Zimbabwe Journal of Educational Research

The ZJER is published three times a year by the University of Zimbabwe, Human Resources Research Centre (HRRC)

Editor: L. M. Nyagura, University of Zimbabwe

ORDERING INFORMATION

	Within Zimbabwe		Outside Zimbabwe	
	Annual	Three year	Annual	Three year
Individual	Z$ 80	Z$220	US$40	US$100
Institution	Z$130	Z$350	US$65	US$170

Single Copies Z$30 US$20

ZJER
HRRC, Faculty of Education
University of Zimbabwe
P O Box MP 167
Mount Pleasant
Harare
Zimbabwe

Volume 4 Number 2 July 1992 ISSN 1013-3445

CONTENTS

Zimbabwe Journal of Educational Research

The ZJER is published three times a year by the University of Zimbabwe, Human Resources Research Centre (HRRC)

Editor: L. M. Nyagura, University of Zimbabwe

ORDERING INFORMATION

	Within Zimbabwe		Outside Zimbabwe	
	Annual	Three year	Annual	Three year
Individual	Z$ 80	Z$220	US$40	US$100
Institution	Z$130	Z$350	US$65	US$170

Single Copies Z$30 US$20

ZJER
HRRC, Faculty of Education
University of Zimbabwe
P O Box MP 167
Mount Pleasant
Harare
Zimbabwe

Volume 4 Number 2 July 1992 ISSN 1013-3445

CONTENTS

Problem Solving, Beliefs About Mathematics, And The Long Arm Of Examinations

David K. J. Mtetwa
Department of Science and Mathematics Education
University of Zimbabwe

ABSTRACT

The recent, almost global, shift in emphasis from computation towards problem solving skills in mathematics education curricula has opened up fresh areas of research. As one of the factors now widely acknowledged as having a tremendous influence on the course and quality of the problem solving process, beliefs about mathematics have been the subject of a number of studies including the present one.

The main objective of this study was to explore and uncover the kinds of beliefs Zimbabwean secondary school students hold concerning the nature of mathematics, the learning of mathematics, and the doing of mathematics. The study focused on Form 4 students (11th graders, typically 16 years old) and used in-depth individual interviews of 10 students (4 of them males) to gather data. A preliminary survey was used to structure the interviews, and video-taped observations of classroom sessions were done to explore the relationship between the beliefs and the context in which most of the mathematics is learned.

Analysis uncovered 46 beliefs. The nature of the beliefs suggests that the students simultaneously and mostly subconsciously hold two distinct views of mathematics. The views, which can be characterized as "discipline" mathematics and "examination" mathematics, overlap to varying degrees in different individuals and have conflicting characteristics in some aspects. Furthermore, the views appear to be strongly influenced and dominated largely by an evaluation effect originating from the practice and culture of summative national examinations and, to some extent, by the nature of the mathematics curriculum and a lack of exposure to genuine problem solving activities in the students' learning experiences.

Introduction

Much talk and research in mathematics education during the last decade has been dominated by problem solving (Kilpatrick, 1985; Romberg et al, 1986; and Silver, 1985, 1987). This shift in the curriculum trend from the "back to basics" emphasis in the seventies was influenced by 2 main factors: the proliferation of computing devices, in particular the computer; and the declining impact of associanist/behaviorist theories as the psychological bases guiding learning and instruction.

The relatively easy accessibility of computers and calculators of all sorts has made educators re-consider the objectives and modes of learning on the one hand, and the role of these computing devices on the other hand. Computing devices are capable of performing fast any computational procedure they are instructed to perform. This should free the learner from a felt need to concentrate on becoming proficient in performing standard numerical and even symbolic procedures such as factoring polynomials - regarded as basic and essential proficiencies in the back to basics curriculum. The learner can then concentrate on higher level competencies such as designing solution schemes that can be translated into programs for the computing devices to carry out the required computational procedures. In short, emphasis should be more on problem solving competence and less on computational competence. This would also be consonant with the kind of everyday life competencies now becoming increasingly necessary for an individual to live and survive in this modern information age ushered in by the silicon chip technology.

