A Novel Approach to Measure Product Quality in Sustainable Supplier Selection

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Abstract

A gap remains due to the intangible and qualitative criteria used to measure product quality for supplier evaluation and selection. Improving product quality is a crucial strategy for achieving reduce, reuse, recycle, and recovery. Quality characteristics are described as functional relationships (called profiles), and with the advancements in measurement technology, high dimensional data are collected. Nonetheless, prior studies have not addressed sustainable supplier selection where a nonlinear profile characterizes the product quality. Hence, this study aims to provide a novel approach to measure product quality using the process yield index, presents multiple comparisons with the best and difference test statistics and proposes a Bonferroni correction method. This study applies a Monte Carlo simulation to find the selection power and the required number of profiles. The statistical properties are investigated, and a comparison study is performed. The results show that multiple comparisons with the best outperform the Bonferroni method regarding the sample size requirement and power, and the number of levels and profiles were found to impact the power of the statistical tests. The required number of profiles and the critical value are tabulated for decision-makers.

Keywords: sustainable supplier selection; nonlinear profiles; multiple comparisons with the best; Bonferroni method; process yield indices

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1. Introduction

Sustainable supplier selection (SSS) is an essential but complex issue and consists of attributes and variables (Zhang et al., 2012). Selecting the right suppliers can reduce costs and provide high-quality products (Gören, 2018). Inferior quality components supplied by upstream suppliers influence the quality of the final product and incur economic, environmental, and social losses. SSS is a crucial process in supply chain management (Li et al., 2019). Manufacturing firms are required to have the ability to choose the right suppliers, but most problems come from manufacturers selecting the wrong ones (Wang and Tamirat, 2016). Different manufacturers have distinct requirements, and hundreds of criteria have been suggested (Zimmer et al., 2016). Govindan et al. (2015) and Ansari and Kant (2017) presented a multicriteria method to assist decision-makers in SSS and found ambiguity in the criteria after applying a qualitative method. The shortcomings of the qualitative approach and intangible criteria need to be readdressed.

For instance, Gören (2018) used the Taguchi loss function to rate suppliers. Chen et al. (2019) employed a yield index for supplier selection and argued that improved product quality results in reduction, reuse, recycling, and recovery (4Rs) benefits. Product quality has been ranked among the top two factors and primary criteria for SSS in manufacturing firms (Govindan et al., 2015; Luthra et al., 2017). Improving product quality has been identified as a crucial strategy for achieving the 4Rs (Chen et al., 2019). Still, quality is measured using qualitative and intangible approaches such as the "six sigma program," "ISO system installed," "low defective rates," and "quality award," which require subjective decisions or are based on system standards (Ho et al., 2012). The process yield is the percentage of units passing inspection and reflects the product quality with respect to the design tolerance (Pearn and Wu, 2013). Process yield indices (PCIs) are standard quantitative criteria for quality measurement in the manufacturing industry. PCIs measure the process potential and process performance, which are necessary for supplier selection (Lin and Kuo, 2014). A quality measurement method is a necessity, and the process yield has been proposed as a measure of a supplier's process to reduce the ambiguity resulting from the use of broad and intangible criteria. Hence, PCIs are applied to measure the quality of the components and raw materials from upstream suppliers.

Industries demand that their products are high quality with the least number of defects due to the rapid improvement in manufacturing technology (Liu and Wu 2015). Manufacturers have to assess, compare, and choose the suppliers with the best capabilities and utilize the advancements in quality measurement technology to do so (Chou et al., 2014; Lin et al. 2018). Assessments are very frequently made in a given space or time, and the quality of products or processes are described by the relationships between a dependent variable and one or more independent variables, which are known as profiles (Maleki et al., 2018). The collection of profile data is common in industry practices (Negash, 2019).

This study provides a quantitative method for measuring quality that benefits decision-makers using multicriteria methods. The proposed method reduces the frustration of suppliers affected by the subjective nature of the decisions. This study considers nonlinear profiles with two-sided specifications and applies multiple comparisons with the best (MCB) and difference statistics comparison techniques. Quantitative quality evaluation and selection procedures are adopted to evaluate suppliers. Prior studies have applied PCIs and multiple comparison techniques. For instance, Lin and Pearn (2011) and Pearn and Wu (2013) provided examples using the ratio test statistic; Lin and Kuo (2014) found that MCB is superior to the ratio method, and Wang and Tamirat (2016) employed MCB to study CPU fans, which are laptop parts. Lin et al. (2018) implemented the Bonferroni method to manage the cumulative error rate when comparing multiple processes. These studies are limited to linear profiles or utilize traditional sampling methods (Cheng and Yang, 2018; Lin et al. 2018). Occasionally, profiles are better explained by a nonlinear equation rather than by a linear one (Guevara and Vargas, 2015; Maleki et al., 2018).

Prior studies do not provide an approach to address the problems related to SSS where a nonlinear profile characterizes the product quality and imprecise quality measurements exist (Luthra et al., 2017; Wang and Tamirat, 2016). This study uses a nonlinear profile with two-sided specifications and applies multiple comparison methods: MCB and difference statistics. It develops two quantitative product evaluation and selection methods. To find the statistical properties of the novel methods, a 100,000 replication Monte Carlo simulation study was performed. This study utilizes the Bonferroni method to reduce the error in the case of difference methods. A novel product quality evaluation and selection method using the PCIs where the nonlinear profile describes the quality characteristic is proposed. The number of profiles and critical values are provided for practitioners. Hence, the objectives of this study are as follows:

- To provide a quantitative supplier evaluation and selection method.
- To apply high dimensional and complex data represented by nonlinear profiles to measure quality.
- To determine the power of the proposed selection methods.

A numerical example is utilized to show the decision-making steps for the new techniques. The statistical properties are investigated using a Monte Carlo simulation, and the two methods are compared. The results show that the MCB is more efficient than the difference test statistics. The remaining part of this study is organized as follows. A literature review is presented in section 2. Section 3 presents the proposed methods. In section 4, a simulation study is performed to determine the power and required sample size. Section 5 shows a numerical example to illustrate the application of the new methods. The conclusions are put forward in the last part.

2. Literature review

This section includes sustainable supplier evaluation and selection, nonlinear profiles, and the process yield index for nonlinear profiles.

2.1 Sustainable supplier evaluation and selection

A high-quality product can avoid economic, ecological, and social losses, and supplier selection is an essential element for building strong sustainable supply chain management (Chen et al., 2019; Li et al., 2019). Gören (2018) argues that choosing the right supplier who can comply with requirements is essential in sustainable supply chain systems to reduce costs, increase productivity, and provide high-quality products. Bastas and Liyanage (2018) observed that with the rapid improvement in manufacturing technology, rising consumer power, and stiff competition in the market, poor product quality has the potential to cause economic, environmental, and social losses for manufacturing firms. Further, with a substantial rise in outsourcing initiatives, product quality is hugely tied with the raw materials and components from suppliers. Chai et al. (2013) discuss the supplier selection problem using multiple-objective and multiple-attribute decision-making. Govindan et al.

(2015) observed that the imprecise nature of the decision criteria causes uncertainty and lack of trust in the outcomes.

Quality is a critical criterion for supplier evaluation and selection in manufacturing firms. Ho et al. (2012) noticed that 87% of peer-reviewed studies consider quality in supplier selection. Nonetheless, quality-related attributes are highly susceptible to subjective judgments. For instance, Deng et al. (2014) utilized the "rejection rate of the product," "increase lead time," "quality assessment," and "remedy for quality problems." These subjective and historical criteria may not reflect the current status. Li et al. (2019) consider ISO certification, among other subjective criteria. Memari et al. (2019) suggested technical capability and reputation as a measure of product quality. There is a need for further clarifying how to objectively measure quality.

PCIs have been used as standard criteria in the manufacturing industry for quality measurement, and prior studies have indicated that PCIs are relevant to supplier evaluation (Pearn et al., 2004; Wang and Tamirat, 2016; Chen et al., 2019). For instance, Pearn et al. (2004) provided an example of the super twisted liquid crystal display manufacturing process and implemented a two-phase procedure. Linn et al. (2006) proposed price information and PCIs for multiple suppliers in a single chart. Polansky (2006) provided a method based on a permutation test when there are two or more suppliers. Wu et al. (2008) applied the bootstrap technique. Lin and Pearn (2011) presented group selection among multiple two-sided manufacturing lines using the ratio test statistic and provided an example of evaluating power inductor production. Tai and Wu (2012) compared two suppliers with multiple quality characteristics and selected the best one for the LED assembly process. Pearn and Wu (2013) provided an example of supplier selection in TFT-LCD manufacturing processes using the ratio test statistic.

Also, using multiple comparisons with the best (MCB), Lin and Kuo (2014) performed a simulation study and found that MCB is superior to the ratio method, especially when the number of suppliers is large or the second-best supplier is nearly as good as the best supplier. Wu et al. (2015) developed an approach called the subtraction method with multiple independent characteristics for two-sided processes and suggested considering replacing a supplier only if the process capability of the competing supplier is better than that of the existing one. Wang and Tamirat (2016) employed MCB and provided an example related to a product called a CPU fan. Pearn and Tai (2016) investigated a group supplier selection problem for multiple line gold bumping processes and found that the subtraction method is more powerful than the ratio method. Pearn et al. (2018) considered group selection, applied the Bonferroni method, and found that the power of group selection increases when the number of production lines increases. Lin et al. (2018) implemented the Bonferroni method to manage the cumulative error rate when comparing multiple processes. However, prior studies are based on traditional data collection methods or linear profiles, ignoring the opportunity to use high dimensional and complex data as a form of nonlinear profiles.

