



---

Generalized Q Models for Investment

Author(s): Marzio Galeotti and Fabio Schiantarelli

Source: *The Review of Economics and Statistics*, Vol. 73, No. 3 (Aug., 1991), pp. 383-392

Published by: The MIT Press

Stable URL: <https://www.jstor.org/stable/2109562>

Accessed: 08-01-2020 14:24 UTC

---

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <https://about.jstor.org/terms>



JSTOR

*The MIT Press* is collaborating with JSTOR to digitize, preserve and extend access to *The Review of Economics and Statistics*

## GENERALIZED $Q$ MODELS FOR INVESTMENT

Marzio Galeotti and Fabio Schiantarelli\*

*Abstract*—We extend the  $Q$  theory of investment to allow for adjustment costs for labor, under the additional assumption that the firm is a monopolistic competitor in the output market. The issue of nonconstant returns to scale is also discussed. We show that the standard  $Q$  model is a special case of a more general model involving testable parameter restrictions. Estimates for the U.S. manufacturing sector suggest that the departure from the assumption of perfect competition and lack of adjustment costs for labor receive empirical support in the data.

### I. Introduction

IN recent years  $Q$  models have become the standard paradigm used to analyze investment decisions (see, for instance, Summers (1981), Hayashi (1982), Poterba and Summers (1983), Abel and Blanchard (1986)). A fair summary of their performance suggests that theoretical neatness has been rarely matched by empirical success. This has led to several attempts to extend the basic framework in order to provide a better explanation for the actual fluctuations of investment over time and/or for its variations across firms. For example, the specification of the technology has become richer and multiple capital inputs have been allowed for (Wildasin (1984), Chirinko (1984), and Hayashi and Inoue (1990)), and attention has been paid to the relationship between investment and financing decisions, especially when there are capital market imperfections (Hayashi (1985), Chirinko (1987), and Fazzari et al. (1988)). Finally, the implications of the assumption of monopolistic competition in

the product market for the specification and estimation of  $Q$  models have been examined by Schiantarelli and Georgoutsos (1990) and by Chirinko and Fazzari (1988). Most of these extensions invalidate the simple and appealing equality between marginal and average  $Q$  that holds under the assumption of linear homogeneity of the production and adjustment cost functions and perfect competition in the product market, as shown in Hayashi (1982).

Another natural extension that we explore in this paper is to assume that it is costly to change not only the capital stock but also employment. The idea that both labor and capital are costly to adjust and that such costs may be interrelated is an old one (see Lucas (1967) and Nadiri and Rosen (1969)) and has been a main feature of most recent work in dynamic factor demand (see Prucha and Nadiri (1986) for a thorough review). One of the approaches has been to estimate the Euler equations for the dynamic optimization problem as in Pindyck and Rotemberg (1983), McIntosh (1983), and Shapiro (1986). Our generalization of the  $Q$  model to the case of convex adjustment costs for labor is closer in spirit to these papers, while maintaining a relationship between investment and average  $Q$ , albeit in a modified form. In addition to treating labor as a quasi-fixed factor, we also allow for monopolistic competition in the output market. The issue of nonconstant returns to scale is discussed as well.

In section II we first derive the relationship between the shadow prices of capital and labor and the market value of the firm under these assumptions. By substituting out the unobservable shadow prices of capital and labor, we show how to obtain a simple dynamic equation containing only actual values or expectations of observable variables. This equation clarifies the relationship between investment, on the one hand,

Received for publication January 26, 1990. Revision accepted for publication December 27, 1990.

\*University of Brescia and Boston University, respectively.

We would like to thank the participants of seminars at Boston University, Brescia University, Bocconi University, the Institute for Fiscal Studies, and at the 1988 European Meeting of the Econometric Society, and two anonymous referees. M. Galeotti acknowledges financial support from Ministero della Pubblica Istruzione.

and average  $Q$ , output, and the cost of adjusting employment, on the other.

Lack of adjustment costs for labor, perfect competition in the output market, and constant returns to scale are special cases of our general model involving testable parameter restrictions. In section III estimates for the U.S. manufacturing sector are presented. The results show that the assumption of a monopolistically competitive firm facing adjustment costs for capital and labor receives more empirical support than the standard version of  $Q$  type of investment models. The estimates of the demand and adjustment cost parameters implied by our results are also discussed in this section.

**II. The Model**

The two basic assumptions of the model developed in this section are that the firm is a monopolistic competitor and that it faces convex internal costs in adjusting capital and labor. We assume, moreover, that the adjustment costs for changing the number of workers are far more important than the costs associated with changing the number of hours so that the latter can be set to zero in our production function. This assumption is quite plausible on a priori grounds and is supported by empirical evidence (Shapiro (1986)). However, changes in hours may affect the average cost of a worker, due to the existence of an overtime premium.

