

A Theory of Classification

Abstract

The decision to classify line items as special items or core earnings is designed to signal item persistence. Using information other than persistence, such as the sign of the line item, is known as classification shifting. Although classification shifting is widely seen as earnings management, we demonstrate in a simple classification and reporting game that the commonly observed reporting patterns arise when no misreporting occurs. In fact, signaling persistence rather than misreporting explains additional empirical findings that a misreporting story cannot address. Overall, strategic classification imposes a surprising amount of structure on reports, and we argue that it would be ill-advised to set policy based on a presumption that classification shifting is earnings management.

Keywords—classification shifting; earnings management; signaling

1 Introduction

Firms have some discretion in how they classify items, and they use it. Losses are more likely to appear as special items, gains as top-line revenue. This asymmetry is widely presented as evidence of earnings manipulation (for early arguments along these lines, see Gonedes, 1975, Ronen and Sadan, 1975, Barnea et al., 1976; more recent examples include McVay, 2006, Fan et al., 2010, and Poonawala and Nagar, 2019). The Securities and Exchange Commission appears to find this evidence convincing, as it has issued several Accounting and Auditing Enforcement Releases based on apparent classification shifting (see McVay, 2006; Alfonso et al., 2015; Abdalla, 2016; Abdalla and Clubb, 2016, for examples and discussion).

Attempts to find direct evidence of misleading classifications, however, point to a different story. Riedl and Srinivasan (2010) find that signaling, rather than deceiving, better explains firms' classification decisions. Others have found that Big-N auditors do not systematically alter classifications (Abernathy et al., 2014), and that corporate governance does not show a consistent relationship with classifications (Joo and Chamberlain, 2017). These findings starkly contrast with observed effects of stricter audit and tighter governance on earnings manipulations, which are associated with reduced apparent accruals manipulation (Klein, 2002; Chen et al., 2015) and substitution with harder to detect real earnings management Cohen et al. (2008). In what follows, we demonstrate that firms signaling their performance through classification decisions leads to the asymmetry that the empirical literature documents. To state this more pointedly, ignoring isolated examples, classification shifting is not earnings management.

We present our argument in a game between a firm and a representative investor. The

firm's line items are hard information, which the firm releases truthfully. Additionally, the firm has soft information about how persistent its items are. The firm ranks each item by persistence, and decides on a threshold. Items less persistent than the threshold are placed in a different section of the income statement, labeled noncore earnings (which we sometimes call special items). Those at least as persistent as the threshold are labeled core earnings. The firm can choose any threshold it likes, but cannot credibly disclose its threshold. Upon seeing the report, the investor prices the firm at its expected value.

It is easy to see that a firm maximizes its market value by convincing the investor that its positive items are more persistent than its negative ones. Firms with more persistent good news therefore try to separate from those with less persistent good news. We show that a surprising degree of separation occurs in equilibrium.

Two assumptions in our analysis are worth discussing here. First, the restriction to binary classifications is typical in practice (on this point, see Dye, 2002), and matches common empirical specifications. Moreover, we show below that this assumption is far less restrictive than it might seem.

The second crucial assumption in our argument is that persistence is soft information, but an outside authority can make relative comparisons. For example, an outside authority might not know how persistent sales revenue is, but could judge sales revenue to be more persistent than gains on disposals of fixed assets. By treating the firm in our model as choosing a threshold, we are assuming an outside authority can require a firm's classifications to respect the persistence ranking, a property we call *coherence*. Requiring coherence is in keeping with Generally Accepted Auditing Standards (GAAS), which require auditors to treat any classifications that mislead about expected values as a misstatement (see Nelson et al., 2002). Similarly, McVay (2006) observes that deliberate misclassification violates

Generally Accepted Accounting Principles (GAAP). In our setting, deliberate misclassification corresponds to violating coherence, and as our goal is to study strategic classification that does not violate GAAP, we therefore impose coherence throughout.

It turns out that requiring only coherence, rather than a bright-line standard, is a weak requirement in the following sense: as the firm's number of items increases, a report that is coherent always provides more information to investors than a bright-line standard. An investor who knows that two given items are both more persistent (or both more transient) than a given threshold does not receive any information on the ranking of these items. We show that knowing the ranking is more valuable to investors than knowing which side of a bright-line each item is on unless the number of items is quite small. In fact, the relative precision of information to investors from a bright-line standard, relative to one that is only coherent, becomes arbitrarily small for sufficiently many items.

The structure of the rest of this paper is as follows. Section 2 provides background on classification shifting. Section 3 describes the model and formally defines coherence. Section 4 compares two benchmarks. The first shows what the investor learns if the firm could commit to an absolute persistence threshold, thought of as a bright-line standard. The second shows what the investor would learn if the firm could fully reveal the persistence-ranking of its items. Section 5 shows the amount the firm can signal if it classifies coherently. Section 6 provides concluding remarks. All proofs are in Appendix A.

