How uncertain is the market about managers' reporting objectives? Evidence from structural estimation

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Abstract

Theory suggests that the market's uncertainty about managers' reporting objectives is an important source for reporting biases (Fischer and Verrecchia 2000), yet little empirical work exists on gauging such uncertainty. We derive a simple structural estimator of this uncertainty, incorporating cross-sectional properties of prices, earnings and restatements. This approach enables us to assess an average level of uncertainty. We show that investors' uncertainty about reporting incentives, albeit non-zero, are generally small. Given the link between uncertainty and reporting biases, our large sample evidence also supports the conjecture that earnings management is not as rife as what prior accounting academic publications would make one believe (Ball 2013). We also characterize the variation in the magnitude of uncertainty across industries and subsamples of firm size and growth.

Keywords: reporting bias, signaling, reporting

JEL Codes: D83; G14; M4.

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1 Introduction

Theory suggests that the market's uncertainty about managers reporting objectives affects biases in financial reports (Dye, 1988; Fischer and Verrecchia, 2000). The intuition is that when investors have imprecise knowledge about a manager's incentives, they are unable to back out biases, thereby allowing the manager to add noise to reports. For example, Fischer and Verrecchia (2000) posit that investors could not always know the precise nature of a manager's compensation, time horizon, psychic costs related to biases, tolerance towards litigation and reputation costs, etc. A growing empirical literature provides compelling arguments for the existence of such uncertainty (Fang, Huang, and Wang 2017; Ferri, Zheng, and Zou 2018; Samuels, Taylor, and Verrecchia 2018), showing that factors tied to incentives and monitoring intensity relate to the quality of accounting information. Despite this active research agenda, there is still limited work on quantitatively assessing the structural implications of this framework.

The purpose of our study is to fill this void and quantify investors' uncertainty about managers' incentives. A vast accounting literature suggests that earnings manipulation is pervasive (Graham, Harvey, and Rajgopal 2005). This widely held belief among accounting researchers draws harsh criticism from other disciplines on the integrity or competence of the accounting profession (Bazerman, Lowenstein, and Moore, 2002). However, the sole existence of earnings manipulation is still controversial. Ball (2013, p. 851) asks: "[i]f a searcher has reliable evidence of financial statement manipulation, is there not a moral obligation to inform the parties affected by it as well as those responsible for acting on it?" The near-absence of academic whistleblowers leads him to conjecture that manipulation may not be as rampant as what academic publications imply. Because transparency about managers' incentives helps investors detect biases and keeps manipulation in check, a large sample empirical assessment of the market's uncertainty could inform the ongoing debate about the magnitude of reporting biases. Consequently, our exercise may shed light on what the accounting research profession should do next (or more precisely, should not continue to do), both for theorists and for empiricists.

Ex-ante, it is unclear whether the market's uncertainty about a manager's reporting objectives is as large and pervasive as what the aforementioned arguments would suggest. There are several reasons that such opacity could be low. First, corporate filings provide extensive disclosures about executive biography, compensation, trading activities, as well as management discussion and analysis that reveal managers' incentives. Second, the Sarbanes-Oxley Act of 2002 provides a litigation environment less prone to earnings manipulation. Third, advance in internet search, social media and social network facilitate the access to information about managers' personal lives, which further reduces the market's uncertainty about their incentives. Finally, the burgeoning compensation and proxy advisors also help investors assess managers' incentives. In sum, managers' scope to maneuver and hide their incentives from investors is limited. To gauge this latent construct of uncertainty, we derive an easy-to-use theory-based measure that remains closely anchored to the classic models (Dye 1988; Fischer and Verrecchia 2000) and embeds information inherent in the cost-benefit trade-off facing the manager.

A structural model is well-suited to examine this problem in that it uses theory to connect unobservable drivers of incentives and unobservable manipulation choices to observable empirical features. We do not observe incentives formally. If all incentives (and other sources of distortions) were perfectly known by the market but for true fundamentals, investors would be able to perfectly back out any private information known to the manager from accounting reports (Dye 1988; Stein 1989). Without directly observing incentives as researchers or market participants, we need to answer this question within the scope of a theoretical framework that connects unobservable incentives to empirical facts.

In this paper, we provide one attempt to answer such a question by fitting empirical facts about reported earnings and restatements using the framework of Fischer and Verrecchia (2000). Managers have reporting incentives unobserved to the market, and reported earnings reflect both uncertain incentives and fundamentals - implying that the bias introduces noise as investors can only correct for the bias in expectation. We enrich this model along various interesting empirical dimensions. First, we extend the theoretical model to involve predictions about misstatement detection and misstated amounts, and use these properties to identify the model. Second, we estimate from the data a non-linear relation between price and earnings, and in one of the models, obtain a distribution of true earnings without pre-assuming any distributional form. Third, our model allows us to obtain various measurements, such as the effect of manipulation on expected value to investors, the effect of manipulation on manager welfare or the loss of information due to accounting noise, as well as various counterfactuals in response to increases or decreases in incentives.

We adopt two estimation approaches in the analysis. The 'parametric' model assumes both earnings and incentives are normally distributed. The second approach is semi-parametric because it only requires reporting incentives to be normally distributed while the true earnings distribution is recovered from the data. Both estimation procedures suggest the uncertainty about the manager's reporting objective is small, i.e., 1.94% of lagged assets in the parametric model and 0.52% in the semi-parametric model, while the average reporting incentive is close to zero. The average manipulation bias is small about one quarter of a percent upward, measured in ROA surprises in the parametric model while closer to zero in the semi-parametric model. To analyze the real effects of these estimates, we evaluate the following three aspects: the information loss due to the uncertainty about the manager's reporting incentive, the welfare change for investors and the ex-ante expected benefit for the manager. The information loss due to the uncertainty about the manager's reporting incentive is about one standard-deviation of true earnings surprise in the parametric model and about half of a standard- deviation in the semi-parametric model. The welfare loss measured as the average difference between the actual price and the price if earnings had been reported truthfully attains about 0.2% of lagged assets in the parametric model, that is, about one third of the standard-deviation of true earnings. The semi-parametric model exhibits a lower loss of about 10% of a standard deviation of the true earnings. Lastly, we find managers benefit from the reporting uncertainty at about 85% of the standard deviation in the manager's utility in presence of uncertainty in the parametric model and about 40% in the semi-parametric model.

To capture the potential difference in the distribution across different settings, we estimate the two models for various industries, firm sizes and growth opportunity groups. We find that larger firms have higher true earnings, higher mean of reporting incentives, but also higher regulatory intensity. Firms with higher growth opportunity have higher reporting incentives but also larger reporting uncertainty. High growth firms exhibit a lower regulatory intensity. Our estimation by industry shows that Business Equipment has the most incentives to manipulate and a large reporting uncertainty.

Two recent studies are very closely related to our model because they specifically focus on estimating a model of noisy manipulation. Bird, Karolyi, and Ruchti (2016) examine, similarly to our study, a model in which a manager subject to noisy incentives manipulates against market price. Like us, they estimate the price response function and we build on their approach when we use simulated data to fit the model and data. The results are also consistent with their findings: managers manipulate away from where the price response is the steepest. However, there are several key differences between the two studies. First, they focus on a 'sharp' identification via a discontinuity around zero rather than the entire distribution of reports - this comes with benefits but also trade-offs since the core of their analysis is concentrated around zero. Second, we also bring information about detected misstatements into the model and estimation, which allows us to narrow down the identification via actual detections. Third, we model the mapping between true earnings and price and extract the welfare implication of noise. Additionally, we perform several counterfactuals by endogenously adjusting the price function to new parameter estimates. Lastly, we develop a semi-parametric approach that requires little knowledge of the distribution of true earnings.

Another study related to ours is Beyer, Guttman, and Marinovic (2018) which, to our knowledge, is the only other study focusing on noisy signalling, albeit in the framework of the (related) Dye and Sridhar (2004) in which it is the uncertainty about the acceptable GAAP earnings that introduce accounting noise. Yet our approaches are very different. They focus on a linear-normal model in which prices are linear in reported earnings. Linearity implies a simple closed-form identification of the structural parameters even though the manager is forward-looking (which we cannot accommodate in our model). Identification in their model relies on a relationship between price and the net discounted value of true earnings. By contrast, we do not make this assumption but instead rely on detections to identify manipulation. Linearity in this approach also precludes implications of noise on ex-ante price. Also, as is usual in learning processes, only a ratio of accounting noise to fundamentals can be identified. Interestingly, albeit a very different approach, their estimates of accounting noise are within the same magnitudes as ours, therefore, suggesting that different methods appear to converge in terms of principal magnitudes.

There are other studies focusing on manipulation and reviewing them in their entirety goes beyond our current focus. A precursor to this study is Bertomeu, Cheynel, Li, and Liang (2018) and focuses on estimating a version of this model when there is no incentive noise and without using misstatement information. Other studies apply structural estimation to other environments, such as conservatism (Breuer and Windisch, 2019), investment in intangibles (Terry, Whited, and Zakolyukina, 2018), contracting (Li, 2018), auditing (Gerakos and Syverson, 2015), general equilibrium (Choi, 2018), disclosure (Zhou, 2017) and analyst forecasts (Xiao, 2015). Relative to this literature, we hope that our study contributes to building more knowledge about using structural estimation in classic accounting problems.

2 Model

The model is adapted from the noisy reporting framework developed in Fischer and Verrecchia (2000) and Frankel and Kartik (2019). We briefly summarize this setting below without any formalism. A manager privately observes two random variables: a value-relevant true earnings signal and a personal incentive to increase market value. Then, the manager reports accounting earnings for a cost that depends on the difference between accounting and true earnings. The market observes the report and forms expectations about the value of the firm rationally. Our interest here is to estimate this framework using observables about the distribution of earnings and restatements and the empirical relationship between prices and earnings. The details of the the timeline and notations are given below.

Environment. Consider a one-period game with a firm and a mass of risk-neutral investors. There is an unobserved value-relevant state of the world $\tilde{s}$. if a realization $\tilde{s} = s$ were perfectly observable, this economic state would map into market value according to some function $\alpha(s)$. In practice, however, the true state of the world will not be fully known at a point in time where market prices form. If there is uncertainty, we assume that the market can price the firm using the expected state, i.e., $\alpha(\mathbb{E}(\tilde{s}|\mathcal{I}))$, where $\mathcal{I}$ is a public information set. This assumption is borrowed from Ganuza and Penalva (2010) and can be thought as a linear approximation in which decisions can be made based on the first moment of the state. Various decision problems that yield this representation are given in Bertomeu, Cheynel, and Cianciaruso

(2018) and Marinovic, Skrzypacz, and Varas (2018).

