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RESERVE FLEET INDEXED TO EXOGENOUS COST VARIABLES

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Abstract. Identifying the optimal time to replace a passenger bus in a buses fleet has implications on the size of the reserve fleet. Such calculations rest on endogenous and exogenous economic variables: the former include operating and maintenance costs and bus depreciation; the latter include market imponderables such as the inflation and real discount rates, as well as energy costs, particularly fuel. The authors have created models for the withdrawal/replacement of buses using endogenous economic variables. The models include standard econometric models reflecting the influence of maintenance policies, especially Condition Monitoring (CM) or predictive maintenance, and the size of the reserve fleet. The paper deals with exogenous economic variables, specifically the influence of the cost of money, the inflation and real discount rates rate and the cost of fuel. Both variables fluctuate over time. The paper proposes analytical models for determining the influence of those variables on the withdrawal time and the size of the reserve fleet. It then comprehensively summarizes the variables in a global model, showing its relevance to the dimensioning of the reserve fleet and the withdrawal time.

Keyword: life cycle cost (LCC), reserve fleet, maintenance, econometric models, economic life, lifespan.

Introduction

In the passenger transport sector, the determination of the optimum time for bus replacement has an effect on both the efficient use of assets and the global costs of the company.

A company needs to know the right time to replace a bus to reduce costs, guarantee the quality of service, and ensure customer satisfaction. Accordingly, the objective of this paper is to define a methodology to determine the best time to replace a bus.

The paper evaluates the influence of financial costs, specifically, the value or cost of money. To reach this goal, it considers the inflation and real discount rates rate and the price of fuel, two costs to be expected to change significantly over time. It presents models of analysis for determining the influence of these variables on the time of bus withdrawal and on the size of the reserve fleet.

The value of money is directly linked to time: the later an asset is withdrawn from use, the greater the action of external agents or the influence of macroeconomic factors in relation to purchasing power. The inflation rate in any capitalist economic system is a decisive factor in the relationship between money and time: a certain amount of money in a certain month of a certain year does not have the same value in the same month of the following year. In addition, the price of fuel is variable and conditioned by various external factors worldwide. This obviously has an effect on transportation costs.

A reserve fleet is defined as the number of vehicles ready to perform the service for which they have been designed, that is, not immobilized by breakdown or in planned maintenance, in order to maximize the availability of the operating fleet. For the companies of the transport sector, the efficient use of physical assets is linked to a well-structured policy of evaluation and fleet replacement. In national and international road transport companies, there is a wide range of suggested ratios for reserve fleets. However, the recommended size of a fleet reserve specified by the US Federal Transit Administration (FTA) in FTA Circular C 9030.1A (1987) Appendix A is 20% (Simões 2011).

This paper presents an approach about the influence of the cost of money – particularly with regard to the infla-

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tion and real discount rates rate and the costs associated with fuel. It suggests models of analysis for determining the influence of these variables on the withdrawal time and the size of the reserve fleet. The paper also provides a comprehensive summary of the preceding variables through a global model, demonstrating its relevance to the analysis of dimensioning the reserve fleet and determining the withdrawal time.

Following the contributions previously published by the authors, this paper presents some additional contributions, concerning the maintenance politics, namely the Condition Monitoring (CM) with prediction to the dimensioning of the reserve fleet. This paper emphasises the influence of exogenous cost variables to those models, specifically fuel price, inflation and real discount rates, to the time of the withdrawal and the dimensioning of the reserve fleet.

The paper is structured as follows:

- chapter 1 describes the current state-of-the-art analysis;
- chapter 2 synthesizes some approaches;
- chapter 3 describes the influence of inflation rate variables on withdrawal time;
- chapter 4 discusses the influence of fuel cost variables on withdrawal time;
- chapter 5 discusses a condition based maintenance versus reserve fleet;
- chapter 6 presents a global discussion of the models and an integrative approach;
- conclusions and future developments are presented.

1. State-of-the-art analysis

The concept of CM was introduced in the late 20th century, around 1970–1980. Briefly stated, CM represented a new approach to preventive maintenance based on knowledge of the health of equipment as determined by a CM system (Cabral 2006). The maintenance of a passenger bus is a strategic activity to maximize the asset's life cycle. It involves a combination of management, technical and economic actions to achieve high availability at reasonable costs (Aoudia *et al.* 2008; Assis 2014; Assis, Julião 2009; Bescherer 2005; Lindholm, Suomala 2005; Korpi, Ala-Risku 2008) and it frequently relies on CM.

The cost of the life cycle of an asset is the sum of all capital spent in support of that asset from design and manufacturing, through operation until the end of its life (Assis 2014). The Life Cycle Cost (LCC) can be significantly higher than the value of the initial investment and, in many cases, is set in the design phase (Assis, Julião 2009).

The analysis of the cost of the life cycle is a prediction of the future. Several methods can be used for cost estimates, as, for example, Activity-Based Costing (ABC) (Durairaj *et al.* 2002; Emblemsvag 2001). Certain standards such as those specified in ASTM E917-17; PAS 55-1:2008; PAS 55-2:2008 – support the cost analysis of the life cycle. The rules on asset management specified in PAS 55-1:2008, PAS 55-2:2008, i.e. ISO 55000:2014, ISO 55001:2014, ISO 55002:2014 – are good guidelines for asset management and can be applied in any sector.

Although the above points are well-known, there is a lack of systematic study on the area. There is a need to apply and create new equipment management models that can bring added value to companies, allowing them to improve their productivity and quality of service, while taking into account environmental sustainability, including quality management standards, environment, safety, maintenance and energy (Farinha 2011). Many companies keep equipment in operation, even when their operation is no longer economically viable, simply because they do not consider their economic cycle (Farinha 2011). This has exogenous implications in several areas, including the size of the reserve fleet.

According to Sullivan *et al.* (2002), traditional production systems are built on the principle of scale economy. The authors illustrate an equipment replacement problem in the context of lean thinking.

Rogers and Hartman (2005) refer to technological change as a motivator for equipment replacement and that is commonly assumed that technology is continually developing according to a well-defined function. Hritonenko and Yatsenko (2007) combine discrete and continuous models in time to show that the replacement time for equipment is less when the technology is more advanced.

According to Assaf Neto (2014), "the evaluation of an asset is established by the cash flows expected from future benefits referred to the present value by a discount rate that reflects the risk of the decision". Consequently, methods considering the value of money over time are the most suitable. In the view of Casarotto Filho and Kopittke (2010), the method of annual cost uniform equivalent is suitable for the analysis of operational activities of a company with investments that can be repeated. Furthermore, the standardization of investment based on annual equivalent values facilitates the analysis required for decision-making. By using this method, it is possible to determine which year has the lowest equivalent annual cost. This, in turn, indicates the best technical replacement period (Casarotto Filho, Kopittke 2010). The calculation of the equivalent annual cost is based on the capital recovery factor. Using this, it is possible to compare two or more investment opportunities and to determine the best time for equipment replacement, taking into account such information as: (1) value of the investment or acquisition, (2) resale value or residual value at the end of each year, (3) operating costs, (4) the cost of capital or the attractive minimum rate (Vey, Rosa 2004).

To determine the economic life of equipment with the objective of finding the best replacement time, four situations are applicable (Motta, Calôba 2002):

- when the asset is already unsuitable for work;
- when the asset has reached its lifespan;
- when the asset is already obsolete due to technological advances;
- when more efficient methods are more economical.

Analysis must also consider the following (Farinha 2011):

- availability of new technology;
- compliance with safety standards or other mandatory requirements;
- availability of spare parts;
- obsolescence that may limit its use.

When the equipment enters the final phase of its economic life cycle, it is important to have calculation methods able to determine the appropriate time to withdraw it from use. Several variables are important in such calculations: (1) purchase price, (2) withdrawal value, (3) operating costs, (4) maintenance costs, (5) inflation rate, (6) real discount rate. The values of most of variables are available in the asset's history, except for the withdrawal value. In this case, it is necessary to know the market value for each particular equipment. Unfortunately, this may prove to be difficult for many assets. In such cases, several types of devaluation can be simulated (Oliveira 1982):

- linear method of depreciation – the decline in equipment value is constant over the years;
- sum of digits method – the annual depreciation is not linear;
- exponential method – the annual depreciation decreases over the life of the equipment.

Another common method is to calculate the economic lifespan that ends when the maintenance costs exceed the cost of maintenance plus the capital amortization of a new equivalent equipment. According to Farinha (2011) there are several methods to determine the economic cycle for equipment replacement. The most common are:

- Method of Uniform Annual Income (MUAI);
- Method of Minimizing the Total Average Cost (MMTAC);
- Method of Minimizing the Total Average Cost – Reduced to Present Value (MMTAC–RPV).

Feldens *et al.* (2010) illustrate the efficient use of physical assets as one of the main objectives of urban passenger transport companies. In the road transport sector, the efficient use of assets is linked to a well-structured policy of fleet evaluation and replacement. Some cases of fleet replacement applied to urban buses are reported in Keles and Hartman (2004), Khasnabis *et al.* (2002), Jin and Kite-Powell (2000), Scarf and Bouamra (1999), Leung and Cheng (2000), Beichelt (2001), Zohrul Kabir (1996), Wijaya *et al.* (2012), Raposo *et al.* (2014).

Campos *et al.* (2010) present a proposal for a generic model of a stochastic process based on neural networks. The proposed neural stochastic process can be applied to problems involving phenomena showing stochastic behaviour with periodic characteristics. Through neural network models, the behaviour of the historical series of phenomena is modelled without requiring *a priori* information about the series, by generating synthetic time series also adaptable to time series. Some cases of use of neural networks and stochastic models are reported in Campos *et al.* (2010), Amaya *et al.* (2007), Figueiredo (2009), Zhao (2009), Luna *et al.* (2006), Müller (2007), Reis *et al.* (2010), Araujo and Bezerra (2004), Huang and Yao (2008), Vujanović *et al.* 2012, Gurney (1997).

