

**MODELADO CINEMÁTICO, DINÁMICO Y VALIDACIÓN DE UN DISPOSITIVO ROBÓTICO
ASISTENCIAL PARA REHABILITACIÓN DE RODILLA**



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Kinematic and Dynamic Modeling and Validation of an Assistive Robotic Device for Knee Rehabilitation

Modelado Cinemático, Dinámico y Validación de un Dispositivo Robótico Asistencial para Rehabilitación de Rodilla

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Abstract

The knee joint is frequently exposed to injuries in people of all ages. In all cases, physical therapy is prescribed to recover the strength and mobility of a patient. The robotic assistance devices are gaining the community attention and aim to improve the quality of life of patients. In this article, we propose the mechanical design of a 5-bar-linkage knee rehabilitation device based on the definition of the physical parameters of Colombian and/or Latin-American population, according to anthropomorphic data. We obtain the complete dynamic model of the proposed rehabilitation system and perform the respective comparisons of movement with the real prototype in order to develop and evaluate appropriate control strategies in future work. For this purpose, we present the kinematic formulation of the device and then we derive the dynamics using two approaches to validate the model; we obtain the motion equation using the Lagrange approach and an algebraic method that simplifies modeling. Both approaches yield a unique model, which is validated either in simulation and by experimental trials, showing the functionality of the system and the validity of the models when performing rehabilitation routines.

Keywords: Assistive Robotics, rehabilitation robotics, kinematics modeling, dynamics modeling.

Resumen

La articulación de la rodilla está frecuentemente expuesta a lesiones en personas de todas las edades. En todos los casos, la terapia física se prescribe para recuperar la fuerza y la movilidad de un paciente. Los dispositivos de asistencia robótica están ganando la atención de la comunidad y apuntan a mejorar la calidad de vida de los pacientes. En este artículo, se propone el diseño mecánico de un dispositivo de rehabilitación de rodilla de enlace de 5 barras basado en la definición de los parámetros físicos de la población colombiana y/o latinoamericana, de acuerdo a los datos de antropometría. Se obtiene el modelo dinámico completo del sistema de rehabilitación propuesto y se realizan las comparaciones respectivas de movimiento con el prototipo real para desarrollar y evaluar estrategias de control apropiadas en trabajos futuros. Para este propósito, se presenta la formulación cinemática del dispositivo y luego se deriva la dinámica utilizando dos enfoques para validar el modelo; se obtiene la ecuación de movimiento utilizando la aproximación de Lagrange y un método algebraico que simplifica el modelado. Ambas aproximaciones producen un modelo único, que se valida en simulación y en ensayos experimentales, mostrando la funcionalidad del sistema y la validez de los modelos cuando se realizan rutinas de rehabilitación.

Palabras clave: Robótica asistencial, robótica de rehabilitación, modelado cinemático, modelado dinámico.

1 Introduction

Knee rehabilitation therapy is a very important process to recover the functional stability, mobility and flexibility of the knee after injury or surgery. Treatments are prescribed also to reduce the adhesion of the knee. Part of the rehabilitation process consists in performing exercises regularly in a controlled way [1, 7, 15]. In several cases, physical therapy is assisted or supervised by a physiatrist or a physiotherapist [15, 22].

Knee injuries are common in people of all ages. The main causes are muscular atrophy due to aging, damage induced by

exercise, labor accidents, and ignoring ergonomic principles during work [15, 28]. Physical rehabilitation usually takes several weeks or even months until full range of motion and joint flexibility are recovered. However, satisfactory results are reached only if the patient performs the exercises regularly. Regarding physiotherapy, there has been an increasing interest in developing assistive devices that can be used for rehabilitation with the purpose of improving the patients and the therapist's quality of life [15,12].

These devices should provide feedback to the patient and the therapist, to allow the evaluation of the patients' progress [15].

Moreover, it is desirable to have a device which may be also used at home. On the other hand, actuation systems are also crucial in devices that are intended to be used by people. Safety and natural motions are required in human-robot interaction. For this reason, in the design that we propose, we consider to use compliant actuators. This is a novel technology that exploits intrinsic advantages of compliant elements to provide more natural and safe motions [30, 11]. Among compliant actuators we find serial elastic actuators (SEA) and variable stiffness actuators (VSA) which will be considered for the design of the assistive robotic device proposed. Both of these actuation systems have an elastic transmission in series to the motor's shaft. The difference among SEA and VSA is that in the former the stiffness is constant while in the latter it can be mechanically adjusted. Due to the compliant element, the rotor's and the link's angular position are decoupled. For further reference on this system, the reader is encouraged to review [10, 29, 30].

Several devices have been designed to assist patients that require physical therapy. For example, in [18] authors present a design that combines a conventional knee brace system with a new type of hinge mechanism consisting of a double gear system that imitates the motion of the knee. The idea is to actively help knee rehabilitation for anterior cruciate ligament (ACL) post-surgery treatment. However, the mechanism is implemented with gears, which are usually rigid. This fact may be a disadvantage compared to compliant elements for this kind of applications.

For assisted rehabilitation purposes, in literature we find Exoskeletons (see e.g. [9, 23]), orthoses (see e.g. [20, 24]) and other devices (e.g. [27]) that have been developed for upper and lower limbs. For instance, a robotic device for knee rehabilitation therapy is presented in [15]. This device is aimed to improve patellar mobility with feedback for the patient and for the therapist. The difference among this approach and ours is that the device that we present is modular and uses compliant actuation.