2.2 Nonlinear profiles

Jin and Shi (1999) introduced profile applications to the force of the stamping process, and profile monitoring continues to receive a lot of attention (Chang et al., 2012). In various circumstances, products or processes are often described by a function known as a profile (Cano et al., 2015). Profile data involve a response attribute referred to as *Y* and one or more independent attributes that are referred to as *X* (Williams et al., 2007). Chou et al., 2014, indicated that profiles could be categorized as linear profiles and nonlinear profiles. Wang and Tamirat (2016) stated that profiles represented by a simple linear regression model are the most investigated. A simple linear profile is given as follows.

$$y_{ij} = \alpha + \beta x_i + \varepsilon_{ij} \tag{1}$$

where *α* and *β* are the intercept and slope parameters, respectively; x_i is the i^{th} level of the independent variable; $\varepsilon_{ij} \sim N(0,\sigma^2)$; *i* = 1, 2, 3, 4, …, *I* and *j* = 1, 2, 3, 4, …, *J*.

Kang and Albin (2000) showed an example of monitoring a process in semiconductor manufacturing. Mahmoud and Woodall (2004) provided a case regarding a calibration process. Zou et al. (2007) proposed a multivariate exponentially weighted moving average scheme to monitor the linear profile. Cheng and Yang (2018) provided an example of a device called a “Babyfinder” designed to find an event of particular concern, like a stolen bicycle or heart failure for patients with a heart problem. However, in practice, profiles cannot always be represented by linear regressions (Guevara and Vargas, 2015). An alternative technique is a nonlinear model (Maleki et al., 2018). Negash (2019) explained that with an advanced measurement system that consists of sensors and transducers, profile data are collected at a high frequency and transformed into high-dimensional data. A nonlinear profile is modeled by the nonlinear function and an error term as follows.

$$y_{ij} = f(x_{ij}, \beta) + \varepsilon_{ij} \tag{2}$$

where *f(·)* is a nonlinear regression, x_{ij} is a single regressor variable, *β* is a vector of $p \times 1$ parameters, and $\varepsilon_{ij} \sim N(\mu,\sigma^2)$. The nonlinear function *f(·)* is given as follows.

$$f(x_{ij}, \beta) = \begin{cases} a_1(x_{ij} - c)^{b_1} + d, x_{ij} > c \\ a_2(-x_{ij} + c)^{b_2} + d, x_{ij} \le c \end{cases} \tag{3}$$

where $\beta = (a_1, a_2, b_1, b_2, c, d)$; *i* = 1, 2, 3, …, *I* and *j* = 1, 2, 3, …, *J*. Williams et al. (2007) proposed the mean squared error to measure the within-profile variability

$$MSE_i = \sum_{j=1}^{J} \frac{(y_{ij} - \hat{y}_{ij})}{(J - p)} \tag{4}$$

where $\hat{y}_{ij}$ is the predicted value of y_{ij} and is based on the nonlinear function.

2.3 Process yield index

Process capability indices are standard criteria for performance measurement in the manufacturing industry, such as process precision, process performance, and process accuracy (Lin and Kuo, 2014). Specification limits are used for the examination, and units are separated into two categories, namely, rejected or nonconforming and passed or conforming (Wu et al., 2009). The required fractions of rejected units or nonconformities are often counted in parts per million and are usually less than 0.01% (Pearn et al., 2018). For an advanced manufacturing system, evaluating yields by counting the number of nonconformities is not possible since any reasonably sized sample is most likely to have no defective units (Pearn et al., 2018). Hence, PCIs are used instead. For a nonlinear profile, the exact value of the PCI is defined as follows by Wang and Guo (2014).

$$S_{pkA} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}(1+P)\right] = \frac{1}{3}\Phi^{-1}\left\langle \frac{1}{2}\left\{1 + \frac{1}{I}\sum_{i=1}^{I}\left[2\Phi(3S_{pki}) - 1\right]\right\}\right\rangle \tag{5}$$

where $P = \frac{1}{I}\sum_{i=1}^{I} p_i = \frac{1}{I}\sum_{i=1}^{I}[2\Phi(3S_{pki}) - 1]$, $p_i = \Phi\left(\frac{USL_i - \mu_i}{\sigma_i}\right) - \Phi\left(\frac{LSL_i - \mu_i}{\sigma_i}\right) = \Phi\left(\frac{USL_i - \mu_i}{\sigma_i}\right) + \Phi\left(\frac{\mu_i - LSL_i}{\sigma_i}\right) - 1$, $S_{pki} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL_i - \mu_i}{\sigma_i}\right) + \frac{1}{2}\Phi\left(\frac{\mu_i - LSL_i}{\sigma_i}\right)\right\}$, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, $\Phi^{-1}(\cdot)$ is the inverse of $\Phi(\cdot)$, USL_i is the upper tolerance limit, LSL_i is the lower tolerance limit, and μ_i and σ_i are the mean and the standard deviation, respectively. To estimate the PCI S_{pkA}, for a stable process, Wang and Guo (2014) used the estimator $\hat{S}_{pkA}$.

$$\hat{S}_{pkA} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}(1 + \hat{P})\right] = \frac{1}{3}\Phi^{-1}\left\langle\frac{1}{2}\left\{1 + \frac{1}{I}\sum_{i=1}^{I}[2\Phi(3\hat{S}_{pki}) - 1]\right\}\right\rangle \tag{6}$$

where $\hat{P} = \frac{1}{I}\sum_{i=1}^{I}\hat{p}_i = \frac{1}{I}\sum_{i=1}^{I}[2\Phi(3\hat{S}_{pki}) - 1]$, $\hat{S}_{pki} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL_i - \hat{\mu}_i}{\hat{\sigma}_i}\right) + \frac{1}{2}\Phi\left(\frac{\hat{\mu}_i - LSL_i}{\hat{\sigma}_i}\right)\right\}$ is acquired at the i^{th} level and $\hat{\mu}_i$ and $\hat{\sigma}_i$ represent the mean and the standard deviation of the sample, respectively. Wang and Tamirat (2016) found the simpler form of the distribution, and it is given as follows.

$$\hat{S}_{pkA} \sim N\left(S_{pkA}, \frac{G^2[\phi(3G)]^2}{2I^2 J[\phi(3S_{pkA})]^2}\right) \tag{7}$$

where

$$G = \frac{1}{3}\Phi^{-1}\left\{\frac{I[2\Phi(3S_{pkA}) - 1] - (I - 2)}{2}\right\} \tag{8}$$

3. Proposed Method

This study uses MCB, difference test statistics, and the Bonferroni method to evaluate product quality and select the best supplier. In addition, it considers processes in which quality is described by nonlinear profiles with two-sided specifications. Figure 1 shows the procedure for this study. Section 3 provides the supplier selection procedures and decision rules.

Insert Figure 1 here

3.1. Multiple comparisons with the best

Considering K $(K \geq 2)$ suppliers, MCB constructs a joint confidence interval at the specified confidence level for the vector of differences from the unknown best population parameter (Horrace and Schmidt, 2000). MCB provides the confidence interval of the difference between the PCIs of each supplier and the best supplier (Lin and Kuo, 2014). The higher the process yield indices are, the better the supplier (Wang and Tamirat, 2016). If decision-makers consider the yield index and other criteria together, the confidence intervals can be used to evaluate whether the yield index is large enough to compensate for other criteria (Lin and Kuo, 2014).

Assume that $S_{pkA,(l)}$ is the PCI of supplier l, where $1 \leq l \leq K$; and it is no more than C, where C is a constant value (Lin and Kuo, 2014). The decision-making procedure for the MCB method is given in five steps.

Step 1: Collect *n* profiles from each supplier, and then calculate $\hat{S}_{pkA}$ using Equation (6).

Step 2: After calculating the $\hat{S}_{pkA}$ of K suppliers, sort the estimators in ascending order as $\hat{S}_{pkA,(1)} \leq \hat{S}_{pkA,(2)} \leq \cdots \leq \hat{S}_{pkA,(K)}$.

Step 3: A subset, called subset S, is constructed to resolve the SSS problem, which contains the suppliers with estimated PCIs that are only slightly smaller than that of the best supplier (Wang and Tamirat, 2016).

$$S = \left\{ l: \hat{S}_{pkA,(l)} \geq \hat{S}_{pkA,(K)} - h_{\alpha,K} \sqrt{\frac{G_l^2[\phi(3G_l)]^2}{2I^2 J[\phi(3C)]^2}}, 1 \leq l \leq K \right\} \tag{9}$$

where $\hat{S}_{pkA,(K)}$ is the supplier with the highest process yield, $h_{\alpha,K}$ is the critical value that controls the overall confidence level with the minimum of $1-\alpha$, and $G = \frac{1}{3}\Phi^{-1}\left\{\frac{I[2\Phi(3C)-1]-(I-2)}{2}\right\}$. The critical value $h_{\alpha,K}$ is defined as Equation (10). Table 1 shows the value of $h_{\alpha,K}$ when comparing two to ten suppliers where α = 0.01, 0.025, 0.05, and 0.10.

$$\int_{-\infty}^{\infty} \left[\Phi\left(z+\sqrt{2}h_{\alpha,K}\right) - \Phi\left(z-\sqrt{2}h_{\alpha,K}\right)\right]^{K-1} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = 1-\alpha \tag{10}$$

Insert Table 1 here

Step 4: The comparison is made between the PCIs of the supplier or suppliers in S with the yield index of all suppliers (Wang and Tamirat, 2016). At a confidence level of at least $1-\alpha$, the proposed simultaneous confidence intervals become the following:

$$LCB_l \leq S_{pkA,(l)} - \max_{m=1,2,\ldots,K} S_{pkA,(m)} \leq UCB_l, \text{ for } l = 1, 2, \ldots, K \tag{11}$$

where

$$LCB_l = \min\left(0, \max_{m \in S} LCB_l^m\right) \tag{12}$$

$$UCB_l = \min\left(0, \min_{m \neq l} UCB_l^m\right) \tag{13}$$

$$LCB_l^m = \begin{cases} 0, & m = l \\ \hat{S}_{pkA,(m)} - \hat{S}_{pkA,(l)} - h_{\alpha,K}\sqrt{\frac{G_l^2[\phi(3G_l)]^2}{2I^2 J[\phi(3C)]^2}}, & m \neq l \end{cases} \tag{14}$$

$$UCB_l^m = \begin{cases} 0, & m = l \\ \hat{S}_{pkA,(m)} - \hat{S}_{pkA,(l)} + h_{\alpha,K}\sqrt{\frac{G_l^2[\phi(3G_l)]^2}{2I^2 J[\phi(3C)]^2}}, & m \neq l \end{cases} \tag{15}$$

Step 5: Make a decision. l is the best supplier with the highest PCI or S_{pkA} with a given significance level of α if $LCB_l = 0$. Otherwise, *l* is the inferior supplier if $LCB_l < 0$. There is only one supplier in S if $LCB_l = UCB_l = 0$.