Within this framework, value-relevant earnings must be signals about the expected state by construction. Therefore, we assume that *true* earnings $\tilde{v} \equiv \mathbb{E}(\tilde{s}|\tilde{\mathcal{I}})$, where $\tilde{\mathcal{I}}$ is a latent random variable indicating information collected by the firm, before any management bias. Thereafter, we work directly with $\tilde{v}$ and assume that it has a p.d.f. $f(.)$ with realizations denoted v. It then follows that the market price of a firm conditional on true earnings $\tilde{v} = v$ is $\alpha(v)$, and the corresponding market price for any noisy signal $\tilde{r} = r$ about true earnings, from the law of iterated expectations, is:

$$\gamma(r) \equiv \alpha(\mathbb{E}(\tilde{s}|r)) = \alpha(\mathbb{E}(\mathbb{E}(\tilde{s}|\mathcal{I})|r)) = \alpha(\mathbb{E}(\tilde{v}|r)), \tag{1}$$

so that the market prices a signal in terms of posterior expectations about earnings.

Note that we assume no a-priori theoretical knowledge about the true mapping $\alpha(.)$ given that different decision problems would imply different functional forms. We will seek to identify this function from observations about *biased* reports and price, i.e., from observing $\gamma(.)$. This requires additional theoretical structure about observed accounting reports, which we discuss below.

Manager's Problem. The manager privately observes true earnings $\tilde{v}$ and a random preference shock $\tilde{x}$ which is normally distributed with a p.d.f. $h(x)$, mean μ_x and variance σ_x^2. The manager chooses a report $R(v, x)$ to maximize

$$R(v, x) \in \arg\max_r x\gamma(r) - \frac{(r - v)^2}{2}. \tag{2}$$

The preference shock $\tilde{x}$ represents the marginal benefit from the market's valuation relative to the marginal cost of earnings management by imposing one extra unit of bias in the report. Following Fischer and Verrecchia (2000), we do not rule out negative values of x. For example, a manager might be willing to increase the price prior to a security offering, or manage earnings downwards to reduce the exercise price on a new grant (Aboody and Kasznik 2000).

Market Pricing. The market conjectures a reporting strategy $R(v, x)$. Upon observing a report r, the price $\gamma(r)$ rationally infers a posterior expectation $\mathbb{E}(v|r = \bar{R}(\tilde{v}, \tilde{x}))$, that is,

$$\gamma(r) = \alpha(\mathbb{E}(\tilde{v}|R(\tilde{v}, \tilde{x}) = r)). \tag{3}$$

Note that Fischer and Verrecchia (2000) assume that $\alpha(.)$ is linear and $(\tilde{v}, \tilde{x})$ are normally distributed, which can be shown to imply the existence of a linear conjecture $\gamma(.)$ - that is, market prices are proportional to both true earnings and reported earnings. In our model, we allow for non-linear functions $\alpha(.)$ and $\gamma(.)$ to accommodate features of the data in the observed $\gamma(.)$ is non-linear. For example, the mapping between price and earnings can also be S-shaped, when there is uncertainty in the precision of earnings (Subramanyam, 1996; Kirschenheiter and Melumad, 2002).

Equilibrium. A Bayesian equilibrium is such that the manager reporting strategy $R(v,x)$ is optimal given the market pricing function and the market pricing function $\gamma(r)$ correctly conjectures the manager's reporting strategy:

(i) The manager optimally selects $R(v,x)$ by solving the optimization problem in Equation (2) for any v and x.

(ii) The market conjecture manager's reporting strategy correctly and prices the firm at $\gamma(r)$ according to Equation (3).

To use additional information from observed misstatements, note that $(r-v)^2/2$ in the manager's problem is the total perceived cost by the manager. We decompose this total cost in terms of a probability of detection $1-e^{-d|r-v|}$ and an incurred cost conditional on detection $\psi(r-v)$ where

$$\frac{(r-v)^2}{2} = (1-e^{-d|r-v|})\psi(r-v).$$

3 A Closed-Form Approach

We develop next a simplified parametric approach that follows closely (i.e., to the notation) the linear-normal framework of Fischer and Verrecchia (2000) but will also illustrate some of the estimation steps further developed in the general model. Assume that the relationship between true earnings and value is linear, that is, $\alpha(v) = kv$, where k may be interpreted as a true earnings multiple which is positive and not directly observable. True earnings $\tilde{v} \sim N(\mu_v, \sigma_v^2)$ and incentives $\tilde{x} \sim N(\mu_x, \sigma_x^2)$ are independently and normally distributed.

Denote $\overline{r}$ as the expected accounting report in order to express the model in returns and earnings surprises. We know from Fischer and Verrecchia (2000) that there is a unique linear equilibrium with

$$\gamma(r) - \gamma(\overline{r}) = \beta k(r-\overline{r}) \quad (4)$$

$$R(v,x) - \overline{r} = v - \mu_v + \beta k(x-\mu_x), \quad (5)$$

Using that

$$\beta(r-\overline{r}) = \mathbb{E}(v|r = v-\mu_v+\beta k(x-\mu_x)) - \mu_v = \frac{\sigma_v^2(r-\overline{r})}{\sigma_v^2+(\beta k)^2\sigma_x^2}$$

and identifying coefficients yields a third-order polynomial $(\beta k)^3\frac{\sigma_x^2}{\sigma_v^2}+\beta k - k = 0$ which can written as

$$\frac{\sigma_x^2}{\sigma_v^2} = \frac{k-\beta k}{(\beta k)^3}. \tag{6}$$

Similarly, the variance of accounting earnings $Var(R(v,x)-\overline{r})$ can be derived in the model using (5) and (6) as

$$\underbrace{Var(R(v,x)-r_0)}_{V_R} = \sigma_v^2+(\beta k)^2\sigma_x^2 = \frac{(\beta k)^2}{1-\beta}\sigma_x^2. \tag{7}$$

Note that βk can be directly estimated from a regression of returns on earnings surprise and V_R can be estimated as the unconditional variance of earnings surprises. However, there are only two equations (6) and (7) for four unknowns $(k,\sigma_v,\sigma_x,\beta)$ so the model is not identified by these equations alone.

Let us assume that for any report $r = R(v,x)$, the probability of detection is given by $1-e^{-d|r-\overline{r}-(v-\mu_v)|}$. For the purposes of mapping this analysis into conventional statistical methods, let us assume, for any r that is sufficiently large: $|r-\overline{r}-(v-\mu_v)| \approx r-\overline{r}-(v-\mu_v)$.[1] Note that:

$$\begin{aligned}\mathbb{E}(v-\mu_v|r=R(v,x)) &= \gamma(r)/k = \beta(r-\overline{r}) &(8)\\ Var(v-\mu_v|r=R(v,x)) &= \frac{\beta^2k^2\sigma_v^2\sigma_x^2}{\sigma_v^2+\beta^2k^2\sigma_x^2} = (1-\beta)\sigma_v^2. &(9)\end{aligned}$$

Therefore, the average probability of detection conditional on $r = R(v,x)$ is

$$\begin{aligned}\xi(r) &\approx 1-\mathbb{E}(e^{-d(r-\overline{r}-(v-\mu_v))|r}) &(10)\\ &\approx 1-e^{-d(1-\beta)(r-\overline{r})+\frac{d^2}{2}(1-\beta)\sigma_v^2} &(11)\\ &\approx \frac{1}{1+e^{-(\xi_0+\xi_1 r)}}. &(12)\end{aligned}$$

where $\xi_0 = -\frac{d^2}{2}(1-\beta)\sigma_v^2$ and $\xi_1 = d(1-\beta)$ can be estimated via a logistic regression of misstatements on earnings surprises on the upper quantiles of the distribution of earnings. Then the other parameters can be

[1] The detection probability is slightly different from the baseline model, in that it is stated in terms of surprises; while not critical for the analysis, this formulation implies a simpler estimation procedure in the linear model.

recovered in closed-form as

$$\beta = -\frac{2\xi_0}{2\xi_0 - V_R\xi_1^2} \tag{13}$$

$$\sigma_v^2 = \max(\frac{2\xi_0 V_R}{2\xi_0 - V_R\xi_1^2}, 0) \tag{14}$$

$$\sigma_x^2 = \max(\frac{V_R^2\xi_1^2}{(\beta k)^2(2\xi_0 - V_R\xi_1^2)}, 0) \tag{15}$$

$$d = \max(\xi_1 - \frac{2\xi_0}{V_R\xi_1}, 0), \tag{16}$$

and, finally, k can be recovered by dividing the estimated βk by the β in (13).

We fit in Table 1 below this set of equations for accounting earnings in the top quintile, subject to the model restriction that $\xi_0 \leq 0 \leq \xi_1$ in the logit regression. We can also estimate the model when r is small, in which case the analysis is similar except that $|r - \overline{r} - (v - \mu_v)| \approx v - \mu_v - (r - \overline{r})$, and thus $\xi_0' = -\xi_0$ and $\xi_1' = -\xi_1$.[2] All these estimates imply a corner solution with limited uncertainty about reporting incentive σ_x, i.e., noise unobserved to investors but observable to the manager. For example, it could be that investors have a reasonably good idea of managerial incentives. As we will see in the general model, part of this observation is due to the simplified nature of the linear-normal model but this provides a general insight of accounting noise.

Table 1: Linear-Normal Model

Model	β	σ_v	σ_x	d	k	corner
Top quintile earnings	1.0001	0.0125	0.0001	0	1.0354	yes
Bottom quintile earnings	.9952	0.0125	0	1,078	1.0406	yes

Data used in the estimation are summarized in Table 2.

4 Sample Construction

Our model implies three observable variables: the accounting report r, the market price $\gamma(r)$ and the occurrence of a misstatement d. We construct an empirical sample for these variables below.

We obtain financial reporting data from Compustat, analyst forecast and companies non-GAAP earnings from IBES, stock price data from CRSP and restatement data from AuditAnalytics. Although the Audit Analytics database started collecting restatements prior to 2000, collections are only systematically conducted afterwards. Further, we start the sample in 2003 to focus on the post Sarbanes-Oxley regulatory environment.

