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## PARAMETRIC AND SEMI-PARAMETRIC MODELLING OF VACATION EXPENDITURES R20 $\begin{array}{lcl}\text { by Bertrand Melenberg } & 330 . / 15 \cdot 12 \\ \text { and Arthur van Soest } & 79\end{array}$

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# Parametric and semi-parametric modelling of vacation expenditures 

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#### Abstract

We analyse several limited dependent variable models explaining the budget share that Dutch families spend on vacations. To take account of the substantial number of zero shares in our cross-section data, two types of models are used. The first is the single-equation censored regression model, of which the Tobit model is a spectal case. We estimate and test several parametric and semi-parametric extensions of the Tobit model. Although the parametric models appear to be misspectfied, estimated income elasticities of vacation expenditures according to the Tobit model and its extensions do not differ substantially. Secondly, we consider two-equation models, in which the decision whether or not to go on vacation and the decision on the amount to spend are treated separately. The first decision is modelled as a binary choice equation, whereas the second is treated as a conditional regression equation. For both equations, we estimate, test and compare parametric and semi-parametric specifications. Simulations are used to investigate whether apparently misspecified specifications indeed lead to biased conclusions about income elastictties of vacation expenditures.


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## 1. Introduction

The aim of this paper is to analyse total annual vacation expenditures of Dutch families. Using cross-section data, we study the impact of family income, household composition, and several other family characteristics on the decision which share of the family budget is spent on vacations.

A typical characteristic of expenditures on vacations compared to other expenditure categories such as food, clothing, etc., is that many families do not spend anything on vacations. In our sample, this is the case for $37 \%$ of all observations. Various limited dependent variable models can be used to account for these zero observations. See, for example, Maddala (1983). In particular, for the problem at hand, two types of models seem adequate: The one-equation censored regression model, of which the familiar Tobit model is the standard special case, and a two-equation model, which consists of a binary choice part explaining the participation decision, 1) and a conditional regression equation explaining the expenditure level if it is positive.

Standard estimators for these types of models are based on maximum likelihood or a Heckman type two-step procedure. Consistency of these estimators generally requires a complete and correct specification of a parametric family of the error distribution. As a consequence, specification testing and model selection are more crucial than in the standard regression model. In this paper, we focus on a thorough treatment of the choice of the econometric model. Following the strategy which Horowitz (1991) uses for his analysis of the binary choice model, we study various parametric and semiparametric models, comparing estimates and performing formal and graphic tests.

In section 2 we consider the single-equation censored regression model. Our starting point is the Tobit model, which is characterised by the assumptions that error terms are homoskedastic and normally distributed. Pseudo maximum likelihood estimates for the parameters of the censored regression model obtained by maximising the Tobit likelihood are generally inconsistent if either the homoskedasticity or the normality assumptions are violated. Appropriate tests for these assumptions are discussed by, for instance, Chesher and Irish (1987) and Newey (1985). In our case, both homoskedasticity and normality are rejected. Our next step is to account for parametric forms of heteroskedasticity and nonnormality, but neither of the extensions are accepted by chi-squared diagnostic tests. We then turn to semi-parametric estimation and present two estimators which allow for both
heteroskedasticity and asymmetry: The censored least absolute deviations estimator (Powell, 1984) and an efficient two-step estimator given by Newey and Powell (1987). Parametric and semi-parametric estimates are compared by looking at estimated confidence bounds for resulting income elasticities.

Apart from many observations with zero expenditures on vacations, our data are also characterised by a lack of observations with small positive vacation expenditures. This is not captured in the usual (zero threshold) censored regression model. In Van Soest and Kooreman (1987), a model with unobserved random thresholds was used to take account of this phenomenon, similar to Nelson (1977). For such a model, no semi-parametric estimators are yet available. In this paper, we therefore use an alternative model which has the same flexibility as the Nelson model. In section 3 , the vacation expenditures decision is modelled in two steps. We use a binary choice equation to model the participation decision. For the non-zero observations, we use a regression equation to explain the (positive) level.

We compare and test various parametric and semi-parametric specifications of the binary choice model. Using standard specification tests, we hardly find any evidence against a simple Probit model, contrary to, for instance, Horowitz (1991). The Probit model is accepted against parametric alternatives allowing for heteroskedasticity or nonnormality, and by nearly all chi-squared diagnostic tests. It is also accepted by the specification test against a nonparametric alternative suggested by Horowitz (1991).

Subsequently, we estimate the binary choice model using the estimator proposed by Klein and Spady (1989). We assume that the model can be written as a single index model: $\mathrm{P}(\mathrm{y}=1 \mid \mathrm{X})=\mathrm{F}\left(\mathrm{X}^{\prime} \alpha\right)$ and present estimates of $\alpha$ as well as (nonparametric) estimates of the unknown function F. The Probit model and the single index model estimates can thus be compared by comparing corresponding predicted probabilities. We find that the outcomes according to the Klein-Spady estimates are quite close to those obtained for the Probit model.