Denote output, capital, employment, average hours, investment and gross hiring respectively by  $Y, K, L, H, I, X$ . The net production function is given by  $Y = F(K, L, H, I, X)$ , where  $F_I < 0$ ,  $F_{II} < 0$ ,  $F_X < 0$ , and  $F_{XX} < 0$ . The firm maximizes the present value of cash flow and it is assumed to use retention financing at the margin. For the purpose of showing the relationship between investment, hiring, and the average value of the firm and output, it is convenient to write the objective function as

$$V_t = E_t \left\{ \sum_{j=0}^{\infty} \beta_t^{t+j} [(1 - \tau_{t+j})P_{t+j} \times F(K_{t+j}, L_{t+j}, I_{t+j}, H_{t+j}, X_{t+j}) - \bar{w}_{t+j}X_{t+j} - (1 - u_{t+j})P_{t+j}^I I_{t+j}] + A_t - B_t \right\} \tag{1}$$

$\beta_i^k = \prod_{s=i}^k (1 + \rho_s)^{-1}$  is the discount rate and  $\rho$  is the appropriate rate of return assumed to be nonstochastic.  $P$  is the own price and it is a choice variable for the firm.  $P^I$  is the price of new investment goods,  $\tau$  is the corporate tax rate,  $u$  the present value of tax savings associated with depreciation allowances and tax credits.  $A$  is the present value of tax savings on depreciation allowances for investment made before period  $t$ . A similar breakdown has been used for labor costs because it clarifies the derivation of the relationship between the shadow values of capital and labor and the market value of the firm.  $\bar{w}_{t+j}$  represents the present value of labor costs on workers hired from time  $t + j$  onward:

$$\bar{w}_{t+j} = \sum_{v=t+j}^{\infty} \beta_{t+j+1}^v (1 - \tau_v) w_v (1 - \gamma)^{v-(t+j)} \tag{2a}$$

where  $w$  is the labor cost per man and  $\gamma$  the exogenous quit rate for workers, which for expositional simplicity is assumed to be constant. Note that  $\beta_{t+j+1}^{t+j}$  equals one by definition.  $B_t$  is, instead, the present value of labor costs for workers hired before time  $t$  and is defined as

$$B_t = E_t \left\{ \sum_{j=0}^{\infty} \beta_t^{t+j} \sum_{v=t-1}^{-\infty} w_{t+j} (1 - \tau_{t+j}) \times (1 - \gamma)^{t+j-v} X_v \right\} \tag{2b}$$

$$= \frac{(1 - \gamma)}{(1 + \rho_t)} L_{t-1} E_t \bar{w}_t.$$

$B_t$  is obviously predetermined for the firm at time  $t$ . The maximization problem is subject to the equations of motion for capital and employment:

$$K_t = I_t + (1 - \delta)K_{t-1} \tag{3a}$$

$$L_t = X_t + (1 - \gamma)L_{t-1}. \tag{3b}$$

$\delta$  is the rate of capital depreciation. Denote with  $\lambda_t^K$  and  $\lambda_t^L$  the Lagrange multipliers associated with equations (3a) and (3b), respectively. Denoting with  $\epsilon$  the price elasticity of output demand, the first order conditions for investment, hiring, the capital stock, and the stock of workers can be

written as

$$E_t \left\{ (1 - \tau_t) P_t \left( 1 - \frac{1}{\epsilon_t} \right) F_I - (1 - u_t) P_t^I + \lambda_t^K \right\} = 0 \quad (4a)$$

$$E_t \left\{ (1 - \tau_t) P_t \left( 1 - \frac{1}{\epsilon_t} \right) F_X - \bar{w}_t + \lambda_t^L \right\} = 0 \quad (4b)$$

$$E_t \left\{ (1 - \tau_t) P_t \left( 1 - \frac{1}{\epsilon_t} \right) F_K - \lambda_t^K + \frac{1 - \delta}{1 + \rho_{t+1}} \lambda_{t+1}^K \right\} = 0 \quad (4c)$$

$$E_t \left\{ (1 - \tau_t) P_t \left( 1 - \frac{1}{\epsilon_t} \right) F_L - \lambda_t^L + \frac{1 - \gamma}{1 + \rho_{t+1}} \lambda_{t+1}^L \right\} = 0. \quad (4d)$$

What is the relationship between the shadow value of capital and labor and the market value of the firm in this model? As in conventional  $Q$  models of investment, we start by assuming that the net production function is homogeneous of degree one in  $K, L, I,$  and  $X$ . It is easy to prove that the following relationship holds:<sup>1</sup>

$$(1 - \gamma) L_{t-1} \lambda_t^L + (1 - \delta) \lambda_t^K K_{t-1} = [V_t - A_t + B_t - C_t](1 + \rho_t) \quad (5)$$

where

$$C_t = E_t \left\{ \sum_{j=0}^{\infty} \beta_t^{t+j} \frac{1 - \tau_{t+j}}{\epsilon_{t+j}} P_{t+j} Y_{t+j} \right\}. \quad (6)$$

Equation (5) says that the shadow price of the quasi-fixed factors, multiplied by their respective quantities, equals the market value of the firm,  $V_t$ , adjusted for (i) the present value of tax savings due to depreciation allowances on past investment,  $A_t$ ; (ii) the present value of wage costs for workers hired in the past,  $B_t$ , given in (2b); (iii) the present value of the loss in monopoly profits due to the price reduction necessary, *ceteris paribus*, to generate the demand to absorb an increase in production,  $C_t$ , given in (6). If

there is perfect competition in the output market,  $\epsilon_{t+j} = \infty$  and  $C_t = 0$ . The expressions in (5) and (6) extend to the case of monopolistic competition and labor as a quasi-fixed factor the result obtained by Wildasin (1984) for multiple capital inputs.

