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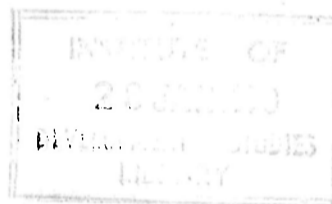
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A FURTHER NOTE ON LABOUR MIGRATION

by

John R. Harris and Frank Mitchell

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John R. Harris and Frank Mitchell [*]

Massell and Yotopoulos (M & Y), in their note [1] on Michalopoulos' migration article [2], found that his conclusions were either trivial (if remittances by migrants were excluded) or wrong (if they were included). We agree. However, we would like to point out that although M & Y's algebra is correct, their geometrical presentation and some of the conclusions they claim to draw from diagram and algebra are wrong. After documenting these points, we comment on an error made by Michalopoulos overlooked by M & Y.

We write M & Y's system with slightly simplified notation as : --

$$Q = f(S_h, U), \quad f_{S_h}, f_U > 0, \tag{1}$$

$$R = g(E, \alpha), \quad g_E > 0, \tag{2}$$

$$E = S - S_h \tag{3}$$

$$Y = Q + R \tag{4}$$

$$L = S_h + U \quad \text{and} \tag{5}$$

$$V = Y/L \tag{6}$$

where Q = domestic output, R = remittances, E = number of emigrants, S_h = skilled labour employed at home, S = total

skilled labour, a = an unspecified shift parameter,
U = unskilled labour, Y = total income, L = total
domestically employed labour force, and V = income per
domestically employed worker.

This system can be solved for the six variables
Q, R, E, Y, L and V given preassigned values of U, S and S_k.
Assuming that S_h is subject to policy control we can find
the value of S_h that will maximise V. The condition for
such a maximum is

$$\frac{dV}{dS_h} = \frac{(S_h + U)(f_{s_h} - g_R) - (f + g)}{(S_h + U)^2} = 0 \qquad (7)$$

where subscripts refer to partial derivatives. This
condition will hold when

$$f_{s_h} - g_R = \frac{(f + g)}{S_h + U} \qquad \text{or} \qquad M_h - M_r = V \qquad (8)$$

where following N & Y, M_h is the marginal product of
skilled labour at home (f_{s_h}) and M_r is the marginal
remittance of skilled emigrants (g_R).

It is useful at this juncture to examine the economic
meaning of equation (8) before looking at the geometry.
If one would-be migrant is forced to remain at home
domestic income is raised by the value of his marginal
product less the remittances he would have sent had he
emigrated. Thus the left hand side of (8) is the marginal
gain to the domestic economy of retaining one additional
skilled worker. Then, the equation states the familiar
condition that an average is maximised when it is equated
to a marginal.

Our Fig. 1 correctly depicts the alternatives open
to such an economy. Skilled domestic labour is measured

2 — the X-axis and average and marginal products on the Y-axis. Total skilled labour available is OS. The curve $M_h = (f_{s_h})$ is the marginal product of skilled labour employed at home and the curve Q/L is average domestic output per maximum output per domestically employed (skilled and unskilled) worker. This curve (which seems to be the same as "V" in M & Y's Figure 1) reaches its maximum at E (corresponding to E in M & Y's Figure 1) where marginal and average domestic product are equal. V, income per domestically employed worker, differs from Q/L by the amount of remittances received per domestically employed worker (R/L). The curve R/L has been drawn under the assumption that the marginal remittance (Mr) is constant. Then the curve V can be drawn by adding R/L and Q/L. It reaches its maximum where it intersects the curve $M_h - M_r$ (drawn by subtracting the constant marginal remittance curve M_r from the M_h curve) at point F. Thus the optimum amount of domestic employment of skilled labour is OA and emigration is equal to AS. The marginal product of skilled labour at this point is AC, marginal remittance is FC (not CB as M & Y suggest), average domestic product is AB, average remittance is AC = BF, and average income per domestically employed worker is maximized at level AF.

If there were no remittances, then the optimal employment of skilled labour would be OD; domestic product and income per domestically employed worker would be equal to DE and marginal and average products would be equated. (This condition is obvious from equation 7). Thus the existence of remittances increases the optimal level of emigration.

