MPI-PhT/94-56
TUM-T31-69/94
August 1994

Limitations in Measuring the Angle β by using $SU(3)$ Relations for B-Meson Decay-Amplitudes *

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Abstract

Flavour $SU(3)$ symmetry of strong interactions and certain dynamical assumptions have been used in a series of recent publications to extract weak CKM phases from B-decays into $\{\pi\pi, \pi K, K\bar{K}\}$ final states. We point out that irrespectively of $SU(3)$-breaking effects the presence of QCD-penguin contributions with internal u- and c-quarks precludes a clean determination of the angle β in the unitarity triangle by using the branching ratios only. This difficulty can be overcome by measuring in addition the ratio x_d/x_s of $B_d^0-\bar{B}_d^0$ to $B_s^0-\bar{B}_s^0$ mixings. The measurement of the angle γ is unaffected by these new contributions. Some specific uncertainties related to $SU(3)$-breaking effects and electroweak penguin contributions are briefly discussed.

*Supported by the German Bundesministerium für Forschung und Technologie under contract 06 TM 732 and by the CEC science project SC1–CT91–0729.

Recently in a series of interesting publications [1]-[5], $SU(3)$ flavour symmetry of strong interactions [6]-[10] has been combined with certain dynamical assumptions (neglect of annihilation diagrams, etc.) to derive simple relations among B-decay amplitudes into $\pi\pi$, πK and $K\bar{K}$ final states. These $SU(3)$ relations should allow to determine in a clean manner both weak phases of the Cabibbo-Kobayashi-Maskawa-matrix (CKM-matrix) [11] and strong final state interaction phases by measuring *only* branching ratios of the relevant B-decays. Neither tagging nor time-dependent measurements are needed!

In this note we would like to point out certain limitations of this approach. Irrespectively of the uncertainties related to $SU(3)$-breaking effects, which have been partially addressed in [1]-[5], the success of this approach depends on whether the penguin amplitudes are fully dominated by the diagrams with internal top-quark exchanges. As we will show below, sizable contributions may also arise from QCD-penguins with internal up- and charm-quarks. The main purpose of our letter is to analyze the impact of these new contributions on the analyses of refs. [1]-[5].

Interestingly enough the determination of the angle γ in the unitarity triangle as outlined in [1, 4, 5] is not affected by the presence of QCD-penguins with internal u- and c-quarks. Unfortunately these new contributions preclude a clean determination of the angle β by using the branching ratios only. We show however that the additional knowledge of the ratio x_d/x_s of $B_d^0-\bar{B}_d^0$ to $B_s^0-\bar{B}_s^0$ mixings would allow a clean determination of β except for $SU(3)$-breaking uncertainties.

In order to discuss these effects, let us denote, as in [1]-[5], the amplitudes corresponding to $b\to d$ and $b\to s$ QCD-penguins by $\bar{P}$ and $\bar{P}'$, respectively, and those representing the CP-conjugate processes by P and P' (these amplitudes can be obtained easily from $\bar{P}$ and $\bar{P}'$ by changing the signs of the weak CKM-phases). Then, taking into account QCD-penguin diagrams with internal u-, c- and t-quarks, we get

$$\begin{aligned}\bar{P} &= \sum_{q=u,c,t} V_{qd}^*V_{qb}P_q = v_c^{(d)}(P_c-P_u)+v_t^{(d)}(P_t-P_u)\\ \bar{P}' &= \sum_{q=u,c,t} V_{qs}^*V_{qb}P_q = v_c^{(s)}(P_c'-P_u')+v_t^{(s)}(P_t'-P_u'),\end{aligned} \tag{1}$$

where we have employed unitarity of the CKM-Matrix and have defined the CKM-factors as

$$\begin{aligned}v_c^{(q)} &= V_{cq}^*V_{cb}\\ v_t^{(q)} &= V_{tq}^*V_{tb}.\end{aligned} \tag{2}$$

Applying the Wolfenstein parametrization [12] gives

$$\begin{aligned} v_c^{(d)} &= -\lambda|V_{cb}|\left(1+\mathcal{O}(\lambda^4)\right) \\ v_t^{(d)} &= |V_{td}|\exp\left(i\beta\right) \end{aligned} \tag{3}$$

and

$$\begin{aligned} v_c^{(s)} &= |V_{cb}|\left(1+\mathcal{O}(\lambda^2)\right) \\ v_t^{(s)} &= -|V_{cb}|\left(1+\mathcal{O}(\lambda^2)\right), \end{aligned} \tag{4}$$

where the estimate of non-leading terms follows ref. [13]. In order to simplify the presentation we will omitt these non-leading terms in λ in our analysis.