[2] The results are identical for other quantiles, or estimating the exponential form directly without the logistic approximation. These are corner solutions where some of the parameters lie at the boundary of the parameter space.

We divide earnings by lagged assets, and subtract the analyst consensus by dividing the IBES earnings consensus by lagged assets - with a slight abuse in language, we denote this as ROA surprise or, in short, ROA.

Note that we scale by assets (versus earnings) to avoid firms dropped due to negative equity (i.e., following histories of losses) or, if scaling by lagged market values, to somewhat mitigate possible effects of earnings announcement drift or momentum. Another downside of using market values as a scaling variable in our model is that a structural model is much more sensitive to noise in variables than a linear regression (since the noise need not average out even with a large sample). As will be shown later, we project returns on accounting variables, so when measuring $\gamma(r)$ the noise in stock price can be averaged out in the pre-estimation step. For the corresponding stock reaction, we use a three-day cumulative value-weighted portfolio-adjusted abnormal return. In order to make sure the restatement data reflects reporting bias as in our model, we only use restatement cases that have income effect, i.e., the variable "include_in_income_calculations" equals one in AuditAnalytics. A misstatement that is a simple misclassification or with some reporting issue that is distinct from income is classified as not detected in the model, so our model does not speak to this type of restatement. We calculate *Restated ROA* as *ROA* plus restatement amount scaled by lagged total assets in the sample of restated years. The *Corrected ROA* is defined as *Restated ROA* if there was restatement occurrence and equals *ROA* if not. Naturally, *Corrected ROA* need not be equal to true ROA if a misstatement was not detected.

We keep in the sample both large *and* small restatements. This requires some discussions as it is known in the literature that many restatements are not legally classified as intentional frauds, i.e., they do not have large effects on the firm and do not come with verifiable evidence of management intent (Hennes, Leone, and Miller 2008). Within the context of our model, we cannot remove small restatements because the model predicts a large fraction of small restatements. More importantly, the model does not presume that there is a legal case against management or even that the restatement itself, while possibly too conservative or liberal relative to GAAP, would be against the law. For this reason, it is important to interpret our results in terms of accounting quality lost due to discretion, and what we call restatements are not all serious frauds that would involve criminal prosecution or even SEC investigation and fines. As is implied by theories such as Fischer and Verrecchia (2000) and Dye and Sridhar (2004), this does not mean that small restatements are unimportant given that, if widespread, they can reduce the ability of investors to understand the true state.

We report summary statistics in Table 2. The median firm in our sample is moderately large, with 721 million USD in assets, but there is significant variation in size with a lower quartile at 217 million USD and an upper quartile at 2.54 billion USD. The average ROA is close to zero which indicates that the IBES

Table 2: Descriptive Statistics

Variable	*N*	*Mean*	*Std. Dev*	*Lower Quartile*	*Median*	*Upper Quartile*
Total Assets	27,806	4,504.437	21,883.488	217.408	721.769	2,540.300
ROA	27,806	-0.001	0.013	-0.002	0.001	0.003
CAR	27,806	0.000	0.078	-0.043	-0.003	0.041
Restated ROA	3,067	-0.005	0.021	-0.006	-0.001	0.003
Corrected ROA	27,806	-0.001	0.015	-0.002	0.000	0.003
BTM	27,617	0.518	86.409	0.259	0.439	0.690
Mktcap	27,622	4949.626	20136.266	256.037	782.953	2598.43

ROA is the return on asset, calculated using the company reported ROA minus analyst forecast consensus. *CAR* is the difference between the buy-and-hold return of a firm and that of CRSP value-weighted market portfolio over the three-day window from Day -1 to Day +1 centered on annual earnings announcement dates. *Restated ROA* is the change of ROA due to restatement, calculated as change in net income scaled by lagged total asset. *Corrected ROA* is the sum of *ROA* and *Restated ROA* (zero if no detected misstatement). *BTM* is book-to-market equity ratio, computed as book value of common equity scaled by market capitalization ($CEQ/(CSHO \times PRCC\_F)$). The last variable *Mktcap* is the market capitalization in the end of the fiscal year. It is the product of common shares outstanding (Compustat item *CSHO*) and fiscal year end closing price (Compustat item *PRCC_ F*).

consensus does not have a very strong bias and the average 3-day return is almost zero, as expected from the expected abnormal return in a 3-day period. On average, we find that the restated ROA is lower than the reported ROA but, overall in the entire sample, the correction to ROA is not large. The distribution of restatements shows some negative skewness with a larger proportion of negative restatements than positive ones.

In univariate analyses presented in Table 3, we show that, as expected, ROA surprises are positively associated to returns - a key reason to manipulate accounting reports in our model. We do not find a directional association between *D_misstate* and *ROA*, which is consistent with restatements occurring at intermediate levels of earnings but with no clear pattern over high or low reported earnings. *Corrected ROA* is negatively associated to *D_misstate*, indicating that restatements tend to be income decreasing. Finally, we observe restatement amount *Restated_ROA* is positively correlated with *ROA*, suggesting low performance firms tend to bias upward and high performance firms on average bias downward.

Table 3: Pearson Correlation

	ROA	*Restated_ROA*	*D_misstate*	*Corrected_ROA*	*CAR*
ROA	1				
	(27,806)				
Restated_ROA	0.5818*	1			
	(3,067)	*(3,067)*			
D_misstate	-0.0048	.	1		
	(27,806)	.	*(27,806)*		
Corrected_ROA	0.8941*	0.9417*	-0.1021*	1	
	(27,806)	*(3,067)*	*(27,806)*	*(27,806)*	
CAR	0.1657*	0.1195*	-0.0024	0.1528*	1
	(27,806)	*(3,067)*	*(27,806)*	*(27,806)*	*(27,806)*

This table presents the pairwise correlation coefficients between variables in each column and row. *D_misstate* is an indicator variable with value being 1 if there is a restatement in the firm-year observation and 0 otherwise. The rest variables are defined in Table 2. Number of observations are in parentheses. Asterisks specify the significance levels of correlation coefficients: * $p < .001$.

5 Estimation Method

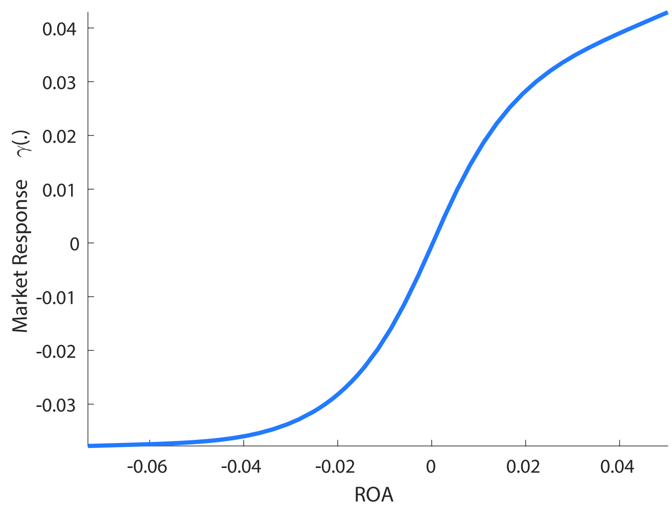
This section discusses the empirical procedure to estimate the model. The dataset is composed of observations $(r_i, d_i, \gamma_i)_{i=1}^N$ where r_i is an accounting report, d_i is the occurrence of a restatement and γ_i is the corresponding return. Each of these observables corresponds to an endogenous variable in the theoretical model but the realizations of the primitives that caused these observables, i.e., the true earnings v_i and the price incentive x_i are not observable to the econometrician. In what follows, we use hats to indicate estimated objects.

First, we estimate two theoretical constructs directly from the data and model restriction without knowledge of the parameters to be estimated. The market response $\gamma(r)$ is estimated from a non-parametric fit of values of γ_i on r_i. We use cubic spline to fit this relationship in order to ensure that the fitted market response has continuous first and second derivatives.

Figure 1 presents the price response function to earnings surprise. This plot is consistent with a non-linear empirical relation between accounting reports and stock price. Consistent with the prior literature (Burgstahler and Dichev, 1997; Degeorge, Patel, and Zeckhauser, 1999; Bird, Karolyi, and Ruchti, 2016), manipulation incentives provided by capital market are highest around zero earnings surprise. Although we do not assume monotonicity, the empirical mapping suggests a monotonic relation between earnings and price reaction. Furthermore, the price response function has a distinctive S-Shape, steeper in the middle and flatter for extreme earnings. This shape may be driven by two forces: (i) the price response function to the true earnings can be itself S-shaped, in line with theories in Freeman and Tse (1992) and Subramanyam

(1996); (ii) manipulation might contribute to an S-shape if manipulation to meet the threshold is interpreted as stronger fundamentals. To distinguish the two potential explanations, we will later examine the non-linear mapping between true earnings and price from the theoretical model.

Figure 1: Earnings Response Functions



The graph displays the fitted earnings-response function ($\hat{\gamma}$) that maps ROA to CAR. ROA is the ROA surprise adjusted by analyst forecast consensus. CAR is the three-day abnormal return surrounding earnings earnings announcement.