In case of the regression equation, which is used to explain the positive budget share spent on vacation, the hypotheses of normality and homoskedasticity are rejected. Imposing the weak moment restriction $\mathrm{E}(\varepsilon \mid \mathrm{X})=$ 0 ensures consistency but not efficiency of the 0LS-estimator. We therefore also estimate the regression equation using the estimator proposed by Robinson (1987), which achieves the asymptotic efficiency bound corresponding to the condition $\mathrm{E}(\varepsilon \mid \mathrm{X})=0$. This specification of the regression equation is tested in several ways and cannot be rejected.

Parametric and semi-parametric specifications of the complete two-equation model are compared by calculating predicted budget shares and by performing some simulations. We do not find substantial differences between the various specifications.

In section 4, we briefly evaluate the results. Although the models in section 3 are more flexible than their section 2 counterparts, neither the parametric nor the semi-parametric models are nested. A formal test to compare the semi-parametric models in sections 2 and 3 is not available. Using less formal arguments, we motivate our preference for the twoequations model.

## 2. The Censored Regression Model

In this section we use a censored regression model to explain family expenditures on vacations as a function of total expenditures and family characteristics. The specification of the model is as follows.

$$
\begin{equation*}
y_{i}^{*}=X_{i}^{\prime} \alpha+\varepsilon_{i}, y_{i}=\max \left(y_{i}^{*}, 0\right) \tag{2.1}
\end{equation*}
$$

Here $y_{i}$ is the budget share spent on vacations for family $i(i=1, \ldots, N)$, i.e., annual expenditures on vacations as a percentage of total annual family expenditures, and $X_{i}=\left(X_{1 i}, \ldots, X_{7 i}\right)^{\prime}$ is a vector of covariates. In our case, we use $X_{1 i}=1, X_{2 i}$ is the logarithm of total annual family expenditures, $X_{3 i}$ is the logarithm of family size, $X_{4 i}$ is an age indicator. $X_{5 i}$ indicates education, $X_{6 i}$ is the degree of urbanisation, and $X_{7 i}=X_{2 i}^{2}$. ) $\alpha=\left(\alpha_{1}, \ldots, \alpha_{7}\right)^{\prime}$ is a vector of unknown parameters. Note that if $\alpha_{7}=0$, then the Engel curves correspond to the Almost Ideal Demand System (cf. Deaton and Muellbauer, 1980). $\varepsilon_{i}$ denotes the error term. The models discussed in this section differ with respect to the assumptions on the conditional distribution of $\varepsilon_{i}$, given $X_{i}$. Throughout this paper, we assume that the $\left(X_{i}, \varepsilon_{i}\right), i=1, \ldots, N$, are i.i.d.

We use a cross-section of $\mathrm{N}=1815$ families with at least two adults, taken from the Consumer Expenditure Survey drawn in 1981 by the Netherlands Central Bureau of Statistics. In appendix 1 , we present details on the definitions of the variables involved and statistics for the whole sample and for the subsamples with zero and positive vacation expenditures. Basically, the same data were used by Van Soest and Kooreman (1987).

## Parametric models

The most common assumptions about the distribution of error terms in (2.1) are

$$
\begin{equation*}
\varepsilon_{i} \mid x_{i} \sim N\left(0, \sigma^{2}\right) \tag{2.2}
\end{equation*}
$$

(2.1) and (2.2) yield the well-known Tobit model. Error terms are normally distributed and independent of the regressors. Asymmetry and heteroskedasticity are not allowed for.

Maximum likelihood estimates for the Tobit model and corresponding standard errors are mentioned in the left panel of table 2.1. The estimates for $\alpha_{2}$ and $\alpha_{7}$ imply that the budget share spent on vacations is increasing as a function of total expenditures as long as total expenditures do not exceed Dfl 111,000 , i.e., in the whole sample range, implying that vacation is a luxury. The estimated ceteris partbus difference between $X_{i}^{\prime} \alpha$ for families with total expenditures Df1 50,000 and Df1 20,000 is 0.052 . This number can be interpreted as the difference between budget shares spent on vacations for such families, assuming that $\varepsilon_{i}$ and $X_{2 i}, \ldots, X_{6 i}$ are such that, for both families, vacation expenditures are positive. Young families spend significantly more on vacations than older families. The education level of the family head has a significant positive impact on vacation expenditures. Families in big cities spend more than those living in the country. The impact of family size is insignificant.

The ML estimates under the Tobit assumptions will generally be inconsistent if the error terms in the censored regression model are heteroskedastic or non-normally distributed (see, e.g., Hurd (1979) and Goldberger (1983), respectively). These assumptions can be tested using, for example, the score or Lagrange Multiplier tests based on generalised residuals described by Chesher and Irish (1987). The test results are as follows.

Heteroskedasticity: realisation 137.0 with critical values $x_{27 ; 0.05}^{2}=40.1$ and $x_{27 ; 0.01}^{2}=47.0$. This leads to rejection of the null that errors are homoskedastic (under the maintained assumption of normality).

Non-normality: realisation 22.3 with critical values $x_{2 ; 0.05}^{2}=6.0$ and $x_{2 ; 0.01}^{2}=9.2$. This leads to rejection of the null of normally distributed errors (under the maintanied hypothesis of homoskedasticity).

