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# Multipliers for bounded convergent double sequences

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**Abstract.** In this paper, we investigate multipliers for bounded convergence of double sequences and study some properties and relations between $\ell_\infty^2$, $c^2(b)$ and $c_0^2(b)$.

**Keywords:** Double Sequences, Multiplier
**PACS:** 02.30.Lt

## INTRODUCTION

Hill [8] was the first who applied methods of functional analysis to double sequences. Also, Kull [10] applied methods of functional analysis of matrix maps of double sequences. A lot of useful developments of double sequences in summability methods can be seen in [1, 9, 12, 15].

The study of the multipliers of one sequence space into another is a well-established area of research and has been the object of several investigations over the last fifty years. Demirci and Orhan [3] studied the bounded multiplier space of all bounded $A$-statistically convergent sequences, and using the "$\beta N$ program" they gave an analogue of a result of Fridy and Miller [6] for bounded multipliers. Connor, Demirci and Orhan [2] studied multipliers and factorizations for bounded statistically convergent sequences and a related result. Dündar and Sever [5] studied multipliers for bounded statistical convergence of double sequences in $\mu_2$-density. Yardımcı [16] studied multipliers for bounded $\mathscr{I}$-convergent sequences. Also, Dündar and Altay [4] investigated analogous results of multipliers for bounded $\mathscr{I}_2$-convergent double sequences.

In this paper, we investigate multipliers for bounded convergence of double sequences and study some properties and relations between $\ell_\infty^2$, $c^2(b)$ and $c_0^2(b)$.

## DEFINITIONS AND NOTATIONS

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers while $\mathbb{R}$ represents the set of all real numbers.

Now, we recall the concepts of double sequence, Pringsheim's convergence, multiplier for bounded convergence of the double sequences [1, 4, 7, 8, 11, 13, 14].

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be convergent to $L \in \mathbb{R}$ if for any $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbb{N}$ such that

$$|x_{mn} - L| < \varepsilon,$$

whenever $m, n > N_\varepsilon$. In this case we write

$$\lim_{m,n\to\infty} x_{mn} = L.$$

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be bounded if there exists a positive real number $M$ such that

$$|x_{mn}| < M,$$

for all $m, n \in \mathbb{N}$, that is

$$\|x\|_\infty = \sup_{m,n} |x_{mn}| < \infty.$$

Note that in contrast to the case for single sequences, a convergent double sequence need not be bounded.

By $\ell_\infty^2$, $c^2(b)$ and $c_0^2(b)$, we denote the spaces of all bounded, bounded convergent and bounded null double sequences, respectively.

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Let $E$ and $F$ be two double sequence spaces. A multiplier from $E$ into $F$ is a sequence $u = (u_{mn})_{m,n\in\mathbb{N}}$ such that

$$ux = (u_{mn}x_{mn}) \in F,$$

whenever $x = (x_{mn})_{m,n\in\mathbb{N}} \in E$. The linear space of all such multipliers will be denoted by $m(E,F)$.

If $E = F$, then we write $m(E)$ instead of $m(E,F)$.

Now we begin with quoting the lemmas due to Dündar and Altay [4] which are needed throughout the paper.

**Lemma 1.** *[4, Theorem 3.2] If E and F are subspaces of $\ell^2_\infty$ that contain $c^2_0(b)$, then*

$$c^2_0(b) \subset m(E,F) \subset \ell^2_\infty.$$

**Lemma 2.** *[4, Lemma 3.4]* $m\big(c^2_0(b)\big) = \ell^2_\infty$.

## MAIN RESULTS

In this section, we deal with the multipliers on or into $\ell^2_\infty$, $c^2(b)$ and $c^2_0(b)$.

**Theorem 3.** $m(\ell^2_\infty) = \ell^2_\infty$.

*Proof.* Let $u = (u_{mn}), x = (x_{mn}) \in \ell^2_\infty$. Then, we have

$$\|u\|_\infty = \sup_{m,n} |u_{mn}| < \infty,$$

$$\|x\|_\infty = \sup_{m,n} |x_{mn}| < \infty.$$

Now, let $z = ux$. Then, we have

$$\|z\|_\infty = \sup_{m,n} |z_{mn}| = \sup_{m,n} |u_{mn}x_{mn}| \leq \sup_{m,n} |u_{mn}| \sup_{m,n} |x_{mn}| < \infty$$

and so $u \in m(\ell^2_\infty)$. This implies that

$$\ell^2_\infty \subset m(\ell^2_\infty).$$

Conversely, since $e \in \ell^2_\infty$ ($e$ is the sequence of all 1's), we have

$$m(\ell^2_\infty) \subset \ell^2_\infty.$$

This completes the proof of the theorem. □

**Theorem 4.** $m\big(\ell^2_\infty, c^2_0(b)\big) = c^2_0(b)$.

*Proof.* Let $u \in c^2_0(b)$ and $\theta \neq x \in \ell^2_\infty$. Then, we have

