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero. Hence, they prefer DCA.

Although this result merely shows that only students in the neighborhood of type 0 need to have these kinds of preferences, explicit equilibrium calculations for many examples (such as the markets we study in our experiments) result in a significant proportion of low-ability students preferring DCA. We provide a depiction of equilibrium effort levels and interim expected utilities for a specific example in Figure 1. In this example, the equilibrium effort levels are predicted to be lower in DCA than in CCA for a wide range of abilities.

Moreover, we establish the reverse ranking for the high-ability students. That is, the high-ability students prefer CCA in the following single-crossing sense: if a student who applies to college 2 in DCA prefers CCA to DCA, then all higher ability students have the same preference ranking.

Proposition 3. *Let c be the equilibrium cutoff in DCA. We have (i) if $EU^C(a) \geq EU^D(a)$ for some $a > c$, then $EU^C(a') > EU^D(a')$ for all $a' > a$, and (ii) if $EU^C(a) < EU^D(a)$ for some $a > c$, then $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$.*

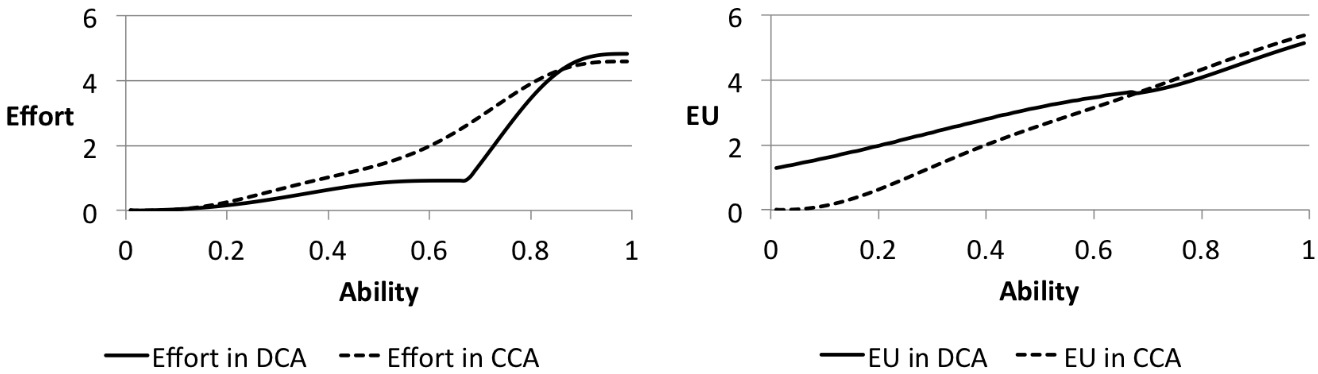

Figure 1: Efforts (left) and expected utility (right) under CCA and DCA

Note: The figures were created with the help of simulations for the following parameters: $n = 12$, $(q_1, q_2) = (5, 4)$, and $(v_1, v_2) = (5, 20)$. The equilibrium cutoff under DCA is calculated as $c = 0.675$.

Proof. Let us define

$$\begin{aligned} K(a) &\equiv v_2 F_{n-q_2,n-1}(a), \\ L(a) &\equiv v_1 \left(F_{n-q_1-q_2,n-1}(a) - F_{n-q_2,n-1}(a)\right), \\ M(a) &\equiv K(a) + L(a), \\ N(a) &= v_2 \left(\sum\nolimits_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum\nolimits_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a)\right). \end{aligned}$$

Then we have

$$EU^C(a) = M(a) - \frac{\int_0^a M'(x)\, x dx}{a}.$$

By integration by parts, we obtain

$$EU^C(a) = \frac{\int_0^a M(x)\, dx}{a}.$$

Similarly,

$$EU^D(a) = N(a) - \frac{\int_0^a N'(x)\, x dx}{a},$$

and by integration by parts, we obtain

$$EU^D(a) = \frac{\int_0^a N(x)\, dx}{a}.$$

Note that, for $a > c$, we have

$$N(a) = K(a).$$

Students whose ability levels are greater than c apply to college 2 in DCA, and therefore a seat is granted to a student with ability level $a > c$ if and only if the number of students with ability levels greater than a is not greater than q_2. This is the same condition in CCA, which is given by the expression $K(a)$. (Also note that we have $N(a) \neq K(a)$ for $a < c$, in fact we have $N(a) > K(a)$, but this is irrelevant for what follows.)

Now, for any $a > c$, we obtain

$$\begin{aligned}\frac{d}{da}\left(aEU^C(a)\right) &= M(a) \\ &= K(a) + L(a)\end{aligned}$$

and

$$\begin{aligned}\frac{d}{da}\left(aEU^D(a)\right) &= N(a) \\ &= K(a).\end{aligned}$$

Since $L(a) > 0$, for any $a > c$, we have

$$\frac{d}{da}\left(aEU^C(a)\right) > \frac{d}{da}\left(aEU^D(a)\right),$$

or

$$EU^C(a) + a\frac{d}{da}EU^C(a) > EU^D(a) + a\frac{d}{da}EU^D(a).$$

This means that for any $a > c$, whenever $EU^C(a) = EU^D(a)$, we have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. Then we can conclude that once $EU^C(a)$ is higher than $EU^D(a)$, it cannot cut through $EU^D(a)$ from above to below and $EU^C(a)$ always stays above $EU^D(a)$. To see this suppose $EU^C(a) > EU^D(a)$ and $EU^C(a') < EU^D(a')$ for some $a' > a > c$, then (since both $EU^C(a)$ and $EU^D(a)$ are continuously differentiable) there exists $a'' \in (a, a')$ such that $EU^C(a'') = EU^D(a'')$ and $\frac{d}{da}EU^C(a'') < \frac{d}{da}EU^D(a'')$, a contradiction. Hence (i) is satisfied. Moreover, (ii) is obviously satisfied since whenever $EU^C(a) < EU^D(a)$, we have to have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. $\square$

Intuitively, since high-ability students (i) can only get a seat in the good college in DCA whereas they can get a seat in both the good and the bad college in CCA, and (ii) their equilibrium probability of getting a seat in the good college is the same across the two mechanisms, they prefer CCA.[19]

We should also note that Proposition 3 establishes a single-crossing result in the sense that for types above the cutoff, the graph of the expected utility of DCA crosses that of CCA from below

[19]More specifically, we observe that the derivative of the "interim expected utility multiplied by type" is equal to the allocation probabilities. Moreover, in CCA the probability of getting a seat is larger than in the DCA. This is because in CCA, a student can get into the bad school if he does not get into the good school, whereas in DCA he cannot. This would imply that the derivatives of the "interim expected utilities times type" are ranked for all students $a > c$, but the derivatives of just the interim expected utilities are not necessarily ranked. So there is a slight difference from the standard intuition for auctions. We would like to thank an anonymous referee for pointing this out.

at most once. But it may as well be the case that this crossing never happens. For instance, in Figure 3 where we provide the equilibrium expected utility comparisons, DCA is preferred for all types in market 2. One may also wonder whether there is a general ex-ante utility ranking of DCA and CCA. It turns out that examples where either DCA or CCA result in higher ex-ante utility (or social welfare) can be found. Specifically, markets 1 and 2 in our experimental sessions result in higher social welfare in CCA and DCA, respectively.

Lastly, we briefly discuss the comparison between DCA and CCA with regard to the sorting of students. Clearly, in CCA the students are perfectly sorted in the sense that in equilibrium students with the highest ability levels are placed in the better college, those of medium ability levels are placed in the worse college, and those of the lowest ability levels are not placed. On the other hand, since mixed strategies are used in DCA, the sorting is not perfect. More specifically, among the students who have abilities lower than the cutoff it may be possible that a student of lower ability is placed in the good college and a student of higher ability is placed in the worse college. Moreover, DCA may result in empty seats together with unassigned students–again due to the use of mixed strategies and randomness of how many students are above and below the cutoff. Nevertheless, this does not mean that DCA always results in lower social welfare than CCA. This is because DCA may entail lower equilibrium effort levels than CCA and since lower efforts increase the utility of the students, DCA can be ex-ante preferable to CCA.

6 Extensions

In this section, we consider three extensions of the model. In the first, we allow for more than two colleges, again ranked in terms of quality. The second extension looks at a larger market in the following sense: as before, a setup is studied with two types of colleges resulting in utilities v_1 and v_2 and with capacities q_1 and q_2, but there are k colleges of each type and there are $k \times n$ students. Finally, in the third extension we consider the case of two colleges and a continuum of students.

6.1 The case of ℓ colleges

Let us consider ℓ colleges, $1, ..., \ell$, where each college k has the capacity $q_k > 0$ and each student gets the utility v_k from attending college k ($v_\ell > v_{\ell-1} > ... > v_2 > v_1 > 0$).

We conjecture that in the decentralized mechanism there will be a symmetric Bayesian Nash equilibrium $((\gamma_k)_{k=1}^{\ell}, \beta^D, (c_k)_{k=0}^{\ell})$:[20] (i) $c_0, \ldots, c_\ell$ are cutoffs such that $0 = c_0 < c_1 < \ldots < c_{\ell-1} < c_\ell = 1$; (ii) β^D is an effort function where each student with ability a makes an effort level of $\beta^D(a)$; (iii) $\gamma_1, \ldots, \gamma_\ell$ are mixed strategies such that for each $k \in \{1, \ldots, \ell-1\}$, each student with ability $a \in [c_{k-1}, c_k)$ applies to college k with probability $\gamma_k(a)$ and college $k+1$ with probability $1-\gamma_k(a)$.

[20]As explained below, the strategies are not formally shown to be an equilibrium since we do not have a proof to show that global deviations are not profitable.

Moreover, each student with ability $a \in [c_{\ell-1}, 1]$ applies to college ℓ, equivalently, $\gamma_\ell(a) = 1$. The equilibrium effort levels can be identified as follows.

Let $k \in \{1, \ldots, \ell\}$ be given. Let $\pi^k(a)$ denote the ex-ante probability that a student has a type less than or equal to a and she applies to college k. Then, $\pi^1(a) = \int_0^a \gamma_1(x)dF(x)$. For $k \in \{2, \ldots, \ell\}$ and $a \in [c_{k-2}, c_k]$,

$$\pi^k(a) = \begin{cases} \int_{c_{k-2}}^{a}(1 - \gamma_{k-1}(x))dF(x) & \text{if } a \leq c_{k-1}, \\ \int_{c_{k-2}}^{c_{k-1}}(1 - \gamma_{k-1}(x))dF(x) + \int_{c_{k-1}}^{a} \gamma_k(x)dF(x) & \text{if } a \geq c_{k-1}. \end{cases}$$

We define H^k to be the probability that a type is less than or equal to a, conditional on the event that she applies to college k:

$$H^k(a) = \frac{\pi^k(a)}{\pi^k(c_k)}.$$

In this equilibrium, each student with ability $a \in [c_{k-1}, c_k]$ exerts an effort of

$$\beta^D(a) = \beta^D(c_{k-1}) + \int_{c_{k-1}}^{a} x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k))h^k_{m-q_k+1,m}(x)dx$$

where $\beta^D(0) = 0$ and $h^k_{m-q_k+1,m}$ is the density of $H^k_{m-q_k+1,m}$. Similar to Theorem 1, it is possible to determine the formulation for cutoffs $c_1, \ldots, c_{\ell-1}$ and mixed strategies $\gamma_1, \ldots, \gamma_\ell$ using the indifference conditions (see the online appendix, part C). This set of strategies can be shown to satisfy immunity with respect to local deviations, but prohibitively tedious arguments to check for immunity with respect to global deviations (as we have done in Appendix B) prevent us from formally proving that it is indeed an equilibrium.

By supposing an equilibrium of this kind, we can actually show that propositions 2 and 3 hold for ℓ colleges. Proposition 2 trivially holds, as students with the lowest ability levels get zero utility from CCA and strictly positive utility from DCA. We can also argue that Proposition 3 holds since the students with ability levels $a \in [c_{\ell-1}, 1]$ only apply to college ℓ. This can be observed by noting that a seat is granted to these students in college k if and only if the number of students with ability levels greater than a is no greater than q_ℓ, which is the same condition in CCA. Hence, even in this more general setup of ℓ colleges, we can argue that low-ability students prefer DCA whereas high-ability students prefer CCA.

6.2 A k-replication of the economy

Consider an environment in which we have (i) k type-1 colleges: $C_1^1, ..., C_1^k$ such that each of them has q_1 seats and gives a utility of v_1 to students, (ii) k type-2 colleges: $C_2^1, ..., C_2^k$ such that each of them has q_2 seats and gives a utility of v_1 to students, and (iii) $k \times n$ students. In other words, in this extension we consider a "k-replication" of our model.

With this extension, in CCA it is easy to see that there is a monotone equilibrium very similar to the original equilibrium. The students will list all type-2 colleges above all type-1 colleges (in an arbitrary fashion), students with the top $k \times q_2$ effort levels will get one of the type-2 college seats, and students with the next top $k \times q_1$ effort levels will get one of the type-1 college seats. In this equilibrium a student with type a will choose the effort

$$\beta^{C(k)}(a) = \int_0^a x\{f_{kn-kq_2,kn-1}(x)\, v_2 - (f_{kn-kq_2-kq_1,kn-1}(x) - f_{kn-kq_2,kn-1}(x))\, v_1\}dx.$$

Moreover, we have that $\beta^{C(k)}(a)$ will be very close to $\beta^{C}(a)$ for all $k = 2, ..., \infty$. In fact, when F is uniform we have

$$\begin{aligned}\beta^{C(k)}(a) &= a\left(\frac{n-q_2}{n}v_2 - \left(\frac{n-q_1-q_2}{n} - \frac{n-q_2}{n}\right)v_1\right)\\ &= a\left(v_2 + \frac{q_1 v_1 - q_2 v_2}{n}\right)\end{aligned}$$

for all $k = 1, 2, ..., \infty$. Hence, for uniform distributions, any k-replica economy bidding function is the same as in the no-replica economy.

In DCA, on the other hand, one can observe that the equilibrium of the k-replica economy essentially remains the same as in the no-replica economy: the cutoff c and equilibrium effort functions will be the same. The only differences would be that (i) each student of ability lower than c will apply to each type-1 college with probability $\frac{\gamma(a)}{k}$ and each type-2 college with probability $\frac{1-\gamma(a)}{k}$, and (ii) each student of ability higher than c will apply to each type-2 college with probability $\frac{1}{k}$.

Hence, if there are many students and many colleges (belonging to one of the two types), our predictions remain valid.

6.3 Continuum of students

For this extension, we consider only two colleges, but introduce a continuum of students applying to these two colleges. Consider a measure 1 continuum of students whose ability levels (a) are independently drawn from F over $[0, 1]$, a good college (college 2) with measure μ_2 of seats, and a bad college (college 1) with measure μ_1 of seats. Getting a seat in college $C \in \{1, 2\}$ gives a utility of v_C. We assume $v_2 > v_1 > 0$ and $\mu_1, \mu_2, \mu_1 + \mu_2 \in [0, 1]$. Students choose effort levels (e) where the cost of effort is $-\frac{e}{a}$ and then they are ranked by their efforts. In the centralized problem, the highest measure μ_2 of effort performers get seats in college 2, and the next measure μ_1 get seats in college 1. In the decentralized problem, students decide where to apply, and colleges consider only their own applicants and allocate the seats according to the ranking by effort.