2 Background and Framework

Generally accepted accounting principles (GAAP) allow firms some discretion in classifying items in the financial statements. Discretion, in turn, has become synonymous with earnings

management. To clarify, we mean earnings management in the sense of Schipper (1989, 92), who defines earnings management as “purposeful intervention in the external financial reporting process, with the intent of obtaining some private gain.” Taking an informational rather than a true earnings perspective, Schipper views earnings as information provided to external end users, and earnings management as interventions that signal-jam, in contrast to interventions that facilitate the financial reporting process.

There are three behaviors that are widely considered methods of earnings management: Accrual-based earnings management, real transaction-based earnings management, and classification shifting. We focus on the third method: classification shifting.

Unlike accrual-based management and the manipulation of real activities, classification shifting does not change GAAP earnings. Because classification shifting does not affect the bottom line, and because it depends on item persistence (which is soft information), it is widely argued that auditors and other authorities do not scrutinize the composition of GAAP earnings as closely as total bottom-line earnings.¹ As evidence firms taking advantage of this lack of scrutiny, studies commonly point to the fact that special items are disproportionately negative (e.g., Gonedes, 1975; Ronen and Sadan, 1975; Barnea et al., 1976; McVay, 2006; Fairfield et al., 2009; Barua et al., 2010; Fan et al., 2010; Lee, 2012; Behn et al., 2013; Abernathy et al., 2014; Lail et al., 2014; Baik et al., 2016; Fan and Liu, 2017; Malikov et al., 2018; Cain et al., 2019; Fan et al., 2019).

To the extent that classification decisions are not subject to any outside discipline, the classification itself might simply be viewed by investors as a form of cheap talk. If so, classification shifting could not qualify as earnings management in Schipper’s sense. Yet classification

¹See Nelson et al. (2002); McVay (2006); Barua et al. (2010); Behn et al. (2013); Abernathy et al. (2014); Alfonso et al. (2015); Malikov et al. (2018), and Poonawala and Nagar (2019) for this assertion.

decisions empirically appear to be informative to investors, despite shifting. Empirical studies present evidence that investors weight special items differently from core earnings (Haw et al., 2011; Alfonso et al., 2015; Baik et al., 2016), and that investors understand that firms generally report more persistent items closer to sales on the income statement (Lipe, 1986; Elliott and Hanna, 1996; Bradshaw and Sloan, 2002; McVay, 2006). In sum, investors view classifications as communicating information about persistence. Knowing this, firms have incentive to manipulate how they classify items.

For these reasons, classification shifting is widely viewed as deliberate misclassification of items within a financial statement.² The concept of misclassification, however, requires some attention. If an item’s persistence is subjective or otherwise soft information, it may be difficult to say what it means for the firm to have misclassified it. One approach is to make relative comparisons, which require may be far more objective. An outside authority could assess whether one item is likely to be more persistent than another, and can use this relative comparison. We say that the income statement is *coherently* classified if it respects the persistence ranking. That is, if the firm classifies a given item as noncore, then the firm must also classify all items likely to be less persistent as noncore.

Once we have clarified what it means for a firm not to misclassify, we turn to our main question: if a firm classifies coherently—that is, if it does not misclassify—do its equilibrium classification decisions match the patterns observed in the empirical literature? Indeed, they do. Negative items are more likely to show up as noncore earnings, and have a greater impact on earnings than positive noncore items.

The driving force is signaling. If possible, firms with the same net income but different

²See McVay (2006); Barua et al. (2010); Fan et al. (2010); Haw et al. (2011); Lee (2012); Abernathy et al. (2014); Alfonso et al. (2015); Fan and Liu (2017); Malikov et al. (2018), and Poonawala and Nagar (2019) for this assertion.

levels of persistence of their line items classify their items differently. Similar to Milgrom and Roberts (1986); Okuno-Fujiwara et al. (1990); Shin (1994, 2003), firms classify in order to distinguish themselves from firms with comparable income but less persistent good news, and investors interpret the classifications skeptically. By pushing bad news into special items and good news into core earnings, a firm indicates that its bad news is less persistent than its good news. A firm with the same income but more persistent bad news could not mimick without violating coherence. For this reason, there is no pooling equilibrium, and although full separation may not always be possible, firms in equilibrium separate as much as they can. As long as an auditor or court or other authority can impose coherence, differences in classifications make investors more informed, and the investor responses to classifications are a result of reduced information asymmetry.³

3 The model

There are two players, a representative investor and a firm. The investor aims to value the firm at its expected net present value, given the firm's financial report and discount rate $r \in (0, 1)$. The firm wants to maximize its market value. Both players are risk neutral and have rational expectations.

The firm's information has two parts. First, the firm has n current period items, x_1, \dots, x_n , each equal to 1 or -1 (similar to the coding in Arya et al., 2000)). Keeping the magnitude of the items the same is without loss of generality: if we had $x_i = 2x_j$, we could split x_i

³Several studies have likewise questioned whether discretionary accruals are evidence of earnings management, or if signaling is often a more plausible explanation. See Subramanyam (1996); Ball (2013); Bertomeu et al. (2019). For a related analysis showing that patterns commonly associated with earnings manipulation arise as statistical artifacts, see Hemmer and Labro (2019).

into two identical items, each of the same magnitude as x_j .

