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Absorption and Recoil of Fundamental String by D-String

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Inclusive absorption cross section of fundamental IIB string to D-string is calculated perturbatively. The leading order result agrees with estimate based on stringy Higgs mechanism via Cremmer-Scherk coupling. It is argued that the subleading order correction is dominated by purely planar diagrams in the large mass limit. The correction represents conversion of binding energy into local recoil process of the fundamental string and D-string bound state. We show their presence explicitly in the next leading order.

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1. Introduction

Dirichlet branes [1] in string theory are *quantum BPS solitons* that are coupled minimally to the Ramond-Ramond (RR) gauge fields. In unraveling nonperturbative aspects of string theories, it has been essential to understand spectra of D-brane bound states. For example, strongly coupled Type IIA string and partonic description via M(atrix) theory relies heavily on the existence of threshold bound state of D zero-branes for arbitrary charge. Likewise, conifold transitions in Type II strings compactified on Calabi-Yau three-fold assumes no threshold bound state for wrapped two-branes and three-branes near the the singularity. (Non)existence of either types of bound states are extensively studied [2]. A more nontrivial class of D-brane bound states are those of non-threshold type, for which binding effect has to be properly taken into account. The most interesting one of this kind arises in Type IIB string theory. The ten-dimensional Type IIB string theory exhibits $SL(2, \mathbf{Z})$ S-duality symmetry [3]. There are two types of rank-two antisymmetric tensor potentials, one $B^{(NS)}_{MN}$ from NS-NS sector and another $B^{(RR)}_{MN}$ from Ramond-Ramond sector. Fundamental string (F-string) and D-string are electric sources that couples minimally to the two tensor potentials respectively. Under the $SL(2, \mathbf{Z})$ S-duality, the pair of antisymmetric tensor potential as well as pair of the F- and the D-strings transform as doublets. The $SL(2, \mathbf{Z})$ S-duality predicts existence of infinite orbits of (m, n) strings carrying NS-NS charge m and RR charge n, with m and n relatively prime [4]. The (m, n) strings are BPS configurations annihilated by sixteen supercharges and have a tension

$$T_{(\mathrm{m,n})} = T\sqrt{m^2 + \frac{n^2}{g^2_{IIB}}}. \tag{1.1}$$

Here, $T \equiv 1/2\pi\alpha'$ is the F-string tension and g_{IIB} denotes Type IIB string coupling parameter. Investigation of the (m, n) string bound states so far has been focused mainly on kinematical aspects such as spectra and degeneracy. Many interesting dynamical issues such as formation/dissociation of the bound states, recoil and radiation and conformal field theory of (m, n) string collective coordinates are largely unexplored.

In this paper, we initiate systematic study of D-brane *dynamics* and study formation of (m, n) string bound state and subsequent conversion of the released binding energy into local recoil. Consider a macroscopic F-string of charge $(m, 0)$ approaching a macroscopic D-string of charge $(0, n)$ at arbitrarily slow relative velocity. Once their wave functions overlap, the F-string can fuse inside the D-string worldsheet by converting itself into homogeneous electric flux. The conversion is nothing but stringy Higgs mechanism [5] of $B^{(NS)}_{MN}$ field mediated via Cremmer-Scherk coupling [6] present in the Dirac-Born-Infeld

(DBI) worldsheet action of D-string. In section 2, we calculate *inclusive* absorption cross section of F-string by D-string and show that the cross section is of order $g_{\rm IIB}$ in the limit $m \gg n$ but of order $g^2_{\rm IIB}$ for finite m, n. Once the (m, n) bound state string is formed, binding energy has to be either released via radiation of closed string states or converted into internal excitations via local recoil of the string. In string perturbation theory, such information is encoded in the higher order contributions to the cross section. Restricting to the former limit $m \gg n$, in section 3, we argue that the dominant higher order corrections come from local recoil of bound state string rather than radiation of closed string states. In section 4, based on the observation of section 3, we calculate leading order correction (annulus diagram) to the absorption cross section. We show explicitly that rigid recoil effect is suppressed in the kinematical regime considered but internal excitation via local recoil deformation is nonvaninshing and provides subleading correction to the total absorption cross section. In section 5, we conclude with implications of the result to other $p-(p+2)$ D-brane nonthreshold bound states that are related to the (m, n) string bound state by a series of S- and T-dualities.

2. Inclusive Absorption Cross Section of Macroscopic F-String

Consider Type IIB string theory compactified on a circle, say, X^9 direction of radius R. $L = 2\pi R$ is the period in X^9 direction. We will take $R \gg \sqrt{\alpha'}$ so that strings wrapped on it are macroscopic. Arrange F-string with winding number m and D-string with winding number n, $m, n \gg 1$ around the X^9 circle. To calculate absorption cross section of the F-string by D-string (or vice versa), we bring the F-string adiabatically to the D-string. For example, we let the F-string approach the D-string with arbitrarily small velocity, $v \ll 1$. As the details of final states are not of direct interest to us, we consider *inclusive* absorption cross section, hence, *total* bound state formation rate. By unitarity and optical theorem, the inclusive cross section is related to the forward Compton scattering amplitude. The full process can then be visualized as follows: the winding F-string meets the D-string target, split by fusing into the D-string worldsheet and then rejoin back to the winding F-string. The leading order parton diagram associated with this forward Compton scattering amplitude is then given by two Type IIB winding string vertex operators on a disk diagram with Dirichlet boundary condition. (A closely related disk amplitude but with Neumann boundary condition has been considered previously [7] in the context of decaying macroscopic bosonic string.)