Figure 1
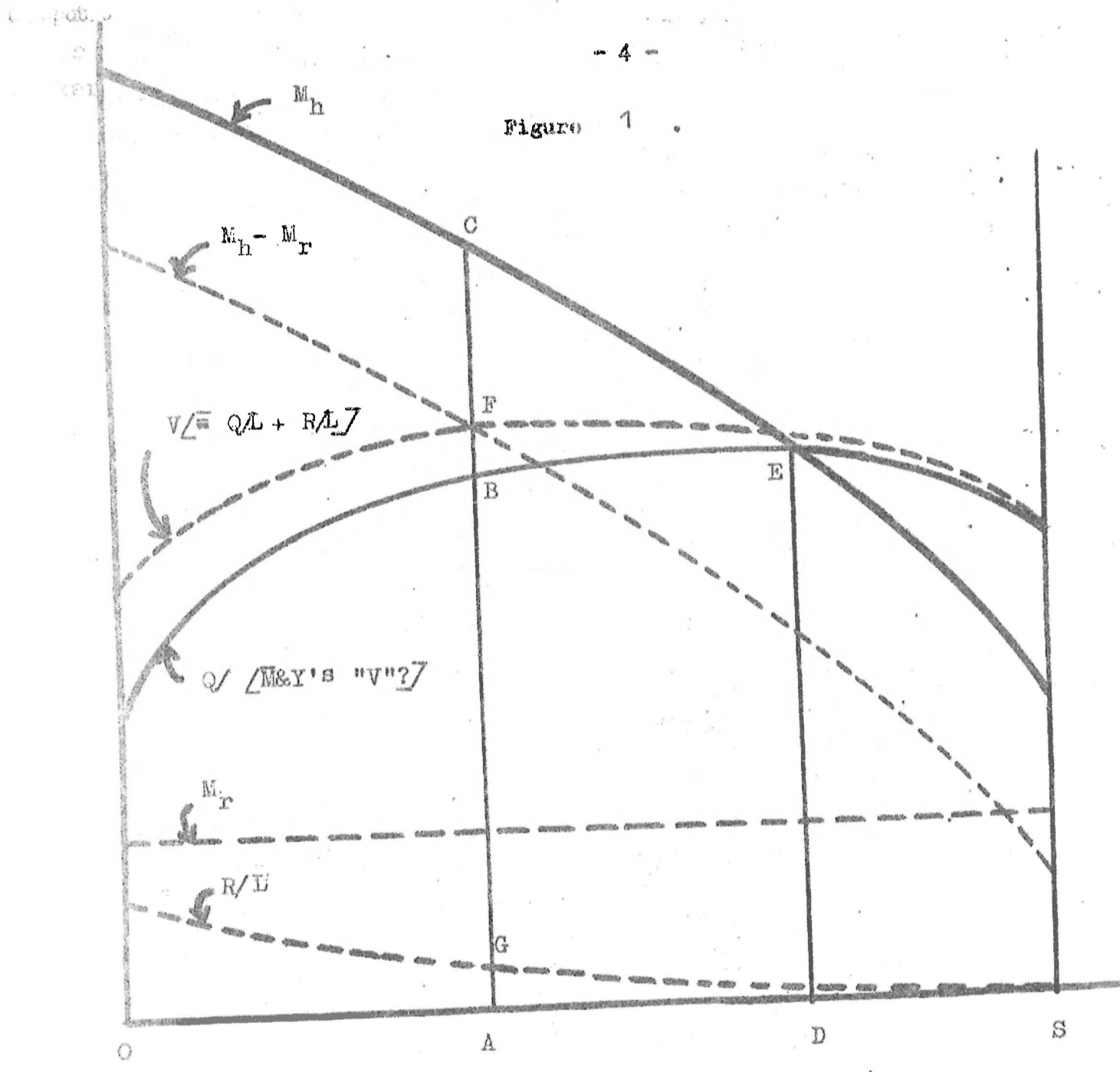
Skilled Labour Employed at Home (Sh) ⟶ ⟵ Skilled emigrants
(S-Sn)

M & Y claim to draw three comparative - static
i   :   ions from the analysis, none of which is necessarily
true.