Compliant actuation can also be reached by pneumatic technologies that resemble artificial muscles. These systems have been designed to support the patients' muscles when there is a lack of strength. For instance, in [5] authors use an antagonistic configuration based on a four-bar link mechanism to help the patient's mobility and strength. A robotic rehabilitation and assistive device for people with severe disabilities is presented to carry out automated re-habilitation training in daily activities. To perform the training with body weight support, a lower extremity exoskeleton is integrated with a mobile platform. Furthermore, the design and manufacturing of a gait rehabilitation robot, which consists in a robotic orthosis for treadmill training, is reported in [22]. In the mentioned work, authors define some important criteria for the design such as low inertia of robot components, back-drivability, and high safety. We take into account these criteria in our design. Nevertheless, this robot is different from ours because the former aims to recover patients normal walking gait, while our design is oriented to repetitive routines for recovering strength and mobility range.

Control systems are highly important for accomplishing properly the rehabilitation routines carried out using assistive devices. In [26], we have already proposed a general control structure, based on a single pendulum dynamic model approach, that would serve as a basis for controlling the

designed rehabilitation device. However, the complete model of the structure is required to design, enhance and adjust a controller for this rehabilitation device. Doing so will guarantee the proper execution of the rehabilitation routines and will prevent damages to the patient due to undesired behaviors. In order to control and command a robotic device, it is mandatory to formulate properly its kinematic and dynamic models. Moreover, once the system is designed according to biomechanical constraints, the validation of the structure and the kinematic and dynamic model are mandatory.

In this paper, we present the definition of the physical parameters for the design of a five-bar-linkage assistive device for knee rehabilitation and its kinematic and dynamic formulation. We present as well the validation of the system and of the modeling. The modeling was partially presented in [33] in a theoretical way. However, a complete analysis of the model and of the validity of the structure was missing, as well as experimental trials of the designed system, which are the focus of this work. The 5-bar configuration was chosen so it could be used in lying and sitting position. Moreover, the design proposed is aimed to prevent efforts generated by the action of the actuators. In this way, the mechanism designed avoids the risk of harming the patient. This is a novel system that can be reconfigured to attend a wide range of patients according to their height. We use soft actuation to help motion at the knee joint, provided the aforementioned advantages of these actuators. Moreover, the actuators are not directly placed on the knee joint to prevent unwanted loading. In this paper, we present some improvements to the design proposed first in [21]. The main difference here is the possibility of attending a wide range of patients, considering their height. This consideration introduces a new variable which is taken into account in the kinematic formulation presented in [33].

In this paper first, we present some theoretical background that allows to define the bio-mechanical constraints of the system derived from anthropomorphic data [6, 13]. Then, we define and present the mechanical design of the device. Afterwards we present the most important facts of the kinematic and dynamic modeling of the five-bar-linkage assistive device, respectively based on rigid body mechanics [25], and Lagrangian formulation [25]. The functionality tests and an initial validation of the system are carried out by performing a dynamic simulation in Matlab. The results of these tests are compared with the results obtained from the behavior of the physical system during the experimental trials for the same routines used in simulation.

2 Theoretical Background

The knee is one of the most frequently injured joint due to its daily use [14]. For example, several injuries may occur when practicing high impact sports such as running, or jogging; other problems are caused by the wrong choice of footwear, and so on. On the other hand, injuries can be derived from traffic or labor accidents. Moreover, osteoarthritis is a very common condition that currently affects mainly the elderly population, but can also appear at an early age [1].

Table 1.
Most frequent injuries in the knees by age and sex.

Age	Male	Female
0-12	Discoid meniscus	Discoid meniscus
12-18	Osteochondritis dissecans. Osgood- Schlatter	Patella luxable
18-30	Meniscus tear	Patella Luxable, Patellar chondromalacia, Infrapatellar fat injury
30-50	Rheumatoid arthritis	Rheumatoid arthritis
45-55	Meniscal degeneration	Meniscal degeneration
+45	Meniscal degeneration	Meniscal degeneration

Source: [2,16]

2.1 Common knee injuries

Sports injuries are frequently associated with problems in the knee, meniscus injuries, ligaments or tendinopathies [16,17]. According to [17], sports injuries affect between 50% and 86% of the lower extremities; the most affected joints are the ankle and knee. In these cases, injuries occur mainly when sudden changes in direction or rotation occur. For high performance athletes, based on the epidemiology of sports injuries, traumatic injuries are more common [17].

Besides, according to a study carried out by the Colombian health entity [8], more than 80% of people over 55 years suffer of osteoarthritis. Of this population, from 10% to 20% are limited in their daily activities by the disease. The most affected joint is the knee [14]. There are effective treatments for osteoarthrosis which include weight loss, aerobic exercise, and analgesics [19].

Furthermore, age and sex of the population are related to the possibility of having a knee injury [1]. In Table 1, a classification of the most frequent injuries by sex and age is presented, according to [2,16].

2.2 Physical Rehabilitation

Physical rehabilitation routines consist on repetitive exercises such as knee extension, hamstring stretching, adductors contraction, leg lifting, standing up, balancing with one leg, leg lateral elevation, calf stretching, and so on. In some cases, the patients may use elements such as elastic bands and weights to stretch and strengthen the muscles involved in the knee joint mobility [4, 7]. The routines vary according to the patient and the diagnosis.

To design an assistive device capable of executing physical rehabilitation routines, we take into account the parameters related to the patients' condition; i.e. the weight, height, age, and the injury. Therefore, we constrain the design to mean Colombian population from ages 18 to 45, that will perform physical therapy to strengthen and improve range of motion of knee joint. It is worth to remark that the design methodology can be adapted for different population characteristics.

