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THE ROLE OF MATHEMATICS IN ECONOMICS

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In his discussion of the role of mathematics "in the pure theory of our science" Schumpeter wrote: "But the use of figures ... or of formulae ... or even the restatement in algebraic form of some result of non-mathematical reasoning does not constitute mathematical economics: a distinctive element enters only when the reasoning itself that produces the result is explicitly mathematical"[1]. I am not sure if this definition would be easy to apply (for example to Sir John Hicks' work), but in any case I wish to set my bounds wider. I want to discuss the uses of mathematics and statistics including the elementary level; and in relation to teaching and applied studies as well as "the pure theory". The case of mathematics and statistics are, it is true, somewhat different, in that the use of statistics has not been an issue much disputed recently. But I include them together as I want to finish up with a suggested minimum level of necessary quantitative techniques. This general approach will mean that much of what I say will be superficial but it will, I hope, remind you of the main issues and provide a stimulus to discussion, particularly in relation to teaching possibilities here.

No one will argue, I assume that mathematics and statistics have not become increasingly important in economics. But since we may as well be quantitative here as elsewhere I will cite the interesting table prepared by Stigler.[2] This analysis comprises 1726 articles appearing in five journals (Q.J.E.; J.P.E.; A.E.R.; R.E.S.; Econometrica) in pairs of years from 1892/93 to 1962/63.

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(1) J. A. Schumpeter: "History of Economic Analysis," p. 954/5.

(2) G. J. Stigler: "Statistical Studies in the History of Economic Thought", p. 48 in "Essays in the History of Economics".

Note. This paper was given at a seminar of the Institute of Development Studies and the Department of Economics. Minor revisions have been made, but no attempt has been made to alter its "oral" form.

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<u>The level of technique in articles (% age)</u>

Years	No special technique	Geometry	Algebra	Calculus or more
1892/93	95	3	2	...
1902/03	92	1	6	...
1912/13	98	1	1	...
1922/23	95	1	2	2
1932/33	80	1	8	10
1942/43	65	8	6	21
1952/53	56	6	7	31
1962/63	33	8	13	46

An article "is ranked by its most elaborate technique, so the use of geometry in 7 per cent of the articles in 1962 - 63 means that only this share used geometry without also using algebra or calculus". We may note that the use of mathematics scarcely changed from 1892/3 to 1922/3, but that there was a rapid increase in the proportion using mathematics during the period 1932/3 to 1962/3 - from 20% to 67%. Further the importance of calculus has grown rapidly: it was scarcely used until the thirties, but now almost half the articles printed employ it.(3)

If we turn our attention to quality, an impressive list of economists who used (or supported the use of) mathematics and statistics can be compiled, even if we exclude current practitioners: Jevons, Walras, Marshall, Edgeworth, Pareto, Barone, Wicksell, Fisher, Moore, Schumpeter, and Keynes.

But despite this there is still a lingering antipathy among some economists, particularly British economists, to the use of mathematics.

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(3) The use of this table needs several qualifications, and thus provides an illustration of the difficulties that often occur in the use of statistics. The table is discussed from this angle in the Appendix.

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This may be due partly to the influence of Marshall. It is well known that although much of his own work depended on a mathematical framework, he tried to discourage its use - the letters to Bowley printed in the Memorials may be cited. More surprisingly, Keynes was also not enthusiastic: in the debate with Tinbergen his tone seems to be destructively not constructively critical, although of course a number of his particular points were well taken. Coming nearer to our times a leading British economic statistician (in, of all places, his Newmarch Lectures) opens his discussion with an avowal that "I have never drawn up a linear programme or inverted a matrix: I am a stranger to model building; and I cannot oscillate a time series or properly analysis a variance".(4) Is it my imagination, or does one hear a note of mock humility, of inverted snobbery?

However, this is not entirely a British matter. According to his biographer, Wicksell (who was initially trained as a mathematician - as were Marshall and Keynes) finally "forbad himself to read mathematics, being afraid to carry the mathematical treatment of economic problems too far".(5) And earlier, when Wicksell and Cassell were applicants for the same chair, the "expert" report opened with an objection to both of them: "They come to economics from mathematics. History, law, or even (sic!) practical business experience are to be preferred".(6)

Before discussing the advantages and disadvantages of the use of mathematics we may dispose of two straw men. First, opposition from persons who are not well placed to judge and who rest their opinion on that of someone else. Thus Fay: "I am a fool at mathematics; and on the one occassion, when we talked about it, he (Marchall), the great mathematical economist, declared with impatience that this part of economics was now-a-days much overdone. The tonic has lasted me from that day to this".(7) I find this second-hand denigration particularly distasteful

(4) E. Devons "Essays in Economics" p. 105

(5) T. Gerdlund: "Life of Knut Wicksell".