It thus becomes clear that at least one of the two assumptions is violated.

Table 2.1. Estimation results parametric models
(standard errors in parentheses)

|  | I |  | II |  | III |  | IV |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\alpha_{1}$ | -292.11 | $(94.83)$ | -515.21 | $(119.01)$ | -608.87 | $(125.64)$ | -492.87 | $(102.36)$ |
| $\alpha_{2}$ | 50.43 | $(18.35)$ | 92.37 | $(22.55)$ | 110.18 | $(23.78)$ | 88.49 | $(19.37)$ |
| $\alpha_{3}$ | -0.91 | $(0.56)$ | -0.38 | $(0.49)$ | -0.82 | $(0.38)$ | -0.39 | $(0.43)$ |
| $\alpha_{4}$ | 0.22 | $(0.06)$ | 0.14 | $(0.06)$ | 0.14 | $(0.06)$ | 0.14 | $(0.05)$ |
| $\alpha_{5}$ | 0.27 | $(0.17)$ | 0.34 | $(0.15)$ | 0.29 | $(0.13)$ | 0.32 | $(0.13)$ |
| $\alpha_{6}$ | 0.56 | $(0.10)$ | 0.39 | $(0.09)$ | 0.43 | $(0.09)$ | 0.39 | $(0.08)$ |
| $\alpha_{7}$ | -2.16 | $(0.88)$ | -4.12 | $(1.07)$ | -4.96 | $(1.14)$ | -3.95 | $(0.92)$ |
| $\sigma$ | 6.23 | $(0.10)$ |  |  |  |  |  |  |


| $\beta_{1}$ | 18.46 | $(12.45)$ | 1568.26 | $(269.99)$ | 9.69 | $(14.02)$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $\beta_{2}$ | -2.86 | $(2.36)$ | -26.48 | $(4.69)$ | -1.21 | $(2.66)$ |
| $\beta_{3}$ | -0.387 | $(0.065)$ | -8.23 | $(1.17)$ | -0.315 | $(0.078)$ |
| $\beta_{4}$ | 0.014 | $(0.006)$ | 0.13 | $(0.04)$ | 0.016 | $(0.008)$ |
| $\beta_{5}$ | -0.047 | $(0.020)$ | -0.51 | $(0.14)$ | -0.035 | $(0.024)$ |
| $\beta_{6}$ | 0.044 | $(0.011)$ | 0.29 | $(0.10)$ | 0.045 | $(0.014)$ |
| $\beta_{7}$ | 0.124 | $(0.112)$ | 0.11 | $(0.02)$ | 0.044 | $(0.126)$ |
| $\gamma_{1}$ |  |  |  |  | -0.061 | $(0.010)$ |

$\begin{array}{llll}\text { loglik. }-4233.86 & -4155.45 & -4157.73 & -4144.15\end{array}$

## Explanation

The censored regression model (equation (2.1)) and the regressors are described at the beginning of this section.
I: Tobit model (2.2)
II: Exponential heteroskedasticity and normality (2.3)
III:Random coefficients and normality (2.4)
IV: Exponential heteroskedasticity and nonnormality (2.5)

Explicit incorporation of parametric forms of heteroskedasticity is straightforward. Estimation results for the following two parametric forms are presented in table 2.1:

$$
\begin{align*}
& \varepsilon_{i} \mid X_{i} \sim N\left(0, \exp \left(X_{i}^{\prime} \beta\right)^{2}\right)  \tag{2.3}\\
& \varepsilon_{i} \mid X_{i} \sim N\left(0,\left|X_{i}: \sum X_{i}\right|\right), \text { where } \Sigma=\operatorname{Diag}\left(\beta_{1}, \ldots, \beta_{7}\right) \tag{2.4}
\end{align*}
$$

In (2.3) heteroskedasticity is modelled in a multiplicative way. This specification has the advantage that the variance is guaranteed to be positive for all values of $\beta=\left(\beta_{1}, \ldots, \beta_{7}\right)^{\prime}$. Gabler et al. (1990) use (2.3) to test for the presence of heteroskedasticity in a binary choice model.

From an economic point of view, (2.4) is more attractive than (2.3), at least, if all components of $\beta=\left(\beta_{1}, \ldots, \beta_{7}\right)^{\prime}$ are nonnegative. In this case, the latent model can be interpreted as a random coefficient model, with diagonal covariance matrix of the vector of coefficients. (2.4) is used by Horowitz (1991) to allow for heteroskedasticity in a binary choice model.

Estimation results for the model with exponential heteroskedasticity (2.3) and the random coefficients model (2.4) are mentioned in the second and third panel of table 2.1. The signs of the estimates for the $\alpha_{i}^{\prime} s$ are the same as in the Tobit model. Differences in magnitude and significance level with the Tobit model estimates do not seem too large either. The estimated difference between the budget share spent on vacations by a family with total expenditures Df1 50,000 and Dfl 20,000 is 0.064 and 0.068 , for the exponential heteroskedasticity and the random coefficients model, respectively.