The firm publicly reports these items before the investor prices the firm. Denote the income statement as $x := (x_1, \dots, x_n)$ and net income as $I := \sum_{i=1}^n x_i$. Because the report is public and truthful, the prior distribution of \tilde{x} is inconsequential for most of what follows. Nevertheless, imposing a prior will be useful below for some comparisons, and we accordingly assume that $Pr(\tilde{x}_i = 1) = 1/2$ and that the \tilde{x}_i are independently drawn.

The second part of the firm's information is a private, n -dimensional signal $\tilde{\alpha}$ about the persistence of each item. For each $i \in \{1, \dots, n\}$, $\alpha_i \in [1, 1 + 1/r]$. If $\alpha_i = 1$, then x_i is completely transitory, and if $\alpha_i = 1 + 1/r$, then x_i is a perpetuity. Generically, α_i is the present value factor of the i^{th} item, including the current period's contribution.

For simplicity and with no loss of generality, normalize the firm's initial value to zero. To focus on the information content in each item, we treat the $\tilde{\alpha}_i$ as independent and identically distributed, and for convenience we assume a uniform distribution. Given x and α , the firm's present value v is

$$v = \sum_{i=1}^n \alpha_i x_i$$

Let \tilde{v} represent the investor's prior value of v . After receiving the report x , the investor forms posterior beliefs about $\tilde{\alpha}$ and updates its estimate of \tilde{v} . The true value of α is soft information, which the firm cannot credibly disclose. However, the ranking of the individual components $(\tilde{\alpha}_1, \dots, \tilde{\alpha}_n)$ can be confirmed.⁴ Although in principle the firm could disclose this ranking, we assume that for exogenous reasons it cannot do so, in order to make our

⁴Cianciaruso and Sridhar (2018) also combine hard information with verifiable properties of soft information. They address a different issue from ours: they are interested in the interaction of voluntary and mandatory disclosures, whereas our focus is mandatory disclosure with discretion in classifications.

classifications comparable to those in the empirical literature.

Instead, the firm signals information about α by classifying items in x . To do this, the firm partitions $\{1, \dots, n\}$ into two subsets, H and L . It lists $\{x_i | i \in H\}$ at the top of the income statement and $\{x_i | i \in L\}$ at the bottom of the income statement. We interpret H as core income and L as special items. This corresponds to the empirical finding that investors weight items based on income statement placement (Lipe, 1986; Elliott and Hanna, 1996; Fairfield et al., 1996; Bradshaw and Sloan, 2002; Burgstahler et al., 2002; McVay, 2006; Bartov and Mohanram, 2014). We require the classification to be *coherent*, i.e., every core item must be more persistent than every special item.

Definition 1. *The partition $\{L, H\}$ of $\{1, \dots, n\}$ is coherent if, for every $i \in L$ and $j \in \{1, \dots, n\}$, if $\alpha_j \leq \alpha_i$, then $j \in L$.*

Definition 1 implies that, for any $i \in H$ and any $j \in \{1, \dots, n\}$, if $\alpha_j \geq \alpha_i$, then $j \in H$. So, a classification is coherent if the firm sorts the entries of α and cuts the sorted list.

The investor cannot distinguish the items except by their classification and whether they are positive or negative. Therefore, the total gains and losses in each classification are sufficient statistics for the information the investor receives:

$$x = (x_H, x_L), \quad \text{where } x_H := \left(\sum_{i \in H} \max\{x_i, 0\}, \sum_{i \in H} \min\{x_i, 0\} \right)$$

$$\text{and } x_L := \left(\sum_{i \in L} \max\{x_i, 0\}, \sum_{i \in L} \min\{x_i, 0\} \right)$$

For example, suppose a firm with 10 positive and 10 negative items classifies 7 positive and 4 negative items as core. Then its report is $((7, -4), (3, -6))$. Total firm income is

$10 - 10 = 0$, core income is $7 - 4 = 3$, and net special items are $3 - 6 = -3$.

Given the income statement and classification, the investor values the firm at

$$E[\tilde{v}|x_H, x_L]$$

Figure 1 summarizes the sequence of events.

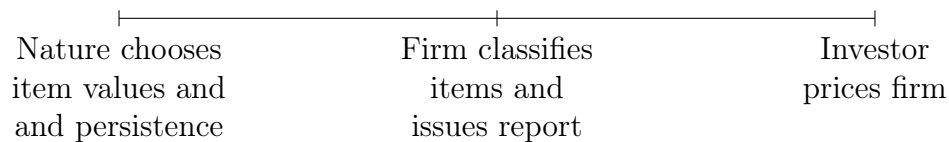


Figure 1 – Sequence of events.

4 Benchmarks

4.1 Bright-line threshold

We begin by providing a benchmark under which the firm can credibly commit to classifying based on an absolute persistence threshold, that is, under a bright-line standard. We limit attention to what is essential for comparisons. Dye (2002) provides a thorough treatment of the consequences of imposing an absolute threshold if the firm can manipulate its classifications at cost. Dye and Verrecchia (1995) discuss the benefits of a uniform versus a discretionary standard in an agency-theoretic setting. We do not revisit these issues and refer the interested reader to the respective articles.