2.3 Knee Bio-mechanics

The knee joint has two degrees of freedom (DoF), and performs movements in two perpendicular planes, i.e. flexo-extension in the sagittal plane (frontal axis), and internal-external rotation in the frontal plane (vertical axis). Knee flexion reaches on average 130°, considering 0° when the leg is completely extended. The maximum limit of amplitude is greater, when the motion is assisted. In general, for the knee joint, the ranges of motion considered normal are: flexion from 130° to 140°; internal rotation from 30°; and External rotation: 40° [1, 2]. In the proposed design of the assistive device, only flexo-extension movements will be considered [2]. This choice is done because the knee joint is the most frequently injured, and in terms of mobility, the DoF considered is the most affected. Moreover, the assisted physiotherapy is mostly focused on flexo-extension movements of the knee [28]. The latter also involves hip motion, so common rehabilitation exercises include raising the entire leg, therefore a 2-DoF device is necessary.

3 Mechanical Design Formulation

In this section we present the mechanical design of the five-bar assistive device for knee rehabilitation, based on the parameters and considerations tackled in section 2.

As previously mentioned, the main parameters that are involved in the performance of the physical therapy are the mass (in Kg) and the height (in m). Both parameters are variable according to the subject and are taken into account in the design and analysis. It is worth to mention that the reconfigurability of the device is done for patient's heights $1.40\text{ m} < h < 1.90\text{ m}$ according to mean population data [13]. Similarly, we consider normal weights (i.e. not overweight nor underweight) according to body mass index (BMI)¹. According to the anthropomorphic data and body proportions [6], the patient's height determines the lengths of the thigh and the leg. In this way, these two lengths will determine the mechanical design of the structure and therefore will be of great importance for the calculations of the kinematic and dynamic model. To carry out the physiotherapy routines, the joints motions are constrained to the allowed normal ranges mentioned before. These ranges of motion are included in the physical therapy protocols that physcists and physiotherapists establish for treating their patients. In general, these protocols may change according to the health center or the professional². We have taken into account several routines, for knee flexion/extension and the corresponding ranges of motion of the hip and knee joints as well. As mentioned before, the system is reconfigurable according to a range of patients' height and weight, which are the design parameters presented in Table 2. These parameters determine the constraints to the construction framework of the device.

¹Consider normal as $20 < BMI < 25$, $BMI = mass/height^2$

²We have based our analysis in the protocol of the orthopedics department of the Central Military Hospital-Bogotá.

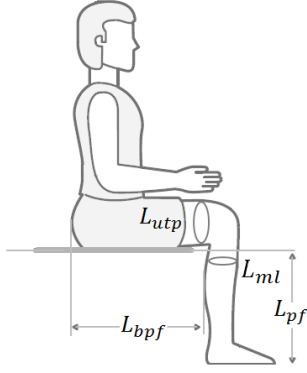


Figure 1. Layout of physical parameters: L_{utp} is the upper thigh perimeter, L_{ml} is the middle leg perimeter, L_{bpf} is the length of the buttock popliteal fossa, and L_{pf} is the height of the popliteal fossa.

Source: [6].

Table 2.

Physical parameters of design restricted to device and focused on the population group where knee injuries are more frequent.

Dimensional physical parameters	Functional physical parameters
Population variation of thigh length between 0.416 to 0.513 m minimum	Flexo-extension of the thigh in the sagittal plane: 0° when the patient is lying down with the joints extended to $90^\circ \pm 10^\circ$ in flexion.
Population variation of leg length between 0.342 to 0.465 m minimum	Flexo-extension of the leg in the sagittal plane: 0° when the patient is lying down with the articulations extended to $130^\circ \pm 10^\circ$ in flexion.
Thigh width variation between 0.473 to 0.639 m minimum	Width adjustment of variation of the thigh according to the patient
Leg width variation between 0.30 to 0.408 m minimum	Adjustment of leg width according to the patient.
	Device for patients with max. body weight $89.9 < W_M < 120$ Kg and max. height between 1.811 and 1.90 m.

Source: [6]

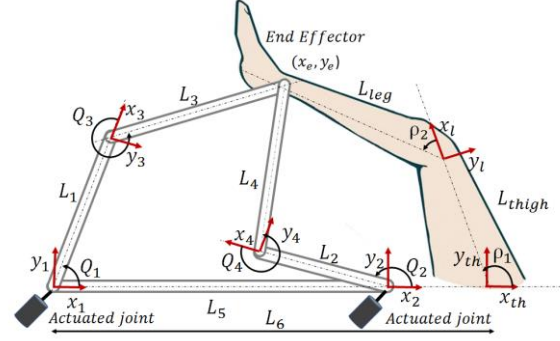
Refer to Fig. 1, where the physical parameters of the leg and the variables involved are defined. The mass of the lower limb is required and can be calculated from the whole-body mass M_b (in Kg). According to anatomical proportions in [13], the lower limb mass is $M_b/7$. Moreover, the mass of the lower limb segments M_t and of the leg M_l can be experimentally determined as

$$M_t = 0.1032M_b + 12.76L_{bpf}L_{utp}^2 - 1.023 \quad (1)$$

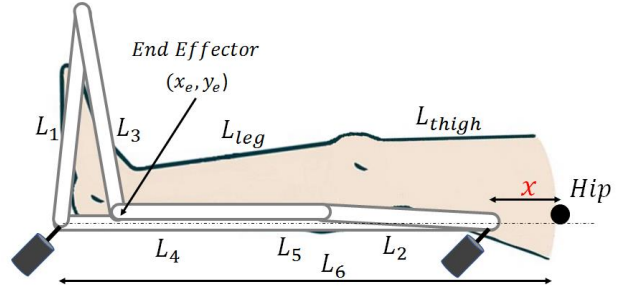
$$M_l = 0.0226M_b + L_{pf}L_{ml}^2 - 0.016 \quad (2)$$

and $M_t + M_l = M_b/7$, approximately.

