Universität Bielefeld

Faculty of Business Administration and Economics

Working Papers in Economics and Management

No. 14-2019
December 2019

Sustainable Debt Policy Rules and Growth in a Small Open Economy Model: Is a balanced government budget worthwhile?

Fabienne Lara Dascher

Bielefeld University
P.O. Box 100 131
33501 Bielefeld – Germany

ISSN 2196–2723

www.wiwi.uni-bielefeld.de

Sustainable Debt Policy Rules and Growth in a Small Open Economy Model: Is a balanced government budget worthwhile?*

Fabienne Lara Dascher

Faculty of Economics, Bielefeld University, f.dascher@wiwi.uni-bielefeld.de.

December 19, 2019

Keywords: Public Debt, Fiscal Sustainability, Economic Growth, Small Open Economy Model

JEL Classification: E62; F43; H63

Abstract

In this paper our objective is to analyze sustainable debt policy rules and economic growth using a model of endogenous economic growth theory. For the government it is possible to run into debt, but, the primary surplus is a positive linear function of the debt-to-GDP ratio which guarantees that public debt is sustainable. We analyze different sustainable debt policies in this small open economy model of endogenous growth with public capital accumulation, as well we take transitions into account. We investigate the characteristics and stability of the steady state and we analyze the effects on welfare for the different debt policies.

*I would like to thank Alfred Greiner and Christiane Clemens for the inspiring ideas and the support received in various meetings. I am indebted to António Afonso, Matthias Schön, Paolo Pasimeni and other participants for very competent and detailed comments during the INFER Workshop on New Challenges for Fiscal Policy which helped to improve the paper.

1 Introduction

The financial crisis that began as a sub-prime crisis in the USA in 2007 and turned into a public debt crisis led to deep economic recessions and to a drastic increase of national debt-to-GDP ratios in some countries. In the euro area, the crisis is still far from being solved: Public debt affects the evolution of market economies, several EU members cannot repay or refinance their government debt and economies are reliant on the support by other countries. The recent dispute on the draft budget between Italy and the EU Commission shows that fiscal policy is still a challenge for Europe and that there are still frictions and pressure among the Member States of the EU and European countries in general.

Even the countries participating in the European Economic and Monetary Union that have signed the Maastricht treaty which states that the public deficit and the public debt relative to GDP must not exceed 3% and 60% to ensure compliance with budgetary discipline, these countries had, and a part of them still have problems with their debt service and some even had to be bailed out by the European Stability Mechanism to prevent bankruptcy (Greiner & Fincke, 2015). During the crisis, different approaches have been implemented such as counter-cyclical expansionary fiscal policy in the US and in Europe, countries embarked on austerity, pro-cyclical policies that aimed at lowering sovereign debt and the interest rate (Arčabić et al., 2018). The EU reaction to the crisis was the introduction of strong fiscal consolidations under the surveillance of the European Commission. Expenditure cuts, notably lower public investments, led to a reduction of public deficits. The debt reduction benchmark announced by the reform of the Stability and Growth Pact (SGP)(also called "Six-Pack") in December 2011 states that Member States whose current debt-to-GDP ratio is higher than the 60% threshold must curtail the distance to 60% by an average rate of one-twentieth per year (Morimoto et al., 2017). The reason for this is the thought that persistent budget deficits lead to problems for the economy and the government, especially due to the government's borrowing associated with a budget deficit that "crowds out" private investment spending and decreases long run growth of the economy. Financial pressure on the government's future budgets and the debt sustainability are problems that occur due to these deficits (Huang et al., 2017). Accordingly, the medium- to long-term growth rate is the smaller the higher the public debt of a country. However, this does not mean that deficit-financed public investments always lead to low growth rate. It depends on the public investments. Productive public investments ensure a rise of GDP and as a consequence a rise in tax revenue which will reduce the long-term debt-to-GDP ratio. Therefore the rise of the debt-to-GDP ratio due to the deficit-financed public investments is only temporary.

The empirical literature is controversial about the effect of high public debt-to-GDP ratios. On the one hand, economists such as Checherita-Westphal & Rother (2012) identify that the effect of public debt on economic growth is hump-shaped which means that up to a certain threshold, higher public debt-to-GDP ratios go along with higher economic growth, however after this threshold, there is a negative correlation between public debt and economic growth. An important contribution in this context was the paper by Reinhart & Rogoff (2010) who stated that a debt-to-GDP ratio of over 90 percent has a significant negative impact on long-run economic growth, which gained attention and initiated a debate in research and policy (Arčabić et al., 2018). On the other hand, there exist empirical studies that do not find that nonlinear effect. Panizza & Presbitero (2014) study the negative correlation between public debt and economic growth, however the effect of debt on GDP depends on the structure and accumulation of debt and the link between debt and growth disappears when they correct for endogeneity. Fincke & Greiner (2015) describe a statistically negative effect for developed countries, but the relation between debt and economic growth is satistically positive for fast growing developing economies. Based on the empirical studies, the relation between public debt and economic growth seems to be negative for developed economies, although there is not a high effect and especially the effects are different between countries. Arčabić et al. (2018) illustrate a survey of empirical studies that aims at analyzing the effects of public debt on economic growth or the other way around. Hence, empirically, there is no final clarification how public debt impacts economic growth.

In our paper, we do not want to focus on debt thresholds, but rather is our objective to analyze sustainable debt policy rules (Balanced Government Budget and Permanent Public Deficits) and economic growth using a model of endogenous economic growth theory. Fiscal sustainability means that the government is able to repay its debt at some point in the future. In this context, an important aspect for the so-called golden rule of public finance is the intergenerational redistribution. Our analysis starts with the intertemporal budget constraint of the government to which the government must stick. Then, the primary surplus (ratio) plays a crucial role. The rise of public debt in an economy implies an increase of future primary surpluses to fulfill its intertemporal budget constraint. When the government does not raise the future primary surpluses, it accepts a default that can be followed by social turbulences which can be a danger for the political system. Raising taxes, reducing public spending or a rise in GDP that leads to more tax revenues can lead to higher primary surpluses and can prevent social tubulences. Naturally a combination of these three measures is also possible to raise the primary surplus. Raising taxes and reducing public spending can have a negative impact on the economy's growth rate. This means that the government stimulates growth if it reduces public debt relative to GDP. We concentrate on public spending and public revenues and neglect in this paper the central bank of an economy since normally governments should not count on money creation by central banks that raises the money supply and leads to a higher inflation rate to reduce the real value of public debt because central banks are independent and assisting in pursuing sustainable debt policies is not one of their main tasks (Greiner & Fincke, 2015).

In our paper, it is possible for the government to run into debt, but, the primary surplus relative to GDP is a positive linear function of the debt-to-GDP ratio which guarantees that public debt is sustainable. This concept is based on the tests by Bohn (1995). The economic intuition behind it says that if governments run into debt today they must take corrective actions in the future by increasing the primary surplus. If the government does not act in this way, public debt will not be sustainable (Greiner & Fincke, 2015). The crisis led to a renewed twofold interest in governments' reaction to the debt accumulation dynamics: Certain theoretical and empirical papers (Bohn, 1995, 1998; Fincke & Greiner, 2012; Greiner et al., 2006) tried to support the existence and the sign and size of this reaction. Whereas, in other papers the nature of governments' reactions to the accumulation of debt in terms of sustainability was of higher importance (Beqiraj et al., 2018). The optimal degree of fiscal policy responsiveness to rising public debt and changing macroeconomic conditions induced intense discussion and diverging views. Bohn (1998) stated that fiscal consolidation in case of a rapidly increasing public debt level is an approach to restore fiscal sustainability. Other authors, nevertheless, studied the risks of dampening economic activity and of facing fiscal fatigue (Ghosh et al., 2013a,b), in particular large and sustained fiscal consolidation to ensure sustainability (Eichengreen & Panizza, 2016). Fiscal reaction functions and its use in fiscal sustainability analysis currently gained noticeable attention. Among others, Ghosh et al. (2013a,b) used fiscal reaction functions to estimate public debt sustainability thresholds and public debt limits (Beqiraj et al., 2018). In this context, it is also crucial to investigate the policy behavior. After illustrating the cointegration of the public debt-GDP ratio and the structural primary surplus, Beqiraj et al. (2018) identified a sort of general empirical law of policy behavior valid for the OECD countries where the long-term governments' reaction to the accumulation of debt in terms of sustainability is negative. This means that fiscal policies are not sustainable. However, in the short term, when the output gap is positive, the policy makers interfere with a new deficit and debt, though it does not establish a symmetrical correction for the reversed situation. This may be taken as evidence for politicians, at least in most OECD countries considered, to have rather a short-sighted perspective. The short-sighted perspective of politicians can be explained by the politician's will to be re-elected. Nevertheless, the present rules and behaviors of fiscal and monetary policies should focus on asymmetrical long-sighted monetary and fiscal policies.