The ground state configuration of a winding F-string comes from the massless modes in the original uncompactified theory, hence, it is characterized by a polarization vector

$\epsilon_{\mu\nu}$. Momentum quantum number of the winding F-string measured in the rest frame of the winding D-string is then given by

$$\begin{aligned}
&p_R = (E, mR/\alpha', \mathbf{p}_\perp), \quad p_L = (E, -mR/\alpha', \mathbf{p}_\perp), \\
&E^2 - \mathbf{p}_\perp^2 = (mR)^2/\alpha'^2.
\end{aligned} \tag{2.1}$$

p_R and p_L are the usual right-moving and left-moving momenta which appear in the zero-mode part of $X(z, \overline{z})$. Define two (dimensionless) kinematic invariants

$$\begin{aligned}
&s \equiv \alpha' p_\parallel^2 = -\alpha' E^2 = -[(mR)^2/\alpha' + \alpha' \, \mathbf{p}_\perp^2] \\
&t \equiv \frac{\alpha'}{2} p_1 \cdot p_2.
\end{aligned} \tag{2.2}$$

The invariant t is defined out of incoming (p_1) and outgoing (p_2) momenta. Since we are interested in the forward scattering limit, we will take $t \to 0$ in the end. Note also that $s \gg 1$ for $m \gg 1$.

The forward Compton scattering disk amplitude is given by

$$\mathcal{A}_{D_2} = n(2\pi^2 T_1 L) \left(\frac{\kappa}{2\pi\alpha' \sqrt{L}} \right)^2 \int_{D_2} \frac{d^2 z_1 d^2 z_2}{V_{\rm CKV}} \langle V_{-1}^\dagger(z_1, \overline{z}_1) V_0(z_2, \overline{z}_2) \rangle_{D_2}. \tag{2.3}$$

The notation is as follows: T_1 is the D-string tension and κ is the 10-dimensional gravitational coupling. The normalization constant $\kappa/2\pi\alpha'$ is for the massless closed string vertex operators. The normalization constant for the disk vacuum amplitude, $2\pi^2 T_1$, depends on the convention for conformal killing volume. We use the infinitesimal form of the standard representation of $SL(2, \mathbf{R})$ on upper half plane, $\delta z = \alpha + \beta z + \gamma z^2$, where α, β and γ are real. We replace three real integrals into integrals over (α, β, γ), compute an appropriate Jacobian and drop the (α, β, γ) integral. The factors of L in (2.3) are inserted to account for the compact direction volume. As such the amplitude is proportional to the Kaluza-Klein zero-brane tension and the gravitational coupling in compactified 9-dimensional Type II string. From the point of view of 9-dimensional Type II string theory, the process under consideration is interpreted as scattering a massive particle of mass $M = m\,R/\alpha'$ off a fixed zero-brane target.

The winding string state is described by vertex operators V_{-1} and V_0 in -1 and 0 ghost number picture. In ten-dimensional notation they are given by

$$\begin{aligned}
&V_{-1}(z_1, \overline{z}_1) = \epsilon_{\mu\nu} : V_{-1}^\mu(p_1, z_1) :: \overline{V}_{-1}^\nu(p_1, \overline{z}_1) : \\
&V_0(z_2, \overline{z}_2) = \epsilon_{\mu\nu} : V_0^\mu(p_2, z_2) :: \overline{V}_0^\nu(p_2, \overline{z}_2) :,
\end{aligned} \tag{2.4}$$

where

$$
\begin{aligned}
V_{-1}^{\mu}(p,z) &= e^{-\phi(z)}\psi^{\mu}(z)\, e^{ip\cdot X(z)} \\
V_{0}^{\mu}(p,z) &= (\partial X^{\mu}(z) + ip\cdot\psi(z)\psi^{\mu}(z))\, e^{ip\cdot X(z)}.
\end{aligned}
\tag{2.5}
$$

When considering scattering off D-p-branes, it is useful to use *doubling method* [10][11]and replace $\overline{X}^{\mu}(\overline{z})$ and $\overline{\psi}^{\mu}(\overline{z})$ by $D^{\mu}_{\nu}X^{\nu}(\overline{z})$ and $D^{\mu}_{\nu}\psi^{\nu}(\overline{z})$. The tensor D^{μ}_{ν} is defined as $D^{\mu}_{\nu} \equiv \mathrm{diag}(1,\cdots,1,-1,\cdots,-1)$, where the first $p+1$ entries are 1. The bosonic and fermionic propagators are given by

$$
\begin{aligned}
\langle X^{\mu}(z)X^{\nu}(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w) \\
\langle \psi^{\mu}(z)\psi^{\nu}(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}\frac{1}{z-w}
\end{aligned}
\tag{2.6}
$$

Using the above propagators, it is straightfoward to evaluate the forward Compton scattering amplitude. Closely related scattering amplitudes have been studied extensively already [10][11]. Keeping track of normalization factors carefully, we obtain

$$
\mathcal{A}_{D_2} = \frac{nT_1\kappa^2}{8}\frac{\Gamma(t)\Gamma(s)}{\Gamma(1+s+t)}(sa_1 - ta_2) \tag{2.7}
$$

a_1 and a_2 are complicated kinematic factors. For our purposes, it is enough to note that $a_1 = s\cdot Tr(\epsilon\cdot\epsilon^{\dagger})$ plus terms that vanish in the forward scattering limit, $t\to 0$. Moreover, $Tr(\epsilon\cdot\epsilon^{\dagger}) = 1$ for the polarization tensor normalization adopted above.