Other tools may contribute to the development of a new model for optimization of bus replacement, such as fuzzy logic and support vector machine (Tsoukalas *et al.* 1997; Yager, Zadeh 1992; Campello *et al.* 2001; Couellan *et al.* 2015; Chen *et al.* 2015; Pooyan *et al.* 2015).

For predictive maintenance, more specifically oil analysis, several mathematical models can be used (Cabral 2006; Farinha 2011; Makridakis *et al.* 1998; André 2008). In the study of the influence of a CM or predictive maintenance policy applied to a reserve fleet, it is important to use the appropriate key performance indicators (KPI's) (Cabrita, Cardoso 2015). Here, the NP EN 15341:2009 – "Maintenance – Maintenance Key Performance Indicators" – is a very important standard.

2. Synthesis of some approaches

2.1. Econometric models

This section summarizes work on equipment replacement models relevant to this paper (Farinha 2011; Raposo *et al.* 2014). According to (Farinha 2011), equipment can be replaced by several reasons. From a financial point of view, two common criterions are (1) the equipment economic cycle and (2) the lifespan, as discussed in the previous section.

To analyse equipment replacement, two variables should be taken into account:

- real discount rate i;
- inflation rate θ.

These rates are related by the following equation:

$$i_A = i + \theta + i \cdot \theta, \quad (1)$$

where: i_A – apparent rate.

The apparent rate is an auxiliary variable that conjugates the real discount and the inflation rates in order to help calculating the Net Present Value (NPV).

According to (Farinha 2011) there are various methods for determining the economic cycle. One of these, the MUAI, makes use of the following data: (1) equipment acquisition cost, (2) withdrawal values (calculated in accordance with the methods above), (3) maintenance and operation costs over time, (4) apparent rate.

The market equipment value suffer depreciation along time. However, for many-used equipment it is not possible to know their market value. When this value is unknown, it is necessary to simulate the equipment depreciation. The equipment under study in this paper are buses for which this situation happens. To solve this problem, this paper uses the exponential method to calculate the withdrawal value of a bus. This method was chosen because it seems more adequate than the linear one, as happens with other types of vehicles. The formula that calculates the annual depreciation cost along the equipment life is expressed by:

$$d_l = VC_{l-1} \cdot \left(1 - \sqrt[N]{\frac{VC_N}{CA}}\right); \quad (2)$$

$$V_n = VC_{l-1} - d_l, \tag{3}$$

where: d_l – annual depreciation quota; CA – cost of acquisition; N – Time of life corresponding to VC_N; VC_N – residual value of the equipment at the end of N periods of time; $l = 1, 2, 3, \ldots, N$; V_n – equipment value in period $n = 1, 2, 3, \ldots, N$.

The NPV per year n (NPV_n) is expressed as:

$$NPV_n = CA + \sum_{j=0}^{n} \frac{CM_j + CO_j}{\left(1+i_A\right)^j} - \frac{V_n}{\left(1+i_A\right)^j}, \tag{4}$$

where: CN_j – cost of maintenance per year $j = 1, 2, 3, \ldots, n$; CO_j – cost of operation per year $j = 1, 2, 3, \ldots, n$.

The Uniform Annual Income (UAI_n) is written as:

$$UAI_n = \frac{i_A \cdot \left(1+i_A\right)^j}{\left(1+i_A\right)^j - 1} \times$$

$$\left(CA + \sum_{j=0}^{n} \frac{CM_j + CO_j}{\left(1+i_A\right)^j} - \frac{V_n}{\left(1+i_A\right)^j} \right); \tag{5}$$

$$UAI_n = \frac{i_A \cdot \left(1+i_A\right)^j}{\left(1+i_A\right)^j - 1} \cdot NPV_n. \tag{6}$$

The UAI indicates the period [years], when a bus ought to be replaced. This value is equivalent to the minimum annual cost of the bus.

To apply the models, we consider variables related to direct operating costs and maintenance costs, as well as the relevant economic indicators, such as inflation and interest rates.

Initially, we performed a survey of the operating data for LCC from a bus fleet belonging to a medium-sized public urban transport company.

The fleet bus company has a total of 115 diesel buses, operating in 84 lines, in a total of 556 km length of road network, carrying annually about 14 million passengers.

The buses were distributed in homogeneous groups; the buses were 21, 18, 16, 12, and 11 years old. The values are in Euro [€] being divided by one thousand to facilitate their presentation; for example, 3345482 € is represented by 33.45K.

With the goal to validate the replacement models, we created simulation models based on a bus data. Among the several possible approaches based on the sample, due to limitations of space of the paper, we chose to use the same historical data, of the same bus (bus 115), in a period from 1993 to 2014.

In the first step, taking into account the exponential depreciation method and using the MUAI to determine the economic cycle, we computed the results shown in Table 1 and Figure 1.

In order to support the results inside the Table 1 and Figure 1, the values that were calculated for the first two periods are shown below:

Table 1. UAI (bus 115)

Bus 115									*VC* [€]	*NPV* [€ year *n*]	*UAI* [€ year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	(1 + $i_{A,j}$)	*CM* [€]	*CO* [€]	Σ_1 [€]	*VP* [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K	8								
1994	1		8	1.08	0.98K	11.22K	12.20K	121.94K	87.74K	34.20K	36.99K
1995	2		8	1.08	1.02K	10.71K	11.73K	131.97K	69.57K	62.40K	35.07K
1996	3		8	1.08	1.12K	10.46K	11.58K	141.12K	55.16K	85.96K	33.45K
1997	4		8	1.08	1.27K	10.48K	11.76K	149.71K	43.74K	105.97K	32.11K
1998	5		8	1.08	1.49K	10.76K	12.25K	157.99K	34.68K	123.31K	31.01K
1999	6		8	1.08	1.77K	11.31K	13.07K	166.15K	27.50K	138.65K	30.14K
2000	7		8	1.08	2.10K	12.12K	14.22K	174.37K	21.80K	152.56K	29.46K
2001	8		8	1.08	2.50K	13.19K	15.69K	182.74K	17.29K	165.45K	28.97K
2002	9		8	1.08	2.95K	14.53K	17.48K	191.37K	13.71K	177.66K	28.63K
2003	10		8	1.08	3.64K	15.54K	19.18K	200.13K	10.87K	189.26K	28.41K
2004	11		8	1.08	3.91K	18.46K	22.37K	209.57K	8.62K	200.95K	28.37K
2005	12		8	1.08	5.97K	20.75K	26.72K	219.99K	6.83K	213.16K	28.52K
2006	13		8	1.08	5.13K	22.01K	27.15K	229.78K	5.42K	224.36K	28.64K
2007	14		8	1.08	5.40K	21.73K	27.13K	238.83K	4.30K	234.53K	28.71K
2008	15		8	1.08	6.06K	26.30K	32.37K	248.81K	3.41K	245.40K	28.95K
2009	16		8	1.08	7.05K	17.92K	24.97K	255.93K	2.70K	253.22K	28.90K
2010	17		8	1.08	10.06K	17.99K	28.05K	263.32K	2.14K	261.18K	28.94K
2011	18		8	1.08	8.61K	21.46K	30.07K	270.64K	1.70K	268.95K	29.02K
2012	19		8	1.08	6.38K	27.52K	33.90K	278.28K	1.35K	276.93K	29.17K
2013	20		8	1.08	8.72K	25.96K	34.68K	285.50K	1.07K	284.44K	29.32K
2014	21		8	1.08	9.36K	26.83K	36.19K	292.47K	0.85K	291.63K	29.47K

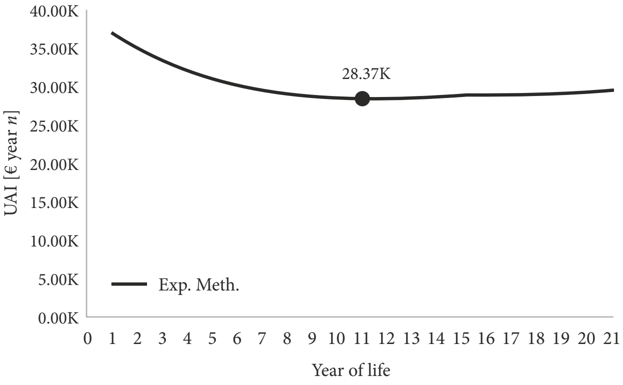


Figure 1. UAI (bus 115)

$$NPV_1=\left(\frac{110.66}{\left(1+0.08\right)^0}+\frac{0.98+11.22}{\left(1+0.08\right)^1}\right)-\frac{94.90}{\left(1+0.08\right)^1}=34.20\text{ €};$$

$$UAI_1=\frac{0.08\cdot\left(1+0.08\right)^1}{\left(1+0.08\right)^1-1}\cdot 34.20=36.99\text{ €};$$

$$NPV_2=\left(\frac{110.66}{\left(1+0.08\right)^0}+\frac{0.98+11.22}{\left(1+0.08\right)^1}+\frac{1.02+10.71}{\left(1+0.08\right)^2}\right)-\frac{81.39}{\left(1+0.08\right)^2}=62.40\text{ €};$$

$$UAI_2=\frac{0.08\cdot\left(1+0.08\right)^2}{\left(1+0.08\right)^2-1}\cdot 62.40=35.07\text{ €}.$$

As the Table 1 and Figure 1 show, there is a clear replacement point in the 11th year. The value of the UAI is 28.37K. It should be noted that these calculations use a constant apparent rate of 8%.

Another method to determine the economic cycle of equipment replacement is the Method of Minimization of Total Average Cost (MMTAC). This method allows to determine the lowest average cost of bus ownership that corresponds to the optimal replacement time. The capital cost and the inflation rate are not considered. The calculation procedure is as follows:

$$C'_n=\frac{\sum_{j=0}^{n}CM_j+CO_j}{n}; \tag{7}$$

$$C''_n=\frac{CA-V_n}{n}; \tag{8}$$

$$C_{n(MMTAC)}=C'_n+C''_n=\frac{\sum_{j=0}^{n}CM_j+CO_j}{n}+\frac{CA-V_n}{n}, \tag{9}$$

where: n – number of years, $n\in\{1,2,3,\ldots,N\}$; C'_n – auxiliary variable; C''_n – auxiliary variable; $C_{n(MMTAC)}$ – method of minimization total average cost.