The five-bar mechanism for knee rehabilitation, with the corresponding coordinate notation is shown in Fig. 2. L_1, L_2, L_3, L_4, L_5 , are the lengths of the system links which are fixed; L_{thigh} and L_{leg} are the lengths of the segments of the patient's leg. Instead, L_6 is a variable length, which makes the system reconfigurable.



(a) Five-bar linkage rehabilitation device. Kinematic definitions used in the derived model.



(b) Five-bar linkage Rehabilitation device.

Figure 2. Five-bar linkage Rehabilitation device parameters, representation of angular positions and lengths.

Source: The authors.

Notice that L_{thigh} and L_{leg} are variable values according to each user. To allow the adjustment and reconfigurability of the system to the defined range of patients' heights, and therefore to take into account the variability of L_{thigh} and L_{leg} , let us refer to Table 2. The mathematical model includes an additional variable, associated with the placement of the hip in the device, which can be obtained from

$$L_6 = x + L_{thigh} + L_{leg} \quad (3)$$

where x is a variable defined when the patient has the leg relaxed horizontally, thus establishing a reference point and avoiding mechanical singularities that may occur during the motion.

As shown in Fig. 2, $Q = [Q_1, Q_2, Q_3, Q_4]$ is the vector of angular positions of each joint in the designed structure. These angular positions are determined using the Denavit-Hartenberg convention [25]. Q_1 is the angle between the links L_1 and L_5 , Q_2 is the angle between the links L_2 and L_5 , Q_3 is the angle between the projection of the link L_1 and L_3 , and finally Q_4 is the angle between the projection of the link L_2 and L_4 . The rotations counterclockwise are positive, according to the convention. The actuated joints are Q_1 and Q_2 , that connect links L_1 and L_2 respectively with L_5 .

4 Kinematic Formulation

In this section we will focus on deriving the kinematics model of the five-bar rehabilitation system shown in Fig.2. Also, an

algebraic formulation is made using as a basis of analysis the theory and kinematics of rigid bodies and kinematics [3]. Let us define $C_i = \cos(Q_i)$ and $S_i = \sin(Q_i)$; on the basis of a closed chain mechanism, analyzing the vectorial components of each link, the kinematics model of the system is

$$L_1C_1 + L_3C_{13} - L_5 - L_2C_2 - L_4C_{24} = 0 \quad (4)$$

$$L_1S_1 + L_3S_{13} - L_2S_2 - L_4S_{24} = 0 \quad (5)$$

4.1 Cartesian position of the foot

We calculate the end effector cartesian position (X_e, Y_e) , as

$$\begin{aligned} X_e &= L_{thigh}C_{\rho_1} + L_{leg}C_{\rho_1\rho_2} + L_6, \\ Y_e &= L_{thigh}S_{\rho_1} + L_{leg}S_{\rho_1\rho_2} \end{aligned} \quad (6)$$

where ρ_1 and ρ_2 are the angular positions of the hip and knee. These two angles will determine the position of the end effector, i.e. the foot, as well as the angular positions of each joint of the mechanism.

Based on the Standardized Anthropometric Technique [6], we can obtain the lengths of the leg as $L_{thigh} = L_{bpf}$ and $L_{leg} = L_{pff}$. The angular positions of the knee and hip, ρ_1 and ρ_2 can be obtained with an angular measurement instrument, e.g. a goniometer. Consider that the coordinates obtained are attached to a point associated to the heel, which represents the starting point of the end effector (foot).

4.2 Joints angular position

The vector of angular positions $Q = [Q_1, Q_2, Q_3, Q_4]$ is obtained from X_e and Y_e . The joints of the angles Q_1 and Q_2 are actuated, this means that the velocities of these actuated joints will be independent, and that the motion of the other joints ($Q_3, Q_4, End\ Effector$) will depend on the whole motion of the independent joints. The angular positions described in Fig. 2 can be calculated by dividing the mechanism into two open chains where the point in common will be the final effector, then we have the expressions that describe these chains. For the first open kinematic chain, we have:

$$\begin{aligned} X_e &= L_1C_1 + L_3C_{13}, \\ Y_e &= L_1S_1 + L_3S_{13} \end{aligned} \quad (7)$$

The same point $\{X_e, Y_e\}$ calculated by analyzing the second open kinematic chain

$$\begin{aligned} X_e &= L_5 + L_2C_2 + L_4C_{24}, \\ Y_e &= L_2S_2 + L_4S_{24} \end{aligned} \quad (8)$$

From (7), and (8), after some algebra we define

$$\begin{aligned} Q_1 &= 2 \tan^{-1} \left(\frac{2L_1Y_e + \alpha}{L_1^2 + 2L_1X_e - L_3^2 + X_e^2 + Y_e^2} \right), \\ Q_2 &= 2 \tan^{-1} \left(\frac{2L_2Y_e + \gamma}{L_2^2 + 2L_2P_a - L_4^2 + P_a^2 + Y_e^2} \right), \end{aligned} \quad (9)$$

$$\begin{aligned} Q_3 &= -2 \tan^{-1} \left(\frac{\alpha\sqrt{\beta}}{\beta} \right), \\ Q_4 &= -2 \tan^{-1} \left(\frac{\gamma\sqrt{\delta}}{\delta} \right) \end{aligned}$$