Note that there is a subtle difference between allowing for multiple capital inputs on the one hand, and treating both the homogeneous capital stock and labor as quasi-fixed inputs on the other. Contrary to buying a new capital good, hiring a worker generates a sequence of wage payments until the worker is made redundant or quits. This explains the presence of the  $B_t$  term on the right hand side of (5), capturing the predetermined nature of the present value of wage costs for workers hired before  $t$ . If both  $B_t$  and  $\lambda_t^L$  are set to zero, one obtains the discrete time analogue of the relationship between marginal and average  $Q$  under monopolistic competition derived originally by Hayashi (1982) and exploited by Schiantarelli and Georgoutsos (1990).

The extension to the case of nonconstant returns to scale is straightforward. There are two options here and it is difficult to choose between them on a priori grounds.<sup>2</sup> If it is assumed that the net production function,  $F(\cdot)$ , is homogeneous of degree  $(1 + \theta)$ , then we need only to modify the definition of  $C_t$  in (6) by replacing  $1/\epsilon_{t+j}$  with  $[1 - \theta(\epsilon_{t+j} - 1)]/\epsilon_{t+j}$ . Assume now that adjustment costs are additively separable so that  $F(K, L, I, H, X) = f(K, L, H) - G(I, X, K, L)$ , where the function  $f(\cdot)$  denotes the gross production function and  $G(\cdot)$  the adjustment cost function. If  $f(\cdot)$  is homogeneous of degree  $(1 + \theta)$  while adjustment costs are still linear homogeneous in their arguments, then on the right hand side of (5) we should also include an additional term,  $M_t$ , containing present and future adjustment costs. More specifically:

$$M_t = E_t \left\{ \sum_{j=0}^{\infty} \beta_t^{t+j} \frac{\theta_{t+j} (\epsilon_{t+j} - 1)}{\epsilon_{t+j}} \times (1 - \tau_{t+j}) P_{t+j} G(t + j) \right\}. \quad (7)$$

<sup>1</sup> In order to obtain (5), multiply (4a) by  $I_t$ , (4b) by  $X_t$ , (4c) by  $K_t$ , (4d) by  $L_t$ , add the resulting equations together and use the linear homogeneity of the production function. Details on all the derivations are contained in an appendix available from the authors upon request.

<sup>2</sup> In the dynamic factor demand literature returns to scale are usually specified with respect to the net production function (see, e.g., Morrison (1988)). In the  $Q$  literature an example is given by Chirinko and Fazzari (1988) and Chirinko (1989).

Even if the presence of  $\lambda_t^L$ ,  $B_t$ ,  $C_t$ , and  $M_t$  introduces a wedge between the marginal and average shadow value of capital adjusted for taxation, it is possible to use the first order conditions for investment and hiring, (4a) and (4b), to obtain a dynamic equation linking investment to average  $Q$ , output, and the marginal adjustment cost for employment. Such an equation contains only observable variables and disposes of the summation of infinite terms contained in  $B_t$ ,  $C_t$ , and  $M_t$ .

Focusing on the constant returns to scale case, use the first order condition for hiring, (4b), to substitute out  $\lambda_t^L$  in (5). Using (2b) and solving for the shadow price of capital,  $\lambda_t^K$ , one obtains:

$$\frac{\lambda_t^K}{(1 - \tau_t)P_t} = \frac{[V_t - A_t - C_t](1 + \rho_t)}{(1 - \delta)(1 - \tau_t)P_t K_{t-1}} - \left(\frac{1 - \gamma}{1 - \delta}\right) \left(1 - \frac{1}{\epsilon_t}\right) G_X(t) \frac{L_{t-1}}{K_{t-1}} \tag{8}$$

Lead (8) one period, multiply by

$$\psi_t = \frac{(1 - \tau_{t+1})P_{t+1}K_t}{(1 + \rho_{t+1})(1 - \tau_t)P_t K_{t-1}}$$

and subtract from (8). This yields:

$$E_t \left\{ D \left( \frac{\lambda_t^K}{(1 - \tau_t)P_t} \right) - D \left( \frac{(V_t - A_t)(1 + \rho_t)}{(1 - \delta)(1 - \tau_t)P_t K_{t-1}} \right) + \frac{Y_t}{\epsilon_t(1 - \delta)K_{t-1}} + \frac{1 - \gamma}{1 - \delta} D \left( \left(1 - \frac{1}{\epsilon_t}\right) G_X(t) \frac{L_{t-1}}{K_{t-1}} \right) \right\} = 0 \tag{9}$$

where for ease of notation we have introduced the quasi forward difference operator  $D$ , such that, for any function of time,  $h(t)$ ,  $D(h(t)) = h(t) - \psi_t h(t + 1)$ . Using the first order condition for investment, (4a), to substitute out  $\lambda_t^K$  in (9), one obtains:

$$E_t \left\{ D \left( \left(1 - \frac{1}{\epsilon_t}\right) G_I(t) \right) - DQ_t + \frac{1}{\epsilon_t} \frac{Y_t}{(1 - \delta)K_{t-1}} + \left(\frac{1 - \gamma}{1 - \delta}\right) \times D \left( \left(1 - \frac{1}{\epsilon_t}\right) G_X(t) \frac{L_{t-1}}{K_{t-1}} \right) \right\} = 0 \tag{10}$$

where

$$Q_t = \left[ \frac{(1 + \rho_t)(V_t - A_t)}{(1 - \delta)P_t^I K_{t-1}} - (1 - u_t) \right] \times \frac{P_t^I}{(1 - \tau_t)P_t} \tag{11}$$