Examining the value of LCB_l is enough to find the best supplier. The value of UCB_l is extra information, and the lower the value of LCB_l, the worse is the supplier (Lin and Kuo, 2014, and Wang and Tamirat, 2016).

3.2. Bonferroni method

Multiple tests are necessary to evaluate and select a better supplier, but multiple tests can cause a significantly inflated overall type I error (Pearn et al., 2018). The Bonferroni method is a practical approach to solve the error inflation problem (Lin and Pearn, 2011). It is widely used in experimental contexts, such as comparing different groups versus the baseline and studying the relationships between attributes (Armstrong, 2014). The Bonferroni method adjusts the p-values by dividing the p-values by the total number of tests performed. The purpose is to maintain the type I error at a certain level and minimize the probability of a type I error during multiple testing (Gelman et al., 2012, and Pearn et al., 2018).

Assume that there are a total of g tests and that E_i represents falsely rejecting the ith test, where $1 \le i \le g$. If the significance level of the individual test is α/g, the likelihood of falsely rejecting any test is less than or equal to α using the Bonferroni inequality (Pearn et al., 2018).

$$P\left(\bigcup_{i=1}^{g} E_i\right) = 1 - P\left(\bigcap_{i=1}^{g} E_i^c\right) = 1 - \left(1 - \frac{\alpha}{g}\right)^g \le g \times \frac{\alpha}{g} = \alpha \tag{16}$$

There are five steps in the supplier selection procedure.

Step 1: Collect n samples from each supplier, and then calculate $\hat{S}_{pkA}$ using Equation (6).

Step 2: Sort the estimators in ascending order as $\hat{S}_{pkA,(1)} \le \hat{S}_{pkA,(2)} \le \cdots \le \hat{S}_{pkA,(K)}$.

Step 3: Calculate the test statistic W_i, where $W_i = \hat{S}_{pkA,(K)} - \hat{S}_{pkA,(m)}$, $1 \le m \le k$ and $\hat{S}_{pkA,(m)} < \hat{S}_{pkA,(K)}$.

Step 4: Hence, supplier K has the highest estimated value of $\hat{S}_{pkA,(K)}$. The proposed selection method compares supplier K with all other suppliers. The testing hypotheses are $H_0: \hat{S}_{pkA,(K)} - \hat{S}_{pkA,(m)} \le 0$ and $H_1: \hat{S}_{pkA,(K)} - \hat{S}_{pkA,(m)} > 0$, where $m = 1, 2, \ldots, K-1$.The testing is conducted after calculating the estimated yield indices (Lin et al., 2018). The test statistic $W_i = \hat{S}_{pkA,(K)} - \hat{S}_{pkA,(m)}$ is used to decide whether supplier m is classified into the subset or not. The asymptotic sampling distribution and the probability density function of W_i are defined as follows:

$$\begin{aligned} W_i &= \hat{S}_{pkA,(K)} - \hat{S}_{pkA,(m)} \\ &\approx N\left(S_{pkA,(K)} - S_{pkA,(m)}, \frac{{G_K}^2[\phi(3G_K)]^2}{2I^2J\left[\phi\left(3S_{pkA,(K)}\right)\right]^2} + \frac{{G_m}^2[\phi(3G_m)]^2}{2I^2J\left[\phi\left(3S_{pkA,(m)}\right)\right]^2}\right) \end{aligned} \tag{17}$$

$$\begin{aligned} f_{W_i}(w_i) &= \frac{1}{\sqrt{2\pi\left(\frac{{G_K}^2[\phi(3G_K)]^2}{2I^2J\left[\phi\left(3S_{pkA,(K)}\right)\right]^2} + \frac{{G_m}^2[\phi(3G_m)]^2}{2I^2J\left[\phi\left(3S_{pkA,(m)}\right)\right]^2}\right)}} \\ &\times exp\left(-\frac{\left[w_i - \left(S_{pkA,(K)} - S_{pkA,(m)}\right)\right]^2}{2 \times \left(\frac{{G_K}^2[\phi(3G_K)]^2}{2I^2J\left[\phi\left(3S_{pkA,(K)}\right)\right]^2} + \frac{{G_m}^2[\phi(3G_m)]^2}{2I^2J\left[\phi\left(3S_{pkA,(m)}\right)\right]^2}\right)}\right) \end{aligned} \tag{18}$$

where $G = \frac{1}{3}\Phi^{-1}\left\{\frac{I[2\Phi(3C)-1]-(I-2)}{2}\right\}$, I is the total number of the levels, and J is the total number of profiles.

By adjusting the significance level of each test at α to $\alpha/[K(K-1)]$, the total error rate is given since $\hat{S}_{pkA,(K)}$ is the largest and confirmed to be less than or equal to α/K. Hence, the critical value is calculated using the following equation.

$$P\left(W_i \geq c_\alpha \middle| \hat{S}_{pkA,(m)} = \hat{S}_{pkA,(K)} = C, n\right) = \frac{\alpha}{[K(K-1)]} \qquad (19)$$

Step 5: Make a decision. If W_i is greater than the critical value c_α, there is inadequate information to determine whether supplier *m* is significantly better than supplier *K*. For practitioners, Table 2 offers the critical values when comparing three to six suppliers with α = 0.05, *n* = 20(10)100, *I* = 4, and *C* = 1(0.1)2.

Insert Table 2 here

4. Statistical analysis and simulation study

4.1. Power analysis

The power is the probability of rejecting H_0 when it is false, and it relies upon the number of suppliers (*K*), the levels (*I*), the profiles (*J*), and the significance level (α) (Wang and Tamirat, 2016). To analyze the statistical power of the new methods, a simulation study was performed. The R programming language is utilized to write computer codes, and the nonlinear profile with a two-sided specification is employed to generate the data (see Equation 20).

$$y_{ij} = A + \frac{F - A}{1 + \left(\frac{x_{ij}}{D}\right)^B} + \varepsilon_{ij} \qquad (20)$$

where A = 0.8955, B = 2.022, D = 0.0525, and F = 0.3911. Williams et al. (2007) use dose-response profiles, where A is the maximal response parameter, B is the rate parameter that specifies how fast the response changes from the minimum response to the maximum response, D is the dose required to elicit a 50% response, and F is the minimal response parameter that is commonly used in bioassay experiments, as described in Equation (20). Table 3 shows the lower and upper tolerance limits of the dependent variable at eight levels of the independent variable.

Insert Table 3 here

Each combination was simulated 100,000 times with the given significance level $\alpha = 0.05$, the number of profiles $J = 100$, and the largest yield index value of $S_{pkA} = 1.50$. Four combinations of the process yield index S_{pkA} were examined: (1) for $K = 3$, the combination is $(S_{pkA} - 0.1, S_{pkA}, 1.5)$; (2) for $K = 4$, the combination is $(S_{pkA} - 0.2, S_{pkA} - 0.1, S_{pkA}, 1.5)$; (3) for $K = 5$, the combination is $(S_{pkA} - 0.3, S_{pkA} - 0.2, S_{pkA} - 0.1, S_{pkA}, 1.5)$; and (4) for $K = 6$, the combination is $(S_{pkA} - 0.4, S_{pkA} - 0.3, S_{pkA} - 0.2, S_{pkA} - 0.1, S_{pkA}, 1.5)$.

The power curves of MCB are shown in Figure 2 for $K = 3(1)6$ when there are different levels, where $I = 4, 8$. For example, when $K = 5$ and $S_{pkA} = 1.0, 1.1, 1.2, 1.3,$ and 1.5, the power increases by 6.93% when the number of levels increases from 4 to 8. Similarly, the power curves for the MCB are given in Figure 3 for $K = 3(1)6$ when there are various numbers of profiles, where $J = 100, 150,$ and 200. For example, when K = 3 and S_{pkA} = 1.2, 1.3, and 1.5, the power values for $J = 100, 150,$ and 200 are 0.3061, 0.5439, and 0.6799, respectively. The results indicate that both the number of profiles and the number of levels impacted the power of the statistical test. That is, increasing the number of profiles and the number of levels improves the power of the statistical test.

Insert Figures 2 - 3 here

Figure 4 illustrates the power of the Bonferroni technique for $K = 3(1)6$ with different levels $(I = 4, 8)$. For instance, when $K = 3$ and $S_{pkA} =$ 1.1, 1.2, and 1.5, the power difference between 4 levels and 8 levels becomes 5.66%. Figure 5 shows the power curves of the Bonferroni technique for $K = 3(1)6$ with different numbers of profiles, where $J = 100, 150,$ and 200. For instance, when K = 4, and S_{pkA} = 1.1, 1.2, 1.3, and 1.5, the power for $J = 100$ to $J = 200$ improves by 11.65%. The results indicate that the ability of the statistical test is affected by the number of levels and the number of profiles. Hence, the higher the number of levels is, the higher the power of the statistical analysis. Further, increasing the number of profiles improves the power of the statistical test.

Insert Figures 4- 5 here

4.1.1 Power comparison

To compare the MCB and Bonferroni methods' power, the number of best suppliers (K_{NB}) and the magnitude difference (h) are considered (Pearn et al. 2018). To compare the MCB and Bonferroni methods, multiple scenarios are considered. For example, when there are four suppliers$(K = 4)$, the scenarios considered are the following: (1) one best supplier $(K_{NB} = 1)$ and three inferior suppliers, (2) two best suppliers $(K_{NB} = 2)$ and two inferior suppliers, and (3) three best suppliers $(K_{NB} = 3)$ and one inferior supplier. Table 4 presents the power comparison of the MCB and Bonferroni methods when $C = 1.33$, $h = 0.33$, $I = 4$, $J = 100$, and $K = 3(1)6$.