Since this is a continuum economy, even when the students use mixed strategies to decide on effort levels (and for the decisions about where to apply), in equilibrium there will be "minimum required effort levels" for each college. Moreover, nobody will waste any effort, hence the students will choose the minimum effort required to get a seat.

Let us first consider the decentralized problem. Let e_C denote the equilibrium minimum required effort level for college C. Then, each type a would choose among the three options: (i) choose effort e_2 and apply to college 2, (ii) choose effort e_1 and apply to college 1, and (iii) choose effort 0. The corresponding utilities for these three cases are (i) $u_2(a) \equiv v_2 - \frac{e_2}{a}$, (ii) $u_1(a) \equiv v_1 - \frac{e_1}{a}$, and (iii) $u_0(a) \equiv 0$. Moreover, for this to be an equilibrium, the measure of students who choose (i) is equal to μ_2, and the measure of those who choose (ii) is equal to μ_1.

It is easy to see that: $u_1(a) \geq u_0(a)$ iff $a \geq \frac{e_1}{v_1}$; $u_2(a) \geq u_0(a)$ iff $a \geq \frac{e_2}{v_2}$; and $u_2(a) \geq u_1(a)$ iff $a \geq \frac{e_2 - e_1}{v_2 - v_1}$. From these observations, it follows that in equilibrium students with the highest ability apply to college 2, those with a medium ability apply to college 1, and those with the lowest ability apply to no college. We can put this more formally by denoting $c_2 := \frac{e_2 - e_1}{v_2 - v_1}$ and $c_1 := \frac{e_1}{v_1}$. A type a greater than c_2 applies to college 2 as $u_2(a) \geq u_1(a)$; a type a between c_1 and c_2 applies to college 1 as $u_1(a) \geq u_2(a)$ and $u_1(a) \geq u_0(a)$; a type smaller than c_1 applies to no college (or applies to any college; since her effort level is zero, she will not receive a seat anyway), as $u_0(a) \leq u_1(a)$. In equilibrium, we then have

$$1 - F(c_2) = \mu_2 \text{ and } F(c_2) - F(c_1) = \mu_1.$$

That is, $c_2 = F^{-1}(1 - \mu_2)$ and $c_1 = F^{-1}(1 - \mu_1 - \mu_2)$. Thus, by the definition of c_1 and c_2, we have e_1 and e_2 given by

$$\begin{aligned} e_1 &= F^{-1}(1 - \mu_1 - \mu_2)\, v_1, \\ e_2 &= F^{-1}(1 - \mu_2)(v_2 - v_1) + F^{-1}(1 - \mu_1 - \mu_2)\, v_1. \end{aligned}$$

For these specifications of e_1, e_2, c_1, and c_2, it is evident that the following is an equilibrium: types in $[c_2, 1]$ apply to college 2 and choose e_2, types in $[c_1, c_2)$ apply to college 1 and choose e_1, types in $[0, c_1)$ choose zero effort.

Moreover, for CCA the same effort levels are also an equilibrium: types in $[c_2, 1]$ choose e_2, types in $[c_1, c_2)$ choose e_1, types in $[0, c_1)$ choose zero effort. We can quickly verify that no types would like to deviate to any other effort level. Hence, the two mechanisms result in the same allocation when we have a continuum of students.

This result about the continuum economy raises the question of how the equilibrium of the finite economy approaches the equilibrium of the continuum model.[21] More specifically, does the equilibrium with a finite number of students studied in sections 3 and 4 approach the equilibrium

[21] There is a small literature that asks under which conditions equilibria of a finite economy converge to equilibria of a continuum economy as the economy grows. For notable examples see Green (1984) and Carmona (2003).

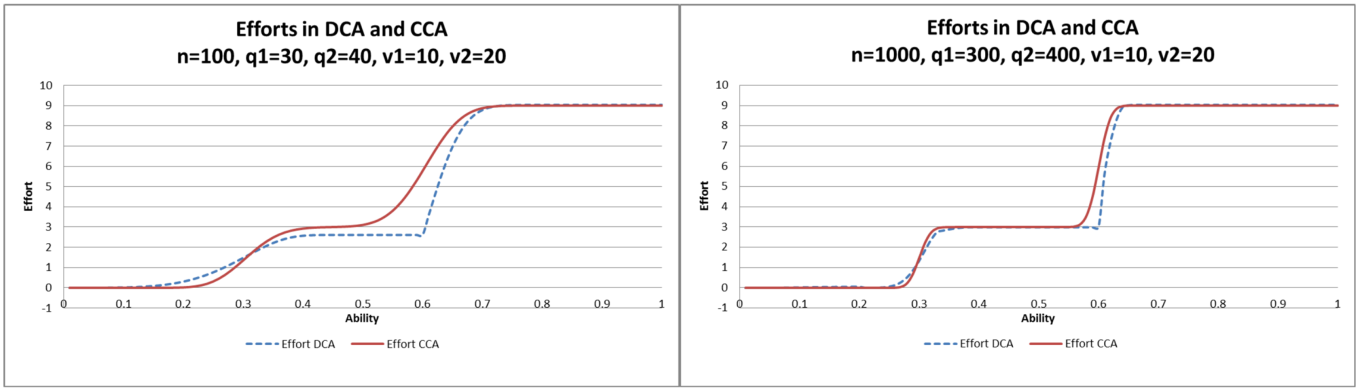

Figure 2: Equilibrium efforts with 100 students (left) and 1,000 students (right)

Note: The full lines indicate the equilibrium efforts for CCA and the dotted lines for DCA.

in section 6.3 as the number of students grows? We conjecture that such a convergence holds. Although—unfortunately—we do not have a formal proof, we numerically observe that as the number of students gets larger, the effort function of the equilibria of both centralized and decentralized problems converges to the same three-level step function. This can be seen in Figure 2 where we consider the number of students being 100 and 1,000 for the same parameter values of q_1, q_2, v_1, and v_2.

7 The Experiment

In the experimental literature on all-pay-auctions, overexertion of effort is a common finding. Our experiment is designed to explore whether our theoretical predictions regarding the comparison of the two admission systems are robust to this observation: we expect to observe an overexertion of effort in CCA, and the question is whether effort levels are close to the equilibrium in DCA or not. Thus, we study which of the mechanisms leads to higher (interim and ex-ante) student welfare, higher efforts of the students, and how the mechanisms affect the sorting of students by ability.

7.1 Design of the experiment

In the experiment that is framed in terms of university choice, participants can apply to college 1 or college 2. There are 12 students who apply for positions, and these students differ with respect to their ability. Every student learns her ability a_s that is drawn from the uniform distribution over the interval from 1 to 100. Students choose an effort level e_s that determines their success in the application process. The cost of effort is determined by $100\frac{e_s}{a_s}$. (Note that we use the range of abilities from 1 to 100 instead of 0 to 1 in order to simplify the calculations for subjects. Accordingly, we scaled up the cost function by a constant of 100.)

In the centralized college admissions mechanism (CCA), all students simultaneously choose

Table 1: Overview of market characteristics

	Number of seats at [value of] college 2	college 1	Predicted utility higher	Predicted effort higher
Market 1	6 [2000]	6 [1000]	CCA	depends; DCA in expectation
Market 2	2 [2000]	2 [1000]	DCA	no diff. in expectation
Market 3	2 [2000]	8 [1000]	depends; DCA in expectation	CCA
Market 4	3 [2000]	9 [1800]	CCA	DCA
Market 5	9 [2000]	1 [1000]	no diff. in expectation	no diff. in expectation

Notes: In some markets, one of the two mechanisms dominates the other for all students. In other markets the ranking of the mechanisms depends on the students' ability, in which case we compare the expected values.

an effort level. Then the computer determines the matching by admitting the students with the highest effort levels to college 2 up to its capacity q_2 and the next best students, i.e., from rank $q_1 + 1$ to rank $q_1 + q_2$, to college 1. All other students are unassigned.

In the decentralized college admissions mechanism (DCA), the students simultaneously decide not only on their effort level but also on which college to apply to. The computer determines the matching by assigning the students with the highest effort among those who have applied to college C, up to its capacity q_C.

We implemented five different markets that differ with respect to the total number of open slots ($q_1 + q_2$), the number of slots at each college (q_1 and q_2) as well as the value of the colleges for the students (v_1 and v_2), see Table 1. This allows us to investigate behavior under different market conditions. The parameters in each market were chosen so as to generate clear-cut predictions regarding the two main outcome variables, effort and the expected utility of each student.

Figure 3 shows the interim expected utility of students in equilibrium. CCA dominates DCA with respect to the interim expected utility of students in market 1, and the reverse holds in market 2. Figure 4 shows the equilibrium effort levels given abilities for each market. Effort is higher for all types in CCA in market 3 while the reverse holds for market 4. The fifth market is designed to make the two mechanisms as similar as possible.

In order to provide a valid comparison of the observed average effort and utility levels in the markets where there is no dominance relationship, i.e., the cells in Table 1 for which the predicted difference depends on the ability of the applicant, we compute the equilibrium effort and utility levels for the realizations of abilities in our experimental markets. We then take expected values given the realized abilities.

An important distinction for the theoretical predictions and for the intuition behind the predicted differences is whether the number of students is equal to the number of seats (markets 1 and 4) or whether there are more students than seats (markets 2, 3, and 5). As illustrated by Figure 4, in markets 1 and 4 with an equal number of seats and applicants, a strictly positive effort level in CCA is only exerted by those who can expect to get into the good college. In DCA, efforts are overall higher in these two markets because of the risk of miscoordination. In markets 2 and 3 in

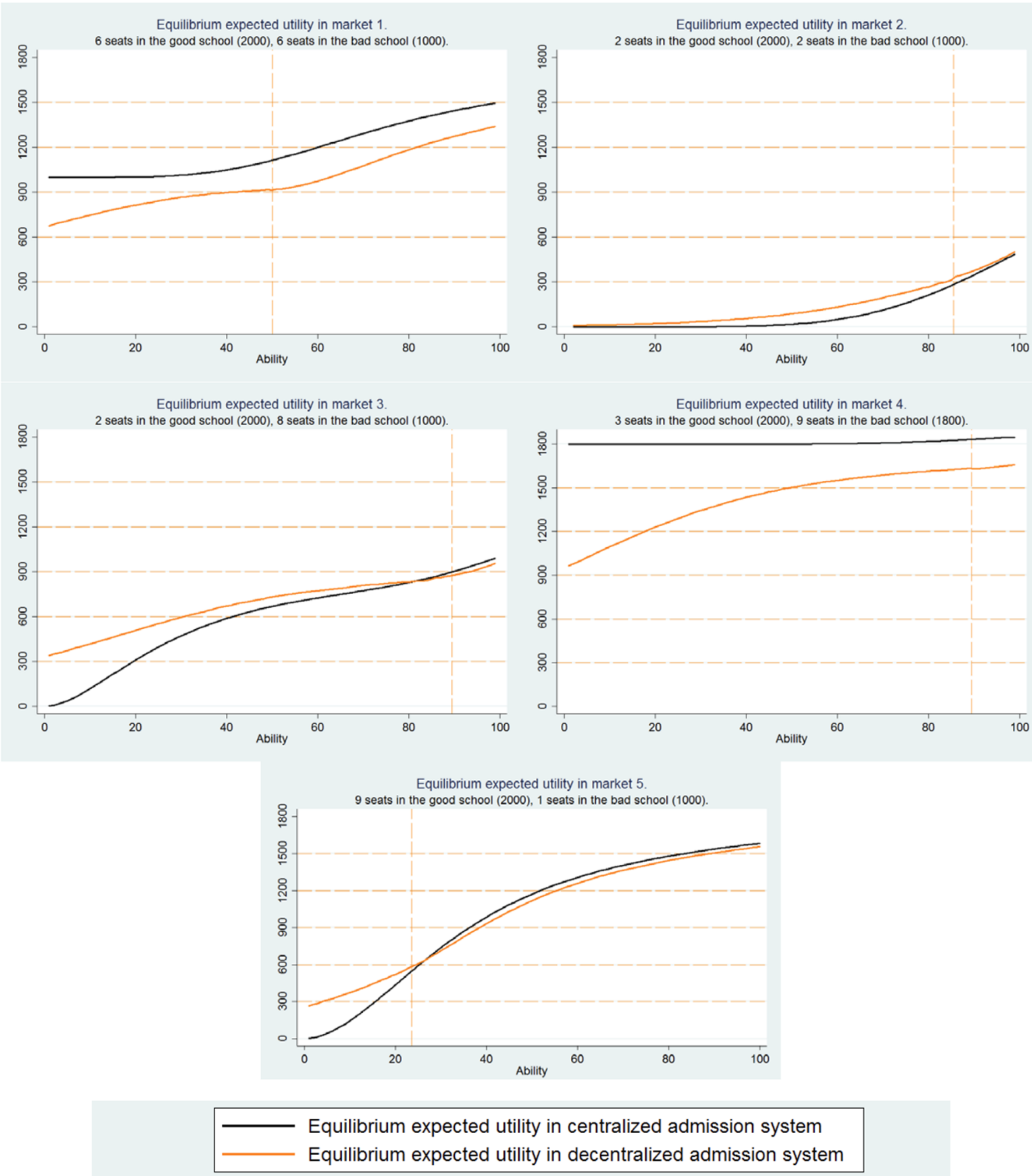

Figure 3: Equilibrium expected utility by ability

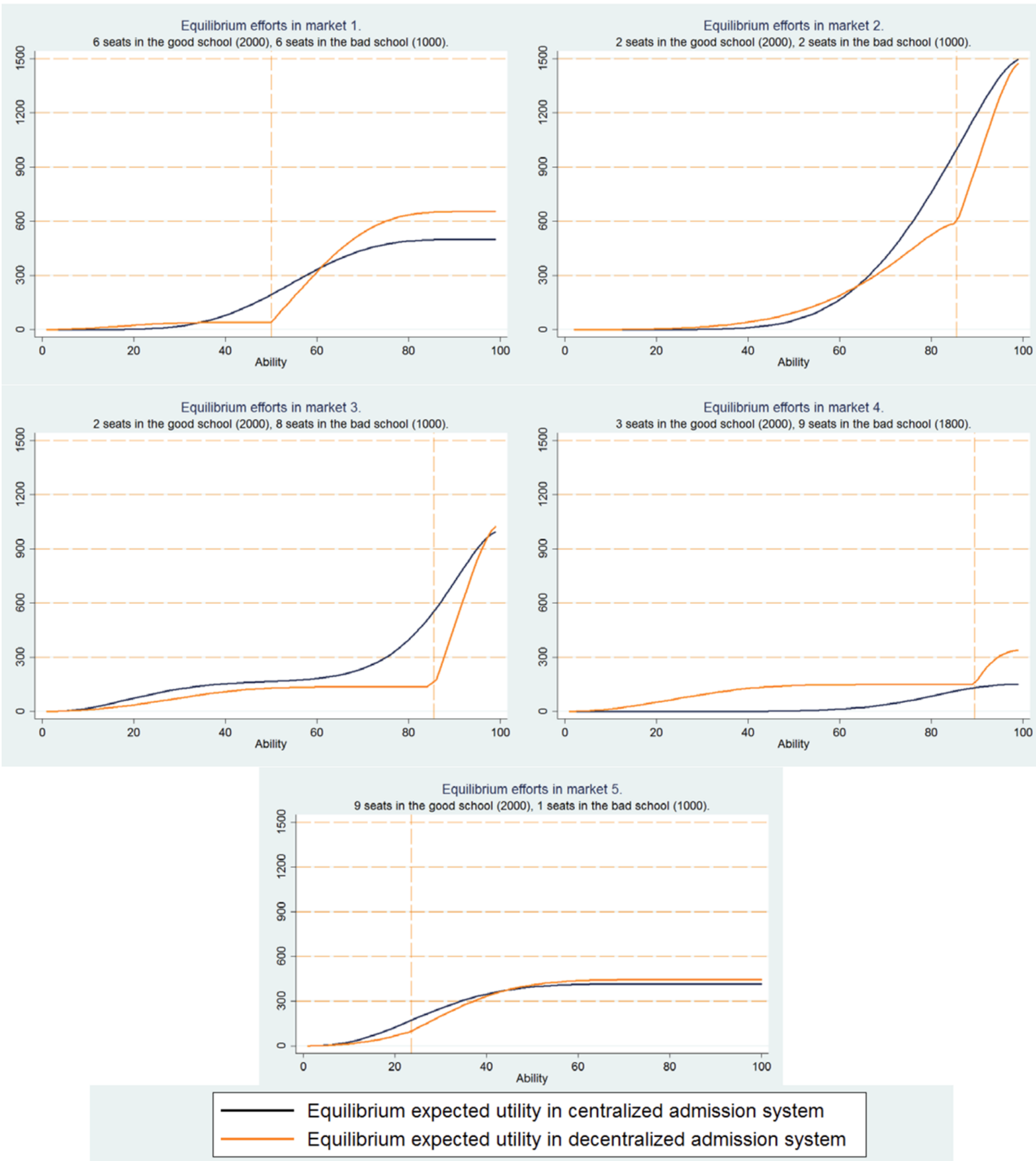

Figure 4: Equilibrium efforts by ability

contrast, high-ability students tend to exert less effort in DCA than in CCA because the expected return is higher in CCA: in CCA one can obtain v_2, v_1 and 0, while in DCA only v_2 (or v_1) and 0 are achievable.