In the limit $s \gg 1$ and $t \to 0$, we can Taylor expand the amplitude in t and use Stirling's formula for $\Gamma(s)$. Recall also that $\Gamma(t) \to 1/t$ as $t \to 0$. In this approximation, the amplitude simplifies to

$$
\mathcal{A}_{D_2} \approx \frac{nT_1\kappa^2}{8}\Big(- s\ln s + \frac{s}{t}\Big) + \text{regular}. \tag{2.8}
$$

The t-channel pole comes from the dilaton and graviton tadpoles and is irrelavant. The imaginary part of $\mathcal{A}$ comes from the branch cut:

$$
\mathrm{Im}(\mathcal{A}_{D_2}) = \frac{\pi n T_1 \kappa^2 s}{8} \tag{2.9}
$$

.

The optical theorem relates $Im(\mathcal{A})$ to the *total absorption* cross section in the following way:

$$
\sigma_{\text{absorp}} = \frac{2\,\mathrm{Im}(\mathcal{A}_{D_2})}{2E|v_{\perp}|} = \frac{\pi n T_1\kappa^2}{8|p_{\perp}|}\left(\frac{(m\,R)^2}{\alpha'} + \alpha'\,\mathbf{p}_{\perp}^2\right), \tag{2.10}
$$

where the denominator takes into account the flux of the F-string in the standard relativistic normalization for one particle state. Finally, we express (2.10) in terms of dimensionless string coupling g_s:

$$\sigma_{\rm absorp} = \frac{4 g_s \pi^7 \alpha'^3 n}{|\mathbf{p}_\perp|} \left(\frac{(m\, R)^2}{\alpha'} + \alpha' \, \mathbf{p}_\perp^2 \right), \tag{2.11}$$

using the following well-known relations [1]

$$T_p \kappa = \sqrt{\pi} (2\pi \sqrt{\alpha'})^{3-p}, \quad \kappa = 8 \pi^{\frac{7}{2}} \alpha'^2 g_s. \tag{2.12}$$

A remark is in order. For a fundamental string with a finite length, $m =$ finite, the forward Compton scattering disk amplitude exhibits poles associated with one particle intermediate states only in the complex momentum plane. As such, the absorption cross section vanishes identically. For a macroscopically long fundamental string $m \gg 1$, however, the infinitely many one particle state poles collapse down densely along a real axis and creates a branch cut effectively. The inclusive absorption cross section we have calculated is precisely in this limit. In fact, the fact that the absorption cross section is $\mathcal{O}(g_s)$ can be understood alternatively from the absolute value square of transition amplitude mediated by Cremmer-Scherk coupling between $B^{(NS)}_{MN}$ and DBI gauge field strength F_{MN}.

3. Higher Order Corrections and Local Recoil Deformation

As we have emphasized repeatedly, the leading order absorption cross section of the previous section is exact only in the limit $m \to \infty$. However, simple kinematical consideration shows that the binding energy left out upon formation of non-threshold bound state out of F-string and D-string scattering state should be converted into other forms appropriately. From the same kinematical consideration, it is straightforward to see that the binding energy is converted either into internal excitation of bound state (m, n) string or release away by radiation of massless modes such as graviton, dilaton or NS-NS and RR antisymmetric tensor fields.

In this section, using $SL(2, \mathbf{Z})$ S-duality of Type IIB string, we argue that the dominant channel of binding energy release is into internal excitation of the bound state (m, n) string. A similar argument in related context has been given by Verlinde [12] recently and we adopt some of his arguments gratefully in out context. As in section 2, we consider the limit $m \gg n$ and, moreover, take $n = 1$ for simplicity. This combination of F- and D-strings is $SL(2, \mathbf{Z})$ dual to a combination of a F-string and m D-strings, hence, we first focus on the dynamics of the latter.

In the large m limit, the dynamics of m D-string is described by (1+1)-dimensional $\mathcal{N}=8$ supersymmetric gauge theory with gauge group $U(m)$. The worldsheet Lagrangian is given by

$$S=\int d^2x\,\mathrm{Tr}\Big[\frac{1}{4g_{\rm YM}^2}F_{\alpha\beta}^2+(D_\alpha\mathbf{X}^i)^2-\frac{g_{\rm YM}^2}{4}[\mathbf{X}^i,\mathbf{X}^j]^2\Big]. \tag{3.1}$$

The $\mathbf{X}^i$ $(i=1,\cdots,8)$ are eight transverse coordinates of the D-string in the light-cone gauge. The disk amplitude can then be viewed as a Wilson loop average of the D-string. At a fixed string coupling, higher order interactions modifies the worldsheet topology by creating many handles and holes. They are just fishnets of Feynman diagrams associated with the above $U(m)$ supersymmetric gauge theory. By the 't Hooft power counting in the large m limit, planar diagrams will dominate, viz. disk diagram with handles attached on it are suppressed compared to that with holes only.