Insert Table 4 here

The higher the statistical power is, the lower the probability of failure when rejecting the null hypothesis. If the statistical power is low, it can impact the validity of the conclusion. Table 4 shows that with a given number of profiles, MCB possesses higher power than the Bonferroni technique. MCB has a lower probability of failure when rejecting the null hypothesis. For the Bonferroni method, when the number of best suppliers is equal to or larger than 3, the power increases and gets closer to that of the MCB. Additionally, the lowest power for MCB always happens when $K_{NB} = [K/2]$ and the lowest power for the Bonferroni method occurs when$K_{NB} = 1$. For example, when there are three suppliers, the minimum power of MCB happens with two best suppliers$(K_{NB} = 2)$, and when there are five suppliers, the lowest power of MCB occurs with three best suppliers$(K_{NB} = 3)$. Figure 6 presents the power comparison of the MCB and Bonferroni methods when there are K_{NB} best suppliers with $C = 1.33$, $h = 0.33$, $I = 4$, $J = 100$, and $K = 3(1)6$. This study determines that MCB achieves higher power than the Bonferroni technique when dealing with the SSS problem. To attain the same power as MCB, the Bonferroni method needs many more profiles.

Insert Figure 6 here

4.2. *Required sample size*

The sample size in this study is the number of profiles. Computer programs are written in the R language, and each combination was simulated 100,000 times. For MCB, to calculate the required number of profiles, the least favorable condition (LFC) is considered, where $\lceil K/2 \rceil$ is the upper limit of $K/2$ and $\lfloor K/2 \rfloor$ is the lower limit of $K/2$ (Wang and Tamirat, 2016). The lowest power occurs with $\lceil K/2 \rceil$ best suppliers (Lin and Kuo, 2014). To identify all of the suppliers that have S_{pkA} less than the best according to the magnitude of h, the minimum required number of profiles with the given power is found using the following equation.

$$Pr\{LCB_i > 0, i = 1,2, \dots, \lfloor K/2 \rfloor | S_{pkA1} + h = \cdots = S_{PkA\lfloor K/2 \rfloor} + h = S_{pkA\lceil K/2 \rceil} = \cdots = S_{pkAK}, S_{pkAK} = C\} \geq 1 - \beta \tag{21}$$

Table 5 provides the required number of profiles given the significance level $\alpha = 0.05$; different combinations of power = 0.7, 0.8, and 0.9; different yield indices $C = 1.00, 1.33, 1.5,$ and 2.0; and different magnitude differences $h = 0.1(0.1)0.5$. For example, when $K = 3$, $C = 1.33$, $h = 0.2$, and the power is 0.7, the required number of profiles is 293.

Insert Table 5 here

For the Bonferroni method, the lowest power occurs when K_{NB} is equal to one, which is when only one best supplier exists (Lin et al., 2018). The number of profiles required for the Bonferroni method is calculated based on the setting when there is only one best supplier. All suppliers that are selected as the best suppliers are assumed to have the same process yield (C), and the inferior suppliers are assumed to have a different equal process yield $(C - h)$. The minimum number of profiles required is obtained using Equation (22).

$$P(W_i \geq c_\alpha, i = 1,2, \dots, K - 1 | S_{pkA1} = S_{pkA2} = \cdots = S_{pkAK-1} = C - h, S_{pkAK} = C) \geq 1 - \beta \tag{22}$$

Tables 6 - 9 show the number of profiles required given the significance level $\alpha = 0.05$; different combinations of power = 0.7, 0.8, and 0.9; different yield indices $C = 1.0, 1.33, 1.5,$ and 2.0; and distinct magnitude differences $h = 0.1(0.1)0.5$. For example, when $K = 5$, $C = 1.0$, $h = 0.1$, and the power is 0.9, the minimum required number of profiles is 1646 with a critical value of 0.0514. Additionally, the supplier would be considered to be a best supplier candidate if the value of W_i is less than 0.0514.

Insert Tables 6 - 9 here

Tables 5-9 present the results for the MCB and Bonferroni methods, and the results are as follows: 1) the higher the value of C, the greater is the required number of profiles; (2) the greater the number of suppliers, the higher is the required number of profiles; (3) the higher the power, the higher is the required number of profiles; (4) the smaller the magnitude difference h, the higher is the required number of profiles; and (5) the minimum required number of profiles for the Bonferroni method is higher than the required number of profiles for MCB. More required profiles results in more information. However, more required profiles costs more effort, money, and time. Therefore, having a sufficient required number of profiles is essential to be able to make decisions without wasting any resources.

5. Numerical Example

In the following, to demonstrate the application of the new methods, a numerical example is presented. The data are collected from a firm that assembles personal computers. To take advantage of cost and quality differences, the firm sources components from locations around the globe. This example focuses on one of its key components, a central processing unit cooling fan. The company has five suppliers for one of its models. Laboratory testing is used to collect the data from a laptop computer. The quality characteristic of interest is the relationship between the input voltage and speed as measured by the revolutions per minute (RPM). In the laboratory testing, the voltages are set at four levels (2.2, 2.5, 4.0, and 5.0 volts). For a quality product, the corresponding results for the speeds are expected to be 2400±200, 2700±200, 3700±200, and 4200±200 RPM, respectively. Assuming that the suppliers are aware that the significance level is 0.050, the maximum yield index value is C = 1.50, which is equivalent to 7 defective items from one million units. Eighty random profiles are collected from each of the five suppliers' processes.

The $\hat{S}_{pkA}$s for the five suppliers are 1.48, 1.37, 1.21, 1.05, and 1.00, respectively. The critical value for MCB, as mentioned in Table 1, is 2.4420. The critical value for the Bonferroni technique, as shown in Table 2, is 0.4118. For MCB, the suppliers can be categorized as a best supplier with the highest process yield index at a given significance level of α if their LCB is equal to zero. For the Bonferroni technique, the suppliers can be categorized as a best supplier if the value of their testing statistic W_i is less than 0.4118. Table 10 presents the decisions made by MCB and the Bonferroni method.

Based on Table 10, the lower confidence bounds for supplier 1 and supplier 2 are equal to zero. Therefore, suppliers 1 and 2 are considered to be the best suppliers by MCB. The values of the test statistic W_i for supplier 1, supplier 2, and supplier 3 are all below 0.4118. Based on the Bonferroni method, supplier 1, supplier 2, and supplier 3 are considered to be the best suppliers. The result shows that MCB can reject more suppliers with a lower yield index than the Bonferroni method. Thus, this result is consistent with the conclusion in the previous section 4.1. That is, MCB possesses more power than the Bonferroni technique, and to reach the same power level as the MCB, the Bonferroni needs more profiles.

Insert Table 10 here

6. Implications for practices and methodology

This study presents a quantitative supplier selection methodology. The results address the gap in the literature because product quality is primarily measured using intangible and qualitative measures. It is crucial to work with the right supplier to make high-quality products, and quality is an essential criterion in manufacturing firms for SSS. In sustainable supply chain management, supplier evaluation and selection is a critical process, and quality plays an essential role. The proposed novel supplier selection methods guarantee that only high-quality products are sourced from upstream suppliers. Hence, the proposed methods play crucial roles in avoiding economic, environmental, and social losses.

With a substantial rise in outsourcing initiatives, managers are more dependent on suppliers, supplier selection is increasingly emphasized in outsourcing, and component quality is a critical factor for manufacturers to succeed in the 4Rs. The imprecise nature of the measurement causes a lack of trust and uncertainty in the ability to choose the right suppliers. With the proposed methods, decisions are made statistically at a desired significance level. That is, the difficulties resulting from the use of intangible and qualitative methods are mitigated.

The quality of the supplier process is quantitatively assessed by the number of defective units produced; however, with modern manufacturing systems, a sensible sized sample is unlikely to have a faulty item (Lin et al., 2018). Hence, process yield indices are more suitable measures. For example, a yield index $\hat{S}_{pkA} = 1.33$ indicates that there will be 66 defective items from one million units. Figure 7 presents a novel method to assess the performance of the supplier process. Data are collected using smart data sensors, and a nonlinear profile describes the quality. MCB and the difference statistic methods are applied. The efficiency of the MCB is found to be higher. MCB has a lower probability of failure when rejecting the null hypothesis when it is not true. The power of the statistical test is affected by the number of competing suppliers, the number of levels, and the profiles; however, it requires more profiles costs more effort, money, and testing time. Yet, carefully choosing the desired significance level is essential to be able to make decisions without wasting resources. This study contributes to enhancing the knowledge related to supplier evaluation and selection. If a manager has a specific requirement, the developed computer programs can be quickly adopted.

The proposed methods are useful for monitoring the quality program implementation and quality improvement activities of suppliers. They can lead to efficiency improvements due to waste reduction in terms of reduce, reuse, recycle, and recover; can lead to reduce, scrap and rework activities; and can decrease required purchases by extending the useful lifetime of a product. Reusable, quality products can be sold or rented second hand. By recycling and recovering all components or only the critical components, consumers can repair the product if an element is damaged rather than buying a replacement. Hence, the proposed methods form an essential aspect for building strong sustainable supply chain management.

** Insert Figure 7 here **

7. Conclusions

Manufacturers are required to be able to produce high-quality products in a competitive and uncertain environment. Sustainable supplier selection (SSS) is the initial step in the process of creating high-quality products. It is a critical attribute for manufacturers who want to succeed in creating sustainable supply chain partnerships. Quality is an important variable in SSS; however, prior studies measure product quality using intangible and qualitative approaches. This creates ambiguity in the interpretation of quality and often frustrates suppliers. This study proposes a quantitative measure of the supplier's process. This study takes advantage of technological advancements in measurement technology to employ high-dimensional and complex data represented by nonlinear profiles to measure quality. This study fills the gaps in prior studies using linear profiles and presents product quality evaluation and selection methods for processes using nonlinear profiles.