Introducing the notation

$$P_{q_1q_2} \equiv |P_{q_1q_2}|\exp\left(i\delta_{q_1q_2}\right) \equiv P_{q_1} - P_{q_2} \tag{5}$$

with $q_1, q_2 \in \{u,c,t\}$ and combining eqs. (3) and (4) with (1) yields

$$\bar{P} = \left[-\frac{1}{R_t}\frac{|P_{cu}|e^{i\delta_{cu}}}{|P_{tu}|e^{i\delta_{tu}}} + e^{i\beta}\right]|V_{td}||P_{tu}|e^{i\delta_{tu}} \tag{6}$$

$$\bar{P}' = \left[-\frac{|P'_{cu}|e^{i\delta'_{cu}}}{|P'_{tu}|e^{i\delta'_{tu}}} + 1\right]e^{i\pi}|V_{cb}||P'_{tu}|e^{i\delta'_{tu}}. \tag{7}$$

R_t is given by the CKM-combination

$$R_t \equiv \frac{1}{\lambda}\frac{|V_{td}|}{|V_{cb}|} \tag{8}$$

and represents the side of the so-called unitarity triangle that is related to B_d^0–$\bar{B}_d^0$ mixing. From present experimental data, we expect R_t being of $\mathcal{O}(1)$ [13].

Assuming $SU(3)$ flavour symmetry of strong interactions, the "primed" amplitudes $|P'_{q_1q_2}|$ and strong phase shifts $\delta'_{q_1q_2}$ are equal to the "unprimed" ones [3]-[5]. Consequently, the penguin-amplitudes (6) and (7) can be expressed in the form

$$\bar{P} = \left[-\frac{1}{R_t}\Delta P + e^{i\beta}\right]|V_{td}||P_{tu}|e^{i\delta_{tu}} \tag{9}$$

$$\bar{P}' = \left[-\Delta P + 1\right]e^{i\pi}|V_{cb}||P_{tu}|e^{i\delta_{tu}}, \tag{10}$$

where ΔP is defined by

$$\Delta P \equiv |\Delta P|e^{i\delta_{\Delta P}} \equiv \frac{|P_{cu}|e^{i\delta_{cu}}}{|P_{tu}|e^{i\delta_{tu}}} \tag{11}$$

and describes the contributions of the QCD-penguins with internal u- and c-quarks. Notice that ΔP suffers from large hadronic uncertainties, in particular from strong final state interaction phases parametrized by δ_{cu} and δ_{tu}. In the limit of degenerate u- and c-quark masses, ΔP would vanish due to the GIM mechanism. However, since $m_u \approx 4.5$ MeV, whereas $m_c \approx 1.3$ GeV, this GIM cancellation is incomplete and in principle sizable effects arising from ΔP could be expected.

In order to investigate this issue quantitatively, let us estimate ΔP by using the perturbative approach of Bander, Silverman and Soni [14]. To simplify the following discussion, we neglect the influence of the renormalization group evolution from $\mu = \mathcal{O}(M_W)$ down to $\mu = \mathcal{O}(m_b)$ and take into account QCD renormalization effects only approximately through the replacement $\alpha_s \to \alpha_s(\mu)$. Then, the low-energy effective penguin Hamiltonian is given by (see, e.g., refs. [15]-[18])

$$\begin{aligned}\mathcal{H}_{\rm eff}^{\rm pen}(\Delta B=-1) &= -\frac{G_{\rm F}}{\sqrt{2}}\frac{\alpha_s(\mu)}{8\pi}\sum_{q=d,s}\Big[v_c^{(q)}\left\{G(m_c,k,\mu)-G(m_u,k,\mu)\right\} \qquad (12)\\ &\quad +v_t^{(q)}\left\{E(x_t)+\frac{2}{3}\ln\left(\frac{\mu^2}{M_W^2}\right)-G(m_u,k,\mu)\right\}\Big]P^{(q)},\end{aligned}$$