Bus 115 is again presented as example, taking into account the method of exponential depreciation using the MMTAC to determine the vehicle's economic cycle (Table 2 and Figure 2).

In order to support the results inside the Table 2 and Figure 2, the values that were calculated for the first two periods are shown below:

$$C_{1(MMTAC)}=\frac{0.98+11.22}{1}+\frac{110.66-94.90}{1}=27.96\text{ €};$$

$$C_{2(MMTAC)}=\frac{0.98+11.22}{2}+\frac{1.02+10.71}{2}+\frac{110.66-81.39}{2}=26.60\text{ €}.$$

The Table 2 and Figure 2 show an evident replacement point in the 9th year of life. The minimum bus ownership cost is 22.54K.

Finally, we use the MMTAC–RPV. The calculation procedure is the same as the one above but this one considers capital costs and inflation rate. The various maintenance and withdrawal values over time are reduced to the present value, using the following procedure:

$$C'_n=\frac{1}{n}\cdot\sum_{j=1}^{n}\frac{CM_j+CO_j}{\left(1+i_A\right)^j}; \tag{10}$$

$$C''_n=\frac{CA-\dfrac{V_n}{\left(1+i_A\right)^j}}{n}; \tag{11}$$

$$C_{n(MMTAC-RPV)}=C'_n+C''_n=\frac{1}{n}\cdot\sum_{j=1}^{n}\frac{CM_j+CO_j}{\left(1+i_A\right)^j}+\frac{CA-\dfrac{V_n}{\left(1+i_A\right)^j}}{n}, \tag{12}$$

where: $C_{n(MMTAC-RPV)}$ – method of minimization total average cost – reduced to present value.

Finally, Table 3 and Figure 3 take into account the exponential depreciation method using the MMTAC–RPV to determine the economic cycle of bus 115.

In order to support the results inside the Table 3 and Figure 3, the values that were calculated for the first two periods are shown below:

$$C_{1(MMTAC-RPV)}=\frac{1}{1}\times\left(\frac{0.98+11.22}{\left(1+0.08\right)^1}+\frac{110.66-\dfrac{94.90}{\left(1+0.08\right)^1}}{1}\right)=34.09\text{ €};$$

$$C_{2(MMTAC-RPV)}=\frac{1}{2}\times\left(\frac{0.98+11.22}{\left(1+0.08\right)^1}+\frac{1.02+10.71}{\left(1+0.08\right)^2}\right)+\frac{110.66-\dfrac{81.39}{\left(1+0.08\right)^2}}{2}=31.12\text{ €}.$$

Table 2. Minimization of total average cost (bus 115)

Bus 115			C' [€]			C'' [€]	$C_{n(MMTAC)}$ [€ year n]
Year	Year j	CA [€]	CM [€]	CO [€]	Σ_1 [€]	Exp. Meth.	Exp. Meth.
1993	0	110.66K					
1994	1		0.98K	11.22K	12.20K	15.76K	27.96K
1995	2		1.02K	10.71K	11.97K	14.63K	26.60K
1996	3		1.12K	10.46K	11.84K	13.62K	25.46K
1997	4		1.27K	10.48K	11.82K	12.70K	24.52K
1998	5		1.49K	10.76K	11.90K	11.86K	23.77K
1999	6		1.77K	11.31K	12.10K	11.11K	23.20K
2000	7		2.10K	12.12K	12.40K	10.41K	22.82K
2001	8		2.50K	13.19K	12.81K	9.78K	22.60K
2002	9		2.95K	14.53K	13.33K	9.21K	22.54K
2003	10		3.64K	15.54K	13.92K	8.68K	22.60K
2004	11		3.91K	18.46K	14.69K	8.20K	22.89K
2005	12		5.97K	20.75K	15.69K	7.76K	23.45K
2006	13		5.13K	22.01K	16.57K	7.36K	23.93K
2007	14		5.40K	21.73K	17.32K	6.98K	24.31K
2008	15		6.06K	26.30K	18.33K	6.64K	24.97K
2009	16		7.05K	17.92K	18.74K	6.32K	25.07K
2010	17		10.06K	17.99K	19.29K	6.03K	25.32K
2011	18		8.61K	21.46K	19.89K	5.76K	25.65K
2012	19		6.38K	27.52K	20.63K	5.51K	26.14K
2013	20		8.72K	25.96K	21.33K	5.28K	26.60K
2014	21		9.36K	26.83K	22.04K	5.06K	27.10K

Table 3. MMTAC–RPV (bus 115)

Bus 115					C' [€]				C'' [€]	$C_{n(MMTAC-RPV)}$ [€ year n]
Year	Year j	CA [€]	i_A [%]	$(1 + i_{A,j})$	CM [€]	CO [€]	Σ_1 [€]	Σ_2 [€]	Exp. Meth.	Exp. Meth.
1993	0	110.66K	8							
1994	1		8	1.08	0.98K	11.22K	11.30K	11.30K	22.79K	34.09K
1995	2		8	1.08	1.02K	10.71K	10.68K	11.08K	20.44K	31.12K
1996	3		8	1.08	1.12K	10.46K	10.18K	10.96K	18.42K	28.60K
1997	4		8	1.08	1.27K	10.48K	9.80K	10.94K	16.66K	26.46K
1998	5		8	1.08	1.49K	10.76K	9.51K	11.02K	15.14K	24.65K
1999	6		8	1.08	1.77K	11.31K	9.29K	11.20K	13.82K	23.11K
2000	7		8	1.08	2.10K	12.12K	9.15K	11.48K	12.66K	21.81K
2001	8		8	1.08	2.50K	13.19K	9.07K	11.86K	11.65K	20.71K
2002	9		8	1.08	2.95K	14.53K	9.03K	12.34K	10.75K	19.78K
2003	10		8	1.08	3.64K	15.54K	9.02K	12.89K	9.96K	18.98K
2004	11		8	1.08	3.91K	18.46K	9.07K	13.60K	9.26K	18.33K
2005	12		8	1.08	5.97K	20.75K	9.20K	14.53K	8.64K	17.84K
2006	13		8	1.08	5.13K	22.01K	9.26K	15.34K	8.09K	17.35K
2007	14		8	1.08	5.40K	21.73K	9.26K	16.04K	7.59K	16.85K
2008	15		8	1.08	6.06K	26.30K	9.32K	16.97K	7.15K	16.46K
2009	16		8	1.08	7.05K	17.92K	9.19K	17.35K	6.74K	15.94K
2010	17		8	1.08	10.06K	17.99K	9.10K	17.86K	6.38K	15.48K
2011	18		8	1.08	8.61K	21.46K	9.01K	18.42K	6.05K	15.06K
2012	19		8	1.08	6.38K	27.52K	8.95K	19.10K	5.75K	14.70K
2013	20		8	1.08	8.72K	25.96K	8.87K	19.75K	5.48K	14.35K
2014	21		8	1.08	9.36K	26.83K	8.79K	20.40K	5.23K	14.02K

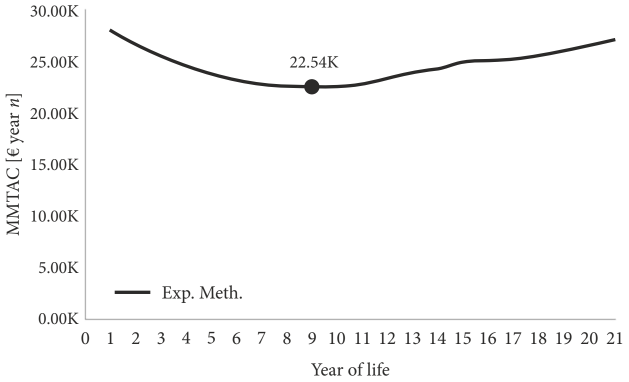


Figure 2. Minimization of total average cost (bus 115)

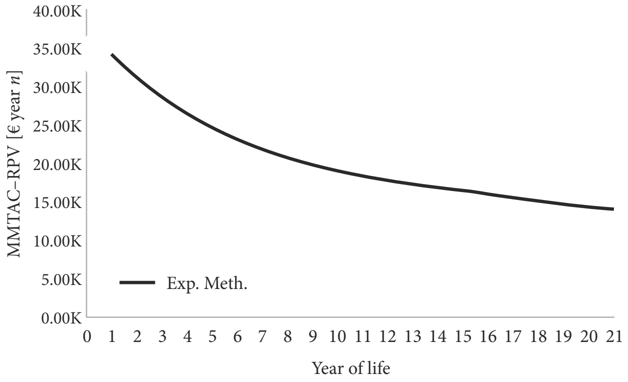


Figure 3. MMTAC–RPV (bus 115)

Neither the table nor the graphic show an evident point of lower average cost of equipment ownership; the average cost of ownership continues to decline and there is no reversal point in the path of the ownership cost within the time interval considered.

To evaluate the efficiency of an investment, one possible measure is the Return On Investment (ROI), which depends on ensuring the maximum availability of the assets at the lowest cost. ROI measures the amount of return on an investment, in percentage, relative to the investment's cost. In the case discussed in this paper, it is used a variable ROI_a, in absolute value, that corresponds to the following formula:

$$ROI_a = \left(-\frac{CA}{\left(1+i_A\right)^j} + \frac{Pf_j}{\left(1+i_A\right)^j} \right) - \frac{\left(C'_n + C''_n\right)}{\left(1+i_A\right)^j}. \qquad (13)$$

The Tables 4, 5 and Figures 4, 5 shown below illustrate these values for a hypothetic bus.

The last two numerical examples allow to cross the ROI_a value with the econometric models used to evaluate the withdrawal time, including the simulation of the past and future values for the several life cycle variables. The past values of the variables were simulated by fixed values and were afterwards corrected by the apparent rate.

