MODELADO CINEMÁTICO Y DINÁMICO DE UN DISPOSITIVO ASISTENCIAL DE 5 BARRAS PARA REHABILITACIÓN DE RODILLA

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Kinematic and Dynamic Modeling of a 5-Bar Assistive Device for Knee Rehabilitation

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***Abstract*—Knee injuries are frequent in people of all ages. In all cases, physical therapy is prescribed to recover strength, and range of motion. Robotic assistive devices are gaining the attention of the community and aim to improve the patients' quality of life. In this paper, we propose a 5-bars-linkage knee rehabilitation device. We are interested in obtaining the complete dynamic model of the proposed rehabilitation system, in order to develop and evaluate adequate control strategies in future work. With this purpose, we present the kinematics formulation of the device and then, we derive the dynamics using two approaches, in order to validate the model; i.e. we obtain the motion equation using Lagrange approach and an algebraic method which simplifies the modeling. These models are simulated and compared with the physical behavior of the system, showing the functionality of the system and the validity of the models when performing a rehabilitation routine.**

***Index Terms*—Robotics assistive rehabilitation device, kinematics, dynamics modeling**

I. Introduction

Knee injuries are common in people of all ages. The main causes are muscular atrophy due to aging, damage induced by exercise, labor accidents, and ignoring ergonomic principles during work [19], [9]. Physical therapy is prescribed to recover joint functions that have been lost or diminished by the injuries [9], [13]. Physical rehabilitation usually takes several weeks or even months until full range of motion and joint flexibility are recovered. However, satisfactory results are reached only if the patient performs the exercises with regularity. Usually, this process involves a series of repeated physical movements assisted by physiotherapists [9]. Regarding physiotherapy, there has been an increasing interest in developing assistive devices that can be used for rehabilitation with the purpose of improving the patients and the therapists quality of life [9], [7].

In literature, there are already several robotic devices for assisted motion and with rehabilitation purposes that are used for upper limb and off course for lower limbs either. In general we find Exoskeletons (see e.g. [5], [14]), orthoses (see e.g. [11], [15]) and other devices (see e.g. [18]) that have been developed for upper and lower limb rehabilitation, with different focus. For instance, a one-degree-of-freedom assistive platform to augment the strength of upper limbs with variable stiffness actuation is presented in [10]. Furthermore, a robotic device for knee rehabilitation therapy is introduced in [9]; this device is oriented to improve patellar mobility with feedback for the patient and therapist. The difference among this approach and ours is that the device that we present is modular and uses soft actuation, which provides several advantages such as low energy consumption, natural motions and safety [20], [6].

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On the other hand, the design and manufacturing of a gait rehabilitation robot, which consists in a robotic orthosis for treadmill training, is reported in [13]. In the mentioned work, authors define some important criteria for the design such as low inertia of robot components, backdrivability, and high safety, which they accomplish with the presented robot, and that we also take into account in our design. However, this robot is different from ours because the first aims to recover patients normal walking gait, while our design is oriented to repetitive routines for recovering strength and mobility range. Regarding the current state of the art, other lower limb rehabilitation devices are presented in [11] or [21].

Control systems are highly important for accomplishing properly the rehabilitation routines carried out using assistive devices. In [17], we have already proposed a general control structure, based on a single pendulum dynamic model approach, that would serve as a basis for controlling the designed rehabilitation device. However, the complete model of the structure is required to design, enhance and adjust a controller for this rehabilitation device. Doing so will guarantee the proper execution of the rehabilitation routines and will prevent damages to the patient due to undesired behaviors. In order to control and command a robotic device, it is mandatory to formulate properly its kinematic and dynamic models.

In this paper, we present the kinematic and dynamic modeling of a five-bar-linkage assistive device for knee rehabilitation. The 5-bar configuration was chosen so it could be used in lying and sitting position, in addition, the movement of this kind of mechanism due to the efforts generated by the action of the actuators minimizes the risk of changing the anatomical and functional integrity of the patient and above all to avoid harming the patient. This is a novel system that can be reconfigured to attend a wide range of patients according to their height. We use soft actuation to help motion at the knee joint, provided the aforementioned advantages of these actuators. Moreover, the actuators are not directly placed on the knee joint, to prevent unwanted loading. A proof of concept of the proposed rehabilitation system has already been reported in [12]. In this paper, we present some improvements to the first design and the results obtained correspond to the model that is currently under construction. The main difference here is the possibility of attending a width range of patients, considering their height. This consideration introduces a new variable which is taken into account in the kinematic formulation presented.