As explained, during the crisis, some economists recommend to cut back the size of the public sector and reduce the government debt for countries holding large external debt and with low rates of economic growth. As a consequence, more private initiative and investment should boost the rate of economic growth. These austere policies would lead to a smaller foreign indebtedness and thus a reduction in foreign debt service. In the long run this cut in the ratio of government debt to domestic income could cause more private consumption. However, the literature does not offer a logically consistent model that is able to explain these arguments. As an example, a balanced-budget rule was critised by Schmitt-Grohé & Uribe (1997) who show that such a rule can be destabilizing because it generates indeterminacy and expectations-driven fluctuations, in a neoclassical model where expenditures are financed with distortionary income taxes and labor income tax rates are endogenous. Even if there is a substantial and expanding literature on endogenous growth (Romer, 1986; Barro, 1989), there is a lack of explanations of the effects of budgetary policies for economic growth in open econonomies (van der Ploeg, 1996). As in reality, the private sector and the government are able to lend and borrow their assets in the foreign capital market, countries like Greece, Italy, Ireland, and Portugal hold large levels of debt and thereof as well large external debt. Hence, it is crucial to analyze the sustainable debt policy rules in a model of an open economy (Morimoto et al., 2017). The literature states some results of the analysis of open economies. van der Ploeg (1996) analyses a small open economy with overlapping generations of households, investment with adjustment costs, externalities in production, a stock market, and a risk premium on foreign debt and shows that supply-side shocks generate a positive correlation between foreign indebtedness and the rate of economic growth while demand-side shocks cause a negative correlation between these variables. Alogoskoufis (2014) demonstrates that openness results in higher steady state growth and a negative net external position if the world real interest rate is lower than the real interest rate under autarky and the other way around. Turnovsky (1996a,b) analyzes a representative household endogenous growth models and shows the existence of a balanced growth path for a small open economy only if the world real interest rate is equal to the pure rate of time preference plus the growth rate of domestic output, a crucial knife edge condition. Elsewise domestic output and domestic consumption grow at different rates. This is often a problem, if either capital market imperfections are postualted (Barro et al., 1992), or by assuming an endogenous labor supply (Turnovsky, 2000). Alogoskoufis (2014) investigates a simple endogenous growth intertemporal model of a small open economy, with adjustment costs for investment and overlapping generations. Huang et al. (2017) propose that integrated world capital and goods markets can be important for the stabilization of the economy, if the government counts on changes in income tax rates to achieve budget balance. Futagami et al. (2008) analyze productive public spending and public debt in an endogenous growth model where government debt has to converge to a certain exogenously given debt ratio asymptotically. Morimoto et al. (2017) investigate this model for a small open economy and indicate that sufficiently low debt-to-GDP ratios can avoid indeterminate equilibrium paths. In the model, endogenous limit cycles can emerge.

As in Greiner & Fincke (2015), in our paper, the government invests in a productive public capital stock that has positive externalities on goods production and raises aggregate production possibilities. Futagami et al. (1993) and Turnovsky (1997) show the positive effects on private goods production of public capital accumulated through public investment. Other studies on public debt finance in endogenous growth models in which the inputs for private goods production are government services or public capital analyze the policy impacts of public debt finance on equilibrium dynamics, long-run growth, and welfare (Greiner & Semmler, 2000; Ghosh & Mourmouras, 2004; Greiner, 2007, 2012; Yakita, 2008)

Here, the expenditures of the government are financed by levying an income tax and by issuing bonds. We investigate the effects of different sustainable debt policies in this small open economy model of endogenous growth with public capital accumulation, where the government can borrow from abroad to finance its debt, on the dynamics of the economy with respect to the characteristics and stability of steady state and with respect to welfare. The economy in this model consists of three sectors: A household sector, a productive sector and the government. Our small open economy does not follow an independent interest rate policy and in the long run, on the SBGP, it does not have an influence on its rate of economic growth. We define economic growth as the medium-to long-term development of GDP. Therefore, we do not analyze the short-term impacts of a government's debt policy, as after Keynes they are used for smoothing economic fluctuations. Therefore, we assume that all markets are in their equilibrium, in particular the labour market with a natural employment rate. We analyze the welfare effects on the sustainable balanced growth path of the different scenarios. Furthermore, we examine the welfare effects along the transition path, that is the effects resulting from a transition from one scenario to the other scenario. Our aim is to answer the question: Is in this model a sustainable balanced government budget worthwhile? Which effects do transitions from one scenario to the other scenario have on welfare? What are the different macroeconomic consequences of these different sustainable debt policies?

In section 2, we introduce the model and present the model structure by explaining the household, the productive sector and the government. Further, we illustrate the model dynamics by describing the equilibrium conditions and the (sustainable) balanced growth path. In section 3, we analyze the two different debt policies, Balanced Government Budget and Permanent Public Deficits, on the SBGP. In 4, we study the welfare effects on the sustainable balanced growth path of the different scenarios and along the transition path. Section 5 summarizes our results.

2 Model Structure and Model Dynamics

The structure of our model is essentially the same as in Morimoto et al. (2017), considering a small open economy model of ongoing growth with public capital accumulation. The economy consists of three sectors: A household sector, a productive sector and the government. For the production of a single final good, labor, private capital and public capital (infrastructure) is used. Private capital and the final good can be traded freely between the countries. Though, public capital cannot cross border and migration of individuals is not possible. For the construction of social infrastructure, public investments of the government are necessary. First, we describe the household and the productive sector.

2.1 The household and the productive sector

The household sector is represented by one household, a representative household that maximizes the discounted stream of utility arising from per-capita consumption, C(t), over an infinite time horizon subject to its flow budget constraint(, taking factor prices as given). The population size is normalized to one and the household inelastically supplies one unit of labor, L(t), at each moment of time. The maximization problem of the household can be written as [1]

$$\max_{C} \int_{0}^{+\infty} \left(\frac{C^{1-\sigma} - 1}{1-\sigma} \right) e^{-\rho \cdot t} \, \mathrm{d}t, \tag{1}$$

subject to

$$\dot{A} = (1-\tau)I - C. \tag{2}$$

$\rho \in (0,1)$ is the household's rate of time preference or the subjective discount rate. The emergence of the parameter σ in the utility of the household reflects the intertemporal elasticity of substitution of consumption. The household's income is denoted by $I = r \cdot A + w$, where r is the world interest rate, w is the wage rate and A is the asset holdings of the household. The asset holdings are classified into three types of assets, private capital (K), government bonds (B) and foreign assets (FA), hence $A = K + B + FA$. $FA < 0$ means that some fractions of private capital or government bonds are owned by foreign agents. As shown in equation

[1] From now on we omit the time argument t if no ambiguity arises.

(2), the government imposes a constant tax on the household's income, $\tau \in (0,1)$ which takes the residence base form meaning that nonresidents' income is not taxed. A must satisfy the no-Ponzi game (NPG) condition, $\lim_{t\to\infty} A_t e^{-r \cdot t} \geq 0$. The dot gives the derivative with respect to time and we neglect depreciation of (private) capital. The household maximizes equation (1) subject to equation (2). To solve this problem we formulate the current-value Hamiltonian for this optimization problem written as

$$\mathcal{H} = \left(\frac{C^{1-\sigma} - 1}{1-\sigma}\right) + \lambda((1-\tau)(rA + w) - C), \tag{3}$$

where λ is the co-state variable or the shadow price of asset holdings. The necessary optimality conditions are given by

$$C^{-\sigma} = \lambda, \tag{4}$$

$$\dot{\lambda} = \rho\lambda - \lambda(1-\tau)r. \tag{5}$$

From the household's maximization of equation (1) subject to equation (2), we get

$$\dot{C} = \frac{\{(1-\tau)r - \rho\}C}{\sigma} \tag{6}$$

and the transversality condition $\lim_{t\to\infty} e^{-\rho \cdot t} A_t \cdot \lambda_t = 0$.

The productive sector is represented by one firm that behaves competitively, i.e. taking the interest rate as given. In addition, the representative firm maximizes static profits. The production function of the firm is given by

$$Y = K^{\alpha}(LK_g)^{1-\alpha}, \tag{7}$$

with $\alpha < 1$.