3. Influence of apparent rate

This section discusses the influence of the inflation and real discount rates on the economic cycle of bus replacement, along the bus life cycle. It uses the same bus data to facilitate comparison. Figures 6 and 7 show charts with inflation and real discount rates, respectively, in Portugal between 1993 and 2014.

Figure 7 illustrates the variation in real discount rate in Portugal in the previous year's range.

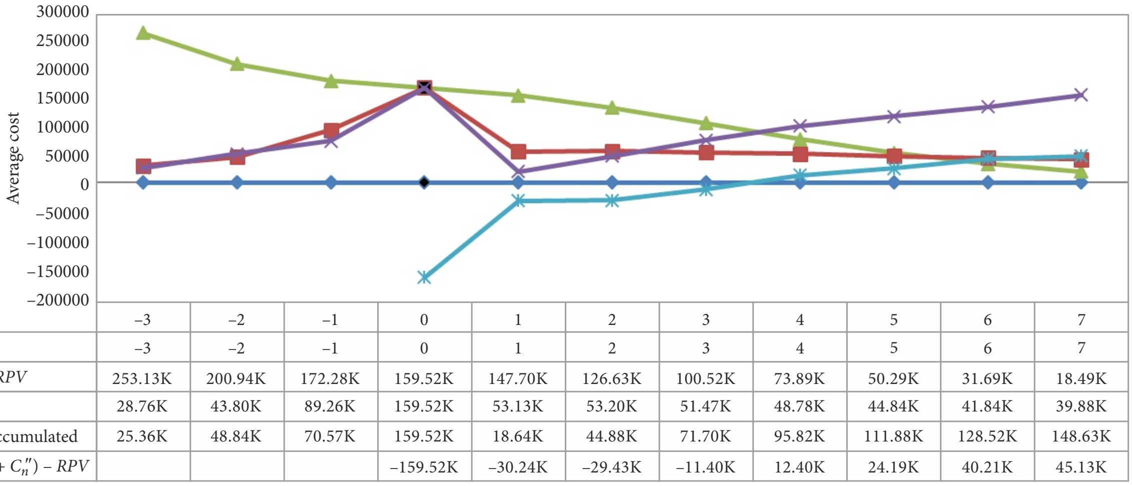
Finally, Figure 8 illustrates the evolution of the apparent rate resulting from the two previous rates for the time considered time interval.

Table 4. MMTAC–RPV: $ROI_a + \left(C'_n + C''_n\right)$

Years	–3	–2	–1	0	1	2	3
Purchase, *CA*				159.52K			
Purchase, *CA* (*RPV*)	253.13K	200.94K	172.28K	159.52K	147.70K	126.63K	100.52K
Exploration cost, *EC*	18.30K	18.30K	18.30K		18.30K	27.82K	30.72K
Exploration cost, *EC* – 10% incr.	20.13K	20.13K	20.13K		20.13K	30.60K	33.79K
Exploration cost, *EC* – *RPV*	25.36K	23.48K	21.74K		18.64K	26.24K	26.83K
C'	8.45K	24.42K	70.57K		18.64K	22.44K	23.90K
Cession, VC_n	175.00K	170.00K	165.00K		135.03K	114.30K	96.75K
Cession VC_n – *RPV*	220.45K	198.29K	178.20K		125.03K	97.99K	76.80K
C''_n	20.31K	19.39K	18.68K	159.52K	34.49K	30.76K	27.57K
$C'_n + C''_n$	28.76K	43.80K	89.26K	159.52K	53.13K	53.20K	51.47K
EC – *RPV* accumulated	25.36K	48.84K	70.57K	159.52K	18.64K	44.88K	71.70K
Profit, *Pf*					197.00K	200.00K	210.00K
Pf – *RPV*					182.41K	171.47K	166.70K
$ROI_a + \left(C'_n + C''_n\right) - RPV$				–159.52K	–30.24K	–29.43K	–11.40K

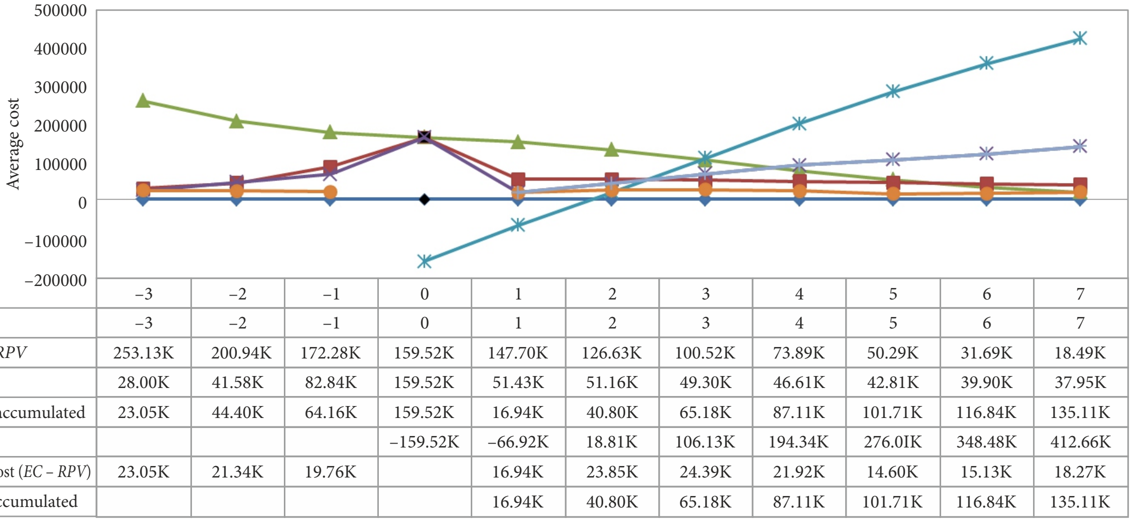
Table 5. MMTAC–RPV: ROI_a and life with accumulated costs

Years	−3	−2	−1	0	1	2	3
Purchase, CA				159.52K			
Purchase, CA (RPV)	253.13K	200.94K	172.28K	159.52K	147.70K	126.63K	100.52K
Exploration cost, EC	18.30K	18.30K	18.30K		18.30K	27.82K	30.72K
Exploration cost, $EC - RPV$	23.05K	21.34K	19.76K		16.94K	23.85K	24.39K
C'	7.68K	22.20K	64.16K		16.94K	20.40K	21.73K
Cession, VC_n	175.00K	170.00K	165.00K		135.03K	114.30K	96.75K
Cession $VC_n - RPV$	220.45K	198.29K	178.20K		125.03K	97.99K	76.80K
C''_n	20.31K	19.39K	18.68K	159.52K	34.49K	30.76K	27.57K
$C'_n + C''_n$	28.00K	41.58K	82.84K	159.52K	51.43K	51.16K	49.30K
$EC - RPV$ accumulated	23.05K	44.40K	64.16K	159.52K	16.94K	40.80K	65.18K
Profit, Pf					100.00K	100.00K	110.00K
$Pf - RPV$					92.59K	85.73K	87.32K
$ROI_a - RPV$				−159.52K	−66.92K	18.81K	106.13K
$EC - RPV$ accumulated					16.94K	40.80K	65.18K



	−3	−2	−1	0	1	2	3	4	5	6	7
Years	−3	−2	−1	0	1	2	3	4	5	6	7
Purchase – RPV	253.13K	200.94K	172.28K	159.52K	147.70K	126.63K	100.52K	73.89K	50.29K	31.69K	18.49K
$C'_n + C''_n$	28.76K	43.80K	89.26K	159.52K	53.13K	53.20K	51.47K	48.78K	44.84K	41.84K	39.88K
$EC - RPV$ accumulated	25.36K	48.84K	70.57K	159.52K	18.64K	44.88K	71.70K	95.82K	111.88K	128.52K	148.63K
$ROI_a + (C'_n + C''_n) - RPV$				−159.52K	−30.24K	−29.43K	−11.40K	12.40K	24.19K	40.21K	45.13K

Figure 4. MMTAC–RPV: $ROI_a + \left(C'_n + C''_n\right)$



	−3	−2	−1	0	1	2	3	4	5	6	7
Years	−3	−2	−1	0	1	2	3	4	5	6	7
Purchase – RPV	253.13K	200.94K	172.28K	159.52K	147.70K	126.63K	100.52K	73.89K	50.29K	31.69K	18.49K
$C'_n + C''_n$	28.00K	41.58K	82.84K	159.52K	51.43K	51.16K	49.30K	46.61K	42.81K	39.90K	37.95K
$ROI - RPV$ accumulated	23.05K	44.40K	64.16K	159.52K	16.94K	40.80K	65.18K	87.11K	101.71K	116.84K	135.11K
$ROI - RPV$				−159.52K	−66.92K	18.81K	106.13K	194.34K	276.0IK	348.48K	412.66K
Exploration cost $(EC - RPV)$	23.05K	21.34K	19.76K		16.94K	23.85K	24.39K	21.92K	14.60K	15.13K	18.27K
$EC - RPV$ accumulated					16.94K	40.80K	65.18K	87.11K	101.71K	116.84K	135.11K