While the dissatisfaction with behaviorism grew, a number of competing perspectives of cognitive psychology gained ground as the new foundational theories for learning. Although the influence of the behaviorist theories was evident in things like drill and mastery approaches to mathematics learning, popular in the seventies, the relevance of the theories was nevertheless later found to be limited to low level competencies such as knowledge of particular facts. The theories could not satisfactorily explain higher order processes such as insight, strategic planning, and evaluation - important ingredients for problem solving competence, and processes cognitive psychologists claim to take into account.

Problem Solving

The adoption of problem solving as the main goal of mathematics education has spurned a lot of research in various aspects of the curriculum including learning and instruction [see for example Silver, 1985.] Here it is important to explain what is meant by problem solving, since the term may be interpreted differently by different individuals, even those within the mathematics education community. In the context of mathematics education, problem solving is widely accepted to mean the process of resolving a mathematical situation that is problematic in that the incumbent wants to move from a given state to a goal state, has both an interest and a reasonable chance of accomplishing the movement, but is not aware of a definite procedure of doing so (Mayer, 1985). Thus, a "problem" is a mathematical task that gives rise to such a problematic situation. When such a task, however, can be resolved by a direct application of a procedure, formula, or algorithm known by the solver, the task becomes merely an "exercise" rather than a problem. In other-words, resolution of a problem calls for *productive* mental behaviors while *re-productive* behaviors may be all the incumbent needs to do an exercise.

Shoenfeld (1985) has described 5 such productive mental behaviors as:

(1) resources: knowledge of relevant facts and procedures;
(2) heuristics: knowledge of general strategies;
(3) control: the self-regulating system;
(4) affect: emotional dispositions; and
(5) beliefs: psychological orientation.

A sixth category, metacognition, which essentially refers to knowledge and control of all the above categories was later added to the list (Garofalo and Lester, 1985). According to Shoenfeld, these components interact in complex ways to produce the sum effect of orienting and guiding the problem solving process in a given situation and, to a large extent, of determining the extent to which the problem solving enterprise is successful. Some researchers, such as Cobb (1986), have investigated the role of beliefs in problem solving while others, such as Thompson (1985), Lester, Garofalo, and Kroll (1989), and Frank (1985), have investigated the nature of student and teacher beliefs about mathematics. Research on beliefs, then, grew out of research on problem solving.

Beliefs and Their Influence

By a belief about mathematics, we mean a proposition concerning some general or specific aspect of the mathematics discipline that an individual, consciously or unconsciously, holds to be true. For instance, many individuals are known to hold beliefs such as "males understand mathematics better than females," and "every mathematical problem requires a definite formula or algorithmic procedure to solve it." Some studies in the United States have uncovered a variety of interesting student beliefs about mathematics. The belief that "in subtraction, numbers to be subtracted must be reasonably close in size," for example, was found to be held by sixth graders (Mtetwa and Garofalo, 1989). The belief that "the correctness of an answer is confirmed by the teacher or textbook was found to be common among young high school female students (Confrey, 1984). Shoenfeld's (1985) study of problem solving with college students led him to conclude that the students believed "all mathematics problems can be solved in a very short time, say, 10 minutes or less."

Such beliefs can have a considerable influence on problem solving performance and, ultimately, on mathematics learning. According to Price (1969, p. 98), "We need beliefs for guiding our actions and practical decisions ... [and] use them (where relevant) as premises in our practical reasoning." In the context of problem solving, Shoenfeld (1985, p. 14) concluded, "Beliefs about math, whether consciously held or not, are responsible for *establishing the psychological context within which students do mathematics* [emphasis added]." Thus, beliefs about maths affect problem solving performance by influencing the decisions made during the problem solving process (Lester et al, 1989); and like a sieve, by filtering the student's mathematical knowledge (Confrey, 1984; Shoenfeld, 1987). Finally, the important influence of beliefs on problem solving was emphasized and described by Cobb (1986, 1987) as that of helping to determine (and modify as necessary) the overall goals and specific subgoals of actions in a given problem solving situation.

To illustrate the "guiding" effect of beliefs, consider a student who believes "one cannot solve a mathematics task whose type is unfamiliar to the individual." Faced with a real "problem," the student's thinking in that event will likely be characterized by frantic attempts to recall from memory a standard procedure for solving a task perceived to be similar to the one at hand and, presumably, encountered before. The recalling effort becomes the student's specific subgoal in that particular situation.

