

PAPER • OPEN ACCESS

## Effect of nonlinearities and objective function in optimization of an energy harvesting device

To cite this article: C D Gatti *et al* 2018 *J. Phys.: Conf. Ser.* **1052** 012098

View the [article online](#) for updates and enhancements.

### Related content

- [Effect of Nonlinearity on Scattering Dynamics of Solitary Waves](#)  
Wen Zhen-Ying, Zhao Hong, Wang Shun-Jin *et al.*
- [Some metal oxides and their applications for creation of Microsystems \(MEMS\) and Energy Harvesting Devices \(EHD\)](#)  
K Denishev
- [A nonlinear stretching based electromagnetic energy harvester on FR4 for wideband operation](#)  
Dhiman Mallick, Andreas Amann and Saibal Roy



**IOP | ebooks™**

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

# Effect of nonlinearities and objective function in optimization of an energy harvesting device

C D Gatti<sup>1</sup>, J M Ramirez<sup>1</sup>, M Febbo<sup>2</sup> and S P Machado<sup>1</sup>

<sup>1</sup>Grupo de Investigación en Multifísica Aplicada, Universidad Tecnológica Nacional FRBB and CONICET. 11 de abril 461, 8000 Bahía Blanca, Argentina.

<sup>2</sup>Instituto de Física del Sur, CONICET and Departamento de Física. Universidad Nacional del Sur (UNS). Av Alem 1253, 8000 Bahía Blanca, Argentina.

**Abstract.** This work presents a study of the impact of the linearity assumption of the mechanical model in the overall performance of an energy harvesting piezoelectric beam. Also, a brief assessment of geometrical optimization solutions using different objective functions is presented. The mechanical model of the harvester is based on both linear and nonlinear variants of the electrical and mechanical constitutive equations for the piezoelectric material. The nonlinear elastic, damping and electromechanical coupling parameters are obtained via least squares identification using physical experimentation; the experimental tests are performed at different ground excitation accelerations. The computational optimization of the harvester is done using the genetic algorithm implemented in Matlab. Different objective functions are tested, i.e. broadband maximum peak power, maximum power at a particular frequency and broadband mean power; the influence of the selection of each of them in the total recovery of the power of the device is analyzed. The most suitable function to recover the vibratory energy from conventional transport vehicles is found.

## 1. Introduction

A large number of works present devices based on piezoelectric materials that recover energy from vibrations taking advantage of material deformation [1-4]. This type of devices have a wide number of applications [5, 6], our main interest are the ones that operate excited by four stroke engine vibrations.

In the design and development of energy harvesting devices the most important variable is the efficiency of the energy conversion. Regardless of the device type and the power source used, a certain configuration of geometrical, mechanical and electrical parameters maximizes the total energy recovered. One of the most effective methods to define this configuration is the use of optimization algorithms [7]. This necessarily implies the definition of variables and objective functions to be optimized. In most works [8-11], a linear mechanical model and a peak harvested power based objective function is used. As it will be shown in this paper, this choice of objective function does not always maximize the total energy recovered. Also, the linearization of the mechanical model entails a not negligible error in the system response.

This work presents two main developments; firstly a study of the impact of the linearity assumption of the mechanical model in the overall performance of an energy harvesting piezoelectric beam, lastly a brief assessment of geometrical optimization solutions using different objective functions. The most suitable function to recover the vibratory energy from conventional transport vehicles in the usual frequency range is found.



First, the mechanical model of the harvester based on linear and nonlinear variants of the electrical and mechanical constitutive equations for the piezoelectric material is presented. In the nonlinear model, the nonlinear parameters are identified using the least squares method fed with experiments performed in the laboratory subjecting the device to different amplitudes of ground acceleration. Then, the computational optimizations performed with genetic algorithms implemented numerically in Matlab are presented. Three different objective functions are tested: broadband maximum peak power, maximum peak power at the RMS frequency of usual use and broadband mean power in the range of frequency use. The influence of the objective function choice in the total energy recovery of the device is analyzed and the most suitable for the application at hand is determined. At last, conclusions regarding the inadequate use of a linear model to optimize are presented.