Let $\hat{\alpha}$ be the firm's bright-line cutoff. That is, suppose for this section that

$$\forall i \in \{1, \dots, n\} \quad \begin{cases} i \in L, & \text{if } \alpha_i < \hat{\alpha} \\ i \in H, & \text{if } \alpha_i \geq \hat{\alpha} \end{cases}$$

The investor's mean squared error in estimating \tilde{v} given cutoff $\hat{\alpha}$ is

$$\begin{aligned} \eta(\hat{\alpha}) &:= E[(\tilde{v} - E[\tilde{v}|x, \hat{\alpha}])^2|x, \hat{\alpha}] \\ &= E \left\{ \left[\sum_{i \in L} \left(\tilde{\alpha}_i - \frac{1 + \hat{\alpha}}{2} \right) x_i + \sum_{j \in H} \left(\tilde{\alpha}_j - \frac{1 + 1/r + \hat{\alpha}}{2} \right) x_j \right]^2 \right\} \end{aligned} \quad (1)$$

The following proposition provides our benchmark:

Proposition 1. *The investor's mean squared error $\eta(\hat{\alpha})$ is minimized at the prior mean of the $\tilde{\alpha}_i$, i.e., at*

$$\bar{\alpha} := 1 + \frac{1}{2r}$$

At this cutoff, the mean squared error given the report is

$$\eta(\bar{\alpha}) = \frac{n}{48r^2}$$

Proposition 1 gives a bound on how much information the investor could receive from a perfectly enforced bright-line standard. The investor's estimation error increases linearly in the number of items n .

4.2 Persistence ranking

We now consider an alternative benchmark, under which the firm is not held to a bright-line standard, but can reveal the persistence ranking of its items.

For convenience, we introduce the following notation:

$$\begin{aligned}
 g &= \sum_{i=1}^n \max\{x_i, 0\} && = \text{total gains (positive items)} \\
 \ell &= n - g = -\sum_{i=1}^n \min\{x_i, 0\} && = \text{total losses (negative items)} \\
 x_{H,1} &= \text{first coordinate of } x_H && = \text{core revenues} \\
 x_{H,2} &= \text{second coordinate of } x_H && = \text{core expenses} \\
 x_{L,1}, x_{L,2} &= \text{first and second coordinates of } x_L && \text{analogous to } x_{H,1}, x_{H,2} \\
 k &= x_{H,1} - x_{H,2} && = \text{core items} \\
 n - k &= x_{L,1} - x_{L,2} && = \text{net special items} \\
 \tilde{\alpha}_{(i)} &= i^{\text{th}} \text{ persistence order statistic}
 \end{aligned}$$

Example 1. Let $x = ((7, -2), (4, -6))$. Then

$$\begin{array}{ll}
 g = 11 & \ell = 8 \\
 x_{H,1} = 7 & x_{H,2} = -2 \\
 x_{L,1} = 4 & x_{L,2} = -6 \\
 k = 9 & n - k = 11
 \end{array}$$

Net income $I = 3$, core income is $x_{H,1} + x_{H,2} = 5$, and net special items are $x_{L,1} + x_{L,2} = -2$.

The sample order statistics $\tilde{\alpha}_{(i)}$ are the values from sorting $\tilde{\alpha}$ in ascending order:

$$\tilde{\alpha}_{(1)} \leq \tilde{\alpha}_{(2)} \leq \dots \leq \tilde{\alpha}_{(n)}$$

The following lemma characterizes the mean and variance associated with the $\alpha_{(i)}$:

Lemma 2. *For each $i \in \{1, \dots, n\}$,*

$$r(\tilde{\alpha}_{(i)} - 1) \sim \text{Beta}(i, n - 1 + 1)$$

Consequently,

$$E[\tilde{\alpha}_{(i)}] = 1 + \frac{i}{r(n+1)}$$

$$\text{Var}[\tilde{\alpha}_{(i)}] = \frac{i(n-i+1)}{(n+1)^2(n+2)r^2}$$

The following result shows that, unless the total number of items is small, the investors learn more if the firm reveals the $\tilde{\alpha}_{(i)}$ than if it reveals whether each $\tilde{\alpha}_i$ is above or below a bright-line threshold. In fact, we get a sharp cutoff at 7 items.

Theorem 3. *If the firm can credibly reveal the $\tilde{\alpha}_{(i)}$, then the mean-square error in the investor's estimate of \tilde{v} , conditional on the report, is*

$$\frac{n}{6(n+1)r^2}$$

Therefore, investors are weakly (resp. strictly) better off from knowing the persistence ranking than they are from knowing if each item is more or less persistent than a bright-line threshold if and only if $n \geq$ (resp. $>$) 7.