There is little or no attempt at *reasoning it out*, for example: exploring possible strategies, examining and analyzing conditions, conjecturing, and guessing and checking. And if that student has encountered a similar task before but cannot recall the solution procedure; or, if the type of task is totally unrecognizable, the student will most likely "bail out'- to use Frank's (1985) expression for giving up without a fight.

The importance of mathematical beliefs in the mathematics learning enterprise, therefore, cannot be overemphasized. In addition to finding out the variety and nature of student beliefs about mathematics, more research on specific effects of particular beliefs and their implications for learning and instruction is needed. The first of such studies to be done in Zimbabwe was exploratory in nature and aimed at uncovering the kinds of beliefs about mathematics found among Zimbabwean secondary school students (Mtetwa, 1991).

Methodology

Assessing the presence or absence of a belief in a particular aspect of mathematics in an individual requires one to corroborate the individual's behaviors, some of which may be covert, with the individual's cognitive and affective responses to various manifestations of that aspect of mathematics. The quality of an investigation that entails such kind of assessment can be enhanced by a methodology that provides opportunities for one to observe the individual's behavior and to probe his or her thought processes such as goals and subgoals at the same time.

With that consideration, in-depth individual interviews of 10 Form 4 (11th grade) students, 6 female and 4 male, were used for this study. A preliminary survey of 463 Form 4 students from 11 selected schools in 3 provinces of Zimbabwe was done in order to structure the interviews. The interviewees were selected from 3 Form 4 classes at a twelfth school. The selection of the interviewees was based on their responses to another mini-questionnaire constructed from the interview protocol guide, and was done in consultation with their mathematics teacher.

For the survey, the respondents completed a 75-item questionnaire. Each item was a single statement such as, "In mathematics you cannever know if your answer is right or wrong unless you check with the teacher or textbook," which had both a Likert-type response scale and a provision for an open ended elaboration of the response.

The interview protocol guide that emerged from an analysis of the survey responses aimed at probing beliefs about: (a) the nature of mathematics, (b) mathematics problems, (c) mathematics ability, (d) mathematics learning and problem solving, (e) the value of mathematics, and (f) out-of-school mathematics. The interviewing process included a problem solving task performance by the students. All the interviews were audio and videotaped, and each student was interviewed twice: the first time for between 80 and 120 minutes; the second time for clarification and elaboration, 2 weeks later for between 45 and 60 minutes. In addition, 9 live mathematics classroom lessons, three of which were taught by the interview students' teacher, were observed and videotaped in order to examine the possible contribution of the classroom context in nurturing some of the beliefs.

Discussion

Analysis of the interview and observation data resulted in identification of 46 beliefs concerning mathematics. One of the major findings of the study is that the nature of the beliefs indicates the students have two views of mathematics: the *"discipline view"* (D-Maths) and the *"examination view"* (E-Maths); which overlap to varying degrees in different students.

The Discipline View (D-Maths)

The students expressed beliefs such as:

(1) Memorizing is not an appropriate way of learning mathematics in order to understand it well.

(2) One can be creative and discover one's own formulas, methods, and facts, even if they may be already known to others.

(3) If no particular method to be used is expected, one can solve a problem using any method that gives a correct result.

(4) In mathematics, not every word problem requires a formula to solve it.

(5) Mathematics is a subject which deals with "numbers" and operations and requires thinking.
(6) Mathematics can't exist on earth without humans because it is a creation of the human mind.

Such beliefs are consistent with a world view of mathematics shared - we can assume - by most "experts" in the field of mathematics. The beliefs describe the kinds of ideas most mathematics educators and professional mathematicians would agree are characteristic of mathematics as a human endeavor. A D-Maths view or (Experts' view), then, is what most educators would want students to develop.

Examination View (E-Maths)

The students also expressed the following kinds of beliefs:[1]

(1) One can learn and understand O' Level mathematics[2] well by rotely memorizing formulas, facts, etc., initially and then "understanding" them through constant reviewing, practice, and application.

(2) Student discovered methods and facts may be mathematically correct, but are not valid if examiners and teachers do not know or expect the methods.

