Soil porosity from seismic velocities

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TECHNICAL NOTE

Soil porosity from seismic velocities

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INTRODUCTION

By analysing the wave propagation phenomenon in the framework of porous media theory, it is possible to account for the interaction between the soil skeleton and the pore fluid, and to establish the dependence of seismic velocities on elastic parameters of the two phases and on the soil porosity (Foti *et al.*, 2002).

Several empirical relationships have also been suggested to correlate porosity and the velocity of propagation of compressional wave in fluid-saturated porous media (Wyllie *et al.*, 1956; Raymer *et al.*, 1980; Castagna *et al.*, 1985; Han *et al.*, 1986; Ederhart-Phillips *et al.*, 1989; Klimentos & McCann, 1990; Krief *et al.*, 1990). A good review on the topic is provided by Berryman (1995), who also discusses analytical approaches based on mixture theory.

Biot (1956a, 1956b) laid down the theory of seismic wave propagation in saturated porous media, and his work is widely recognised as a milestone in this field. Biot's theory is based on a macroscopic approach that models the binary continuum as a superposition of a fluid and a solid phase occupying simultaneously the same regions of space. In this context, porosity can be seen as a parameter upscaling information from micro to macro scale (Lancellotta, 2002).

Assuming an isotropic, linear elastic soil skeleton saturated by a non-dissipative compressible fluid, Biot was able to prove the existence of two different compressional waves, namely of the first and second kind. The existence of the compressional wave of the second kind, also known as the Biot wave, has been proven by Plona (1980) and, more recently, by Nakagawa *et al.* (1997). The Biot wave is very difficult to observe experimentally because, as it is slower than the compressional wave of the first kind, the first arrival in seismic signals is always associated with the latter.

Considering the timescale of wave propagation and the permeability of soils, the assumption that no relative motion actually occurs between the solid and the fluid phases holds for the low-frequency range (Santamarina *et al.*, 2001). Introducing the additional hypothesis of incompressible soil grains, it is possible to obtain the following explicit relationship for soil porosity n, as shown by Foti *et al.* (2002):

$$n = \frac{\rho^{\mathrm{S}} - \sqrt{(\rho^{\mathrm{S}})^2 - \left\{4(\rho^{\mathrm{S}} - \rho^{\mathrm{F}})K^{\mathrm{F}} / \left[V_{\mathrm{P}}^2 - 2\left(1 - \nu^{\mathrm{SK}}\right)/\left(1 - 2\nu^{\mathrm{SK}}\right)V_{\mathrm{S}}^2\right]\right\}}}{2(\rho^{\mathrm{S}} - \rho^{\mathrm{F}})} \quad (1)$$

where V_{p} and V_{S} are the velocities of propagation of compressional and shear waves respectively. The above relationship is dependent only on properties that assume rather standard values (grain density ρ^{S}, water density ρ^{F}, water bulk modulus K^{F}) and on the Poisson ratio of the soil skeleton, ν^{SK}. ν^{SK} has a limited range of variability in soils (0·1–0·4), and it can be shown to have negligible influence on the estimated values of porosity (Foti *et al.*, 2002). Moreover, it is possible to show that the hypothesis of incompressible soil grains can be removed with a relatively small influence on the final results (Foti *et al.*, 2002), even if in this case the porosity evaluation requires the solution of a more cumbersome mathematical inverse problem.

As already stated, the above solution is restricted to low-frequency perturbations, because it assumes that there is no relative motion between the soil skeleton and the fluid phase. Indications about the range of validity of this assumption can be found in Miura *et al.* (2001); it can be observed that sources typically adopted for geophysical in-situ tests are such that the predominant frequencies in the seismic signals are well below the threshold value for all kind of soils (Foti *et al.*, 2002).

DESCRIPTION OF THE DATASET

Hunter (2003) recently reported a large dataset of seismic velocities and porosity data from 12 boreholes in Canada. A subset of his data is used in the present paper, referring only to high-quality data (six boreholes) (Hunter, 2003, personal communication). The porosity data of this subset have been obtained from undisturbed samples, processed on the specialised GSC laboratory on the same day as retrieval, except for one case in which the data were analysed in the laboratory of Carleton University. Great care was taken to preserve the original water content during transportation of the samples. A detailed description of the procedure and of all the cautions taken to obtain reliable estimates is reported in Hunter (2003).

Details of in-situ measurements of P and S wave velocities, performed in down-hole tests, are also reported in Hunter (2003).

Table 1 gives a description of the six boreholes. The available data are typically for depths up to 60 m, except for one borehole (JA01-4) for which data to a depth of 140 m are available. Most of the data are related to clays, with the exception of a few datapoints related to sands or tills.

