

Meson Dynamics and the resulting “3-Nucleon-Force” diagrams: Results from a simplified test case

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Abstract. A simplified 1D (one-dimensional) model for a generalized $3N$ system is considered, as a testing ground for the explicit treatment of the meson dynamics in this system. We focus attention on the irreducible diagrams generated by the pion dynamics in the $3N$ system, and in particular to a new type of three-nucleon force discussed recently in the literature, and generated by the one-pion-exchange mechanism in presence of a nucleon-nucleon correlation. It is found that these new terms in the simplified model have an approximately 30% effect compared to the standard three-nucleon force terms in a ‘Triton’ binding energy calculation. It is suggested that this effect should also not be ignored in realistic calculations.

INTRODUCTION

An important aspect in modern three-nucleon dynamics is the explicit inclusion of mesonic degrees of freedom that can not be described by conventional two-nucleon potentials. These additional effects can be described by three-nucleon forces (3NF), which describe the addition of possible irreducible mesonic contributions. The construction and implementation of these terms is not trivial and care has been taken to avoid double-contributions. The standard 3NF approaches turned out to be able to account for the underbinding of the three-nucleon bound state. However, so far they could not fully explain the puzzle of the vector analyzing powers in nucleon-deuteron scattering at low energies.

Recently, starting from the rigorous four-body theory of Grassberger-Sandhas [1] and Yakubovsky [2], a new method to describe the coupled $NNN - \pi NNN$ system has been developed [3]. In a subsequent paper, an approximation scheme for this new method was derived by the authors [4]. It is based on reasonable physical and mathematical approximations and a procedure to freeze-out the pion channel.

After the approximations were performed and the pionic channel was projected out, it was shown that the residual pion-dynamics produced correction terms to the standard Faddeev-Alt-Grassberger-Sandhas $3N$ equation. These correction terms can be interpreted as three-nucleon force diagrams (3NF). The approach developed has the distinctive feature that these irreducible diagrams naturally appear in the Faddeev equation, and the corresponding 3NF is generated consistently with the $3N$ dynamical equation used for the actual calculations. To our knowledge, this level of consistency has here been

obtained for the first time.

A simplified one-dimensional model was developed, which describes in first approximation the dynamics of the three-nucleon system [5]. It is based on a spinless one-dimensional scattering theory with a potential which is the $1D$ analogue of the standard Malfliet-Tjon potential. The simplicity of the model allowed a straightforward investigation of the effects due to the residual pion dynamics.

THE SIMPLIFIED TEST CASE

An approximation scheme for reducing the $\pi - NNN$ system to a tractable set of equations has been recently developed [4]. At the lowest order, the residual effects of the pion dynamics result in irreducible corrections to the driving term of the Lovelace type $3N$ equation. The driving term now reads

$$Z_{ab} = Z_{ab}^{AGS} + Z_{ab}^{\pi} \quad (1)$$

where a, b are the standard Faddeev labels. The first term has the usual structure of an AGS-type driving term, namely it is vanishing for $a = b$, while for $a \neq b$ it is

$$Z_{12}^{AGS} = \langle N_1 (N_2 N_3) | g_0 | N_2 (N_1 N_3) \rangle \quad (2)$$

and the other term corresponds to the correction that comes from the residual pion dynamics. Another type of correction term results from projecting out the pion channel. It describes the intermediate propagation of a pion under the presence of a correlated three nucleon cluster. In this study such an additional correction is not taken into account.

To the lowest order, the correction term Z_{ab}^{π} contains two topologically different contributions, for $a \neq b$,

$$Z_{12}^{\pi} = \langle N_1 (N_2 N_3 \pi); N_1 N_2 (N_3 \pi) | \tau_{(N_3 \pi)} | N_2 (N_3 N_1 \pi); N_1 N_2 (N_3 \pi) \rangle \quad (3)$$

and, for $a = b$,

$$Z_{11}^{\pi} = \langle N_1 (N_2 N_3 \pi); N_1 (N_2 N_3) \pi | \tau_{(N_2 N_3)} | (N_1 \pi) (N_2 N_3); N_1 N_3 (N_1 \pi) \rangle \quad (4)$$

$$+ \langle (N_1 \pi) (N_2 N_3); N_1 (N_2 N_3) \pi | \tau_{(N_2 N_3)} | N_1 (N_2 N_3 \pi); N_1 N_3 (N_1 \pi) \rangle$$

The static approximation of the first term corresponds to a correction term usually attributed to a three-nucleon force diagram of the Fujita-Miyazawa type [6]. The second term corresponds to a topologically different type of correction term that describes the propagation of an intermediate pion under the presence of a correlated two nucleon cluster. Furthermore, it should be noted that the first correction term has contributions to the off-diagonal elements of the driving term only. On the other hand, the second type has contributions to the diagonal elements of the driving term only, which in the standard AGS driving term are always zero.