$Q_t$  captures the tax-adjusted ratio between the market value of capital and its replacement cost and is used as an explanatory variable in standard  $Q$  models of investment (see Summers (1981) and Poterba and Summers (1983)). The output per unit of capital variable appears because of the assumption of monopolistic competition. The last term is present in equation (10) because there are adjustment costs associated with changing employment. More precisely it reflects the marginal adjustment costs for changing the level of employment multiplied by the labor-capital ratio.

For the purpose of empirical implementation we will assume that adjustment costs are quadratic. Initially, purely for ease of exposition, it will also be assumed that there are no cross-adjustment costs. The case of interrelated adjustment costs can be easily accommodated and is discussed below. The adjustment cost function can be written as

$$G = \frac{a}{2} \left( \frac{I_t}{K_t} - b \right)^2 K_t + \frac{c}{2} \left( \frac{X_t}{L_t} - d \right)^2 L_t \tag{12}$$

where  $a, b, c, d$  are positive constants. If the price elasticity of demand is also constant, then equation (10) becomes

$$E_t D \left( \frac{I_t}{K_t} \right) = b[1 - E_t \psi_t] + \frac{\epsilon}{a(\epsilon - 1)} E_t DQ_t - \frac{1}{a(\epsilon - 1)} \left( \frac{Y_t}{(1 - \delta)K_{t-1}} \right) - \frac{c}{a} \left( \frac{1 - \gamma}{1 - \delta} \right) E_t D \left( \frac{X_t}{L_t} \frac{L_{t-1}}{K_{t-1}} \right) + \frac{dc}{a} \left( \frac{1 - \gamma}{1 - \delta} \right) E_t D \left( \frac{L_{t-1}}{K_{t-1}} \right) \tag{13}$$

This model encompasses the case of perfect competition and of lack of adjustment costs for

labor. If there is perfect competition in the product market,  $\epsilon = \infty$ . The coefficient in front of the output term becomes zero and the one in front of the  $Q$  terms becomes  $1/a$ . If labor is a variable factor, both  $c$  and  $d$  equal zero, so that the last two terms drop out of equation (13). All these restrictions are easily testable and this is one of the advantages of this formulation. Note, moreover, that the equation is just identified and all the structural parameters can be recovered. If there are nonconstant returns to scale affecting the gross production function, then the coefficient in front of output becomes  $[1 - \theta(\epsilon - 1)]/[a(\epsilon - 1)]$  and the term  $(\theta/2)(1 - \delta)((I_t/K_t) - b)^2(K_t/K_{t-1})$  must be added to the right hand side of (13). On the other hand, when the departure from linear homogeneity affects only the net production function, no term has to be added to (13). In this case the structural parameters  $\epsilon$  and  $\theta$  cannot be separately recovered, based on estimation of (13) alone. However, if a demand equation is estimated jointly with (13), unique estimates for  $\epsilon$  and  $\theta$  can be obtained.

In deriving equation (13) we have assumed that the rate of capital depreciation and the quit rate are fixed, but the derivations go through when this assumption is relaxed. Finally, the interrelated nature of adjustment costs can also be easily allowed for by including, for instance, the term

$$s \frac{I_t}{K_t} \frac{X_t}{L_t} K_t^\alpha L_t^{1-\alpha} \quad (0 \leq \alpha \leq 1) \quad (14)$$

in the adjustment cost function,  $G(\cdot)$ . Under this assumption, as can be seen from (10), an additional regressor representing the interaction term in  $G(\cdot)$  would appear in (13). Given the parameterization adopted in (12), the additional regressor is

$$-\frac{s}{a} \left\{ E_t D \left( \frac{X_t}{L_t} \left( \frac{L_t}{K_t} \right)^{1-\alpha} \right) + \left( \frac{1-\gamma}{1-\delta} \right) E_t D \left( \frac{I_t}{K_t} \left( \frac{K_t}{L_t} \right)^\alpha \frac{L_{t-1}}{K_{t-1}} \right) \right\}. \quad (15)$$

In our empirical work we concentrate on (13) and use the framework it provides to test different versions of  $Q$  models of investment. Al-

though the normalization embodied in (13) is not the only possible one, it is the most convenient one for this purpose. The estimated model will also be used to draw inferences on the structure of adjustment costs, the nature of the output market, and the degree of returns to scale. In the next section we will discuss the empirical results obtained when the model is estimated using yearly data for U.S. total manufacturing.

### III. Econometric Specification and Empirical Results

In reporting and discussing the empirical results we have chosen to proceed from the "specific to the general" in terms of model specification. The simplest case we report corresponds to the standard  $Q$  model with perfect competition in the output market commonly found in the literature. We then show how the empirical performance of the model changes when moving to more general specifications. More specifically, we first introduce monopolistic competition in the investment equation, we then discuss the role of nonconstant returns to scale, and finally we allow for the impact of adjustment costs for labor on investment decisions.