First, we define and present the mechanical design of the device; the physical parameters are derived from anthropomorphic data [8], [4]. Then, we carry out the kinematic modeling of the five-bar-linkage assistive device, based on rigid body mechanics [16]. Furthermore, we derive the dynamic equations of the device, using two different formulation approaches, i.e.

Langrange formulation [16], and an algebraic approach [22]. In this way we test the functionality of the system and validate the model, showing that the behavior of the model obtained with the two approaches is the same, and corresponds to the dynamic simulation performed in Solidworks.

II. Mechanical Design Formulation and Parameters

The knee joint has two degrees of freedom (DoF), and performs movements in two perpendicular planes, i.e. flexo-extension in the sagittal plane (frontal axis), and internal-external rotation in the frontal plane (vertical axis). In the proposed design of the assitive device, only flexo-extension movements will be considered [2], because the knee joint is the most frequently injured and in terms of mobility, the one considered is the most affected DoF. Moreover, the assisted physiotherapy is mostly focused on flexo-extension movements [19], e.g. consider the rehabilitation of injuries related to the anterior cruciate ligament (LCA).

Knee flexion reaches on average 130°, considering 0° when the leg is completely extended. The maximum limit of amplitude is greater, when the motion is assisted. In general, for the knee joint, the ranges of motion considered normal are: flexion from 130° to 140°; internal rotation from 30°; and External rotation: 40° [1], [2]. In our system, as we defined previously, we just consider the range of flexion. Flexio-extension movements of the knee also involve the hip motion, so common rehabilitation exercises include raising the entire leg, therefore a 2-DoF device is necessary.

As the device will be used to assist the motion of the leg, to improve the pathological condition of the knee, the mechanical analysis is done in terms of the physical (anthropomorphic) characteristics of the users. The main parameters that are involved in the performance of the physical therapy are the mass (in Kg) and the height (in m). Both parameters are variable according to the subject and will be taken into account in the design and analysis. It is worth to mention that the reconfigurability of the device is done for patients heights 1.40 m $< h <$ 1.90 m according to mean population data [8]. Similarly, we consider normal weights (i.e. not overweight nor underweight) according to body mass index (BMI)[1]. According to the anthropomorphic data and body proportions [4], the patient's height determines the lengths of the thigh and the leg. In this way, these two lengths will determine the mechanical design of the structure and therefore will be of great importance for the calculations of the kinematic and dynamic model. To carry out the physiotherapy routines, the joints motions are constrained to the allowed normal ranges mentioned before. These ranges of motion are included in the physical therapy protocols that physicists and physiotherapists establish for treating their patients. In general, these protocols may change according to the health center or the professional[2]. We have taken into account several routines, for knee flexion/extension and the corresponding ranges of motion of the hip and knee joints as well. As mentioned before, the system is reconfigurable according to a range of patients' height and weight, which are the design parameters presented in Table I. These parameters determine the constraints to the construction framework of the device.

[1]Consider normal as $20 < BMI < 25$, $BMI = mass/height^2$

[2]We have based our analysis in the protocol of the orthopedics department of the Central Military Hospital-Bogota.

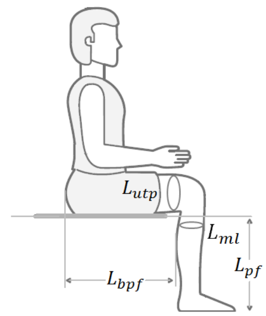


Fig. 1: Layout of physical parameters: L_{utp} is the upper thigh perimeter,L_{ml} is the middle leg perimeter, L_{bpf} is the length of the buttock popliteal fossa, and L_{pf} is the height of the popliteal fossa.

