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N = 2 SUPER W_∞ ALGEBRA AND ITS NONLINEAR REALIZATION THROUGH SUPER KP FORMULATION

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Abstract

A nonlinear realization of super W_∞ algebra is shown to exist through a consistent superLax formulation of super KP hierarchy. The reduction of the superLax operator gives rise to the Lax operators for $N = 2$ generalized super KdV hierarchies, proposed by Inami and Kanno. The Lax equations are shown to be Hamiltonian and the associated Poisson bracket algebra among the superfields, consequently, gives rise to a realization of nonlinear super W_∞ algebra.

It has been known since the inception of Zamolodchikov, Fateev and Lykyanov's [1] nonlinear realization of extended conformal symmetry that W_n algebra incorporates in its calssical limit the Hamiltonian structure of nonlinear integrable systems *viz.*, generalized KdV hierarchy [2]. Subsequently, there were important attempts to classify conformal field theory via the Hamiltonian structure of generalized KdV hierarchies [3]. The role of the generalized KdV hierarchies became apparent later when it was seen that equations of motion and symmetries in quantum 2D gravity and noncritical string theory at least with $c < 1$ can be formulated in terms of integrable nonlinear equations of KdV type. (There is an exhustive literature in this direction; for example see [4]). Since all the KdV hierarchies are contained into the larger integrable system *viz.*, KP hierarchy [5] it has been conjectured [6] that it may provide a universal framework exhibiting the underlying structure of 2D quantum gravity. In this connection and also from the point of view Lie algebra, the algebraic structure of the large n limit of W_n algebra namely, $W_{1+\infty}$ and W_∞ has been probed within the framework of KP hierarchy [7]. These are universal W algebras containg all the conformal spins. All these algebras have so far been linear Lie algebras. However, owing to the ambiguity inherent in the large n limits, a nonlinear realization of universal W algebra, $\hat{W}_\infty$ has been constructed and identified with the second Hamiltonian structure of KP hierarchy [8].

Later on, Manin and Radul [9] provided a consistent supersymmetric extention of the KP hierarchy and subsequently the physically relevant even parity superLax formulation of super KP hierarchy containing linear universal W algebra (super $W_{1+\infty}$) has been developed [10]. While linear $N = 2$ universal super W algebra [10] was shown to exist, Inami and Kanno [11, 12] have shown that $N = 2$ super KdV hierarchies can arise in association with affine Lie algebras. This is an important step towards $N = 2$ super analogue of Drinfeld Sokolov formulation [2]. Clearly, it hints that, as a generalization of Inami and Kanno's work [11, 12], there must be a consistent $N = 2$ superLax formulation of super KP hierarchy which should be Hamiltonian with respect to super Gelfand Dikii bracket of second kind and should reduce to the Lax operators, considered in [12] under siutable reduction. This observation, in fact, enables us to show the existence of a nonlinear realization of super W_∞ algebra through super KP formulation.

In this letter we formulate an even parity superLax operator which, under suitable reductions, becomes isomorphic to the Lax operators of $N = 2$ super KdV hierarchies [12]. Subsequently, we show that a consistent superLax formulation corresponding to this superLax operator leads to $N = 2$ super KP hierarchy. We also show the Lax euqations are Hamiltonian. For this purpose, we obtain the Poisson bracket algebra among the coefficient fields corresponding to the superLax operator following Gelfand Dikii method. We observe that the full Poisson bracket algebra has $N = 2$ superconformal algebra as a subalgebra and the bosonic and fermionic componets of the superfields carry correct conformal weights. As a consequence, we claim that the full Poisson bracket algebra among the superfields gives rise to a realization of $N = 2$ nonlinear super W_∞ algebra.

We begin with the even parity superLax operator of the form

$$L = D^2 + \sum_{i=0}^{\infty} u_{i-1}(X) D^{-i} \tag{1}$$

where, D is the superderivative with $D^2 = \frac{d}{dx}$ and $u_{i-1}(X)$ are superfields in $X = (x, \theta)$ space, θ being Grassman odd coordinate. The grading of $u_{i-1}(X)$ is $|u_{i-1}| = i$ so that u_{2i-1} are bosonic superfields whereas u_{2i} are fermionic ones.