All the models involve expected values of future variables. Since we assume that expectations are rational, we can replace them by their realizations and use the method of instrumental variables (McCallum (1976)). Any subset of information available to the agents represents a legitimate instrument set when the estimating equation contains only white noise expectational errors. If we include, in addition to the constants  $b$  and  $d$ , two serially uncorrelated additive stochastic terms in the adjustment cost function, the composite disturbance in the estimating equation will exhibit a moving average structure of order one (MA(1)). In order to take this possibility into account, we use instruments dated  $t - 1$  or earlier. In addition, we report the serial correlation test proposed by Breusch and Godfrey (1981) for AR(1)/MA(1) errors and the Sargan (1964) misspecifications test on the correlation between the residuals and the instruments.<sup>3</sup>

<sup>3</sup> Since we always reject the presence of MA(1)/AR(1) residuals, the use of instruments dated  $t$  or earlier may be legitimate. When contemporaneous instruments are used, the results remain very similar to the ones presented in the tables and are not reported here.

TABLE 1.—*Q* MODELS OF INVESTMENT: PERFECT AND MONOPOLISTIC COMPETITION IN THE OUTPUT MARKET; LABOR AS A VARIABLE INPUT

Explanatory Variables	SAMPLE PERIOD: 1951–1985; DEPENDENT VARIABLE: $D(I_t/K_t)$	
	(1)	(2)
$1 - \psi_t$	0.185 (3.826)	0.295 (4.038)
$DQ_t$	0.054 (2.306)	0.084 (2.633)
$Y_t/(1 - \delta)K_{t-1}$	—	-0.0042 (2.214)
$DDUM_t$	0.025 (2.568)	0.030 (2.616)
$a$	18.657 (2.306)	12.531 (2.567)
$b$	0.185 (3.826)	0.295 (4.038)
$\epsilon$	—	20.00 (2.289)
$H$	0.00126	0.00077
Sargan	13.150 [9]	5.416 [8]
Breusch-Godfrey	0.964	-0.222

Notes: (i) Column (1) corresponds to the perfect competition case; column (2) corresponds to the monopolistic competition case. (ii)  $DUM_t$  is a dummy variable equal to one in 1973 and 1974 and zero otherwise (see Holland and Myers (1984), table 2B2b note d). (iii)  $H$  is the minimized value of the objective function; Sargan is a misspecification test, distributed as  $\chi^2$  with  $(m - n)$  degrees of freedom ( $m$  = number of instruments;  $n$  = number of regressors); Breusch-Godfrey is a test for MA(1)/AR(1) errors, distributed as a standardized normal. Asymptotic robust  $t$ -statistics in round brackets; degrees of freedom in square brackets. (iv) The instrument set for both equations is constant,  $DDUM_t$ ,  $I_{t-1}/K_{t-2}$ ,  $K_{t-1}$ ,  $\tau_{t-1}$ ,  $u_{t-1}$ ,  $A_{t-1}/K_{t-2}$ ,  $I_{t-2}/K_{t-2}$ ,  $R_{t-1}$ ,  $DQ_{t-2}$ ,  $Y_{t-1}/(1 - \delta)K_{t-2}$ ,  $Y_{t-2}/(1 - \delta) \times K_{t-3}$ .

The models have been estimated for the period 1951–1985 using yearly U.S. manufacturing data.<sup>4</sup> In the theoretical section materials were omitted from the production function for ease of notation. However, if they are included and the production function is assumed to be homogeneous in  $K$ ,  $L$ ,  $I$ ,  $X$ , and materials, then all the derivations go through. Moreover,  $Y$  should be interpreted as output and not value added in the definition of  $C_t$  in equation (6). For this reason we have used gross output data in estimation.<sup>5</sup>

In table 1, column (1), we report the estimates of the equation relating investment to tax-adjusted  $Q$ , obtained under the assumption of constant returns to scale, perfectly competitive output market, and labor as a variable factor, so that  $\epsilon = \infty$  and  $c = d = 0$  in equation (13). Both the dependent variable and the regressors are expressed as quasi-forward differences. The depreciation rate for capital is set to a value of 0.0965. We include in the equation also a dummy variable to account for the fact that the value of the firms in the manufacturing sector, derived from the capitalization of dividends for the years 1973–74, may be seriously mismeasured (see Holland and Myers (1984)).

<sup>4</sup> Strictly speaking, estimation of the model on aggregate data is appropriate under the assumption that all firms are identical. More loosely, the estimated parameters can be interpreted as averages for the entire manufacturing sector.

<sup>5</sup> An appendix describing the sources of data and construction of variables is available from the authors upon request.

In column (2) we allow for a monopolistic output market. The results suggest that the output–capital ratio enters the equation in the way suggested by the theory, its coefficient has the expected sign and it is quite significant. Although the departure from perfect competition is one of the reasons that justifies the presence of the output–capital ratio in the equation, it is not the only one. For instance, assume that the market value of the firm is an imperfect and noisy measure for expected future profits. In this case the significance of output may also depend upon its role as a predictor of future profitability. The size of the coefficient of  $Q$  increases and the goodness of fit statistic improves, using as a criterion the value of the objective function that is minimized by the instrumental variable estimator. The Sargan test is borderline at conventional levels in the case of the model in column (1) whereas it does not indicate misspecification for the model of column (2). This is consistent with the omission of the output variable from the more restricted regression.