First, they claim "that the optimal level of
migration is an increasing function of the level of marginal
remittances." (p. 333).   While this result is the most
plausible empirically, it need not be true.   This case can
be analyzed by considering a change in α such that $g_{E\alpha} > 0$
-- the marginal remittance rate is increased.   Assuming that
condition (7) has been solved for the optimal $S_h = S_h^*$, it
can be implicitly differentiated to obtain the result

$$\frac{dS_h^*}{d\alpha} = \frac{g_\alpha + g_{E\alpha}\ (S_h + U)}{d^2 V/dS_h^2} \tag{9}$$

The second order condition for a maximum requires that
$d^2 V/dS_h^2 < 0$.   Hence $\dfrac{dS_h^*}{d\alpha} \underset{<}{\overset{>}{\sim}} 0$ as $\dfrac{1}{(S_h+U)}\ g_\alpha + g_{E\alpha} \underset{>}{\overset{<}{\sim}} 0.$

Since $g_{E\alpha} > 0$, $\dfrac{dS_h^*}{d\alpha} < 0$ if $g_\alpha \geq 0.$ This is the result
~~posited~~
posited by M & Y -- increased marginal remittance leads to
more migration.   However, it is clear that if g_α is
sufficiently negative -- that is that the average remittance
falls even though marginal rises -- higher marginal remittances
will be associated with a smaller optimum level of migration.
This is quite clear from our Figure 1.   Higher marginal
remittances at the previous optimum (F), lowers the $M_a - M_r$
curve at that point.   If V remains constant or shifts up,
the maximum point on F must move to the left.   However, if
V shifts down (because the R/L shifts down) the maximum
point can move to the right.   This will occur only if the
M_r curve "twists" so that marginal remittances falls at
lower levels while they rise at higher levels of migration.

M & Y's second proposition is "if the marginal remit-
tance curve is horizontal, the optimal level of skilled
employment will be independent of the size of the total labour
force, as can be seen from equation (7) or Figure 1". (p.333).
This result cannot be seen from either their equation (7)
$\int$our equation (8)$\int$ nor from their figure 1 as asserted.
In fact the statement is wrong.

There are several ways in which the labour force can
be changed.

1. Assume the total labour force is changed by altering
the value of U. Implicitly differentiating the condition
for maximum V (7), simplifying and assuming that the
second order condition for maximum V holds ($d^2V/dSh^2 < 0$),
we find $\dfrac{dS_h^*}{dU} \gtrless 0$ as $V + f_{S_hU}(S_h + U) \gtrless f_u$ (10)
where S_h^* is the level of skilled employment at which V
is maximized. Hence, the slope of the marginal remittance
curve does not affect the direction of change of the optimal
skilled labour force employed at home when the unskilled
labour force changes. However, the optimum home employed
skilled labour force does depend on the number of unskilled
persons employed, independent of that slope, contra the state-
ment cited above.

It is worth providing some interpretation of (10).
Since in general $f_u < V$, (that is, the marginal product of
unskilled labour will be less than the average product of
skilled and unskilled labour plus average remittances), and
$f_{S_hU} > 0$, (that is, skilled and unskilled labour are comple-
mentary inputs) it follows that $\dfrac{dS_h^*}{dU} > 0$. Thus, the effect
of an increase in the total unskilled labour force will be to
reduce the optimal amount of emigration.

Referring to our diagram, an increase in U will shift M_h and $M_h - M_r$ up, and will reduce V on the assumption that unskilled labourers have a lower marginal product than the average product of all labour. Hence, the intersection of these curves, which represents the optimum, will shift to the right, meaning that more skilled labour will be held at home.

2. What happens when only the amount of skilled labour increases? Implicitly differentiating (7), we find

$$\frac{dS_h^*}{dS} = \frac{g_E}{\frac{L}{d^2 V/d S_h^2}} < 0. \qquad (11)$$

That is, an increase in the skilled labour force will result in a smaller amount of skilled labour being employed at home. This is obvious from the diagram. The increase in the skilled labour force will leave the $M_h - M_r$ curve unaffected (assuming a constant marginal remittance rate), but R/L and hence V will shift up for each amount of skilled labour employed because of a higher number of emigrants, and the intersection of the two will shift to the left.