Where

$$\begin{aligned} \alpha &= \sqrt{\beta(L_1^2 + 2L_1L_3) + L_3^2 - X_e^2 - Y_e^2}, \\ \beta &= -L_1^2 + 2L_1L_3 - L_3^2 + X_e^2 + Y_e^2, \\ \gamma &= \sqrt{\delta(L_2^2 + 2L_2L_4) + L_4^2 - P_a^2 - Y_e^2}, \\ \delta &= -L_2^2 + 2L_2L_4 - L_4^2 + P_a^2 + Y_e^2, \text{ and} \\ P_a &= X_e - L_5. \end{aligned}$$

4.3 Velocity components of each link

The centroid velocities \dot{X}_i and \dot{Y}_i for links $i = 1, 2, 3, 4$, are calculated from the centroid positions of the $i - th$ link L_{ci} , assuming for symmetry that it is located in the middle of the link,

$$\begin{aligned} X_1 &= L_{c1}C_1 & Y_1 &= L_{c1}S_1 \\ X_2 &= L_{c2}C_2 & Y_2 &= L_{c2}S_2 \\ X_3 &= L_{c3}C_{13} + L_1C_1 & Y_3 &= L_{c3}S_{13} + L_1S_1 \\ X_4 &= L_{c4}C_{24} + L_2C_2 & Y_4 &= L_{c4}S_{24} + L_2S_2 \end{aligned}$$

Then, the differential kinematics are defined by the velocity components of each link, which are obtained from the centroid coordinates of each link, and can be written as

$$\begin{aligned} \dot{X}_1 &= -L_{c1}S_1\dot{Q}_1, & \dot{Y}_1 &= L_{c1}C_1\dot{Q}_1 \\ \dot{X}_2 &= -L_{c2}S_2\dot{Q}_2, & \dot{Y}_2 &= L_{c2}C_2\dot{Q}_2 \\ \dot{X}_3 &= -L_{c3}S_{13}(\dot{Q}_1 + \dot{Q}_3) - L_1S_1\dot{Q}_1, \\ \dot{Y}_3 &= L_{c3}C_{13}(\dot{Q}_1 + \dot{Q}_3) + L_1C_1\dot{Q}_1 \\ \dot{X}_4 &= -L_{c4}S_{24}(\dot{Q}_2 + \dot{Q}_4) - L_2S_2\dot{Q}_2, \\ \dot{Y}_4 &= L_{c4}C_{24}(\dot{Q}_2 + \dot{Q}_4) + L_2C_2\dot{Q}_2 \end{aligned}$$

These terms will be useful to derive the dynamic equations in the next section.

5 Dynamic Formulation

In this section we derive the dynamic model of the five-bar linkage device. A first approach considers the Lagrangian formulation to obtain the dynamic equations. Alternatively, another modeling approach based on Lagrangian formulation, that relies on an algebraic method can also be used. We show that both methods yield a unique valid model, which allows to validate in an analytical manner our methodology.

5.1 Lagrangian approach

The general equations of motion of a mechanical linkage system can be obtained from Lagrange equations [25]. The application of Lagrange mechanics yields to differential equations corresponding to the generalized coordinates Q_i . This method deals with kinetic (K) and potential (P) energies that are scalar quantities, defined respectively as

$$K = \frac{1}{2} \sum_{i=1}^4 [I_i \dot{Q}_i + m_i (\dot{X}_i + \dot{Y}_i)], \quad (10)$$

$$P = \frac{1}{2} \sum_{i=1}^4 m_i g Y_i \quad (11)$$

Where I_i is the inertia of the i th-link, m_i the mass of the i th-link; X_i and Y_i are the horizontal and vertical components of the i th-link centroid position, respectively, and g is the acceleration due to gravity.

Let us define the partial derivatives of the kinetic energy (dK_i) and potential energy (dP_i) w.r.t the generalized coordinates (Q_i), for $i = 1, 2$ which correspond to the actuated joints, as

$$\begin{aligned} dK_1 = & L_1 L_{c4} M_3 \dot{Q}_1 \dot{Q}_4 S_{2-14} - L_{c3} L_{c4} M_4 \dot{Q}_2 \dot{Q}_3 S_\phi \\ & - L_{c3} L_{c4} \dot{Q}_3 \dot{Q}_4 S_\phi (M_3 + M_4) \\ & - L_{c3} L_{c4} M_3 \dot{Q}_1 \dot{Q}_4 S_\phi - L_2 L_{c3} M_4 \dot{Q}_2 \dot{Q}_3 S_{13-4} \end{aligned}$$

$$\begin{aligned} dK_2 = & L_{c3} L_{c4} M_3 \dot{Q}_1 \dot{Q}_4 S_\phi - L_2 L_{c4} M_4 \dot{Q}_2^2 S_2 \\ & + L_{c3} L_{c4} M_4 \dot{Q}_3 S_\phi (M_4 \dot{Q}_2 + M_3 \dot{Q}_4) \\ & + L_{c3} L_{c4} M_4 \dot{Q}_3 \dot{Q}_4 S_\phi - L_1 L_{c4} M_3 \dot{Q}_1 \dot{Q}_4 S_{2-14} \\ & - L_2 L_{c4} M_4 \dot{Q}_2 \dot{Q}_4 S_2 \end{aligned}$$

$$dP_1 = g M_1 L_{c1} C_1 + M_3 L_{c3} C_{13} Q_1 + L_1 C_1$$

$$dP_2 = g (M_1 L_{c2} C_2 + M_4 L_{c4} C_{24} Q_2 + L_2 C_2)$$

According to the Lagrangian formulation, the dynamic equations are obtained from

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{Q}_i} \right) - \frac{\partial K}{\partial Q_i} + \frac{\partial P}{\partial Q_i} = \tau_i \quad (12)$$