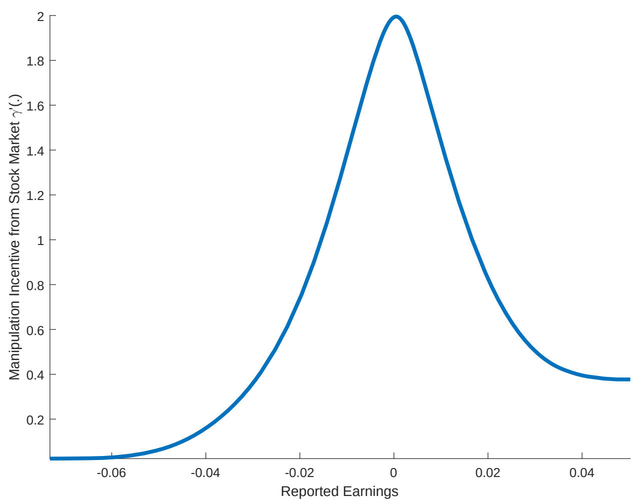
According to Figure 1, when reported earnings are zero, that is meeting analyst forecast, three-day abnormal returns are very close to zero, at 0.41%. On one hand, markets strongly penalize reported earnings missing the consensus by only 2% by yielding an abnormal return of -2.82%. Reported earnings missing the consensus by 5% are associated with an even lower three day abnormal return of -3.25%. On the other hand, when firms beat the consensus by 2%, markets reward them with a three day abnormal return of 2.83%. If they beat the consensus by 5%, the reward is even higher attaining abnormal returns of 4.76%,

To gain additional intuition as to how this market response produces incentives to manipulate, differentiating the manager's problem in Equation (2) and using the estimated market response yields

$$\underbrace{r - v}_{\text{reporting bias}} = x\hat{\gamma}'(r). \tag{17}$$

Hence, the bias is proportional to the realized reporting incentive x and the slope of the market response γ'. We plot the first derivative of the earnings response function $\gamma'(.)$, as in Figure 2. The market incentive peaks at a value of 2 when reported earnings is zero. This derivative remains positive over the range of reported earnings but flattens for reported earnings in the tails. The derivative is up to 10 times lower for accounting reports that lie away from the consensus indicating muted incentives to manipulate.

Figure 2: Market Incentive Functions



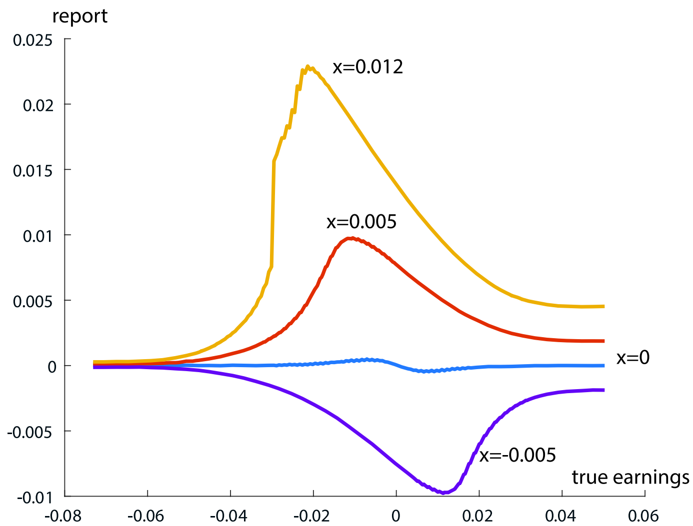
The graph displays the first derivative of the earnings-response function ($\hat{\gamma}'$) . ROA is the ROA surprise adjusted by analyst forecast consensus and CAR is the three-day abnormal return surrounding earnings earnings announcement.

Building on this intuition, for the next step of the analysis, we numerically solve for the agent's reporting strategy

$$\hat{R}(v, x) \in \arg\max x\hat{\gamma}(r) - \frac{(r - v)^2}{2}$$

by choosing a grid of (v_j, x_j), computing $\hat{R}(v_j, x_j)$ and interpolating for the $R(v, x)$ that are not on the grid. The resulting reporting functions are plotted next, for different values of the private incentive x.

Figure 3: Reporting functions



The graph displays the numerical solution to the manager's problem for various v (horizontal axis) and different incentives ($x = -0.005$, $x = 0$, $x = 0.005$ and $x = 0.012$).

Note that the manager over-reports when $x > 0$ and under-reports when $x < 0$. Interestingly, for any $x \neq 0$, the bias is not necessarily maximal near zero true earnings because managers with earnings significantly below the steepest point may attempt to reach positive earnings. Indeed, the maximal bias is reached for lower true earnings when x is higher. The reverse occurs when $x < 0$ as managers with higher true earnings choose more downward bias as x increases.

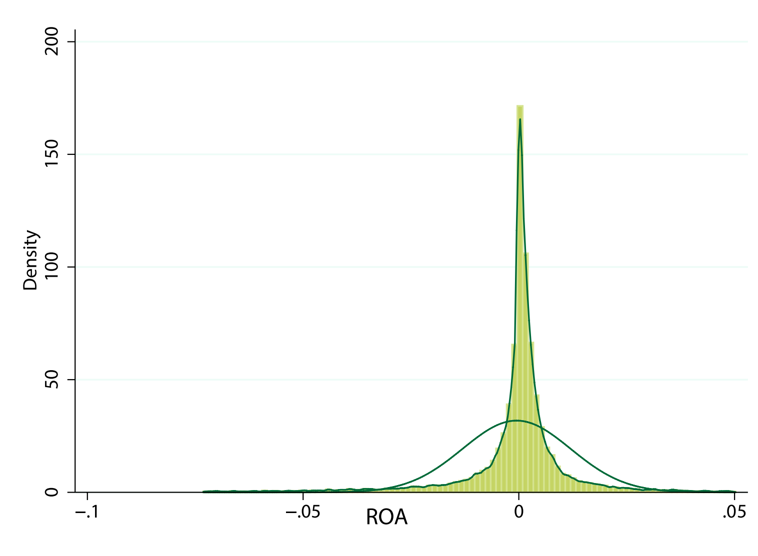
Having recovered $(\hat{\gamma}, \hat{R}(.))$, we are now equipped to recover the parameters of the distributions of true earnings $\tilde{v}$ and reporting incentives $\tilde{x}$. We present first a fully parametric approach to recover the distribution of earnings and incentives. Assume that $\tilde{v} \sim N(\mu_v, \sigma_v^2)$ and $\tilde{x} \sim N(\mu_x, \sigma_x^2)$ are independent and normally distributed random variables, so that the possible parameters of the model are vectors $\theta = (\mu_v, \sigma_v, d, \mu_x, \sigma_x)$. Our estimation procedure will be to simulate a dataset of reports and restatement occurrences for a given θ and find the parameters such that the joint distribution of these variables in the simulation closely matches the distribution of these variables in the empirical sample.

Formally, for a given θ, we draw true earnings v_i^θ and reporting incentives x_i^θ, and compute an accounting report $r_i^\theta = \hat{R}(v_i^\theta, x_i^\theta)$. We then draw a restatement d_i^θ from a Bernoulli distribution with success rate $1 - e^{d|r_i^\theta - v_i^\theta|}$. This procedure generates, for any given θ, a simulated dataset (r_i^θ, d_i^θ). We use the method of moments to identify the five parameters of interest, matching the following moments: the mean and standard-deviation of reported earnings, the average probability of detection and the mean and standard-deviation of detected restatements.

One limitation of the parametric model is that the distribution of reports as predicted by the model will be generally inconsistent with the distribution of reports in the data because true earnings are unlikely to be normally distributed (Hemmer and Labro 2019; Breuer and Windisch 2019). This may cause the model to attribute manipulation to any non-normality in the original distribution of true earnings.

To illustrate this point further, we plot below the distribution of reported earnings in the sample, and compare a kernel fit of the density to the best normal approximation. As can be seen, reported earnings appear to feature both more skewness and kurtosis than the normal distribution, and have heavier tails, as shown by the much larger probability mass near zero or small positive earnings jointly with many extreme observations. Within our framework, the mismatch between the two distributions can be explained by manipulation which, given that actual restatements cannot be too large due to the moderate probabilities of restatements (and their size), translates into a significant amount of potential misspecification.

Figure 4: Histogram of ROA surprises

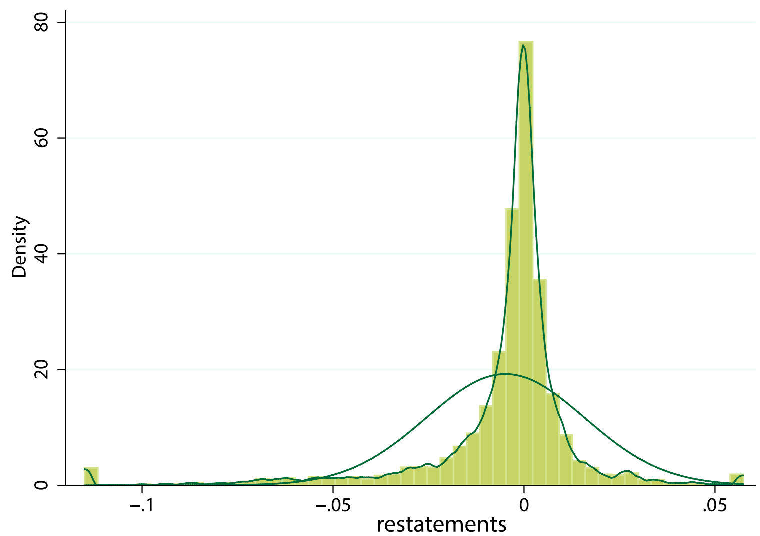


The graph displays a histogram of the reported ROA surprises, and overlays a non-parametric kernel fit as well as a parametric normal distribution.

We develop next an alternative empirical approach that draws information from the observed distribution of earnings. A brief description of this method is given below. We proceed iteratively starting from a guess about the distribution of true earning, estimating the parameters of the model for this given distribution, and then updating the distribution of true earnings by inverting the distribution of observed reports given the estimated parameters. We compare the updated distribution to the current guess and, if the updated distribution has not converged yet, we repeat the procedure with the new distribution of true earnings (until a convergence criterion is satisfied).

Formally, the procedure is iterative starting at iteration $i = 0$. At this point, we initialize the distribution of true earnings $\hat{f}_0$ to the distributed of restated earnings. Note that this starting point is intuitive since, for example, if detections are sufficiently accurate, only small manipulations will be undetected and the remaining restated earnings will be mapped back to true earnings by subtracting the manipulation (observed when there is a restatement). Interestingly, observed manipulations also feature heavy tails, as can be seen in the following histogram of restatements. Figure 5 reports the proportion of restatements as a function of the magnitude of the restatements. Figure 5 exhibits dispersion in incentives around zero in Fischer and Verrecchia (2000) since restatements can be positive or negative.

Figure 5: Histogram of restatements (ROA)



The graph displays a histogram of restatement in % of ROA.

The next step of our analysis is identical to the parametric approach except that $\tilde{v}$ is drawn from the empirical distribution $\hat{F}_0$ instead of $N(\mu_v, \sigma_v^2)$. As in the previous approach, we estimate the model using (generalized) method of moments, matching the mean and standard deviation of earnings and restatements, and the probability of detection. There are fewer parameters than moments, so we weight moments using the bootstrapped variance-covariance matrix of the moments.