α is the private capital share and $(1-\alpha)$ gives the labour share. K_g denotes stock of infrastructure and reflects the external effects. K_g is labour augmenting. The production function satisfies the standard neoclassical characteristics, particularly the constant returns to scale with respect to K and LK_g. We define $x = K/K_g$, $k_g = K_g/Y$ and $\omega = (1-\alpha)x^{\alpha}$.

Using that labour is normalized to one, perfect competition and profit maximization yields

$$w = (1 - \alpha)K^{\alpha}K_g^{1-\alpha}, \tag{8}$$

$$r = \alpha K^{\alpha-1}K_g^{1-\alpha}. \tag{9}$$

Resorting to equations (4), (5) and (8), (9), which must hold in equilibrium, the growth rate of consumption is derived as

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\{(1-\tau)\alpha K^{\alpha-1}K_g^{1-\alpha} - \rho\}. \tag{10}$$

The growth rate of consumption is determined exogenously and thus constant.

2.2 The government

The government in our economy receives tax revenues from income taxation and has revenues from issuing government bonds. Further, the government sets the primary surplus such that it is a positive linear function of public debt which guarantees that public debt is sustainable, as in Greiner & Fincke (2015).

The accounting identity describing the accumulation of public debt in continuous time is given by:

$$\dot{B} = rB - S, \tag{11}$$

where B stands for real public debt (real public net debt), r is the real world interest rate (as above), and S is real government surplus exclusive of interest payments.

The intertemporal budget constraint of the government is fulfilled if

$$\lim_{t\to\infty} e^{-r\cdot t}B(t) = 0 \tag{12}$$

holds, which is the no-Ponzi game condition (see e.g. Blanchard & Fischer (1989)).

Now, assume that the ratio of the primary surplus to GDP is a positive linear function of the debt-to-GDP ratio and of a constant.

The primary surplus ratio, then, can be written as

$$\frac{S}{Y} = \phi + \psi \frac{B}{Y}, \tag{13}$$

where Y is the gross domestic income and $\phi \in \mathbb{R}$, $\psi \in \mathbb{R}_{++}$ are constants. The parameter ψ determines how strongly the primary surplus reacts to changes in public debt and therefore, can be considered as a feedback parameter of public debt[2]. This feedback parameter of public debt can vary over time, but has to be positive on average and greater than the difference between the world interest rate and the growth rate of GDP, so that the debt-to-GDP ratio converges to a constant.

ϕ determines whether the level of the primary surplus rises or falls with an increase in GDP. Using that equation the differential equation describing the evolution of public debt can be written as

$$\dot{B} = (r - \psi)B - \phi Y. \tag{14}$$

From Greiner & Fincke (2015) we know that, given the rule assumed in equation (13), a positive linear dependence of the primary surplus to GDP ratio on the debt ratio on average, guarantees that the intertemporal budget constraint of the government is met. Further, Greiner & Fincke (2015) shows that there is empirical evidence that developed countries raise their primary surplus as public debt rises. Therefore, we consider that the government sets the primary surplus according to (13) implying that public debt is sustainable, so that the evolution of public debt is given by (14). In principle, public debt could be negative signifying that the government is creditor. In this paper, however, we assume throughout that public debt is positive. In (13), the primary surplus is not only determined by public debt, it also depends on the level of GDP in a country which capture the total tax revenue and temporary government spending. Still sustainability of public debt is independent of the effect of GDP on the primary surplus as long as the primary surplus is a positive linear function of public debt.

To construct infrastructure, the government makes public investments. The amount of public investments is denoted by G. Then, stock of infrastructure accumulates according to $\dot{K}_g = G$.

[2]If ψ takes a large (small) value, the government adjusts $b = \frac{B}{Y}$ at a fast (slow) pace.

We define $g = \frac{G}{Y}$. As stated in the beginning, the government finances its expenditure in two ways, by levying an income tax and by issuing bonds.

Thus, the budget constraint of the government can be written as

$$rB + G = \dot{B} + \tau(r \cdot A + w). \tag{15}$$

Since the government sticks to the rule defined in equation (13), the government adjusts its debt-GDP ratio, $b = \frac{B}{Y}$, according to the following rule derived from (14)

$$\dot{b} = (r - \psi - \gamma)b - \phi, \tag{16}$$

where $\gamma = \frac{\dot{Y}}{Y}$ is the growth rate of GDP.

The budgetary rule (13) imposes a constraint on the possibility of the government to control public investment. This holds because a rise in public debt, for whatever reasons, implies that public investment must decrease, for given values of the parameters ϕ and ψ and for a given tax revenue. The reason is that the government must raise the primary surplus such that a fiscal policy is sustainable when public debt rises. This leads to an internal crowding-out effect of public debt (crowding-out of public investment as a result of public debt). The government conducts fiscal policy by setting the parameters ϕ and ψ. Certainly, the government can also set the tax rate τ, however variations in τ have been studied frequently and therefore we will fix this fiscal parameter.

Public debt is a state variable which evolves gradually. Over time, however, a higher deficit implies a higher debt and the primary surplus must rise in order to keep fiscal policy sustainable.

In the next sections, we will study in more detail the effects of the parameters and we will get more insights in the dynamics of the model.

2.3 Equilibrium conditions and the (sustainable) balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (8) and (9)), the household solves (1) subject to (2) and the budget constraint of the government (15) is fulfilled with the primary surplus set according to (13).

A sustainable balanced growth path (SBGP) is defined as a path where the economy is in equilibrium and on which the following endogenous variables grow at the same strictly positive constant growth rate, i.e. $\dot{K}/K = \dot{C}/C = \dot{A}/A$ holds, and the intertemporal budget constraint of the government is fulfilled, that is equation (12) must hold. Note that the SBGP is dynamically efficient[3] and the transversality condition of the household is fulfilled. Since we have posited that the government sets the primary surplus according to (13) with $\psi > 0$ any path which satisfies $\dot{K}/K = \dot{C}/C = \dot{A}/A = \dot{B}/B$ is associated with a sustainable public debt. To make this clear we speak of a sustainable balanced growth path.

To analyze our economy around a SBGP we define the new variables $c = C/Y$ and $a = A/Y$. Here, Y is given by equation (7), K_g is a state variable, and $L = 1$ holds in equilibrium. We then have to treat Y as a predetermined variable. Then, a and b are state variables and c is a jump variable. From (7) and $\dot{K}_g = G$, we have $g = k_g\gamma$. From (2), (6), (8), (15) and the definitions of I and γ, we obtain

$$\dot{b} = (r - \gamma)b + g - \tau(r \cdot a + \omega k_g), \tag{17}$$

$$\dot{a} = \{(1 - \tau)r - \gamma\}a - c + (1 - \tau)\omega k_g, \tag{18}$$

$$\dot{c} = \left\{ \frac{(1 - \tau)r - \rho}{\sigma} - \gamma \right\} c. \tag{19}$$

Defining the debt-GDP ratio as $b = \frac{B}{Y}$, we merge the evolution of the public debt-GDP ratio, $\dot{b}$, resulting from equation (14), and the evolution of the debt-GDP ratio, $\dot{b}$, coming from (15).

[3] The world interest rate exceeds the growth rate of GDP on average.

Solving for γ using $g = k_g\gamma$, we obtain

$$\gamma(= \frac{g}{k_g}) = \frac{\tau(ra + \omega k_g) - \psi b - \phi}{k_g} = \gamma(a, b, \cdot). \tag{20}$$

This equation shows that the parameters ψ and ϕ have an impact on γ in the short run. Substituting equation (20) into (18) and (19), a three dimensional system of differential equations with its initial values $b(0)$ and $a(0)$ for $t = 0$ is given by

$$\dot{b} = (r - \psi - \gamma)b - \phi, \tag{21}$$

$$\dot{a} = \{(1 - \tau)r - \gamma(a, b, \cdot)\}a - c + (1 - \tau)\omega k_g, \tag{22}$$

$$\dot{c} = \left\{ \frac{(1 - \tau)r - \rho}{\sigma} - \gamma(a, b, \cdot) \right\} c. \tag{23}$$

A solution of $\dot{b} = \dot{a} = \dot{c} = 0$ with respect to b, a, c gives a SBGP for this model and the corresponding ratios $b^\star, a^\star, c^\star$ on the SBGP.