In the model with exponential heteroskedasticity, four out of six of the heteroskedasticity slope parameters (the $\beta_{j}^{\prime} s$ ) are significantly different from 0 on a $5 \%$ level. In the random coefficients model, all heteroskedasticity parameters are significantly different from 0 . Three of them are negative, contradicting the random coefficients interpretation. For two observations, the estimated value of $X_{i}^{\prime} \Sigma X_{i}$ is negative and taking the absolute value in (2.4) becomes necessary.

According to the log-likelihoods mentioned at the bottom of table 4.1, the Tobit model is strongly rejected as a special case of either one of the two more general models. In terms of likelihoods, the exponential heteroskedasticity model does slightly better that the random coefficients model.

Both heteroskedasticity and nonnormality can be incorporated in a parametric framework, using, for example, the following family of distributions for $\varepsilon_{i} \mid X_{i}$ :

$$
\begin{equation*}
P\left[\varepsilon_{i}<t \mid X_{i}, \beta\right]=F\left(t \mid X_{i}, \beta\right)=G\left(t / h\left(X_{i}, \beta\right)\right) \tag{2.5}
\end{equation*}
$$

with

$$
\begin{equation*}
G(s)=\Phi\left(\gamma_{0}+s+\gamma_{1} s^{2}+\gamma_{2} s^{3}\right) \tag{2.6}
\end{equation*}
$$

Here $h\left(X_{i}, \beta\right)$ is a twice differentiable nonnegative function. For the exponential case we have $h\left(X_{i}, \beta\right)=\exp \left(X_{i}^{\prime} \beta\right)$. The distribution function $G$ was proposed by Ruud (1984) as a family of probability distributions generalising the standard normal. $\Phi$ denotes the standard normal distribution function. If $\gamma=\left(\gamma_{0}, \gamma_{1}, \gamma_{2}\right)^{\prime}=0$ then the conditional distribution of $\varepsilon_{i}$ is normal with zero mean and standard deviation $h\left(X_{i}, \beta\right)$. In general, $\gamma$ must be such that the probability density function corresponding to $G$ can only take on nonnegative values. This density is given by

$$
\begin{equation*}
g(s)=\left(1+2 \gamma_{1} s+3 \gamma_{2} s^{2}\right) \varphi\left(\gamma_{0}+s+\gamma_{1} s^{2}+\gamma_{2} s^{3}\right) \tag{2.7}
\end{equation*}
$$

where $\varphi$ denotes the standard normal density function. Thus, $g(s) \geq 0$ for all $s \in R$, is equivalent to

$$
\begin{equation*}
\gamma_{2} \geq \gamma_{1}^{2} / 3 \tag{2.8}
\end{equation*}
$$

An identifying restriction on the conditional distribution of $\varepsilon_{i}$ (or $\left.\varepsilon_{i} / h\left(X_{i}, \beta\right)\right)$ is necessary, implying that $\gamma_{0}$ can be written as a function of $\gamma_{1}$ and $\gamma_{2}$. Two natural identifying restrictions can be imposed:
a. $F\left(0 \mid X_{i}, \beta\right)=1 / 2$ (zero conditional median) : $\gamma_{0}=0$.
b. $E\left(\varepsilon \mid X_{i}, \beta\right)=0$ (zero conditional mean): $\gamma_{0}=\psi\left(\gamma_{1}, \gamma_{2}\right)$, where $\psi$ is some intricate function with the properties

$$
\begin{equation*}
\psi(0,0)=0, \quad \frac{\partial \psi}{\partial \gamma_{1}}(0,0)=-1, \quad \frac{\partial \psi}{\partial \gamma_{2}}(0,0)=0 \tag{2.9}
\end{equation*}
$$

These conditions can easily be verified by substituting $\psi\left(\gamma_{1}, \gamma_{2}\right)$ for $\gamma_{0}$ and differentiating the equality $\int s \mathrm{~g}(\mathrm{~s}) \mathrm{ds}=0$ with respect to $\gamma_{1}$ and $\gamma_{2}$.

In the homoskedastic case, a. en b. are equivalent and necessary and sufficient to identify the constant term $\alpha_{1}$ in (2.1). In case of heteroskedasticity, the equivalence no longer holds. The log-likelihood contribution (conditional on $X_{i}$ ) of an observation ( $y_{i}, x_{i}$ ) is as follows.