Figure 5. MMTAC–RPV: ROI_a and life with accumulated costs

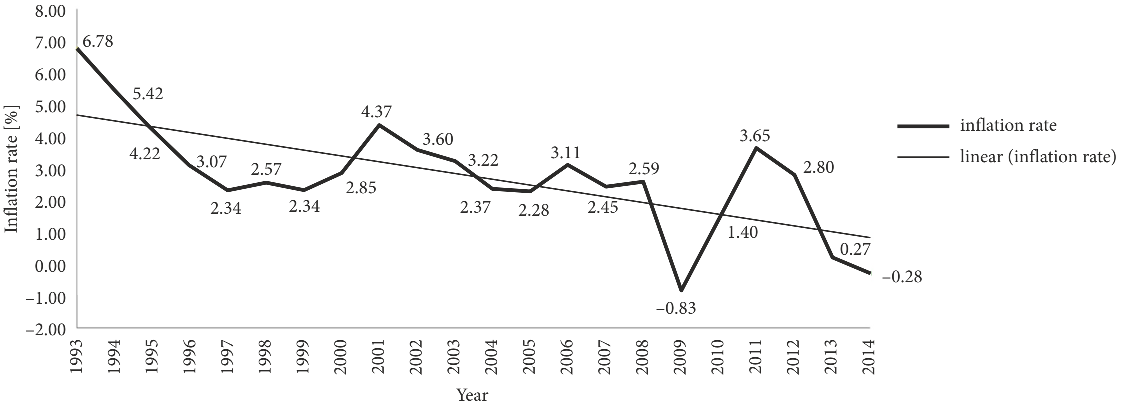


Figure 6. Change in inflation rate between 1993 and 2014

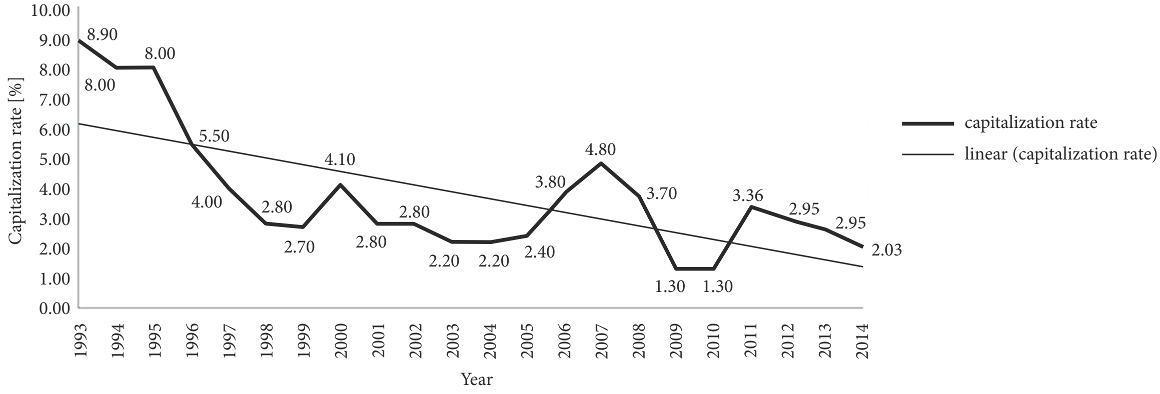


Figure 7. Change in real discount rate between 1993 and 2014

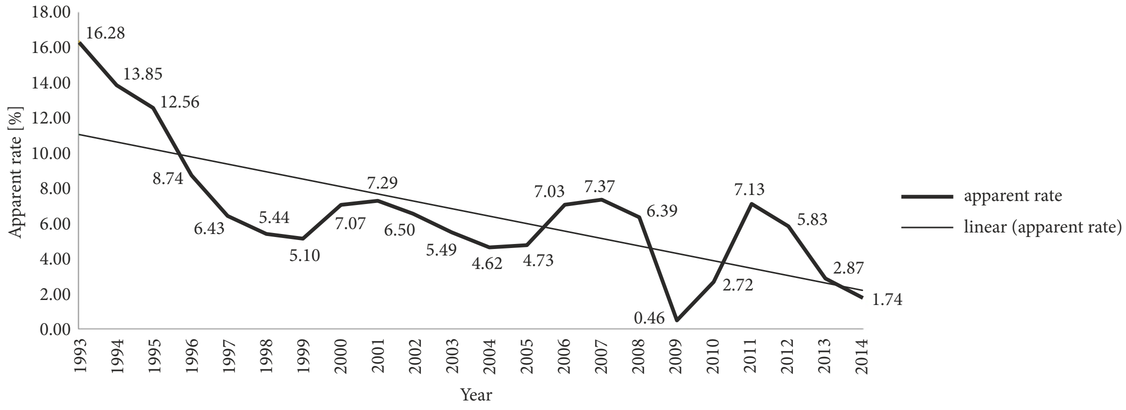


Figure 8. Change in apparent rate between 1993 and 2014

The first consideration is the effect of apparent rate on the UAI, using real data. Table 6 and Figure 9 show these calculations for bus 115 – the same bus used in the previous section.

It can be verified, through the Table 6 and Figure 9, some oscillations of the UAI over the years. In this case, it can be observed a lot of oscillations that cannot be indexed to the apparent rate variation, but a not good quality of the company's maintenance policy, what do not make clear the bus withdrawal time.

Because of the precedent situation, it will be used some theoretical data in order to exemplify the importance of the apparent rate in the withdrawal time, as will be discussed next. Table 7 and Figure 10 show these theoretical calculations.

As the Table 7 and Figure 10 show, the replacement point is located in the 17th year of life. The UAI is 27.50K.

As said before, there is big influence in the increase of the apparent rate on the MUAI. The results appear in Table 8 and Figure 11, again for bus 115.

As the Table 8 and Figure 11 show, the replacement point is in the 12th year of life, so the increase in the inflation rate and or the real discount rate over time increases the apparent rate. The value of the UAI is 31.53K. In other words, the rise in the apparent rate substantially increases the UAI of the bus.

Table 6. Actual apparent rate: MUAI (bus 115)

Bus 115									*VC* [€]	*NPV* [€ Year *n*]	*UAI* [€ Year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	$(1 + i_{A,j})$	*CM* [€]	*CO* [€]	Σ_1 [€]	*VP* [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K	16								
1994	1		14	1.14	1.00K	10.00K	11.00K	120.32K	83.35K	36.97K	42.09K
1995	2		13	1.13	1.05K	11.00K	12.05K	129.94K	64.24K	65.70K	39.16K
1996	3		9	1.09	1.10K	12.00K	13.10K	141.15K	54.29K	86.87K	34.16K
1997	4		6	1.06	1.15K	13.00K	14.15K	153.52K	46.65K	106.87K	31.15K
1998	5		5	1.05	1.20K	14.00K	15.20K	165.79K	39.39K	126.40K	29.55K
1999	6		5	1.05	1.25K	15.00K	16.25K	178.82K	32.66K	146.16K	28.89K
2000	7		7	1.07	1.30K	16.00K	17.30K	185.20K	23.41K	161.79K	30.09K
2001	8		7	1.07	1.35K	17.00K	18.35K	195.02K	18.44K	176.58K	29.91K
2002	9		7	1.07	1.40K	18.00K	19.40K	208.91K	15.75K	193.15K	29.02K
2003	10		5	1.05	1.45K	19.00K	20.45K	225.76K	13.96K	211.81K	28.09K
2004	11		5	1.05	1.50K	20.00K	21.50K	244.35K	12.43K	231.93K	27.37K
2005	12		5	1.05	1.55K	21.00K	22.55K	256.41K	16.73K	239.68K	26.64K
2006	13		7	1.07	1.60K	22.00K	23.60K	246.79K	6.21K	240.58K	28.83K
2007	14		7	1.07	1.65K	23.00K	24.65K	252.96K	4.76K	248.20K	29.01K
2008	15		6	1.06	1.70K	24.00K	25.70K	273.14K	4.37K	268.77K	28.38K
2009	16		0	1.00	1.75K	25.00K	26.75K	399.61K	8.81K	390.81K	25.39K
2010	17		3	1.03	1.80K	26.00K	27.80K	362.90K	5.15K	357.75K	26.56K
2011	18		7	1.07	1.85K	27.00K	28.85K	290.26K	2.02K	288.24K	28.93K
2012	19		6	1.06	1.90K	28.00K	29.90K	322.05K	2.04K	320.02K	28.31K
2013	20		3	1.03	1.95K	29.00K	30.95K	411.80K	2.91K	408.89K	27.15K
2014	21		2	1.02	2.00K	30.00K	32.00K	469.04K	3.06K	465.98K	26.69K

Table 7. Decline in apparent rate: MUAI (bus 115)

Bus 115									*VC* [€]	*NPV* [€ year *n*]	*UAI* [€ year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	$(1 + i_{A,j})$	*CM* [€]	*CO* [€]	Σ_1 [€]	*VP* [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K	8								
1994	1		8	1.08	1.00K	10.00K	11.00K	120.85K	87.91K	32.94K	35.56K
1995	2		8	1.08	1.05K	11.00K	12.05K	131.25K	70.11K	61.14K	34.16K
1996	3		8	1.08	1.10K	12.00K	13.10K	141.84K	56.13K	85.71K	32.98K
1997	4		7	1.07	1.15K	13.00K	14.15K	152.63K	45.11K	107.52K	31.98K
1998	5		7	1.07	1.20K	14.00K	15.20K	163.52K	36.39K	127.13K	31.11K
1999	6		7	1.07	1.25K	15.00K	16.25K	174.80K	29.48K	145.32K	30.41K
2000	7		7	1.07	1.30K	16.00K	17.30K	186.22K	23.97K	162.25K	29.80K
2001	8		7	1.07	1.35K	17.00K	18.35K	197.90K	19.56K	178.33K	29.29K
2002	9		6	1.06	1.40K	18.00K	19.40K	209.87K	16.03K	193.84K	28.87K
2003	10		6	1.06	1.45K	19.00K	20.45K	222.17K	13.19K	208.99K	28.52K
2004	11		6	1.06	1.50K	20.00K	21.50K	234.85K	10.89K	223.96K	28.23K
2005	12		6	1.06	1.55K	21.00K	22.55K	247.96K	9.03K	238.93K	28.00K
2006	13		5	1.05	1.60K	22.00K	23.60K	261.54K	7.51K	254.03K	27.82K
2007	14		5	1.05	1.65K	23.00K	24.65K	275.67K	6.28K	269.39K	27.68K
2008	15		5	1.05	1.70K	24.00K	25.70K	290.41K	5.27K	285.14K	27.59K
2009	16		5	1.05	1.75K	25.00K	26.75K	305.83K	4.44K	301.40K	27.53K
2010	17		5	1.05	1.80K	26.00K	27.80K	322.03K	3.75K	318.28K	27.50K
2011	18		4	1.04	1.85K	27.00K	28.85K	339.08K	3.18K	335.89K	27.51K
2012	19		4	1.04	1.90K	28.00K	29.90K	357.09K	2.71K	354.38K	27.54K
2013	20		4	1.04	1.95K	29.00K	30.95K	376.18K	2.32K	373.86K	27.61K
2014	21		4	1.04	2.00K	30.00K	32.00K	396.46K	1.99K	394.47K	27.69K