Note that our design aims at comparing the two mechanisms. We do not study the comparative statics of the equilibria of CCA and DCA by systematically varying one parameter. This would require a completely different design that is beyond the scope of this study.

We employed a between-subjects design. Students were randomly assigned either to the treatment with CCA or the treatment with DCA. In each treatment, subjects played 15 rounds with one market per round. Each of the five different markets was played three times by every participant, and abilities were drawn randomly for every round. These draws were independent, and each ability was equally likely. We employed the same randomly drawn ability profiles in both treatments in order to make them as comparable as possible. Markets were played in blocks: first, all five markets were played in a random order once, then all five markets were played in a random order for a second time, and then again randomly ordered for the last time. We chose this sequence of markets in order to ensure that the level of experience does not vary across markets. Participants faced a new situation in every round as they never played the same market with the same ability twice. They received feedback about their allocation and about the points they earned after every round.

At the beginning of each round of the experiment, students received an endowment of 2,200 points. At the end of the experiment, one of the 15 rounds was randomly selected for payment. The points earned in this round plus the 2,200 endowment points were paid out in Euro with an exchange rate of 0.5 cents per point. The experiment lasted 90 minutes and the average earnings per subject were EUR 14.10.

The experiment was run at the experimental economics lab at the Technical University Berlin. We recruited student subjects from our pool with the help of ORSEE by Greiner (2004). The experiments were programmed in z-Tree, see Fischbacher (2007). For each of the two treatments, CCA and DCA, independent sessions were carried out. Each session consisted of 24 participants that were split into two matching groups of 12 for the entire session. In total, six sessions were conducted, that is, three sessions per treatment, with each session consisting of two independent matching groups of 12 participants. Thus, we end up with six fully independent matching groups and 72 participants per treatment.

At the beginning of the experiment, printed instructions were given to the participants (see Appendix E). Participants were informed that the experiment was about the study of decision-making, and that their payoff depended on their own decisions and the decisions of the other participants. The instructions were identical for all participants of a treatment, explaining in detail the experimental setting. Questions were answered in private. After reading the instructions, all individuals participated in a quiz to make sure that everybody understood the main features of the experiment. Moreover, subjects were provided with a calculator on the screen, which returned the payoff given their effort choice for each possible outcome, namely not being accepted, being accepted at the bad and at the good college. Subjects could use it as many times as they wanted before submitting their effort in every round.

7.2 Experimental results

We first present the aggregate results in order to compare the two mechanisms. In a second step, we study behavior in the two mechanisms separately to test the point predictions and to shed light on the reasons for the aggregate findings. The significance level of all our results is 5%, unless otherwise stated.

7.2.1 Treatment comparisons: Aggregate results

We compare the two mechanisms with respect to three properties, summarized in results 1 to 3. The first comparison concerns the utility of students in the two mechanisms which is equal to the number of points earned, due to the assumption of risk neutrality. Second, we investigate whether one of the mechanisms induces higher effort levels than the other mechanism. And third we ask whether individuals of different abilities prefer different mechanisms.

Result 1 (Average utility): In markets 1 and 4 (where the number of seats equals the number of students), the average utility of students in CCA is significantly higher than in DCA, as predicted by the theory. In markets 2 and 3 (where there are fewer seats than applicants), the average utility of students in DCA is not significantly higher than in CCA, in contrast to the theoretical predictions. In market 5, there is no significant difference both in theory and in the data.

Support. Table 2 presents the average number of points or the average utility of the participants in the two mechanisms in all five markets. Column (3) provides the equilibrium prediction as to which mechanism, CCA or DCA, leads to a higher utility of the students. To generate this prediction, we compute the equilibrium utilities given the realized draws of abilities in the experiment for both mechanisms. We then test for each market whether these equilibrium utilities are significantly different between the two mechanisms. Thus, column (3) displays the preferred mechanism and the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of equilibrium utilities. While columns (4) and (5) display the observed average utilities, the table also provides a normalized measure of the utility in columns (6) and (7) to evaluate the relative efficiency of the different markets. For the normalization, we fix the minimum possible payoff by assuming that all students spend their entire endowment on effort. The maximum possible payoff is reached when zero effort is exerted. We abstract from the possibility of payoffs lost due to miscoordination in DCA.[22]

[22]The minimum payoff including the possibility of miscoordination would require some ad hoc assumptions regarding the application behavior of students in DCA, which we want to avoid. The exact measure of normalized efficiency that we use is the following:

Table 2: Average utility

Market	Predicted utility higher for all students (2)	Predicted average utility higher for realized types (p-values) (3)	Observed average utility in CCA (4)	Observed average utility in DCA (5)	Observed normalized utility in CCA (6)	Observed normalized utility in DCA (7)	Observed utilities different in CCA and DCA (p-values) (8)
1	CCA	CCA (0.00)	1001	716	0.77	0.64	(0.02)
2	DCA	DCA (0.02)	-122	-169	0.72	0.70	(0.75)
3	N/A	DCA (0.00)	342	305	0.70	0.68	(0.63)
4	CCA	CCA (0.00)	1507	1014	0.84	0.62	(0.00)
5	N/A	N/A (0.63)	809	797	0.65	0.64	(1.00)

Notes: Columns (3) and (8) show the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on the averages of the six matching groups per treatment. Columns (6) and (7) display the normalized efficiency where maximum and minimum payoffs are given by zero and maximum effort levels, respectively, and no miscoordination regarding the choice of colleges.

In markets 1 to 4, we expect that the utility of students in the two mechanisms is significantly different. Column (8) provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the observed number of points earned in the two mechanisms. First note that the average utilities are always higher in CCA than in DCA. In markets 1 and 4, the equilibrium predictions for the comparison of utilities of students are consistent with the experimental data, since the average utility in CCA is significantly higher in both markets, in line with the theory. Thus, with an equal number of applicants and seats, CCA is preferable to DCA if the goal is to maximize the utility of the students. This is due to the potential miscoordination of applicants in DCA in these markets, inducing higher effort levels. However, we fail to observe the superiority of DCA in both markets where this is predicted, namely markets 2 and 3. The relationship is even reversed, with the average utility being higher in CCA than in DCA in both markets. Note also that the average utility is negative in the competitive market 2 (with only four seats for 12 students) such that, contrary to the prediction, the subjects earn less than the 2,200 points they are endowed with.

The normalized measure in columns (6) and (7) shows that in treatment CCA, markets 1 and 4 have the highest normalized efficiency. These markets have enough seats for all applicants such that the effort levels are relatively low, and thus the loss of efficiency due to effort costs is smaller than in the other markets. In DCA, markets 2 and 3 have the highest normalized efficiency. These markets are competitive, and it is in environments such as these that DCA is relatively beneficial since the risk of unfilled seats is low and effort incentives are dampened.

Next, let us turn to the effort choices of the students.

Result 2 (Average effort): In markets 1 and 4 (where the number of seats equals the number

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$$\text{Normalized efficiency} = \frac{\sum(\text{actual payoff}) - \text{Minimum possible total payoff}}{\text{Maximum possible total payoff} - \text{Minimum possible total payoff}}$$

.

of students), the average effort level of students in DCA is significantly higher than in CCA. This is in line with the predictions. In market 3, the average effort levels of students in CCA are not significantly higher than in DCA, in contrast to the theoretical prediction. In markets 2 and 5, there is no significant difference in effort between the two mechanisms, as predicted.

Support. Table 3 presents the average effort levels of the participants by different mechanisms and markets. Analogous to Table 2, the third column displays the equilibrium prediction regarding which mechanism leads to significantly higher effort levels. For this prediction, we compute the equilibrium effort levels given the realization of abilities in the five markets. The third column also indicates the p-values of the Wilcoxon rank-sum test regarding the difference between equilibrium efforts in CCA and DCA. We expect effort to differ significantly between the two mechanisms only in markets 3 and 4 (with a marginally significant difference in market 1). The last column provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the observed effort levels in the two mechanisms. The equilibrium predictions regarding the comparison of efforts in markets 1 and 4 are confirmed by the data because observed average effort is significantly higher in DCA. In market 3 average efforts are higher in CCA than in DCA as predicted, but the difference is not significant.

Table 3: Average effort

Market	Predicted effort higher for all students	Predicted average effort higher for realized types (p-values)	Observed average effort in CCA	Observed average effort in DCA	Observed effort different in CCA and DCA (p-values)
1	N/A	DCA (0.06)	276	362	(0.04)
2	N/A	N/A (0.15)	389	410	(0.75)
3	CCA	CCA (0.00)	397	354	(0.42)
4	DCA	DCA (0.00)	191	340	(0.02)
5	N/A	N/A (0.75)	400	395	(1.00)

Notes: Columns 3 and 6 show the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups per treatment.

Taking results 1 and 2 together, we observe that in markets without a shortage of seats (market 1 and market 4) students are on average better off in CCA where they exert less effort. In market 5 the results are also in line with the theoretical predictions with almost identical effort and expected utility levels in both mechanisms. In the two remaining markets with a surplus of students over seats, markets 2 and 3, the results do not support the equilibrium comparison of the two systems. Markets 2 and 3 should lead to a higher average utility of the students in DCA than in CCA, which is not observed in the lab. Therefore, the overall results suggest that with respect to the utility of students, CCA performs better than predicted relative to DCA.

Next, we turn to the question of whether students of different abilities prefer different mechanisms by providing an experimental test of propositions 2 and 3. According to Proposition 2, low-ability students prefer DCA over CCA if there are more applicants than seats in the market, as in our markets 2, 3, and 5. Proposition 3 implies that if any student who is above the cutoff in DCA prefers CCA over DCA then all students with a higher ability must also prefer CCA. (Remember that in markets 1 and 4, all students prefer CCA, and we therefore do not consider these markets here.)

Result 3 (Utility of low- and high-ability students): In markets 2 and 3 (with fewer seats than applicants), the average utilities of students with low abilities are higher in DCA, and the average utilities of students with high abilities are higher in CCA. However, significantly fewer students than predicted prefer DCA to CCA. There is no significant difference between the average utilities of students in DCA and CCA in market 5.

Support: In three of our markets, namely 2, 3, and 5, low-ability students prefer DCA in equilibrium. We refer to the *predicted switching point* as the maximum ability at which students prefer DCA in equilibrium. The predicted switching points by markets are represented in Figure 5 by the intersection of the broken lines. For market 2, the switching point is 100, for market 3 it equals 81, and for market 5 it equals 26. Figure 5 also shows the *observed switching points* as the intersection of the solid lines in markets 2, 3, and 5. The figure reveals that in markets 2 and 3, the observed switching points are substantially lower than the predicted switching points. This suggests that fewer students than predicted prefer DCA to CCA in these markets.

To assess the statistical significance of these differences in switching points, we use bootstrapping. That is, we sample from the dataset with replacement to generate new samples and calculate the bootstrap confidence intervals of the observed switching points in markets 2 and 3.[23] Before turning to the bootstrap confidence intervals, we first use the bootstrap samples to assess the theoretical prediction of a unique switching point with ability types above the switching point preferring CCA in all markets. The vast majority of the bootstrap samples indeed produce a unique switching point in the predicted direction, i.e., lower ability students prefer DCA while students with abilities above the switching point prefer CCA (Proposition 3).[24]

When restricting attention to bootstrap samples with a unique switching point in the predicted direction, the average switching point in market 2 is 48.4 with a 95% confidence interval of [30.5,

[23] Bootstrap confidence intervals are calculated by the percentile method (Efron, 1982). We perform block resampling to account for the dependence of observations within matching groups (see Davison, 1997). For each of 50,000 bootstrap samples, we draw six random matching groups with replacement and calculate the bootstrap switching point for each market based on the polynomial smoothing of the observed utilities (we use lpoly in STATA with bandwidth 15 both for the bootstrap and for producing Figure 5) in the online appendix, part C. We did not calculate bootstrap confidence intervals for market 5 because there is no significant difference in the expected utility for high- and low-ability students in the two systems, as predicted.

[24] In market 2, 77.5% of the bootstrap samples yield a unique bootstrapped switching point in the predicted direction. In this market, a number of draws resulted in two switching points. This can be explained by the fact that in DCA two students with very low abilities of 2 and 6, respectively, took dominated effort choices by spending all their endowment, which resulted in a utility of -2200 points. In some bootstrap samples, these two observations shift the smoothed line of the expected utility in DCA below the line for CCA for the lowest ability types. In market 3, 80.1% of the bootstrap samples provide a bootstrapped switching point that is unique and in the predicted direction. See Table 8 in the online appendix, part C, for detailed results of bootstrapping and the number of switching points.