(3) If a particular method to be used for solving a problem or exercise is not stated, the most correct method to use is the one which is taught by the teacher or one that is in a textbook.

(4) In mathematics, everything is either right or wrong [the right/wrong dichotomy belief]

(5) In general, doing maths requires fast thinking: exams or no exams.

---

[1] The list is illustrative, not exhaustive

[2] Based on the British model of school curricular, 'O' Level is the mathematics course covered during the first 4 years of secondary school (Forms 1-4), i.e., Grades 8-11.

(6) Neatness when writing down maths is very important for one's "understanding" of maths.

(7) Knowing when and how to use and apply formulas, facts, and rules is more important than knowing when and how those things come about.

These beliefs are clearly no longer consistent with the D-Maths view of mathematics described above. They appear to be characterizing a view of mathematics evolving from a mathematics curriculum centered around mathematics topics and tasks popular in examinations. The students' conceptions of the nature of such topics and tasks, how the topics are organized and learned and the tasks performed, and the significance of all these activities in the students' lives all crystalize into a definite examination based view of mathematics I have called E-Maths.

Relationship Between D-Maths and E-Maths Views

The first three D-Maths beliefs are in direct conflict with the corresponding first three E-Maths beliefs. D-Maths belief 1 is a recognition that "understanding," rather than memorizing the mathematical relationships, is the appropriate way to learn mathematics well. On the contrary, E-Maths belief 1 suggests one can learn O' Level maths with understanding by rote memorization followed by constant rehearsal and application of the memorized things. D-Maths belief 2 recognizes the validity of student created facts which are mathematically correct. On the contrary, E-Maths belief 2 denies validity to such facts, unless they are first approved by authorities. D-Maths belief 3 allows one to use any mathematically correct method to solve a problem at hand. On the contrary, E-Maths belief 3 uses the criterion that the method be first taught by a teacher or be illustrated in a textbook for the method to be permissible to use to resolve a school maths task.

With respect to E-Maths belief 3, it is interesting to note that the students expressed this belief despite their current teacher's revelation that she makes it clear to her students that a method she uses is her own favorite one and that they were free to use any method of their choice. The teacher made the revelation in my discussions with her during classroom observations. Thus, there is a mismatch between what the teacher said she does for instruction and what the students believe is true about the same aspect of mathematics learning. The mismatch suggests the students may have developed the belief earlier and it was ingrained in them by the time they got to this teacher. Alternatively, the students may have considered the teacher's advice as appropriate for D-Maths and not for E-Maths, particularly because a special examining board rather than the teacher sets the final examination.

While D-Maths beliefs 4, 5, and 6 are recognizable as features of discipline mathematics to most mathematics educators at least, E-Maths beliefs 4, 5, 6, and 7 are not characteristic of a D-Maths view of maths. In D-Maths, it is inappropriate to say that all mathematical relationships are either right or wrong because conjectures, for example, by definition contain a degree of uncertainty. Also, in D-Maths, thinking fast and neatness of written work are not intrinsically essential for one to learn or do mathematics with understanding. Finally, in the D-Maths view, a greater premium is placed on how and why a particular relationship emerges, rather than on the mechanics of using the relationship, if the intent is to understand that relationship well.

Thus, D-Maths and E-Maths reflect different conceptions or world views of mathematics as a discipline (or subject). Every interviewee expressed beliefs, some of which are consistent with D-Maths and some with E-Maths, on different aspects of mathematics. When the students were asked if they perceived two different views of mathematics, most students claimed only one discipline of mathematics exists. Yet, there were instances when students expressed conflicting beliefs, for example, D-Maths belief 2 and E-Maths belief 2, almost at the same time. When the contradiction arising from the conflicting nature of the beliefs was pointed out to the students (on the assumption the students possessed only one view of maths), the students sometimes recognized the contradiction. In such cases, they suggested there is maths "for school" and maths as "part of life or a job." This corresponds roughly to E-Maths and D-Maths respectively. Other times the students explained the contradiction away by suggesting, for example, that it is one mathematics in different situations. In other words, they could not see the contradictions as arising out of two different but related conceptions of mathematics.