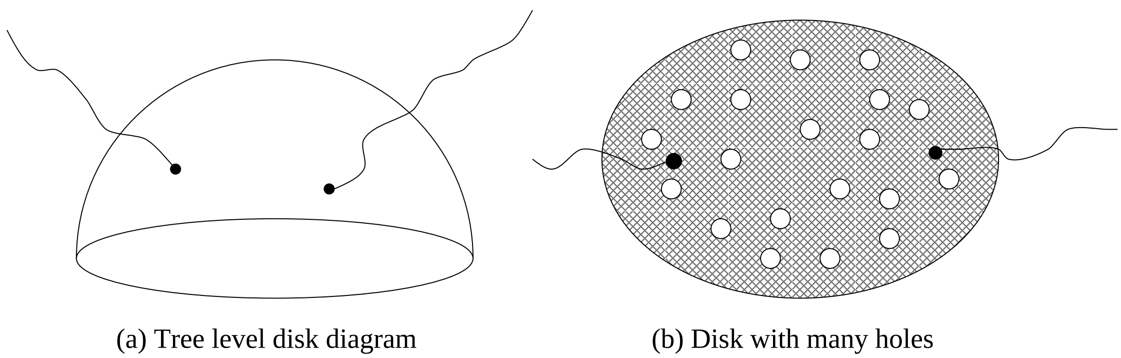

Fig. 1: Planar diagram with many holes

We now apply $SL(2,\mathbf{Z})$ S-duality and reinterpret the above (1+1)-dimensional $\mathcal{N}=8$ supersymmetric gauge theory with gauge group $U(m)$ as a M(atrix) theory description of m multiply winding Type IIB F-string. In this case, the strong coupling limit of gauge theory corresponds to weakly coupled Type IIB string theory, the limit we have assumed in the absorption cross section calculation in section 2. To see this, consider the large m limit at a fixed string coupling. The bound state string tension (1.1) is then approximated as[3]

$$T_{(m,1)}=\frac{1}{\alpha'}\sqrt{m^2+\frac{1}{g_{\rm IIB}^2}}\approx\frac{m}{\alpha'}+\frac{(1/g_{\rm IIB}^2\alpha')}{2m}. \tag{3.2}$$

In the M(atrix) string theory picture, the second term is the energy density associated with electric flux $E/L=g_{\rm YM}^2/2N$ and is interpreted as contribution of D-string. We thus

[3] In this section and ref. [12], α' denotes 2π times what we usually call α'

obtain the identification

$$g_{\rm IIB}^2 \alpha' = \frac{1}{g_{\rm YM}^2}. \tag{3.3}$$

Note that the D-string tension has been 'renormalized' by a factor $2mg_{\rm IIB}$.

The large-m argument for the dominance of planar diagrams, viz. disk amplitude in which many holes are nucleated but not handles should then hold in the $SL(2, \mathbf{Z})$ S-dual description as well. Applying this to the problem we are interested in, any higher order corrections in string loop expansion to the forward Compton scattering disk amplitude are dominated by holes rather than handles on it. From this observation follows the first of our claim: in the large m limit the release of binding energy into radiation of Type IIB closed string massless modes is completely suppressed. What about the planar diagram contributions? The boundaries of holes are all with boundary interactions

$$S_{\rm boundary} = \oint d\sigma \, [A_\alpha(X^0, X^1)\partial_\sigma X^\alpha + \Phi_i(X^0, X^1)\partial_\tau \mathbf{X}^i]. \tag{3.4}$$

They represent gauge field fluctuation on the worldsheet and collective coordinate fluctuation of the transverse light-cone coordinates [13]. As such, imaginary part of the forward Compton scattering amplitude comes from various corners of moduli space of the holes. Physically, these degeneration limit is represented by disk amplitude with lower numbers of holes and insertion of vertex operators representing gauge field fluctuation and local recoil of D-string trajectory. This is intuitively transparent. Overall rigid recoil of the $(m, 1)$ string bound state is not possible as is easily seen from simple kinematical consideration. On the other hand, the bound state can support internal excitations in the form of local recoil of each bound state string bit. The local recoil is quite complicated, hence, *exclusive* absorption cross section would be technically far more involved to calculate. On the other hand, the *inclusive* absorption cross section we study in this paper is summed over all phase space, hence, can be straightforwardly calculated from the forward Compton scattering amplitude.

From the $SL(2, \mathbf{Z})$ S-dual point of view, the large m limit corresponds to the limit in which the DBI field strength approaches the *maximal* value $F_{01} = 1/\alpha'$. Physically this means that, once the Type IIB string is splitted on the D-string worldsheet, the two oppositely charged ends are pulled apart very far away from each other. In other words, actual Type IIB worldsheet is stretched to a macroscopic size compared to typical string scale α'. In turn, the Type IIB string tension is effectively reduced by a large factor, leading to a new effective tension $T_{\rm eff} = 1/(2mg_{\rm IIB}\alpha')$ [12].

With the above intuitive understanding of the absorption processs, in the next section, we compute the leading order perturbative correction to the absorption cross section we have calculated in section 2.