The increase in the coefficient of the  $Q$  variable is a hopeful sign but still implies a slow speed of adjustment of investment to exogenous shocks. The elasticity of investment with respect to changes in  $Q$  ranges between 0.20 and 0.31. This is to be compared with values in the range of 0.1–0.3 in Abel and Blanchard (1986), values of 0.14–0.23 in Summers (1981), and a value of 0.27

TABLE 2.—Q MODELS OF INVESTMENT: MONOPOLISTIC COMPETITION IN THE OUTPUT MARKET; LABOR AS A QUASI-FIXED FACTOR

SAMPLE PERIOD: 1951–1985; DEPENDENT VARIABLE:  $D(I_t/K_t)$

Explanatory Variables	(1)	(2)
$1 - \psi_t$	0.104 (1.804)	—
$DQ_t$	0.037 (3.916)	—
$Y_t/(1 - \delta)K_{t-1}$	-0.0041 (3.268)	—
$[(1 - \gamma)/(1 - \delta)]D((X_t/L_t)(L_{t-1}/K_{t-1}))$	-4.393 (2.003)	—
$[(1 - \gamma)/(1 - \delta)]D(L_{t-1}/K_{t-1})$	3.792 (4.246)	—
$DDUM_t$	0.011 (5.009)	0.009 (3.638)
$a$	30.083 (3.432)	80.682 (3.476)
$b$	0.104 (1.804)	—
$\epsilon$	9.177 (2.433)	5.933 (3.905)
$c$	132.14 (2.845)	2781.5 (2.843)
$d$	0.863 (1.584)	0.093 (4.135)
$s$	—	-816.3 (1.707)
$\alpha$	—	0.296 (2.933)
$H$	0.00074	0.00015
Sargan	15.903 [10]	11.189 [9]
Breusch-Godfrey	1.548	1.636

Notes (see also table 1): (i) Column (1) corresponds to the case of no interrelated adjustment costs; column (2) corresponds to the interrelated case. (ii) The instrument set used in table 1 is, for both equations, augmented by the following variables:  $X_{t-1}/L_{t-1}$ ,  $D((X_{t-2}/L_{t-2})(L_{t-3}/K_{t-3}))$ ,  $D(L_{t-2}/K_{t-2})$ ,  $Y_{t-2}/(1 - \gamma)L_{t-3}$ .

in Hayashi (1982).<sup>6</sup> The elasticity of demand implied by the estimates is rather large. We have remarked that if there are nonconstant returns to scale of degree  $(1 + \theta)$  in the net production function, it is not possible to recover  $\theta$  and  $\epsilon$  separately. In general, if there are increasing (decreasing) returns to scale, the assumption of constant returns ( $\theta = 0$ ) maintained here leads to overestimating (underestimating) the demand elasticity. A way to tackle this problem is to estimate a demand equation for output jointly with the investment equation, under the assumption that the elasticity of demand faced by each individual firm equals the elasticity of demand for aggregate manufacturing output. We have pursued empirically this option by assuming (Shapiro (1987)) that the ratio between manufacturing output,  $Y$ , and GNP,  $Z$ , is a loglinear function of the ratio between the manufacturing price level,  $P$ , and the aggregate price level,  $V$ . The inverse demand function can be written in terms of growth rates as

$$\Delta \log \left( \frac{P_t}{V_t} \right) = -\frac{1}{\epsilon} \Delta \log \left( \frac{Y_t}{Z_t} \right) + \Delta u_t \quad (16)$$

where  $\Delta u$  is a taste shock. When (16) is estimated jointly with (13), the value of the demand elasticity decreases to 4.0 and the returns to scale

<sup>6</sup> See p. 266 in Abel and Blanchard (1986), table 5 p. 93 in Summers (1981), and p. 222 in Hayashi (1982).

parameter  $\theta$  is equal to 0.26 with a  $t$ -ratio of 1.29.<sup>7</sup> Although the point estimate suggests the possibility of increasing returns to scale, the evidence is weak and the hypothesis of constant returns cannot be rejected.<sup>8</sup>

If nonconstant returns to scale affect only the gross production function, then the term  $(\theta/2)((I_t/K_t) - b)^2(K_t/(1 - \delta)K_{t-1})$  should be added to the right hand side of (13) and the model is just identified, even without specifying a demand function. When implementing this model, however, the variability in the data was insufficient to yield reliable estimates of the returns to scale parameter together with the other structural coefficients. However, a Lagrange multiplier test of the significance of  $\theta$  (distributed  $\chi^2$  with one degree of freedom) was performed, yielding a value of 0.655. Hence, the hypothesis of constant returns in gross production cannot be rejected.<sup>9</sup>

<sup>7</sup> There are no relevant changes instead in the estimated values of the adjustment cost parameters.

<sup>8</sup> See Morrison (1988) and Chirinko (1989) for a similar procedure in the context of models based on parameterizations of both technology and demand, in which evidence is found in favor of increasing returns.