If $y_{i}=0: \log F\left(-X_{i}^{\prime} \alpha \mid X_{i}, \beta\right)=\log G\left(s_{i}\right)$, with $s_{i}=-X_{i}^{\prime} \alpha / h\left(X_{i}, \beta\right)$.
If $y_{i}>0: \log f\left(y_{i}-X_{i}^{\prime} \alpha \mid X_{i}, \beta\right)=-\operatorname{logh}\left(X_{i}^{\prime} \beta\right)+\log g\left(s_{i}\right)$, with $s_{i}=\left(y_{i}-X_{i}^{\prime} \alpha\right) / h\left(X_{i}, \beta\right)$.
The Chesher and Irish (1987) test for nonnormality in the Tobit model can be interpreted as a Lagrange multiplier test on $\left(\gamma_{1}, \gamma_{2}\right)^{\prime}=(0,0)^{\prime}$ for the homoskedastic case of (2.5)-(2.6). To obtain a test for nonnormality which remains appropriate in case of a parametric form of heteroskedasticity (e.g. exponential heteroskedasticity), it is possible to estimate the model (2.5)(2.6) by maximum likelihood, without imposing homoskedasticity but imposing normality, and subsequently perform a Lagrange multiplier test on $\left(\gamma_{1}, \gamma_{2}\right)^{\prime}=(0,0)^{\prime}$. The test statistic can straightforwardly be derived and, as in Chesher and Irish (1987), be rewritten in terms of generalised standardised residuals

$$
e_{i}^{(k)}=E_{\gamma=0}\left\{\left(\varepsilon_{i} / h\left(X_{i}^{\prime} \beta\right)\right)^{k} \mid y_{i}, x_{i}, \beta\right\}-\mu(k), \text { where } \mu(k)=\Gamma(k) /\left\{2^{k / 2-1} \Gamma(k / 2)\right\} .
$$

The result depends on which restriction is imposed, zero conditional median (case a) or zero conditional mean (case b). In either case, the test statistic can be obtained as the explained sum of squares in a regression of a vector $(1, \ldots, 1)^{\prime} \in \mathbb{R}^{N}$ on the columns of an $N x m$ matrix, where $N$ is the number of observations and $m$ is 2 plus the number of free parameters in $\alpha$ and $\beta$. For the exponential heteroskedasticity case, the typical row entries in this matrix are

$$
\begin{aligned}
& e_{i}^{(1)} \frac{x_{j i}}{\exp \left(X_{i}^{\prime} \beta\right)}, \quad e_{i}^{(2)} x_{j i} \quad(j=1 \ldots, .7), \\
& \text { either }-e_{i}^{(3)}+2 e_{i}^{(1)} \text { (case a.) or }-e_{i}^{(3)}+3 e_{i}^{(1)} \text { (case b.). } \\
& \text { and } \quad-e_{i}^{(4)}+3 e_{i}^{(2)},
\end{aligned}
$$

where the unknown parameters are replaced by their ML estimates under the null $(\gamma=0)$. Under the null, the test statistic asymptotically follows a $x_{2}^{2}$ distribution.

In case of homoskedastic error terms (i.e., $\boldsymbol{\beta}_{\mathrm{j}}=0$ for $\mathrm{j}>1$ ) the two variants a. and $\underline{b}$. of the test statistic coincide and are identical to the standard test statistic in, for example, Chesher and Irish (1987) and Newey (1985). This test statistic can also be derived using White's information matrix comparison (cf. Chesher and Irish, 1987), starting from moment restrictions as in Newey (1985), or as a Lagrange multiplier test obtained by embedding the standard normal distribution in a Pearson family (cf. Bera et al., 1984).

In case of heteroskedastic error terms however, variants a. and b. are not identical, since the transformed covariates $X_{j i} / \exp \left(X_{i}^{\prime} \hat{\beta}\right)$ will generally not contain a constant term.

The empirical results of the test for the exponential heteroskedasticity case are given below. In order to take account of the inequality restriction (2.9), we also present the test results for one parameter restriction only. The conclusion is clear: normality, and in particular the restriction $\gamma_{1}=0$, are rejected at the usual significance levels. The difference in the outcomes of variants a. and b. is very small. Since the conditional distribution is symmetric if and only if $\gamma_{1}=0$, the results suggest that the error terms follow an asymmetric distribution. Note that for the third test $\gamma_{1}=0$ is a maintained hypothesis and the results for the second test strongly indicate that this assumption is violated, implying that the third test is not feasible.

Tests on normality assumptions in the exponential heteroskedasticity model

| $\mathrm{H}_{0}$ | degrees of <br> freedom | test statistic <br> test a. |  | test b. | critical value |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| level 0.05 | level 0.01 |  |  |  |  |  |

It is straightforward to estimate (2.5)-(2.6) allowing for exponential heteroskedasticity and without imposing normality, maximising the loglikelihood based on (2.10). Results are mentioned in the righthand panel of table 2.1. Restriction (2.8) was imposed and appeared to be binding.

According to the standard t-test, the restriction $\gamma_{1}=0$ is rejected at the usual significance levels. The same conclusion is obtained using a likelihood ratio test. Still, the estimates for both $\alpha$ and $\beta$ and their estimated standard errors are quite close to those in the second panel of the table.

The implied ceteris partbus pattern of expenditures on vacations as a function of total expenditures is quite similar to the pattern found before: the budget share spent on vacations is maximal if total expenditures are Dfl 73,000. The difference between the predicted shares if total expenditures are Df1 50,000 and Df1 20,000 is 0.061 .