TABLE I: Physical parameters of design restricted to device and focused on the population group where knee injuries are more frequent

Dimensional physical parameters	Functional physical parameters
Population variation of thigh length between 0.416 to 0.513 m minimum	Flexo-extension of the thigh in the sagittal plane: 0° when the patient is lying down with the joints extended to 90°±10° in flexion
Population variation of leg length between 0.342 to 0.465 m minimum	Flexo-extension of the leg in the sagittal plane: 0° when the patient is lying down with the articulations extended to 130°± +10° in flexion
Thigh width variation between 0.473 to 0.639 m minimum	Width adjustment of variation of the thigh according to the patient
Leg width variation between 0.30 to 0.408 m minimum	Adjustment of leg width according to the patient
	Device for patients with max. body weight $89.9 < W_M <$ 120 Kg and max. height between 1.811 and 1.90 m.

Refer to Fig. 1, where the physical parameters of the leg and the variables involved are defined. The mass of the lower limb is required and can be calculated from the whole body mass M_b (in Kg). According to anatomy proportions in [8], the lower limb mass is $M_b/7$. Moreover, the mass of the lower limb segments can be experimentally determined as

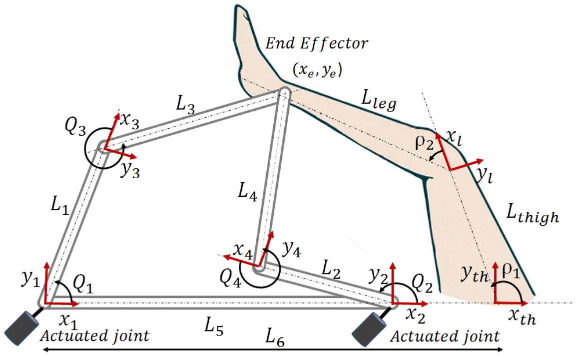
$$M_t = 0.1032M_b + 12.76L_{bpf}\ L_{utp}^2 - 1.023\,, \quad (1)$$

and the leg mass M_l is obtained as

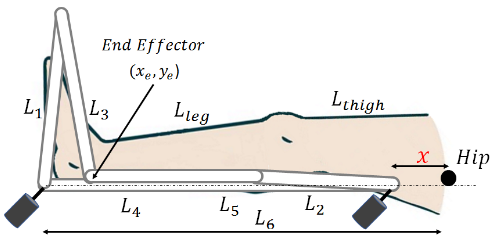
$$M_l = 0.0226M_b + L_{pf}L_{ml}^2 - 0.016\,. \quad (2)$$

where $M_t + M_l$ corresponds approximately to $M_b/7$.

The five-bar mechanism for knee rehabilitation, with the corresponding coordinate notation is shown in Fig. 2. Where L_1, L_2, L_3, L_4, L_5, are the lengths of the system links which are fixed; L_{thigh} and L_{leg} are the lengths of the segments of the patient's leg. Instead, L_6 is a variable length, which makes the system reconfigurable. Notice that L_{thigh} and L_{leg}



(a) Five-bar linkage rehabilitation device. Kinematic definitions used in the derived model.



(b) Five-bar linkage Rehabilitation device.

Fig. 2: Five-bar linkage Rehabilitation device parameters, representation of angular positions and lengths.

are variable values according to each user. To allow the adjustment and reconfigurability of the system to the defined range of patients' heights, and therefore to take into account the variability of L_{thigh} and L_{leg}, let us refer to Table I. Now, the mathematical model includes an additional variable, namely x, associated with the placement of the hip in the device. Then, this variable length can be calculated as

$$L_6 = x + L_{thigh} + L_{leg}\,, \tag{3}$$

where x is defined when the patient has the leg relaxed horizontally, thus establishing a reference point and avoiding mechanical singularities that may occur during the motion.

As shown in Fig. 2, $Q = [Q_1, Q_2, Q_3, Q_4]$ is the vector of angular positions of each joint in the designed structure. These angular positions are determined using the Denavit-Hartenberg convention [16], where Q_1 is the angle between the links L_1 and L_5, Q_2 is the angle between the links L_2 and L_5, Q_3 is the angle between the projection of the link L_1 and L_3, and finally Q_4 is the angle between the projection of the link L_2 and L_4. The rotations counterclockwise are positive, according to the convention. Here, the actuated joints are Q_1 and Q_2, that connect links L_1 and L_2 respectively with L_5.