To get insight into our model we derive the ratios $b^\star, a^\star, c^\star$ on the SBGP by solving simultaneously the three dimensional system of differential equations. Here, we set $\dot{c}/c = 0$ to get a $c^\star \neq 0$. As the analytical model turns out to become complex, we just state

$$b^\star = \frac{\sigma\phi}{r(\sigma + \tau - 1) + \rho - \sigma\psi}, \tag{24}$$

$$\gamma^\star = \frac{(1 - \tau)r - \rho}{\sigma}. \tag{25}$$

$a^\star$ and $c^\star$ on the SBGP exist. To ensure positive growth, we assume that $\gamma^\star = \frac{(1-\tau)r-\rho}{\sigma} > 0$.

The policy parameters ψ and ϕ have no effects on the long-run growth rate, $\gamma^\star$.

We see that $b^\star$ contains $\gamma^\star$ and we can rewrite $b^\star$ by

$$b^\star = \frac{\phi}{r - \gamma^\star - \psi}. \tag{26}$$

From this expression, we can see that in the case $\phi < 0$, to get a positive $b^\star$, the feedback parameter of public debt ψ has to be larger than the difference between r and $\gamma^\star$, this means $\psi > r - \gamma^\star$. In the case $\phi > 0$, $\psi < r - \gamma^\star$ must hold to get a positive $b^\star$.

b on the SBGP can be positive or negative since it is an endogenous variable. However, from an economic point of view a positive value of government debt is more realistic since most countries are characeterized by debt. So, we assume that the government is a debtor, that is $b > 0$ holds.

To examine the stability of the unique SBGP, we linearize the three dimensional system of differential equations around the SBGP characterized by $b^\star, a^\star, c^\star$ and $\gamma^\star$, and then we obtain

$$\begin{bmatrix} \dot{b} \\ \dot{a} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} r - \psi(1 - \frac{b^\star}{k_g}) - \gamma^\star & -\frac{\tau r}{k_g} b^\star & 0 \\ \frac{\psi}{k_g} a^\star & (1-\tau)r - \frac{\tau r}{k_g} a^\star - \gamma^\star & -1 \\ \frac{\psi}{k_g} c^\star & -\frac{\tau r}{k_g} c^\star & 0 \end{bmatrix} \begin{bmatrix} b - b^\star \\ a - a^\star \\ c - c^\star \end{bmatrix}$$

Thanks to the Jacobian Matrix with SBGP values we determine the eigenvalues, the determinant and the trace of this Jacobian Matrix and thus indicate the stability of the SBGP. In this context, it is crucial to indicate the growth rate of the asset holdings of the household and the growth rate of public debt that are given by

$$\frac{\dot{A}}{A} = (1-\tau) \cdot (\alpha K^{\alpha-1} K_g^{1-\alpha}) + \frac{(1-\tau) \cdot (1-\alpha)}{a} - \frac{c}{a}, \tag{27}$$

$$\frac{\dot{B}}{B} = ((\alpha K^{\alpha-1} K_g^{1-\alpha}) - \psi) - \frac{\phi}{b}. \tag{28}$$

We see that the growth rate of the asset holdings of the household is influenced by the variables a and c and the growth rate of public debt is affected by the variable b and by the policy parameters ψ and ϕ, in the short run. Fluctuations of the variables b, a, c have an impact on these growth rates. We have taken the model from Morimoto et al. (2017) and have integrated the sustainable debt policy rule presented for instance in Greiner & Fincke (2015). In the next section, we analyze this model with the sustainable debt policy rule used to investigate two different scenarios and study the effects on the SBGP.

3 Model Analysis: Debt policies on the SBGP

To analyze our model further, we resort to simulations. We do so because the analytical model turns out to become too complex to derive further results. As a benchmark for our simulations we set the income tax rate to thirty percent, i.e. $\tau = 0.3$, the private capital share is also set to thirty percent, i.e. $\alpha = 0.3$. The household's rate of time preference is set to 1.5 percent $\rho = 0.015$. Interpreting one time period as 3 (5, 10) years then gives an annual rate of time preference of 0.5 (0.3, 0.15) percent. The world interest rate is set to $r = 0.05$. Those parameters are left unchanged throughout our simulations. We only change the parameter σ and the policy parameters ψ and ϕ.

In the tables we report the results of our simulations for values of σ in combination with values for the policy parameters ψ and ϕ. The long-run growth rate, $\gamma^\star$ is indicated and $b^\star, a^\star, c^\star$ are also stated. Unstable means that at least two eigenvalues are positive or have positive real parts.

3.1 Balanced Government Budget

To model the balanced budget rule, we set $\phi = 0$ and $\psi = r$. From equation (14) one immediately realizes that this implies $\dot{B} = 0$, that is a balanced budget which indicates that the debt-to-GDP ratio converges to zero in the long-run. Such a situation is sustainable and we can even speak of strong sustainability in this case since the government balances its budget. It should be noted that the debt-to-GDP ratio asymptotically converges to zero (Greiner & Fincke, 2015). Setting $\dot{B} = 0$ implies that the ratio of public debt to GDP equals zero on the SBGP, i.e. $b^\star = 0$ holds. Applying the parameter setting stated above with $\sigma = 2$, $\phi = 0$ and $\psi = r$, we see that there exists a unique SBGP for the balanced budget scenario. We get the following values for $b^\star = 0$, as stated above due to $\dot{B} = 0$, $a^\star = -13.691$ and $c^\star = 0.148$. The positive $c^\star$ states that on the SBGP for the balanced budget scenario we have a positive ratio of consumption to GDP and the negative $a^\star$ signifies that on the SBGP the ratio of asset holdings of the household to GDP is negative, which means that foreign agents hold some fractions of private capital or government bonds of the household, i.e. the household is indebted abroad. As the policy parameters ψ and ϕ have no effects on the long-run growth rate, $\gamma^\star$, for the stated parameter setting with $\sigma = 2$, we get $\gamma^\star = 0.01$. To analyze stability of the SBGP, we calculate the eigenvalues of the Jacobian evaluated at the sustainable balanced growth path. We then see that the unique sustainable balanced growth path is saddle point stable (one positive and two negative eigenvalues).

We recall that the balanced budget scenario implies $\psi = r$ so that a high value for r implies a high value for ψ, too. Now, applying the parameter setting stated above with $\sigma = 0.7$, $\phi = 0$ and $\psi = r$, we see that there exists a unique SBGP for the balanced budget scenario. We get the following values for $b^\star = 0$, as stated above due to $\dot{B} = 0$, $a^\star = -13.116$ and $c^\star = 0.406$. The positive $c^\star$ states that on the SBGP for the balanced budget scenario we have a positive ratio of consumption to GDP and the negative $a^\star$ signifies that on the SBGP the ratio of asset holdings of the household to GDP is negative which means that the household is indebted abroad. As the policy parameters ψ and ϕ have no effects on the long-run growth rate, $\gamma^\star$, for the stated parameter setting with $\sigma = 0.7$, we get $\gamma^\star = 0.029$. We see that the unique sustainable balanced growth path is saddle point stable (one positive and two negative eigenvalues).

For the parameter setting with $\sigma < 1$, our model generates a higher growth rate of GDP on the SBGP, a higher $a^\star$ and a higher $c^\star$ for the BGB-scenario compared to the parameter setting with $\sigma > 1$ for the the BGB-scenario. Next, we study our model where the government has permanent public deficits.

3.2 Permanent Public Deficits

In this subsection, we allow permanent public deficits where the government debt grows at the same rate as all other endogenous variables in the long-run. Since the government sets the primary surplus according to equation (13) it does not play a Ponzi game in this case but fulfills the intertemporal budget constraint. This situation can be called weak sustainability since it only guarantees that the government does not play a Ponzi game but public debt grows at the same rate as GDP in the long-run, i.e. $\dot{B}/B = \dot{K}/K = \dot{C}/C = \dot{A}/A = \gamma^\star$ holds (Greiner & Fincke, 2015). We limit the analysis to the case $\psi > 0$, in order to not violate the intertemporal budget constraint of the government along a balanced growth path (see Greiner & Fincke (2015)). In equation (26) and its explanations, we can identify how the policy parameters ψ and ϕ have effects on the debt-to-GDP ratio on the SBGP. As the model becomes rather complex, such that it is difficult to gain analytical results, we resort to simulations in order to gain insights into our model economy.

We study how public debt policy via the reaction coefficient ψ affects the main indicators on the SBGP and the stability of the SBGP.