Naturally, this need not be the correct parameter if the distribution of true earnings is different from our starting conjecture - which, as noted earlier, is only valid to the extent that there aren't too many large undetected misstatements. However, the estimate $\hat{\theta}_0$ can be used to recover an improved estimate of the distribution of true earnings. Specifically, for any sample report r_i, we can form a simulated true earnings

$$v_i^s = r_i - b_i^{\hat{\theta}_0} \tag{18}$$

where the bias b_i^θ is drawn randomly from the simulated conditional distribution $r_i^{\hat{\theta}_0} - v_i^\theta | r_i^{\hat{\theta}_0} = r_i$.[3]

It is clear that, if $\hat{\theta}_0$ is equal to the true parameter, the distribution of v_i^s will be the distribution of v_i even though it is not possible to recover the true v_i in the sample without knowing the true x_i. From this property, a *necessary* condition for θ_0 to be the true parameter is the c.d.f. of v_i^s, which we denote $\hat{F}_1(.)$ should coincide with the c.d.f. $\hat{F}_0(.)$. We use a stopping condition such that the maximal difference between the two c.d.f. should be less than 0.001. If this is not the case, we update the conjecture about the distribution of true earnings to $F_1(.)$ and repeat the steps above, simulating $\tilde{v}$ from $F_1(.)$ and re-estimating $\hat{\theta}_1$, updating the distribution of true earnings to $F_2(.)$. We repeat this procedure iteratively until the stopping condition is met.[4]

6 Estimation Results

Table 4 reports estimation results for the two estimation approaches. In the first model ("parametric"), earnings and incentives are assumed to be normally-distributed, while in the second model ("semi-parametric"), only the incentive noise is assumed to be normally-distributed and the distribution of true earnings is recovered from the estimation procedure described in section 5.

In the parametric model, the mean of true earnings is $\hat{\mu}_v = -0.28\%$. Given that reported earnings are expressed in surprises, this suggests a small upward bias of a quarter of a percent, which is about one third of the estimated standard deviation of true earnings of $\hat{\sigma}_v = 0.78\%$. The regulation intensity is $\hat{d} = 8.89$, which roughly corresponds to a manipulation of 1% being detected with probability 8.5%. The mean of the incentive $\hat{\mu}_x = 0.08\%$ is very close to zero and suggests that on average managers are about equally likely to be biasing upwards or downwards; hence, most of the manipulation is driven by the noise component $\hat{\sigma}_x = 1.94\%$.

Perhaps unsurprisingly, the semi-parametric model implies less evidence of bias. In the parametric model results, the structural model explains deviations from the normal distribution in terms of earnings management choices which may overstate the actual level of earnings management if the true distribution is not normal for reasons entirely separate from managerial discretion (Hemmer and Labro 2019). We find that the average reporting bias is close to zero and the incentive noise is about one fourth of the level found in the parametric model, at $\hat{\sigma}_x = 0.52\%$. To fit the observed likelihood of detection, the intensity of detection

[3]Specifically, we simulate 10,000 draws of $(r_j^{\hat{\theta}_0}, v_j^{\hat{\theta}_0})$, choose the draw i with minimal $|r_i^{\hat{\theta}_0} - r_i|$ and compute $b_i^{\hat{\theta}_0} = r_i^{\hat{\theta}_0} - v_i^{\hat{\theta}_0}$.

[4]While, in principle, this procedure yields an empirically-driven distribution of true earnings, it does come with a downside. Because the procedure is a fixed point over a distribution, this problem could have multiple solutions that depend on the starting point. While we did not find convergence to other plausible solutions, this possibility cannot be fully ruled out computationally. Having noted this, we believe that the initializing true earnings to restated earnings provides a plausible starting point and could not find convergence to other empirically reasonable distributions with other starting point.

is also higher at $\hat{d} = 23.95$. For a manipulation of 1%, this estimated intensity maps to a probability of detection of 21.3%.

Table 4: Estimation Result

Model	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
Parametric Model	0.0008 *(0.0004)*	0.0194 *(0.0017)*	8.8876 *(0.4890)*	-0.0028 *(0.0004)*	0.0078 *(0.0009)*
Semi-parametric Model	-4.40E-05 *(0.0003)*	0.0052 *(0.0005)*	23.9540 *(1.9010)*		

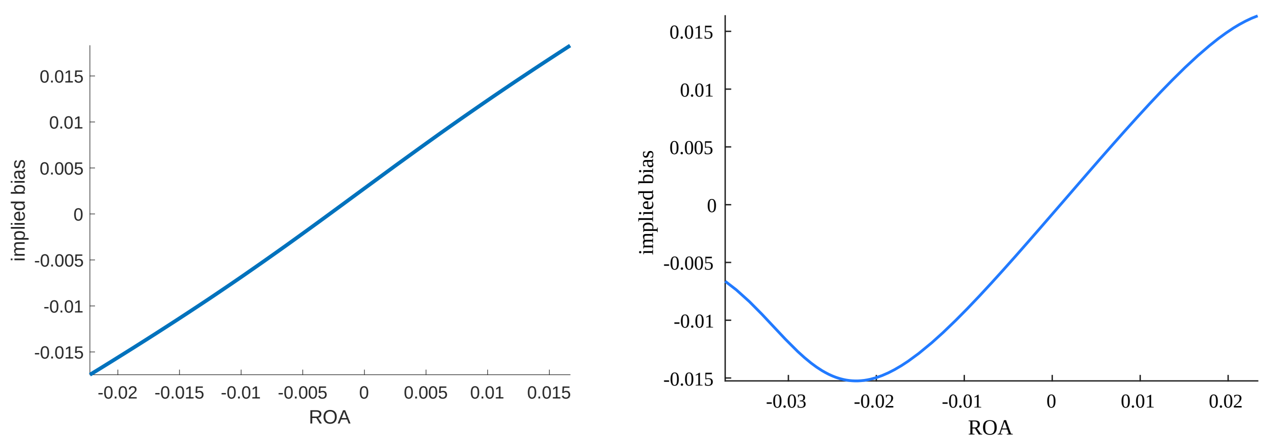
Bootstrapped standard errors are in parenthesis.

We report the conditional reporting bias $r - \mathbb{E}(\tilde{v}|r)$ in Table 6 in both models. The plot shows that, even if the bias is on average positive, there is a fair amount of variation across biases with, typically, positive (negative) surprises implying positive (negative) biases as implied by observed bunching around zero earnings surprise. As expected from Fischer and Verrecchia (2000), the parametric model implies a relationship between bias and reported earnings that is close to linear. In the semi-parametric model, by contrast, the bias is U-shaped and has a lower magnitude given large losses. For such losses, the market incentives are usually too low to manipulate earnings to become positive implying that firms with large losses tend to report more truthfully.

Figure 6: Implied Manipulation

Panel A: Parametric Model

Panel B: Semi-parametric Model



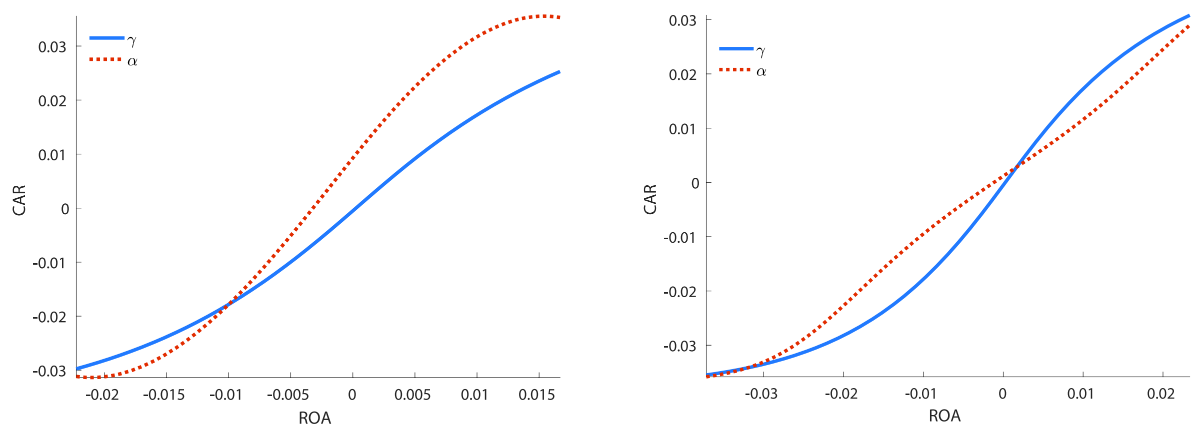
The x-axis is reported earnings, measured by ROA surprise and y-axis is the predicted implied manipulation given reported earnings.

In Figure 7, we plot the estimated relation between true earnings and stock price $\hat{\alpha}(v)$, which characterizes how prices would form if there were no manipulation. This function is recovered in several steps. First, we simulate a sample $(v_i, x_i, r_i, \gamma_i)_{i=1}^{N}$ at the estimated parameters of the model, where $r_i \equiv \hat{R}(v_i, x_i)$ is the optimal report and $\gamma_i \equiv \hat{\gamma}(r_i)$ is the estimated market response to the manager report, as plotted in Figures 3 and 1 respectively. We then fit v_i on r_i to estimate $\overline{v}_i \equiv \mathbb{E}(\tilde{v}|r_i)$ and finally, fit γ_i on $\overline{v}_i$ to recover the mapping between true earnings and stock price $\alpha(.)$ since $\gamma(r) = \alpha(\mathbb{E}(\tilde{v}|r))$. In the parametric model, the non-linearity in the price response requires non-linearities in $\alpha(.)$, implying that this relation tends to be markedly S-shaped. In the semi-parametric model, interestingly, we find that the mapping is much more linear.

Figure 7: Observed and underlying market response functions $\gamma(.)$ and $\alpha(.)$

Panel A: Parametric Model

Panel B: Semi-Parametric Model



The x-axis of the figures is the ROA surprise and y-axis is three-day CAR.The solid blue curve is the fitted market response function $\gamma(r)$, which is the same as in Figure (1). and the dotted curve is the fitted mapping between true earnings and stock price $\alpha(v)$.

Next, we use the estimates of the model to answer three questions. First, how much information is lost due to the co-mingling of the incentive noise into the accounting report (Frankel and Kartik 2019)? Second, does accounting noise appear to benefit or hurt investors? Third, in equilibrium, do managers benefit ex-ante from manipulation?