Comparing Proposition 1 and Theorem 3, we see that the investor's mean-squared error in estimating \tilde{v} given the $\tilde{\alpha}_{(i)}$ asymptotically approaches a constant of $1/(6r^2)$, but that the estimation error from a bright-line standard grows linearly in n . The relative precision of the investor's information from the order statistics, compared with the bright-line classification, is

$$\frac{6(n+1)r^2/n}{48r^2/n} = \frac{n+1}{8}$$

which becomes arbitrarily large.

We end this section by showing that binary classifications suffice to reveal the persistence ranking, deferring discussion of strategic issues to the next section. The following result shows the number of ways to estimate \tilde{v} given these weights is never larger than what the firm can communicate with a binary classification system.

Theorem 4. *A binary classification of the income statement is a large enough message space to communicate every possible valuation based on the $\tilde{\alpha}_{(i)}$. In particular, given g gains and $\ell = n - g$ losses, there are*

$$g \cdot \ell + 1 \text{ possible values of } E[\tilde{v}|x, \tilde{\alpha}_{(1)}, \dots, \tilde{\alpha}_{(n)}], \text{ and}$$

$$g \cdot \ell + 1 + n \text{ possible classifications of } x = (x_H, x_L)$$

Moreover, exactly $n + 1$ classifications are consistent with the worst possible ordering (and exactly one possible expected firm value consistent with the worst possible ordering).

5 Coherent classification

We now focus attention on equilibria in which the firm is restricted to classifying coherently (see Definition 1). We note that there are many Bayesian Nash equilibria in our game. Even if we refine the equilibria by the intuitive criterion (Cho and Kreps, 1987), the equilibrium is not unique. Nevertheless, we can characterize strategies that are coherent and not strictly dominated, in a sense we elaborate on below. This characterization is the best that can reasonably be hoped for, as the firm's objective is in essence a knapsack problem embedded in a signaling game.⁵

Example 2 illustrates how the investor uses the expected values of the $\tilde{\alpha}_{(i)}$:

Example 2. *Suppose $n = 2$. There are three possible values of net income I :*

- $I = 2$. *Regardless of the classification, the investor estimates the firm's value as*

$$E[\tilde{v}|x_H, x_L] = E[\tilde{\alpha}_{(1)}] \cdot 1 + E[\tilde{\alpha}_{(2)}] \cdot 1 = 2 + \frac{\sum_{i=1}^2 i}{3r} = 2 + \frac{1}{r}$$

- $I = -2$. *Regardless of the classification, the investor estimates the firm's value as*

$$E[\tilde{v}|x_H, x_L] = -2 - \frac{1}{r}$$

- $I = 0$. *Then there are four possible reports:*

$$(x_H, x_L) \in \{((0, -1), (1, 0)), ((1, 0), (0, -1)), ((1, -1), (0, 0)), ((0, 0), (1, -1))\}$$

⁵The knapsack problem, which dates to at least Mathews (1897), is a constrained partitioning problem. It is known to be NP-hard.

Consider each:

Case 1 $x_H = (0, -1), x_L = (1, 0)$. By coherence, $\tilde{\alpha}_{(1)}$ is associated with the positive item in x_L , and $\tilde{\alpha}_{(2)}$ is associated with the negative item in x_H . The investor values the firm at

$$E[\tilde{v}|(0, -1), (1, 0)] = 1 + \frac{1}{3r} - 1 - \frac{2}{3r} = -\frac{1}{3r}$$

Case 2 $x_H = (1, 0), x_L = (0, -1)$. By a symmetric argument to Case 1,

$$E[\tilde{v}|(1, 0), (0, -1)] = \frac{1}{3r}$$

Cases 3 and 4 Either $x_H = (0, 0)$ or $x_L = (0, 0)$. Coherence does not restrict the sample order statistics of $\tilde{\alpha}$. Depending on the firm's strategy and whether it randomizes, the expected value of the firm can be anything in $[-\frac{1}{3r}, \frac{1}{3r}]$.

In Example 2, there is full separation in every equilibrium. If both items have the same sign, then the classification does not matter. If the items have opposite signs and the positive item is more persistent, then the firm classifies as $x_H = (1, 0), x_L = (0, -1)$. If the items have opposite signs and the negative item is more persistent, then the firm can classify as any of $\{x_H = (0, 0), x_L = (1, -1); x_H = (1, -1), x_L = (0, 0); x_H = (0, -1), x_L = (1, 0)\}$. In this last case, classifying all items as core or all items as special items does not violate the intuitive criterion; hence, the intuitive criterion does not select a unique equilibrium.

We further note that, in any rational expectations equilibrium, if the items have different signs and the positive item is more persistent, then the positive item always ends up as core income and the negative one as a special item. This does not necessarily occur when the

positive item is less persistent. Signaling alone generates the asymmetry that the empirical literature documents.⁶

The next example shows that complete separation is not always possible.