The findings of the Monte Carlo simulation study indicated that the difference test statistics method possesses inferior performance compared to MCB. It required more profiles; and more profiles costs more effort, money, and time. For MCB, the lowest power happens when the number of best suppliers is equal to the upper limit of K/2. The minimum power happens when there is only one best supplier for the Bonferroni technique. In addition, increasing the number of levels of profiles is found to improve the selection power.

The contributions are multifold: (1) to reduce the ambiguity resulting from broad and intangible criteria, a process yield index S_{pkA} has been proposed to provide a numerical measure; (2) a single numerical index is used to compare the supplier's product quality, and decisions are statistically made using a desired significance level; and (3) two multiple comparison methods, the MCB and the Bonferroni methods, are proposed. The MCB considers the uncertainty of the best supplier, and the Bonferroni method maintains the overall error rate. To make the results convenient for decision-makers, tables are provided that gives the critical values and the minimum number of profiles. The new methods are simple to understand and implement and can help practitioners to deal with SSS problems with qualitative criteria in an effective way.

This study has multiple limitations. The nonlinear profiles are limited to a single quality characteristic. Multiple or a vector of quality characteristics needs to be investigated in the future with an emphasis on correlation or autocorrelation. Quality is described by a nonlinear profile with two-sided specifications. The result may not be generalizable to product quality with one-sided tolerance limits, and profile analysis is performed assuming the independence of consecutive observations.

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Table 1. The critical values of $h_{\alpha,K}$, K = 2(1)10 at α = 0.010, 0.0250, 0.050, 0.100.

α/K	2	3	4	5	6	7	8	9	10
0.010	2.5760	2.7940	2.9150	2.9980	3.0600	3.1110	3.1520	3.1880	3.2190
0.025	2.2420	2.4780	2.6070	2.6950	2.7610	2.8140	2.8580	2.8960	2.9290
0.050	1.9600	2.2120	2.3490	2.4420	2.5120	2.5670	2.6130	2.6520	2.6860
0.100	1.6450	1.9160	2.0620	2.1600	2.2340	2.2920	2.3410	2.3820	2.4170

Table 2. The critical values for K = 3(1)6, C = 1(0.1)2, n = 20(10)100, I = 4, and α = 0.05.

K	n	C										
		1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	20	0.3973	0.4601	0.5218	0.5827	0.6429	0.7023	0.7612	0.8196	0.8776	0.9352	0.9924
	30	0.3244	0.3756	0.4261	0.4758	0.5249	0.5735	0.6216	0.6692	0.7166	0.7636	0.8103
	40	0.2810	0.3253	0.3690	0.4121	0.4546	0.4966	0.5383	0.5796	0.6206	0.6613	0.7017
	50	0.2513	0.2910	0.3300	0.3686	0.4066	0.4442	0.4815	0.5184	0.5551	0.5915	0.6277
3	60	0.2294	0.2656	0.3013	0.3356	0.3712	0.4055	0.4395	0.4732	0.5067	0.5399	0.5730
	70	0.2124	0.2459	0.2789	0.3115	0.3436	0.3754	0.4069	0.4381	0.4691	0.4999	0.5305
	80	0.1987	0.2301	0.2609	0.2914	0.3215	0.3512	0.3806	0.4098	0.4291	0.4676	0.4962
	90	0.1873	0.2169	0.2460	0.2747	0.3031	0.3311	0.3589	0.3864	0.4137	0.4409	0.4678
	100	0.1800	0.2058	0.2334	0.2606	0.2875	0.3141	0.3405	0.3666	0.3925	0.4182	0.4438
	20	0.4379	0.5070	0.5751	0.6422	0.7084	0.7740	0.8389	0.9033	0.9671	1.0202	1.0936
	30	0.3575	0.4140	0.4695	0.5243	0.5785	0.6320	0.6850	0.7375	0.7897	0.8415	0.8930
	40	0.3096	0.3585	0.4066	0.4541	0.5010	0.5473	0.5932	0.6387	0.6839	0.7287	0.7733
	50	0.2770	0.3207	0.3637	0.4062	0.4481	0.4895	0.5306	0.5713	0.6117	0.6518	0.6917
4	60	0.2528	0.2927	0.3320	0.3708	0.4090	0.4469	0.4844	0.5215	0.5584	0.5950	0.6314
	70	0.2341	0.2710	0.3074	0.3433	0.3787	0.4137	0.4484	0.4828	0.5170	0.5509	0.5846
	80	0.2190	0.2535	0.2876	0.3211	0.3542	0.3870	0.4195	0.4517	0.4836	0.5153	0.5468
	90	0.2064	0.2390	0.2711	0.3027	0.3340	0.3649	0.3955	0.4258	0.4559	0.4858	0.5156
	100	0.1959	0.2268	0.2572	0.2872	0.3169	0.3462	0.3752	0.4040	0.4325	0.4443	0.4891
	20	0.4584	0.5394	0.6118	0.6832	0.7538	0.8235	0.8926	0.9610	1.0290	1.0202	1.1636
	30	0.3804	0.4405	0.4996	0.5579	0.6155	0.6724	0.7288	0.7847	0.8402	0.8953	0.9501
	40	0.3294	0.3814	0.4327	0.4831	0.5330	0.5823	0.6312	0.6796	0.7276	0.7754	0.8228
	50	0.2947	0.3412	0.3870	0.4321	0.4767	0.5208	0.5645	0.6078	0.6508	0.6935	0.7359
5	60	0.2690	0.3115	0.3533	0.3945	0.4352	0.4755	0.5153	0.5549	0.5941	0.6331	0.6718
	70	0.2490	0.2884	0.3271	0.3652	0.4029	0.4402	0.4521	0.5137	0.5500	0.5861	0.6220
	80	0.2330	0.2697	0.3059	0.3416	0.3769	0.4118	0.4463	0.4805	0.5145	0.5483	0.5818
	90	0.2196	0.2543	0.2885	0.3221	0.3554	0.3882	0.4208	0.4531	0.4851	0.5169	0.5485
	100	0.2084	0.2413	0.2737	0.3056	0.3371	0.3683	0.3992	0.4298	0.4602	0.4904	0.5204
	20	0.4584	0.5641	0.6398	0.7144	0.7882	0.8611	0.9333	1.0049	1.0760	1.0202	1.2167
	30	0.3978	0.4606	0.5224	0.5833	0.6436	0.7031	0.7621	0.8205	0.8785	0.9362	0.9935
	40	0.3445	0.3989	0.4524	0.5052	0.5573	0.6089	0.6600	0.7106	0.7608	0.8108	0.8604
	50	0.3081	0.3568	0.4046	0.4519	0.4985	0.5446	0.5903	0.6356	0.6805	0.7252	0.7695
6	60	0.2813	0.3257	0.3694	0.4125	0.4551	0.4972	0.5389	0.5802	0.6212	0.6620	0.7025
	70	0.2604	0.3015	0.3420	0.3819	0.4213	0.4603	0.4521	0.5372	0.5752	0.6129	0.6504
	80	0.2436	0.2821	0.3199	0.3572	0.3941	0.4306	0.4667	0.5025	0.5380	0.5733	0.6084
	90	0.2297	0.2659	0.3016	0.3368	0.3716	0.4060	0.4400	0.4738	0.5072	0.5405	0.5736
	100	0.2179	0.2523	0.2861	0.3195	0.3525	0.3851	0.4174	0.4494	0.4812	0.5128	0.5442

Table 3. Specification limits at eight levels.

j	1	2	3	4	5	6	7	8
x	0.003	0.009	0.028	0.084	0.25	0.76	2.27	6.8
USL_j	0.6	0.62	0.64	0.9	0.98	1	1.05	1.1
LSL_j	0.2	0.22	0.24	0.4	0.48	0.5	0.65	0.7

Table 4. Power comparison of MCB and Bonferroni having K_{NB} best suppliers with C = 1.33, h = 0.33, I = 4, J = 100, and K = 3(1)6.

K	Method	$K_{NB} = 1$	$K_{NB} = 2$	$K_{NB} = 3$	$K_{NB} = 4$	$K_{NB} = 5$
3	MCB	0.85698	0.81697			
	Bonferroni	0.41762	0.74071			
4	MCB	0.77465	0.68619	0.76679		
	Bonferroni	0.22112	0.45574	0.70764		
5	MCB	0.70543	0.59314	0.57541	0.70372	
	Bonferroni	0.12690	0.27182	0.44827	0.67503	
6	MCB	0.64571	0.48600	0.46689	0.50837	0.64455
	Bonferroni	0.07764	0.16921	0.28340	0.43339	0.64810

Table 5. Number of profiles for MCB.

K	C	1.00					1.33					1.50					2.00				
	$1-\beta$ | h	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5
3	0.7	450	129	51	27	16	1209	293	132	73	45	1620	377	199	114	69	3180	731	357	211	131
	0.8	517	154	63	31	21	1292	340	161	90	50	1818	461	212	126	77	3741	908	393	234	169
	0.9	606	166	80	41	22	1596	372	178	109	54	2255	532	265	153	94	4621	1034	480	280	192
4	0.7	597	149	72	35	21	1576	371	166	96	58	2157	505	216	128	84	4445	1019	446	269	160
	0.8	679	168	78	39	22	1693	406	190	106	66	2393	574	267	148	97	4772	1176	487	278	193
	0.9	791	192	88	47	27	2082	453	202	119	77	2897	700	280	159	110	5117	1197	564	360	208
5	0.7	637	169	79	39	23	1690	413	193	108	67	2414	586	267	149	98	4830	1179	489	278	198
	0.8	741	193	89	45	25	2049	451	201	118	75	2775	680	278	159	105	5147	1215	575	312	203
	0.9	912	209	96	51	29	2333	507	212	127	80	3304	743	354	172	121	6369	1556	596	376	213
6	0.7	779	190	87	46	25	2039	473	201	112	70	2819	640	271	162	102	5200	1313	590	331	204
	0.8	877	203	91	51	29	2119	519	220	122	76	3128	737	321	166	112	5833	1503	619	365	226
	0.9	969	222	102	55	32	2430	590	260	133	86	3462	806	374	190	121	6597	1602	710	402	262

Table 6. Number of profiles and critical values for the Bonferroni method at C = 1.0.