Additional data have been collected in Italy, using cross-hole tests for the seismic measurements and standard laboratory procedures for the determination of porosity (Jamiolkowski, 2003, personal communication). This dataset is related to silty clays and clayey silts, with PI ranging between 15% and 35%.

In order to have results relative to sandy soils, data from laboratory tests, reported in the literature by Bates (1989) and Tsukamoto *et al.* (2002), have also been considered.

MEASURED AND PREDICTED POROSITY

Using the available experimental results, porosity has been estimated from the seismic velocities using equation (1). As mentioned above, the Poisson ratio of the soil skeleton, ν^{SK}, has a very limited influence on the results; in the present

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Table 1. Results for the Canada dataset (experimental data by Hunter, 2003)

Borehole ID	Range of depth: m		Difference between predicted and measured soil porosity: %	
	Min	Max	Mean value	Median value
AR93-1	11	50	10·9	7·8
LV96-1	4	31	13·4	8·8
LV96-2	17	36	10·0	5·7
JA-4	9	140	6·9	5·6
JA-5	4	35	13·2	13·6
JA-6	3	64	5·8	4·8

work the estimate has been repeated considering different values of ν^{SK} ranging from 0·1 to 0·4, and the corresponding results have been averaged to get a single estimate of porosity. The following values have been considered for the other parameters: $\rho^F = 1{\cdot}0\ \mathrm{Mg/m^3}$; $K^F = 2{\cdot}15$ GPa; $\rho^S = 2{\cdot}65\ \mathrm{Mg/m^3}$ for sands and $2{\cdot}72\ \mathrm{Mg/m^3}$ for clays.

The results obtained for two representative boreholes from the Canada dataset are reported in Figs 1 and 2, with profiles showing:

(*a*) measured P-wave and S-wave velocities
(*b*) measured porosity (laboratory data) and predicted porosity (from equation (1))
(*c*) the difference between measured and predicted porosity, and the associated mean and median values.

The trends of the measured and predicted porosity values with depth show defined similarities: in particular, borehole JA01-4 is the only one in which very small values of porosity (around 0·2) were measured in the laboratory for samples retrieved at depth between 130 and 140 m. The seismic velocities measured at this depth are consistently larger than all the values measured at other depths and in other boreholes (V_p higher than 2000 m/s and V_s around 800 m/s), and the predicted porosity is close to the measured one. It is important to note also that these values of seismic velocity and porosity are very consistent with the values for dense sands and gravels reported by Foti *et al.* (2002).

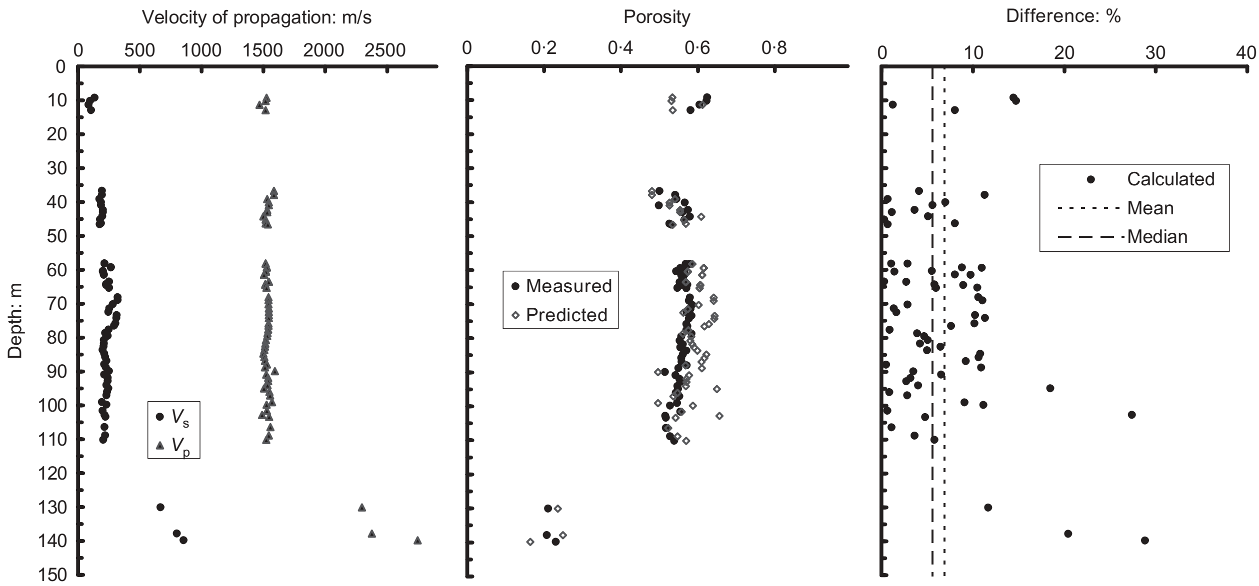

Fig. 1. Results for borehole JA01-4; experimental data from Hunter (2003)