3. What will happen when the labour force expands through proportional expansion of both skilled and unskilled labour? That S and U rise in proportion can be described by

$$S = a.U, \qquad (12)$$

where 'a' is some constant. Substituting this expression into the system, we again implicitly differentiate (7) and find that

$$\frac{dS_h^*}{dU} \gtreqless 0 \quad \text{as} \quad (fg_h u - ag_{EE})(\ L\ ) + V \gtreqless f_u + ag_E, \qquad (13)$$

assuming a constant marginal remittance rate ($g_{EE} = 0$), a comparison with (10) shows that while it remains likely that home employment of skilled labour will rise, it is somewhat

less likely than when unskilled labour alone increases because of the term ag_E. As in that case, the rise in unskilled labour will raise the marginal product of skilled labour, tending to lead to greater employment of skilled labour at home. However, this effect is counterbalanced to some extent by a rise in the R/L curve, as occurred in case 2 when only skilled labour rose.

It is more interesting to ask whether, when skilled and unskilled labour forces expand proportionally, home skilled employment will rise more than in proportion to the growth of the labour force. The condition for a rise in the proportion of skilled to unskilled labour employed at home is found by implicitly differentiating (7), still assuming (12). After simplifying, the condition can be written as

$$\frac{d\log S_h}{d\log U} = \frac{d}{dU}\frac{S_h}{S_h} \gtreqless 0 \text{ as } L. \ f_{S_h U} + V \gtreqless ag_E - f_{S_h S_h} \cdot \frac{L}{U} + f_U \quad (14)$$

when $g_{SE} = 0$. There is no simple interpretation of the condition but it is readily seen that it will depend heavily on the relative magnitudes of $f_{S_h S_h}$ and $f_{S_h U}$. The stronger the degree of diminishing returns to skilled labour, ($f_{S_h S_h}$

< 0 and large in absolute value) and the less complementary are skilled and unskilled labour in production ($f_{S_h U} > 0$ but small) the greater will be the optimal proportion of skilled labour emigrating.

Of one thing we can be certain, however: the amount of skilled employment at home will in general not be independent of the size of the labour force when the marginal remittance curve is held constant.

The third proposition that M & Y claim to derive from the model is that "the optimal level of migration will be less

if the objective is to maximize income per total population
(including emigrants)." (footnote 2 p. 333). This proposi-
tion too is wrong. The opposite can be shown to be true.
we can formalize this version of the model with the following
equations:

$$Q = f(S_h, U) \qquad (1)$$

$$T = t(E) \qquad (15)$$

$$E = S-S_h \qquad (3)$$

$$Z = Q+T \qquad (16)$$

$$N = S + U \qquad (17)$$

$$W = Z/N = \frac{Q + T}{S + U} \qquad (18)$$

where T is total earnings of emigrants and N is the total
citizen population residing both at home and abroad. W is
then the average income of all citizens and the objective
of the system is to maximize W. Note that remittances in
this model are merely transfers between individuals who are
equally weighted in the implicit welfare function. The
condition for a maximum of W is:

$$\frac{dW}{dS_h} = \frac{1}{N} (f^*_{S_h} - t_E) = 0 \qquad (19)$$

$f^*_{S_h}$ is starred to distinguish the optimal marginal product
of skilled labour when W is maximized compared with when V
is maximized. The condition is obvious -- migration takes
place until marginal earnings of skilled labour is equated
between home and foreign employment. Will maximization
of W result in more or less migration than maximization of
V ? We can find the answer by comparing the optima
represented by equations (8) and (19) Rewriting,

$$f_{S_h} - V - g_E = 0 \qquad (8)$$

$$f^*_{S_h} - t_E = 0 , \qquad (19)$$

and subtracting (19) from (8) we find

$$(f_{s_h} - f^*_{s_h}) = V + g_E - t_E = V - (M_t - M_r) \qquad (20)$$

where Mt is the marginal earnings abroad of an emigrant.