Table 1: Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for the stated parameter setting with $\sigma = 2$ and $\phi = -0.04$

		$\sigma = 2$		$\phi = -0.04$	
ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	-1.333	-17.246	0.059	0.01	Real Eigenvalues (+,+,-)
$\psi = 0.02$	-2.	-19.024	0.014	0.01	Real Eigenvalues (+,+,-)
$\psi = 0.03$	-4.	-24.357	0.119	0.01	Eigenvalues (0.554+0i, 0.005 ±0.007i)
$\psi = 0.04$	Complex	$-2.951 * 10^{16}$	Complex	0.01	not solvable
$\psi = 0.1$	0.667	-11.913	0.192	0.01	Eigenvalues (0.545+0i, -0.026 ±0.0053i)
$\psi = 0.2$	0.25	-13.024	0.1644	0.01	Real Eigenvalues (+,-,-)
$\psi = 1$	0.042	-13.58	0.151	0.01	Real Eigenvalues (-,+,-)
$\psi = 11$	0.004	-13.681	0.148	0.01	Real Eigenvalues (-,+,-)

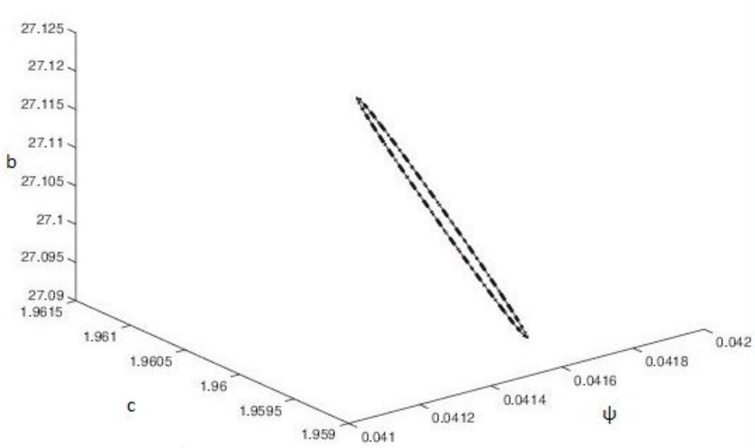
Table 1 shows how $b^\star, a^\star, c^\star$, $\gamma^\star$ and the stability of the SBGP react when the government changes the reaction coefficient ψ, that means when it changes the weight on stabilizing public debt, where $\sigma = 2$, $\phi = -0.04$ are set. Even if we assumed that public debt is positive, for the sake of completeness and to understand the system, in table 1 we consider $b^\star < 0$.

As stated above, table 1 demonstrates that the policy parameters ψ and ϕ have no effects on the long-run growth rate, $\gamma^\star$ and therefore $\gamma^\star$ does not change when increasing the reaction coefficient ψ. Of course, $b^\star$ declines as the government puts more weight on stabilizing public debt, that is when it increases the reaction coefficient ψ but only under $b^\star > 0$. We can also see that under $b^\star > 0$, raising the reaction coefficient ψ implies a decrease of $c^\star$. The economic mechanism behind that result is that a stricter public debt policy leads to less consumption in the economy on the SBGP. Table 1 shows that under $b^\star > 0$, raising the reaction coefficient ψ reduces $a^\star$. A negative $a^\star$ states that on the SBGP the ratio of asset holdings of the household to GDP is negative, signifying that foreign agents hold some fractions of private capital or government bonds of the household, i.e. the household is indebted abroad. Under $b^\star > 0$, a higher reaction coefficient of the government ψ leads to a lower $a^\star$, therefore to more influence of foreign agents on the asset holdings of the household on the SBGP. If the government puts more weight on stabilizing public debt, on the SBGP the ratio of asset holdings of the household to GDP becomes smaller due to higher negative foreign assets which means that the ratio is affected by higher influence of foreign agents.

For each ψ with the given parameter values, the eigenvalues of the Jacobian matrix evaluated at the sustainable balanced growth path of the system are calculated and the stabilities of the SBGP are indicated. In table 1 we can observe that if $\psi = r - \gamma^\star$, the system cannot be solved which is also observable in equation (26). Analyzing the eigenvalues of each ψ with the given parameter setting, in table 1 we can see that the economy is stable with one pair of complex conjugate eigenvalues and one positive real eigenvalue for some specific values of ψ. For the given parameter setting and between $\psi = 0.041$ and $\psi = 0.0415$, there is the possibility that the real part of the pair of complex conjugate eigenvalues equals zero, since between these two ψ-values we can identify a change of sign of the real part of the pair of complex conjugate eigenvalues. We realize that for $0.041 < \psi^{crit} < 0.0415$ the SBGP undergoes a subcritical Hopf bifurcation and leads to unstable limit cycles. For a slightly different value of σ, the model reacts in a sensitive way with respect to σ, namely for $\sigma = 2.01$, a supercritical Hopf bifurcation can be observed for $\psi = \psi^{crit} = 0.041525$ which leads to stable limit cycles since the first Lyapunov coefficient L_1 is negative, $L_1 = -2.805939e - 001$. In this case, there exists an interval of values of ψ with strictly positive measure for which the economy does not converge to the SBGP but converges to persistent cycles. From an economic point of view, this means that the economy is characterized by sustained fluctuations around the SBGP and that the economy is not characterized by a constant growth rate at which all variables grow but the growth rates are cyclically fluctuating over time.

Figure 1 shows the limit cycle in the (b-c-ψ) phase space for this specific interval of ψ-values where we can see that increasing the reaction coefficient ψ implies a decrease of consumption in the economy and naturally it reduces public debt, if public debt-to-GDP ratio is positive. When the public debt-to-GDP ratio has fallen enough for a specific ψ value and consumption relative to GDP has been reduced, public investment rises again that increases the debt-to-

Figure 1: Stable limit cycle in the (b-c-ψ) phase space for $\sigma = 2.01$



GDP ratio of the government, increases the consumption relative to GDP ratio and spurs economic growth. This generates a cyclical evolution and generates the limit cycle for this specific interval of ψ-values. To conduct the respective policy rule, ψ is fixed and is not varied, but b and c fluctuate over time to their new SBGP-values.

We note from table 1 that after the (unstable) limit cycle, with a higher ψ value, we get a stable SBGP. For ψ values higher than ψ^{crit} and with some distance from this ψ^{crit} value, the SBGP becomes stable. As described in Greiner & Fincke (2015), the reaction coefficient should be positive, sufficiently large on average and should exceed the positive difference between the interest rate and the GDP growth rate on average to guarantee that the debt-to-GDP ratio converges to a constant. The debt-to-GDP ratio should remain bounded in the long-run due to the primary surplus which cannot become larger than GDP.

In the appendix, we find further tables as the one described above. One with the stated parameter setting for $\sigma = 2$ and $\phi = 0.04$ which is not relevant due to instability of the SBGP for $b^{\star} > 0$. For the same reason the table with the stated parameter setting for $\sigma = 0.7$ and $\phi = 0.04$ is not explained in detail. The table with the stated parameter setting for $\sigma = 0.7$ and $\phi = -0.04$ does not give hints for (unstable/stable) limit cycles.

In section 4, we investigate the welfare effects on the sustainable balanced growth path of the different scenarios and along the transition path.

4 Welfare Analysis of Debt policies

In this section, we analyze the welfare effects on the SBGP of the two different scenarios: the Permanent Public Deficits scenario (PPD-scenario) and the Balanced Government Budget scenario (BGB-scenario). Furthermore, this section examines the welfare effects along the transition path, that is the effects resulting from a transition from the Permanent Public Deficits scenario (PPD-scenario), where public debt grows at the same rate as all other endogenous variables in the long-run, to the Balanced Government Budget scenario (BGB-scenario) on welfare. To get further insights of the model, we investigate it the other way around as well, i.e. the effects resulting from a transition from the Balanced Government Budget scenario (BGB-scenario) to the Permanent Public Deficits scenario (PPD-scenario).

To analyze the welfare effects along the transition, we assume that the economy is originally on the SBGP when the government decides to change to the new scenario on the SBGP from t=0 onwards. Therefor, we set the policy parameters ψ and ϕ as explained in section 3. In section 3, we set $\phi = 0$ and $\psi = r = 0.05$ for the BGB-scenario and for the PPD-scenario we set $\phi = -0.04$ and $\psi = 0.2$ since for these policy parameters and for once $\sigma = 2$ and then $\sigma = 0.7$, the model yields a positive $b^\star$, is characterized by a saddle point and the Jacobian Matrix with SBGP values above has two negative real eigenvalues (see table 1).