To motivate the first question, note that the conditional reporting bias can always be adjusted by the market, so that a report can be debiased by subtracting the conditional bias r. While it may be important to do so if the true earnings need to be used in a fundamental valuation model or for the purposes of deciding the merits of an enforcement case, investors can debias true earnings in models such as Dye (1988) or Stein (1989). But in our framework, reported earnings are contaminated by incentive noise, implying that reported earnings are less precise than the true earnings.

Following Beyer, Guttman, and Marinovic (2018) and Bertomeu, Ma, and Marinovic (2019), we measure the information loss in terms of the residual uncertainty about true earnings

$$\sigma_{v|r} \equiv \frac{\sqrt{Var[\tilde{v} - \mathbb{E}(\tilde{v}|\tilde{r})]}}{\sigma_v} \tag{19}$$

and the residual uncertainty about the firm value

$$\sigma_{\alpha(v)|r} \equiv \sqrt{\frac{Var[\alpha(\tilde{v}) - \alpha(\mathbb{E}(\tilde{v}|\tilde{r}))]}{Var(\alpha(\tilde{v}))}}. \tag{20}$$

The results are reported in Table 5. The information loss is larger in the parametric model than in the non-parametric model. In the latter, a little less than half of the information is lost due to accounting noise. These are large but note that, because of true earnings are unobservable, they describe a theoretical counter-factual where enforcement might perfect and the information known to managers would be always perfectly known to the market.

Table 5: Information Loss

Model	Info loss (earnings) $\sigma_{v\|r}$
Parametric Model	0.86
	(0.003)
Semi-Parametric Model	0.40
	(0.001)

bootstrap standard errors are in parenthesis.

To motivate the second question, agents in our model do not directly value the information loss but may indirectly benefit or be hurt by the accounting noise. We first calculate a welfare metric, we define the absolute change in expected price as

$$\Delta W = \mathbb{E}[\gamma(\tilde{r}) - \alpha(\tilde{v})], \tag{21}$$

which corresponds as the average difference between the actual price $\gamma(r)$ and the price if earnings had been reported truthfully. We can estimate this expression by simulating (r_i, v_i) and applying the estimated $\hat{\gamma}(.)$ and $\hat{\alpha}(.)$. We also report a scaled version of this number, dividing ΔW by the estimated standard error of $\alpha(\tilde{v})$. Note that, in Fischer and Verrecchia (2000), the linearity in $\gamma(.)$ and $\alpha(.)$ implies that $\Delta W = 0$ since the expected price $\mathbb{E}(\gamma(\tilde{r}))$ must equal the expected fundamental $\mathbb{E}(\alpha(\tilde{r}))$. If $\alpha(.)$ is convex, accounting noise will increase expected price, while if $\alpha(.)$ is concave, accounting noise will decrease expected price. Several models with decision-making can provide examples in which the price may be convex (Bertomeu, Cheynel, and Cianciaruso 2018), or neither concave nor convex (Friedman, Hughes, and J Michaeli 2019).

In our model, while we do not specifically need to model the underlying decision problem, we empirically recover the form of $\alpha(.)$.

Table 6 reports the estimates. We show that accounting noise actually causes a (small) reduction in firm value in the parametric model, at about 0.2%, that is, about one third of the standard-deviation of true earnings. The semi-parametric model reveals a much smaller estimate, which is consistent with both the lower estimated incentive noise and the more pronounced linearity in $\alpha(.)$. In this model, the loss is about 0.1%, that is, 10% of a standard deviation of earnings.

Table 6: Investor and Manager Welfare

Model	ΔW	$\Delta W/\sigma_v$	ΔM	$\Delta M/\sigma_M$
Parametric Model	-0.002	-0.293	2.726e-04	0.856
	(0.0012)	*(0.1579)*	*(3.78e-05)*	*(0.0413)*
Semi-Parametric Model	-0.001	-0.098	3.176e-05	0.403
	(0.0001)	*(0.0417)*	*(5.21e-06)*	*(0.0224)*

σ_M is the standard deviation of the manager's utility in presence of reporting uncertainty. Bootstrap standard errors are in parenthesis.

To motivate the third question, we ask whether manager might be interested in lobbying for less informative earnings. Fischer and Verrecchia (2000) show that managers are better-off with incentive noise than they would be without noise, since they can strategically manipulate when x is large. In our model, however, this benefit creates a decrease in expected price as noted in Table 6. We define the manager payoff as

$$\Delta M = \mathbb{E}\big(\tilde{x}\gamma(\tilde{r}) - \frac{1}{2}(\tilde{r} - \tilde{v})^2\big) - \mu_x \mathbb{E}\big(\alpha(\tilde{v})\big), \tag{22}$$

that is the difference between the expected utility of the manager in the equilibrium and the expected utility of the manager in a model where fundamentals must always be reported truthfully. Consistent with Fischer and Verrecchia (2000), we show that managers remain better-off with accounting noise despite the lower expected price, in both the parametric and semi-parametric models. The benefit of noise to the manager is about 85% of the standard deviation in the manager's utility in presence of uncertainty in the parametric model, and about 40% in the semi-parametric model, indicating small to moderate private benefits of noise.

7 Model Fit

We examine the goodness of fit by examining the data moment and simulated moments by both models evaluated at the estimates. Table 7 reports the empirical moments that we target and the predicted values from the two models, with their standard errors in parentheses. The moment include the mean of reported earnings (ROA surprise), standard deviation of reported earnings, ratio of observations with a restatement to total observations, mean of the restated amount and standard deviation of restated amount. Both models show good fits between sample data and the model-implied moments.

Table 7: Model and Data Moments

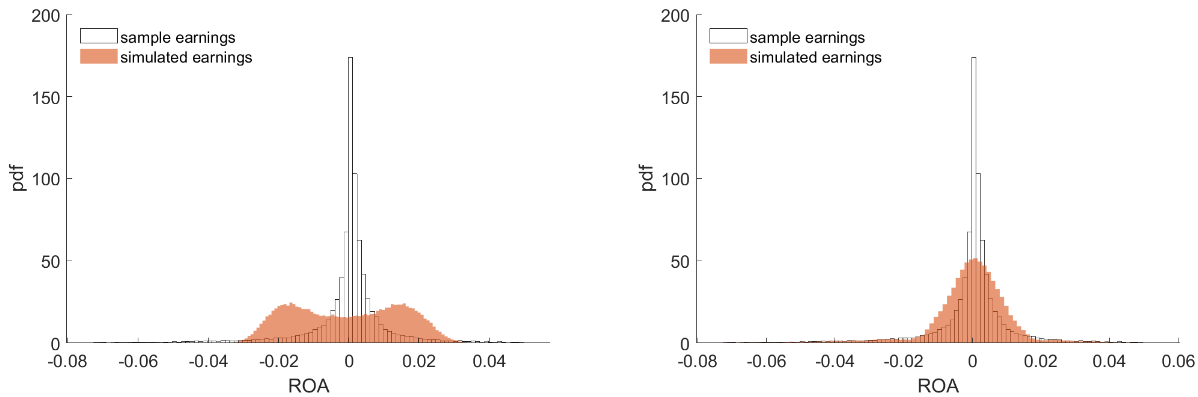
#	Moments	Data	Parametric Model	Semi-Parametric Model
1	Mean of reported earnings	-0.0005	-0.0006	-0.0007
		(0.0000)	*(0.0004)*	*(0.0002)*
2	Std of reported earnings	0.0125	0.0155	0.0131
		(0.0002)	*(0.0005)*	*(0.0002)*
3	Ratio of restatement cases	0.1103	0.1103	0.1097
		(0.0036)	*(0.0036)*	*(0.0042)*
4	Mean of restated amount	-0.0049	-0.0045	0.0004
		(0.0005)	*(0.0006)*	*(0.0008)*
5	Std of restated amount	0.0207	0.0182	0.0087
		(0.0009)	*(0.0005)*	*(0.0005)*

bootstrap standard errors are in parenthesis.

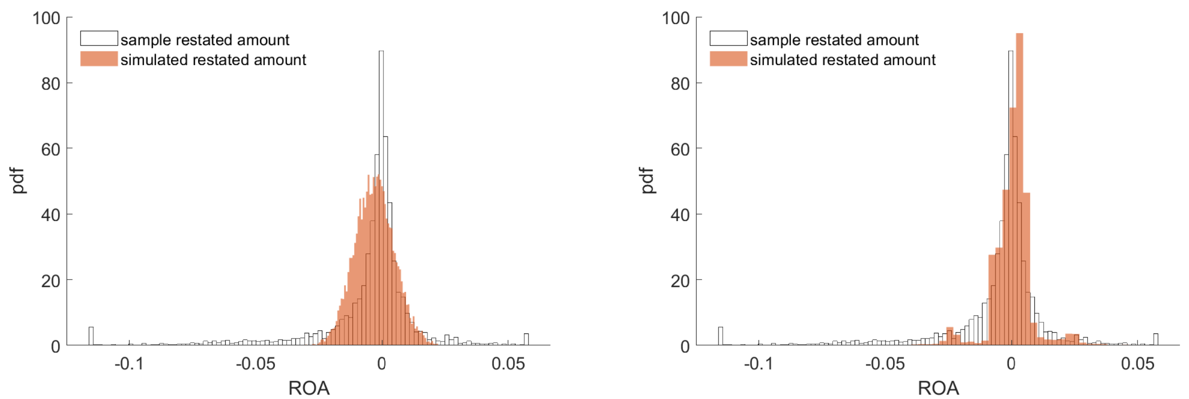
Figure 8 compares the simulated distributions of various types of earnings using the estimates in the two models. Panel A plots the histogram of reported earnings simulated by the two model estimates, with sample reported earnings as a benchmark. Ideally, we expect the simulated distribution of reported earnings similar to the sample distribution if the model is a good fit. The simulated data distribution using parametric model is more dispersed than the sample reported earnings distribution. We do not find the peak on zero earnings, and instead the distribution has two peaks around zeros. Note that the parametric model assumes a normal distribution for true earnings as shown in Figure 4, which as discussed before, inherently translates a significant amount of misspecification. Compared to a normal distribution, the simulated earnings distribution indicates misreporting has translate the earnings around zero to the adjacent areas around -0.02 and 0.02. The non-parametric approach produces a distribution much closer to the sample distribution, more skewed than a normal distribution and closer to the sample distribution.