To test whether the parametric models fit the data, without having in mind a specific parametric alternative, we performed several chi-squared goodness of fit tests developed by Andrews (1989). These tests are based upon classifying observations into cells and comparing estimated cell probabilities (conditional on the covariates) with empirical probabilities based upon the sample data. Andrews (1989) indicates how the cells can be chosen for the Tobit model and his strategy can straightforwardly be generalised to censored regression models with more general (parametric) specifications of the error distribution. We used a partitioning based upon the endogenous variable into five cells: $y_{i}=0$, and four cells with $y_{i}>0$, distinguished by the value of the transformed error term, where the transformation is such that the transformed error is standard normal. We also used products of these cells with a partitioning into four cells of the space of covariates, based upon the value of $X_{2 i}$ (total expenditures) only, yielding a partitioning into 20 cells.

In case of maximum likelihood estimation, the test statistics can easily be obtained as the explained sum of squares of a regression of a vector $(1, \ldots, 1)^{\prime} \in \mathbb{R}^{N}$ on the vectors of scores and the vectors of differences between predicted and sample probabilities for each of the cells. Under the null hypothesis of no misspecification, the test statistics follow a $x^{2}$ distribution, with 16 or 4 degrees of freedom, depending on whether 5 or 20 cells are used.

The two tests were performed for each of the four parametric specifications. In all cases, the null hypothesis of no misspecification was strongly rejected. The values of the test statistic for the 20 cells case varied from 84.1 (the normal exponential heteroskedasticity model) to 181.6 (the Tobit model). Allowing for heteroskedasticity reduces the chi-squared statistics and the exponential heteroskedasticity model does better than the random coefficients model in this respect. Allowing for the chosen
parametric form of nonnormality in the exponential heteroskedasticity model does not reduce the values of the chi-squared statistics.

## Semi-parametric models

In this subsection we consider semi-parametric estimation of the censored regression model. At present, a number of semi-parametric estimators for model (2.1) are available. The estimators vary with respect to the assumptions on the conditional distribution of $\varepsilon_{i}$ given $X_{i}$ necessary for consistency. For most of these estimators, these assumptions include independence between the error term and the covariates, i.e., the conditional distribution does not depend on $X_{i}$. Examples are the estimators given by Duncan (1986), Fernandez (1986), Horowitz (1986), and Ruud (1986). The empirical results of the previous subsection, however, strongly suggest the presence of heteroskedasticity, which the independence assumption does not allow for. Moreover, the significance of the nonnormality parameter $\gamma_{1}$ suggests that imposing symmetry of the error terms distribution should be avoided. This excludes the use of Powell's symmetrically trimmed least squares estimator (Powell, 1986).

The estimators we do apply are characterised by the weak identifying restriction that the conditonal distribution of $\varepsilon_{i}$ has zero median:

$$
\begin{equation*}
\operatorname{Med}\left(\varepsilon_{i} \mid X_{i}\right)=0 \tag{2.11}
\end{equation*}
$$

Two estimators thus are appropriate. The first one is Powell's Censored Least Absolute Deviations (CLAD-) estimator (Powell, 1984). The CLAD estimate for $\alpha$ solves the minimisation problem

$$
\begin{equation*}
\operatorname{Min}_{\alpha} S_{N}(\alpha) \text {, where } S_{N}(\alpha)=\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-\max \left\{0, X_{i} \alpha\right\}\right| \tag{2.12}
\end{equation*}
$$

This estimator is consistent and asymptotically normal if condition (2.11) together with some mild regularity conditions on the distribution of the covariates are satisfied. Powell (1984) also derives a consistent estimator for its asymptotic covariance matrix. The CLAD estimator is not efficient: its asymptotic covariance matrix does not attain the asymptotic efficiency bound. ${ }^{3)}$

The second estimator is proposed by Newey and Powell (1987). They show that, under mild regularity conditions, the efficient score corresponding to (2.11) satisfies

$$
\begin{equation*}
s\left(Z_{i}\right)=2 f\left(0 \mid x_{i}\right) I_{(0, \infty)}\left(-\alpha^{\prime} X_{i}\right) \operatorname{sgn}\left(y_{i}-\alpha^{\prime} X_{i}\right) \cdot X_{i} \tag{2.13}
\end{equation*}
$$

Here $Z_{i}=\left(y_{i}, X_{i}^{\prime}\right)^{\prime}, f\left(. \mid X_{i}\right)$ denotes the conditional density function of $\varepsilon_{i}$ given $X_{i}$ with respect to the Lebesgue measure on $R$, and $\operatorname{sgn}\left(\varepsilon_{i}\right)=1$ if $\varepsilon_{i}>0$, $\operatorname{sgn}\left(\varepsilon_{i}\right)=0$ if $\varepsilon_{i}=0$, and $\operatorname{sgn}\left(\varepsilon_{i}\right)=-1$ if $\varepsilon_{i}<0$.