To investigate the effects of a change from one scenario to the other we study the solution of the linearized system of equations (21) - (23) which is given by

$$b(t) = b^\star + C_1 v_{11} e^{(ev_1)t} + C_2 v_{21} e^{(ev_2)t}, \tag{29}$$

$$a(t) = a^\star + C_1 v_{12} e^{(ev_1)t} + C_2 v_{22} e^{(ev_2)t}, \tag{30}$$

$$c(t) = c^\star + C_1 v_{13} e^{(ev_1)t} + C_2 v_{23} e^{(ev_2)t}, \tag{31}$$

with v_{jl} the l-th element of the eigenvector belonging to the negative real eigenvalue ev_j, $j = 1, 2$. C_j, $j = 1, 2$, are constants determined by the initial conditions $a(0)$ and $b(0)$. Setting $t = 0$ gives C_j, $j = 1, 2$, as a function of $a(0)$ and $b(0)$. Inserting these C_j, $j = 1, 2$, in equation (31) with $t = 0$ gives the unique $c(0)$ on the stable manifold leading to the SBGP in the long-run. It is the initial value for c on the stable branch onto the saddle point.

We are interested in the following policy experiments (as explained above): First, we calculate welfare for the different scenarios. Second, we study the question of whether switching from the BGB-scenario to the PPD-scenario, where the debt ratio is positive in the long-run, raises welfare. Then, as a third aim, we analyze welfare effects of switching from the PPD-scenario, where the government runs into debt, to the BGB-scenario. By comparing these different scenarios and transitions, it is possible to identify which scenario or which transition is preferable to choose over the other(s), assuming this specific parameter setting in this model.

To compute welfare effects we numerically calculate the expression

$$F = \max_{C(t)} \int_0^{+\infty} \left(\frac{C(t)^{1-\sigma} - 1}{1 - \sigma} \right) e^{-\rho \cdot t} \, \mathrm{d}t. \tag{32}$$

We remember that $c(0) = \frac{C(0)}{Y} = \frac{C(0)}{x^\alpha K_g(0)}$, where we set $K_g(0) = 1$. We get $c(0)$ from the solution of the linearized system of equations (21) - (23), which is given above by equations (29)-(31). In the beginning, we defined x^α, thus we can solve for $C(0)$, the value for consumption at $t = 0$.

From equation (32) we see that the welfare effects depend on the parameter σ, the intertemporal elasticity of substitution of consumption of the household. Hence, we once set $\sigma < 1$ and then $\sigma > 1$. In doing so, we have to take into account the following (limiting transversality) condition of the household: $\rho > \gamma^\star \cdot (1 - \sigma)$.

Now, we compute equation (32) for the different scenarios and transitions with the particular parameter setting (once $\sigma = 2$ and then $\sigma = 0.7$).

Table 2: Welfare in PPD-scenario and welfare resulting from a transition to BGB-scenario for $\sigma = 2$.

	PPD-scenario	From PPD-scenario to BGB-scenario
F	-46.2259	-123.383

Table 3: Welfare in BGB-scenario and welfare resulting from a transition to PPD-scenario for $\sigma = 2$.

	BGB-scenario	From BGB-scenario to PPD-scenario
F	-58.962	50.0439

Table 4: Welfare in PPD-scenario and welfare resulting from a transition to BGB-scenario for $\sigma = 0.7$.

	PPD-scenario	From PPD-scenario to BGB-scenario
F	276.578	202.338

Table 5: Welfare in BGB-scenario and welfare resulting from a transition to PPD-scenario for $\sigma = 0.7$.

	BGB-scenario	From BGB-scenario to PPD-scenario
F	275.822	495.988

The tables report the outcomes of the policy experiments. When analyzing the transitions, we switch from the sustainable balanced growth path of one scenario to the sustainable balanced growth path of the other scenario. As to the parameter values we use for the different scenarios the parameter setting as stated above, as an example for $\sigma = 0.5$ the limiting transversality condition of the household is not fulfilled, thus we use once $\sigma = 2$ and then $\sigma = 0.7$.

The tables show that independent of the parameter σ, which we once set greater and then smaller one, the PPD-scenario, where public debt grows at the same rate as all other endogenous variables in the long-run, leads to higher welfare than the welfare of the BGB-scenario. Thus, it is shown as well that the transition from the PPD-scenario to the BGB-scenario brings the smallest welfare.

The reason for this outcome is that the level of consumption at $t = 0$ in the BGB-scenario is less than in the PPD-scenario. Since the policy parameters ψ and ϕ do not affect the growth rate of GDP on the sustainable balanced growth path, $\gamma^\star$, none of the scenarios leads to a higher growth rate of GDP on the sustainable balanced growth path, i.e. referring to $\gamma^\star$, permanent public deficits or accumulating a stock of debt do not have a negative effect for the government. $\gamma^\star$ is not influenced by consumption. The real world interest rate, r, is exogenous and determines with other parameters $\gamma^\star$. Changing the scenarios does not show any reaction of the growth rate of GDP on the sustainable balanced growth path.

As well, we identify that in the PPD-scenario, $b^\star > 0$, $c^\star$ on the SBGP is greater than in the BGB-scenario. This is also the case for $a^\star$ on the SBGP, i.e. it is greater in the PPD-scenario than in the BGB-scenario. This means that foreign agents hold less fractions of private capital or government bonds of the household in the PPD-scenario under the SBGP. If the government holds permanent public deficits, the household is less indebted abroad under the SBGP. The policy parameters ψ and ϕ have an impact on $b^\star$, $c^\star$ and $a^\star$ on the SBGP. Therefore, if these policy parameters rise, which is the case for the PPD-scenario, not only $b^\star$ rises but also $c^\star$ and $a^\star$. Since we set $\phi = 0$ and $\psi = r = 0.05$ for the BGB-scenario and for the PPD-scenario we set $\phi = -0.04$ and $\psi = 0.2$ (once $\sigma = 2$ and then $\sigma = 0.7$), we get higher values in the PPD-scenario compared to the BGB-scenario.

By far, the highest welfare is achieved by switching from the BGB-scenario to the PPD-scenario. We identify that the level of consumption at $t = 0$ is here higher than in the other policy experiments and higher than in the previous BGB-scenario on the SBGP. It is even greater than 1, $c(0) > 1$, which states that at $t = 0$, consumption C is greater than GDP, Y. At $t = 0$, the consumption comes into the economy from abroad, as the economy is small and open, this is possible. c is a jump variable and the unique $c(0)$ on the stable manifold leads to the SBGP in the long-run. It is the initial value for c on the stable branch onto the saddle point. Later, the consumption reduces and adapts to the $c^\star$ on the SBGP of the new PPD-scenario.

Another possibility to analyze welfare of the debt policies is the equivalent variation expressed in relative consumption units, i.e. the maximum amount a consumer would be willing to pay to avoid the change of policy, which is calcuated by $\frac{(C(0)_{new} - C(0)_{old})}{C(0)_{old}}$. Setting as $C(0)_{new}$, the $C(0)$ of the transition from the BGB-scenario to the PPD-scenario and as $C(0)_{old}$, the $C(0)$

of the BGB-scenario on the SBGP, then we get the value 0.07% for $\sigma = 2$ and then 0.02% for $\sigma = 0.7$. Therefore, we identify that the maximum amount which the consumer would be willing to pay to avoid the change of policy is less than 0.1% for the both σ-values.

Taking for $a(0)$ and $b(0)$ arbitrary randomly chosen initial condition that are not too far away from the SBGP values of the new scenario, i.e. a situation of randomly chosen initial debt and household assets to each scenario, we also find out that the PPD-scenario performs better than the BGB-scenario with respect to welfare, independently of σ. For an extended robustness analysis, we analze the model with a changed parameter setting that can be found in the appendix, and then we get from a qualitative point of view, the same outcomes which confirms the results above. As the PPD-scenario fulfills the intertemporal budget constraint of the government and with the policy parameters ψ and ϕ changes the primary surplus ratio according to equation (13), it is a sustainable policy scenario associated with a sustainable public debt.

Combining the budget constraint of the household with that of the government gives the economy-wide resource constraint

$$Y + r \cdot FA = G + C + \dot{K} + \dot{FA} \tag{33}$$

In this small open economy model, the revenues through GDP and the product of the world interest rate and foreign assets[4] are used for public investments, consumption, the accumulation of private capital and of foreign assets. To understand the mechanism of this model the combination of equation (14) and equation (15) gives the following

$$G = \tau(r \cdot A + w) - \psi B - \phi Y \tag{34}$$

From equation (34) we can see that if the real public debt (real public net debt), B, increases which is the case in the PPD-scenario, the government reacts due to equation (13), the feedback of the government, by reducing the amount of public investments, G[5]. Hence, equation (33) could give insights why C ($C(0)$) rises in the PPD-scenario when the amount

[4]i.e. foreign agents have to pay the world interest rate on the fractions of private capital or government bonds that they own

[5]The income tax rate is fixed.

of public investments, G, is reduced.
In this section 4, we studied the welfare effects on the sustainable balanced growth path of the different scenarios and along the transition path. We identified that the Permanent Public Deficits scenario performs better with regard to welfare than the Balanced Government Budget scenario. The highest welfare for a specific parameter setting is achieved by switching from the BGB-scenario to the PPD-scenario. Nevertheless, as every model, this one has its limitations and assumptions, as stated above, that have effects on the results. Therefore, one should take care when interpreting the results. As an example, we have not taken into account uncertainty or risk premium.