(6) E. Lindahl - Introduction (p.21) to K. Wicksell: "Selected Papers on Economic Theory".

(7) A.C. Pigou (ed.) Memorials of Alfred Marshall p. 77.

Another argument we may mention only to discard is that economics must shun methods that might weaken its effect on the general public. This is a particularly Marshallian point: as Edgeworth put it, he "deferred to the prejudices of those whom he wished to persuade".[8] This attitude would obviously cripple the development of the subject; but in any case much of non-mathematical discussion would now be disqualified as well.[9]

What then are the advantages of using mathematics and quantitative methods? First mathematics is an extremely powerful method of extracting all the "juice" out of a set of assumptions. Allied to this is the facility it gives in following through long chains fo reasoning. Shove, in his well known article on Marshall, shows how much of the structure of the Principles can be related to putting the Ricardian theory into mathematics, carrying the argument forward and filling the gaps. Similarly much of the later development with Cobb-Douglas production functions has come from manipulation of the basic formula.

Mathematics frequently leads to formulations which make for fruitful classification. This idea of elasticity with the ranges between 0, 1, and infinity is an example. Another example is the setting of conditions for the equilibrating or non-equilibrating tendency of a dynamic system.

Another advantage is ease in handling complicated systems, particularly when more than two variables are concerned. It is now common to see comments such as this exchange at a recent I.E.A. conference: Professor Johnson "agreed with Sir Roy's general point, but insisted that a third factor analysis would require explicit mathematical treatment Sir Roy Harrod "agreed on the need for mathematics".[10] The mathematical method also has advantages when compound rates of change have to be dealt with.

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(8) Ibid. p. 67

(9) But the attitude is still very much alive. Cf. "Professor Johnson stressed that we wanted a simple theory, not least because we had to explain to others". Report of debate at the I.E.A. Conference, 1961: "International Trade Theory in a Developing World" (ed. Harrod and Hague)p.401.

(10) Ibid. p. 423.

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Mathematics is also unambiguous. It can be followed step by step, and the points at which the various assumptions become crucial can be seen clearly. The mathematical method generally leads to a more explicit treatment of assumptions, although this is not always so. Stigler discussing the controversy on the exhaustion of the product under marginal productivity theory (which was one of the first economic arguments formulated almost entirely in mathematical reasoning) considered that the "entire argument rested on differences between the implicit assumptions of the various participants".(11) It has even been argued that too plain a setting out of assumptions is dangerous,(12) but this seems to confuse procedures which are needed to get an argument right with those that might be used to get it accepted at a lower level of sophistication.

This clarity of mathematics leads to ease in communications - see for example the correspondence between Walras and Pareto and Walras and Barone in the recent monumental edition of Walras' correspondence prepared by Professor Jaffe. Further, given the basic mathematical knowledge among students, it facilitates teaching.

The advantages of the use of statistics may be summed up by saying they confront the theory with the real world (or at least that part of it that can be put in statistics - a print to which we shall return). The possibility of using statistics tends to increase as theory is cast in a mathematical form, but there is not a necessary connection.

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(11) G. J. Stigler.

(12) "There is of course, always a danger of losing sight of one's implicit assumptions in analytical uses of a body of theory, but the reactions to this section of Douglas' Theory of Wages (i.e. that where he sets out ten implicit assumptions) have perhaps illustrated that there are concomitant dangers in making all assumptions explicit". A. M. Cartter "Theory of Wages and Employment" p. 39.

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What are the disadvantages? Some of the criticisms deal rather with misuses of the method than the method itself. Of course mathematics does not guarantee correctness or relevance - we have all ruefully contemplated von Thünen and his tombstone engraved with $\sqrt{ap}$ - but neither does any other method of logic. Further, techniques which are acceptable at one stage have to be reconsidered subsequently: moving averages cannot be employed now as they were before Slutsky showed they could introduce fluctuations:[13] users of regression and correlation have to consider the possibilities of multi-collinearity.[14] But no science is born fully mature.