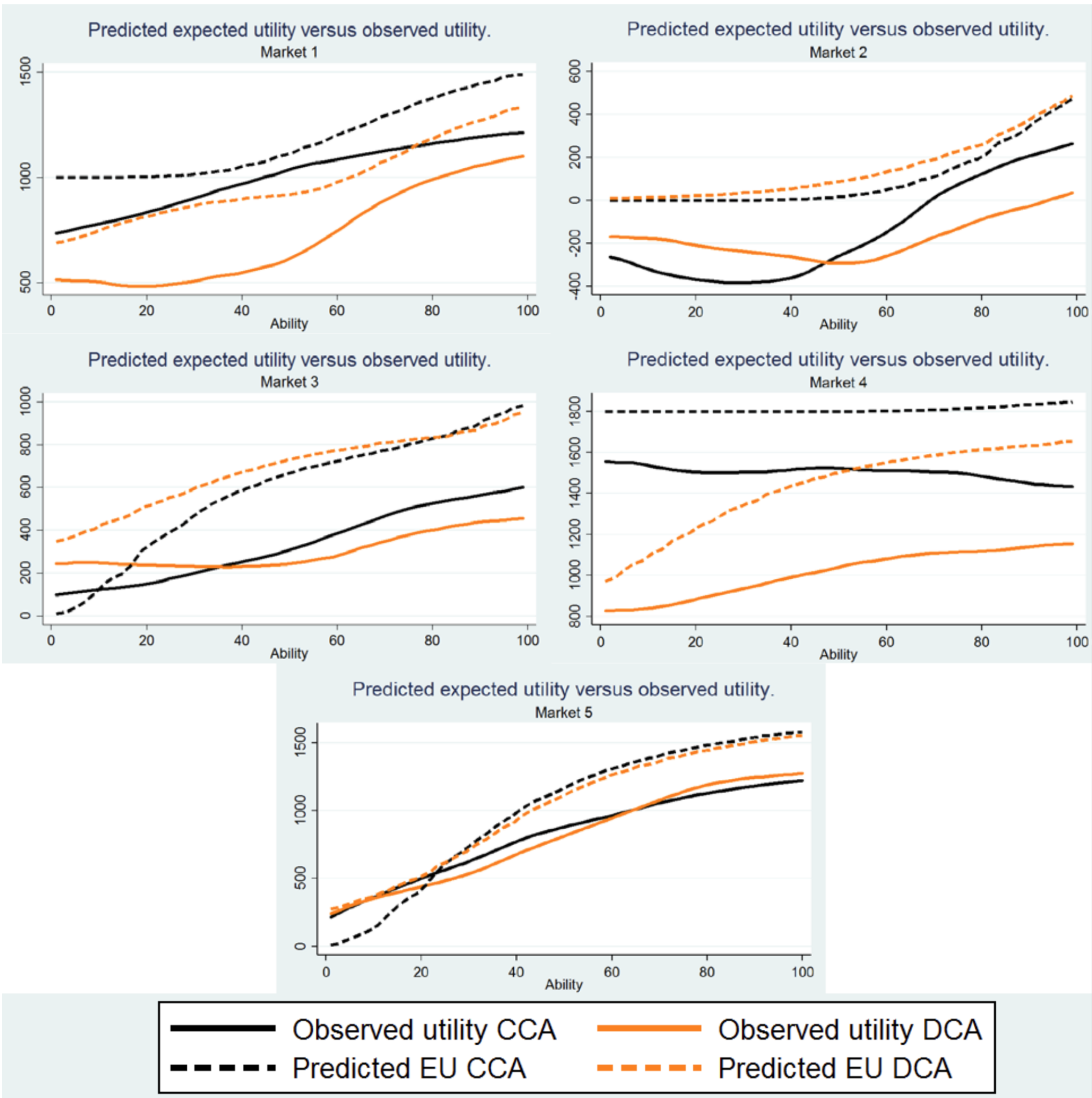

Figure 5: Predicted expected utilities and kernel regression of observed utilities by abilities.

68.4]. Thus, the observed switching point is below the theoretical prediction of 100. In market 3, the average switching point is 37.6 with a 95% confidence interval of [9.2, 69.3], also indicating that the observed switching point is below the prediction of 81. We conclude that the observed switching points are significantly lower than the predicted switching points in markets 2 and 3, implying that students from a smaller range of abilities prefer DCA than suggested by the equilibrium comparison of the two systems.[25]

7.2.2 Point predictions for effort choices and utility

We hypothesized that the overexertion of effort observed in previous experiments of all-pay auctions such as CCA may limit the predictive power of our theory with respect to the comparison between CCA and DCA. However, we also observe overexertion in DCA. To gain a better understanding of the deviations from the predicted outcomes, in particular the relatively poor performance of DCA with respect to student utility, we test the point predictions regarding the utility and effort levels in CCA and DCA.

Result 4 (Average utility and effort by markets): (i) The average utility is significantly lower than predicted across all markets and mechanisms. (ii) Average effort levels in the experiments are higher than the equilibrium efforts in all 10 markets. This overexertion of effort is significant in all five markets in DCA and in three out of five markets in CCA.

Table 4: Average utility and effort by markets

	Average equilibrium utility (1)	Average observed utility (2)	p-value obs.=pred. (3)	Average equilibrium efforts (4)	Average observed efforts (5)	p-value obs.=pred. (6)
CCA						
Market 1	1173	1001	0.04	230	276	0.19
Market 2	107	-122	0.01	364	389	0.54
Market 3	609	342	0.01	280	397	0.02
Market 4	1809	1507	0.01	35	191	0.01
Market 5	1011	809	0.02	305	400	0.05
DCA						
Market 1	975	715	0.01	262	362	0.02
Market 2	152	-169	0.00	309	410	0.00
Market 3	699	305	0.00	195	354	0.00
Market 4	1430	1014	0.00	125	340	0.00
Market 5	1019	797	0.00	307	395	0.00

Notes: Column (3) [(6)] shows the p-values for the significance of the constant when regressing the difference between (1) and (2) [(4) and (5)] on a constant, with standard errors clustered at the level of matching groups.

Support: Table 4 displays the equilibrium and observed averages for utility and effort levels by

[25]See Figure 8 in the online appendix, part C, for a histogram of the bootstrapped switching points.

markets. The average utility of subjects is significantly lower than predicted in all five markets under both mechanisms. This is consistent with the fact that in all markets and mechanisms, average effort levels are higher than predicted, as can be taken from a comparison of columns (4) and (5) in Table 4.[26] In CCA the difference is significant for three out of five markets (markets 3, 4, and 5) while in DCA it is significant for all five markets. Thus, DCA leads to significant overexertion in more markets than CCA.[27] Note that these findings are in line with the findings of Hakimov (2016) in the field who found higher average efforts in DCA than in CCA, while theory predicts similar efforts for both treatments.

Table 5: Observed effort choices

	(1) Efforts all rounds	(2) Efforts all rounds	(3) Efforts last block
Equilibrium effort	.74***	.33***	.39***
	(.04)	(.08)	(.12)
Dummy for CCA	-47.75	-61.62*	-59.23*
	(29.92)	(28.38)	(30.93)
Equilibrium effort in CCA	.01	.08	.03
	(.08)	(.09)	(.11)
Time played	-47.73***	-49.05***	
	(13.08)	(12.12)	
Ability		5.23***	4.11***
		(.61)	(.85)
Constant	290.34***	126.57***	30.28
	(33.82)	(31.76)	(23.86)
Observations	2160	2160	720
No. of clusters	12	12	12
R^2	.3167	.3897	.3815
F-test	111.41	147.69	59.78

Notes: OLS estimation of effort levels based on clustered robust standard errors at the level of matching groups. Equilibrium effort in CCA is an interaction of the CCA dummy and equilibrium effort. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level. Standard errors in parentheses.

Our findings for both mechanisms are similar to those in previous all-pay auction experiments where the observed efforts are higher than predicted. Moreover, it turns out that overexertion is even stronger in DCA. Despite the negative results regarding the point predictions, the equilibrium effort levels have significant predictive power. This emerges from an OLS estimation of observed efforts based on clustered robust standard errors at the level of matching groups, see Table 5.

[26]Figure 7 in the online appendix, part C, depicts the observed efforts of individuals, the kernel regression estimation of efforts, and the equilibrium predictions for each of the markets and mechanisms. All 10 panels for the 10 markets show that the kernel of effort increases in ability, as predicted. Moreover, the observed effort levels typically lie above the predicted values, except for high-ability students in a few markets.

[27]We also tested the equilibrium prediction that students with abilities below the cutoff choose the same effort irrespective of their college choice. We find that participants tend to exert higher effort when applying to the good as compared to the bad college (see column 4 of Table 10 in the online appendix, part C). Thus, relative overbidding in DCA goes along with students conditioning their effort choice on the choice of the college.

Furthermore, there is no significant difference between the two mechanisms with respect to the predictive power of the equilibrium efforts, as the interaction of the predicted effort and the dummy for CCA is not significant. The regression also confirms that efforts decrease over time and that there is, on average, less overexertion in CCA than in DCA, since the dummy for CCA is significant (at the 10% level) when controlling for equilibrium efforts.

Since the aggregate welfare of students is affected by their choice of effort levels, overexertion of effort in DCA relative to CCA explains why DCA never dominates CCA from the point of view of the students, in contrast to the predictions.[28]

7.2.3 Sorting of students

In a next step, we study how students sort across colleges with respect to their ability. In particular we ask whether the best students end up at the good college 2, the lower-ability students receive a seat at the bad college 1, and the students with the lowest ability are unassigned. In equilibrium, sorting by ability is always perfect in CCA while it is likely to be imperfect in DCA. Equilibrium miscoordination in DCA is due to the mixed strategy of low-ability students and the possibility that the number of students with realized abilities below and above the cutoff does not correspond to the number of seats in the two colleges. As a consequence, miscoordination in DCA can lead to more unassigned students and less sorting by ability than in CCA.

Before investigating the average ability levels at the colleges, we study the choice of participants to apply to college 1 or college 2 in DCA. Recall that the symmetric Bayesian Nash equilibrium characterized in Theorem 1 has the property that students with an ability above the cutoff should always apply to the better college (college 2) whereas students with an ability below the cutoff should mix between the two colleges.

Result 5 (Choice of college in DCA): In DCA, students above the equilibrium ability cutoff choose the good college 2 more often than students below the cutoff. However, high-ability students apply to the good college significantly less often than predicted in all markets while low-ability students apply to the good college more often than predicted (significant in three markets).

Support: Table 6 displays the equilibrium cutoff ability for each market in column (1). In column (2) it provides the average equilibrium probability of choosing the good college 2 for students with abilities below the cutoff in the respective markets. This average is calculated given the actual realization of abilities in the experiment. It can be compared to the observed frequency of choosing the good college by these students in column (3) and the 95% confidence intervals with standard errors clustered at the level of matching groups in column (4). It emerges that subjects below the cutoff choose the good college 2 more often than predicted in all five markets (significant for

[28]Inspecting the observed and predicted average effort levels in the two markets where DCA should be preferable for students (markets 2 and 3, see Table 4), it emerges that overbidding is more pronounced in DCA. In market 2, average observed efforts and equilibrium efforts differ by (389-364)=25 points in CCA while the difference is 101 in DCA; similarly for market 3 with average overbidding of 117 in CCA and 159 in DCA.

markets 1, 3, and 5). Column (5) displays the proportion of subjects above the cutoff applying to college 2, followed by the 95% confidence interval with standard errors clustered at the level of matching groups in column (6). In equilibrium these high-ability students should apply to college 2 with certainty, but we can reject this hypothesis in all five markets.[29] Finally, the last column of Table 6 presents the p-values for the Wilcoxon rank-sum test of equality of the distributions of the choice of college 2 below and above the market-specific equilibrium cutoff based on averages of six matching groups. In all markets except market 4, the differences are significant at the 1% significance level, and the difference is marginally significant for market 4. This is evidence of the predictive power of the model. Further evidence is provided by a probit regression of observed choices of college 2. The coefficient for the equilibrium probability of choosing the good college is significant.[30] Thus, we conclude that the choices of the subjects reflect the predicted equilibrium pattern, but that the point predictions fail.

Table 6: Proportion of choices of good college 2

	Equilibrium ability cutoff	Equ. prop. of choices of college 2 below the cutoff	Obs. prop. of choices of college 2 below the cutoff		Obs. prop. of choices of college 2 above the cutoff		p-values for equality of proportions above and below the cutoff
			mean	95% conf. int	mean	95% conf. int	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Market 1	50	13%	33%	[25%-44%]	85%	[75%-92%]	0.00
Market 2	85.5	43%	51%	[41%-61%]	92%	[77%-98%]	0.00
Market 3	85.5	15%	27%	[20%-36%]	68%	[49%-82%]	0.00
Market 4	89.5	16%	17%	[11%-27%]	42%	[21%-67%]	0.07
Market 5	23.5	51%	64%	[54%-72%]	91%	[84%-95%]	0.00

Notes: Column (7) displays the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups. Confidence intervals are estimated with standard errors clustered at the level of matching groups.

In order to better understand why the point predictions fail, we investigate the application decisions of students by ability. Figure 6 presents the choices of subjects in DCA by markets and ability quantiles, together with the equilibrium predictions. Students above the equilibrium cutoff in markets 1, 2, and 5 choose the good college 2 almost certainly, in line with the theory. The proportions of choices of students with low ability are also close to the equilibrium mixing probabilities. The biggest difference between the observed and the equilibrium proportions is due to students who are slightly above or below the cutoff. This finding is particularly evident in markets 1, 2, and 4. Remember that the equilibrium is characterized by a discontinuity regarding

[29]In markets 1, 2, and 5 the observed proportions are close to the equilibrium. In market 3 fewer high-ability students choose the good college, which may be due to the large bad college (eight seats) relative to the good college (two seats). In market 4, the relatively low proportion of high-ability students applying to the good college may be driven by the similarity of payoffs for both colleges (1,800 points versus 2,000 points).

[30]See Table 9 in the online appendix, part C. The same table shows that there is no gender difference in the choice of the good college.

the probability of the choice of college 2: students with abilities just above the cutoff have a pure strategy of choosing college 2, while students just below the cutoff choose college 1 with almost 100% probability. Not surprisingly, the choices of universities by our subjects are smooth around the cutoff. In line with this, we also do not observe the predicted kink in the effort choices shown in Figure 7 in the online appendix, part C. These findings could be due to the fact that students with an ability level around the cutoff under- or overestimate the cutoff, which would result in the observed smoothing.

We can also investigate whether students below the cutoff randomize individually.[31] In total, we have three decisions per subject in each market, but since we can only consider types below the cutoff for whom mixing is predicted in equilibrium, we lose part of the sample. (Moreover, we have to drop market 5 from this analysis, since we do not have enough observations due to the low value of the cutoff.) Among the subjects who took at least two decisions as a low-ability type in a given market, 43% chose different colleges in different decisions. For the sake of comparison, we generate two additional distributions of strategies based on 1,000 random draws: The first distribution is based on the assumption that subjects play a pure strategy of choosing the bad college and make a mistake and choose the good college with a 5% probability.[32] The second distribution is based on the assumption that subjects use mixed strategies according to the predicted probabilities. We reject the hypothesis that subjects follow the pure strategy of choosing the bad college for all four markets considered. We also reject the hypothesis of mixing in line with the predicted probabilities in markets 1 to 3. This is due to too many low-ability subjects choosing the good college. Only for market 4 we cannot reject the hypothesis of the predicted mixed strategies.[33]

As a final step, we compare CCA and DCA with respect to the resulting average abilities of the students in each college. Panels A, B, and C of Table 7 present the equilibrium and observed average abilities of students assigned to the good and the bad college, and of unassigned students, respectively.[34] Panel D presents the equilibrium and the observed percentage of unfilled seats by markets.

Result 6 (Composition of colleges): (i) (Good college) There is no significant difference in the average ability of students in CCA and DCA. This is in line with the theory except for markets 3 and 4 where a significantly higher ability of students in CCA is predicted. (ii) (Bad college) Ability levels are not significantly different in markets 1 and 5, as predicted. The average ability of students in DCA is significantly lower than predicted than in CCA in markets 2 and 3.

[31] We thank an anonymous referee who pointed this out.

[32] We focus on the pure strategy of choosing the bad college, since in equilibrium the probability of choosing the bad college is higher than the probability of choosing the good college in all markets.

[33] For the numbers and test results see Table 11 in the online appendix, part C.

[34] The equilibrium assignment in CCA is straightforward to calculate given the ability draws. For DCA the choice of the college is random for students below the ability cutoff. We generate one realization of the choice of the college for all abilities below the cutoff, given the equilibrium probabilities. The resulting equilibrium allocation is determined and used for the calculation in this table.