The interesting consequence of the choice of the superLax operator (1) is that under suitable choice of reduction it reduces to the superLax operator for generalized $N = 2$ KdV hierarchies, proposed by Inami and Kanno. Let us first define the n*th.* reduction of L as

$$\tilde{L}_n = L^n_{>0} \tag{2}$$

where, '> 0' implies the +ve part of L^n without D^0 term. Using definition (2) for the superLax operator (1), $\tilde{L}_n$ can be expressed in the following general form

$$\tilde{L}_n = L^n_{>0} = D^{2n} + \sum_{i=1}^{2n-2} \mathcal{U}_i^{(n)} D^i \tag{3}$$

In (3) we have used super Liebnitz rule [9] for multiplication of the operators L and the superfields $\mathcal{U}_i^{(n)}$ are functions of $u_{i-1}(X)$ and their superderivatives. $\tilde{L}_n$ in (3) is precisely the superLax operator considered in [12]. For example, for $n = 2$

$$\tilde{L}_2 = D^4 + 2u_{-1} D^2 + 2u_0 D \tag{4}$$

which is precisely $N=2$ super KdV Lax operator [11]. Similarly, for $n=3$

$$\tilde{L}_3 = D^6 + 3u_{-1}D^4 + 3u_0D^3 + 3(u_1 + u_{-1}^{[2]} + u_{-1}^2)D^2 + 3(u_2 + u_0^{[2]} + 2u_{-1}u_0)D \tag{5}$$

is isomorphic to the Lax operator corresponding to $N=2$ super Boussinesq hierarchy [11]. Notice that we have chosen the reduction prescription as in definition (2), since D^0 term is absent in the superLax operators (3, 4, 5), considered in [11, 12]. This observation is, in fact, a compelling evidence to propose the superLax operator in (1) as a right candidate for describing $N=2$ super KP hierarchy.

It is clear from (4, 5) that the presence of the superfield u_{-1} in (1) is essential to obtain superLax operators for $N=2$ generalized super KdV hierarchies by reduction. Hence u_{-1} must have nontrivial dynamics. In order to have nontrivial flow of u_{-1} for each time t_n we now define the Lax equation as

$$\frac{dL}{dt_n} = [L^n_{>0}, L] \tag{6}$$

We will show later that the presence of u_{-1} renders the Gelfand Dikii Poisson bracket of second kind local whereas the absence of u_{-1} field makes the same nonlocal [13].

We give below first three evolution equations which follow from (1) and (6).

$$\frac{du_{i-1}}{dt_1} = u_{i-1}^{[2]} \tag{7a}$$

$$\begin{aligned}\frac{du_{i-1}}{dt_2} =& 2u_{i+1}^{[2]} + u_{i-1}^{[4]} + 2u_{-1}u_{i-1}^{[2]} + 2u_0u_{i-1}^{[1]} - 2\begin{bmatrix} i+1 \\ 1 \end{bmatrix} u_iu_{-1}^{[1]} - 2(1+(-1)^i)u_0u_i \\ &+ 2\sum_{m=0}^{i-1}\begin{bmatrix} i \\ m+1 \end{bmatrix}(1-)^{i+[-\frac{m}{2}]}u_{i-m-1}u_0^{[m+1]} \\ &+ 2\sum_{m=0}^{i-1}\begin{bmatrix} i+1 \\ m+2 \end{bmatrix}(1-)^{[\frac{m}{2}]}u_{i-m-1}u_{-1}^{[m+2]}\end{aligned} \tag{7b}$$