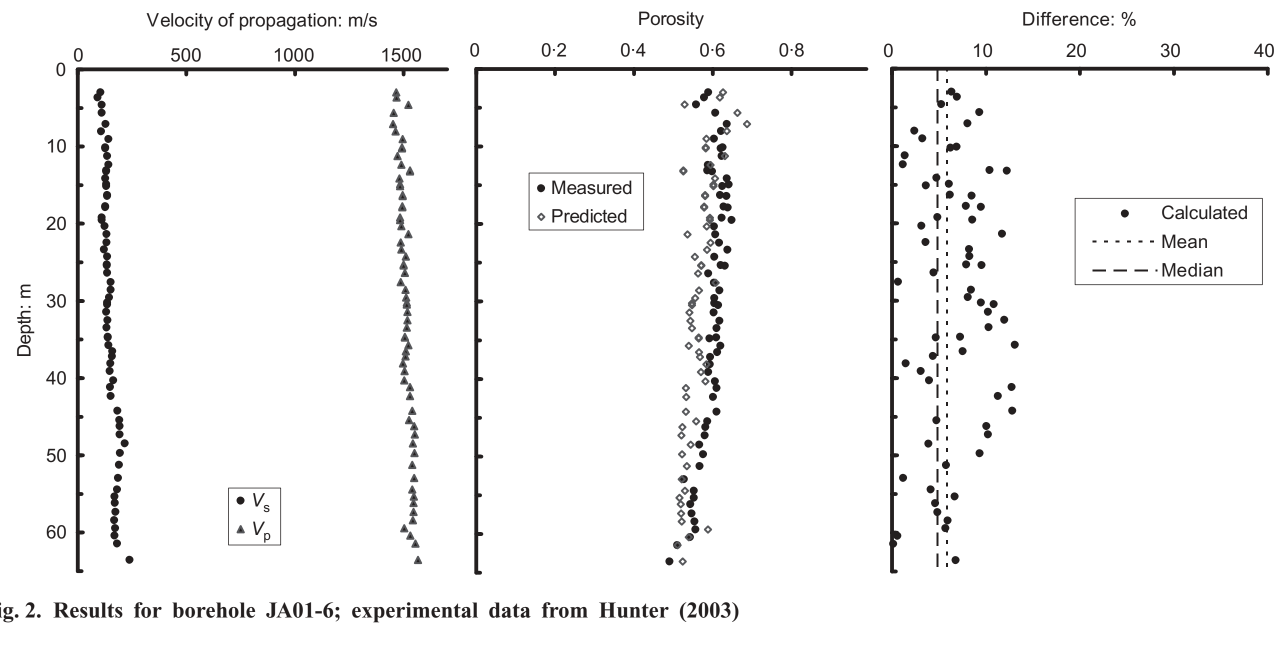

Fig. 2. Results for borehole JA01-6; experimental data from Hunter (2003)

A summary of the statistics of the difference between measured and predicted porosity for the six boreholes is reported in Table 1. On average, the difference between measured and predicted porosity is less than 9%, but boreholes JA01-4 and JA01-6 are even better, with an average difference of about 5%.

The same procedure has been applied to the other available datasets. A summary of the results is reported in Table 2. A reduced number of datapoints is available in these datasets, but the quality of the estimate is supported, the difference being on average of the order of 7%. The profiles with experimental data and porosity predictions for the Florence site are reported in Fig. 3.

Figure 4 reports the results obtained for the different datasets. The predicted porosity on the basis of seismic velocities is plotted against the measured porosity obtained from undisturbed samples or from the properties of the samples for laboratory testing.