Example 3. *Suppose $n = 4$ and $g = 2$. There are 6 orderings of x from least to most persistent:*

Table 1 – Possible orderings, 4 items, 2 positive

Permutation	Items, least to most persistent	$E[\tilde{v} \text{persistence ranking}]$
1	(1, 1, -1, -1)	$-\frac{4}{5r}$
2	(1, -1, 1, -1)	$-\frac{2}{5r}$
3a	(-1, 1, 1, -1)	0
3b	(1, -1, -1, 1)	0
4	(-1, 1, -1, 1)	$\frac{2}{5r}$
5	(-1, -1, 1, 1)	$\frac{4}{5r}$

There are 9 possible classifications (x_H, x_H) . Among these:

- $((2, 0), (0, -2))$ is coherent if and only if x is permutation 5. The firm optimally classifies permutation 5 this way.
- $((2, -1), (0, -1))$ is coherent if and only if x is permutation 3a, 4, or 5. The firm optimally classifies permutation 3a this way.
- $((1, 0), (1, -2))$ is coherent if and only if x is permutation 3b, 4, or 5. The firm optimally classifies permutation 3b this way.
- $((1, -1), (1, -1))$ is coherent if and only if x is permutation 2, 3a, 3b, or 4. The firm optimally classifies permutation 2 this way.

⁶For supportive empirical results see Lipe (1986); Dechow and Ge (2006); Riedl and Srinivasan (2010); Abdalla (2016); Abdalla and Clubb (2016). Similar findings about firms using principles-based standards to signal are in Folsom et al. (2017). A theoretical argument consistent with Folsom et al. is in Dye and Sridhar (2008).

- The remaining 5 classifications are coherent for permutation 1. The firm can choose any of these for permutation 1.

For permutation 4, the firm optimally classifies as any mixture between $((2, -1), (0, -1))$ and $((1, 0), (1, -2))$.

Example 3 shows a common feature of the equilibria. As in Milgrom and Roberts (1986); Okuno-Fujiwara et al. (1990), and Shin (1994, 2003), the firm selects a classification that maximizes the most pessimistic interpretation of its news. This most pessimistic interpretation need not be unique, and some permutations (such as Permutation 4 in the example) may not be the most pessimistic interpretation of any values of (x_H, x_L) .

One complication in the current setting, not present in Milgrom and Roberts (1986); Okuno-Fujiwara et al. (1990); Shin (1994, 2003), is that the possible classifications may be incompletely ranked. Figure 2 shows the ordering of possible core income reports $(x_{H,1}, x_{H,2})$ for Example 3, in which there are two positive and two negative items.

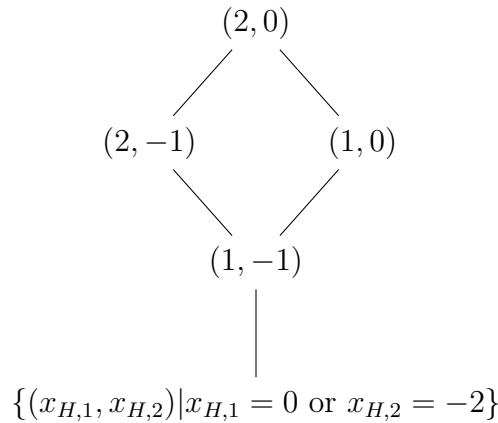


Figure 2 – Ranking of core income reports for firm with 2 positive and 2 negative items. Both $(2, -1)$ and $(1, 0)$ indicate core income of 1 (and therefore net special items of -1).

From Figure 2, we can see a natural condition that provides a strict dominance ranking.

We state this below:

Definition 2. Fix n and g . Then core income report $x_H = (x_{H,1}, x_{H,2})$ strictly dominates $x'_H = (x'_{H,1}, x'_{H,2})$ if every permutation $\{(x_{(i)}, \alpha_{(i)})\}_{i=1}^n$ for which report x is coherent implies at least as high a present value of v as any permutation for which x' is coherent, with at least one strict inequality.

Proposition 5. Let x_H, x'_H be two core income reports for fixed n, g . Suppose $x_{H,1} \geq x'_{H,1}, x_{H,2} \geq x'_{H,2}$, and at least one inequality is strict. Then x_H strictly dominates x'_H .

To understand Proposition 5, refer again to Figure 2. Core income reports $(2, -1)$ and $(1, 0)$ both strictly dominate core income report $(1, -1)$ and are both strictly dominated by core income report $(2, 0)$.

An immediate consequence of Proposition 5 is that in any rational expectations equilibrium, a firm will never classify in a way in which both core revenues and core expenses can both be improved. This generates a pattern of firms classifying in a way that increases core income.

Our last result provides some quantification of the amount of pooling that can occur in equilibrium. We show that in a sense the amount of pooling is small.

Theorem 6. For any given $n > 2$, the number of undominated coherent classifications is always less than $1/4$ the total number of coherent classifications.

6 Discussion and conclusion

Classifications are designed to provide information to the end users of financial reports. Our requirement of coherence states that these classifications can change on the fly and do not need to depend on any fixed notion of how transient an item must be to count as a special item. All that is required is that the firm's financial report can be held to its own standard. Saying that one item is so transient that it must count as a special item prevents a firm from saying an even more transient item is part of core earnings.

Our results explain the empirical findings that investors react to classifications, that firms tend to classify losses as special items and gains as core earnings, and that variation in auditor and in corporate governance does not systematically affect the amount of discretionary classification.