$$\begin{aligned}\frac{du_{i-1}}{dt_3} =& 3u_{i+3}^{[2]} + 3u_{i+1}^{[4]} + u_{i-1}^{[6]} + 6u_{-1}u_{i+1}^{[2]} + 3u_{-1}u_{i-1}^{[4]} \\ &- 3\begin{bmatrix} i+3 \\ 1 \end{bmatrix}u_{i+2}u_{-1}^{[1]} + 3\begin{bmatrix} i+3 \\ 2 \end{bmatrix}u_{i+1}u_{-1}^{[2]} + 3\begin{bmatrix} i+3 \\ 3 \end{bmatrix}u_iu_{-1}^{[3]} \\ &- 3(1+(-1)^i)u_0u_{i+2} + 3u_0u_{i+1}^{[1]} - 3(-1)^iu_0u_i^{[2]} + 3u_0u_{i-1}^{[3]} \\ &+ 3\begin{bmatrix} i+2 \\ 1 \end{bmatrix}(-1)^iu_{i+1}u_0^{[1]} - 3\begin{bmatrix} i+2 \\ 2 \end{bmatrix}(-1)^iu_iu_0^{[2]}\end{aligned}$$

$$+ 3(u_1 + 2u_{-1}^{[2]} + u_{-1}^2)u_{i-1}^{[2]} - 3\begin{bmatrix} i+1 \\ 1 \end{bmatrix} u_i(u_1 + u_{-1}^{[2]} + u_{-1}^2)^{[1]}$$

$$+ 3(u_2 + 2u_{-1}u_0 + u_0^{[2]})u_{i-1}^{[1]} - 3(1 + (-1)^i)(u_2 + 2u_{-1}u_0 + u_0^{[2]})u_i$$

$$- 3\sum_{m=0}^{i-1} \begin{bmatrix} i+3 \\ m+4 \end{bmatrix} (-1)^{[\frac{m}{2}]} u_{i-m-1} u_{-1}^{[m+4]}$$

$$- 3\sum_{m=0}^{i-1} \begin{bmatrix} i+2 \\ m+3 \end{bmatrix} (-1)^{i+[-\frac{m}{2}]} u_{i-m-1} u_0^{[m+3]}$$

$$+ 3\sum_{m=0}^{i-1} \begin{bmatrix} i+1 \\ m+2 \end{bmatrix} (-1)^{[\frac{m}{2}]} u_{i-m-1}(u_1 + u_{-1}^{[2]} + u_{-1}^2)^{[m+2]}$$

$$+ 3\sum_{m=0}^{i-1} \begin{bmatrix} i \\ m+1 \end{bmatrix} (-1)^{i+[-\frac{m}{2}]} u_{i-m-1}(u_2 + 2u_{-1}u_0 + u_0^{[2]})^{[m+1]} \qquad (7c)$$

With the identification of $t_1 = x$ and $t_2 = y$, (7a) resembles the consistency condition, whereas (7b) becomes constraint equation. The time variables, therefore, may be identified as $t_3, t_4, ...$ *etc.*

Let us now look for the t_3 time evolution (first time evolution) equations for the superfields u_{-1} and u_0. We may eliminate all other fields from the equations of motion of u_{-1} and u_0 in (7c) by using the constraint (7b). As a consequence, however, the equations of motion for u_{-1} and u_0 become nonlocal and have the form

$$\frac{du_{-1}}{dt_3} = \frac{1}{4}u_{-1}^{[6]} - \frac{1}{2}(u_{-1}^3)^{[2]} + \frac{3}{2}(u_0 u_{-1}^{[1]})^{[2]} + \frac{3}{4}\frac{d^2 u_{-1}^{[-2]}}{dy^2}$$

$$+ \frac{3}{2}u_{-1}^{[2]}\frac{du_{-1}^{[-2]}}{dy} - \frac{3}{2}u_{-1}^{[1]}\frac{du_0^{[-2]}}{dy} - 3u_0\frac{du_{-1}^{[-1]}}{dy} + 3u_0\frac{du_0^{[-2]}}{dy} \qquad (8a)$$

$$\frac{du_0}{dt_3} = \frac{1}{4}u_0^{[6]} + \frac{3}{2}(u_0 u_0^{[1]})^{[2]} - \frac{3}{2}(u_0 u_{-1}^{[2]})^{[2]} - \frac{3}{2}(u_0 u_{-1}^2)^{[2]}$$

$$+ \frac{3}{4}\frac{d^2 u_0^{[-2]}}{dy^2} + \frac{3}{2}(u_0\frac{du_0^{[-2]}}{dy})^{[1]} + \frac{3}{2}u_0^{[2]}\frac{du_{-1}^{[-2]}}{dy} + \frac{3}{2}u_0\frac{du_{-1}}{dy} \qquad (8b)$$

The higher time evolution of this hierarchy also have similar nonlocal terms involving y derivate only.