4. Leading Order Perturbative Corrections

In the previous section, we have argued that the worldsheet picture of fused fundamental string into D-string is a creation of infinitely many holes, viz. fishnet of planar Feynman diagrams from the M(atrix) gauge theory point of view. In this section, we calculate explicitly the leading order correction to the absorption cross section in string perturbation expansions and draw two important conclusions. First, we will show that the rigid recoil effect is completely suppressed in the kinematical regime under consideration. This follows from the fact that logarithmic infrared divergence arising from exchange of massless Dirichlet open string states (Bloch-Nordsieck processes) is kinematically suppressed and vanishes individually. Second, we find there are finite corrections of $\mathcal{O}(g_{\rm st}^2)$ to the inclusive absorption cross section. These are effects associated with local recoil deformation of the F- and D-string bound state.

The next-leading order process in string perturbation expansion is given by an annulus diagram with insertion of two winding F-string vertex operators. We pay particular attention to possible infrared logarithmic divergences (Bloch-Nordsiek divergences) as well as finite contributions. Throughout this section we set $\alpha' = 2$.

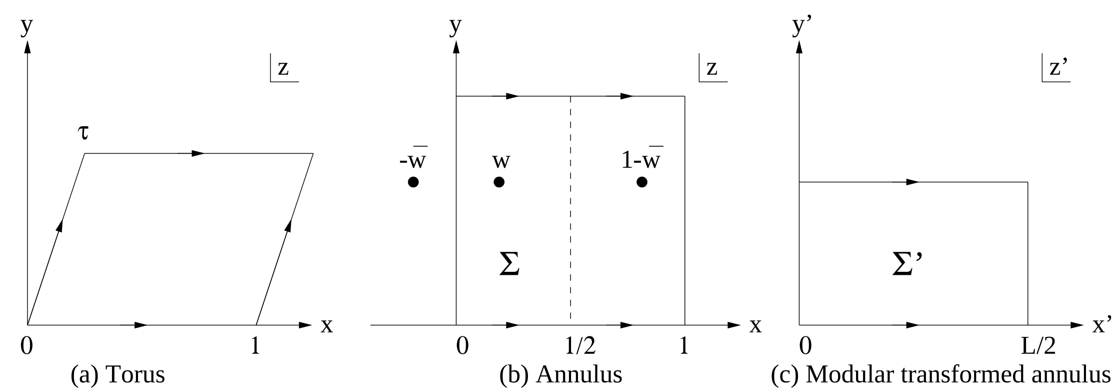

Fig. 2: The world sheet coordinate for torus and annulus

4.1. Correlators on Annulus

The annulus diagram amplitude is conveniently evaluated from $\mathbf{Z}_2$ involution of torus diagram amplitude. On torus, we introduce standard coordinates with a flat metric $z = x + iy$ and complex structure modulus τ characterizing the parallelogram. The coordinate system is depicted in Fig. 2. In the following, $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ will denote the positions of the closed string vertices. For notational convenience, we will sometimes use (z,w) in place of (z_1,z_2). Similarly, we will use (p,q) and (p_1,p_2) interchangebly for the momenta of the closed strings.

The propagator on a torus may be obtained by the method of images. After regularizing the infinite image charge sum, one finds

$$G_T(z,w|\tau) = -\log\left|\frac{\theta_1(z-w|\tau)}{\theta_1'(0|\tau)}\right|^2 + \frac{2\pi}{\tau_2}(\mathrm{Im}(z-w))^2. \tag{4.1}$$

The first term gives rise to the correct short-distance singularity ($G_T \to -\log|z-w|^2$ as $z-w\to 0$) and is periodic in $z-w\to z-w+1$. The second term is needed to ensure periodicity in $z-w\to z-w+\tau$ and flux conservation.

In order to go to an annulus, we introduce a new coordinate σ, such that $z=\sigma_1+\tau\sigma_2$. An annulus is obtained by projecting world sheet fields under the mapping $\sigma_1\to 1-\sigma_1$, $\sigma_2\to\sigma_2$. Clearly, the distance between two points on the world sheet is invariant if and only if $Re(\tau)=0$ and so we can set $\tau=iT$ for a real number T. After the projection, the annulus world sheet (Σ) is parametrized by $z=x+iy$, where $0\le x\le\frac{1}{2}$ and $0\le y\le T$. Note that, unlike the torus, the annulus does not have modular invariance as $\tau\to -1/\tau$ and T ranges from 0 to ∞.

We can impose the boundary conditions on the propagator again by the method of images. Neuman and Dirichlet propagators correspond to even and odd projections, respectively, under the mapping mentioned above,

$$\begin{aligned} G_N(z,w|T) &= G_T(z,w|iT) + G_T(z,-\overline{w}|iT) \\ G_D(z,w|T) &= G_T(z,w|iT) - G_T(z,-\overline{w}|iT). \end{aligned} \tag{4.2}$$

An image charge at $-\overline{w}$ enforces the boundary condition at $x=0$. By the periodicity of G_T on the torus, the same image charge is placed at $1-\overline{w}$, which imposes the same boundary condition at $x=\frac{1}{2}$.

A final remark is in order. In the presence of world sheet boundaries, the self contraction of $X(z,\overline{z})$ and $\psi(z,\overline{z})$ give non-zero contributions. In the case of the annulus diagram, the self contraction is

$$G_s(z|T) = \mp\log\left|\frac{\theta_1(z+\overline{z}|iT)}{\theta_1'(0|iT)}\right|, \tag{4.3}$$

where minus and plus signs correspond to Neuman and Dirichlet boundary condition respectively.