In policy terms, our results suggest that strategic classification is largely benign, at least to the extent that firms classify coherently. Some information is lost to pooling, but the overall amount lost is in a sense small. By contrast, an alternative policy of imposing a bright-line classification standard is clearly not in investors' interest, particularly for large or complex firms.

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A Proofs

Proof of Proposition 1. By independence, we can consider each $\tilde{\alpha}_i$ individually. For any random variable \tilde{y} and any constant γ ,

$$E[(\tilde{y} - \gamma)^2] = E[\tilde{y}^2] - 2\gamma E[\tilde{y}] + \gamma^2$$

Differentiating with respect to γ gives

$$-2E[\tilde{y}] + 2\gamma = 0 \quad \Rightarrow \quad \gamma = E[\tilde{y}]$$

It is immediate that the second derivative is 2, so that $\gamma = E[\tilde{y}]$ gives the unique global minimum, and the squared deviation is the variance of \tilde{y} .

By the uniformity of the $\tilde{\alpha}_i$, each i is equally likely to be in L or H . Therefore, the mean squared error is minimized where the variance of $\tilde{\alpha}_i$ is equal for $i \in L$ as for $i \in H$. This is easily seen to be at the midpoint $\bar{\alpha} = 1 + 1/(2r)$.

Finally, the variance of a uniform draw over $[1, 1 + 1/(2r)]$ (or over $[1 + 1/(2r), 1 + 1/r]$) is $1/(12(2r)^2) = 1/(48r^2)$. Over n independent draws, the total variance is $n/(48r^2)$. \square

Proof of Lemma 2. For each i , $\tilde{\alpha}_i \sim U[1, 1 + 1/r]$, so

$$r(\tilde{\alpha}_i - 1) \sim U[0, 1]$$

and $r(\tilde{\alpha} - 1)_{(i)} = r(\tilde{\alpha}_{(i)} - 1)$, i.e., the order is the same as the ordering of the $\alpha_{(i)}$. The pdf $f_{(i)}(\cdot)$ of the i^{th} sample order statistic of n iid random variables with distribution $F(\cdot)$ and density $f(\cdot)$ is

$$f_{(i)}(t) = \frac{n!}{(i-1)!(n-i)!} F^{i-1}(t) [1 - F(t)]^{n-i} f(t)$$

For a $U[0, 1]$ variable, $F(t) = t$, $1 - F(t) = 1 - t$, and $f(t) = 1$. Therefore, using $\Gamma(k) = (k-1)!$ for positive integer k ,

$$f_{(i)}(t) = \frac{\Gamma(n+1)}{\Gamma(i)\Gamma(n-i+1)} t^{i-1} (1-t)^{n-i}$$

which is the pdf of a $\text{Beta}(i, n-i+1)$ -distributed random variable (see Arnold et al., 2008, for discussion).

The mean and variance of a $\text{Beta}(a, b)$ -distributed random variable are $a/(a+b)$ and $ab/[(a+b)^2(a+b+1)]$. Therefore,

$$\begin{aligned} E[r(\tilde{\alpha}_{(i)} - 1)] &= \frac{i}{n+1} \quad \Rightarrow \quad E[\tilde{\alpha}_{(i)}] = 1 + \frac{i}{r(n+1)} \\ \text{Var}[r(\tilde{\alpha}_{(i)} - 1)] &= \frac{i(n-i+1)}{(n+1)^2(n+2)r^2} \end{aligned}$$

\square

Proof of Theorem 3. From Lemma 2, the investor's mean-squared error in estimating \tilde{v} given a report that reveals the order statistics is

$$\begin{aligned} \sum_{i=1}^n \frac{i(n-i+1)}{(n+1)^2(n+2)r^2} &= \frac{1}{(n+1)^2(n+2)r^2} \left[(n+1) \sum_{i=1}^n i - \sum_{i=1}^n i^2 \right] \\ &= \frac{n(n+1)^2/2 - n(n+1)(2n+1)/6}{(n+1)^2(n+2)r^2} \\ &= \frac{n}{6(n+1)r^2} \end{aligned} \tag{2}$$

From Proposition 1, the investor's mean-squared error in estimating \tilde{v} from a fully revealed bright-line classification standard is $n/(48r^2)$. This error is weakly greater than that of learning the order statistics (2) if and only if

$$\begin{aligned} \frac{n}{48r^2} &\geq \frac{n}{6(n+1)r^2} \\ \Leftrightarrow n &\geq 7 \end{aligned}$$

□

Proof of Theorem 4. An immediate corollary of Lemma 2 is that, for $i \in \{1, \dots, n-1\}$,

$$E[\tilde{\alpha}_{(i+1)}] - E[\tilde{\alpha}_{(i)}] = \frac{1}{r(n+1)},$$

i.e., the expected weights are evenly spaced. The number of possible values of the firm is therefore equivalent to the number of possible ways to sum g integers chosen from $\{1, \dots, n\}$, i.e., to determine the possible weighted sums of g positive items. This total plus knowledge of g and n are sufficient for determining the sum of the negative items and therefore of the firm's present value.