Where $L = K - P$ is the Lagrangian function. The generalized torques $\tau = [\tau_1, \tau_2]^T$ are the actuated joints torques, associated with the generalized coordinates Q , which in this case correspond to the actuated joints. Then, from (12), we derive the vector of generalized torques of the actuated joints, corresponding to the equations of motion of the five-bar-linkage rehabilitation device, as

$$I_1 \ddot{Q}_1 - dK_1 + dP_1 = \tau_1 \quad (13)$$

$$I_2 \ddot{Q}_2 - dK_2 + dP_2 = \tau_2 \quad (14)$$

5.2 Second Formulation Method: Algebraic approach

Alternatively, we use a formulation based on Lagrange formulation with an algebraic method that simplifies the dynamic model derivation and validates the equations. According to the development of the model for hybrid machines (HMs), and an approximate dynamic model of a 5-bar mechanism proposed in [32], and considering the

definitions of dK_i given previously, the generalized torques can be written as

$$D \ddot{Q}_1 + \dot{D} \dot{Q}_1 - dK_1 + G_1 = \tau_1 \quad (13)$$

$$D \ddot{Q}_2 + \dot{D} \dot{Q}_2 - dK_2 + G_2 = \tau_2 \quad (13)$$

Here, the generalized inertia matrix D is defined terms of the angular and linear velocities; the vector of gravity torque is $G = \partial P / \partial Q$ f, i.e., and Q, \dot{Q} are the vectors of angular positions and angular velocities, respectively obtained in the previous section.

The terms D and G include the inertia of the motor armature, the load and the links, as well as the effects of the centripetal torque and gravity torque.

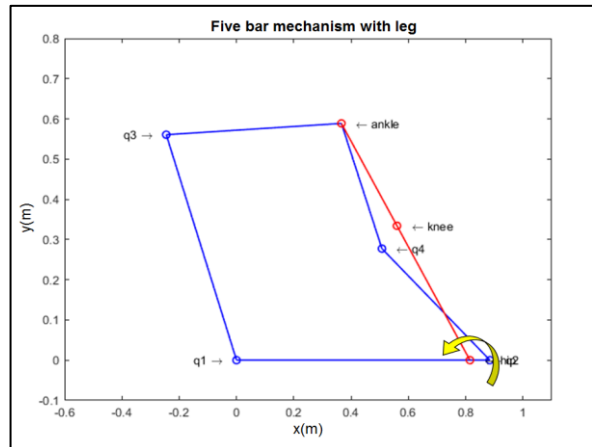
6 Model Validation and Results

To test and validate the kinematic and dynamic model of the mechanism obtained in the previous sections, we first compare the torque obtained with both formulation approaches in simulation. Then we carry out some experimental trials that allow to validate our model. For this two of the most common routines for knee rehabilitation, were chosen, i.e. leg raising and knee flexo-extension, according to physiotherapists criteria.

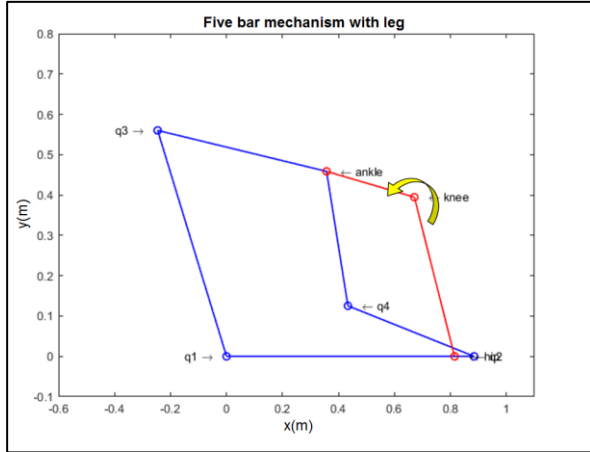
6.1 Simulation tests

For the simulation, we consider a person of height $h = 1.70$ m, so $L_{thigh} = 0.42$ m and $L_{leg} = 0.32$ m. To validate the calculations and the model obtained, we compare the results of the simulations done in Matlab with the dynamic results obtained in the real system. The model parameters that define the mechanical structure are presented in Table 3.

Then we define two desired motions based on the common movement's routines, the first one is where the leg has to be completely extended and the second one where the knee performs a range of movements. Fig. 3. shows the positions where the movements are executed.



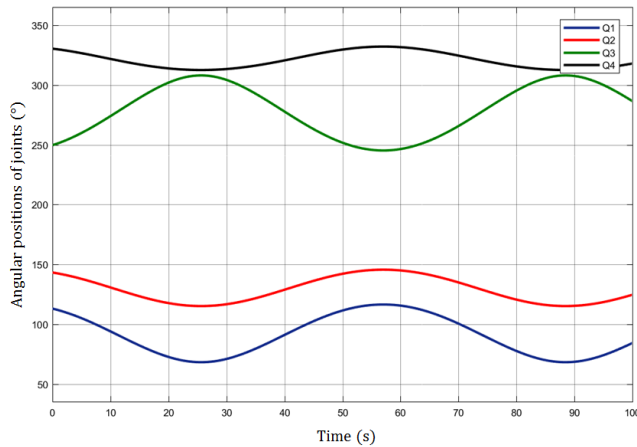
(a) Five-bar linkage device simulated with the first routine in Matlab.