Figure 8: Simulated Distribution

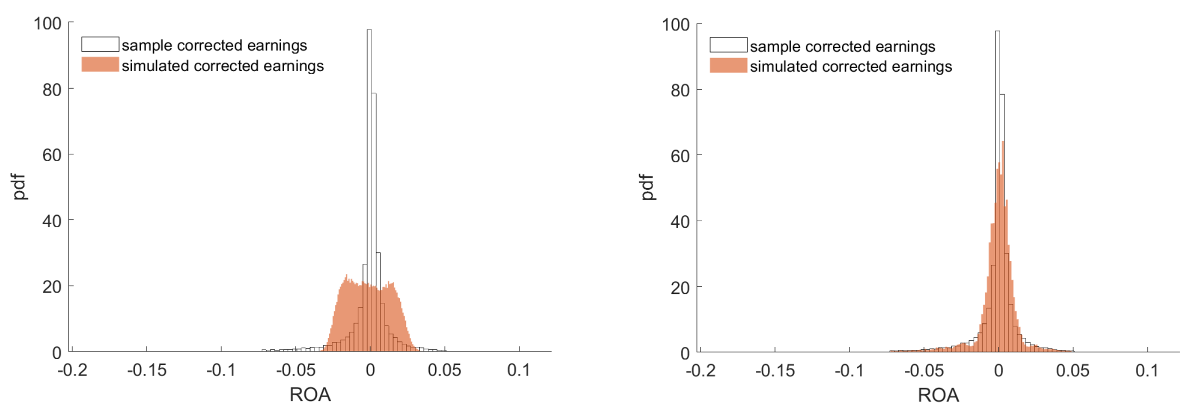
Panel A: Reported Earnings



Panel B: Misstated Earnings



Panel C: Corrected Earnings



We also explore the distribution of misstated amount in Panel B and corrected earnings in Panel C of Figure 8. The semi-parametric model generated data distribution is closer to the sample distribution in both two panels. For the parametric model, both the simulated misstated earnings and corrected earnings are more dispersed than the sample. As for the semi-parametric approach, our estimates simulate very close distribution of misstated amount, shown in Panel B of Figure 8, with a concentrated peak at around 0.005.

We assess the performances of the two models by comparing a range of non-targeted moments. These moments are not utilized in our estimations but they capture important features of the financial reporting and regulatory quantities. Panel A focuses on misstating firms which only contains observations that has a restatement in the specific firm-year, while the non-misstating firms in Panel B only use observations that do not have a restatement. We investigate both the mean and standard deviation of the reported earnings, as well as the covariance of reported earnings and misstated amount.

Table 8: Non-Targeted Moments

Moments	Data	Parametric Model	Semi-parametric model
	Panel A. Misstating Firms		
Mean of reported earnings	-0.00050	-0.0009	-0.0008
	(0.0001)	*(0.0004)*	*(0.0002)*
std of reported earnings	0.01257	0.0153	0.0135
	(0.0002)	*(0.0005)*	*(0.0002)*
cov. reported earnings and misstated earnings	0.0002	-0.0003	-0.0001
	(0.0000)	*(0.0000)*	*(0.0000)*
	Panel B. Non-misstating Firms		
mean of reported earnings	-0.0007	0.0015	0.0003
	(0.0003)	*(0.0008)*	*(0.0003)*
std of reported earnings	0.01224	0.0170	0.0096
	(0.0005)	*(0.0006)*	*(0.0004)*

8 Nested Models

We consider two special cases of the baseline models to study the extent to which our model can explain the data. First, we set the mean of reporting incentive $\mu_x = 0$, so that the chances of a manager preferring a higher or lower stock price are equally half. Second, we change reporting incentive $\tilde{x}$ from random variable to a constant by setting $\sigma_x = 0$ - in this case, as is well-known, the market can reverse engineer the true bias - note that this model is estimated structurally in Bertomeu, Cheynel, Li, and Liang (2018) in the case of normal distributions but without restatement data. Table 9 reports the analysis of the nested models.

Since our baseline estimate for μ_x is close to zero, the results do not change compared to the baseline estimates in both models. By contrast, the constant incentive model fits significantly less. The estimate for μ_x in the parametric model is 0.0083, lower than in the baseline model since all firms must be over-reporting. The distribution of true earnings is also different, with larger mean and standard deviation.

In the semi-parametric model, the estimated detection intensity is 53.51, more than twice of the baseline estimates.

Table 9: Nested Models

Panel A. $\mu_x = 0$					
Model		Std.Dev.$_x$ σ_x	Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
Parametric Model		0.0198	8.8344	-0.0033	0.0071
		(0.0018)	*(0.4844)*	*(0.0005)*	*(0.0008)*
Semi-parametric Model		0.0054	23.2819		
		(0.0005)	*(2.0073)*		
Panel B. $\sigma_x = 0$					
Model	Mean $_x$ μ_x		Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
Parametric Model	0.0083		9.9990	-0.0111	0.0135
	(0.0023)		*(8.2394)*	*(0.0028)*	*(0.0008)*
Semi-parametric Model	-0.0003		53.5146		
	(0.0002)		*(3.6826)*		

Bootstrap standard errors are in parentheses.

9 Counterfactuals

In the model, the market's uncertainty about managers reporting objectives is the key factor on both manager and investor's welfare. In Table 10, we quantify the effect of reporting incentives based on counterfactual analysis. The market response to true earnings $\alpha(.)$ stays unchanged in the counterfactual analysis, while $\gamma(.)$ should change due to the change of parameters. Thus, we first numerically solve for $\gamma(.)$ given the new set of parameters. Second, given the set of parameters and the new $\gamma(.)$, we solve the manager's optimal reporting strategy and calculate the information loss and investor and manager welfare change the same way as in Table 5 and Table 6.

The counterfactual analyses are conducted based on the parameter estimates from Table 4. The baseline column reports the result computed using the estimates in Table 4. We first increase the mean of reporting incentive μ_x by 100 times and Column 'High Incentive' reports the analyses. In this case, $\mu_x = 0.08$ for the parametric model and 0.004 for the semi-parametric model. We find that the residual uncertainty $\sigma_{v|r}$ increases significantly, especially in the parametric model. When the manager has higher incentive to issue a higher report, the information on v and $\alpha(v)$ contained in the report is much lower. Investor welfare becomes negative in the parametric model, but does not change much in the semi-parametric model. Manager's welfare increases by twice in both models. In the last column, we reduce the uncertainty of reporting incentive to only 1% of the estimate. The new σ_x is 0.0002 for the parametric model and less than 0.0001 for the second one. The residual uncertainty on v is 0.623, about 27.8% reduction compared to the baseline, while the residual uncertainty on $\alpha(v)$ is reduced by 73%. We do not find a significant change in investor welfare change (ΔW and $\Delta W/\sigma_v$), but there is a large drop in manager's welfare, as expected from Fischer and Verrecchia (2000) since the manager is exploiting variability in incentives.

Table 10: Counterfactual Analysis

Panel A. Parametric Model				
		Baseline (Estimated)	High Incentive $\mu_x = 100\hat{\mu}_x$	Low Uncertainty $\sigma_x = 0.01\hat{\sigma}_x$
Info loss	$\sigma_{v\|r}$	0.863	0.954	0.623
Investor welfare	ΔW	2.58E-04	-0.002	7.30E-06
	$\Delta W/\sigma_v$	0.033	-0.293	9.41E-04
Manager welfare	ΔM	5.76E-05	2.73E-04	-3.35E-07
	$\Delta M/\sigma_v$	0.493	0.856	-0.052
Panel B. Semi-parametric Model				
		Baseline (Estimated)	High Incentive $\mu_x = 100\hat{\mu}_x$	Low Uncertainty $\sigma_x = 0.001\hat{\sigma}_x$
Info loss	$\sigma_{v\|r}$	0.398	0.425	0.364
Investor welfare	ΔW	-3.09E-04	-4.99E-05	-2.18E-04
	$\Delta W/\sigma_v$	-0.025	-0.004	-0.017
Manager welfare	ΔM	2.52E-06	5.60E-07	9.30E-09
	$\Delta M/\sigma_v$	0.054	0.010	0.024

The table reports information loss and investor and manager welfare changes, computed under alternative parameters. The market response function $\gamma(.)$ is estimated given $\alpha(.)$ and the alternative set of parameters. We then compute the welfare and information changes using the fitted $\gamma(.)$ and alternative parameters. The baseline column results are computed with estimated parameters reported in Table 4. Results in Column 'High Incentive' are calculated with μ_x increases by 100 times. That is $\mu_x = 0.08$ for parametric model and 0.004 for semi-parametric model. The Column 'Low Uncertainty' reports result that reduces σ_x by 100 times, which are very close to zero in both models. The last column reduces the regulation intensity by 100 times. The new d is 0.009 in Panel A and 0.24 in Panel B.

10 Subsamples

In this section we address the potential heterogeneity across firms and industries by dividing our sample into subgroups. We explore two firm characteristics that may influence firms' reporting incentive x: size and growth opportunity. We later estimate the model by industry.

To study whether firms of different sizes exhibit differences in their estimates, we divide the sample into three groups according to their market value of equity (*mktvalue*) each year. Table 11 reports the results. We find that estimates from the two models share some common trends. First, the mean of reporting incentive x is largest in the large firms and lowest in the small firms. This suggests that larger firms overall have higher benefit of misreporting. Second, the trend is the same for regulatory intensity d. In the parametric model, d is 5.99 for small firms portfolio and 15.19 for large firms, more than twice larger. In Semi-parametric model, although d estimates are higher than that of the aggregate sample due to the limit of estimation method, the trend persists with d equal to 54.22 for large firms and 39.04 for smaller firms. This is consistent with the previous finding that larger firms are subject to more stringent regulatory monitoring. Lastly, we notice a monotonic increasing mean of true earnings in the parametric approach. The mean of true earnings for small firms is slightly negative, -0.0058, while larger firms have an average realized true earnings very close to zero.