## 2. Harvester model and parameters identification

The device design takes advantage of the vibration of a piezoelectric beam subjected to the excitation of its base by the transport vehicle's engine. A potential difference between the electrodes of the piezoelectric due to beam deformation is generated.

The mathematical model is based on the scheme shown in Figure 1 which consists of a cantilevered stainless steel beam with a piezoelectric sheet (MFC M8507P2) attached to the upper surface. This beam is excited in the base by means of a shaker (Labworks ET-132), and a triaxial accelerometer (PCB Piezotronics ICP 356A32) is used to measure the accelerations.

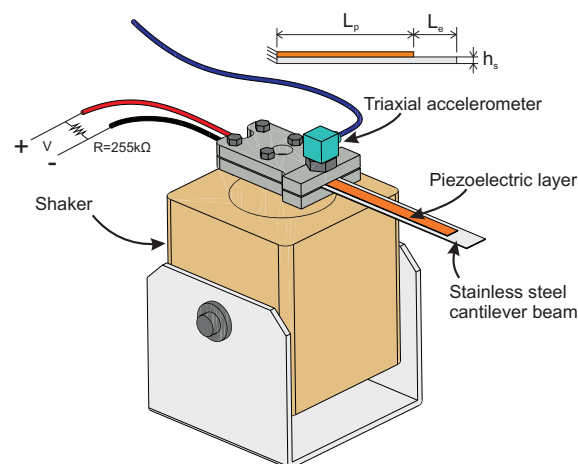
The beam model is based on a Bernoulli-Euler formulation, considering a nonlinear piezoelectric constitutive equations [12], which is presented by the authors in [13]. A Lagrangian approach is used to derive the system of differential equations, which result:

$$M\ddot{q} + (Bq \operatorname{sgn}(q) + B_n q^2) \operatorname{sgn}(\dot{q}) + Kq + K_n q^2 \operatorname{sgn}(q) - (\tilde{\theta} + \tilde{\theta}_n q \operatorname{sgn}(q))v = f, \quad (1)$$

$$C_p \dot{v} + \frac{v}{R_L} + (\tilde{\theta} + \tilde{\theta}_n q) \dot{q} = 0. \quad (2)$$

where  $M$ ,  $B$ ,  $B_n$ ,  $K$ ,  $K_n$ ,  $\tilde{\theta}$ ,  $\tilde{\theta}_n$ ,  $Q$ ,  $f$ ,  $C_p$  and  $R_L$  are the modal mass, linear and nonlinear damping, linear and nonlinear stiffness, linear and nonlinear electromechanical coupling, displacement and modal force, internal capacitance of the piezoelectric unimorph sheet and load resistance. Table 1 presents the numerical values of the main parameters of the model for all study cases.

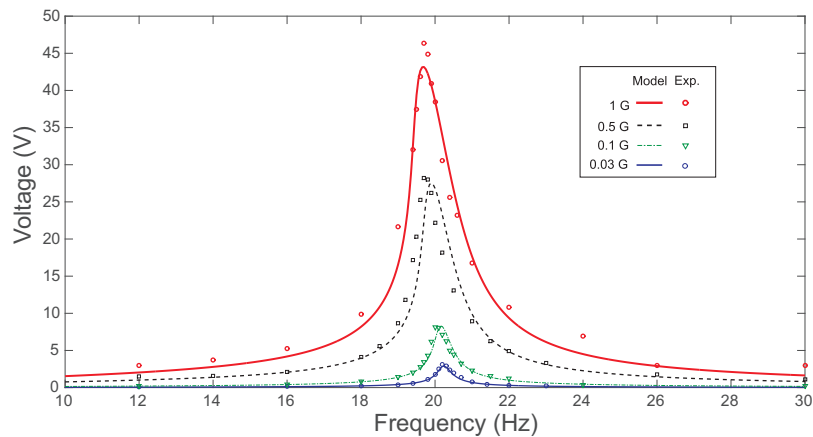
The nonlinear differential equations are solved analytically using harmonic balance method, so that the dynamic response of the system is obtained. From this, it is possible to find the voltage  $v(t)$  to compare with the experiments and to identify the nonlinear parameters that are presented in Table 2 (which are related to those in equations (1,2) in [12]). The identification is performed using least squares method. Figure 2 shows the voltage curves at different levels of acceleration. As a first singular feature, it is possible to observe that the curves bend to the left (softening response) as the amplitude of the base acceleration increases.