(b) Five-bar linkage device simulated with the second routine in Matlab.

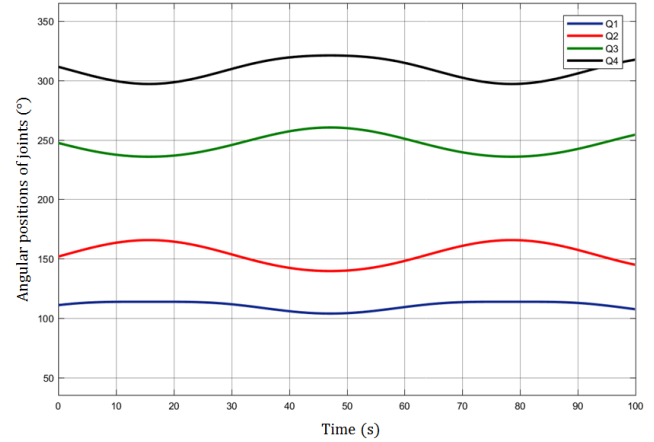
Figure 3. Five-bar linkage Rehabilitation device parameters, representation of angular positions and lengths.
Source: The authors.

During the simulation of the first routine we observe the loading effect when a transition occurs, since the system instantly becomes a 3-bar system with a fixed bar, this happens when L_3 and L_4 are parallel and Q_4 changes the segment of the coordinate axis with respect to Q_2 , therefore the torque needed to return to the initial segment is very high, so for the current application we restrict the movement to avoid undesired loading. Then we have the behavior of the simulated mechanism for the desired motions in Fig. 4.



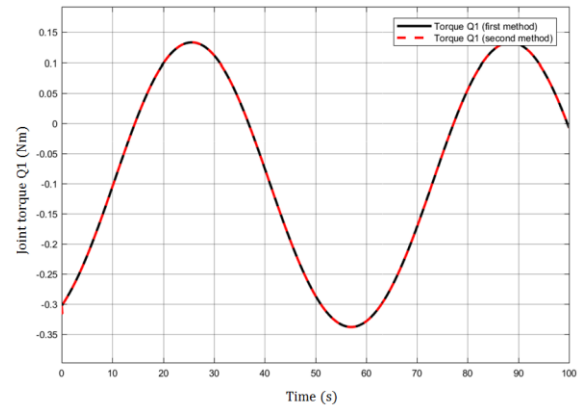
(a) Angular positions for variation of the hip with extended leg (first routine).

To verify the dynamic model, we compare the results obtained from the two formulation approaches used. The normalized torques of the actuated joints Q_1 and Q_2 are shown in Fig. 5. For simulation, we recalculate the lengths of the links to ensure that there are no singularities in the process. Then we obtain the generalized torques for the first rehabilitation routine.

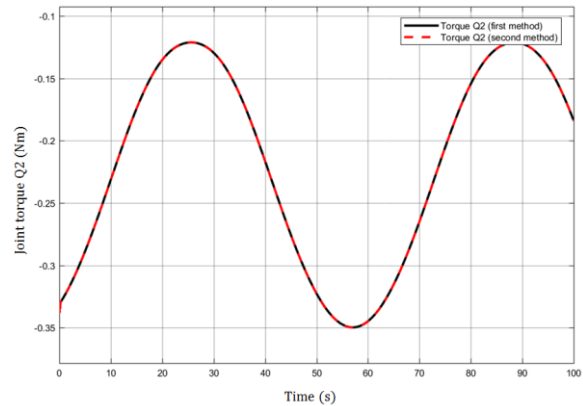


(b) Angular positions for flexo extension of the knee (second routine).

Figure 4. Angular positions for two routines simulated in Matlab.
Source: The authors.



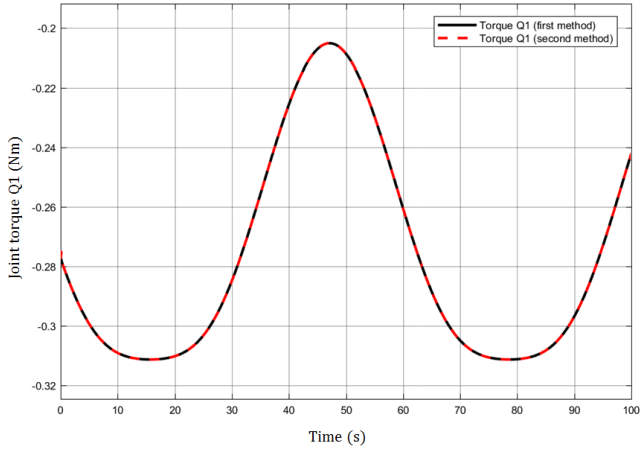
(a) Torque estimation of the actuated joint Q_1 .



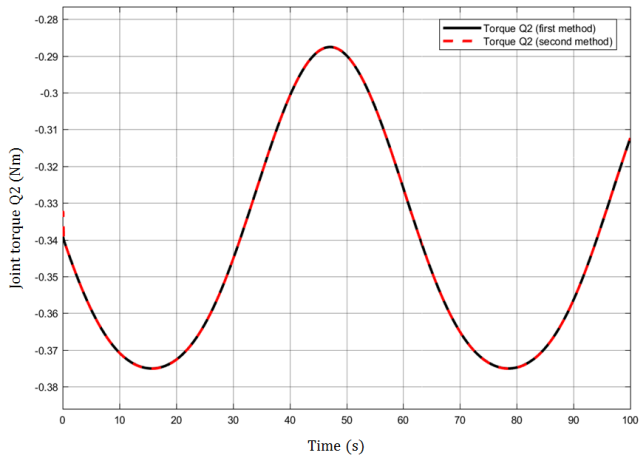
(b) Torque estimation of the actuated joint Q_2 .