H		0.10		0.20		0.30		0.40		0.50	
K	Power	n	C_α	n	C_α	n	C_α	n	C_α	n	C_α
	0.7	976	0.0569	212	0.1221	105	0.1734	51	0.2488	31	0.3192
3	0.8	1048	0.0549	232	0.1167	114	0.1165	58	0.2333	34	0.3048
	0.9	1192	0.0515	278	0.1066	127	0.1557	72	0.2094	42	0.2742
	0.7	1152	0.0577	277	0.1177	123	0.1762	66	0.2411	39	0.3136
4	0.8	1217	0.0562	289	0.1152	128	0.1731	75	0.2261	46	0.2887
	0.9	1475	0.0510	341	0.1061	144	0.1632	81	0.2176	52	0.2716
	0.7	1377	0.0562	302	0.1199	131	0.1821	72	0.2456	43	0.3177
5	0.8	1602	0.0521	360	0.1098	140	0.1761	78	0.2359	51	0.2918
	0.9	1646	0.0514	388	0.1058	165	0.1622	93	0.2161	53	0.2862
	0.7	1463	0.0570	381	0.1117	149	0.1785	80	0.2436	48	0.2980
6	0.8	1575	0.0549	387	0.1108	165	0.1696	88	0.2323	53	0.2993
	0.9	1755	0.0520	404	0.1084	179	0.1629	96	0.2224	58	0.2861

Table 7. Number of profiles and critical values for the Bonferroni method at C = 1.33.

H		0.1		0.2		0.3		0.4		0.5	
K	Power	n	C_α	n	C_α	n	C_α	n	C_α	n	C_α
	0.7	2527	0.0535	525	0.1173	233	0.1761	139	0.2279	87	0.2788
3	0.8	2813	0.0507	600	0.1097	275	0.1621	157	0.2145	106	0.2610
	0.9	3039	0.0488	729	0.0996	339	0.1460	178	0.2014	110	0.2562
	0.7	3009	0.0540	704	0.1116	312	0.1677	163	0.2320	111	0.2811
4	0.8	3200	0.0524	790	0.1054	339	0.1609	178	0.2220	116	0.2750
	0.9	3498	0.0501	911	0.0981	352	0.1579	203	0.2079	131	0.2587
	0.7	3568	0.0528	880	0.1062	360	0.1661	197	0.2245	118	0.2901
5	0.8	3784	0.0513	927	0.1035	393	0.1590	210	0.2174	128	0.2785
	0.9	4035	0.0496	952	0.1022	431	0.1518	227	0.2091	140	0.2663
	0.7	3958	0.0524	973	0.1057	393	0.1662	215	0.2247	133	0.2857
6	0.8	4119	0.0514	1004	0.1040	404	0.1639	222	0.2211	143	0.2755
	0.9	4613	0.0486	1121	0.0984	479	0.1506	259	0.2047	164	0.2573

Table 8. Number of profiles and critical values for the Bonferroni method at C = 1.5.

H		0.1		0.2		0.3		0.4		0.5	
K	Power	n	C_α	n	C_α	n	C_α	n	C_α	n	C_α
3	0.7	3180	0.0557	813	0.1102	374	0.1625	195	0.2250	125	0.2810
	0.8	3483	0.0533	857	0.1073	400	0.1571	206	0.2189	132	0.2734
	0.9	4062	0.0493	1034	0.0977	421	0.1531	234	0.2504	159	0.2491
4	0.7	4501	0.0516	984	0.1104	424	0.1681	259	0.2151	152	0.2808
	0.8	4802	0.0500	1146	0.1023	474	0.1590	269	0.2111	160	0.2737
	0.9	5013	0.0489	1190	0.1004	553	0.1472	281	0.2065	183	0.2556
5	0.7	4964	0.0523	1203	0.1062	486	0.1671	269	0.2246	178	0.2761
	0.8	5084	0.0517	1260	0.1038	513	0.1626	276	0.2217	196	0.2631
	0.9	5487	0.0498	1533	0.0941	607	0.1495	346	0.1980	207	0.2560
6	0.7	5240	0.0532	1336	0.1054	557	0.1632	310	0.2188	191	0.2787
	0.8	5589	0.0516	1427	0.1020	603	0.1569	340	0.2089	206	0.2683
	0.9	6358	0.0483	1524	0.0987	624	0.1542	360	0.2030	220	0.2597

Table 9. Number of profiles and critical values for the Bonferroni method at C = 2.0.

H		0.1		0.2		0.3		0.4		0.5	
K	Power	n	C_α	n	C_α	n	C_α	n	C_α	n	C_α
3	0.7	6780	0.0539	1592	0.1113	691	0.1689	389	0.2551	251	0.2802
	0.8	6939	0.0533	1643	0.1095	818	0.1552	422	0.2161	278	0.2662
	0.9	8756	0.0475	2052	0.0908	864	0.1510	486	0.2014	312	0.2513
4	0.7	8212	0.0540	2033	0.1085	899	0.1632	502	0.2183	299	0.2829
	0.8	8622	0.0527	2257	0.1030	969	0.1572	527	0.2131	343	0.2641
	0.9	9641	0.0499	2509	0.0977	1009	0.1540	588	0.2017	370	0.2543
5	0.7	8900	0.0552	2272	0.1092	950	0.1689	529	0.2263	362	0.2735
	0.8	9963	0.0522	2516	0.1038	1108	0.1564	617	0.2095	392	0.2629
	0.9	11653	0.0483	2632	0.1015	1214	0.1492	699	0.1969	434	0.2498
6	0.7	10265	0.0538	2767	0.1035	1151	0.1604	606	0.2211	393	0.2745
	0.8	11513	0.0508	2828	0.1024	1222	0.1557	629	0.2170	407	0.2698
	0.9	12235	0.0492	3128	0.0973	1333	0.1491	751	0.1986	457	0.2546

Table 10. The decision made by MCB and the Bonferroni method.

K	$\hat{S}_{pkA}$	MCB		Bonferroni method	
		[LCB, UCB]	Decision	W_i	Decision
1	1.480	[0, 0.25]	Best	0	Best
2	1.370	[0, 0.36]	Best	0.11	Best
3	1.110	[0.01, 0.62]	Inferior	0.37	Best
4	1.050	[0.07, 0.68]	Inferior	0.43	Inferior
5	1.000	[0.12, 0.73]	Inferior	0.48	Inferior