where

$$P^{(q)} = -\frac{1}{3}Q_3^{(q)} + Q_4^{(q)} - \frac{1}{3}Q_5^{(q)} + Q_6^{(q)} \qquad (13)$$

is a linear combination of the usual QCD-penguin operators

$$\begin{aligned} Q_3^{(q)} &= (\bar{q}b)_{\rm V-A}\textstyle\sum_{q'}(\bar{q}'q')_{\rm V-A}\\ Q_4^{(q)} &= (\bar{q}_\alpha b_\beta)_{\rm V-A}\textstyle\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{\rm V-A}\\ Q_5^{(q)} &= (\bar{q}b)_{\rm V-A}\textstyle\sum_{q'}(\bar{q}'q')_{\rm V+A}\\ Q_6^{(q)} &= (\bar{q}_\alpha b_\beta)_{\rm V-A}\textstyle\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{\rm V+A}\end{aligned} \qquad (14)$$

and the function $G(m,k,M)$ is defined by [18]

$$G(m,k,M) \equiv -4\int_0^1 dx x(1-x)\ln\left[\frac{m^2-k^2x(1-x)}{M^2}\right]. \qquad (15)$$

The four-vector k denotes the momentum of the virtual gluon appearing in the QCD-penguin diagrams, $x_t = m_t^2/M_W^2$ and

$$E(x) = -\frac{2}{3}\ln x + \frac{x^2(15-16x+4x^2)}{6(1-x)^4}\ln x + \frac{(18-11x-x^2)x}{12(1-x)^3} \qquad (16)$$

is one of the so-called Inami-Lim functions [19]. In eq. (14), q' runs over the quark flavours being active at the scale $\mu = \mathcal{O}(m_b)$ ($q' \in \{u,d,c,s,b\}$) and α, β are $SU(3)_{\rm C}$ colour indices.

Evaluating hadronic matrix elements of $\mathcal{H}_{\text{eff}}^{\text{pen}}(\Delta B=-1)$ and comparing them with eq. (1), we find

$$\Delta P \approx \frac{G(m_c,k,\mu)-G(m_u,k,\mu)}{E(x_t)+\frac{2}{3}\ln\left(\frac{\mu^2}{M_W^2}\right)-G(m_u,k,\mu)}. \tag{17}$$

In this perturbative approximation, the strong phase shift of ΔP is generated exclusively through absorptive parts of the penguin amplitudes with internal u- and c-quarks ("Bander–Silverman–Soni mechanism" [14]). Whereas the μ-dependence cancels exactly in (17), ΔP depends strongly on the value of k^2, as can be seen from Figs. 1 and 2. Simple kinematical considerations at the quark-level imply that k^2 should lie within the "physical" range [17, 18]

$$\frac{1}{4} \lesssim \frac{k^2}{m_b^2} \lesssim \frac{1}{2}. \tag{18}$$

For such values of k^2, we read off from Figs. 1 and 2 that

$$0.2 \lesssim |\Delta P| \lesssim 0.5 \quad \text{and} \quad 70^\circ \lesssim \delta_{\Delta P} \lesssim 130^\circ, \tag{19}$$

respectively. Consequently, ΔP may lead to sizable effects in the $SU(3)$ triangle relations discussed below. We are aware of the fact that the estimate of ΔP given here is very rough. It illustrates however a potential hadronic uncertainty which cannot be ignored.

In refs. [1]-[5], only QCD-penguins with internal top-quarks have been taken into account. This approximation corresponds to $\Delta P=0$ and gives

$$\bar{P}_{\Delta P=0} = a_P e^{i\beta} e^{i\delta_P} \tag{20}$$

$$\bar{P}'_{\Delta P=0} = a_{P'} e^{i\pi} e^{i\delta_P}, \tag{21}$$

where

$$a_P=|V_{td}||P_{tu}|, \quad a_{P'}=a_P/(\lambda R_t) \quad \text{and} \quad \delta_P=\delta_{tu}. \tag{22}$$

Notice that the weak- and strong phase structure of (21) is similar to (10) which can be re-written in the form

$$\bar{P}'=\rho_{P'}a_{P'}e^{i\pi}e^{i(\delta_P-\psi')} \tag{23}$$

with

$$\rho_{P'}=\sqrt{1-2|\Delta P|\cos\delta_{\Delta P}+|\Delta P|^2} \tag{24}$$

and

$$\tan\psi' = \frac{|\Delta P|\sin\delta_{\Delta P}}{1-|\Delta P|\cos\delta_{\Delta P}}. \tag{25}$$

In eq. (23), π represents the CP-violating weak phase, while $\delta_P - \psi'$ denotes the CP-conserving strong phase shift.