4.2. Annulus Diagram for Forward Compton Scattering

Calculation of the forward Compton scattering on the annulus diagram proceeds essentially the same as on the disk diagram. In contracting the two F-string winding vertex operators, the only role played by the correlators of ∂X and ψ is to determine kinematic factors, as is explicitly shown, for example, in [14]. The integrand of the final expression contain only the zero-mode contributions of the partition function and the correlators of the exponential factors, $\exp(ik \cdot X)$. Using the propagators and the self-contraction given in the previous sub-section, one obtains

$$\begin{aligned}
&\langle e^{ip\cdot X(z,\overline{z})} e^{iq\cdot X(w,\overline{w})} \rangle \\
&= \exp[-p_{\|}\cdot q_{\|} G_N(z,w) - p_{\perp}\cdot q_{\perp} G_D(z,w) - (p_{\|}^2 - p_{\perp}^2) G_s(z) - (q_{\|}^2 - q_{\perp}^2) G_s(w)] \\
&= \exp[-t\{G_T(z,w) - G_T(z,-\overline{w})\} + s\{G_T(z,-\overline{w}) - G_s(z) - G_s(w)\}] \\
&= \left|\frac{\langle 12\rangle\langle \overline{1}\overline{2}\rangle}{\langle 1\overline{2}\rangle\langle \overline{1}2\rangle}\right|^t \left|\frac{\langle 1\overline{1}\rangle\langle 2\overline{2}\rangle}{\langle 1\overline{2}\rangle\langle \overline{1}2\rangle}\right|^s \exp\left[-\frac{\pi}{2T} s(z-\overline{z}-w+\overline{w})^2\right],
\end{aligned} \tag{4.4}$$

where we have defined $\langle ij\rangle = |\theta_1(z_i - z_j|iT)|$ and $\langle i\overline{j}\rangle = |\theta_1(z_i + \overline{z_j}|iT)|$.

Using these results, we have found that the forward Compton scattering amplitude on an annulus diagram is given by

$$\mathcal{A}_{C_2} = iA_0 K \int_0^\infty \frac{dT}{T} T^{-(p+1)/2} \int_\Sigma d^4 z \left|\frac{\langle 12\rangle\langle \overline{1}\overline{2}\rangle}{\langle 1\overline{2}\rangle\langle \overline{1}2\rangle}\right|^t \left|\frac{\langle 1\overline{1}\rangle\langle 2\overline{2}\rangle}{\langle 1\overline{2}\rangle\langle \overline{1}2\rangle}\right|^s e^{\frac{2\pi}{T}s(y_1-y_2)^2}, \tag{4.5}$$

where A_0 is a normalization constant defined as

$$A_0 \equiv n^2 (8\pi^2\alpha')^{-\frac{p+1}{2}} L \left(\frac{\kappa}{2\pi\sqrt{L}}\right)^2 \tag{4.6}$$

and $K = sa_1 - ta_2$ is the same kinematic factor as in the disk diagram. The measure for the modulus integral is dT/T. The factor $T^{-(p+1)/2}$ comes from the integration of normalized zero-modes in Neuman directions.

Conformal Killing symmetry is modded out by fixing one of the y coordinates, dropping the integral and multiplying the integrand by T. We choose to set $y_1 = 0$ and let $y \equiv y_2$ vary from $-T/2$ to $T/2$.

4.3. Bloch-Nordsieck Infrared Divergence at $T \to \infty$

The above annulus amplitude (4.5) contains an infrared divergence. The divergence is due to propagation of massless open string states, viz. recoil vertex operators in $T \to \infty$.

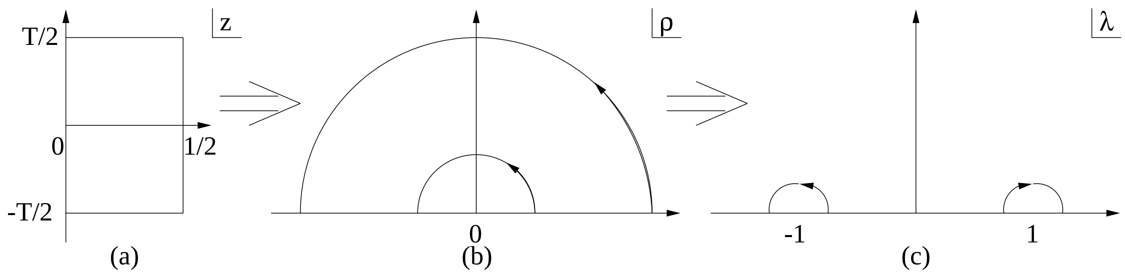

Fig. 3: $T \to \infty$ limit is a disk with a long, thin strip attached.

In order to understand the physics behind the divergence, it is convenient to perform the follwing conformal mappings

$$\rho = \exp(2\pi i z) \;\; \lambda = \frac{\rho - 1}{\rho + 1}. \tag{4.7}$$

as depicted in the following figure.

The conformal λ-plane in Fig. 3(c) is the upper half plane, which is equivalent to a disk with two small semicircles removed and identified. As T goes to infinity, the semicircles shrink to points on the boundary. As discussed above, this is the limit in which the annulus diagram factorizes into a disk diagram with Dirichlet open string states propagating between the two points on the boundary[15].