The smallest possible sum of g integers from $\{1, \dots, n\}$ is

$$\sum_{i=1}^g i = \frac{g(g+1)}{2}$$

The largest possible sum of g integers from $\{1, \dots, n\}$ is

$$\sum_{i=n-g+1}^n i = \sum_{i=1}^n i - \sum_{j=1}^{n-g} j = \frac{n(n+1)}{2} - \frac{(n-g)(n-g+1)}{2}$$

Because the integers in $\{1, \dots, n\}$ are consecutive, any integer sum between the smallest and largest possible sum is attainable. Therefore, the total number of possible sums is one plus the difference between the largest and smallest possible sum. Noting that $\ell = n - g$, this total is (after some substitutions)

$$\frac{g(g+1)}{2} - \frac{n(n+1)}{2} - \frac{\ell(\ell+1)}{2} = g \cdot \ell + 1$$

With a binary classification system, there are $\{0, \dots, g\}$ possible values of $x_{H,1}$ and $\{0, \dots, \ell\}$ possible values of $x_{H,2}$. These fully determine the binary classifications, so there are a total of

$$(g+1)(\ell+1) = g \cdot \ell + g + \ell + 1 = g \cdot \ell + n + 1$$

possible ways to report with a binary classification.

Finally, among the possible classification profiles, there are $g + 1$ with $x_{H,1} = 0$ and $\ell + 1$ with $x_{H,2} = \ell$, and exactly one report ($x_H = (0, 0)$) consistent with both. This gives $n + 1$ classification profiles that do not separate from the worst possible ordering, $g \cdot \ell$ classification profiles that do, and $g \cdot \ell$ possible orderings that are better than the worst possible ordering. \square

Proof of Proposition 5. Suppose $x_{H,1} = x'_{H,1}$ and $x_{H,2} = x'_{H,2} + 1$. Let k be the number of core items in report x , i.e., $x_{H,1} - x_{H,2}$, and define k' similarly for x' . As x has one fewer core expense than x' and the same number of core revenues, we have $k = k' - 1$. Let $x_{(n-k'+1)}$ be the (k') th most persistent item. If $x_{(n-k'+1)} = -1$, then the sequence can be reported as either x or x' . If $x_{(n-k'+1)} = 1$, then sequence can be reported as x but not as x' . Thus every sequence reportable as x' is either also reportable as x or differs by making one negative item more persistent.

The case where $x_{H,1} = x'_{H,1} + 1$ and $x_{H,2} = x'_{H,2}$ is similar, and the general result follows from induction. □

Proof of Theorem 6. We begin by calculating the maximum number of undominated sequences associated with a core income report. Given x, g , and n , the total number of sequences (including dominated ones) that x coherently classifies is

$$\binom{x_{H,1} - x_{H,2}}{x_{H,1}} \binom{n - x_{H,1} + x_{H,2}}{g - x_{H,1}} = \binom{k}{x_{H,1}} \binom{n - k}{x_{L,1}} \quad (3)$$

The reason is as follows: rank the k core income items. Among these, there are $x_{H,1}$ positive items, which can take any of the possible ranked positions without replacement. The reasoning is similar among the $n - k$ noncore items, of which $x_{L,1}$ are positive.

If the lowest-ranked core income item in a sequence is negative, then the firm could have classified its core income as $(x_{H,1}, x_{L,1} + 1)$. Similarly, if the highest ranked non-core item is positive and the lowest-ranked core item is negative, then the firm could have classified its core income as $(x_{H,1} + 1, x_{L,1})$ but not as $(x_{H,1}, x_{L,1} + 1)$. (The condition that $n > 2$ is required here, because of the two items with values that are fixed.) By Proposition 5, the

number of undominated sequences coherently classified as x is therefore

$$\binom{k}{x_{H,1}} \binom{n-k}{x_{L,1}} - \binom{k-1}{x_{H,1}} \binom{n-k}{x_{L,1}} - \binom{k-1}{x_{H,1}-1} \binom{n-k-1}{x_{L,1}-1}$$

which after some tedious algebra reduces to

$$\binom{k-1}{x_{H,1}-1} \binom{n-k-1}{x_{L,1}} \quad (4)$$

Noting that $k = x_{H,1} - x_{H,2}$ and summing (4) over $x_{H,1}$ from 1 to g and $-x_{H,2}$ from 0 to $n - g - 1$ gives the total undominated sequences consistent with n and g as

$$\frac{(n-1)!}{(g-1)!(n-g)!} = \frac{\Gamma(n)}{\Gamma(g)\Gamma(n-g)} \quad (5)$$

Scaling (5) by (3) gives the maximum fraction of sequences that are undominated. This ratio turns out to be

$$\frac{g(n-g)}{n(n+1)}$$

which is maximized at $g = n/2$. Substituting gives an upper bound of

$$\frac{n^2}{4n(n+1)} = \frac{1}{4 + 1/n}$$

Thus the amount of pooling is always less than 1/4 of the opportunities to pool. \square