A simple test model was designed from a one-dimensional scattering theory describing symmetric spinless particles. The strength parameters of the Malfliet-Tjon type potential

were chosen in a way to reproduce the deuteron binding energy. The potential also shows a similar spatial behaviour as observed in more realistic nuclear potentials, which makes it a good candidate for our test-calculation. The t-matrices are described by the Unitary Pole Approximation (UPA), which is expected to be a good approximation due to the simplicity of the toy-model. The ‘deuteron’ binding energies are found using a Sturmian procedure. Subsequently we calculated the ‘triton’ binding energy of this system without the correction terms and found a value of $-7.28MeV$.

In the calculation the first type of correction term was included in a static approximation which allowed the interpretation of the dynamical terms as $3NF$. They were described by the one-dimensional equivalent of the Tucson-Melbourne or Brasil type contact term [7, 8]. The absence of spin in our model reduces all other terms in the Tucson-Melbourne/Brasil $3NF$ to identically zero. The contact term of the TM/B- $3NF$ depends on a freely variable parameter and the πN scattering length. The coefficient a_1 in the contact term, which depends on the πN scattering length is defined in our simplified model by the expression

$$a_1 = -\frac{2\pi\mu}{\hbar^2}t_{\pi N}(p=0, p'=0, E=0) \quad (5)$$

where the threshold πN t-matrix is approximated by the corresponding NN t-matrix times an adjustable parameter $c_{\pi N}$. This parameter $c_{\pi N}$ can be chosen freely over a certain range to recover the triton binding energy. However, we also know that the corresponding threshold scattering lengths differ by a factor of 0.01 and we choose the parameter $c_{\pi N} = 0.01$ as a first guess. With this value we find a ‘triton’ binding energy of $-8.17MeV$. We then varied the parameter $c_{\pi N}$ and found that the correction term depends strongly on this adjustable parameter. It is observed that it is in principle possible to adjust the parameter $c_{\pi N}$ in order to recover the experimental ‘triton’ binding energy.

When calculating the effect of the second type of correction terms a delicate cancellation has to be observed in order to avoid overcounting. This cancellation is described by Canton and Schadow [9] and is of the type $t - v$. Namely, we have to subtract the potential from the full t-matrix and it turns out that this subtraction does not result in a complete cancellation at low energies. For more details on the cancellation we refer to the original publications [9, 10]. We then calculated the ‘triton’ energy including both types of correction terms, where for the adjustable parameter $c_{\pi N}$ a value of 0.01 was chosen, thereby obtaining $-8.48MeV$ for the model ‘triton’ binding energy. It should be noted that the effect of the ‘new’ correction terms in respect to the Fujita-Miyazawa type terms is approximately 30%. If we use the parameter $c_{\pi N}$ that was already adjusted to recover the ‘triton’ binding energy, then the new type of correction term leads to an over binding. It is therefore important to fit the adjustable parameter $c_{\pi N}$ with both types of correction terms present.

CONCLUSIONS

We developed a model that included one pion degree of freedom explicitly in an AGS type three body equation. It was shown that the effect of the pion dynamics resulted

in correction terms in the driving terms of the two-cluster equations. We calculated the explicit effects of these terms, which could be interpreted as 3NF terms, on the ‘triton’ binding energy. In a simplified model calculation we show that the correction terms can account for an underbinding of the ‘three-nucleon’ bound state. However, a new type of correction term also has an approximately 30% effect on the ‘triton’ binding energy. A third topological type of correction term still needs to be investigated, but it is expected that the effect of this term is not as important.

The calculations we presented were done only with a rather simple model. However, one might expect that the new type of correction terms could have an effect also in more realistic calculations. This suggestion has been recently confirmed by Canton and Schadow in a more realistic study [9, 10] on the nd vector analyzing powers, where it was shown that a new 3NF term of tensor structure can be generated by these new correction terms, and could indeed hold the key to solve the A_y -puzzle. In conclusion, a new way of handling the pion dynamics in $3N$ calculations has been developed. Calculations suggest that a newly developed type of 3NF should also be included in realistic three-nucleon calculations.

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