If we further demand that the superfields are independent of y coordinate the evolution equations of u_{-1} and u_0 become local and reduce to the equations,

$$\frac{du_{-1}}{dt_3} = -\left[u_{-1}^{[6]} + 3(u_{-1}^{[1]}u_0)^{[2]} - \frac{1}{2}(u_{-1}^3)^{[2]}\right] \qquad (9a)$$

and

$$\frac{du_0}{dt_3} = -\left[u_0^{[6]} - 3(u_0 u_0^{[1]})^{[2]} - \frac{3}{2}(u_0 u_{-1}^2)^{[2]} + 3(u_0 u_{-1}^{[2]})^{[2]}\right] \tag{9b}$$

after rescaling of $u_{-1} = -\frac{1}{2}u_{-1}$, $u_0 = -\frac{1}{2}u_0$ and $t_3 = -\frac{1}{4}t_3$. (9) are the evolution equations for $N = 2$ super KdV system [11].

Moreover, we show that (7) contain KP equation in the bosonic limit. For this purpose, let us first write down the superfields in the component forms as

$$u_{2i-1}(X) = u_{2i-1}^b(x) + \theta u_{2i-1}^f(x) \tag{10a}$$

$$u_{2i}(X) = u_{2i}^f(x) + \theta u_{2i}^b(x) \tag{10b}$$

where, b and f denote fermion and boson respectively. In particular, it follows from (7) that the equation of motion of $u_0^b(x)$ have the following form

$$\frac{3}{4}\frac{d^2 u_0^b(x)}{dy^2} = \frac{d}{dx}\left(\frac{du_0^b(x)}{dt_3} - \frac{1}{4}\frac{d^3 u_0^b(x)}{dx^3} - 3u_0^b(x)\frac{du_0^b(x)}{dx}\right) \tag{11}$$

after setting the fermionic components of the superfields and $u_{-1}^b(x)$ field to zero. (11) is, indeed, the KP equation [14]. Thus the set of equations (7) is nothing but the super KP equation. In addition, we have shown that (7) reduce to $N = 2$ super KdV equations (9). Hence, this suggests that (1) and (6) describe the dynamics of $N = 2$ super KP hierarchy. Now, it remains to show that the Poisson bracket algebra among the superfields corresponding to (1) and (6) has $N = 2$ superconformal algebra as a subalgebra.

To show that the Lax equations (6, 7) are Hamiltonian, we first calculate the Poisson bracket algebra among the coefficient fields $u_{i-1}(X)$ following the method of Gelfand and Dikii. The super Gelfand Dikii bracket is defined as

$$\{F_P(L), F_Q(L)\} = -Tr\left[\{L(PL)_- - (LP)_- L\}\, Q\right] \tag{12}$$

where, P, Q are auxiliary fields (analogous to the super Volterra operators in the case of super KdV system). We choose P, Q in the form

$$P = \sum_{j=-2}^{\infty} D^j p_j \qquad ; \qquad Q = \sum_{j=-2}^{\infty} D^j q_j \tag{13}$$

with the grading $|p_j| = |q_j| = j$ so that the linear functional $F_P(L)$ (and similarly $F_Q(L)$) becomes

$$F_P(L) = Tr(LP) = \sum_{i=0}^{\infty} \int dX (-1)^{i+1} u_{i-1}(X) p_{i-1}(X) \tag{14}$$

Consequently the L.H.S. of (12) becomes

$$\{F_P(L), F_Q(L)\} = \sum_{i,j=0}^{\infty} \int dX \int dY (-1)^{i+j} p_{i-1}(X) \{u_{i-1}(X), u_{j-1}(Y)\} q_{j-1}(Y) \tag{15}$$