Therefore, the determination of the weak CKM-angle γ through $SU(3)$ triangle relations involving the charged B-meson decays $B^+ \to \{\pi^0 K^+, \pi^+ K^0, \pi^+\pi^0\}$ (and the corresponding CP-conjugate modes) as outlined in refs. [1, 4, 5] is not affected by ΔP at all, since no non-trivial weak phases appear in P' ($\bar{P}'$) even in the presence of QCD penguins with internal u- and c-quarks. However, the strong phase differences $\delta_P - \delta_{T,C}$ are shifted by the angle ψ'. Here δ_T and δ_C denote the strong phases of the "tree" and "colour-suppressed" amplitudes

$$T = a_T e^{i\gamma} e^{i\delta_T} \quad \text{and} \quad C = a_C e^{i\gamma} e^{i\delta_C} \tag{26}$$

contributing to $B^\pm \to \pi^\pm\pi^0$, respectively.

On the other hand, the QCD-penguin contributions with internal u- and c-quarks affect the extraction of the phase β by using the triangle relations [3]-[5]

$$\begin{array}{rcl} A(B_d^0 \to \pi^+\pi^-) + \sqrt{2}A(B_d^0 \to \pi^0\pi^0) & = & \sqrt{2}A(B^+ \to \pi^+\pi^0) \\ (T+P) \qquad + \qquad (C-P) & = & (T+C) \end{array} \tag{27}$$

and

$$\begin{array}{rcl} A(B_d^0 \to \pi^- K^+)/r_u + \sqrt{2}A(B_d^0 \to \pi^0 K^0)/r_u & = & \sqrt{2}A(B^+ \to \pi^+\pi^0) \\ (T+P'/r_u) \qquad + \qquad (C-P'/r_u) & = & (T+C), \end{array} \tag{28}$$

where $r_u = V_{us}/V_{ud}$.

Following the approach outlined in ref. [5], the complex amplitudes P' and P can be determined up to a *common* strong phase shift (and some discrete ambiguities) through a two-triangle construction involving the rates of the five modes appearing in (27) and (28) and two additional rates that determine $|P|$ and $|P'|$ (e.g., $B^+ \to K^+\bar{K}^0$ and $B^+ \to \pi^+ K^0$, respectively). Therefore, the relative angle ϑ between P and P' can be measured. Expressing P in the form

$$P = \rho_P a_P e^{-i\beta} e^{i(\delta_P - \psi)} \tag{29}$$

with

$$\rho_P = \frac{1}{R_t}\sqrt{R_t^2 - 2R_t|\Delta P|\cos(\beta+\delta_{\Delta P}) + |\Delta P|^2} \tag{30}$$

and

$$\tan\psi = \frac{|\Delta P|\sin(\beta+\delta_{\Delta P})}{R_t - |\Delta P|\cos(\beta+\delta_{\Delta P})}, \tag{31}$$

we find using (22), (23) and (29)

$$\frac{1}{r_t}\frac{P'}{P}=\frac{\rho_{P'}}{\rho_P}e^{i(\psi-\psi')}\equiv\frac{\rho_{P'}}{\rho_P}e^{i(\vartheta-\beta)}, \tag{32}$$

where $r_t \equiv V_{ts}/V_{td}$. Note that the deviation of the rhs. of eq. (32) from one represents corrections to the relation between P' and P presented in refs. [2]-[5]. Consequently, ϑ is given by

$$\vartheta=\beta+\psi-\psi'. \tag{33}$$

In contrast to ψ', which is a pure strong phase, ψ is a combination of both CP-conserving strong phases ($\delta_{\Delta P}$) and the CP-violating weak phase β.