III. Kinematic Formulation

In this section we will focus on deriving the kinematics model for the five-bar rehabilitation system shown in Fig.2. Also, an algebraic formulation is made using as a basis of analysis the theory of rigid bodies and kinematics of rigid bodies [3]. On the basis of a closed chain mechanism, analyzing the vectorial components of each link, the kinematics model of the system is defined by

$$L_1C_1 + L_3C_{13} - L_5 - L_2C_2 - L_4C_{24} = 0\,, \tag{4}$$

$$L_1S_1 + L_3S_{13} - L_2S_2 - L_4S_{24} = 0\,. \tag{5}$$

Where we define $C_i = \cos(Qi)$ and $S_i = \sin(Qi)$ to simplify notation. Additionally, ρ_1 and ρ_2 are the angular positions of the hip and knee. These two angles will determine the position of the end effector, i.e. the foot, as well as the angular positions of each joint of the mechanism.

A. Cartesian position of the end effector

We calculate the point $(X_e,\, Y_e)$ which represents the end effector Cartesian position from the variables ρ_1, ρ_2, L_{thigh} and L_{leg}, where

$$\begin{aligned} X_e &= L_{thigh}C_{\rho_1} + L_{leg}C_{\rho_1\rho_2} + L_6\,, \\ Y_e &= L_{thigh}S_{\rho_1} + L_{leg}S_{\rho_1\rho_2} \end{aligned} \tag{6}$$

Based on the Standardized Anthropometric Technique [4], we can obtain the the lengths of the leg as $L_{thigh} = L_{bpf}$ and $L_{leg} = L_{pf}$. The angular positions of the knee and hip, namely ρ_1 and ρ_2 can be obtained with an angular measurement instrument. For instance, in the field of the ostoarticular system, the most popular instrument is an analog goniometer. Consider that the coordinates obtained are attached to a point associated to the heel, which represents the starting point of the end effector (foot).

B. Angular position vector Q

The vector of angular positions $Q = [Q_1, Q_2, Q_3, Q_4]$ is obtained from X_e and Y_e. The joints of the angles Q_1 and Q_2 are actuated, this means that the velocities of these actuated joints will be independent, and that the motion of the other joints (Q_3, Q_4, End Effector) will depend on the whole motion of the independent joints. The angular positions described in Fig. 2 can be calculated by dividing the mechanism into two open chains where the point in common will be the final effector, then we have the expressions that describe these chains, for the first open kinematic chain:

$$\begin{aligned} X_e &= L_1C_1 + L_3C_{13}\,, \\ Y_e &= L_1S_1 + L_3S_{13}\,, \end{aligned} \tag{7}$$

For the second open kinematic chain:

$$\begin{aligned} X_e &= L_5 + L_2C_2 + L_4C_{24} \\ Y_e &= L_2S_2 + L_4S_{24} \end{aligned} \tag{8}$$

From (7), (8), after some algebra we can define

$$\begin{aligned} Q_1 &= 2tan^{-1}\left(\frac{2L_1Y_e + \alpha}{L_1^2 + 2L_1X_e - L_3^2 + X_e^2 + Y_e^2}\right), \\ Q_2 &= 2tan^{-1}\left(\frac{2L_2Y_e + \gamma}{L_2^2 + 2L_2P_a - L_4^2 + P_a^2 + Y_e^2}\right), \\ Q_3 &= -2tan^{-1}\left(\frac{\alpha\sqrt{\beta}}{\beta}\right), \\ Q_4 &= -2tan^{-1}\left(\frac{\gamma\sqrt{\delta}}{\delta}\right), \end{aligned} \tag{9}$$

where
$\alpha = \sqrt{\beta(L_1^2 + 2L_1L_3) + L_3^2 - X_e^2 - Y_e^2)}$,
$\beta = -L_1^2 + 2L_1L_3 - L_3^2 + X_e^2 + Y_e^2$,
$\gamma = \sqrt{\delta(L_2^2 + 2L_2L_4) + L_4^2 - P_a^2 - Y_e^2)}$,
$\delta = -L_2^2 + 2L_2L_4 - L_4^2 + P_a^2 + Y_e^2$, and
$P_a = X_e - L_5$.