Student L, for example, claimed and argued that the nature of mathematics is such that it requires fast thinking even when one is not in a test or examination situation. This indicates an overlap of the two views in the thinking aspect of maths. In fact, not only do the two views overlap, but also the degree of overlap varies with individuals. Further, the students, at least those whom overlap was not nearly complete, were largely unaware of the possibility they held two distinct views of mathematics. This could be inferred from the amount of surprise and sometimes disbelief at a sudden realization of conflict where they thought there was none.

Showing such surprise, student T distinguished the two kinds of mathematics. He cited some of the conventions students use when drawing graphs and setting up work, which he argued are only peculiar to examination requirements and practice. In addition, he expressed the belief that examiners require students to use examiner-prescribed methods for solving problems and cited this belief as an example of one such examination maths rule (E-Maths). He then concluded that although those examiner-determined rules and expectations may not be necessarily meaningful and valid in mathematics generally (D-Maths), following them can mean the difference between passing and failing the examination (E-Maths), (see Mtetwa 1991, for illustrative examples).

There were more instances of students expressing conflicting beliefs. Maths ability, for example, was perceived in two ways. Some defined it purely in terms of performance on tests: "passing with high marks most of the time" (E-Maths). Some defined it in terms of a combination of "marks," knowledge, and conceptual understanding. For example, student C said maths ability means "understanding it [maths] enough to be able to explain it to others in a simplified form" and T defined maths ability in terms of "logical reasoning capabilities." C's and T's characterizations are more consistent with D-Maths than with E-Maths.

Another example concerns mathematical accuracy. The mixed form 51 $^1/4$ can be considered more precise than the corresponding decimal form 51.25 because the latter form represents a range51.245 -51.254. A few of the students were able to perceive the mathematical meaning of precision. For example, student S suggested the representations are equally accurate because they represent the same number except when the decimal recurs, in which case the fraction is more accurate. L suggested the decimal representation is more accurate because one can improve the accuracy to a desired level by increasing the number of decimal places when converting fractions to decimals. But even after expressing such an awareness for the typical D-Maths interpretation of mathematical accuracy, the same students immediately declared that they felt the decimal is more accurate than the fraction. They then cited computational reasons such as:

(1) the decimals' column format makes it easy to decipher,

(2) the decimal is more convenient to use and is standard (calculators use decimals), and

(3) the decimal is easier and less mistake-prone to use in computations and therefore ensures a more accurate answer.

In this case mathematical accuracy for the students means *computational accuracy,* that is, error-free work - an interpretation of mathematical accuracy which is more consistent with the E-Maths view than with the D-Maths view. For some of the students, computational accuracy is the only kind of accuracy they perceived or, at least, expressed, despite repeated attempts to point to them that I was referring to mathematical accuracy. The overlap of the two views is evident here.

Finally, the question of neatness of written work produced mixed interpretations of what "understanding mathematics" means. While some agreed neatness per se cannot make one understand mathematics concepts (D-Maths), others claimed neatness is "essential" for understanding maths. The latter interpretation of understanding maths, however, translates into facilitating task review, detection of computation mistakes in the solution process, and evaluation by the examiner or teacher. These are aspects of E-Maths.

Thus, apart from the discipline view of mathematics, the students possessed a distinct but related view of mathematics (E-Maths view) which emanates from examination culture and practice, and a maths curriculum centered around examination topics. This finding is important in that it alerts educators that examinations and examining may be producing potentially harmful unintended effects. The finding also serves to remind educators of the need to continually re-examine and modify commonly held assumptions about evaluation practices and the curriculum.

Influence of Examinations

The above discussion more than hints that evaluation, in particular the culture of examinations, has had a tremendous impact on students' mathematical experience. Examinations or, rather, the practice of examinations and all that it entails has shaped the way the students think about mathematics and doing mathematics to the extent of engendering a distinct perspective of mathematics in their minds. In some students, narrow E-maths perspective replaces the wider and more appropriate D-Maths view of mathematics. In others, the two views co-exist, albeit subconsciously, one view or the other being activated in a particular context.