Notice that (15) does not involve terms like p_{-2} and q_{-2} since the superfields in (1) starts from $u_{-1}(X)$. To ensure this consistency we have to show that R.H.S. of (12) sets the coefficients of p_{-2} and q_{-2} identically zero. This needs the coefficient of D term in the $V_P(L)$ to be zero, where $V_P(L)$ is defined by

$$V_P(L) = L(PL)_- - (LP)_- L.$$

The above condition thus leads to the constraint

$$p_{-1}(X) = -\sum_{r=0}^{\infty} \sum_{m=0}^{r-1} \begin{bmatrix} r-1 \\ m+1 \end{bmatrix} (-1)^{m(r+1)} \left(p_{r-1}(X) u_{r-m-2}(X)\right)^{[m-1]} \tag{16}$$

In particular, vanishing of the coefficient of D term ensures that R.H.S. of (12) is independent of q_{-2}. Now by using the constraint (16) we can make the coefficient of p_{-2} zero. We remark that the origin of the constraint (16) is due to the absence of D term in the superLax operator (1) itself. Finally, we obtain the Poisson bracket algerba among the superfields by using the relations (12)-(16). Thus we have

$$
\begin{aligned}
&\{u_{j-1}(X), u_{k-1}(Y)\} = \\
&\Bigg[-\sum_{m=0}^{j+1} \begin{bmatrix} j+1 \\ m \end{bmatrix} (-1)^{j(k+m+1)+[\frac{m}{2}]} u_{j+k-m} D^m \\
&+ \sum_{m=0}^{k+1} \begin{bmatrix} k+1 \\ m \end{bmatrix} (-1)^{jm+(k+1)(m+1)} D^m u_{j+k-m} \\
&+ \sum_{m=0}^{j-1} \sum_{l=0}^{k-1} \left(\begin{bmatrix} j \\ m+1 \end{bmatrix} \begin{bmatrix} k \\ l+1 \end{bmatrix} - \begin{bmatrix} j-1 \\ m \end{bmatrix} \begin{bmatrix} k-1 \\ l \end{bmatrix} \right) (-1)^{j(m+1)+k+l+[\frac{m}{2}]} \\
&\qquad\qquad u_{j-m-2} D^{m+l+1} u_{k-l-2} \\
&+ \sum_{n=0}^{k-1} \sum_{l=0}^{k-n-1} \begin{bmatrix} k-n-1 \\ l \end{bmatrix} (-1)^{j(n+l)+(l+1)(n+k+1)} u_{n-1} D^l u_{j+k-n-l-2} \\
&- \sum_{m=0}^{j+k-n-l-1} \sum_{n=0}^{k-1} \sum_{l=0}^{k-n-1} \begin{bmatrix} j-1 \\ m \end{bmatrix} \begin{bmatrix} n+l-1 \\ l \end{bmatrix} (-1)^{j(m+n+l+k+1)+n(l+1)+[\frac{m}{2}]} \\
&\qquad\qquad u_{j+k-m-n-l-2} D^{m+l} u_{n-1} \Bigg] \Delta(X-Y)
\end{aligned}
\tag{17}
$$

Next we define the Hamiltonians, H_n as

$$
H_n = \frac{1}{n} \int dX sRes(L^n)(X) \tag{18}
$$

for $n = 1, 2, 3, ...$ etc. Here '$sRes$' means superresidue,$i.e.$ the coefficient of D^{-1}. To check that first few equations, (7) satisfy Hamilton's equation

$$
\frac{du_{i-1}(X)}{dt_n} = \{u_{i-1}(X), H_n\} \tag{19}
$$

we give explicit form of first three Hamiltonians

$$
\begin{aligned}
H_1 &= \int dX u_0 \\
H_2 &= \int dX (u_2 + u_{-1} u_0) \\
H_3 &= \int dX (u_4 + 2u_2 u_{-1} + 2u_1 u_0 + u_0 u_0^{[1]} + u_{-1} u_0^{[2]} + u_0 u_{-1}^2)
\end{aligned}
\tag{20}
$$

For example, for $n = 1$, (19) becomes

$$
\frac{du_{i-1}(X)}{dt_1} = (-1)^i \int dY \{u_{i-1}(X), u_0(Y)\} \tag{21}
$$

Now by using (17) we have from (21)

$$\frac{du_{i-1}(X)}{dt_1} = u^{[2]}_{i-1}(X)$$

which is nothing but (7a). Similarly for $n = 2, 3$ we have verified by using (17), (19) and (20) that Hamilton's equations exactly match with (7b,c).