C. Velocity components of each link

The centroid velocities $\dot{X}_i$ and $\dot{Y}_i$ for links $i = 1, 2, 3, 4$, are calculated from the centroid positions of the $i - th$ link L_{c_i}, assuming for symmetry that it is located in the middle of the link,

$$\begin{aligned} X_1 &= L_{c1}C_1 & Y_1 &= L_{c1}S_1 \\ X_2 &= L_{c2}C_2 & Y_2 &= L_{c2}S_2 \\ X_3 &= L_{c3}C_{13} + L_1C_1 & Y_3 &= L_{c3}S_{13} + L_1S_1 \\ X_4 &= L_{c4}C_{24} + L_2C_2 & Y_4 &= L_{c4}S_{24} + L_2S_2. \end{aligned}$$

Then, the differential kinematics are defined by the velocity components of each link, which are obtained from the centroid coordinates of each link, and can be written as

$$\begin{aligned} \dot{X}_1 &= -L_{c1}S_1\dot{Q}_1\,, & \dot{Y}_1 &= L_{c1}C_1\dot{Q}_1 \\ \dot{X}_2 &= -L_{c2}S_2\dot{Q}_2\,, & \dot{Y}_2 &= L_{c2}C_2\dot{Q}_2 \\ \dot{X}_3 &= -L_{c3}S_{13}(\dot{Q}_1 + \dot{Q}_3) - L_1S_1\dot{Q}_1\,, \\ \dot{Y}_3 &= L_{c3}C_{13}(\dot{Q}_1 + \dot{Q}_3) + L_1C_1\dot{Q}_1 \\ \dot{X}_4 &= -L_{c4}S_{24}(\dot{Q}_2 + \dot{Q}_4) - L_2S_2\dot{Q}_2\,, \\ \dot{Y}_4 &= L_{c4}C_{24}(\dot{Q}_2 + \dot{Q}_4) + L_2C_2\dot{Q}_2\,. \end{aligned}$$

These terms will be useful to derive the dynamic equations in the next section.

IV. Dynamic Formulation

In this section we show two ways of deriving the dynamic model of the five-bar linkage device. A first approach considers the Lagrangian formulation to obtain the dynamic equations. Alternatively, we use another modeling approach based on Lagragian formulation, but using an algebraic method. At the end we show that both methods yield a unique valid model.

A. First Formulation Method: Lagrangian approach

The general equations of motion of a mechanical linkage system can be obtained from Lagrangian equations [16]. The application of Lagrange mechanics yields to differential equations corresponding to the generalized coordinates Q_i. This method deals with kinetic (K) and potential (P) energies that are scalar quantities, defined as

$$K = \frac{1}{2}\sum_{k=1}^{4}\left[I_i\dot{Q}_i^2 + m_i(\dot{X}_i^2 + \dot{Y}_i^2)\right]\,, \tag{10}$$

$$P = \frac{1}{2}\sum_{k=1}^{4} m_i g Y_i\,. \tag{11}$$

Where I_i is the inertia of the i_{th}-link, m_i the mass of the i_{th}-link; X_i and Y_i are the horizontal and vertical components of the i_{th}-link centroid position, respectively, and g is the acceleration due to gravity.

The Lagrangian function is defined as $L = K - P$. The generalized torques $\tau = [\tau_1 \;\; \tau_2]^T$ are the actuated joints torques, associated with the generalized coordinates Q, which in this case correspond to the actuated joints. Then, according to the Lagrangian formulation, the dynamic equations are obtained from

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_i}\right) - \frac{\partial K}{\partial Q_i} + \frac{\partial P}{\partial Q_i} = \tau_i \tag{12}$$