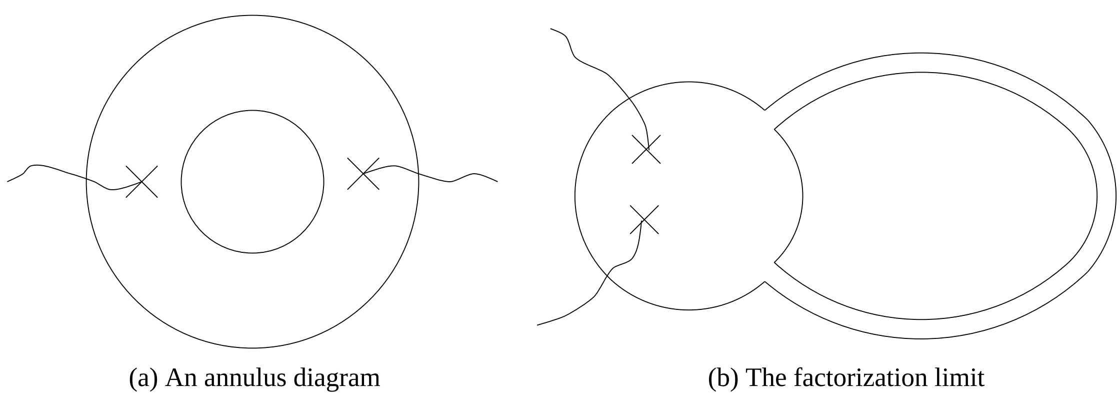

Fig. 4: An annulus diagram and the factorization limit of it.

The T-variable is the proper time of propagating open string state and T-integral gives rise to a propagator for each open string states. Clearly, zero momentum limit of the massless open string state propagator will give rise to spacetime infrared divergence. The zero momentum open string state is nothing but the translational zero mode of the D-brane represented by the vertex operator $V_o = \oint \partial_n \mathbf{X}^i$. From the worldsheet point of view, the divergence is due to violation of conformal invariance due to shrinking annulus open string tadpole.

The violation of conformal invariance is cured by taking into account of recoil of the D-string [16][17]. Recoil of the D-string amounts to fluctuation of transverse position of the D-string and is described by insertion of recoil vertex operator $V_{\rm recoil} = \oint \Phi_i(X^0, X^1)\partial_n \mathbf{X}^i$ to the disk diagram. The disk diagram with recoil vertex operator inserted is not conformally invariant either. However, when summed over, the two different sources of violation of conformal invariance cancel each other via Fischler-Susskind mechanism [18][19][13]. It is in this way energy and momentum conservation for scattering process involving D-branes is maintained.

Fortunately, in our case, all these complications disappear. This is because the inclusive absorption cross section is related to the forward Compton scattering amplitude. As such, there is no momentum transfer from the F-string to the D-string, hence, we expect that the infrared divergence vanishes in this limit. Below, we will explicitly see that this is indeed the case.

It is actually straightforward to separate the potential divergence from (4.5)using the following product representation of the theta function:

$$\theta_1(z|iT) = f(T)\sin\pi z \prod_{n=1}^{\infty}(1 - 2e^{-2\pi nT}\cos 2\pi z + e^{-4\pi nT}). \tag{4.8}$$

Since the integrand depends only on the ratio of theta functions, the prefactor $f(T)$ is irrelavant. Moreover, for fixed positions of z_1 and z_2, the infinite product simply converges to 1. Therefore, the leading order divergence of the annulus amplitude takes the following simple form:

$$\begin{aligned} \mathrm{div}(\mathcal{A}_{C_2}) &= iA_0 K \int_1^{\infty} \frac{dT}{T} T^{-(p+1)/2} \cdot T \times 4I, \\ I &\equiv \int dx_1 dx_2 dy \left| \frac{\sin\pi(x_1 - x_2 + iy)}{\sin\pi(x_1 + x_2 + iy)} \right|^{2t} \left[\frac{\sin 2\pi x_1 \sin 2\pi x_2}{|\sin\pi(x_1 + x_2 + iy)|^2} \right]^s. \end{aligned} \tag{4.9}$$

As before, we have kept the Dirichlet boundary directions arbitrary so that the dependence of divergence on the D-brane dimension p can be read off. If we cut off the proper-time integral at $T = \Lambda$, we find $\sqrt{\Lambda}$ divergence for D-particle ($p = 0$), $\log\Lambda$ for D-string ($p = 1$) and no divergence for $p \geq 2$.

After straightforward calculation, one finds

$$I \sim t\frac{\Gamma(t)\Gamma(s)}{\Gamma(1 + s + t)}. \tag{4.10}$$

We conclude that the coefficient of the divergence is the tree level amplitude times t, the momentum-squared transferred to the D-string. Since we are taking $t \to 0$ in the end, we conclude that there is no violation of conformal invariance, hence, no recoil of the D-string.

4.4. Finite Corrections from Local Recoil Deformation

Having shown that there is no infinities associated with rigid recoil of the D-string, it now remains to calculate finite corrections. In this subsection, we will find finite corrections arising from the $T \geq 1$ region. The $T \leq 1$ region corresponds to a closed string emitted and re-absorbed by the D-brane and is not relevant to our problem. We will continue neglecting the infinite product in the theta functions, which amounts to neglecting all massive open string modes.