5 Conclusion

This paper contributes a small open economy model of endogenous growth with public capital accumulation which analyzes sustainable debt policy rules and economic growth by investigating the dynamics, the characteristics and stability of the steady state, the welfare effects on the SBGP and along the transition path of the different debt policies scenarios (Balanced Government Budget and Permanent Public Deficits). Higher public debt induces the government to raise the primary surplus that is a positive linear function of the debt-to-GDP ratio guaranteeing that public debt is sustainable. This assumption results from empirical studies that identify an increase of the ratio of the primary surplus to GDP as the debt ratio rises. From the economy, consisting of a household sector, a productive sector and the government, we get the following results:
Assuming constant tax rates, we demonstrate for the specific parameter setting that in the BGB-scenario a unique SBGP that is always saddle point stable is generated. In the PPD-scenario, we also get a unique SBGP, independent of the governmental reaction to higher public debt. However, in this PPD-scenario, stability is only given if the feedback parameter of public debt is sufficiently large (see table 1). We have seen that higher values of the feedback parameter of public debt tend to stabilize the economy. A reduction of this ψ value can lead to a loss of stability and for the parameter setting in the Permanent Public Deficits scenario, we identify that for certain critical values of that ψ value the model generates en-

dogenous growth cycles/limit cycles via a Hopf bifurcation. This means that the economy is characterized by cyclical growth rates and not by a constant balanced growth rate. The policy parameters ψ and ϕ have no effects on the long-run growth rate, $\gamma^\star$ and therefore $\gamma^\star$ does not change when increasing the reaction coefficient ψ. $\gamma^\star$ is determined by the real world interest rate, r, which is exogenous and by other parameters.

As regards welfare, we investigate the welfare effects on the sustainable balanced growth path of the different scenarios and we examine the welfare effects along the transition path, that is the effects resulting from a transition from one scenario to the other scenario. We show that the Permanent Public Deficits scenario yields a higher value for welfare than the Balanced Government Budget scenario. The highest welfare for this specific parameter setting is achieved by switching from the BGB-scenario to the PPD-scenario.

Still, as the PPD-scenario fulfills the intertemporal budget constraint of the government and with the policy parameters ψ and ϕ changes the primary surplus ratio according to equation (13), it is a sustainable policy scenario.

An integration of monetary policy or migration in this model would be interesting to analyze current politico-economic questions and challenges.

Appendix

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for the stated parameter setting with $\sigma = 2$ and $\phi = 0.04$

$\sigma = 2$ $\phi = 0.04$					
ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	1.333	-10.135	0.237	0.01	Real Eigenvalues (+,+,-)
$\psi = 0.02$	2.	-8.357	0.281	0.01	Real Eigenvalues (+,-,+)
$\psi = 0.03$	4.	-3.024	0.414	0.01	Real Eigenvalues (+,-,+)
$\psi = 0.035$	8.	7.643	0.681	0.01	Real Eigenvalues (+,-,+)
$\psi = 0.04$	Complex	$2.951 * 10^{16}$	Complex	0.01	not solvable
$\psi = 0.041$	-40.	-120,357	-2,519	0.01	Real Eigenvalues (+,-,+)
$\psi = 0.0415$	-26.667	-84.802	-1.630	0.01	Real Eigenvalues (+,-,+)
$\psi = 0.1$	-0.667	-15.468	0.103	0.01	Real Eigenvalues (+,-,-)
$\psi = 0.2$	-0.25	-14.357	0.131	0.01	Real Eigenvalues (+,-,-)
$\psi = 11$	-0.004	-13.700	0.148	0.01	Real Eigenvalues (-,+,-)

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for the stated parameter setting with $\sigma = 0.7$ and $\phi = -0.04$

$\sigma = 0.7$ $\phi = -0.04$

ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.005$	-2.435	-16.595	0.383	0.03	Real Eigenvalues (+,-,+)
$\psi = 0.01$	-3.5	-18.116	0.374	0.03	Real Eigenvalues (+,-,+)
$\psi = 0.02$	-28	-53.116	0.149	0.03	Real Eigenvalues (+,-,+)
$\psi = 0.021$	-93.333	-146.45	-0.451	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.025$	11.2	2.884	0.509	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.027$	7.18	-2.86	0.472	0.03	Eigenvalues (0.536+0i, -0.012±0.002i)
$\psi = 0.03$	4.667	-6.45	0.448	0.03	Eigenvalues (0.535+0i, -0.014±0.007i)
$\psi = 0.04$	2.154	-10.04	0.425	0.03	Eigenvalues (0.534+0i, -0.018 ±0.012i)
$\psi = 0.05$	1.4	-11.116	0.419	0.03	Eigenvalues (0.533+ 0i, -0.022 ±0.015i)
$\psi = 0.1$	0.509	-12.389	0.410	0.03	Eigenvalues (0.526+0i, -0.044 ±0.005i)
$\psi = 0.2$	0.224	-12.796	0.408	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.3$	0.144	-12.911	0.407	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.5$	0.084	-12.997	0.406	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.9$	0.046	-13.051	0.406	0.03	Real Eigenvalues (-,+,-)

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for the stated parameter setting with $\sigma = 0.7$ and $\phi = 0.04$

$\sigma = 0.7$ $\phi = 0.04$

ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	3.5	-8.116	0.438	0.03	Real Eigenvalues (+,-,+)
$\psi = 0.02$	28	26.884	0.663	0.03	Real Eigenvalues (+,-,+)
$\psi = 0.03$	-4.667	-19.783	0.363	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.05$	-1.4	-15.116	0.392	0.03	Real Eigenvalues (+,-,-)
$\psi = 0.1$	-0.5091	-13.844	0.401	0.03	Real Eigenvalues (+,-,-)

Different parameter setting: $r = 0.035$, $\tau = 0.2$, $\rho = 0.02$, $\alpha = 0.3$

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for a different parameter setting with $\sigma = 0.9$ and $\phi = -0.04$

$\sigma = 0.9$ $\phi = -0.04$					
ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	-2.483	-28.755	0.010	0.009	Real Eigenvalues (+,+,-)
$\psi = 0.02$	-6.545	-43.91	-0.279	0.009	Eigenvalues (0.461+0i, 0.004±0.007i)
$\psi = 0.03$	10.286	18.873	0.921	0.009	Eigenvalues (0.459+0i, -0.0003±0.012i)
$\psi = 0.04$	2.88	-8.751	0.393	0.009	Eigenvalues (0.457+0i, -0.004 ±0.014i)
$\psi = 0.1$	0.541	-17.475	0.226	0.009	Real Eigenvalues (+,-,-)
$\psi = 0.2$	0.23	-18.636	0.204	0.009	Real Eigenvalues (+,-,-)
$\psi = 1$	0.041	-19.341	0.190	0.009	Real Eigenvalues (-,+,-)
$\psi = 11$	0.004	-19.481	0.188	0.009	Real Eigenvalues (-,+,-)

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for a different parameter setting with $\sigma = 0.9$ and $\phi = 0.04$

$\sigma = 0.9$ $\phi = 0.04$					
ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	2.483	-10.233	0.364	0.009	Real Eigenvalues (+,-,+)
$\psi = 0.03$	-10.286	-57.862	-0.546	0.009	Real Eigenvalues (+,-,+)
$\psi = 0.05$	-1.674	-25.740	0.068	0.009	Real Eigenvalues (+,-,-)
$\psi = 0.07$	-0.911	-22.894	0.122	0.009	Real Eigenvalues (+,-,-)
$\psi = 0.1$	-0.541	-21.514	0.149	0.009	Real Eigenvalues (+,-,-)
$\psi = 0.2$	-0.23	-20.352	0.171	0.009	Real Eigenvalues (+,-,-)
$\psi = 0.5$	-0.084	-19.809	0.181	0.009	Real Eigenvalues (-,+,-)
$\psi = 0.7$	-0.059	-19.716	0.183	0.009	Real Eigenvalues (-,+,-)
$\psi = 1$	-0.041	-19.648	0.185	0.009	Real Eigenvalues (-,+,-)
$\psi = 11$	-0.004	-19.508	0.187	0.009	Real Eigenvalues (-,+,-)