<sup>9</sup> The same conclusion is reached by calculating a Wald type of statistic based on testing the significance of the three additional regressors obtained by multiplying out the additional term described in the text. The test statistic (distributed  $\chi^2$  (3)) has a value of 6.42. Finally, these results are also confirmed by the joint estimation of the investment model together with a demand function, along the lines described above.



In table 2 we report the results of estimating equation (13) when there is monopolistic competition and there are adjustment costs in changing the level of employment.<sup>10</sup> We provide two sets of results. In column (1) we present estimates of (13) assuming that there is no interrelated component in adjustment costs; in column (2) we allow for interrelated costs and include the term specified in (15). In this latter case the model is nonlinear in variables and we only present estimates of the structural parameters. Note that in this case we have set the constant in the adjustment cost function for capital,  $b$ , equal to zero because it was insignificant. In column (1) the coefficients of the terms representing marginal adjustment costs for labor have the sign expected a priori and are significant, which is also true for the remaining parameters. In column (2) the interrelation coefficient  $s$  has an asymptotic  $t$ -ratio of 1.707, and the other parameters are again quite precisely determined. The fit of the equation increases, basically because the second term in (15) is highly correlated with the dependent variable. Again, the hypothesis of constant returns to scale affecting the net production function cannot be rejected when a demand equation is also estimated. The value of the parameter  $\theta$  is 0.155 with a  $t$ -ratio of 0.951 for the model with no interrelated adjustment costs, and 0.091 with a  $t$ -ratio of 0.969 for the interrelated costs model. The estimate of the elasticity of demand remains basically unchanged.<sup>11</sup>

It is instructive to discuss the value of the structural parameters implied by our estimates and to compare them with those obtained by other researchers. Allowing for labor as a quasi-fixed factor decreases the estimate of elasticity of demand from 20 in table 1 to 9.2 and 5.9 in

columns (1)–(2) of table 2.<sup>12</sup> This latter evidence implies a markup of price over marginal costs of 12.2%–20.3%. This is quite close to the range of estimates found by Morrison (1988) (11%–23%) and for two-digit manufacturing industries by Hall (1986, 1988).<sup>13</sup> The evaluation of the adjustment costs for capital is a delicate issue since they depend both upon the multiplicative parameter  $a$  and the constant  $b$ . The value of the constant is large enough in table 1 to imply unacceptable negative marginal costs of adjustment for capital, when calculated at sample values. With labor as a quasi-fixed factor but no interrelated adjustment costs, the estimated value of the constant decreases and marginal adjustment costs for capital become positive, on average, although very small (0.1% of output). In the most general model in which the constant is set equal to zero and interrelated adjustment costs are present, adjustment costs for investment are 1.7% of output or 4.2% of value added. This is larger than the figure of 0.7% of value added found by Shapiro (1986). The large value of the constant  $d$  in the adjustment cost function gives rise to negative marginal adjustment costs for employment, when interrelated adjustment costs are not allowed for, as in column (1) of table 2. However, when interrelatedness is permitted, as in column (2) of table 2, the sign is correct and we find the marginal adjustment costs for labor to be, on average, 4.6% of output or 11.5% of value added. As in Shapiro (1986) we find that marginal adjustment costs for labor are higher than for capital. However, our estimates of the former are much higher than the figure of 1.8% of value added found by Shapiro.

#### IV. Conclusions

In the theoretical section of this paper we have analyzed the implications of monopolistic compe-

<sup>10</sup> In both cases we used the average value of the quit rate,  $\gamma$ , for the period 1951–1980. The data were not available after 1980.

<sup>11</sup> Similar evidence emerges for the constant returns to scale hypothesis in the gross production function on the basis of the Wald statistics. The  $W$  test has a value of 5.678 for the model in column (1) and of 1.185 for the one in column (2) (with 6 and 7 degrees of freedom, respectively). Estimation of a demand equation along with the investment model leads to the same result.

<sup>12</sup> The assumption of a constant elasticity of demand is clearly an approximation that may or may not be adequate. The two diagnostic tests we have provided do not suggest any significant form of misspecification.

<sup>13</sup> In order to make our estimates comparable with Hall's, the markup ratios based on gross output must be converted to those based on value added, using the formula in Hall (1986), p. 294. Our markup in terms of value added is in the range of 28%–51%.

tion and labor adjustment costs for  $Q$  models of investment. This has allowed us to derive a generalized  $Q$  model for investment, emphasizing the relationship between investment, hiring, the average market value of the firm and output. Estimates of the adjustment costs and demand parameters can be recovered and the assumption of perfect competition, constant returns to scale, and absence of labor adjustment costs can be tested.

The overall conclusion that can be derived from the empirical results presented in this paper is that standard  $Q$  models can be considerably generalized by adopting different assumptions about the output market and the structure of adjustment costs. Monopolistic competition introduces output in the investment equation, in addition to  $Q$ , and provides a rationale for its frequently observed significance in empirical work. The results also suggest that it is important to treat labor as a quasi-fixed factor and to allow adjustment costs to be interrelated. Empirically, adjustment costs for labor appear to be quantitatively more important than those for capital. Finally, little evidence is found against the hypothesis of constant returns to scale.