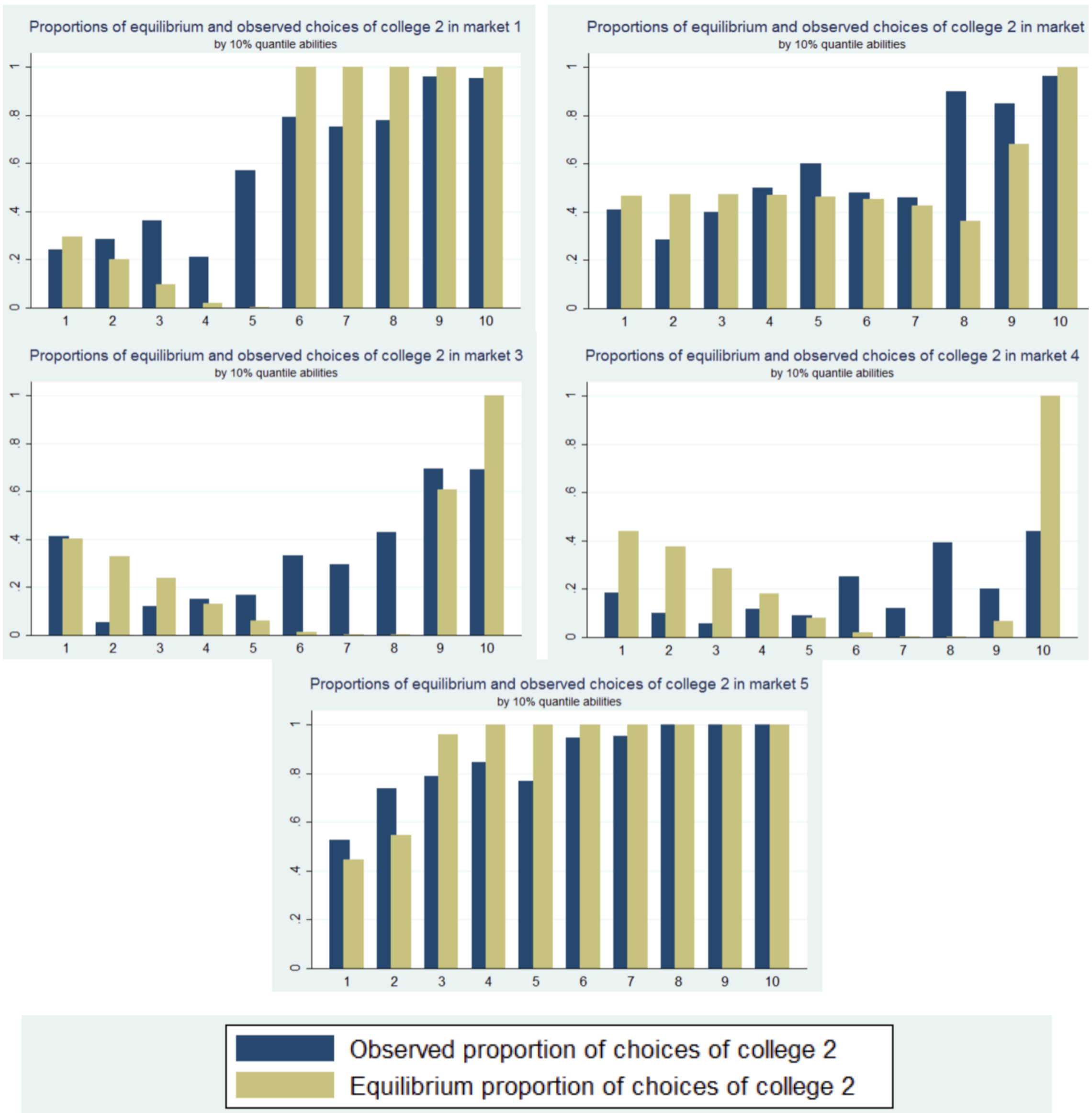

Figure 6: Choice of college in DCA

Table 7: Average abilities of students and unfilled seats by colleges

		Market				
		1	2	3	4	5
Panel A	Assigned to good college, equil.					
	CCA (1)	74.4	91.4	91.9	86.9	62.0
	DCA (2)	73.8	89.6	82.7	54.5	61.2
	CCA=DCA, equil., p-value (3)	*0.69*	*0.69*	*0.05*	*0.00*	*0.93*
	Assigned to good college, observed					
	CCA (4)	66.2	80.2	84.5	65.6	58.2
	DCA (5)	67.2	82.7	80.9	66.1	59.1
	CCA=DCA, observed, p-value (6)	*0.81*	*0.87*	*0.52*	*0.87*	*0.63*
	CCA observed=CCA equil., p-value (7)	*0.07*	*0.02*	*0.02*	*0.00*	*0.04*
	DCA observed=DCA equil., p-value (8)	*0.04*	*0.02*	*0.63*	*0.06*	*0.13*
Panel B	Assigned to bad college, equil.					
	CCA (1)	25.3	77.2	52.4	38.1	24.0
	DCA (2)	27.2	72.3	50.8	51.5	42.0
	CCA=DCA, equil., p-value (3)	*0.63*	*0.08*	*0.26*	*0.00*	*0.34*
	Assigned to bad college, observed					
	CCA (4)	33.5	75.3	52.4	45.2	40.9
	DCA (5)	31.0	43.7	45.5	49.6	34.5
	CCA=DCA, observed, p-value (6)	*0.42*	*0.01*	*0.04*	*0.15*	*0.52*
	CCA observed=CCA equil., p-value (7)	*0.00*	*0.70*	*1.0*	*0.00*	*0.05*
	DCA observed=DCA equil., p-value (8)	*0.40*	*0.00*	*0.05*	*0.29*	*0.04*
Panel C	Not assigned, equil.					
	CCA (1)		35.3	10.6		10.9
	DCA (2)	25.4	37.2	29.7	12.2	16.9
	CCA=DCA, equil., p-value (3)	*N/A*	*0.47*	*0.01*	*N/A*	*0.01*
	Not assigned, observed					
	CCA (4)	-	38.5	18.0	-	18.5
	DCA (5)	39.8	45.8	49.1	25.6	20.5
	CCA=DCA, observed, p-value (6)	*N/A*	*0.04*	*0.01*	*N/A*	*0.52*
	CCA observed=CCA equil., p-value (7)	*N/A*	*0.26*	*0.09*	*N/A*	*0.09*
	DCA observed=DCA equil., p-value (8)	*0.32*	*0.02*	*0.01*	*0.14*	*0.14*
Panel D	Percentage of unfilled seats, equil.					
	CCA (1)	0%	0%	0%	0%	0%
	DCA (2)	12.0%	1.4%	3.3%	5.1%	2.2%
	Percentage of unfilled seats, observed					
	CCA (3)	0%	0%	0%	0%	0%
	DCA (4)	10.2%	1.4%	7.8%	9.3%	2.8%

Notes: Rows (3) and (6) of panels A, B and C display the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups. Rows (7) and (8) of panels A , B and C display the p-values of t-test of equality of the averages of the six matching groups and the predicted constant value.

Support: We consider each market separately and mainly refer to rows (3) and (6) in panels A and B of Table 7. In markets 1 and 5, both the theory and the experimental data show no significant difference between ability levels in the good and bad college when comparing CCA with DCA. In market 4 where the two colleges have almost the same value for the students, the average ability of students in the good college is predicted to be significantly lower in DCA, and conversely, the average ability is predicted to be higher in the bad college in DCA. We fail to observe this significant difference for both colleges because the average ability levels at both colleges are more similar than predicted under both mechanisms. Thus, there is no sorting advantage of CCA in market 4, other than predicted.

In markets 2 and 3, the observed abilities of students assigned to the bad college are significantly higher in CCA than in DCA (see row (6) of Panel B). In equilibrium the difference has the same sign but is much smaller and is not significant. Thus, in DCA low-ability students have a better chance than predicted of being admitted to the bad college in markets 2 and 3, at the cost of some high-ability students who remain unassigned (cf. rows (2) and (5) for markets 2 and 3 in Panel C). The reason abilities are higher at the bad college in CCA than in DCA in these markets is due to a purely mechanical effect: CCA allows high-ability students who are unable to get into the good college to obtain a seat in the bad college. This raises the average ability in the bad college compared to DCA where the students who are unsuccessful at the good college remain unassigned.

Table 7 also reports on the point predictions for each market separately, with test results in rows (7) and (8) of panels A, B, and C. The point predictions are rejected in more than half of the cases, but we refrain from discussing them in detail here since our main focus is on the comparison of the two mechanisms.

One candidate to explain the difference between predicted and observed utility levels in the two mechanisms is the number of unfilled seats in DCA. If students coordinate worse than predicted in equilibrium, the attractiveness of DCA is reduced relative to CCA. Table 7, Panel D presents the equilibrium and observed shares of unassigned seats by markets. The share of unfilled seats in DCA is somewhat higher than in equilibrium only in markets 3 and 4, and the difference is small. Thus, unfilled seats can at best partially explain the lower student welfare in DCA in our experiment relative to the equilibrium predictions.

8 Conclusion

In this paper, we study college admissions exams which concern millions of students every year throughout the world. Our model abstracts from many aspects of real-world college admission games and focuses on the following two important issues: (i) colleges accept students by considering student exam scores, (ii) students have differing abilities which are their private information, and the costs of getting ready for the exams are inversely related to ability levels. We focus on two extreme policies that capture practices in a number of countries. In the centralized model students can freely and without cost apply to all colleges whereas in the decentralized mechanism, students can only apply to one college. We consider a model that is as simple as possible by assuming homogeneous student preferences over colleges in order to derive analytical results as Bayesian Nash solutions to the two mechanisms.

The solution of the centralized admissions mechanism follows from standard techniques in the contest literature. The solution to the decentralized mechanism, on the other hand, has interesting properties such as lower ability students using a mixed strategy when deciding which college to apply to. Our main theoretical result is that low- and high-ability students differ in terms of their preferences between the two mechanisms where high-ability students prefer the centralized mechanism and low-ability students the decentralized mechanism.

We employ experiments to test the theory and to develop insights into the functioning of centralized and decentralized mechanisms that take into account behavioral aspects. While overbidding is a common finding in all-pay auction experiments (see Barut et al. 2002; Noussair and Silver 2006), our results confirm this in the well-known context of a single contest with multiple prizes (CCA), but we also show that it holds in parallel contests (DCA). Our experiments allow us to compare the two mechanisms, leading to our main result of the lower welfare of students in DCA relative to CCA, even in markets where it should be preferred by all students.

Overall, many predictions of the theory are supported by the data, despite a few important differences. We find that in our markets with an equal number of seats and applicants, the centralized mechanism is better for all applicants, as predicted by the theory. Again in line with the theory, we observe that in markets with an overdemand for seats, low-ability students prefer the decentralized admissions mechanism whereas high-ability students prefer the centralized mechanism. However, in these markets the predicted superiority of the decentralized mechanism for the students is weaker than predicted. Thus, only a smaller group of (low-ability) students profits from the decentralized system. This can be ascribed to one robust and stark difference between theory and observed behavior, namely overexertion of effort, which is more pronounced in the decentralized mechanism. Moreover, the decentralized mechanism leads to less sorting by ability and to more high-ability students being unassigned, both compared to the centralized mechanism and compared to the equilibrium prediction. Our findings resonate with a number of countries having moved from a decentralized to a more centralized procedure in the past years, e.g., Russia and other former Soviet states as well as South Korea. The findings are also in line with the culture of ronin students in Japan where a decentralized system is in place. After finishing school, these large groups of students spend several of the following years studying for the entrance exams of the prestigious public universities.

To explain the observed overexertion of effort especially in DCA, risk aversion is a potential candidate. However, the theoretical results regarding risk aversion in contests are quite sensitive to seemingly small differences in the assumptions. But we would like to elaborate on risk aversion as a potential explanation for the difference between CCA and DCA. Fibich et al. (2006) have shown that in a single contest, players with high values bid higher than they would bid in the risk-neutral case (as compared to low-value bidders who will bid less). The intuitive reason for this is that these bidders have more to lose in case of not winning the prize, due to concave utility functions. Let us use this intuition to compare the overexertion of effort in CCA versus DCA. In CCA, a high-ability student can get a high prize (v_2), a low prize (v_1), or no prize (0), whereas in DCA, she would get either a high prize (v_2) or no prize (0). Therefore, in CCA just failing to win a high prize would still give this bidder a low prize, whereas this would result in no prize in DCA. In other words, this bidder has more to lose in a decentralized mechanism. Hence, we can expect that overexertion of effort would be more pronounced in DCA than in CCA.

For the evaluation of the two mechanisms from a welfare perspective, it matters whether the effort spent preparing for the exam has no benefits beyond improving the performance in the exam or whether this effort is useful. If effort is only a cost, then welfare can be measured by the mean utility of the students. In all our markets, the centralized mechanism outperforms the decentralized mechanism with respect to this criterion. However, if the effort exerted by the students increases their productivity then the decentralized mechanism becomes relatively more attractive, where efforts are weakly higher than in the centralized mechanism across markets.

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A Appendix

A.1 Preliminaries

The following lemmata are useful for the results given in the rest of the Appendix.