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for a different parameter setting with $\sigma = 1.5$ and $\phi = -0.04$

$\sigma = 1.5$ $\phi = -0.04$

ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	-2.034	-28.317	-0.082	0.005	Real Eigenvalues (+,+,+)
$\psi = 0.02$	-4.138	-37.234	-0.284	0.005	Eigenvalues (0.465+0i, 0.007±0.007i)
$\psi = 0.03$	120	488.875	11.641	0.005	Eigenvalues (0.463+0i, 0.003±0.012i)
$\psi = 0.04$	3.871	-3.291	0.485	0.005	Eigenvalues (0.461+0i, -0.001 ±0.014i)
$\psi = 0.05$	1.967	-11.359	0.303	0.005	Eigenvalues (0.459+0i, -0.005 ±0.015i)
$\psi = 0.1$	0.569	-17.286	0.168	0.005	Real Eigenvalues (+,-,-)
$\psi = 0.3$	0.148	-19.07	0.128	0.005	Real Eigenvalues (+,-,-)
$\psi = 0.5$	0.085	-19.336	0.122	0.005	Real Eigenvalues (+,-,-)
$\psi = 1$	0.041	-19.522	0.118	0.005	Real Eigenvalues (-,+,-)
$\psi = 11$	0.004	-19.681	0.114	0.005	Real Eigenvalues (-,+,-)

Reaction coefficient ψ and its $b^\star$, $a^\star$ and $c^\star$ on the SBGP for a different parameter setting with $\sigma = 1.5$ and $\phi = 0.04$

$\sigma = 1.5$ $\phi = 0.04$

ψ	$b^\star$	$a^\star$	$c^\star$	$\gamma^\star$	Stability
$\psi = 0.01$	2.034	-11.077	0.309	0.005	Real Eigenvalues (+,+,-)
$\psi = 0.02$	4.138	-2.16	0.511	0.005	Real Eigenvalues (+,-,+)
$\psi = 0.04$	-3.871	-36.102	-0.258	0.005	Real Eigenvalues (+,-,+)
$\psi = 0.1$	-0.569	-22.107	0.059	0.005	Real Eigenvalues (+,-,-)
$\psi = 0.3$	-0.148	-20.324	0.099	0.005	Real Eigenvalues (+,-,-)
$\psi = 0.5$	-0.085	-20.057	0.105	0.005	Real Eigenvalues (-,+,-)
$\psi = 1$	-0.041	-19.871	0.11	0.005	Real Eigenvalues (-,+,-)
$\psi = 11$	-0.004	-19.712	0.113	0.005	Real Eigenvalues (-,+,-)

Welfare in PPD-scenario and welfare resulting from a transition to BGB-scenario for $\sigma = 0.9$.

	PPD-scenario	From PPD-scenario to BGB-scenario
F	-10.637	-39.765

Welfare in BGB-scenario and welfare resulting from a transition to PPD-scenario for $\sigma = 0.9$.

	BGB-scenario	From BGB-scenario to PPD-scenario
F	-14.724	107.682

Welfare in PPD-scenario and welfare resulting from a transition to BGB-scenario for $\sigma = 1.5$.

	PPD-scenario	From PPD-scenario to BGB-scenario
F	-50.928	-97.975

Welfare in BGB-scenario and welfare resulting from a transition to PPD-scenario for $\sigma = 1.5$.

	BGB-scenario	From BGB-scenario to PPD-scenario
F	-65.242	57.337

References

Alogoskoufis, G. (2014). Endogenous growth and external balance in a small open economy. *Open Economies Review*, 25(3), 571–594.

Arčabić, V., Tica, J., Lee, J., & Sonora, R. J. (2018). Public debt and economic growth conundrum: nonlinearity and inter-temporal relationship. *Studies in Nonlinear Dynamics & Econometrics*, 22(1), 145.

Barro, R., Mankiw, N. G., & Sala-i Martin, X. (1992). Capital mobility in neoclassical models of growth.

Barro, R. J., Ed. (1989). *Modern business cycle theory*. Cambridge, Mass.: Harvard Univ. Pr.

Beqiraj, E., Fedeli, S., & Forte, F. (2018). Public debt sustainability: An empirical study on oecd countries. *Journal of Macroeconomics*, 58, 238–248.

Blanchard, O. & Fischer, S. (1989). *Lectures on macroeconomics*. Cambridge, Mass. [u.a.]: MIT Pr.

Bohn, H. (1995). The sustainability of budget deficits in a stochastic economy. *Journal of Money, Credit and Banking*, 27(1), 257.

Bohn, H. (1998). The behavior of u. s. public debt and deficits. *The Quarterly Journal of Economics*, 113(3), 949–963.

Checherita-Westphal, C. & Rother, P. (2012). The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area. *European Economic Review*, 56(7), 1392–1405.

Eichengreen, B. & Panizza, U. (2016). A surplus of ambition: can europe rely on large primary surpluses to solve its debt problem? *Economic Policy*, 31(85), 5–49.

Fincke, B. & Greiner, A. (2012). How to assess debt sustainability? some theory and empirical evidence for selected euro area countries. *Applied Economics*, 44(28), 3717–3724.

Fincke, B. & Greiner, A. (2015). Public debt and economic growth in emerging market economies. *South African Journal of Economics*, 83(3), 357–370.

Futagami, K., Iwaisako, T., & Ohdoi, R. (2008). Debt policy rule, productive government spending, and multiple growth paths. *Macroeconomic Dynamics*, 12(4), 445–462.

Futagami, K., Morita, Y., & Shibata, A. (1993). Dynamic analysis of an endogenous growth model with public capital. *The Scandinavian Journal of Economics*, 95(4), 607.

Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D., & Qureshi, M. S. (2013a). Fiscal fatigue, fiscal space and debt sustainability in advanced economies. *The Economic Journal*, 123(566), F4–F30.

Ghosh, A. R., Ostry, J. D., & Qureshi, M. S. (2013b). Fiscal space and sovereign risk pricing in a currency union. *Journal of International Money and Finance*, 34, 131–163.

Ghosh, S. & Mourmouras, I. A. (2004). Endogenous growth, welfare and budgetary regimes. *Journal of Macroeconomics*, 26(4), 623–635.

Greiner, A. (2007). An endogenous growth model with public capital and sustainable government debt. *The Japanese Economic Review*, 58(3), 345–361.

Greiner, A. (2012). Public capital, sustainable debt and endogenous growth. *Research in Economics*, 66(3), 230–238.

Greiner, A. & Fincke, B. (2015). *Public Debt, Sustainability and Economic Growth: Theory and empirics*. Business, economics. Cham: Springer International Publishing.

Greiner, A., Koller, U., & Semmler, W. (2006). Debt sustainability in the european monetary union: Theory and empirical evidence for selected countries. *Oxford Economic Papers*, 59(2), 194–218.

Greiner, A. & Semmler, W. (2000). Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics*, 22(3), 363–384.

Huang, K. X., Meng, Q., & Xue, J. (2017). Balanced-budget income taxes and aggregate stability in a small open economy. *Journal of International Economics*, 105, 90–101.

Morimoto, K., Hori, T., Maebayashi, N., & Futagami, K. (2017). Debt policy rules in an open economy. *Journal of Public Economic Theory*, 19(1), 158–177.

Panizza, U. & Presbitero, A. F. (2014). Public debt and economic growth: Is there a causal effect? *Journal of Macroeconomics*, 41, 21–41.

Reinhart, C. M. & Rogoff, K. S. (2010). Growth in a time of debt. *American Economic Review*, 100(2), 573–578.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5), 1002–1037.

Schmitt-Grohé, S. & Uribe, M. (1997). Balanced–budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy*, 105(5), 976–1000.

Turnovsky, S. J. (1996a). Fiscal policy, adjustment costs, and endogenous growth. *Oxford Economic Papers*, 48(3), 361–381.

Turnovsky, S. J. (1996b). Fiscal policy, growth, and macroeconomic performance in a small open economy. *Journal of International Economics*, 40(1-2), 41–66.

Turnovsky, S. J. (1997). Fiscal policy in a growing economy with public capital. *Macroeconomic Dynamics*, 1(03).

Turnovsky, S. J. (2000). Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics*, 45(1), 185–210.

van der Ploeg, F. (1996). Budgetary policies, foreign indebtedness, the stock market, and economic growth. *Oxford Economic Papers*, 48(3), 382–396.

Yakita, A. (2008). Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics*, 92(3-4), 897–914.