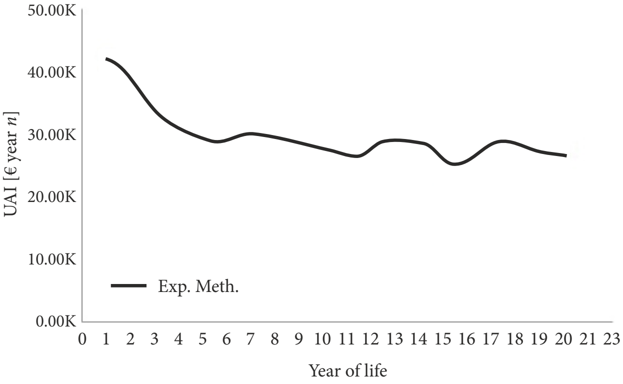


Figure 9. Actual apparent rate: MUAI (bus 115)

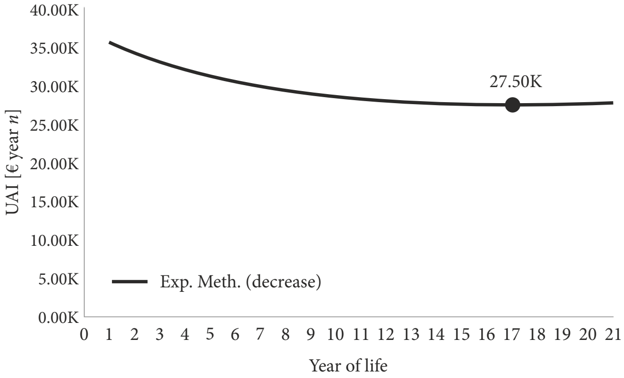


Figure 10. Decline in apparent rate: MUAI (bus 115)

Table 8. Increase in apparent rate: MUAI (bus 115)

Bus 115									*VC* [€]	*NPV* [€ year *n*]	*UAI* [€ year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	$(1 + i_{A,j})$	*CM* [€]	*CO* [€]	Σ_1 [€]	*VP* [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K	8								
1994	1		8	1.08	1.00K	10.00K	11.00K	120.81K	87.57K	33.24K	36.02K
1995	2		9	1.09	1.05K	11.00K	12.05K	131.01K	69.04K	61.97K	35.03K
1996	3		9	1.09	1.10K	12.00K	13.10K	141.13K	54.22K	86.91K	34.20K
1997	4		9	1.09	1.15K	13.00K	14.15K	151.04K	42.42K	108.62K	33.52K
1998	5		9	1.09	1.20K	14.00K	15.20K	160.71K	33.06K	127.65K	32.99K
1999	6		9	1.09	1.25K	15.00K	16.25K	169.82K	25.66K	144.16K	32.53K
2000	7		10	1.10	1.30K	16.00K	17.30K	178.53K	19.85K	158.68K	32.19K
2001	8		10	1.10	1.35K	17.00K	18.35K	186.69K	15.29K	171.39K	31.93K
2002	9		10	1.10	1.40K	18.00K	19.40K	194.25K	11.74K	182.51K	31.74K
2003	10		10	1.10	1.45K	19.00K	20.45K	201.19K	8.98K	192.22K	31.62K
2004	11		10	1.10	1.50K	20.00K	21.50K	207.49K	6.84K	200.65K	31.55K
2005	12		11	1.11	1.55K	21.00K	22.55K	213.13K	5.19K	207.94K	31.53K
2006	13		11	1.11	1.60K	22.00K	23.60K	218.12K	3.92K	214.20K	31.54K
2007	14		11	1.11	1.65K	23.00K	24.65K	222.47K	2.95K	219.52K	31.59K
2008	15		11	1.11	1.70K	24.00K	25.70K	226.20K	2.22K	223.98K	31.67K
2009	16		12	1.12	1.75K	25.00K	26.75K	229.33K	1.66K	227.67K	31.77K
2010	17		12	1.12	1.80K	26.00K	27.80K	231.89K	1.23K	230.65K	31.89K
2011	18		12	1.12	1.85K	27.00K	28.85K	233.92K	0.92K	233.00K	32.02K
2012	19		12	1.12	1.90K	28.00K	29.90K	235.45K	0.68K	234.78K	32.16K
2013	20		12	1.12	1.95K	29.00K	30.95K	236.53K	0.50K	236.03K	32.32K
2014	21		13	1.13	2.00K	30.00K	32.00K	237.20K	0.37K	236.83K	32.48K

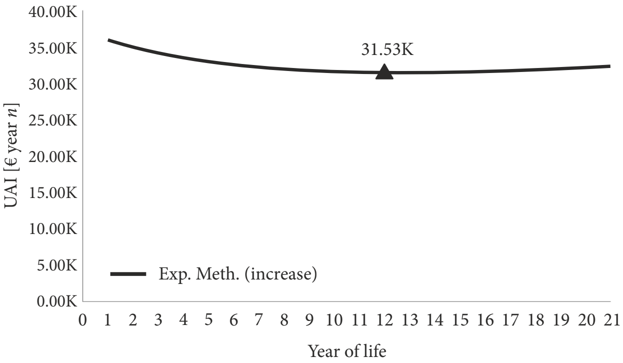


Figure 11. Increase in the apparent rate: UAI (bus 115)

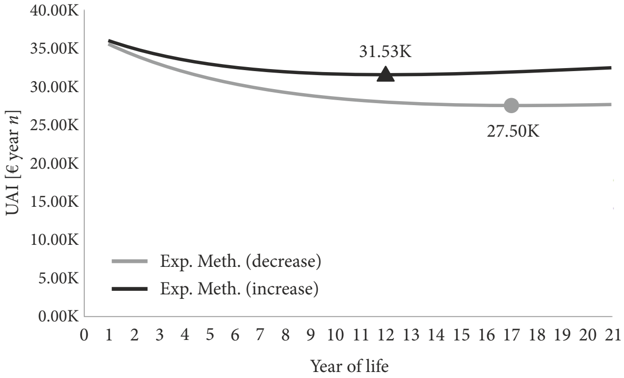


Figure 12. Apparent rate variation influence on the MUAI (bus 115)

Figure 12 verifies the influence of the high apparent rate in the econometric models. An increase or decrease in the apparent rate over time causes the withdrawal point to vary. The replacement point can vary by more than five years; i.e., the replacement point is 12 years if the apparent rate increases over time, but if the apparent rate decreases, the replacement point is 18 years. As mentioned above, the increase in annual income of the bus is also evident. Thus, it can be stated that the replacement period varies considerably with the apparent rate, influencing the final decision of the manager.

4. Influence of fuel cost variables

Variations in oil prices in international markets influence transportation costs, as the costs of most means of transport are directly linked to the price of a barrel of crude oil. Therefore, the cost to transport a particular product can vary greatly depending on the market fuel prices. The price of fuel is conditioned by several external factors worldwide, especially in OPEC countries. The Figures 13 and 14 shows the average price variation of fuel (diesel) from 2003 to 2014.

As Figure 13 suggests, the average price varies greatly, but over the study interval, there is an increasing trend. Figure 14 shows the operating costs of a bus taking into account the price of diesel fuel for 2003–2014.

The Tables 9, 10 and Figures 15, 16 that follow indicate the influence of the price of diesel fuel on bus replacement time. In the first example, Table 9 and Figure 15 show the effect of rising costs on the replacement time of bus 115. Note that the highest costs occur between 2003 and 2014. The cost in 2012 is the average cost.

As the Table and Figure 15 show, the replacement point is located in the 14th year, and the UAI is 27.65K. This calculation demonstrates that an increase in the price of diesel fuel influences the replacement time of an urban bus used for public transport.

We now turn to calculations showing what happens if the cost of diesel drops. Table 10 and Figure 16 show the effect of a decline in the price of diesel fuel on the replacement time for bus 115.

The Table 10 and Figure 16 determine the replacement point as occurring in the 18th year of life, taking into account the price of diesel in 2014, when it is lower by about 0.11 cents than in 2012. The UAI is 27.58K.

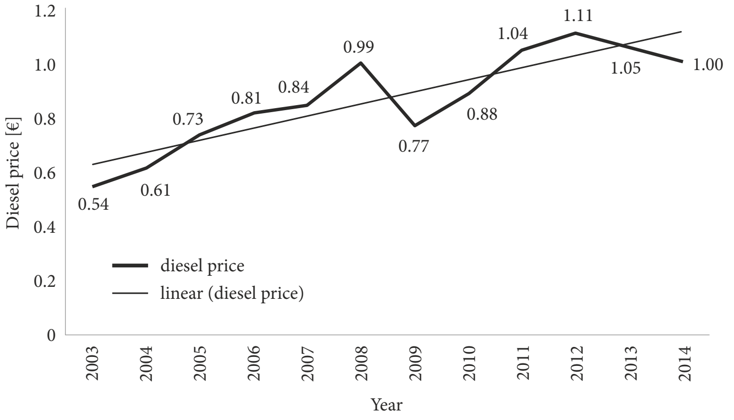
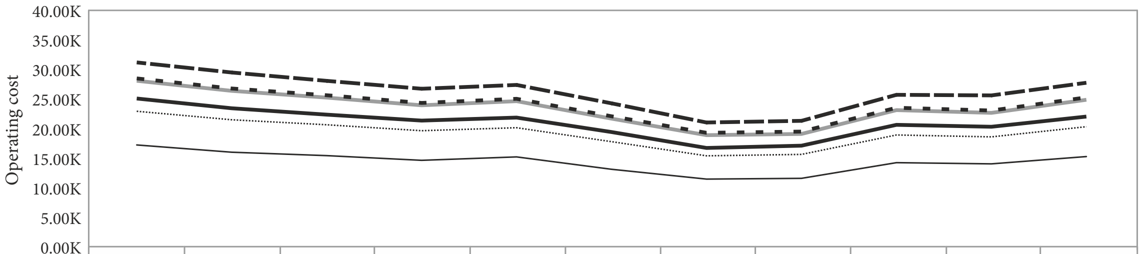