Table 11: Estimation by Size

Panel A. Parametric Model

Portfolio	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
Aggregate	**0.0008**	**0.0194**	**8.8876**	**-0.0028**	**0.0078**
	(0.0004)	*(0.0017)*	*(0.4890)*	*(0.0004)*	*(0.0009)*
Small	-0.0009	0.0362	5.9925	-0.0058	0.0075
	(0.0019)	*(0.0028)*	*(0.4370)*	*(0.0008)*	*(0.0002)*
Medium	0.0005	0.0182	10.8254	-0.0023	0.0003
	(0.0007)	*(0.0027)*	*(0.7464)*	*(0.0004)*	*(0.0044)*
Large	0.0006	0.0076	15.1930	0.0000	0.0055
	(0.0007)	*(0.0037)*	*(1.4022)*	*(0.0006)*	*(0.0045)*

Panel B. Semi-parametric Model

Portfolio	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d
Aggregate	**-4.40E-05**	**0.0052**	**23.9540**
	(0.0003)	*(0.0005)*	*(1.9010)*
Small	2.56E-05	0.000	39.042
	(0.0008)	*(0.0008)*	*(1.3027)*
Medium	6.69E-05	0.001	37.056
	(0.0004)	*(0.0013)*	*(4.9907)*
Large	0.0003	0.000	54.216
	(0.0004)	*(0.0011)*	*(2.9923)*

We next explore whether firms with different growth opportunities share the same reporting incentives and external regulation. We sort the sample into three portfolios by book-to-market equity ratios (*bm*) and update the portfolio each year. We use bm to avoid the situation where book value of equity is zero. The high book-to-market ratio portfolio contains firms with lower growth opportunity, and thus the results are tabulated in the "Low" portfolio. In Panel A, portfolio with high growth opportunity on average has a higher but more dispersed reporting incentive. The high growth opportunity portfolio reports a μ_x of 0.0038, σ_x of 0.0269, compared to the low portfolio with μ_x of -0.0009 and σ_x of 0.0168. We find high growth firms have a relatively smaller regulatory intensity, with d of 7.5461 compared to 10.5764. In terms of the true earnings, we find low growth firms have a much more concentrated distributions with σ_v of 0.0019, four times smaller than the aggregate sample result. Panel B reports the semi-parametric results and the findings are consistent with the parametric estimation. Reporting incentives for high growth firms have a slightly higher mean and higher standard deviation. We also find that high growth firms face less regulatory intensity, as d for high portfolio is 187564 and 30.4650 for low portfolio, consistent with the result in Panel A. Overall, we find that high growth firms have higher and more dispersed reporting incentives and face lower regulatory intensity.

Table 12: Estimation by Growth Opportunity

Panel A. Parametric Model

Portfolio	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
Aggregate	**0.0008**	**0.0194**	**8.8876**	**-0.0028**	**0.0078**
	(0.0004)	*(0.0017)*	*(0.4890)*	*(0.0004)*	*(0.0009)*
Low	-0.0009	0.0168	10.5764	-0.0026	0.0019
	(0.0009)	*(0.0022)*	*(0.7537)*	*(0.0004)*	*(0.0028)*
Medium	0.0006	0.0159	9.4809	-0.0022	0.0103
	(0.0018)	*(0.003)*	*(0.8777)*	*(0.0004)*	*(0.0007)*
High	0.0038	0.0269	7.5461	-0.0035	0.0105
	(0.0024)	*(0.0037)*	*(0.4619)*	*(0.0007)*	*(0.0045)*

Panel B. Semi-Parametric Model

Portfolio	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d
Aggregate	**-4.40E-05**	**0.0052**	**23.9540**
	(0.0003)	*(0.0005)*	*(1.9010)*
Low	-0.0005	0.0040	30.4650
	(0.0003)	*(0.0004)*	*(2.4734)*
Medium	6.69E-05	0.0009	37.0557
	(0.0005)	*(0.0014)*	*(4.8196)*
High	-8.83E-05	0.0072	18.7564
	(0.0006)	*(0.0013)*	*(2.1568)*

Table 13: Estimation by Industry

Panel A. Parametric Model

FF 12 Industries	Industry	Mean $_x$ μ_x	Std.Dev.$_x$ σ_x	Intensity d	Mean$_v$ μ_v	Std.Dev.$_v$ σ_v
	Aggregate Sample	**0.0008**	**0.019**	**8.888**	**-0.0028**	**0.0078**
		(0.0004)	*(0.0017)*	*(0.4890)*	*(0.0004)*	*(0.0009)*
1	Consumer NonDurables	-0.0006	0.0103	12.3145	-0.0022	0.0001
		(0.0062)	*(0.0068)*	*(11.3599)*	*(0.0051)*	*(0.0006)*
2	Consumer Durables	0.0098	0.0656	4.9970	-0.0050	0.0141
		(0.0151)	*(0.0038)*	*(10.9876)*	*(0.01)*	*(0.0101)*
3	Manufacturing	-0.0047	0.0290	9.0231	-0.0039	0.0000
		(0.0096)	*(0.0154)*	*(2.5817)*	*(0.004)*	*(0.0063)*
4	Oil, Gas, and Coal	-0.0014	0.0057	20.3675	-0.0019	0.0050
	Extr. and Prod.	*(0.009)*	*(0.0281)*	*(3.5103)*	*(0.0073)*	*(0.008)*
5	Chemicals and Allied	-0.0003	0.0114	10.8386	-0.0033	0.0114
	Products	*(0.003)*	*(0.0105)*	*(5.1519)*	*(0.0033)*	*(0.0074)*
6	Business Equipment	0.0392	0.0682	3.8445	-0.0105	0.0117
		(0.0176)	*(0.0403)*	*(1.9131)*	*(0.0018)*	*(0.0074)*
9	Wholesale, Retail,	-0.0015	0.0119	10.2768	-0.0012	0.0022
	and Some Services	*(0.008)*	*(0.0093)*	*(5.0219)*	*(0.0043)*	*(0.0048)*
10	Healthcare, Medical	0.0014	0.0147	11.1282	-0.0007	0.0122
	Equipment, and Drug	*(0.0058)*	*(0.0355)*	*(1.1046)*	*(0.0026)*	*(0.0038)*
12	Other	0.0020	0.0135	17.6952	0.0036	0.0000
		(0.0063)	*(0.0507)*	*(2.4614)*	*(0.0027)*	*(0.0054)*

Panel B. Semi-Parametric Model

FF 12 Industries	Industry	Mean_x μ_x	S.D. x σ_x	Intensity d
	Aggregate Sample	**-4.40E-05**	**0.005**	**23.954**
		(0.0003)	*(0.0005)*	*(1.9010)*
1	Consumer NonDurables	-0.0004	0.0033	27.8092
		(0.5022)	*(0.0078)*	*(10.46)*
2	Consumer Durables	-0.0017	0.0181	11.9593
		(0.0013)	*(0.0026)*	*(18.4)*
3	Manufacturing	-0.0012	0.0003	48.2605
		(0.0057)	*(0.0007)*	*(38.27)*
4	Oil, Gas, and Coal	-0.0013	0.0001	53.0732
	Extr. and Prod.	*(0.0132)*	*(0.0091)*	*(11.97)*
5	Chemicals and Allied	4.31E-05	0.0003	29.0716
	Products	*(0.0065)*	*(0.0036)*	*(15.6)*
6	Business Equipment	0.0017	0.0159	11.8253
		(0.0057)	*(0.0069)*	*(17.63)*
9	Wholesale, Retail,	-0.0008	7.71E-05	29.8053
	and Some Services	*(0.0057)*	*(0.0002)*	*(23.52)*
10	Healthcare, Medical	0.0004	3.98E-05	27.6476
	Equipment, and Drug	*(0.0033)*	*(0.0002)*	*(1.83)*
12	Other	-0.0012	0.0001	26.0470
	(0.007)	*(0.0019)*	*(19.08)*	

Lastly we investigate whether certain industries display different behaviors in earnings manipulation. We use Fama-French 12 industry classification and since we exclude firms in financial and regulated industries, there are nine groups of industries in the subsample analysis. Table 13 has the parameter estimates. Some industries stand out and the results are consistent in both models. We find that Business Equipment – Computers, Software, and Electronic Equipment has the highest mean of reporting benefit μ_x among all industries. Both Business Equipment and Consumer Durables have similarly high standard deviation of x. As for regulation intensity, the Oil, Gas and Coal Extraction and Production industry has the highest d in both models.

11 Concluding Remarks

In this study, we develop a simple approach to estimate the noisy earnings management model of Fischer and Verrecchia (2000), bringing together information about reported earnings, price and detected misstatements. The model requires limited a-priori theoretical knowledge about the relation between price and earnings, or the distribution of true earnings, and may allow researchers to interpret cross-sectional evidence about restatements in terms of accounting noise. Our approach adds a few novelties to the literature. First, we try to keep the analysis simple, by offering a model that does not require to numerically solve the model - and, hence, the analysis requires limited code or computational intensity and may be easily fitted to many samples. Second, we try to lift as many assumptions as we can about functional forms, using the observed relation between price and earnings as a building block for the estimation, and, in a semi-parametric model, assuming no knowledge about the distribution of true earnings. Third, we write a model that can bring in this semi-parametric framework information about detection probabilities and actual misstastements, based on the idea that identification is likely to be greatly improved from ex-post information about restatements. Fourth, we use the model to measure the effect of manipulation on average market price or manager welfare, showing who benefits ex-ante from accounting noise. In terms of primary conclusion, we find that reporting uncertainty is real, but its consequences on biases, prices and welfare are not large; we also observe that the average bias is quite small because a very significant of firms each period actually behaves as if they are willing to decrease earnings.

Yet, one may not view this approach as a finalized perspective on estimating manipulation and keeping the model simple is, on our opinion, an invitation to expand on the many simplifications that we have made along this process - some of which we develop next. We have focused on a purely static model in which managers focus on price each year. One could plausibly extend the analysis to estimating the dynamic relation between price and reported earnings and use this relation to solve for a dynamic manipulation strategy; empirically, we do not know yet if such complex dynamic manipulation strategies are likely to be first-order or if managers use heuristics to look at current price. We also left aside the interaction of contracting and manipulation, which suggests that the distribution of the incentive x may be endogenous; it is yet unclear whether boards target manipulation as their main concern when choosing contracts. We have also observed anomalous patterns that are not well-explained by the model, which seems to suggest that some functional forms, specifically relating to manipulation cost or the distribution of incentives may need to be examined and relaxed. The empirical data that would allow us to pin down these functional forms is still to be found.

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