Let us define the partial derivatives of the kinetic energy (dK_i) and potential energy (dP_i) w.r.t the generalized coordinates (Q_i), for $i = 1, 2$ which correspond to the actuated joints, as

$$\begin{aligned} dK_1 &= L_1L_{c4}M_3\dot{Q}_1\dot{Q}_4S_{2-14} - L_{c3}L_{c4}M_4\dot{Q}_2\dot{Q}_3S_\phi \\ &\quad - L_{c3}L_{c4}\dot{Q}_3\dot{Q}_4S_\phi(M_3 + M_4) - L_{c3}L_{c4}M_3\dot{Q}_1\dot{Q}_4S_\phi \\ &\quad - L_2L_{c3}M_4\dot{Q}_2\dot{Q}_3S_{13-4}\,, \\ dK_2 &= L_{c3}L_{c4}M_3\dot{Q}_1\dot{Q}_4S_\phi - L_2L_{c4}M_4\dot{Q}_2^2S_2 \\ &\quad + L_{c3}L_{c4}M_4\dot{Q}_3S_\phi(M_4\dot{Q}_2 + M_3\dot{Q}_4) + L_{c3}L_{c4}M_4\dot{Q}_3\dot{Q}_4S_\phi \\ &\quad - L_1L_{c4}M_3\dot{Q}_1\dot{Q}_4S_{2-14} - L_2L_{c4}M_4\dot{Q}_2\dot{Q}_4S_2\,, \\ dP_1 &= gM_1L_{c1}C_1 + M_3L_{c3}C_{13}Q_1 + L_1C_1\,, \\ dP_2 &= g(M_1L_{c2}C_2 + M_4L_{c4}C_{24}Q_2 + L_2C_2)\,. \end{aligned}$$

Then, from (12), we derive the vector of generalized torques of the actuated joints, corresponding to the equations of motion of the five-bar-linkage rehabilitation device, as

$$I_1\ddot{Q}_1 - dK_1 + dP_1 = \tau_1\,, \tag{13}$$

$$I_2\ddot{Q}_2 - dK_2 + dP_2 = \tau_2\,, \tag{14}$$

B. Second Formulation Method: Algebraic approach

In this section, we use another modeling based on Lagrange formulation by algebraic method that simplifies the dynamic model derivation. This algebraic method is based on the development of the model for hybrid machines (HMs), and an approximate dynamic model of a 5-bar mechanism proposed in [22]. Let us define the generalized inertia matrix D in terms of the angular and linear velocities; the vector of gravity torque from the partial derivative of the potential energy P w.r.t the generalized coordinates Q, i.e. The vector of gravity torque $G = \frac{\partial P}{\partial Q}$ and the kinetic energy $K = \frac{1}{2}\dot{Q}^TD\dot{Q}$. where Q, $\dot{Q}$ are the vectors of angular positions and angular velocities, respectively, obtained in section III-B.

Considering the definitions of dK_i given previously, the generalized torques can be written as

$$D\ddot{Q}_1 + \dot{D}\dot{Q}_1 - dK_1 + G_1 = \tau_1\,, \tag{15}$$

$$D\ddot{Q}_2 + \dot{D}\dot{Q}_2 - dK_2 + G_2 = \tau_2 \tag{16}$$

The inclusion of the variables D and G respond to a variant formulation of the dynamics from the Lagrangian formulation where the inertia of the motor armature, the load and the links are included, as well as the effects of the centripetal torque and gravity torque.

V. Model Validation and Results

We have performed several simulations to test and validate the kinematic and dynamic model of the mechanism obtained in the previous sections. For this, we compare the torque obtained with both methods. For the simulation, we consider a person of height h = 1.70 m, so L_{thigh} = 0.458 m and L_{leg} = 0.386 m. The numerical values for the link parameters used in the simulations are shown in Table I. To validate the calculations and the model obtained, we compare the results of the simulations done in Matlab with the dynamic simulation of the real system carried out in Solidworks, as shown in Fig. 3. The CAD model parameters that define the mechanical structure are presented in Table II. We define a desired motion in which the leg has to be completely extended. This is that the desired hip angular position is $116° < \rho_1 < 180°$, and the desired knee angular position is fixed $\rho_2 = 0°$. Fig. 4.a shows the kinematic and dynamic behavior of the mechanism for this