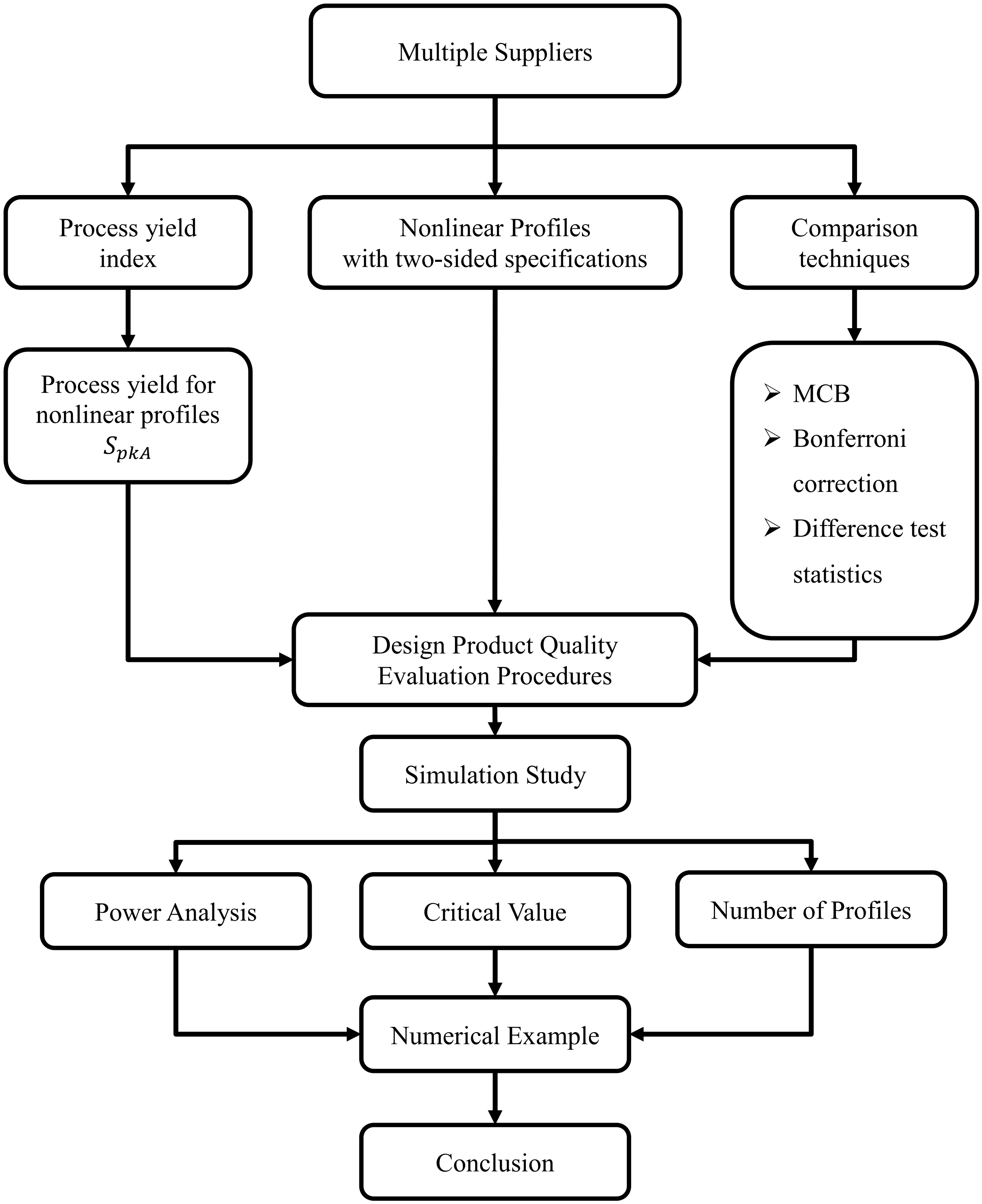

Figure 1. Analytical flow

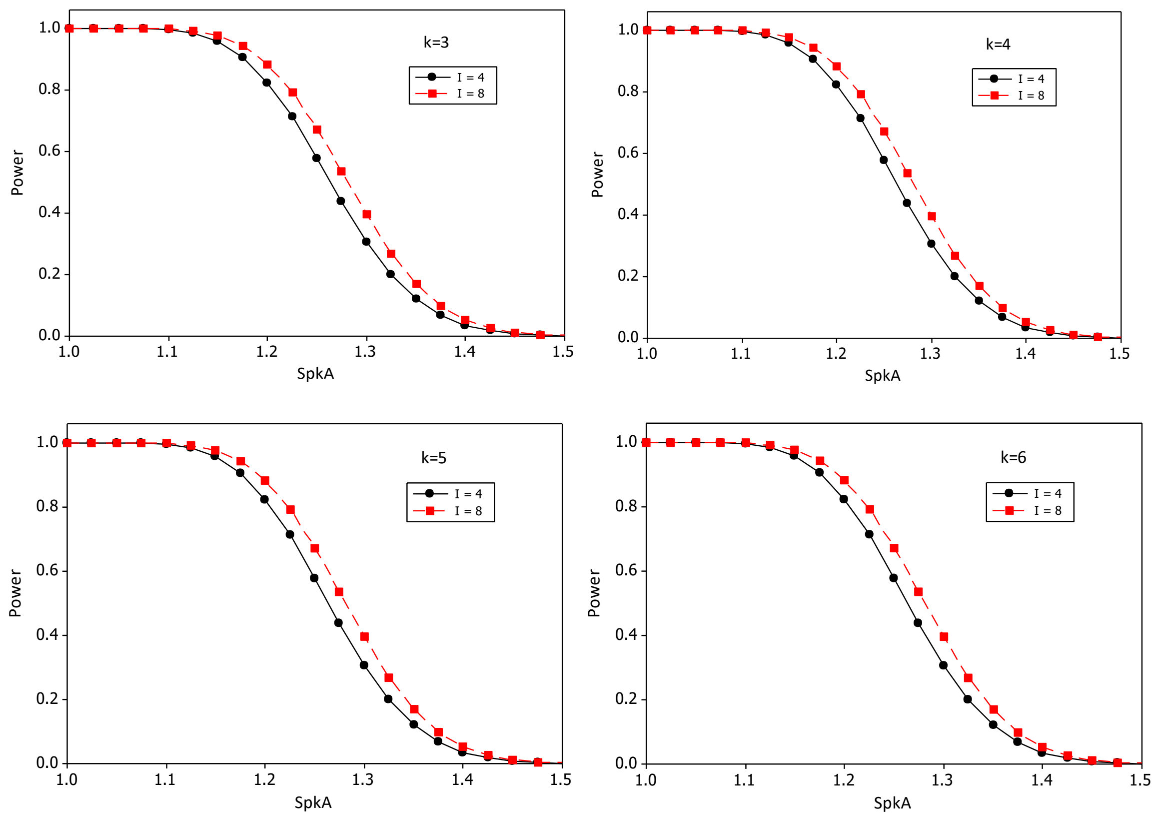

Figure 2. Power analysis of MCB at K = 3(1)6.

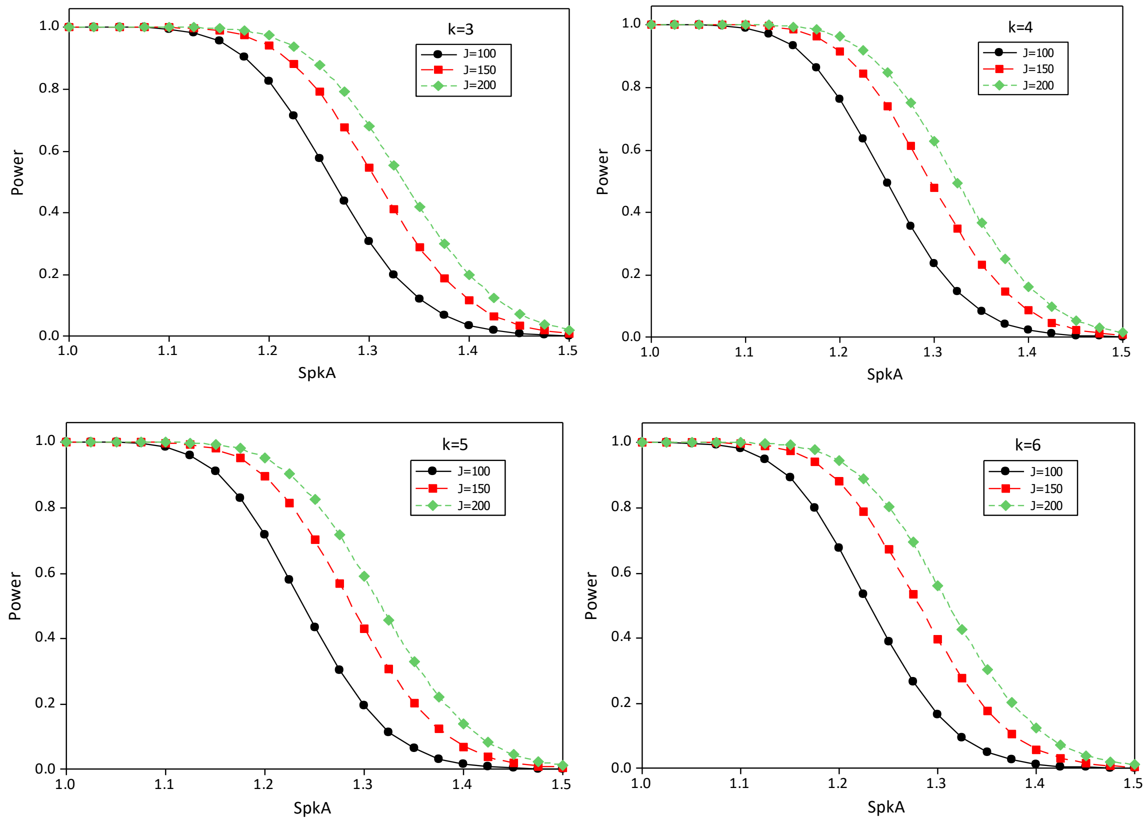

Figure 3. Power analysis with the number of profiles of MCB at K = 3(1)6.

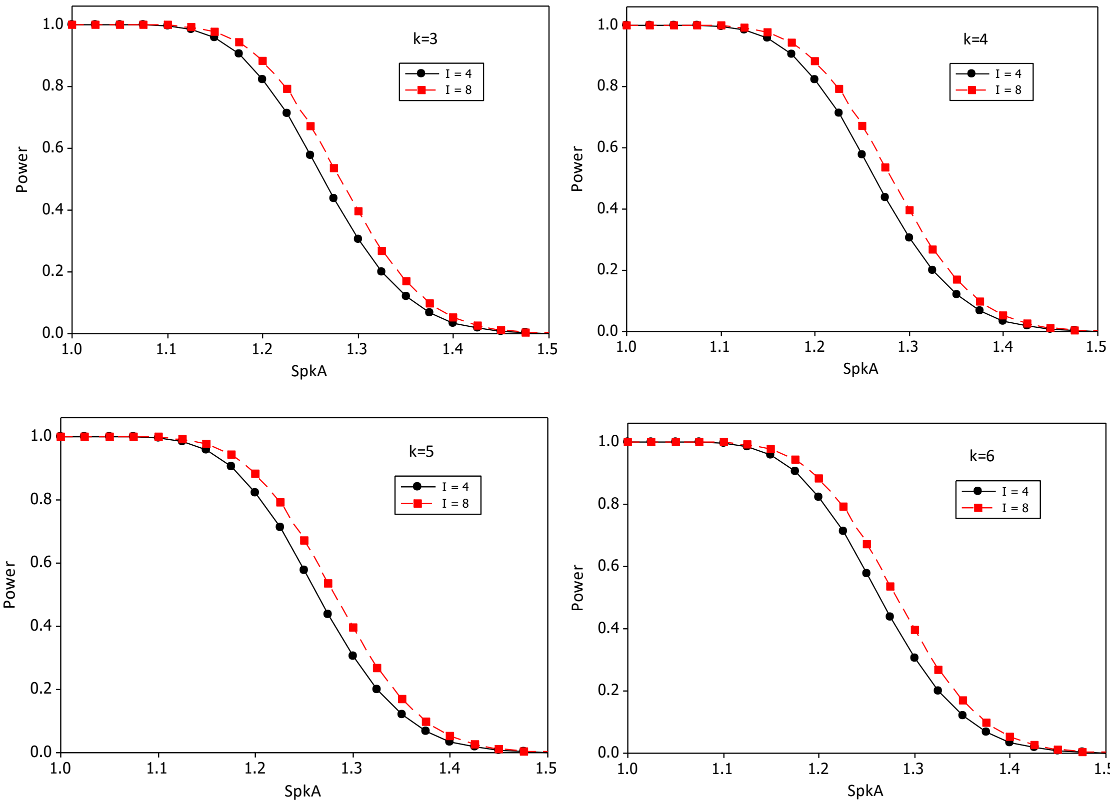

Figure 4. Power analysis of the Bonferroni at K = 3(1)6.