Nevertheless, the power of mathematics and statistical techniques often tends to foster excessive self-confidence amongst their practitioners. A judge once said to a barrister "This Court may sometimes be in error but it is never in doubt". I get the impression sometimes that because mathematical economists are never in doubt they believe they can never be in error. It seems likely for example, that if Moore had not been so enamoured by his methods, he would have noticed that his demand curve for pig iron was in fact a supply curve.

One criticism that/is justified is that since mathematical techniques are likely to have relatively greater effect in some fields than others, then these fields may have a disproportionable amout of effort put into them, especially in so far as mathematical economics becomes a "band wagon", or as it attracts an undue proportion of the brighter graduates. Even within a specific field some aspect or model may be stressed because it is susceptible to mathematical manipulation. For example, the universal employment of the Cobb Douglas production function is rather surprising after all the criticism to which it has been subjected.[15]

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(13) E. Slutsky. "The Summation of Random Causes as the source of Cyclic Processes." Econometrica Vol. 5 No. 2, Apr. 1937.

(14) R. Frisch "Confluence Analysis" 1934.

(15) Recently the C - D function has been replaced by a constant Elasticity of Substitution (CES) model; but it appears that the new model also operates under tight restrictions.

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Another criticism is that mathematical techniques lead to an undue concentration on refinements. This may be partly a reflection of a dispute between mathematician and statistician, and comes over into economics on questions of the adequacy of statistical data for decisions about an economic theory. To cite an example from statistics, some survey statisticians could well spend less time on refining their sampling techniques and more on getting good basic data. There is also a tendency to overlook the deficiencies of the methods too readily and to use inappropriate techniques. As Stigler remarked "The universe became linear in 1946"!

The major argument however is that some important aspects of economic reality cannot be handled in statistical or mathematical form. This is not only a question of misuse or forgetfulness: it is felt that the use of statistics and mathematics predisposes investigators to neglect non-quantifiable aspects or to employ unsuitable statistical strait-jackets.and mathematical models. Thus Devons, in the essay I have already quoted, ridicules research workers who calculated that "the loss due to a death of a person is equal, gross, to the expected production of that person during the remainder of his life had he not been killed ... and the net loss, the difference between this and his expected 'consumption'".(16) Certainly, on this basis, road hogs might be licensed as public benefactors in societies where it has been argued that the marginal productivity of a large number of workers is little more than zero. Similarly the calculations of Enke and others suggesting that you pay people not to have children seem mildly ludicrous - Gilbertian rather than Swiftian. Another example, which has caused serious harm, is the use of per capita national incomes as measures of welfare in an international scale. Usher's calculations(17) suggest that U.K. per

(16) Op. cit. p. 107

(17) D. Usher: "The Thai National Income at U.K. Prices". Bull. of the Ox. Inst. of Ec. & Stats., 25 (3), Aug. 1963.

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capita income is about $2\frac{1}{2}$ times that of the Thai figure: the U. N. figures suggest the U. K. figure is 14 times as large.

It is of course true that the mathematical economist has to abstract and cannot deal with all factors, but the ordinary economist is in a similar position. The criticism made of the mathematical economist by some of his fellow economists is that made of all economists by many non-economists. There does not seem to be strong evidence that the simplifications used by the mathematical economist are, per se, more likely to lead him into error than the simplifications used by other economists: and he usually has less room to wriggle around in when presented with the "inconvenient fact". It is interesting to note that the argument of neglected variables is used by "orthodox" Marxists in the Soviet Union against their "revisionist" colleagues.

But this line of argument has considerable weight in considering the pattern of teaching, since it is more difficult to get over to the student the imponderables than it is to get him to accept the right/wrong answers from mathematics: and he may easily think his mathematics is taking him further than it is.

Finally, a rather peculiar point made by Harrod. "There may", he writes in the Introduction of Economic Essays,(18) "be value in ambiguity. The doctrines of competition purported to relate to a real world ... There was the danger ... that in the very process of sharpening analytical concepts, the mathematical school(19) might render them inapplicable to the real world, or to some part of it." This could be interpreted as a plea for operating with crude tools - vague concepts and unstated assumptions. If so, it is analogous to

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(18) p. viii.