In the previous sub-section, the analysis was simplified because (1) the integration became an infinite strip, which was mapped to the upper half plane and (2) the integrand of the z-integral converged. The finite corrections then come from (1) finiteness of the integration region and (2) the deviation of the integrand from its limiting value. In the following, we compute these two contributions. It turns out that both of them give same result up to an overall constant and correspond to two open string branch cut.

Correction due to the finite length of the strip.

It is again easy to work in the upper half plane (Fig. 3(c)). Clearly, the integral (4.9) over the two semicircles will give the correction. Since $T \geq 1$, $e^{-\pi T}$ is always small and we can use the approximation $\lambda = i \tan z_2 = \pm 1 + re^{i\theta}$, where $0 \leq r \leq e^{-\pi T}$ and $0 \leq \theta \leq \pi$. The other closed string vertex is fixed on the imaginary axis with coordinate $b = \tan x_1$. Note also that the integrals over the two semicircles yield the same value. We have, to the leading order in $e^{-\pi T}$,

$$2 \cdot \frac{1}{2} \int \frac{r dr d\theta db}{r^2(1+b^2)} \left| \frac{1-ib}{1+ib} \right|^{2t} \left[\frac{4r \sin\theta \cdot b}{|1+ib|^2} \right]^s . \tag{4.11}$$

After some algebra, we obtain

$$\frac{s}{32\pi^2} e^{-\pi s T} \left[\frac{\Gamma(s)}{\Gamma(1+s/2)^2} \right]^2 . \tag{4.12}$$

This particular combination of Γ-functions in the square bracket is exactly the one found in the tree level amplitude for a closed string absorbed by the D-string and to excite two massless open string modes [11]. Thus we recognize this correction as coming from two open string branch cut. Integration over T produces a numerical constant, which we do not compute.

Correction due to deviation of integrand from limiting value

Rescale the y coordinate in (4.9) by $y \to yT/2$ so that y ranges between -1 and $+1$. The integrand becomes

$$T \times \left[\frac{\cosh \pi T y - \cos 2\pi(x_1 - x_2)}{\cosh \pi T y - \cos 2\pi(x_1 + x_2)}\right]^t \left[\frac{2 \sin 2\pi x_1 \sin 2\pi x_2 e^{\frac{1}{2}\pi T y^2}}{\cosh \pi T y - \cos 2\pi(x_1 + x_2)}\right]^s . \tag{4.13}$$

For a fixed value of y, the integrand converges to 0 as $T \to \infty$ and non-zero contribution comes from a domain $|y| \le 1/T$. For a large value of T, we have

$$\left[4 \sin 2\pi x_1 \sin 2\pi x_2 e^{-\frac{1}{2}\pi T(2y - y^2)}\right]. \tag{4.14}$$

The factorized $x_{1,2}$ interals yields

$$\Big(\int_0^{1/2} (2 \sin 2\pi x)^s dx\Big)^2 = \Big[\frac{1}{2} s \frac{\Gamma(s)}{\Gamma(1 + s/2)}\Big]^2, \tag{4.15}$$

the combination that appears in the tree level amplitude of a closed string absorbed by the D-string and to excite two massless open string modes. This is again the two open string branch cut. The remaining y and T integrals give rise to a numerical factor, which we again do not compute.

We conclude that the leading order finite corrections come from exchange of two open string massless states viz. gauge fields and translation zero-modes. The correction corresponds to local recoil deformation of the F- and D-string bound state.

5. Discussions

In this paper, we have studied dynamics of bound state formation between fundamental and Dirichlet strings in Type IIB string theory. We have calculated total inclusive absorption cross section via optical theorem from the forward Compton scattering amplitude of an F-string off a D-string target. We have found that the leading order disk diagram gives cross section of order $\mathcal{O}(g_s)$ and agrees with power counting from stringy Higgs mechanism via Cremmer-Scherk coupling. Using M(atrix) string theory and SL(2,**Z**) mapping, we have argued that higher order corrections come from disk diagram with arbitrarily large number of holes and describes conversion of binding energy into local recoil deformation. To check this, we have calculated explicitly leading order corrections from annulus diagram and have found that rigid recoil of the bound state is absent and that two open string state branch cut is present as expected.

By a series of S- and T-dualities, one can relate the (m, n) bound-state string to other configurations of strings and branes. For example, T-duality in X^7 and X^8 directions followed by S-duality turn the (m, n) string into m D-strings and n D-3-branes, which is T-dual to any $p-(p+2)$ bound states [20][21][22]. Additional T-duality in a direction not parallel to coordinate axis give rise to branes intersecting at angles [23][24]. Early studies of these brane configurations have been focused on their mass spectrum, degeneracy and supersymmetry. Recently, there have been some progress in analyzing their dynamics by using scattering of a D-0-brane [20] and low energy Born-Infeld action [21][25][22]. One notable progress was made in Ref. [22], where it was shown that the condensate of tachyonic modes of ND strings connecting $p-$ and $(p+2)-$branes account for the binding energy predicted by the BPS formula. In spite of all the progress, the details of the *dynamics* of non-threshold brane bound states, brane-anti-brane configuration and intersecting branes are still poorly understood. We hope that our analysis will be useful in future investigations on these issues.

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