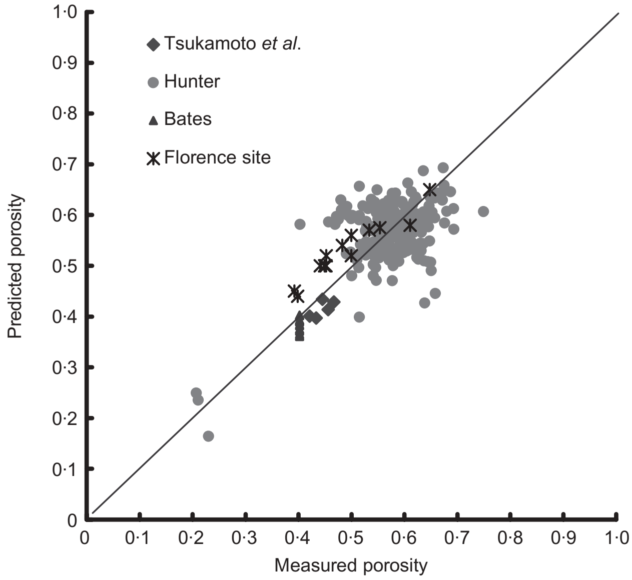

Fig. 4. Predicted against measured porosity for all datasets

CONCLUSIONS

Porosity is a state parameter that plays a relevant role for many geotechnical applications because of its influence on dilatancy and associated phenomena.

The use of seismic velocities for predicting soil porosity on the basis of Biot's theory is very promising. Estimation of porosity for different independent datasets from both in-situ and laboratory tests has given very encouraging results, with average differences from direct measurements below 10%. Moreover, no systematic bias has been observed in the predictions, and good results have been obtained over a wide range of porosity, even if a limited number of points was available for very dense and very loose materials.

A relevant advantage of the method is that by using high-quality seismic measurements (preferably from cross-hole tests) it is also possible to evaluate in detail the porosity profile for coarse materials, which are still difficult to sample.

One of the most relevant applications is related to the evaluation of liquefaction susceptibility. Indeed the possibility of estimating soil porosity from non-destructive testing can lead to substantial improvements compared with current practice.

Table 2. Results for all datasets

Dataset	Technique	Number of datapoints	Average difference: %
Hunter	Site (down-hole)	233	9·0
Florence	Site (cross-hole)	13	8·3
Bates	Laboratory	15	4·7
Tsukamoto	Laboratory	5	6·6

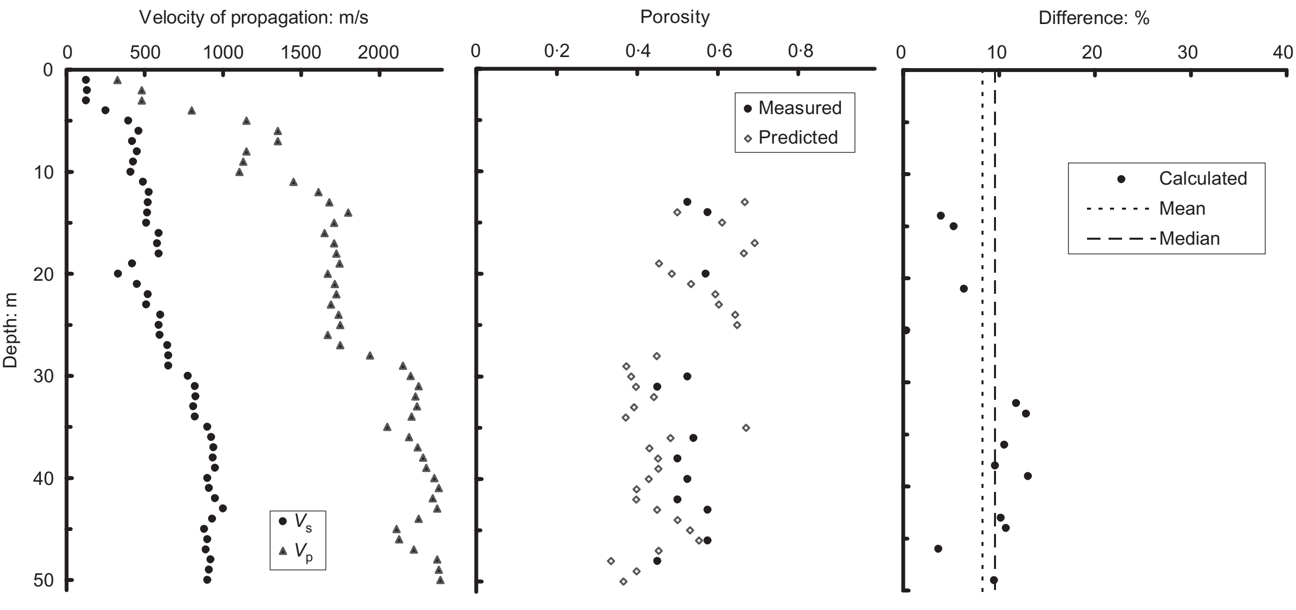

Fig. 3. Results for Florence site; experimental data from Jamiolkowski (2003, personal communication)

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