(19) The development of the perfet competition assumptions is a long story, but they were refined most fully by Clark and Knight who were not mathematical economists.

the argument cited at (12) that it may be dangerous to make your assumptions explicit; and represents what, in Air Force language, might be called the "brute force and ignorance" or "rubber hammer" school. But if when you sharpen your tools (this does not fit the rubber hammer!) you find the relevance of the theory is not as great as you thought, you have to improve your theory: there is little point in carrying on with a crude theory and thinking it has more basis than it in fact has.

To sum up, there may be dangers attendant on the use of the mathematical method - but then there are dangers in crossing the road. The approach is here, and it is here to stay: everything indicates that it will become relatively more important in both theory and practice. What the arguments show is the need for caution.

The acerbity would largely disappear from the dispute if more of the economists using mathematical methods followed Sir John Hick's practice, outline in the Preface to "Capital and Growth": 'The theory, as I understand it, is in essentials a mathematical theory; but I have been anxious that in my statement of it I should keep myself writing economics. I have tried to keep a firm eye on the economic meaning; and to be on the look-out for devices (there are several such that are available if one looks for them) by which the purely mathematical points can be by-passed. Though I have allowed myself a freer use of algebra than would have been appropriate in the old days (such as those of Value and Capital) it is only in a few places that the algebra does more than express, a shade more sharply, what can be (and generally is) also expressed in words. I do not think that interested readers, even those who reckon themselves to be non-mathematicians, will usually find much difficulty with it. There is nevertheless a danger that by adopting this method of exposition, one should lose touch with the mathematical work. In the Appendices (especially B and C) something is done towards building a bridge.'

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As indicated here, to some extent the mathematical work must proceed independently in specialist contributions. What is needed are more "bridge builders" and in the nature of the situation most of them must come from the mathematical side.

In early days one did not need much equipment to be a mathematical economist. Barone said that every normal and normally educated person could acquire what was needed by the spare time work of about six months:[20] and the classical sources - Bowley's "Mathematical Groundwork of Economics" and Pareto's article - are short and use little more than fairly elementary calculus. But this has changed now: partly because of new developments in maximising and minimising techniques(particularly in project appraisal); partly because of greater interest in problems of general rather than partial equilibrium; and partly because of the increase in the statistical data available and the consequent use of more refined statistical analysis.

A more detailed list of the developments that together, and reacting on one another, have caused the extension of the mathematical and statistical background required is:

(1) The invasion by the mathematicians, particularly von Neumann and Wald, and the power of the methods they introduced.

(2) The Keynesian analysis, with its aggregates susceptible to measurement through social accounting.

(3) Input/output models and their relations with social accounting and with Walrasian general equilibrium theory.

(4) The increase in information due to the general development of communications in society, the greater role played by Government, and the establishment of effective sample survey techniques.

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(20) Schumpeter: op. cit. p. 955. And Walras failed the mathematics examination for the Ecole Polytechnique twice.

(5) The experiences of economists in World War II.

(6) Decision theory, particularly linear programming techniques: the explicit treatment of uncertainly leading, most recently, to the introduction of Bayesian decision theory.

(7) The advent of computers.

(8) The importance of development planning, and the growing intervention by Government in economic matters generally.

How can we view teaching possibilities against this background? I should first emphasise that I am looking at the short term. A number of our problems may disappear when students trained in the "new mathematics" start coming along (though, of course, by then we may be needing a still newer approach). At the moment however, we have difficulty in getting students with even the "old mathematics" since most of them so qualified go to the science faculties.

I think students must be divided into two groups. The majority must have courses leading to a carefully selected minimum knowledge. Part of this course should be revision - logarithms, indexes, progressions, simple algebraic manipulations and equations. It will be found some students haven't even dealt with some of these items before. Part should be elementary mathematics related to economics: simple co-ordinate geometry including the hyperbola; total, average, and marginal concepts; tangents: the <u>idea</u> of the differential and its relation to the rate of change and the tangent: the exponential curve; elasticity and logarithms. The statistics section should deal with sources and methods. The methods should emphasise different types of distribution (with particular discussion of skew distributions frequently occurring in economics) and variability. It should also spend a considerable amount of time on index number problems, elementary regression, and elementary interpolation and graduation.

I shoudl stress the teaching of graphical techniques which are quite efficient at this level - for example, it is possible to introduce the basic ideas of multivariate regression this way. Further, the

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graphical methods will frequently be adequate for dealing with problems at a practical level in Government and business.