Figure 13. Variation in cost of diesel between 2003 and 2014



	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
— 2004	18.46K	17.30K	16.55K	15.78K	16.16K	14.26K	12.42K	12.60K	15.19K	15.03K	16.37K
····· 2006	24.55K	23.00K	22.01K	20.99K	21.48K	18.97K	16.51K	16.75K	20.20K	19.99K	21.77K
▬ 2008	30.06K	28.16K	26.95K	25.69K	26.30K	23.22K	20.22K	20.50K	24.72K	24.47K	26.65K
━ 2010	26.75K	25.06K	23.98K	22.86K	23.40K	20.66K	17.99K	18.24K	22.00K	21.77K	23.71K
━ ━ 2012	33.45K	31.34K	29.99K	28.59K	29.27K	25.85K	22.50K	22.82K	27.52K	27.23K	29.66K
- - - 2014	30.26K	28.35K	27.12K	25.86K	26.47K	23.37K	20.35K	20.64K	24.89K	24.63K	26.83K

Figure 14. Operating cost of bus versus price of fuel

Table 9. Increase in price of diesel: MUAI (bus 115)

Bus 115								*VC* [€]	*NPV* [€ year *n*]	*UAI* [€ year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	$(1 + i_{A,j})$	*CM* [€]	*CO* [€]	Σ_1 [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K								
1994	1		1.08	1.00K	11.22K	12.22K	121.96K	87.74K	34.22K	37.01K
1995	2		1.08	1.05K	10.71K	11.76K	132.01K	69.57K	62.44K	35.09K
1996	3		1.08	1.10K	10.46K	11.56K	141.15K	55.16K	85.99K	33.46K
1997	4		1.08	1.15K	10.48K	11.63K	149.65K	43.74K	105.91K	32.09K
1998	5		1.08	1.20K	10.76K	11.96K	157.73K	34.68K	123.05K	30.95K
1999	6		1.08	1.25K	11.31K	12.56K	165.58K	27.50K	138.08K	30.01K
2000	7		1.08	1.30K	12.12K	13.42K	173.32K	21.80K	151.52K	29.26K
2001	8		1.08	1.35K	13.19K	14.54K	181.09K	17.29K	163.80K	28.68K
2002	9		1.08	1.40K	14.53K	15.93K	188.95K	13.71K	175.24K	28.24K
2003	10		1.08	1.45K	15.54K	16.99K	196.70K	10.87K	185.83K	27.90K
2004	11		1.08	1.50K	18.46K	19.96K	205.13K	8.62K	196.51K	27.74K
2005	12		1.08	1.55K	20.75K	22.30K	213.83K	6.83K	206.99K	27.70K
2006	13		1.08	1.60K	22.01K	23.61K	222.34K	5.42K	216.92K	27.69K
2007	14		1.08	1.65K	21.73K	23.38K	230.14K	4.30K	225.84K	27.65K
2008	15		1.08	1.70K	29.27K	30.97K	239.69K	3.41K	236.28K	27.88K
2009	16		1.08	1.75K	25.85K	27.60K	247.56K	2.70K	244.86K	27.95K
2010	17		1.08	1.80K	22.50K	24.30K	253.96K	2.14K	251.82K	27.90K
2011	18		1.08	1.85K	22.82K	24.67K	259.97K	1.70K	258.27K	27.87K
2012	19		1.08	1.90K	27.52K	29.42K	266.60K	1.35K	265.25K	27.94K
2013	20		1.08	1.95K	27.23K	29.18K	272.68K	1.07K	271.61K	27.99K
2014	21		1.08	2.00K	29.66K	31.66K	278.78K	0.85K	277.93K	28.09K

Table 10. Decline in price of diesel: MUAI (bus 115)

Bus 115									*VC* [€]	*NPV* [€ year *n*]	*UAI* [€ year *n*]
Year	Year *j*	*CA* [€]	i_A [%]	$(1 + i_{A,j})$	*CM* [€]	*CO* [€]	Σ_1 [€]	*VP* [€]	Exp. Meth.	Exp. Meth.	Exp. Meth.
1993	0	110.66K	8								
1994	1		8	1.08	1.00K	11.22K	12.22K	121.96K	87.74K	34.22K	37.01K
1995	2		8	1.08	1.05K	10.71K	11.76K	132.01K	69.57K	62.44K	35.09K
1996	3		8	1.08	1.10K	10.46K	11.56K	141.15K	55.16K	85.99K	33.46K
1997	4		8	1.08	1.15K	10.48K	11.63K	149.65K	43.74K	105.91K	32.09K
1998	5		8	1.08	1.20K	10.76K	11.96K	157.73K	34.68K	123.05K	30.95K
1999	6		8	1.08	1.25K	11.31K	12.56K	165.58K	27.50K	138.08K	30.01K
2000	7		8	1.08	1.30K	12.12K	13.42K	173.32K	21.80K	151.52K	29.26K
2001	8		8	1.08	1.35K	13.19K	14.54K	181.09K	17.29K	163.80K	28.68K
2002	9		8	1.08	1.40K	14.53K	15.93K	188.95K	13.71K	175.24K	28.24K
2003	10		8	1.08	1.45K	15.54K	16.99K	196.70K	10.87K	185.83K	27.90K
2004	11		8	1.08	1.50K	18.46K	19.96K	205.13K	8.62K	196.51K	27.74K
2005	12		8	1.08	1.55K	20.75K	22.30K	213.83K	6.83K	206.99K	27.70K
2006	13		8	1.08	1.60K	22.01K	23.61K	222.34K	5.42K	216.92K	27.69K
2007	14		8	1.08	1.65K	21.73K	23.38K	230.14K	4.30K	225.84K	27.65K
2008	15		8	1.08	1.70K	26.47K	28.17K	238.83K	3.41K	235.42K	27.77K
2009	16		8	1.08	1.75K	23.37K	25.12K	245.99K	2.70K	243.29K	27.77K
2010	17		8	1.08	1.80K	20.35K	22.15K	251.83K	2.14K	249.68K	27.67K
2011	18		8	1.08	1.85K	20.64K	22.49K	257.31K	1.70K	255.61K	27.58K
2012	19		8	1.08	1.90K	24.89K	26.79K	263.34K	1.35K	261.99K	27.60K
2013	20		8	1.08	1.95K	24.63K	26.58K	268.88K	1.07K	267.81K	27.60K
2014	21		8	1.08	2.00K	26.83K	28.83K	274.43K	0.85K	273.58K	27.65K

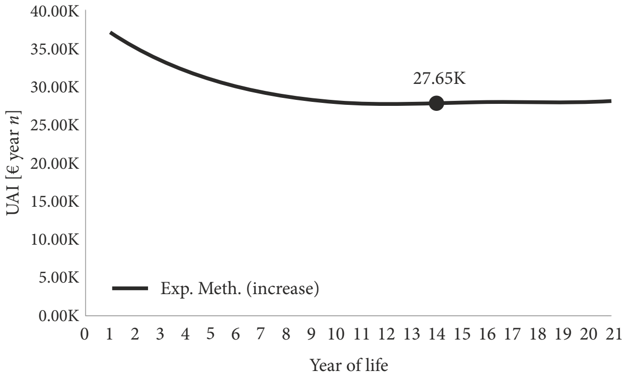


Figure 15. Increase in price of diesel: MUAI (bus 115)

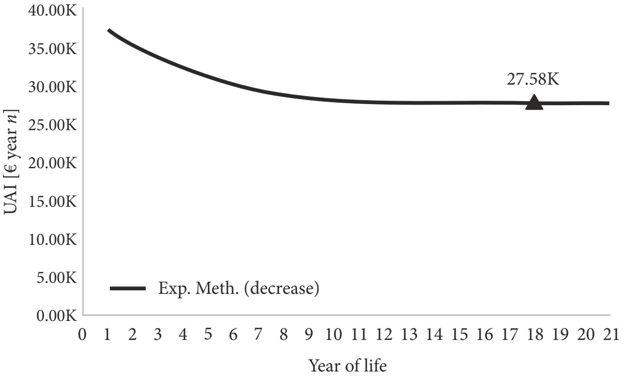


Figure 16. Decline in price of diesel: MUAI (bus 115)

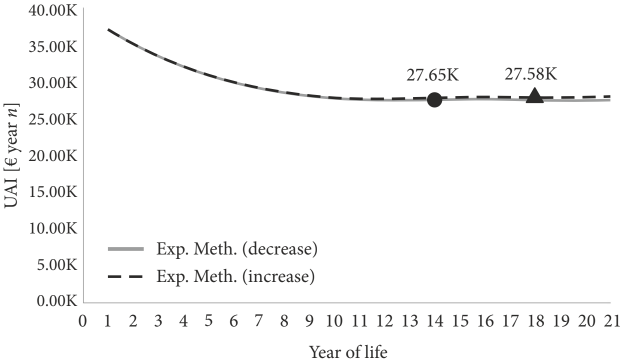


Figure 17. Influences of price of diesel on MUAI (bus 115)

Figure 17 shows the influence of the diesel price on the replacement time for bus 115. An increase or decrease in the cost of diesel over the years under consideration causes the point of withdrawal to vary. According to the Figure 17, the replacement point may vary by four years, if the average diesel prices between 2012 and 2014 are taken as a reference. There is also an increase in the UAI of the bus. For example, the company would have saved 2631.31 € or 2.63K on fuel for that bus in 2012 if the average price of diesel was the same as the average price in 2014.

In other words, the replacement period varies considerably with the price of diesel, affecting the manager's final decision.

5. Condition-based maintenance versus reserve fleet

The development and implementation of a policy to support strategies based on CM and, in particular, predictive maintenance, imply the consolidation of several monitoring systems. For example, a bus's Engine Control Unit (ECU) can monitor several variables, such as engine temperature, gearbox, engine speed, fuel consumption, etc.