Figure 5. Comparison of formulation methods (first routine).
Source: The authors.

The generalized torques for the second routine are shown in Fig. 6.



(a) Torque estimation of the actuated joint Q_1 .



(b) Torque estimation of the actuated joint Q_2 .
Figure 6. Comparison of formulation methods (second routine).
Source: The authors.

The torque estimation with both approaches shows that the formulation methods for both routines are similar, so any of these methods can be applied for the calculation of the normalized torques in the real device.

6.2 Experimental validation

To validate the system behavior experimentally, we compare the simulated model behavior with measurements obtained when performing the rehabilitation routines with the designed system. The mechanism was built from a CAD model in Solidworks with the dimensions defined in Table 3, according to the analysis presented previously.

Table 3.
Estimated parameters of the mechanism.

Link i	$m_i(Kg)$	$L_i(m)$	$L_{ci}(m)$	$J_i(x10^{-2}Kg\cdot m^2)$
1	0.06877	0.61190	0.3060	0.30522
2	0.05575	0.46790	0.2339	0.15516
3	0.06877	0.61190	0.3060	0.30522
4	0.04765	0.34190	0.1709	0.074428

Source: The authors



Figure 7. Physical mechanism in operation.
Source: The authors.

The variables measured will be the angular positions of the joints, Q_1 and Q_2 will be obtained from the integrated sensors of the motors, for the application we use SEA built with Maxon motors. Notice that for the experimental tests we configure the system with high constant stiffness (i.e. rigid), to verify the correct behavior of the system; afterwards we will change the stiffness according to an identification analysis which is out of the scope of this paper. For Q_3 and Q_4 inertial measurement sensors are used, to obtain the following comparisons. Fig. 8 shows the simulated vs. the real angular positions of the mechanism joints when performing the leg rising rehabilitation routine.

The angular positions obtained in Fig.8 shows similarity in their behavior. Q_1 and Q_2 measured follow the same trajectory of their simulated variables with small lags that are negligible when carrying out the routine this due to the execution time of the commands that produces a delay in the execution time of the motors. In the case of Q_3 and Q_4 although the trajectories are similar, the small differences between signals are produced by the inertial sensors in conjunction with the execution times that produce small changes of the measured variables with respect to the actuated points Q_1 and Q_2 . control strategies can correct those decompensations.

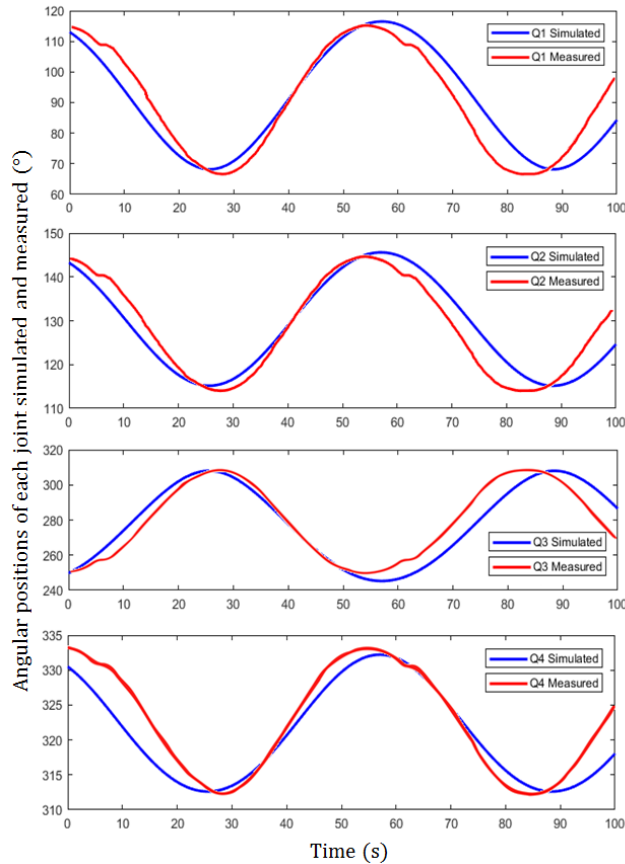


Figure 8. Comparison of angular positions simulated and measured (first routine).

Source: The authors.

7 Conclusions

In this paper, we define the physical parameters to establish the characteristics of the mechanical design deriving the kinematic and dynamic model of a 5-bars-linkage knee rehabilitation device. These models are the key for developing adequate control strategies, which will be carried out in future work. The dynamics is carried out using two approaches. First, by applying the Lagrange formulation, and then by using an algebraic method which has simplified the calculations. Both models were simulated using Matlab showing the convergence of both approaches. Moreover, we compared these results with the physical device by sensing the actuating points Q_1 and Q_2 with encoders and the dependent angles Q_3 and Q_4 with inertial sensors, showing the functionality of the system and the validity of the models when performing two rehabilitation routines.

All of the parameters and constraints that define our device have been obtained from anthropomorphic data and based on specific rehabilitation routines of flexion and extension of the knee joint in order to recover strength and mobility of this joint. Future steps consist on designing and testing the control strategies in the real device on the basis of the modeling presented and on the rehabilitation routines.

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