A few remarks about the Poisson bracket algebra (17) are in order.

(i) The first three terms in algebra are manifestly antisymmetric, but the last two terms are not apparently antisymmetric. This causes obstruction in proving Jacobi identity in a covariant fashion by using the manifestly antisymmetry property and cyclicity in the indices. We have, however, checked for $k = 0, 1, 2, 3$ and arbitrary j and also for $j = 0, 1, 2, 3$ and arbitrary k that Poisson brackets are, indeed, antisymmetric. For convenience, we display the explicit expressions of these Poisson brackets below. From (17) the Poisson brackets between $k = 0, 1, 2, 3$ and arbitrary j are given by

$$\begin{aligned}\{u_{j-1}(X), u_{-1}(Y)\} = \Big(&-u_j + (-1)^j Du_{j-1} \\ &- \sum_{m=0}^{j+1} \begin{bmatrix} j+1 \\ m \end{bmatrix} (-1)^{j(m+1)+[\frac{m}{2}]} u_{j-m} D^m \Big) \Delta(X-Y)\end{aligned} \qquad (22a)$$

$$\begin{aligned}\{u_{j-1}(X), u_0(Y)\} = \Big(&D^2 u_{j-1} \\ &- \sum_{m=0}^{j} \begin{bmatrix} j+1 \\ m+1 \end{bmatrix} (-1)^{j(m+1)+[-\frac{m}{2}]} u_{j-m} D^{m+1} \Big) \Delta(X-Y)\end{aligned} \qquad (22b)$$

$$\begin{aligned}\{u_{j-1}(X), u_1(Y)\} = \Big(&-u_{j+2} + (-1)^j Du_{j+1} - D^2 u_j + (-1)^j D^3 u_{j-1} + (-1)^j u_0 u_{j-1} \\ &- u_{-1} u_j - u_{j-1} Du_{-1} + (-1)^j u_{-1} Du_{j-1} - \sum_{m=0}^{j+1} \begin{bmatrix} j+1 \\ m \end{bmatrix} (-1)^{j(m+1)+[\frac{m}{2}]} u_{j+2-m} D^m \\ &- \sum_{m=0}^{j} \begin{bmatrix} j \\ m \end{bmatrix} (-1)^{j(m+1)+[-\frac{m}{2}]} \big(u_{j-m} D^m u_{-1} - (-1)^j u_{j-m-1} D^m u_0\big) \Big) \Delta(X-Y)\end{aligned} \qquad (22c)$$

$$\begin{aligned}\{u_{j-1}(X), u_2(Y)\} = \Big(&2D^2 u_{j+1} + D^4 u_{j-1} - (-1)^j u_0 u_j + u_0 Du_{j-1} - u_{j-1} D^2 u_{-1} \\ &+ u_{-1} D^2 u_{j-1} - \sum_{m=0}^{j} \begin{bmatrix} j+1 \\ m+1 \end{bmatrix} (-1)^{[-\frac{m}{2}]} \big(u_{j+2-m} D^{m+1} + u_{j-m} D^{m+1} u_{-1}\big) \\ &+ \sum_{m=0}^{j} \begin{bmatrix} j \\ m \end{bmatrix} (-1)^{j+[\frac{m}{2}]} u_{j-m} D^m u_0 - \sum_{m=0}^{j-1} \begin{bmatrix} j \\ m+1 \end{bmatrix} (-1)^{[-\frac{m}{2}]} u_{j-m-2} D^{m+2} u_0 \Big) \Delta(X-Y)\end{aligned} \qquad (22d)$$