The second term on the right hand side of (20) (Mt - Mr) is the marginal ∧retained earnings of emigrants. Thus the sign of $f_{s_h} - f^*_{s_h}$ depends on whether or not the retained earnings abroad of emigrees is greater or less than the average income of all persons remaining at home. Recalling that all emigrees are skilled workers while the income at home is averaged over the mass of unskilled workers, it seems plausible to impose the condition $V - (M_t - M_r) < 0$. This is indeed a rather weak behavioural constraint; skilled workers will only migrate if their standard of living (retained income) abroad is at least as high as the average income of all residents of the home country. If this condition is imposed, then it follows that $f_{s_h} < f^*_{s_h}$ which implies that optimal migration will be greater if the welfare (income) of emigrants is taken into account.

This result sheds some light on the question of whether developing countries need to restrict free emigration. The answer depends on whether or not welfare of citizens living abroad enters the country's welfare function. If it does, and migration of skilled labour takes place so long as earnings are higher abroad, equation (19) demonstrates that free migration will maximize such a welfare function. On the other hand, according to condition (20) free migration will result in average income of residents of the country being lower than it would be if migration were restricted. Thus if only income of residents is important to the government, some restriction of migration will be desirable.

This is, of course, a very simple model. A complete evaluation of the desirability of migration restriction would have to include considerations of the effects of emigration on capital accumulation and appropriate allocation of investment between physical and human capital, as well as on intangible but important considerations such as respect for liberty of the individual.

Finally, we would like to point out that Michalopoulos' conclusion on the optimal size of population when the marginal product of labour abroad exceeds the maximum average product of labour at home is incorrect. Our Figure 2 reproduces his Figure 2, in which AP is the average product of labour (or average per capita income if we assume that the labour/population ratio is fixed) in country B, MP is the marginal product of labour in country B, and YY is the marginal product of labour in country A and labour is assumed to be homogeneous. According to Michalopoulos, "if there were no restrictions to migration, there would be an incentive for B's workers to emigrate until only OJ remained in B" (p. 136). This is clearly incorrect, for while the 'marginal product' of labour in B equals the marginal product in A, it is impossible (without subsidies from abroad, which are excluded from the analysis) that labourers in B will be paid this marginal product. Indeed, if the whole domestic product of B is paid to labour, the average earnings of labours will come to only JM, which is less than what they could earn abroad. In the circumstances shown, the optimal population in B will be nil - the case of Antarctica.

Perhaps it is worthwhile commenting a little more fully upon the use of the diagram, since similar confusions can arise in the analysis of other problems. There are two

basic arguments which might be used to justify the upward
sloping marginal and average product segments.   First, a
variable proportions argument that the amount of land,
capital, technology, etc. is held fixed and that if very
small amounts of labour are applied to this fixed quantity
of complementary resources, the marginal (and average)
products of labour will rise over some range.   On the
assumption of a constant returns to scale production
function, the marginal products of the fixed factors will
be negative in this range.   Insofar as it is possible
for the economy to apply its labour to only some of these
resources (which seems empirically reasonable since most
countries of the world, including even the poorest, contain
some unused land, mineral deposits, etc.), the relevant
average product of labour will be given by NR (corresponding
to the highest average product of labour) up to labour
input OD, beyond which the curves are as shown by Michalopoulos

Figure 2.