Lemma 1. *Let l, m be given integers. Then,*

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= -m\, p_{l,m-l-1}(x) \quad \text{when } 0 \le l < m, \\
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{j,m-j}(x)\Big) &= m\, p_{l-1,m-l}(x) \quad \text{when } 0 < l \le m,
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{m-j,j}(x)\Big) &= m\, p_{m-l-1,l}(x) \quad \text{when } 0 \le l < m, \\
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{m-j,j}(x)\Big) &= -m\, p_{m-l,l-1}(x) \quad \text{when } 0 < l \le m.
\end{aligned}$$

Proof. We use the following equation:

$$\binom{m}{j-1}(m-j+1) = \frac{m!}{(j-1)!(m-j+1)!}(m-j+1) = \frac{m!}{(j-1)!(m-j)!} = \binom{m}{j} j. \quad (7)$$

The first formula: Suppose $0 = l$. Then, $\sum_{j=0}^{l} p_{j,m-j}(x) = p_{0,m}(x) = (1-x)^m$. Its derivative is $-m(1-x)^{m-1} = -m\, p_{0,m-1}(x)$. Thus the formula holds. Consider another case where $0 < l$. Then we have

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= \frac{d}{dx}\Big(\sum_{j=0}^{l} \binom{m}{j} x^j (1-x)^{m-j}\Big) \\
&= \sum_{j=1}^{l} \binom{m}{j} j\, x^{j-1}(1-x)^{m-j} - \sum_{j=0}^{l} \binom{m}{j}(m-j)x^j(1-x)^{m-j-1} \\
&= \sum_{j=1}^{l} \binom{m}{j} j\, x^{j-1}(1-x)^{m-j} - \sum_{j=1}^{l+1} \binom{m}{j-1}(m-j+1)x^{j-1}(1-x)^{m-j} \\
&= \sum_{j=1}^{l} \binom{m}{j} j\, x^{j-1}(1-x)^{m-j} - \sum_{j=1}^{l+1} \binom{m}{j} j\, x^{j-1}(1-x)^{m-j} \quad \text{(by (7))}
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= - \binom{m}{l+1}(l+1)\,x^l(1-x)^{m-l-1} = -\frac{m!}{l!(m-l-1)!}x^l(1-x)^{m-l-1} \\
&= - m\frac{(m-1)!}{l!(m-l-1)!}x^l(1-x)^{m-l-1} = -m\,p_{l,m-l-1}(x).
\end{aligned}$$

<u>The second formula:</u> Suppose $l = m$. Then, $\sum_{j=l}^{m} p_{j,m-j}(x) = p_{m,0}(x) = x^m$. Its derivative is $mx^{m-1} = mp_{m-1,0}(x)$. Thus the formula holds. Consider another case where $l < m$. Then we have

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{j,m-j}(x)\Big) &= \frac{d}{dx}\Big(\sum_{j=l}^{m}\binom{m}{j}x^j(1-x)^{m-j}\Big) \\
&= \sum_{j=l}^{m}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} - \sum_{j=l}^{m-1}\binom{m}{j}(m-j)x^j(1-x)^{m-j-1} \\
&= \sum_{j=l}^{m}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} - \sum_{j=l+1}^{m}\binom{m}{j-1}(m-j+1)x^{j-1}(1-x)^{m-j} \\
&= \sum_{j=l}^{m}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} - \sum_{j=l+1}^{m}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} \quad \text{(by (7))}
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= \binom{m}{l}\,l\,x^{l-1}(1-x)^{m-l} = \frac{m!}{(l-1)!(m-l)!}x^{l-1}(1-x)^{m-l} \\
&= m\frac{(m-1)!}{(l-1)!(m-l)!}\,x^{l-1}(1-x)^{m-l} = m\,p_{l-1,m-l}(x).
\end{aligned}$$

<u>The third formula:</u> By the second formula, we have

$$\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{m-j,j}(x)\Big) = \frac{d}{dx}\Big(\sum_{j=m-l}^{m} p_{j,m-j}(x)\Big) = m\,p_{m-l-1,l}(x).$$

<u>The fourth formula:</u> By the first formula, we have

$$\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{m-j,j}(x)\Big) = \frac{d}{dx}\Big(\sum_{j=0}^{m-l} p_{j,m-j}(x)\Big) = m\,p_{m-l,l-1}(x).$$

□

B On Equilibria of Decentralized College Admissions

B.1 On properties of monotone and symmetric equilibrium of decentralized college admissions

We focus on symmetric and monotone equilibrium. More specifically, each student will use the same probability mixing function $\gamma(a)$, and the same effort function $\beta_i(a)$ while applying to college $i \in \{1, 2\}$. Moreover, for all values β_i is defined (i.e., for all types a which apply to college i with positive probability) $\beta_i(a)$ is increasing in a and $\gamma(a)$ is integrable (continuous except for a zero measure set). We define

$$\pi(a) = \int_0^a \gamma(x) f(x) dx.$$

We then define the conditional distributions

$$H^1(a) = \frac{\pi(a)}{\pi(1)} \text{ and } H^2(a) = \frac{F(a) - \pi(a)}{1 - \pi(1)}$$

We then define the probability of being in the top q_i among applicants to college i by $K_i(a)$ and its corresponding density by $k_i(a)$. That is, we have

$$\begin{aligned}
K_1(a) &= \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(1)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(1)) H^1_{m-q_1+1,m}(a), \\
K_2(a) &= \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(1)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(1)) H^2_{m-q_2+1,m}(a).
\end{aligned}$$

We first prove that there cannot be a pure strategy equilibrium (i.e., $\gamma(a) \in \{0, 1\}$ for all $a \in [0, 1]$).

Proposition 4. *There cannot be a pure-strategy symmetric and monotone equilibrium.*

Proof. By method of contradiction, suppose that there is one. It is easy to see that the measure of both $\{a \in [0, 1] : \gamma(a) = 0\}$ and $\{a \in [0, 1] : \gamma(a) = 1\}$ are strictly positive. Then there exist c, d with $0 < c < d \leq 1$ such that all $a \in [0, c]$ applies to college 1 and all $a \in (c, d]$ applies to college 2. It is then easy to see that (i) $\beta_1(c) > 0$ and $\lim_{a \downarrow c} \beta_2(a) = 0$, and (ii) $v_1 K_1(c) - \frac{\beta_1(c)}{c} = v_2 K_2(c)$. Now, we argue that any $a \in \left(c, \beta_2^{-1}(\beta_1(c))\right)$ would be strictly better off by "mimicking" type c. To see this, consider

$$f(a) \equiv v_1 K_1(c) - \frac{\beta_1(c)}{a} - \left(v_2 K_2(a) - \frac{\beta_2(a)}{a}\right)$$

which represents the gain from mimicking type c. We obtain $f(a) > 0$ for all $a \in \left(c, \beta_2^{-1}(\beta_1(c))\right)$ by noting that $f(c) = 0$ and

$$f'(a) = \frac{\beta_1(c)}{a^2} - \frac{\beta_2(a)}{a^2} > 0$$

by the envelope theorem. $\square$

We then show that in any mixed strategy equilibrium, $\gamma(a) \in (0,1)$ implies that $\beta_1(a) = \beta_2(a)$.

Proposition 5. *If $\gamma(a) \in (0,1)$, then $\beta_1(a) = \beta_2(a)$.*

Proof. Let $a \in [0,1]$ such that $\gamma(a) \in (0,1)$. There is an interval $I = [\underline{a}, \overline{a}]$ such that $a \in I$ and for all $b \in I$, $\gamma(b) \in (0,1)$. Then, for all $b \in I$, since type b is indifferent to applying to colleges 1 and 2, we have

$$EU_1(b) \equiv v_1 K_1(b) - \frac{\beta_1(b)}{b} = v_2 K_2(b) - \frac{\beta_2(b)}{b} \equiv EU_2(b).$$

Since the first-order conditions imply

$$\beta_i(b) = v_i \int_{\underline{a}}^{b} x k_i(x)\,dx + D_i = v_i \left(K_i(b)\, b - K_i(\underline{a})\underline{a} - \int_{\underline{a}}^{b} K_i(x)\,dx \right) + D_i,$$

where D_i is a constant. Thus

$$\begin{aligned} EU_i(b) &= v_i K_i(b) - v_i K_i(b) + v_i K_i(\underline{a})\frac{\underline{a}}{b} + v_i \frac{\int_{\underline{a}}^{b} K_i(x)dx}{b} - \frac{D_i}{b} \\ &= v_i K_i(\underline{a})\frac{\underline{a}}{b} + v_i \frac{\int_{\underline{a}}^{b} K_i(x)dx}{b} - \frac{D_i}{b}. \end{aligned}$$

Then, as $EU_1(\underline{a}) = EU_2(\underline{a})$, we have

$$v_1 K_1(\underline{a}) - \frac{D_1}{\underline{a}} = v_2 K_2(\underline{a}) - \frac{D_2}{\underline{a}}. \tag{8}$$

Moreover, for all $b \in I$, as $EU_1(b) = EU_2(b)$,

$$\begin{aligned} & v_1 K_1(\underline{a})\frac{\underline{a}}{b} + v_1 \frac{\int_{\underline{a}}^{b} K_1(x)dx}{b} - \frac{D_1}{b} = v_2 K_2(\underline{a})\frac{\underline{a}}{b} + v_2 \frac{\int_{\underline{a}}^{b} K_2(x)dx}{b} - \frac{D_2}{b} \\ \Rightarrow\ & v_1 K_1(\underline{a}) + v_1 \frac{\int_{\underline{a}}^{b} K_1(x)dx}{\underline{a}} - \frac{D_1}{\underline{a}} = v_2 K_2(\underline{a}) + v_2 \frac{\int_{\underline{a}}^{b} K_2(x)dx}{\underline{a}} - \frac{D_2}{\underline{a}} \\ \Rightarrow\ & v_1 \frac{\int_{\underline{a}}^{b} K_1(x)dx}{\underline{a}} = v_2 \frac{\int_{\underline{a}}^{b} K_2(x)dx}{\underline{a}} \quad (\because (8)) \\ \Rightarrow\ & v_1 \int_{\underline{a}}^{b} K_1(x)dx = v_2 \int_{\underline{a}}^{b} K_2(x)dx. \end{aligned} \tag{9}$$

Thus, we have

$$\text{for all } b \in I,\ v_1 K_1(b) = v_2 K_2(b). \tag{10}$$

Therefore, using the equalities (8), (9), and (10), we can conclude that for all $b \in I$, $\beta_1(b) = \beta_2(b)$. Hence,

$$\beta_1(a) = \beta_2(a).$$

□

Hence, it is without loss of generality that we focus on the equilibria in the main text: when students mix between applying to colleges, they choose the same effort level while applying to either college.

B.2 On the symmetric equilibrium

We briefly discussed the derivation of the symmetric equilibrium in the main body. In this section, we show the conditions (i) to (iv) in Theorem 1.

Note that for all $a \in [0, c]$, we have

$$G_{m-q_1+1,m}(a) = \sum_{j=m-q_1+1}^{m} p_{j,m-j}\left(\frac{\pi(a)}{\pi(c)}\right) \quad \text{and} \quad H_{m-q_2+1,m}(a) = \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(a)-\pi(a)}{1-\pi(c)}\right).$$

<u>Theorem 1 (i).</u> The equation (6) at $a = 0$ leads to

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)), \tag{11}$$

which is the equation in Theorem 1 (i).

Now we show that there is a unique value $\pi(c)$ that satisfies Equation (11). Define a function $\varphi_1 : [0, 1] \to \mathbb{R}$: for each $x \in [0, 1]$,

$$\varphi_1(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x) - v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x).$$

Differentiate φ_1 at each $x \in (0, 1)$: using Lemma 1, we have

$$\varphi_1'(x) \;=\; v_2(n-1)\, p_{(n-1)-(q_2-1)-1,q_2-1}(x) + v_1(n-1)\, p_{q_1-1,(n-1)-(q_1-1)-1}(x) > 0.$$

Thus, φ_1 is strictly increasing. Moreover, $\varphi_1(0) = -v_1 < 0$ and $\varphi_2(1) = v_2 > 0$. Thus, since φ_1 is a continuous function on $[0, 1]$, there is a unique $x^* \in (0, 1)$ such that $\varphi_1(x^*) = 0$. Thus, since $\varphi_1(\pi(c)) = 0$ by (11), there is a unique $\pi(c) \in (0, 1)$ that satisfies Equation (11).

<u>Theorem 1 (ii).</u> The equation (6) at $a = c$ leads to

$$v_1 = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(c)-\pi(c)}{1-\pi(c)}\right), \tag{12}$$

which is the equation in Theorem 1 (ii).

Given a unique $\pi(c)$, we now show that there is a unique cutoff $c \in (0, 1)$. In Equation (12), since $\pi(c)$ is known by Step 1, the the only unknown is c via $F(c)$. Define a function $\varphi_2 : [\pi(c), 1] \to \mathbb{R}$

as follows: for each $x \in [\pi(c), 1]$,

$$\varphi_2(x) \;=\; v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{x-\pi(c)}{1-\pi(c)}\Big) - v_1.$$

Differentiate φ_2 at each point $x \in (\pi(c), 1)$: using Lemma 1, we have

$$\varphi_2'(x) \;=\; v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\Big(\frac{1}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{x-\pi(c)}{1-\pi(c)}\Big) > 0.$$

Thus, φ is strictly increasing. Moreover, $\varphi_2(1) = v_2 - v_1 > 0$ and

$$\begin{aligned}
\varphi_2(\pi(c)) \;&=\; v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}(0) - v_1 \\
&=\; v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) - v_1 \quad (\because p_{j,m-j}(0) = 0 \text{ for } j \geq m - q_2 + 1 \geq 1) \\
&=\; v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) - v_1 \quad (\because (11)) \\
&<\; 0.
\end{aligned}$$

Therefore, there is a unique $x^* \in (\pi(c), 1)$ such that $\varphi_2(x^*) = 0$. Since $\varphi_2(F(c)) = 0$, $x^* = F(c)$. Thus, since F is strictly increasing, there is a unique cutoff $c \in (F^{-1}(\pi(c)), 1)$ such that $c = F^{-1}(x^*)$.

Theorem 1 (iii). Using (11), we can rewrite Equation (6) as follows. For all $a \in [0, c]$.

$$v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\left(\frac{\pi(a)}{\pi(c)}\right) = v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(a)-\pi(a)}{1-\pi(c)}\right), \tag{13}$$

We show that given unique $\pi(c)$ and c, for each $a \in [0, c)$, there is a unique $\pi(a) \in (0, 1)$ that satisfies (13). Fix $a \in [0, c)$. Define a function $\varphi_3 : [0, F(a)] \to \mathbb{R}$:

$$\varphi_3(x) \;=\; v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)-x}{1-\pi(c)}\Big) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{x}{\pi(c)}\Big).$$

Let us differentiate φ_3 at each $x \in (0, F(a))$ by using Lemma 1:

$$\begin{aligned}\varphi_3'(x) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\Big(-\frac{1}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-x}{1-\pi(c)}\Big)\\ &\quad -v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\Big(\frac{1}{\pi(c)}\Big)\, m\, p_{m-q_1,q_1-1}\Big(\frac{x}{\pi(c)}\Big) < 0.\end{aligned}$$

Thus, φ is strictly decreasing. Moreover,

$$\begin{aligned}\varphi_3(0) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{1-\pi(c)}\Big) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}(0)\\ &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{1-\pi(c)}\Big) \quad (\because p_{j,m-j}(0)=0)\\ &> 0.\end{aligned}$$

and

$$\begin{aligned}\varphi_3(F(a)) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}(0) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{\pi(c)}\Big)\\ &= -v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{\pi(c)}\Big) \quad (\because p_{j,m-j}(0)=0)\\ &< 0.\end{aligned}$$

Thus, there is a unique $x^* \in (0, F(a))$ such that $\varphi_3(x^*) = 0$. Since $\varphi_3(\pi(a)) = 0$, $x^* = \pi(a)$. Hence, there is a unique $\pi(a) \in (0,1)$ that satisfies Equation (13).
Theorem 1 (iv). We derive $\gamma(a)$ for each $a \in (0, c)$. Recall that in (13), $\pi(a) = \int_0^a \gamma(x) f(x) dx$ and $\pi(c)$ and $\pi(a)$ are known by previous steps. Differentiate (13) with respect to a by using Lemma 1:

$$\begin{aligned}& v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\Big(\frac{\gamma(a)f(a)}{\pi(c)}\Big)\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big)\\ &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\Big(\frac{f(a)-\gamma(a)f(a)}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big).\end{aligned} \tag{14}$$

Let us define the following functions:

$$\begin{aligned}A(a) &:= v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big) > 0,\\ B(a) &:= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big) > 0.\end{aligned}$$

Then, we can write (14) as

$$\frac{\gamma(a)f(a)}{\pi(c)}A(a) = \frac{f(a)(1-\gamma(a))}{1-\pi(c)}B(a). \tag{15}$$

Solving for $\gamma(a)$ in (15), we obtain

$$\gamma(a) = \frac{\pi(c)B(a)}{(1-\pi(c))A(a)+\pi(c)B(a)} \in (0,1).$$

By construction, function γ we have derived satisfies Equation (13).