$$\|x\|_\infty = \sup_{m,n\in\mathbb{N}} |x_{mn}| < \infty,$$
$$\|u\|_\infty = \sup_{m,n\in\mathbb{N}} |u_{mn}| < \infty$$

and for $\varepsilon > 0$ there exists $N = N(\varepsilon) \in \mathbb{N}$ such that

$$|u_{mn}| < \frac{\varepsilon}{\|x\|_\infty}$$

for every $m,n > N$. Let $z = xu$. Then, we have

$$\|z\|_\infty = \sup_{m,n\in\mathbb{N}} |z_{mn}| = \sup_{m,n\in\mathbb{N}} |x_{mn}u_{mn}| \leq \sup_{m,n\in\mathbb{N}} |x_{mn}| \sup_{m,n\in\mathbb{N}} |u_{mn}| < \infty,$$



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so $z$ is bounded and

$$|x_{mn}u_{mn}| = |x_{mn}||u_{mn}| < \|x\|_\infty \frac{\varepsilon}{\|x\|_\infty} = \varepsilon$$

for $m,n > N$. Hence, we have $z \in c_0^2(b)$. This shows that

$$c_0^2(b) \subset m\big(\ell_\infty^2, c_0^2(b)\big).$$

Now, since $e \in \ell_\infty^2$ we have

$$m\big(\ell_\infty^2, c_0^2(b)\big) \subset c_0^2(b).$$

This completes the proof of the theorem. □

**Theorem 5.** $m\big(c_0^2(b), \ell_\infty^2\big) = \ell_\infty^2$.

*Proof.* Since $c_0^2(b) \subset \ell_\infty^2$ then by Theorem 3 we have

$$m\big(c_0^2(b), \ell_\infty^2\big) \subset \ell_\infty^2.$$

Now, let $u \in \ell_\infty^2$ and $x \in c_0^2(b)$. Then, it is clear that

$$ux \in \ell_\infty^2$$

and so

$$\ell_\infty^2 \subset m\big(c_0^2(b), \ell_\infty^2\big).$$

Hence, we have $m\big(c_0^2(b), \ell_\infty^2\big) = \ell_\infty^2$. □

**Theorem 6.** $m\big(c^2(b), \ell_\infty^2\big) = \ell_\infty^2$.

*Proof.* Since $c^2(b) \subset \ell_\infty^2$ then by Theorem 3 we have

$$m\big(c^2(b), \ell_\infty^2\big) \subset \ell_\infty^2.$$

Now, let $u \in \ell_\infty^2$ and $x \in c^2(b) \subset \ell_\infty^2$. Then, we have

$$ux \in \ell_\infty^2$$

and so

$$\ell_\infty^2 \subset m\big(c^2(b), \ell_\infty^2\big).$$

This completes the proof of the theorem. □

**Theorem 7.** $m(c^2(b)) = c^2(b)$.

*Proof.* Let $e = (1) \in c^2(b)$. Then, we have

$$ue = u \in c^2(b)$$

for each $u \in m(c^2(b))$ and so

$$m(c^2(b)) \subset c^2(b).$$

Now, let $u \notin c^2(b)$. Since $e \in c^2(b)$, then we have

$$ue = u \notin c^2(b)$$

so $c^2(b) \subset m(c^2(b))$. □

**Theorem 8.** $m\big(c^2(b), c_0^2(b)\big) = c_0^2(b)$.



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*Proof.* Let $u \in c_0^2(b)$ and $e \in c^2(b)$. Then, we have

$$ue = u \in c_0^2(b)$$

and so

$$c_0^2(b) \subset m\big(c^2(b), c_0^2(b)\big).$$

Let $u \notin c_0^2(b)$. Since $e \in c^2(b)$ then,

$$ue = u \notin c_0^2(b)$$

and so

$$u \notin m\big(c^2(b), c_0^2(b)\big).$$

Hence, we have

$$m\big(c^2(b), c_0^2(b)\big) \subset c_0^2(b).$$

This completes the proof of the theorem. □

**Theorem 9.** $m\big(c_0^2(b), c^2(b)\big) = \ell_\infty^2$.

*Proof.* Since $c_0^2(b) \subset \ell_\infty^2$ and $c^2(b) \subset \ell_\infty^2$, by Lemma 1

$$m\big(c_0^2(b), c^2(b)\big) \subset \ell_\infty^2.$$

Conversely, since $c_0^2(b) \subset c^2(b)$, by Lemma 2

$$\ell_\infty^2 \subset m\big(c_0^2(b), c^2(b)\big).$$

Therefore, we have

$$m\big(c_0^2(b), c^2(b)\big) = \ell_\infty^2.$$

□

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