**Figure 1.** Harvester and shaker scheme.

**Table 1.** Numerical values of the main parameters of the model.

Parameter	Stainless steel	Piezoelectric steel
Width (mm)	12.7	8
Thickness (mm)	$h_s$	0.3
$L_e$ (mm)	21.5	-
$L_p$ (mm)	-	$L_p$
Density ( $\text{kg/m}^3$ )	7900	5440
Young's modulus (GPa)	193	15.85
Charge constant $d_{31}$ (pm/V)	-	-170
Electrical permittivity (nF/m)	-	16.81
Electrical capacitance (nF)	-	38.11
Load resistance (kΩ)	255	



**Table 2.** Nonlinear parameters identified.

Parameter	Value
$b_{11}$ (N/m <sup>2</sup> )	$1.11 \cdot 10^9$
$b_{111}$ (N/m <sup>2</sup> )	$6.38 \cdot 10^{12}$
$c_{111}$ (TPa)	-79.42
$e_{311}$ (kC/m <sup>2</sup> )	-113.42

**Figure 2.** Voltage-frequency curves used for the identification of parameters.

Finally, the following power equation used to perform the optimizations is obtained from the voltage:

$$P = \frac{V^2}{R_L}. \quad (3)$$

### 3. Optimizations and results

The geometric optimizations of the harvester are done using a genetic algorithm implemented in Matlab (*ga* command). The variables to be optimized are the piezoelectric length ( $10 \text{ mm} < L_p < 85 \text{ mm}$ ) and the thickness of the stainless steel beam ( $0.1 \text{ mm} < h_s < 1.5 \text{ mm}$ ). Three optimizations with the nonlinear model and three with the linear model ( $B_n = K_n = \tilde{\theta}_n = 0$ ) are performed, proposing three different objective functions: broadband maximum peak power (A), maximum peak power at the RMS frequency (B) and broadband mean power (C). In order to adequately compare the performance of each optimized device, the energy recovered by the device along an urban way of a conventional transport vehicle is evaluated. This energy is obtained by integrating the temporal signal of the electric power ( $P$ ) obtained by the model.

Table 3 shows the different study cases. As can be observed, the parameters optimized with the linear model differ markedly from those optimized with the nonlinear model. In addition, it is observed that the optimization C obtains the device that recovers more energy; this shows that choosing the broadband mean power as objective function is the best option for this type of applications. The recovered energy is shown in Figure 3, comparing the linear and nonlinear model.

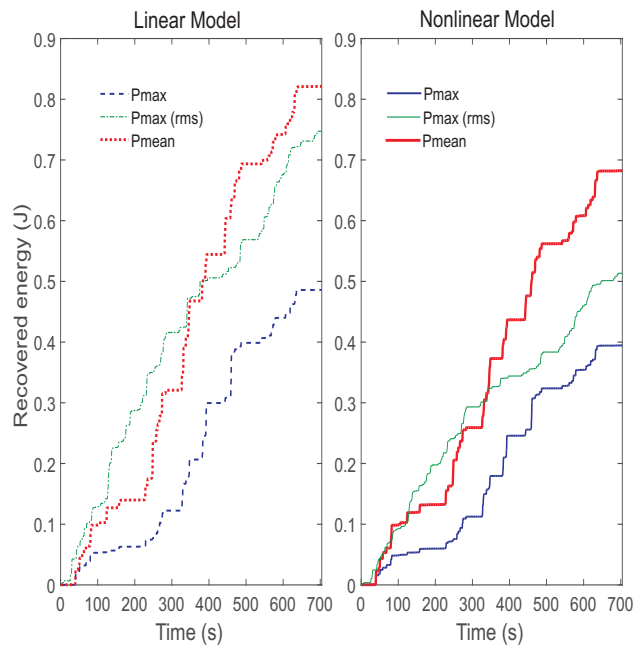
Finally, to evaluate the impact of assuming a linear model to perform the optimization, we introduce the optimized variables by the linear model within the nonlinear model. Case C is used to perform the analysis because it is the one with the most energy recovered. The recovered energy is 0.44 J, approximately 34% less than the power that it would have recovered (see Table 3 and Figure 4, nonlinear model of case C).