It also follows from (17) that the Poisson brackets between $j = 0, 1, 2, 3$ and arbitrary k have the form

$$\begin{aligned}\{u_{-1}(X), u_{k-1}(Y)\} = \Big(&-u_k - u_{k-1}D \\ &+ \sum_{m=0}^{k+1} \begin{bmatrix} k+1 \\ m \end{bmatrix} (-1)^{(k+1)(m+1)} D^m u_{k-m}\Big) \Delta(X-Y)\end{aligned} \qquad (23a)$$

$$\begin{aligned}\{u_0(X), u_{k-1}(Y)\} = \Big(&-(-1)^k u_{k-1} D^2 \\ &- \sum_{m=0}^{k} \begin{bmatrix} k+1 \\ m+1 \end{bmatrix} (-1)^{km} D^{m+1} u_{k-m}\Big) \Delta(X-Y)\end{aligned} \qquad (23b)$$

$$\begin{aligned}&\{u_1(X), u_{k-1}(Y)\} = \big(-u_{k+2} - u_{k+1}D + u_k D^2 + u_{k-1}D^3 + u_{k-1}u_0 \\ &- u_k u_{-1} - u_{k-1}Du_{-1} + (-1)^k u_{-1}Du_{k-1} + \sum_{m=0}^{k+1} \begin{bmatrix} k+1 \\ m \end{bmatrix} (-1)^{(k+1)(m+1)} D^m u_{k+2-m} \\ &- \sum_{m=0}^{k} \begin{bmatrix} k \\ m \end{bmatrix} (-1)^k \left(u_{-1}D^m u_{k-m} - (-1)^m u_0 D^m u_{k-m-1}\right)\Big) \Delta(X-Y)\end{aligned} \qquad (23c)$$

$$\begin{aligned}&\{u_2(X), u_{k-1}(Y)\} = -(-1)^k \big(2u_{k+1}D^2 - u_{k-1}D^4 - u_k u_0 - u_{k-1}Du_0 + u_{k-1}D^2u_{-1} \\ &- u_{-1}D^2 u_{k-1} + \sum_{m=0}^{k} \begin{bmatrix} k+1 \\ m+1 \end{bmatrix} \left(D^{m+1}u_{k+2-m} + u_{-1}D^{m+1}u_{k-m}\right) \\ &+ \sum_{m=0}^{k} \begin{bmatrix} k \\ m \end{bmatrix} u_0 D^m u_{k-m} - \sum_{m=0}^{k-1} \begin{bmatrix} k \\ m+1 \end{bmatrix} (-1)^m u_0 D^{m+2} u_{k-m-2}\Big) \Delta(X-Y)\end{aligned} \qquad (23d)$$

It is now easy to see that (22a) is antisymmetric to (23a) and so on. We have also checked that Jacobi identities are satisfied for the above cases.

(ii) The algebra is local. This enables us to associate it with a superconformal algebra. To make contact the algebra (17) with extended $N = 2$ superconformal algebra, *viz.* super W_∞ algebra, we first consider the Poisson bracket algebra between the superfields $u_{-1}(X)$ and $u_0(X)$. From (17) we have

$$\begin{aligned}\{u_{-1}(X), u_{-1}(Y)\} &= \left(-2u_0(X) - u_{-1}(X)D_X + D_X u_{-1}(X)\right) \Delta(X-Y) \\ \{u_0(X), u_{-1}(Y)\} &= \left(-D_X u_0(X) - u_{-1}(X)D_X^2\right) \Delta(X-Y) \\ \{u_{-1}(X), u_0(Y)\} &= \left(-u_0(X)D_X + D_X^2 u_{-1}(X)\right) \Delta(X-Y) \\ \{u_0(X), u_0(Y)\} &= \left(u_0(X)D_X^2 + D_X^2 u_0(X)\right) \Delta(X-Y)\end{aligned} \qquad (24)$$

which is closed and hence is a subalgebra of the full algebra (17). To show that (24) has $N=2$ superconformal structure it is useful to write (24) in terms of component fields by using (10). If we further redifine the fields as

$$T = u_0^b - \frac{1}{2}(u_{-1}^b)', \qquad U = u_{-1}^b, \qquad G^+ = u_0^f, \qquad G^- = u_0^f - u_{-1}^f \tag{25}$$