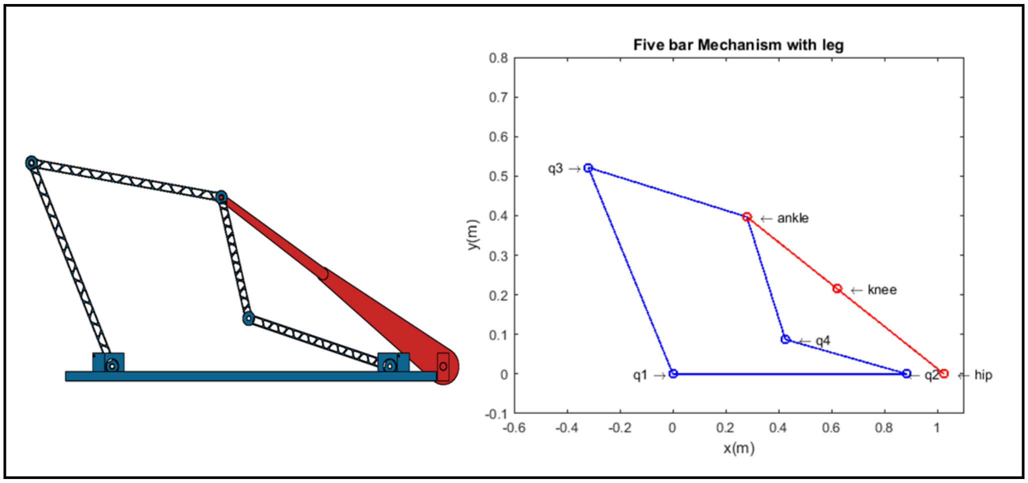
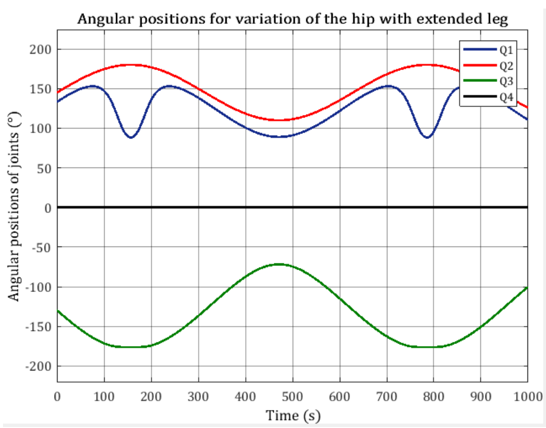
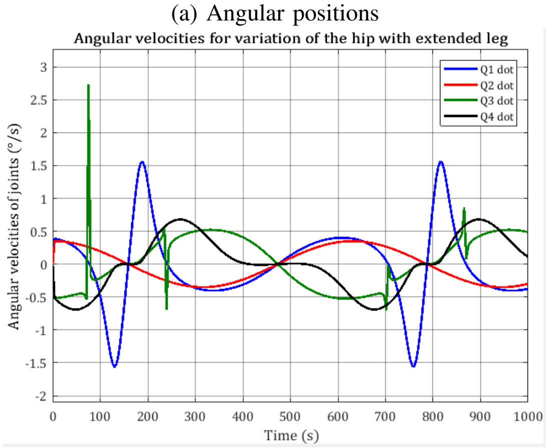


Fig. 3: Simulation environments compared: left, the device simulated in Solidworks; right, the simulated system in Matlab.



(a) Angular positions



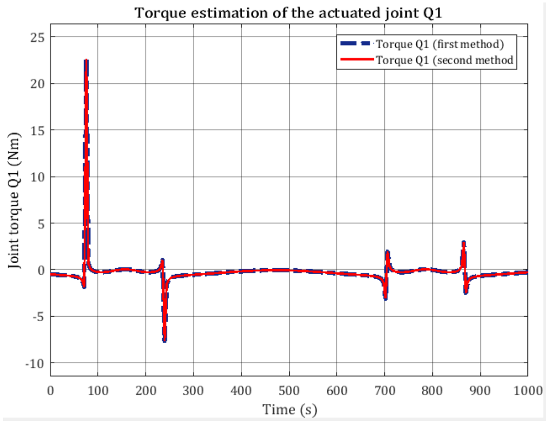
(b) Angular velocities

Fig. 4: Angular positions and velocities with extended leg: the hip angular desired position ρ_1 changes as $116° < \rho_1 < 180°$, and the knee angular desired position is $\rho_2 = 0°$.