If we neglect the QCD-penguins with internal u- and c-quarks, as the authors of refs. [3]-[5], we have $\Delta P=0$ and, thus, ϑ is equal to the CKM-angle β in this approximation. However, as can be seen from Figs. 1 and 2, the perturbative estimates of ΔP indicate that sizable contributions may arise from this amplitude which show up in eq. (33) as the phase difference $\psi-\psi'$. Since both ψ and ψ' contain strong phases, ϑ is not a theoretical clean quantity in general (even if the $SU(3)$ triangle relations were valid exactly!) and this determination of the angle β suffers from hadronic uncertainties in contrast to the assertions made in [3]-[5].

In order to illustrate this point quantitatively, we have plotted the dependence of $\psi-\psi'$ on k^2/m_b^2 arising from (17) for $R_t=1$ and various angles β in Fig. 3. The corresponding curves for $\rho_{P'}/\rho_P$ (see eq. (32)) are shown in Fig. 4. In drawing these figures, we have taken into account that the angle β is smaller than 45° for the present range of $|V_{ub}/V_{cb}|$ [13]. Notice that the hadronic uncertainties in (32) and (33) cancel each other, i.e., $P'=r_tP$ and $\psi'=\psi$, if we choose $R_t=1$ and $\beta=0$. This cancellation is, however, incomplete in the general case.

As an illustration consider a measurement of $\vartheta=15°$. Setting $\Delta P=0$ one would conclude that $\beta=15°$ and $\sin 2\beta=0.50$. With $\Delta P\neq 0$, as calculated here, the true β could be as high as 20° ($\psi-\psi'=-5°$) giving $\sin 2\beta=0.64$. We observe that this uncertainty (in addition to possible $SU(3)$-breaking effects) could spoil the comparison of β, measured this way, with the clean determination of $\sin 2\beta$ in $B_d\to\psi K_{\rm S}$.

We now want to demonstrate that the hadronic uncertainties affecting the determination of β through (33) can be eliminated provided R_t is known. To this end, we consider the "normalized" penguin amplitudes

$$\frac{1}{\lambda|V_{cb}|}P = \left[-\Delta P+R_te^{-i\beta}\right]|P_{tu}|e^{i\delta_{tu}} \tag{34}$$

$$\frac{1}{|V_{cb}|}P' = \left[\Delta P-1\right]|P_{tu}|e^{i\delta_{tu}} \tag{35}$$

and those of the corresponding CP-conjugate processes (see (9) and (10)) which are related to (34) and (35) through the substitution $\beta \to -\beta$. Combining these complex amplitudes in the form

$$z \equiv \frac{P + \lambda P'}{\bar{P} + \lambda \bar{P}'} = \frac{1 - R_t e^{-i\beta}}{1 - R_t e^{i\beta}} = e^{i2\gamma}, \tag{36}$$

we observe that both ΔP and $|P_{tu}| \exp(i\delta_{tu})$, which are unknown, non-perturbative quantities, cancel in the ratio z. The appearance of γ in this ratio can be understood by noting that

$$\bar{P} + \lambda \bar{P}' = -v_u^{(d)} P_{tu} = -|V_{ub}| e^{-i\gamma}(1 + \mathcal{O}(\lambda^2))|P_{tu}| e^{i\delta_{tu}}. \tag{37}$$

Consequently, in the limit of exact $SU(3)$ triangle relations (27) and (28), the angle 2γ, which is related to β through

$$\tan 2\gamma = \frac{2R_t(1 - R_t \cos\beta)\sin\beta}{1 - 2R_t \cos\beta + R_t^2 \cos 2\beta}, \tag{38}$$

can be also here extracted without theoretical uncertainties. If, in addition, R_t is also known, the CKM-phase β can be determined as well. In Fig. 5, we have illustrated the dependence of 2γ on β for various values of R_t. Note that $2\gamma = \pi - \beta$, if $R_t = 1$.

The theoretically cleanest way of measuring R_t without using CP-violating quantities is obtained through

$$R_t = \frac{1}{\sqrt{R_{ds}}}\sqrt{\frac{x_d}{x_s}}\frac{1}{|V_{us}|}, \tag{39}$$

where x_d and x_s give the sizes of $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings, respectively, and

$$R_{ds} = \frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{m_{B_d}}{m_{B_s}} \left[\frac{F_{B_d}\sqrt{B_{B_d}}}{F_{B_s}\sqrt{B_{B_s}}}\right]^2 \tag{40}$$

summarizes the $SU(3)$ flavour-breaking effects. In the strict $SU(3)$ limit, we have $R_{ds} = 1$. The main theoretical uncertainty resides in the values of the B-meson decay constants $F_{B_{d,s}}$ and in the non-perturbative parameters $B_{B_{d,s}}$ which parametrize the hadronic matrix elements of the relevant operators. We believe however that R_{ds} can be more reliably estimated than ΔP.