Obviously, the model has limitations and can be extended in several directions. It would be useful, for example, to distinguish between different types of workers who may be characterized by different adjustment costs. Moreover, the model could be usefully estimated on more disaggregated data in order to allow demand and technology parameters to differ across industries. In future research we plan to address these and other issues. However, we have shown that quasi-fixity of labor and monopolistic competition are critical ingredients in the theoretical and empirical reformulation of generalized  $Q$  models.

## REFERENCES

- Abel, Andrew B., and Olivier Blanchard, "The Present Value of Profits and Cyclical Movements in Investment," *Econometrica* 54 (1986), 249–273.
- Berndt, Ernst, and David Wood, "U.S. Manufacturing Output and Factor Input Price and Quantity Series, 1908–1947 and 1947–1981," M.I.T. Energy Laboratory w.p. 86-010WP (1986).
- Breusch, Trevor, and Leslie Godfrey, "A Review of Recent Work on Testing Autocorrelation in Dynamic Economic Models," in D. Currie, A. R. Nobay and D. Peel (eds.), *Macroeconomic Analysis: Essays in Macroeconomics and Econometrics* (London: Groom Helm, 1981).
- Chirinko, Robert, "Investment, Tobin's  $Q$ , and Multiple Capital Inputs," Cornell University Working Paper No. 328 (1984).
- , "Tobin's  $Q$  and Financial Policy," *Journal of Monetary Economics* 19 (1987), 69–87.
- , "Non-convexities, Labor Hoarding, Technology Shocks, and Procyclical Productivity: A Structural Econometric Approach," paper presented at the 1988 NBER Summer Institute (1989).
- Chirinko, Robert, and Steven M. Fazzari, "Tobin's  $Q$ , Non-constant Returns to Scale, and Imperfectly Competitive Product Markets," *Recherches Economiques de Louvain* 54 (1988), 259–275.
- Fazzari, Steven, R. Glenn Hubbard, and Bruce C. Petersen, "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity* 1 (1988), 141–195.
- Hall, Robert E., "Market Structure and Macroeconomic Fluctuations," *Brookings Papers on Economic Activity* 2 (1986), 285–338.
- , "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy* 96 (1988), 921–947.
- Hayashi, Fumio, "Tobin's Average  $q$  and Marginal  $q$ : A Neoclassical Interpretation," *Econometrica* 50 (1982), 215–224.
- , "Corporate Finance Side of the  $Q$  Theory of Investment," *Journal of Public Economics* 27 (1985), 261–288.
- Hayashi, Fumio, and Tohru Inoue, "The Relationship of Firm Growth and  $Q$  with Multiple Capital Goods: Theory and Evidence from Panel Data on Japanese Firms," *Econometrica* 59 (1991), 731–735.
- Holland, David B., and Stewart C. Myers, "Trends in Corporate Profitability and Capital Costs in the United States," in D. Holland (ed.), *Measuring Profitability and Capital Costs* (Boston: Lexington Books, 1984).
- Lucas, Robert J., "Optimal Investment Policy and the Flexible Accelerator," *International Economic Review* 8 (1967), 78–85.
- McCallum, Bennett T., "Rational Expectations and the Estimation of Economic Models: An Alternative Procedure," *International Economic Review* 17 (1976), 484–490.
- McIntosh, James, "Dynamic Interrelated Factor Demand Systems: The United Kingdom 1950–1978," *Economic Journal* (Supplement, 1983), 79–86.
- Morrison, Catherine J., "Markups in U.S. and Japanese Manufacturing: A Short Run Econometric Analysis," National Bureau of Economic Research Working Paper No. 2799 (1988).
- Nadiri, M. Ishaq, and Sherwin Rosen, "Interrelated Factor Demand Functions," *American Economic Review* 59 (1969), 457–471.
- Pindyck, Robert S., and Julio J. Rotemberg, "Dynamic Factor Demands under Rational Expectations," *Scandinavian Journal of Economics* 85 (1983), 223–238.
- Poterba, James M., and Larry H. Summers, "Dividend Taxes, Corporate Investment, and  $Q$ ," *Journal of Public Economics* 22 (1983), 135–167.
- Prucha, Ingmar, and M. Ishaq Nadiri, "A Comparison of Alternative Methods for the Estimation of Dynamic Factor Demand Models under Non-static Expectations," *Journal of Econometrics* 33 (1986), 187–211.
- Sargan, J. Denis, "Wages and Prices in the U.K.," in P. G. Hart, G. Mills and J. K. Whittaker (eds.), *Econometric*

- Analysis for National Economic Planning* (London: Butterworth, 1964).
- Schiantarelli, Fabio, and Dimitri Georgoutsos, "Monopolistic Competition and the  $Q$  Theory of Investment," *European Economic Review* 35 (1990), 1061–1078.
- Shapiro, Matthew D., "The Dynamic Demand for Capital and Labor," *Quarterly Journal of Economics* 101 (1986), 513–542.
- \_\_\_\_\_, "Measuring Market Power in U.S. Industry," National Bureau of Economic Research Working Paper No. 2212 (1987).
- Summers, Larry H., "Taxation and Corporate Investment: A  $Q$  Theory Approach," *Brookings Papers on Economic Activity* 1 (1981), 67–160.
- Wildasin, David E., "The  $Q$  Theory of Investment with Many Capital Goods," *American Economic Review* 74 (1984), 203–210.