Let

$$
\begin{aligned}
& N_{1}=\operatorname{Int}(N / 2), \text { the largest integer } \leq N / 2, N_{2}=N-N_{1}, \\
& I_{1}=\left\{1, \ldots, N_{1}\right\}, I_{2}=\left\{N_{1}+1, \ldots, N\right\}, \\
& \tilde{\alpha}_{1}, \tilde{\alpha}_{2}: \begin{array}{l}
\text { CLAD-estimators (or other feasible preliminary estimators) } \\
\text { based on subsamples } I_{1} \text { and } I_{2}, \text { respectively, }
\end{array}
\end{aligned}
$$

$\hat{s}_{1}(Z), \hat{s}_{2}(Z)$ : estimators of the efficient score, based on $I_{1}$ and $I_{2}$,

$$
v_{1}^{*}=\left[\sum_{i \in I_{2}} \hat{s}_{1}\left(Z_{i}\right) \hat{s}_{1}\left(Z_{i}\right)^{\prime} / N_{2}\right]^{-1}, v_{2}^{*}=\left[\Sigma_{i \in I_{1}} \hat{s}_{2}\left(Z_{i}\right) \hat{s}_{2}\left(Z_{i}\right)^{\prime} / N_{1}\right]^{-1}
$$

Then the estimator $\hat{\alpha}$ proposed by Newey and Powell is given by

$$
\begin{equation*}
\hat{\alpha}=\frac{1}{2}\left(\tilde{\alpha}_{1}+\tilde{\alpha}_{2}\right)+\frac{1}{N}\left[V_{2}^{*} \Sigma_{i \in I_{1}} \hat{s}_{2}\left(Z_{i}\right)+V_{1}^{*} \Sigma_{i \in I_{2}} \hat{s}_{1}\left(Z_{i}\right)\right] \tag{2.14}
\end{equation*}
$$

This estimator is based on the efficient score. Newey and Powell prove that, under some regularity conditions, it is semi-parametrically efficient, and derive a consistent estimator for its asymptotic covariance matrix.

CLAD-estimates require minimisation of the non-differentiable expression in (2.12). For this purpose, we used the simplex algorithm introduced by Nelder and Mead (1965) and extended by O'Neill (1971). Estimation results are mentioned in table 2.2. Estimating the covariance matrix of the CLAD estimator involves the choice of smoothness parameters $c_{0}$ and $\gamma$, cf. Powell (1984). Following Powell (1984) we chose $\gamma=0.2$ and we tried various values of $c_{0}$. Reported smoothing parameters minimised the covariance matrix. Thus the resulting standard errors might be interpreted as (estimated) lower bounds for the true standard errors. In case of the

Newey-Powell estimator, the choice of the smoothness parameters not only affects the covariance matrix but also the estimates themselves. We tried different values of the smoothing parameters. The choice hardly affected the parameter estimates, but had some impact on the estimated covariance matrix. The reported smoothing parameters yield standard errors which are smaller than the corresponding CLAD-standard errors.

According to the point estimates, the budget share spent on vacations is maximal if total expenditures are Df1 57,000 (CLAD) or 79,000 (NeweyPowell). The estimated difference between the shares for families with total expenditures Df1 50,000 and Df1 20,000 is 0.066 (CLAD) or 0.047 (NeweyPowell). These numbers do not seem to be too much out of line with those based upon the parametric estimates. This is also the case for the estimates of the slope parameters corresponding to family size, age, education, and degree of urbanisation.

Table 2.2. Estimation results semi-parametric model
(standard errors in parentheses)

|  | I |  | II |  |
| :--- | ---: | ---: | ---: | ---: |
| $\alpha_{1}$ | -731.30 | $(180.65)$ | -350.72 | $(175.75)$ |
| $\alpha_{2}$ | 133.99 | $(33.68)$ | 62.92 | $(33.05)$ |
| $\alpha_{3}$ | -0.23 | $(0.90)$ | -1.03 | $(0.44)$ |
| $\alpha_{4}$ | 0.14 | $(0.10)$ | 0.13 | $(0.06)$ |
| $\alpha_{5}$ | 0.14 | $(0.18)$ | 0.01 | $(0.13)$ |
| $\alpha_{6}$ | 0.49 | $(0.11)$ | 0.31 | $(0.08)$ |
| $\alpha_{7}$ | -6.12 | $(1.57)$ | -2.79 | $(1.55)$ |

## Explanation

The model assumptions are given by (2.1) and (2.11). The regressors are described at the beginning of this section.
I: CLAD estimator with smoothness parameters $c_{0}=0.73875$ and $\gamma=0.20$.
II: Newey-Powell estimator with smoothness parameters $c=0.7$ and $k=40$.

The estimated standard errors for the CLAD-estimator are larger than those for the Newey-Powell estimator. Still, the estimated difference between the two covariance matrices fails to be positive definite. As a consequence, a Hausman type specification test could not be performed. This may be a consequence of the choice of smoothness parameters.

In order to compare the results of parametric and semi-parametric model estimates, we used the estimates to compute confidence intervals for elasticities of vacation expenditures with respect to total expenditures (TE). Results are given in table 2.3. Two elasticities were considered: the elasticity of $y_{i}^{*} \mathrm{TE}_{i}$ with respect to $\mathrm{TE}_{i}$ for the average family (EL1), and the elasticity of the average value of $y_{i}^{*} \mathrm{TE}_{i}$ with respect to $\mathrm{TE}_{i}$ with the same percentage for all families (EL2). The censoring was not taken into account, because computation of elasticities of $y_{i} \mathrm{TE}_{i}$ would require a complete specification of the error distribution, which is not provided by the semi-parametric model.