At this point, it should be stressed that the elimination of the hadronic uncertainties arising from ΔP, i.e., the QCD-penguins with internal u- and c-quarks, requires to consider also the CP-conjugate modes to extract "clean" values of

β. Furthermore, R_t has to be known. These complications are very different from the situation in refs. [3]-[5], where it has been emphasized that it was not necessary to measure the charge-conjugate rates in order to determine β.

Assuming *factorization*, $SU(3)$-breaking corrections can be taken into account approximately through the substitutions $r_u \to r_u f_K/f_\pi$ [1]-[5] and $r_t \to r_t f_K/f_\pi$ in eqs. (28) and (32), respectively, where P' and P in eq. (32) are the same as in the triangle relations (27) and (28). Moreover, we have to replace λ in our result (36) by $\lambda f_\pi/f_K$. $SU(3)$-breaking effects must also be taken into account in the determination of $|P|$ and $|P'|$ from the decay amplitudes $|A(B^+ \to K^+\bar{K}^0)|$ and $|A(B^+ \to \pi^+K^0)|$, respectively. Within the framework of factorization we find

$$|P| = \frac{f_\pi}{f_K}\frac{F_{B\pi}(0;0^+)}{F_{BK}(0;0^+)}|A(B^+ \to K^+\bar{K}^0)| \tag{41}$$

$$|P'| = |A(B^+ \to \pi^+K^0)|, \tag{42}$$

where $F_{B\pi}(0;0^+)$ and $F_{BK}(0;0^+)$ are form factors parametrizing the hadronic quark-current matrix elements $\langle\pi^+|(\bar{b}d)_{\mathrm{V-A}}|B^+\rangle$ and $\langle K^+|(\bar{b}s)_{\mathrm{V-A}}|B^+\rangle$, respectively [20]. Unfortunately, hadronic form factors appear in eq. (41) which are model dependent. Using, for example, the model of Bauer, Stech and Wirbel [21], we estimate that the $SU(3)$-breaking factor in (41) should be of $\mathcal{O}(0.7)$.

At present, there is no reliable theoretical technique available to evaluate non-factorizable $SU(3)$-breaking corrections to the relevant B-decays. Since already the factorizable corrections are quite large $((20-30)\%)$, we expect that non-factorizable $SU(3)$-breaking may also lead to sizable effects. In particular, such corrections could spoil the elimination of the QCD-penguins with internal u- and c-quarks through eq. (36). Furthermore, in the presence of a heavy top-quark, electroweak-penguin contributions may also lead to sizable corrections $((10-30)\%$ at the amplitude level) to the penguin sectors of B-decays into final states that contain mesons with a CP-self-conjugate quark content [22]-[24]. Possible impact of electroweak penguins on the approach of refs. [1]-[5] has been recently also emphasized in ref. [25].

In summary, we have shown that QCD-penguins with internal u- and c-quarks may lead to sizable systematic errors in the extraction of the CKM-phase β by using the approach presented in refs. [3]-[5]. However, β can still be determined in a theoretical clean way (up to corrections arising from non-factorizable $SU(3)$-breaking and certain neglected contributions which are expected to be small on dynamical grounds [1]-[5]), if R_t and the rates of the CP-conjugate processes appearing in the corresponding triangle relations are measured. On the other hand, the determination of γ along the lines suggested in [1]-[5] and in (36) in the present paper is not affected by these new QCD-penguin contributions. Its fate depends then only on the ability of estimating $SU(3)$-breaking effects and on the precision with which the relevant branching ratios can be measured one day.

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Figure Captions

Fig. 1: The dependence of $|\Delta P|$ on k^2/m_b^2.

Fig. 2: The dependence of $\delta_{\Delta P}$ on k^2/m_b^2.

Fig. 3: The dependence of $\psi - \psi'$ on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β.

Fig. 4: The dependence of $\rho_{P'}/\rho_P$ on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β.

Fig. 5: The dependence of angle 2γ on the CKM-angle β for various values of R_t.