B.3 Verification: The candidate is an equilibrium

In this appendix, we check for global deviations and confirm that the unique symmetric equilibrium candidate we have derived in Theorem 1 is indeed an equilibrium. As a preliminary notation and analysis, let us calculate the probability, denoted by $P[1,b|c,\gamma,\beta^D]$, that a student who makes effort $e=\beta^D(b)$ and applies to college 1 will end up getting a seat in college 1:

$$P[1,b|\gamma,\beta^D] = \begin{cases} \sum_{m=0}^{q_1-1}\hat{p}_{m,n-m-1}(c) + \sum_{m=q_1}^{n-1}\hat{p}_{m,n-m-1}(c)G_{m-q_1+1,m}(b) & \text{if } b\in[0,c] \\ 1 & \text{if } b\geq c. \end{cases}$$

Obviously, if the student chooses an effort more than $\beta(c)$, he will definitely get a seat in college 1. Otherwise, the first line represents the sums of the probability of events in which e is one of the highest q_1 efforts among the students who apply to college 1.

Similarly, let us calculate the probability, denoted by $P[2,b|\beta,\gamma]$, that a student who makes effort $e=\beta(b)$ and applies to college 2 ends up getting a seat in college 2.

$$P[2,b|\gamma,\beta^D] = \begin{cases} \sum_{m=0}^{q_2-1}\hat{p}_{n-m-1,m}(c) + \sum_{m=q_2}^{n-1}\hat{p}_{n-m-1,m}(c)H_{m-q_2+1,m}(b) & \text{if } b\in[0,1] \\ 1 & \text{if } b\geq 1. \end{cases}$$

Obviously, if the student chooses an effort greater than $\beta(1)$, he will definitely get a seat in college 2.[35] Otherwise, the first line represents the sums of probability of events in which e is one of the highest q_2 efforts among the students who apply to college 2.

Next, denote by $U(r,b|\gamma,\beta^D,a)$ (or $U(r,b|a)$ for short) the expected utility of type a who chooses college 1 with probability r and makes effort $e=\beta^D(b)$ when all of the other students follow the strategy (γ,β^D). We have,

$$U(r,b|a) := rP[1,b|\gamma,\beta^D]v_1 + (1-r)P[2,b|\gamma,\beta^D]v_2 - \frac{e}{a}.$$

We need to show that for each $a\in[0,1]$, each $r\in[0,1]$ and each $b\geq 0$, $\hat{U}(a)\equiv U(\gamma(a),a|a)\geq$

[35] Of course, there is no type b with $b>1$, if a student chooses an effort e strictly greater than $\beta^D(1)$, we represent him as mimicking a type $b>1$.

$U(r, b|a)$. Fix $a \in [0,1]$. It is sufficient to show that $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$, as these two conditions together implies required "no global deviation" condition. Below, we show that for any $a \in [0,1]$, and for $b \geq 0$, both $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$ hold. We consider two cases, one for lower ability students ($a \in [0, c]$), one for higher ability students ($a \in [c, 1]$). As sub-cases, we analyze b to be in the same region (b is low for a low, and b is high for a high), different region (a high, b low; and a low, b high), and b being over 1. The no-deviation results for the same region is standard, whereas deviations across regions need to be carefully analyzed.

Case 1: Type $a \in [0, c]$

Case 1-1: $b \in [0, c]$. Then, by our derivation, we have $U(0, b|a) = U(1, b|a)$ and also $\hat{U}(a) \geq U(1, b|a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Hence, we can conclude that $\hat{U}(a) \geq U(1, e|a) = U(0, e|a)$.

Case 1-2: $b \in (c, 1]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

$$
\begin{aligned}
\hat{U}(a) &\geq U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{\beta^D(b)}{a} \quad (\because \beta^D(c) \leq \beta^D(b)). \\
&= U(1, b|a).
\end{aligned}
$$

Next, we show $\hat{U}(a) \geq U(0, b|a)$.

$$
\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = P[2, c|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{a} \\
&= \left(P[2, \beta^D(c)|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{c}\right) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} = U(0, c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(0, b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} = P[2, b|\gamma, \beta^D]v_2 - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= \left(P[2, b|\gamma, \beta^D] - \frac{\beta^D(b)}{a}\right) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(0, b|a) + \left(\beta^D(b) - \beta^D(c)\right)\left(\frac{1}{a} - \frac{1}{c}\right) \\
&\geq U(0, b|a) \quad (\because \beta^D(b) \geq \beta^D(c),\ a < c).
\end{aligned}
$$

Case 1-3: $b > 1$ (or $e > \beta^D(1)$).

$$
\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&> v_1 - \frac{e}{a} \quad (\because \beta^D(c) \leq \beta^D(1) < e) \\
&= U(1, b|a).
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) \quad \text{(by Case 1-2)} \\
&= v_2 - \frac{\beta^D(1)}{a} \\
&> v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}
$$

Case 2: Type $a \in [c, 1]$

Case 2-1: $b \in [0, c]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

$$
\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= U(\gamma(c), c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(\gamma(b), b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(\gamma(b), b|a) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(1, b|a) + (\beta^D(c) - \beta^D(b)) \left(\frac{1}{c} - \frac{1}{a} \right) \quad (\because U(\gamma(b), b|a) = U(1, b|a)) \\
&\geq U(1, b|a) \quad (\because \beta^D(c) - \beta^D(b) \geq 0,\ c < a).
\end{aligned}
$$

To obtain $\hat{U}(a) \geq U(0, b|a)$, note that in the above inequalities, if we use $U(\gamma(b), b|a) = U(0, b|a)$ in the fourth line, we obtained the desired inequality.

Case 2-2: $b \in (c, 1]$. First, by our derivation, $\hat{U}(a) \geq U(0, e|\gamma, \beta^D, a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Next, we show

$\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= v_1 - \frac{\beta^D(c)}{a} \quad (\because v_2 P[2, c|\gamma, \beta^D] = v_1) \\
&\geq v_1 - \frac{\beta^D(b)}{a} = U(1, b|a) \quad (\because \beta^D(c) \leq \beta^D(b)).
\end{aligned}$$

Case 2-3: $b > 1$ (or $e > \beta^D(1)$)

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{e}{a} \quad (\because e > \beta^D(1) > \beta^D(c)) \\
&\geq U(1, b|a).
\end{aligned}$$

and

$$\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) = v_2 - \frac{\beta^D(1)}{a} \\
&\geq v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}$$

C Additional tables and figures (for online publication)

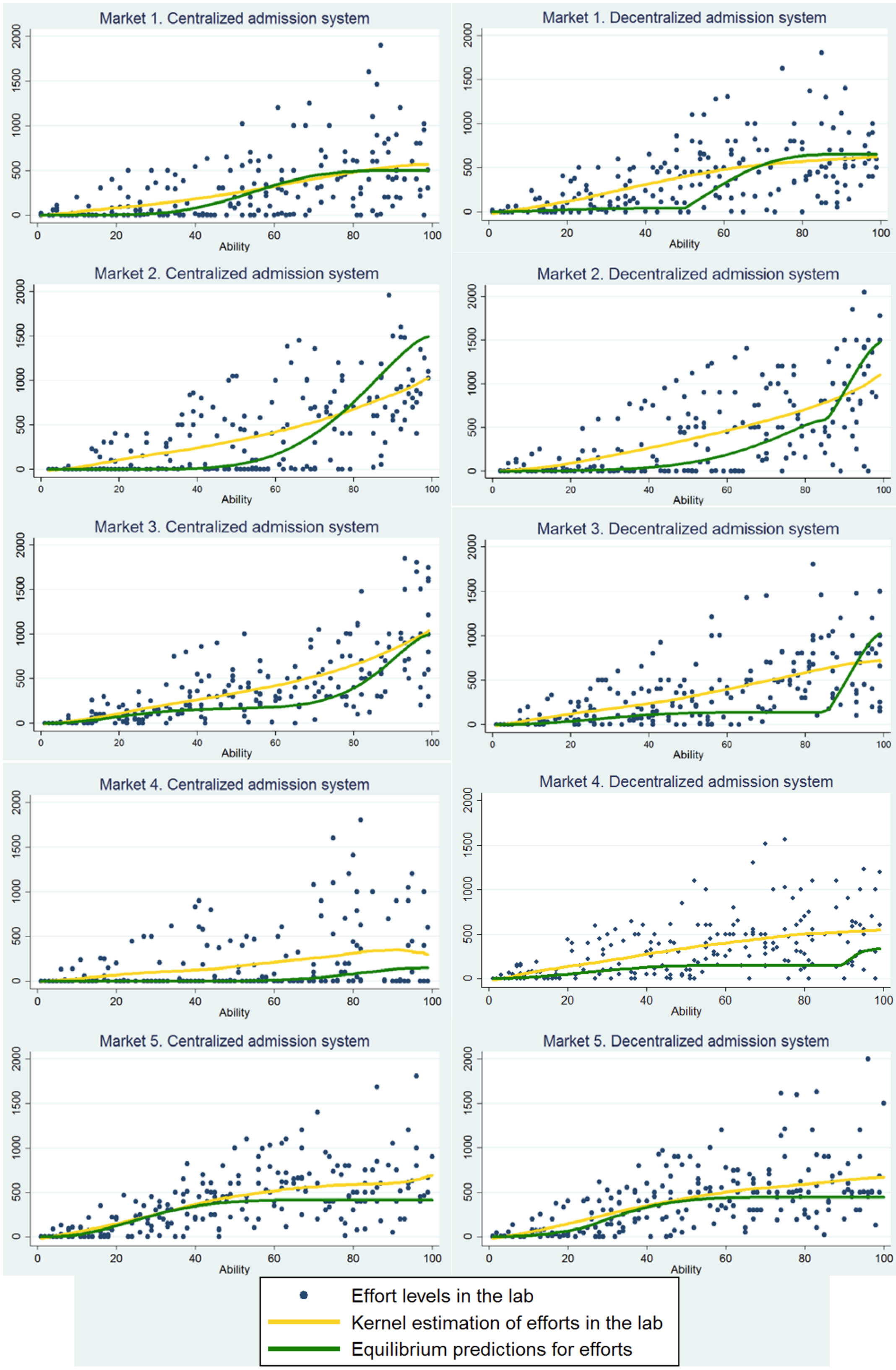

Figure 7: Individual efforts by ability

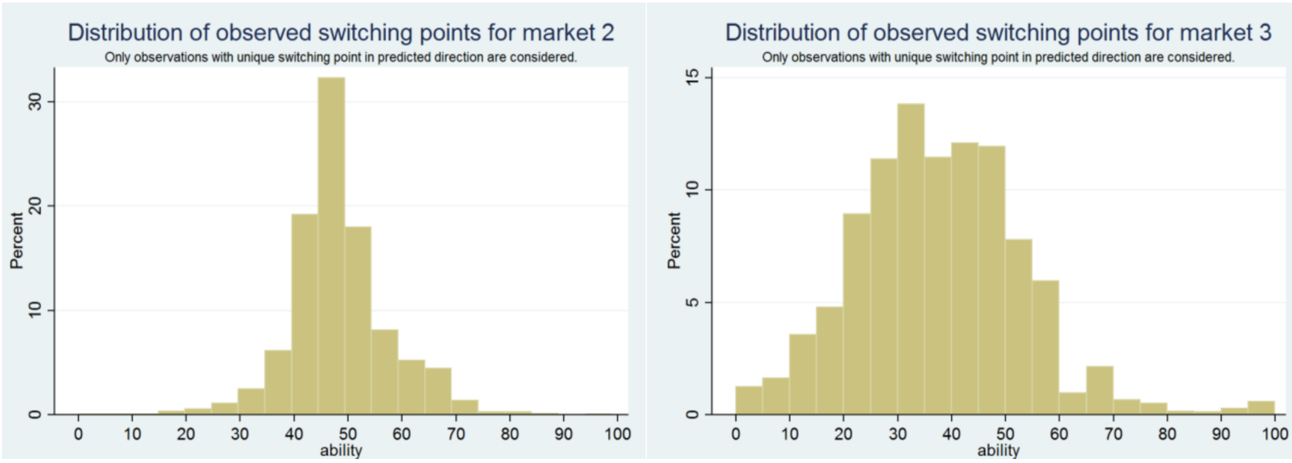

Figure 8: Distribution of observed switching points

Table 8: Number of switching points in the 50,000 bootstrapped samples, by markets

	Market	
	2	3
Unique switching point in the predicted direction	77.5%	80.1%
Two switching points	17.3%	4.5%
Three or more switching points	0.8%	4.6%
No switching points	4.2%	6.9%
Unique switching point in the opposite direction	0.2%	3.9%

Table 9: Choice of the good college 2 in DCA

	(1)	(2)	(3)
Equilibrium probability of choosing the good college	1.684***	1.464***	1.465***
	(.106)	(.118)	(.113)
Ability		.009***	.009***
		(.002)	(.002)
Female dummy			-.016
			(.114)
Constant	-.793***	-1.144***	-1.139***
	(.079)	(.110)	(.115)
Observations	1080	1080	1080
log(likelihood)	-615.461	-596.561	-596.543

Notes: Probit estimation of dummy for the choice of the good college based on clustered robust standard errors at the subject level. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level. Standard errors in parentheses.

Table 10: Average overbidding given the choice of the college in DCA.

		Ability below cutoff				Ability above cutoff			
		CCA (1)	DCA Bad college (2)	DCA Good college (3)	P-value Good= Bad (4)	CCA (5)	DCA Bad college (6)	DCA Good college (7)	P-value Good= Bad (8)
Market 1	N	106	70	36		110	16	92	
	Overbidding	3.2	3.4	7.7	0.02	0.4	-0.9	1.8	0.10
Market 2	N	177	86	91		39	3	36	
	Overbidding	3.2	2.5	4.9	0.10	-4.1	-8.6	-0.8	0.02
Market 3	N	179	130	49		37	12	25	
	Overbidding	2.5	3.5	5.8	0.24	0.7	-1.3	0.3	0.26
Market 4	N	190	157	33		26	15	11	
	Overbidding	3.1	4.0	5.8	0.06	2.2	1.4	6.8	0.10
Market 5	N	44	16	28		172	15	157	
	Overbidding	2.3	-0.5	5.4	0.02	1.6	0.4	1.8	0.35

Notes: Columns (4) and (8) show the p-values for the significance of the dummy variable for applying to the good college when regressing overbidding on the dummy and a constant for abilities below and above the theoretical cutoff in DCA, respectively, with standard errors clustered at the level of matching groups.

Table 11: Mixing of low-ability students between the two colleges

	Market 1			Market 2			Market 3			M4			M5		
	Actual	Pure	Pred	Actual	Pure	Pred	Actual	Pure	Pred	Actual	Pure	Pred	Actual	Pure	Pred
Only bad	37.5%	85.8%	68.1%	27.0%	85.8%	24.2%	48.5%	85.8%	65.9%	67.1%	85.8%	64.4%	16.7%	41.7%	20.1%
Mix	56.3%	14.2%	31.1%	47.6%	14.2%	61.3%	47.0%	14.2%	33.1%	30.0%	14.2%	34.5%	0.0%	14.2%	62.2%
Only good	6.3%	0.1%	0.8%	25.4%	0.1%	14.6%	4.5%	0.1%	1.0%	2.9%	0.1%	1.2%	83.3%	44.2%	17.8%
N	32	2000	2000	63	2000	2000	66	2000	2000	70	2000	2000	6	2000	2000
p-value		0.00	0.00		0.00	0.03		0.00	0.01		0.00	0.32		N/A	N/A

Notes: 'Actual' presents the empirical distribution of strategies, 'Pure' presents the distribution generated under the assumption that subjects choose the bad college and make mistakes with 5% probability, 'Pred' presents the distribution generated given the assumption that subjects play the equilibrium in mixed strategies. The p-values are based on the Chi square test.

D Equilibrium derivation for ℓ colleges (for online publication)

We show how to derive cutoffs, mixed strategies, and cost functions provided there exists an equilibrium as specified in section 6.1. The basic procedure follows the one in Theorem 1.