Labour Force (or Population)
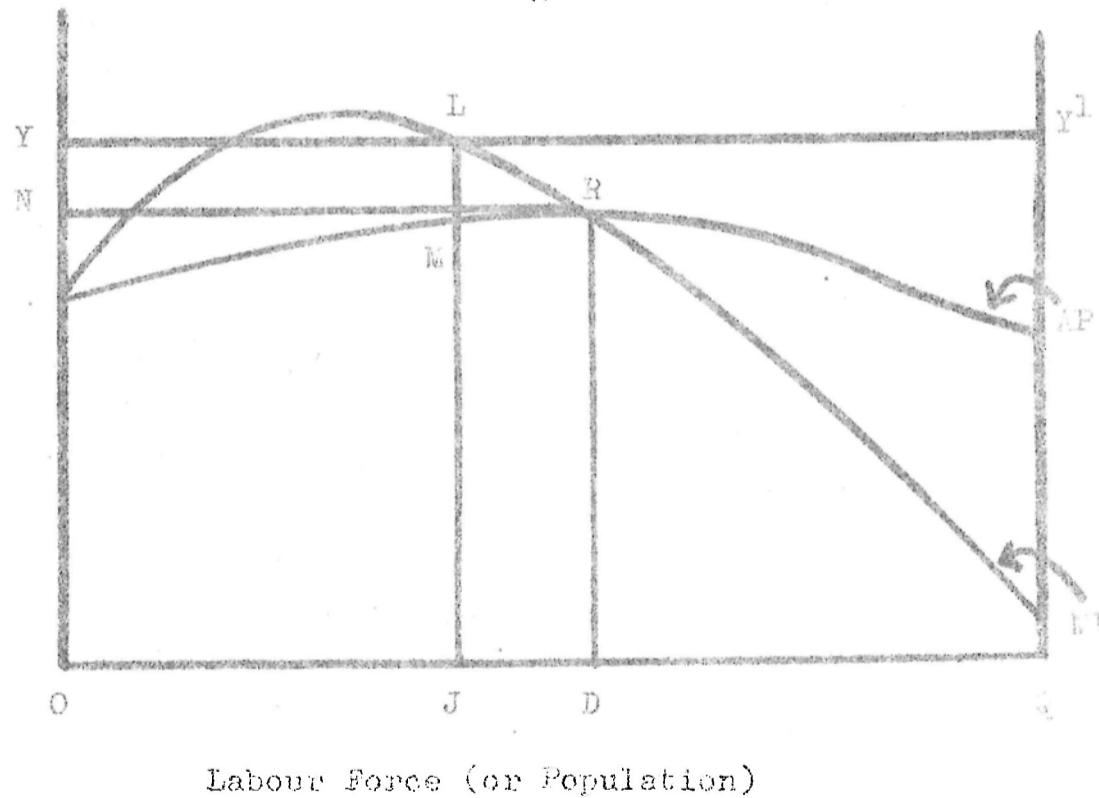
Second, what if the upward rising segments of the marginal and average product curves are explained by "increasing returns to the size of the economy"? There are several stories which could be told to account for this. However, they all have in common the notion that output per bundle of resources will rise as the economy expands. If the economy consists of atomistic decision - making units, no-one will find it worthwhile to take this relationship into account in making his decisions. The relationship will be external to firms (and labourers), but internal to the economy. (The common analogy would be the Marshallian analysis of economies external to the firm, but internal to the industry).

In this case, the MP curve will indicate the increment in output which will be associated with increasing the labour force by one unit (where the labour force is exogenously determined), when the amounts of all other factors adjust to the new amount of labour according to the "economic laws" of the economy under consideration. Its relationship to any observed locus of output/labour points will depend upon the nature of those "laws". [4]   The curve is only of practical relevance if we assume a central decision - maker who can control the labour supply (through birth control and health programmes, immigration and emigration control, etc.).[5]

F O O T   N O T E S

1. Benton F. Massell and Pan A. Yotopoulos. "A Note on
    Labour Migration", Kyklos, Vol. XXII - 1969 -
    Fasc. 2, pp. 331-3.

2. Constantine Michalopoulos, "Labour Migration and
    Optimum Population", Kyklos, Vol. XXI - 1968 -
    Fasc. 1, pp. 130 - 44.

3. A more general analysis of the welfare implications
    of the objective in a model of rural-urban migration
    with unemployment is provided by John R. Harris and
    Michael P. Todaro, "Migration, Unemployment and
    Development: A Two-Sector Analysis", American
    Economic Review, (forthcoming, 1970).

4. The relationship between this curve and the AP curve
    and the "operating" MP and AP curves (shorn of the
    externality element) will be analogous to the
    relationships among long run marginal and average
    cost, and short run marginal and average cost, in
    the conventional analysis of the cost curves of
    the firm.

5. For the policy implications of maximizing an objective
    function similar to that proposed by Michalopoulos,
    under this type of production function, fixed (non-
    labour) factor prices, and additional assumptions abc
    labour supply (supply of effort by residents, degree
    discrimination possible in payment of immigrants, etc
    see Evsey D. Domar, "The Soviet Collective Farm as a
    Producer Co-operative", American Economic Review, Vc
    LVI, No. 4, Part I, September, 1966, pp. 734-57.