the algebra (24) in terms of T, U, G^+ and G^- becomes

$$\begin{aligned}
\{T(x), T(y)\} &= (2T(y)\partial_y + T'(y))\,\delta(x-y)\\
\{T(x), U(y)\} &= (U(y)\partial_y + U'(y))\,\delta(x-y)\\
\left\{T(x), G^{\pm}(y)\right\} &= \left(\frac{3}{2}G^{\pm}(y)\partial_y + (G^{\pm})'(y)\right)\delta(x-y)\\
\left\{G^{\pm}(x), U(y)\right\} &= \pm G^{\pm}(y)\delta(x-y)\\
\left\{G^{+}(x), G^{-}(y)\right\} &= \left(T(y) - U(y)\partial_y - \frac{1}{2}U'(y)\right)\delta(x-y)\\
\left\{G^{\pm}(x), G^{\pm}(y)\right\} &= \{U(x), U(y)\} = 0
\end{aligned} \tag{26}$$

(26) is nothing but the classical analogue of $N=2$ superconformal algebra.

(iii) Let us now calculate the Poisson bracket of the componet fields $u_{2i-1}^b, u_{2i}^b, u_{2i-1}^f$ and u_{2i}^f with the energy momentum tensor T, defined in (25). It follows from (10) and (17) that

$$\begin{aligned}
\left\{T(x), u_{2i-1}^b(y)\right\} = \Big(&(i+1)u_{2i-1}^b(y)\partial_y + (u_{2i-1}^b(y))'\\
&- \sum_{m=0}^{i-2}(-1)^m \binom{i}{m+2} u_{2i-2m-3}^b(y)\partial_y^{m+2}\\
&- \frac{1}{2}\sum_{m=0}^{i-1}(-1)^m \binom{i}{m+1} u_{2i-2m-3}^b(y)\partial_y^{m+2}\Big)\,\delta(x-y)
\end{aligned} \tag{27a}$$

$$\begin{aligned}
\left\{T(x), u_{2i}^b(y)\right\} = \Big(&(i+2)u_{2i}^b(y)\partial_y + (u_{2i}^b(y))'\\
&- \sum_{m=0}^{i-2}(-1)^m \binom{i+1}{m+2} u_{2i-2m-2}^b(y)\partial_y^{m+2}\\
&+ \frac{1}{2}\sum_{m=0}^{i-1}(-1)^m \binom{i+1}{m+1} u_{2i-2m-1}^b(y)\partial_y^{m+2}\Big)\,\delta(x-y)
\end{aligned} \tag{27b}$$

$$\left\{T(x), u_{2i-1}^f(y)\right\} = \Big((i+\frac{3}{2})u_{2i-1}^f(y)\partial_y + (u_{2i-1}^f(y))'$$

$$-\sum_{m=0}^{i-2}(-1)^m \binom{i}{m+2} u^f_{2i-2m-3}(y)\partial_y^{m+2}$$

$$+\frac{1}{2}\sum_{m=0}^{i-1}(-1)^m \binom{i}{m+1} u^f_{2i-2m-3}(y)\partial_y^{m+2}\Bigg)\delta(x-y) \qquad (27c)$$

$$\left\{T(x), u^f_{2i}(y)\right\} = \Bigg((i+\frac{3}{2})u^f_{2i}(y)\partial_y + (u^f_{2i}(y))'$$

$$-\sum_{m=0}^{i-2}(-1)^m \binom{i+1}{m+2} u^f_{2i-2m-2}(y)\partial_y^{m+2}\Bigg)\delta(x-y) \qquad (27d)$$

It is now evident from (27) that u^b_{2i-1}, u^b_{2i} and u^f_{2i-1}, u^f_{2i} are respectively bosonic and fermionic conformal fields with respect to the energy momentum tensor T, introduced in (25).

Hence we cliam that the Poisson bracket algebra (17) is a nonlinear realization of super W_∞ algebra. Notice that similar situation has been observed in the case of Gelfand Dikii bracket of second kind for the bosonic KP hierarchy [8].

To conclude, the superLax operator (1), indeed, corresponds to $N = 2$ super KP hierarchy. The Poisson bracket algebra, we have obtained following Gelfand Dikii method, gives rise to a nonlinear realization of super W_∞ algebra. In alalogy with bosonic KP hierarchy [8] we believe this algebra may be realized as universal super W_∞ algebra.

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