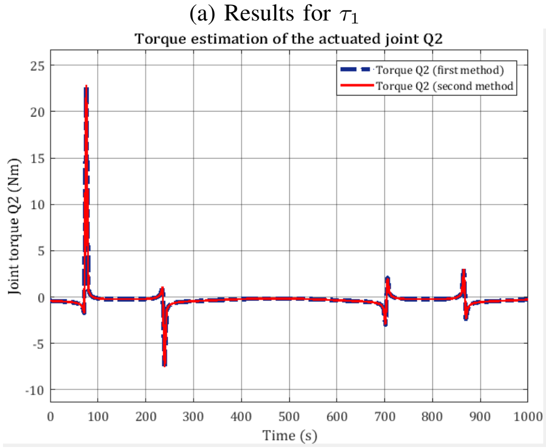
TABLE II: Estimated parameters of the mechanism

Link i	$m_i(Kg)$	$L_i(m)$	$L_{ci}(m)$	$J_i(x10^{-2}Kgm^2)$
1	0.06877	0.61190	0.3060	0.30522
2	0.05575	0.46790	0.2339	0.15516
3	0.06877	0.61190	0.3060	0.30522
4	0.04765	0.34190	0.1709	0.074428

desired motion. We observe the joints angular position Q_1, Q_2, Q_3, and Q_4, that are required to accomplish the motion. Notice that the joint Q_4 is always 0 through time; this happens because the leg moves without changing the angle of the knee and shares the same point of the joint Q_2. If the knee angular position does not change, neither the angular position of joint Q_4. The angular velocities of the joints are shown in Fig. 4.b. Notice that $\dot{Q}_3$ has an overshoot between 70 and 80 seconds approximately. This happens when the link L_3 is parallel to the link L_4 regardless its orientation in the coordinate frame- In that moment, the speed generated is maximum due to the compensation required when a transition of abrupt load occurs, since the system instantly becomes a 3-bar system with a fixed bar. The highest overshoot is generated when the leg is carried down, due to the forces that interact, i.e. the gravity and the weight of the mechanism and the leg. However, this behavior does not occur when lifting the leg, since speed when rising the leg is diminished by the counter-effort made by the motors. The angular velocities of the other joints remain within an accepted range that guarantees that at any time, the joints will not suffer any abrupt change. To verify the dynamic model, we compare the results obtained by applying both approaches presented in section IV. The normalized torques of the actuated joints Q_1 and Q_2 are shown in Fig. 5.a, and 5.b, respectively. As mentioned above, due to the abrupt change in the angular velocity $\dot{Q}_3$, a maximum effort is produced, so the actuators compensate with an opposite action that prevents the system from collapsing. Furthermore, we have recalculated the lengths of the links to ensure that there are no singularities; these lengths are $L_1 = 0.61$ m, $L_2 = 0.47$ m, $L_3 = 0.61$ m, $L_4 = 0.34$ m, $L_{thigh} = 0.46$ m, and $L_{leg} = 0.39$ m, constant at all times. Notice that these values correspond to values in Table II as expected.



(a) Results for τ_1



(b) Results for τ_2

Fig. 5: Comparison of formulation methods.

VI. CONCLUSION

In this paper, we have derived the kinematic and dynamic model of a 5-bars-linkage knee rehabilitation device. These models are the key for developing adequate control strategies, which will be carried out in future work. The dynamics is analyzed using two approaches. First, by applying the Lagrange formulation, and then by using an algebraic method which has simplified the calculations. Both models were simulated using Matlab showing the convergence of both approaches. Moreover, we compared these results with the physical simulation of the system carried out in Solidworks, showing the functionality of the system and the validity of the models when performing a rehabilitation routine. All of the parameters and constraints that define our device have been obtained from anthropomorphic data and based on specific rehabilitation routines of flexion and extension of the knee joint in order to recover strength and mobility of this joint. Future steps consist on designing and testing the control strategies in the real device on the basis of the modeling presented and on the rehabilitation routines.

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