We first show how to obtain the equilibrium cutoffs $c_1, \ldots, c_{\ell-1}$ and the mixed strategy function $\gamma_1, \ldots, \gamma_{\ell-1}$. Let $k \in \{1, \ldots, \ell-1\}$. A necessary condition for this to be an equilibrium is that each type $a \in [c_{k-1}, c_k]$ has to be indifferent between applying to college 1 and college 2. Thus, for all $a \in [c_{k-1}, c_k]$,

$$\begin{aligned}
&v_k\Big(\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k))H^k_{m-q_k+1,m}(a)\Big) \\
&= v_{k+1}\Big(\sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) + \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1}))H^{k+1}_{m-q_{k+1}+1,m}(a)\Big). \quad (16)
\end{aligned}$$

Step 1: Find $\pi^1(c_1), \ldots, \pi^l(c_\ell)$. Equation (16) can be written as

$$\begin{aligned}
&v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi^1(c_1)) = v_2 \sum_{m=0}^{q_2-1} p_{m,n-m-1}(\pi^2(c_2)), \\
&v_{k-2} = v_k \sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) \quad \text{for } k \in \{3, ..., \ell\}, \quad (17)
\end{aligned}$$

where the first equation is Equation (16) at $a = 0$ under $k = 1$, which says that a type $a = 0$ is indifferent between college 1 and 2; the second equation follows from Equation (16) at $a = c_k$ under $k-1$ and k, which says that a type $a = c_{k-2}$ is indifferent between colleges $k-2$ and k. Therefore, $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$ can be obtained by solving Equation (17).

Step 2: Given $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$, find cutoffs $c_1, \ldots, c_{\ell-1}$. We first show the following claim that shows how to obtain $\pi^k(c_{k-1})$ from $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$.

Proof. For $k = 2$: Note that $\pi^1(c_1) = \int_0^{c_1} \gamma_1(x)dF(x)$. Thus $\pi^2(c_1) := \int_0^{c_1}(1-\gamma_1(x))dF(x) = F(c_1) - \pi^1(c_1)$. Suppose that the claim is true up to $k-1$ where $k \geq 3$. Then $\pi^{k-1}(c_{k-1}) := \pi^{k-1}(c_{k-2}) + \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x)$. Thus $\int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x) = \pi^{k-1}(c_{k-1}) - \pi^{k-1}(c_{k-2})$. Hence, by the induction hypothesis, we have

$$\begin{aligned}\pi^k(c_{k-1}) : &= \int_{c_{k-2}}^{c_{k-1}} (1-\gamma_{k-1}(x))dF(x) \\ &= F(c_{k-1}) - F(c_{k-2}) - \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x) \\ &= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + \pi^{k-1}(c_{k-2}) \\ &= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + (F(c_{k-2}) - \sum_{j=1}^{k-2} \pi^j(c_j)) \\ &= F(c_{k-1}) - \sum_{j=1}^{k-1} \pi^j(c_j).\end{aligned}$$

□

Now Equation (16) at $a = c_k$ can be rewritten as, for each $k \in \{1, \ldots, \ell - 1\}$,

$$\begin{aligned} v_k &= v_{k+1} \sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\ &+ v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\Big(\frac{F(c_k) - (\pi^1(c_1) + \ldots + \pi^k(c_k))}{\pi^{k+1}(c_{k+1})}\Big), \end{aligned} \tag{18}$$

where we use the induction claim and

$$H^{k+1}_{m-q_{k+1}+1,m}(c_k) = \sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\left(\frac{\pi^{k+1}(c_k)}{\pi^{k+1}(c_{k+1})}\right).$$

Hence, given $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$, we can find c_k by solving Equation (18).

Step 3: Given $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$ and $c_1, \ldots, c_{\ell-1}$, for each $k \in \{1, \ldots, \ell-1\}$ and each $a \in [c_{k-1}, c_k]$, there is a unique $\pi^k(a)$ that satisfies Equation (19). Moreover, we can get the mixed strategy function $\gamma^k(a)$ by differentiating Equation (19).

Equation (16) at $a \in [c_{k-1}, c_k]$ can be rewritten as, for each $k \in \{1, \ldots, \ell - 1\}$,

$$\begin{aligned} & v_k \sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) \sum_{j=m-q_k+1}^{m} p_{j,m-j}\left(\frac{\pi^k(a)}{\pi^k(c_k)}\right) \\ &= v_{k+1} \sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\ &\quad + v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\left(\frac{F(a) - F(c_{k-1}) - \pi^k(a) + \pi^k(c_{k-1})}{\pi^{k+1}(c_{k+1})}\right) \end{aligned} \tag{19}$$

where we used the following equation: for each $a \in [c_{k-1}, c_k]$, since $\pi^k(a) := \pi^k(c_{k-1}) + \int_{c_{k-1}}^{a} \gamma_k(x)dF(x)$,

$$\begin{aligned}\pi^{k+1}(a) &:= \int_{c_{k-1}}^{a}(1-\gamma_k(x))dF(x)\\ &= F(a)-F(c_{k-1})-\pi^k(a)+\pi^k(c_{k-1}).\end{aligned}$$

Differentiate Equation (19) with respect to a by using Lemma 1:

$$\begin{aligned}& v_k\sum_{m=q_k}^{n-1}p_{m,n-m-1}(\pi^k(c_k))\frac{\gamma_k(a)f(a)}{\pi^k(c_k)}mp_{m-q_k,q_k-1}\Big(\frac{\pi^k(a)}{\pi^k(c_k)}\Big)\\ &= v_{k+1}\sum_{m=q_{k+1}}^{n-1}p_{m,n-m-1}(\pi^{k+1}(c_{k+1}))\frac{f(a)-\gamma_k(a)f(a)}{\pi^{k+1}(c_{k+1})}mp_{m-q_{k+1},q_{k+1}-1}\Big(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})}\Big). \quad (20)\end{aligned}$$

Let us define the following functions:

$$\begin{aligned}A^k(a) &= v_k\sum_{m=q_k}^{n-1}p_{m,n-m-1}(\pi^k(c_k))mp_{m-q_k,q_k-1}\Big(\frac{\pi^k(a)}{\pi^k(c_k)}\Big)>0\\ B^k(a) &= v_{k+1}\sum_{m=q_{k+1}}^{n-1}p_{m,n-m-1}(\pi^{k+1}(c_{k+1}))mp_{m-q_{k+1},q_{k+1}-1}\Big(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})}\Big)>0.\end{aligned}$$

Then we can write (20) as

$$\frac{\gamma_k(a)f(a)}{\pi^k(c_k)}A^k(a)=\frac{f(a)(1-\gamma_k(a))}{\pi^{k+1}(c_{k+1})}B^k(a). \quad (21)$$

Solving for $\gamma_k(a)$ in (21), we obtain

$$\gamma_k(a)=\frac{\pi^k(c_k)B^k(a)}{\pi^{k+1}(c_{k+1})A^k(a)+\pi^k(c_k)B^k(a)}.$$

Step 4: We find the effort function β^D. Consider a student with type $a\in[c_{k-1},c_k]$. A necessary condition is that she does not want to mimic any other type a' in $[c_{k-1},c_k]$. Her utility maximization problem is

$$\max_{a'\in[c_{k-1},c_k]} v_k\Big(\sum_{m=0}^{q_{k-1}}p_{m,n-m-1}(\pi^k(c_k))+\sum_{m=q_k}^{n-1}p_{m,n-m-1}(\pi^k(c_k))H^k_{m-q_k+1,m}(a')\Big)-\frac{\beta^D(a')}{a}.$$

The first-order necessary condition requires the derivative of the objective function to be 0 at $a'=a$. Hence,

$$v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h_{m-q_k+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition at $\beta^D(c_{k-1})$, we obtain

$$\beta^D(a) = \beta^D(c_{k-1}) + v_k \int_{c_{k-1}}^{a} x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h^k_{m-q_k+1,m}(x) dx$$

for all $a \in [c_{k-1}, c_k]$.

E Instructions of the experiment (for online publication)

Welcome! This is an experiment about decision-making. You and the other participants in the experiment will participate in a situation where you have to make a number of choices. In this situation, you can earn money that will be paid out to you in cash at the end of the experiment. How much you earn depends on the decisions that you and the other participants in the experiment make.

During the experiment you are not allowed to use any electronic devices or to communicate with other participants. Please exclusively use the programs and functions that are intended to be used in the experiment.

These instructions describe the situation in which you have to make a decision. The instructions are identical for all participants in the experiment. It is important that you read the instructions carefully so that you understand the decision-making problem well. If something is unclear to you while reading, or if you have other questions, please let us know by raising your hand. We will then answer your questions individually.

Please do not, under any circumstances, ask your question(s) aloud. You are not permitted to give information of any kind to the other participants. You are also not permitted to speak to other participants at any time throughout the experiment. Whenever you have a question, please raise your hand and we will come to you and answer it. If you break these rules, we may have to terminate the experiment.

Once everyone has read the instructions and there are no further questions, we will conduct a short quiz where each of you will complete some tasks on your own. We will walk around, look over your answers, and solve any remaining comprehension problems. The only purpose of the quiz is to ensure that you thoroughly understand the crucial details of the decision-making problem.

Your anonymity and the anonymity of the other participants will be guaranteed throughout the entire experiment. You will neither learn the identity of the other participants, nor will they learn your identity.

General description

This experiment is about students who try to enter the university. The 24 participants in the room have been sorted into two groups of 12 persons each. These 12 participants represent students competing for university seats. The experiment consists of 15 independent decisions (15 rounds), which represent different student admission processes. At the end of each round every student will receive at most one seat in one of the universities or will remain unassigned.

There are two universities that differ in quality. We refer to the best university as University 1. Admission to the best university (University 1) yields a payoff of 2,000 points for the students. Admission to University 2 yields a smaller payoff for the students, which can vary across the rounds. Each university has a certain number of seats to be filled, a factor which can also be different for each of the rounds.

Instructions for CCA

The allocation procedure is implemented in the following way:

At the beginning of each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus, every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

Admission to universities is centralized and is based on the amount of effort that each student puts into a final exam. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) the effort. The higher the ability the easier (cheaper) the effort is. The price of one unit of effort is determined as: 100 divided by the ability, 100/ability. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen. You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding of the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can either be used to exert effort or kept.

After each student has decided how much effort to buy, these effort levels are sent to the centralized clearing house which then determines the assignments to universities. The students who have chosen the highest effort levels are assigned to University 1 up to the capacity of this university. They receive 2,000 points. The students with the next higher levels of effort are assigned to University 2 up to its capacity and receive the corresponding amount of points. All other students who have applied remain unassigned and will receive no points. Participants that have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff from university admission. Thus:

Payoff = Endowment − price of effort∗units of effort + payoff from assignment

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for your actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats. The admission to University 1 yields 2,000 points and the admission to University 2 yields 1,000 points.

Julia has an ability of 25. Thus, the cost of one unit of effort is $100/25 = 4$ points for her. Her endowment is 2,200 points, which means that she can buy a maximum of $2{,}200/4 = 550$ units of effort. Let us imagine that Julia decides to buy 400 units of effort. She thus has to pay $400*4 = 1{,}600$ points and keeps 600 points of her endowment.

Peter has an ability of 50. Thus, for him the cost of effort is $100/50 = 2$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2{,}200/2 = 1100$ units of effort. Let us assume that Peter chooses 600 units of effort. He thus has to pay $600*2 = 1{,}200$ points.

Simon has an ability of 80. For him, the cost of one unit of effort is $100/80 = 1.25$ points for one unit of effort. His endowment is 2,200 points. Thus, he can buy a maximum of $2{,}200/1.25 = 1760$ units of effort. Let us imagine that Simon decides to buy 500 units of effort. He thus has to pay $500*1.25 = 625$ points.

Imagine that the following effort levels were chosen by the other nine participants: 10, 70, 200, 250, 420, 450, 550, 700, 1,200.

Thus, the four students with the highest effort levels are assigned to University 1 and receive a payoff of 2,000 points. These are the students with effort levels 1,200, 700, 600 (Peter), and 550. Of the remaining eight students, five students with the highest levels of efforts are assigned to University 2 and receive a payoff of 1,000 points. These are the students with the efforts levels 500 (Simon), 450, 420, 400 (Julia) and 250.

The students with effort levels 10, 70, and 200 remain unassigned.

Thus, the payoff for Julia is $2,200 - 1,600 + 1,000 = 1,600$, for Peter $2,200 - 1,200 + 2,000 = 3,000$, and for Simon $2,200 - 625 + 1,000 = 2,575$.

Instructions for DCA

The allocation procedure is implemented as follows:

At the beginning of each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus, every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

The admission to universities is decentralized. Students first decide which university they want to apply to. Thus, you have to choose one university you want to apply to. After the decision is made, you will compete only with students who have decided to apply to the same university. The assignment of seats at each university is based on the amount of the effort that each student puts into a final test. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) the effort is. The price of one unit of effort is determined as: 100 divided by the ability, 100/ability. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen. You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding for the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can be used to exert effort or can be kept.

After each student decides how much effort to buy, these efforts are used to determine the assignments to universities. Among the students who apply to University 1, the students with the highest effort levels are assigned to this university up to its capacity and receive 2,000 points. All other students who applied to University 1 remain unassigned. Among those students who apply to University 2, the students with the highest effort levels are assigned a seat up to the capacity of University 2. They receive the corresponding amount of points. All other students who have applied to University 2 remain unassigned. Participants that have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff from university admission. Thus:

Payoff = Endowment − price of effort∗units of effort + payoff from assignment

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for your actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats.

Julia has an ability of 25 and decides to apply to University 2. Thus, the cost of one unit of effort is 100/25 = 4 points for her. Her endowment is 2,200 points, which means that she can buy a maximum of 2,200/4 = 550 units of effort. Let us imagine that Julia decides to buy 400 units of effort. She thus has to pay 400*4 = 1,600 points and keeps 600 points of her endowment.

Peter has an ability of 50. He applies to University 1. Thus, for him the cost of effort is 100/50 = 2 points for one unit of effort. His endowment is 2,200 points. Thus, he can buy a maximum of 2,200/2 = 1,100 units of effort. Let us assume that Peter chooses 600 units of effort. He thus has to pay 600*2 = 1,200 points.

Simon has an ability of 80. He applies to University 2. Thus, the cost of one unit of effort is 100/80 = 1.25 points for one unit of effort. His endowment is 2,200 points. He can thus buy a maximum of 2,200/1.25=1,760 units of effort. Let us imagine that Simon decides to buy 500 units of effort. Thus, he has to pay 500*1.25=625 points.

Imagine that there are an additional four students who decide to apply to University 2 (competing with Julia and Simon), and five students who decide to apply to University 1 (competing with Peter). The following efforts were bought by the four participants who apply to University 2, together with Julia: 10, 70, 450, 550.

Thus, there are six contenders for five seats. All students, but one with the effort of 10, receive a seat at University 2 and thus a payoff of 1,000 points.

The following efforts were bought by the five other participants who applied to University 1, together with Peter: 200, 250, 420, 700, 1,200.

Thus, there are six contenders for four seats. The four students with the highest efforts are assigned to University 1, including Peter, and all receive 2,000 points.

The students with effort levels 200 and 250 remain unassigned.

Thus, the payoff for Julia is $2,200 - 1,600 + 1,000 = 1,600$, for Simon $2,200 - 625 + 1,000 = 2,575$ and for Peter $2,200 - 1,200 + 2,000 = 3,000$. document