Apart from the influence of examinations, some of the beliefs identified but not listed here, are consistent with the E-Maths view although they appear to have their origins in the nature of the mathematics curriculum. For example, the belief that "mathematics problems are solved by recognizing the type of problem on the spot, or by classifying the problem with a familiar topic or previously encountered task, then applying the known method to solve the task at hand," suggests that most of the students' mathematical learning tasks are "exercises" rather than genuine "problems." The learning tasks are probably just schema-driven tasks. Analyzed data from classroom observations support this conclusion. A problem-solving-deficient curriculum is likely to engender such beliefs about solving mathematics problems.

Implications for Learning and Instruction

Because beliefs may be accurate portrayals of the incumbent's actual mathematical experiences, it is best not to refer to beliefs as correct or incorrect, but rather, as appropriate or inappropriate, or as healthy or unhealthy (Cobb, 1985). Healthy beliefs enhance more complete understanding of mathematical concepts and exhort the student to experience genuine mathematical thinking. On the contrary, unhealthy beliefs do not and may, instead, encourage the development of a narrow view of mathematics and lead to a superficial (instrumental in Skemp's (1987) sense) understanding of mathematics.

Here, we can identify the D-Maths kind of beliefs as healthy and the E-Maths kind as unhealthy. Follow up studies to understand exactly how the E-Maths beliefs mediate the problem solving process in students would be useful. So, too, would knowing the kinds of mathematical beliefs teachers share with students. Such studies are already being planned by the author. Our responsibility as mathematics educators is to influence learning and instruction in such a way that the development and subsequent reinforcement of unhealthy beliefs in students is, at worst, minimized and, at best, avoided altogether.

References

Cobb, P. (1986). Contexts, goals, beliefs, and learning mathematics. *For The Learning of Mathematics,* 6(2), 2-9.

Cobb, P. (1987). An investigation of young children's academic contexts. *Educational Studies in Mathematics,* 18, 109-124.

Confrey, J. (1984, April). *An examination of the conceptions of mathematics of young women in high school.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Frank, M.L. (1985). *Mathematical beliefs and problem solving.* Ph.D dissertation. Purdue University, West Lafayete, Indiana.

Garofalo, J. & Lester, F.K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal of Research in Mathematics Education,* 16(3), 163-176.

Kilpatrick, J. (1985). A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 1-16). Hillsdale, N J: Lawrence Erlbaum Associates.

Lester, F.K., Garofalo, J. & Kroll, D.L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem solving behavior. In D.B. McLeod & V.M. Adams (Eds.), *Affect on mathematical problem solving: A new perspective.* New York: Sringer-Verlag.

Mayer, R. E. 1985. Implications of cognitive psychology for instruction in mathematical problem solving. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 1-16). Hillsdale, N J: Lawrence Erlbaum Associates.

Mtetwa, D.K.J. & Garofalo, J. (1989). Beliefs about mathematics: An overlooked aspect of student difficulties. *Academic Therapy,* 24 (5), 611-618.

Mtetwa, D. K. J. (1991). *An investigation of Zimbabwean secondary* school students' mathematical beliefs and classroom contexts. Unpublished doctoral dissertation. University of Virginia, Charlottesville, Virginia.

Price, H.H. (1969). *Beliefs.* London, UK: Allen & Unwin. Romberg, T.A. & Carpenter, T.P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In American Educational Research Association, *Handbook of research on teaching (Third Edition)* (pp. 850-870). N Y, New York: McMillan.

Shoenfeld, A.H. (1985). Mathematical problem solving. Orlando, FL: Academic Press Inc.

Shoenfeld, A.H. (1987). What's all that fuss about metacognition? In A.H. Shoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-216). Hillsdale, N J: Lawrence Erlbaum Associates.

Silver, E.A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247-266). Hillsdale, N J: Lawrence Erlbaum Associates.

Silver, E.A. (1987). Foundations of cognitive theory and research for mathematics problem solving instruction. In A.H. Shoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 33-60). Hillsdale, N J: Lawrence Erlbaum Associates.

Skemp, R. R. (1987). *The psychology of learning mathematics.* Hillsdale, N J: Lawrence Erlbaum Associates.

Thompson, A.G. (1985). Teachers' conceptions of mathematics and the teaching of problem solving. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 281-294). Hillsdale, N J: Lawrence Erlbaum Associates.