**Table 3.** Optimization cases and results obtained.

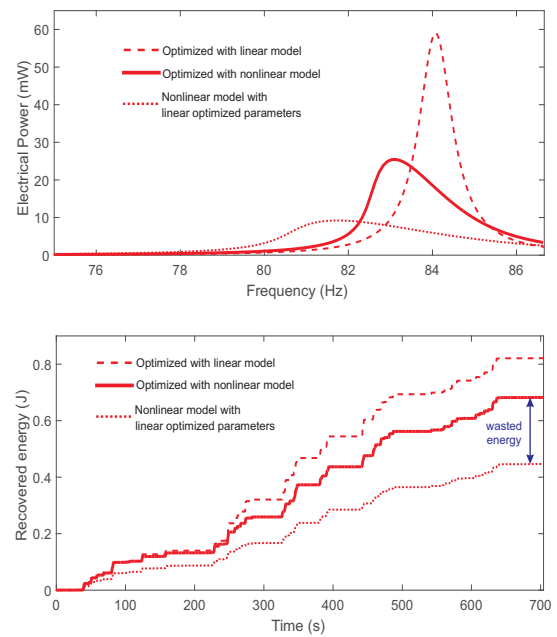
Case	Objective function	Linear model			Nonlinear model		
		$L_p$ (mm)	$h_s$ (mm)	Recovered energy (J)	$L_p$ (mm)	$h_s$ (mm)	Recovered energy (J)
A	Broadband maximum peak power	37	0.3	0.48	85	1.24	0.39
B	Maximum peak power at RMS frequency	60	0.44	0.74	85	0.82	0.51
C	Broadband mean power	52	0.53	0.82	85	1.19	0.68

### 4. Conclusions

From the numerical results two important conclusions are obtained. First, the optimized parameters obtained with the linear and nonlinear models are not the same. The linear model overestimates the



**Figure 3.** Recovered energy with different objective functions.



**Figure 4.** Recovered power and energy: comparison between optimization with linear and nonlinear models.

generated power in comparison with the nonlinear one. Additionally, a low power is obtained when the parameters optimized by means of the linear model are used in the nonlinear model.

Second, there is a great influence of the objective function selected in the optimization algorithm. It is clear that selecting the broadband mean power as objective function, instead of using the other two functions (most frequently used), the total recovered energy is considerably larger.

### Acknowledgments

The authors wish to thank CONICET, UNS, UTN FRBB, ANPCyT under grant PICT 2013-2065 for their financial support.

### References

- [1] Roundy S, Wright P K and Rabaey J 2003 *Comput. Commun.* **26** 1131-44
- [2] Priya S 2007 *J. Electroceram.* **19** 167-84
- [3] Febbo M, Machado S, Ramirez J and Gatti C 2016 *J. Phys. Conf. Ser.* **773** 012058
- [4] Erturk A and Inman D J 2008 *J. Intel. Mat. Syst. Str.* **19** 1311-5
- [5] Xiao H and Wang X 2014 *Fuel Cells* **280** 28
- [6] Febbo M, Machado S, Gatti C and Ramirez J 2017 *Energ. Convers. Manage.* **152** 166-75
- [7] Goldberg D E 1989 *Reading: Addison-Wesley*
- [8] Shafer M W, Bryant M and Garcia E 2012 *Smart Mater. Struct.* **21** 085008
- [9] Park J, Lee S and Kwak B M 2012 *J. Mech. Sci. Technol* **26** 137-43
- [10] Lefeuvre E, Badel A, Richard C and Guyomar D 2005 *J. Intel. Mat. Syst. Str.* **16** 865-76
- [11] Shu Y-C and Lien I 2006 *J. Micromech. Microeng.* **16** 2429
- [12] Leadenham S and Erturk A 2015 *Nonlinear Dynam.* **79** 1727-43
- [13] Gatti C D, Ramirez J M, Machado S P and Febbo M 2016 *J. Phys. Conf. Ser.* **773**