Monitoring the bus "health" by determining the condition of certain variables implies a subsequent decrease or increase in maintenance intervals, usually the latter, with the following implications:

- eliminating unnecessary disassembly for inspection, thereby increasing the availability of the vehicle and decreasing the maintenance costs;
- reducing unplanned maintenance, thus increasing the availability of the vehicle and decreasing the maintenance costs;
- reducing serious breakdowns in service, thereby increasing bus reliability and, consequently, decreasing bus unavailability and immobilization costs;
- detecting problems before they become critical, reducing intervention costs;
- increasing useful life of the components and the bus, thus increasing profitability.

A variable that must be taken into account in econometric models to replace equipment is the cost of maintenance. This variable is very relevant in determining the optimum time of replacement of any equipment, in this case an urban passenger bus. One of the advantages of predictive maintenance based on oil analysis is the reduction of maintenance costs, and maintenance policy influences replacement time. Monitoring the degradation of oil influences the maintenance KPI's, namely, the Mean Time to Repair (MTTR) and Mean Time Between Failure (MTBF): lower MTTR and higher MTBF values indicate that maintenance is supporting well the production operations. The following equations express the availability *A*, the *MTTR* and the *MTBF* respectively:

$$A = \frac{MTBF}{MTBF + MTTR}; \tag{14}$$

$$MTTR = MTBF \cdot \frac{\left(1 - A\right)}{A}; \tag{15}$$

$$MTBF = \frac{MTTR}{\frac{\left(1 - A\right)}{A}}. \tag{16}$$

Table 11 and Figure 18 show that when MTTR decreases, bus availability increases.

The Table 11 and Figure 18 clearly demonstrate that a policy of CM or predictive maintenance leads to a higher MTBF and a lower MTTR. This, in turn, increases the availability of the bus. It is obviously important to demonstrate the effect of the interaction of these indicators on the size of the reserve fleet.

Table 12 and Figure 19 show variations in the size of the reserve fleet according to MTTR.

Table 11. Availability versus MTTR

Availability [%]	MTBF [days]	MTTR [days]
70.0	365	156
75.0	365	122
80.0	365	91
85.0	365	64
90.0	365	41
95.0	365	19
96.0	365	15
97.0	365	11
98.0	365	7
99.0	365	4

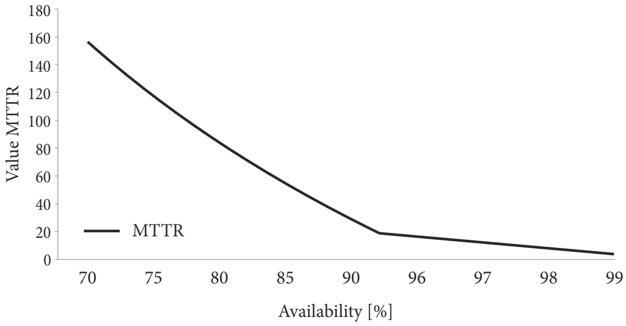


Figure 18. MTTR versus reserve fleet

To determine the size of the reserve fleet, the following calculation can be used:

$$RF = \frac{m \cdot MTTR}{k}, \tag{17}$$

where: RF – reserve fleet; m – number of fleet buses; $MTTR$ – mean time to repair; k – number of days per year.

According to Table 12 and Figure 19, the size of the reserve fleet increases with an increased MTTR. The lower this indicator, the lower the company's investment in a reserve fleet. Clearly, maintenance indicators and the maintenance policies practiced by road transport companies have an enormous impact in it.

6. An integrative approach to evaluate reserve fleet

After the presentation of the several econometric models and the examples based on real data, that allowed to demonstrate their importance, next, we present their synthesis and an integrative approach to them.

First, we presented the global models, namely the UAI, with RPV, and the MMTAC–RPV. From the first and second methods, the years of withdrawal are well defined, within the time interval considered. From the third method the withdrawal time appears outside of it, for the apparent rate considered. It may be concluded that, for this type of equipment, the withdrawal time may vary a lot according to the econometric model considered, what can be verified in practice.

Table 12. MTTR versus reserve fleet

MTTR [days]	Bus fleet [m]	Reserve fleet *RF*	Interval
5	100	1.4	[1, 2]
10	100	2.7	[2, 3]
15	100	4.1	[4, 5]
20	100	5.5	[5, 6]
25	100	6.8	[6, 7]
30	100	8.2	[8, 9]

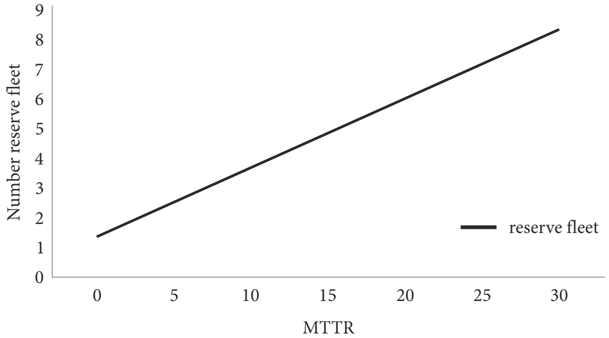


Figure 19. MTTR versus reserve fleet

Next it was studied the influence of the ROI_a versus withdrawal time, in which it can be concluded that may have an enlarged time since the ROI_a crosses the costs curves until that time.

In the next sections the influence of apparent rate, i.e., inflation and real discount rate, and the fuel price in the withdrawal time were discussed. It can be concluded that these variables have a big influence in this time.

Finally, the influence of the maintenance policy in the withdrawal time, namely in the MTTR interval was discussed, not only because its relevance in the buses availability, but also in the reserve fleet. This happens because the MTTR relates directly with the maintenance costs and, obviously, with the withdrawal time. The variation in MTTR have great influence in the fleet reserve dimension and value, because the high cost of each bus. As MTTR decreases it implies to have less buses in the reserve fleet, what represents a diminishing of thousands of monetary units in its cost.

Based on the presented models and variables studied, it is possible to make many other analysis, and to reach results within a wide spectrum of results in time, in order to get the most rational management.

The results show that it is necessary to accompany carefully, and systematically, this type of equipment in order to reach the most rational decision to withdrawal buses and to dimension the reserve fleet.

The global approaches to determining bus withdrawal can be synthesised as follows:

- econometric models for determining the minimum value of LCC:
  - UAI, taking into account the following variables:
    - operating cost:
      - maintenance costs;

      - • fuel costs;
    - replacement value;
    - inflation rate;
    - real discount rate;
  - useful life, taking into account the following variables:
    - operating costs:
      - • maintenance costs;
      - • fuel costs;
    - replacement value;
    - inflation rate;
    - real discount rate;
- conditioning or predictive maintenance models to maximize availability:
  - KPI's.

These approaches can be integrated into a single analytical model to determine the best time for bus withdrawal and, consequently, the size of the reserve fleet indexed to the size of the overall fleet, as the first implies the second.

Note that the integrated approach is valid for both new and used buses, through the monetary correction effect. Note also that the analysis herein does not include technological and environmental aspects.

Figure 20 shows the integrative model to evaluate the size of a reserve fleet. This figure synthesizes the above approaches, and have as a goal to give a global idea of the development followed along the paper, including the numerical analysis done. As the Figure 20 indicates, inflation costs and the real discount rate influence the replacement value, the maintenance, and the fuel costs. These costs, in return, are influenced by the maintenance policies practiced, such as scheduled maintenance indicated by the supplier, or conditioning maintenance with or without prediction. The type of maintenance performed by companies affects the buses LCC and, hence, the withdrawal models. Finally, the variables shown in the figure influence the size of the reserve fleet, if the aim is to maximize operational availability.

Conclusions and future developments

The paper presents an overview of the authors' on-going research into the use of econometric models to determine the most rational time to replace a bus. The economic aspects are guided by indicators associated with acquisition, maintenance, and operation costs, among others. The study presented in this paper enables the equipment life cycle to be monitored.

The paper presents some variations in the most rational time to replace vehicles. Using the MUAI for its calculations, it demonstrates the important influence of some variables, namely inflation and real discount rate, withdrawal value and diesel fuel prices, on the most rational time to replace a bus.

Another aspect emphasized by the paper is the importance of implementing condition-based maintenance policies. This helps to reduce the costs associated with maintenance, contributing to a decrease in overhead costs and providing an additional guarantee of equipment availability.

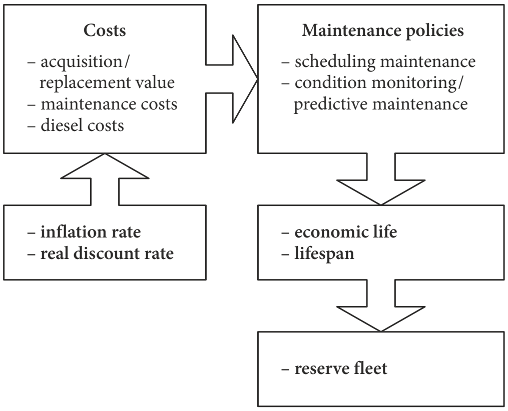


Figure 20. Integrative model for dimensioning a reserve fleet

In addition, the paper suggests the importance of monitoring the bus lifecycle through oil analysis, as well as the use of a condition-based/predictive maintenance policy, as this influences both the cost of equipment and the size of the reserve fleet.

Finally, it proposes integrating all methodologies into a single model as they all support the decision to replace buses and to determine the size of the reserve fleet.

Future developments include the generalization of models used in other types of buses including the new generation ones. The goal of the urban bus company is the validation of the models presented to the new hybrid and electric buses that the company is buying. This made us the challenge to enlarge the analysis and to develop new econometric models to this new type of buses.

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Contribution

Authors are welcome to declare any involvement in writing a manuscript (e.g. conception and design of the work, acquisition of data, or analysis and interpretation of data, drafting the article or revising it critically for important intellectual content, etc.).

Disclosure statement

The authors declare that not they have any competing financial, professional, or personal interests from other parties.

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