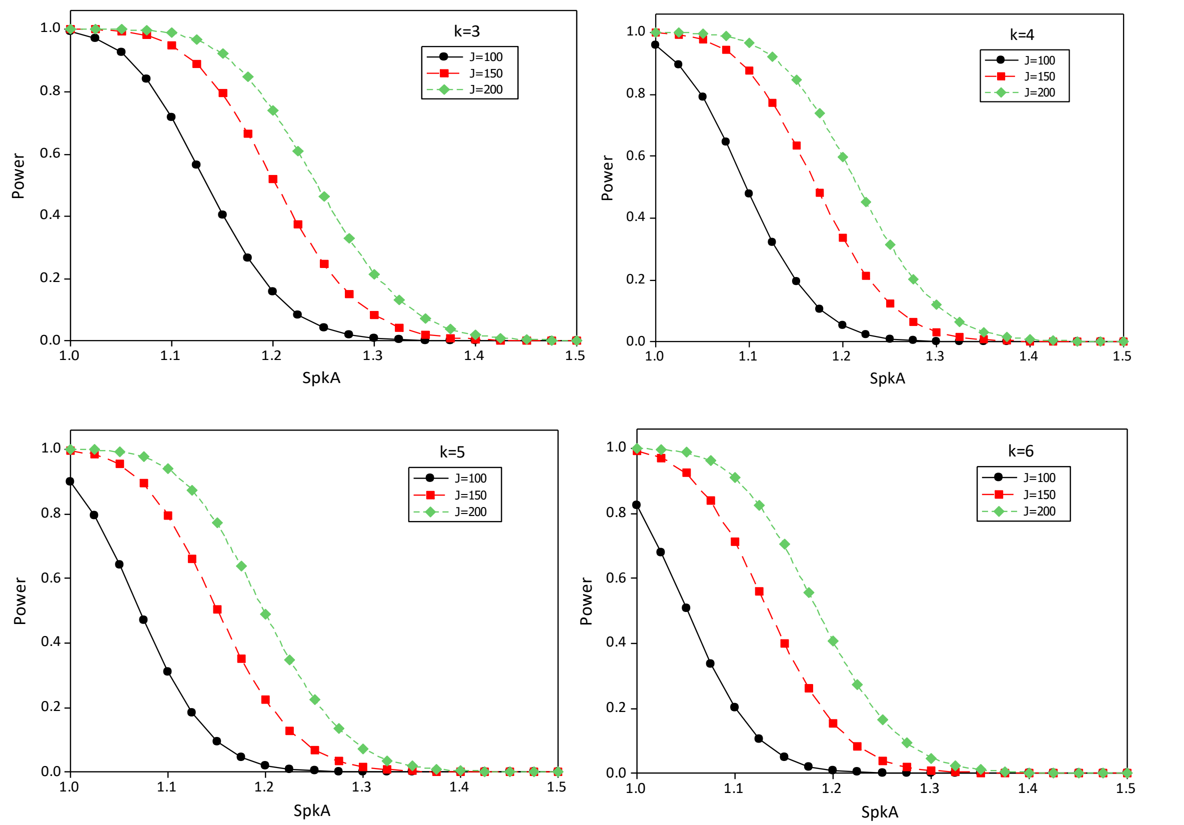

Figure 5. Power analysis with the number of profiles of the Bonferroni at K = 3(1)6.

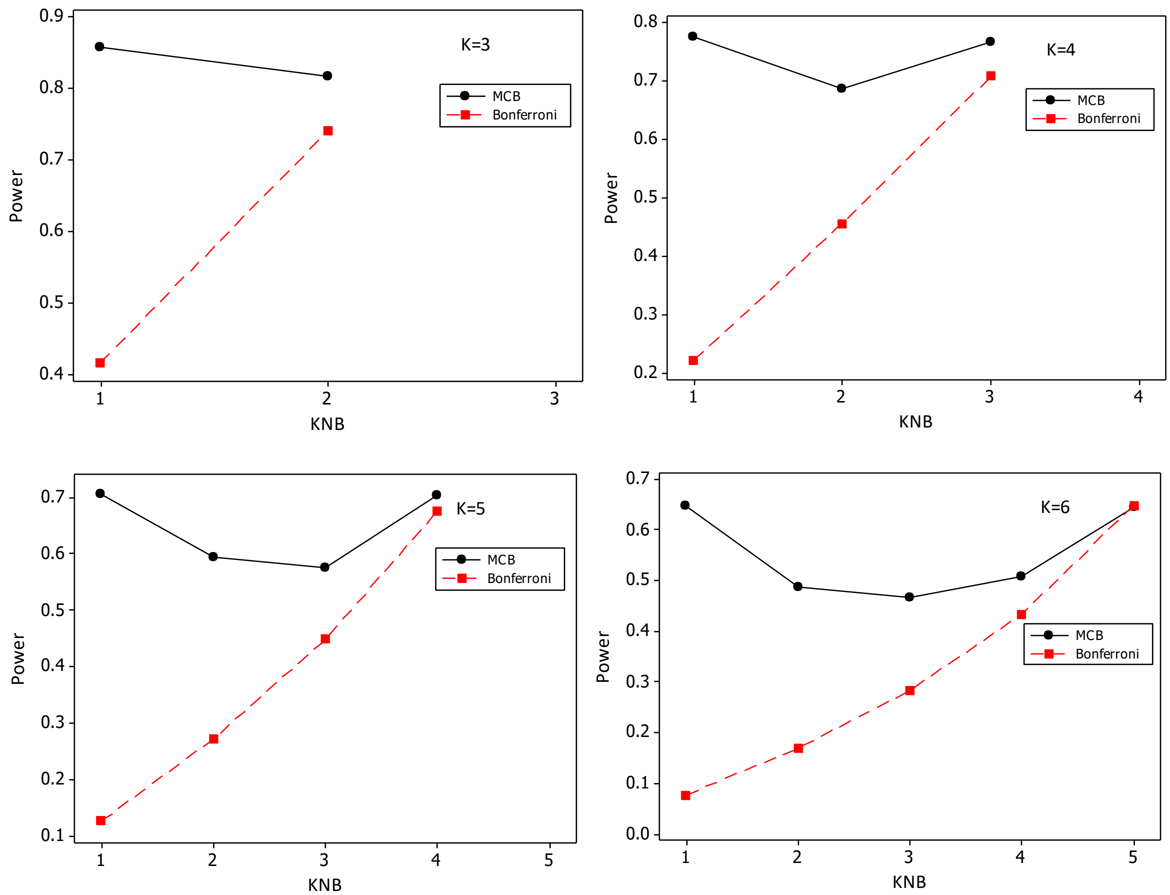

Figure 6. Power comparison of MCB and Bonferroni having KNB best suppliers with C = 1.33, h = 0.33, I = 4, J = 100, and K = 3(1)6.

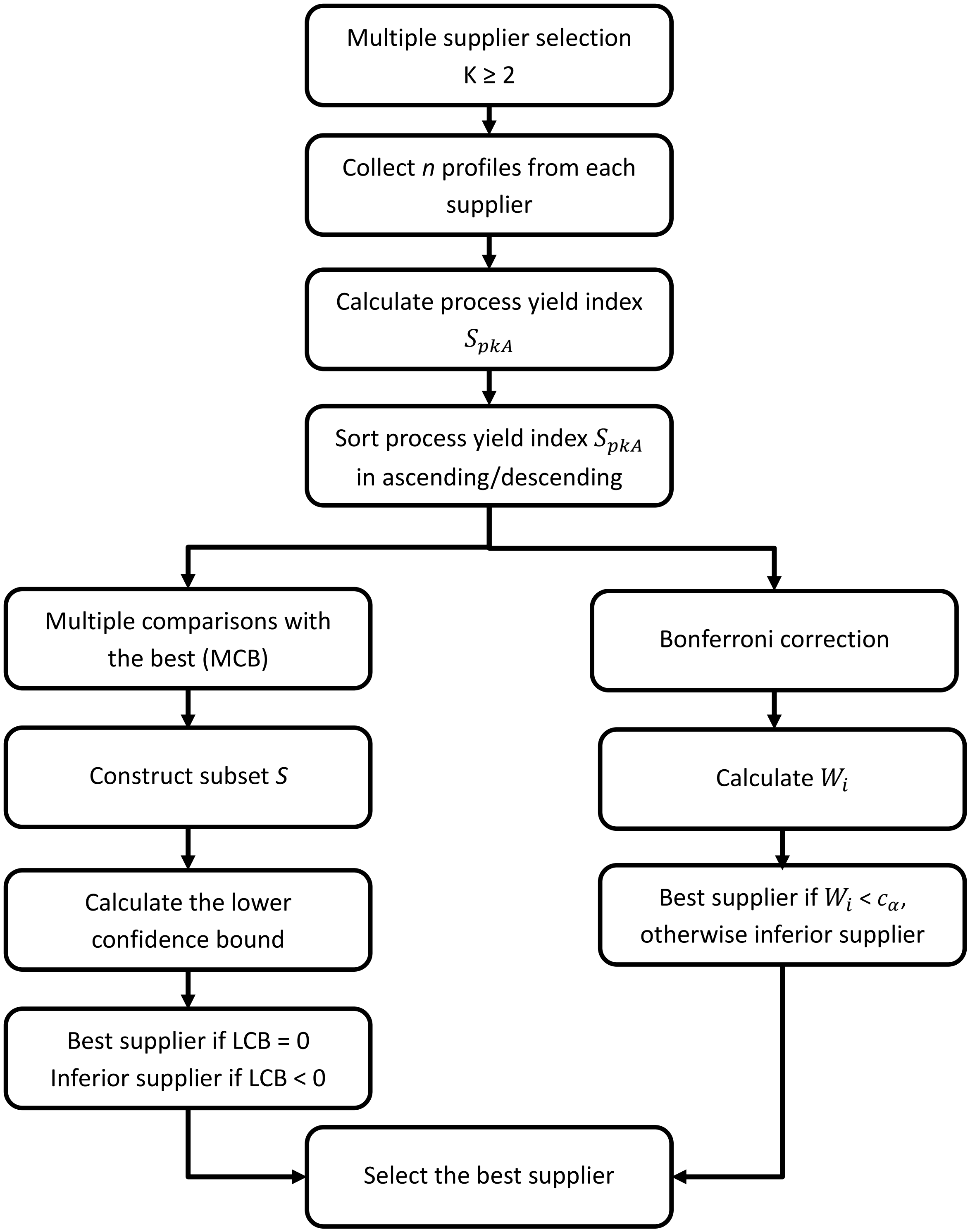

Figure 7. Proposed methods