It will be seen that what I am suggesting is already in embryo in the present teaching arrangements at Nairobi. There are two points: I think quantitative methods to this level should be compulsory. We should not let loose on Government and business, students who haven't at least this ability. Secondly I think people with this background can be very useful - they are likely to be more useful in African conditions than those more highly specialised types who can't get their feet on the ground (note: I am not saying they are necessarily more useful than the specialised types who can and do adjust). What we hope to produce are economists with a feeling for the relative importance of numbers: but with more than the knowledge Keynes suggested to an administrator as sufficient - which was to know where to put the decimal point!

This deals with the majority. The minority - who will have to be carefully selected and should preferably already have A level mathematics - can be taken farther. They should not, I think take the elementary course (except on statistical sources) but should start at a more advanced level from the beginning. In mathematics they need some calculus, linear algebra, difference equations, and probability. In statistics they need - in addition to the obvious basic structure - special attention to multivariate regression, time series, sampling survey techniques, and an introduction to model building.

How far should we hope that what I have prescribed for the minority should eventually become majority fare? Leaving aside the problem of the type of student coming forward, we come up against an issue of the allocation of resources. The current ideal economist should know a considerable amount of mathematics, but he should also know a lot of other things - about history, about institutions etc. Nevertheless I think there is a strong case for mathematics at this stage of his training. Although I have seen no reports of investigations to prove the thesis, it is generally accepted that it is easier to imbibe mathematics when one is young. On the other hand it is probable that one gets more from studying history, sociology, and organisational problems when one is older and has more experience. Mathematics is in any case a key to many of these other studies and thus gives the student a greater degree of choice with regard to his future development.

Further, a danger will confront non-mathematical economists in the future. It is that, no matter what their speciality is, they will find a considerable and increasing part of developments inaccessible. No mathematical training at undergraduate level can of course provide sufficient for current or future requirements - and in any case no - one can be sure which aspects of mathematics may become important in the future. But students should at least be given a sufficient foundation on which they can build as requirements arise in the development of their careers.

It will be noticed that the requirements suggested do not require as much mathematics as that needed by the science or engineering student. But there is not time for more for those students who will not go beyond a first degree. However, for persons who are going to become teachers of the higher school forms and for persons who could clearly go on to a postgraduate degree in economics, there is a lot to be said for a joint degree in economics and mathematics but further discussion, including a suitable syllabus, would take us too long.

Appendix

The table concentrates on journals, not total activity (books, reports, papers etc.) but there is no reason to believe this distorts the picture seriously. The period is acceptable since it covers the years in which Academic Economic Journals have thrived. The selection of the years ending in 2 and 3 in each decade is not likely to produce any but irregular variations in the trend.

There are, however, only 5 journals considered. They are all published in America (although one of them is the organ of an international society). Contributors come from the world over but the selection policy is heavily influenced by the place of publication and the predominantly American character of the editorial boards. There is little doubt that mathematics in economics has become more popular in America than elsewhere, and it is likely that a more widely distributed group of journals would have shown a slower rate of increase.

Further, "Econometrica" did not start until 1933. It is of course the journal which is specifically concerned with the application of mathematics and statistics to economics, and practically all the articles there published use quantitative techniques. The inclusion of this journal half way through the series must contribute to the very rapid change shown in the last 30 years. It might be argued that if this journal had not existed then many of the articles published there would have appeared in the other journals. This may of course be true, but it is not so clear that they would have displaced non-mathematical articles in those journals. Once more, the inference is that the table may overstate the increase in the use of mathematical techniques.

There is one further point: the scale of difficulty used in the categories, "Geometry", "Algebra" and "Calculus or more" may not be entirely appropriate. As indicated briefly in the paper the key feature of mathematical development since the war has been the growing

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use of analytical methods drawn from branches of mathematics other than calculus (linear algebra, set theory, fixed point theorems, and existence proofs). These often require a greater degree of mathematical sophistication than the calculus that is used. The table therefore does not bring out clearly one aspect of the change that has occured.

I am not suggesting that the table is not useful - I would not have cited it if I did not think it meant something. But it seemed worth while to use it as a minor illustration of the way in which it is necessary to supplement the apparent precise results of a statistical analysis with other considerations.