The elasticities are calculated 500 times, for 500 independent draws of the parameter values from the estimated asymptotic distribution of the estimator of the parameter vector. For each case, we present the mean and the median elasticity, and the 0.05 and 0.95 quantiles. The latter two can be interpreted as the bounds of a two-sided $90 \%$ confidence interval.

The difference between elasticities based on CLAD and Newey-Powell estimates are relatively large, although these estimators are consistent under the same assumptions. In particular, the elasticities according to Newey-Powell estimates are small compared to the others. This estimator also yields the smallest confidence intervals. All confidence intervals overlap and in each case the elasticity is significantly larger than one on a $5 \%$ level.

Table 2.3: Elasticities

|  | EL1 | EL2 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mean | median | Q05 | Q95 | mean | median | Q05 | Q95 |
| Tobit model | 3.00 | 3.13 | 2.05 | 4.72 | 2.67 | 2.78 | 1.95 | 3.84 |
| Expon. Heterosk. | 2.95 | 3.10 | 2.05 | 4.64 | 3.75 | 4.11 | 2.40 | 7.03 |
| Exp. Het. \& Nonnorm. | 2.82 | 2.89 | 2.14 | 3.98 | 3.52 | 3.68 | 2.15 | 5.47 |
| CLAD | 2.53 | 2.54 | 1.74 | 3.44 | 3.45 | 3.62 | 2.00 | 5.85 |
| Newey-Powell | 1.70 | 1.70 | 1.17 | 2.25 | 1.89 | 1.95 | 1.05 | 2.81 |

## 3. A Two Equations Model

In the previous section, a single latent variable equation was used to model both the choice between zero and a positive budget share, and the level of the budget share. In this section we consider a model in which these two decisions are separated:

$$
\begin{align*}
& P\left(a_{i}=1 \mid X_{i}\right)=F\left(X_{i}^{\prime} \alpha_{a}\right) ; P\left(a_{i}=0 \mid X_{i}\right)=1-F\left(X_{i}^{\prime} \alpha_{a}\right)  \tag{3.1}\\
& y_{i}^{*}=X_{i}^{\prime} \alpha_{b}+\varepsilon_{b i}  \tag{3.2}\\
& y_{i}=0 \text { if } a_{i}<0 ; y_{i}=y_{i}^{*} \text { if } a_{i}>0 . \tag{3.3}
\end{align*}
$$

Here $F$ is an unknown function (not necessarily a distribution function) with range contained in [0,1]. (3.1) is a single index binary choice equation for the decision whether or not to go on holiday. $X_{i}$ only enters (3.1) through the 'single index' $X_{i}^{\prime} \alpha_{a}$. In order to identify $\alpha_{a}$, some normalisation must be imposed. If it is assumed that $F(z)=\Phi\left(z / \sigma_{a}\right)$, where $\Phi$ denotes the standard normal distribution function, (3.1) becomes the familiar Probit model.

The regression equation (3.2) explains the budget share spent on vacation, conditional upon the decision to go on holiday. We cannot think of economic arguments for excluding regressors from either (3.1) or (3.2). Therefore, in principle, (3.2) and (3.3) may include the same regressors, although in the final empirical specifications, the two sets of regressors will include different cross products. Instead of imposing exclusion restrictions on the regressors in (3.2), we make the following identifying assumption:

$$
\begin{equation*}
E\left(\varepsilon_{b i} \mid x_{i}, a_{i}=1\right)=0 \tag{3.4}
\end{equation*}
$$

This assumption makes it possible to estimate (3.2) separately from (3.1), using only those observations with $y_{i}>0$. 4)

In the remainder of this section, we discuss and compare parametric and semi-parametric estimation results of (3.1) and (3.2) separately. Finally, the estimation results of both equations are used to perform some simulations.

## The Binary Choice equation

The endogenous variable $a_{i}$ in (3.1) represents the decision to go on holiday $\left(a_{i}=1\right)$ or $\operatorname{not}\left(a_{i}=0\right)$. We first consider the Probit model, i.e.

$$
\begin{equation*}
F(z)=\Phi\left(z / \sigma_{a}\right), \tag{3.5}
\end{equation*}
$$

for some $\sigma_{a}>0$. In the vector of covariates $X_{i}$ we included $X_{1 i}, \ldots, X_{6 i}$, described at the beginning of section 2 . On the basis of preliminary
estimation results with squares and cross products of the regressors $X_{1 i}, \ldots, x_{6 i}$, we also included the cross term $X_{4 i} X_{5 i}$, whereas there appeared to be no reason to include $x_{2 i}^{2}$, which was included in the model in the previous section.

The maximum likelihood estimates of the Probit model are presented in the left panel of table 3.1. In order to be able to compare the Probit estimation results with the semi-parametric estimation results discussed below, we have normalised the coefficient $\alpha_{a 2}$, corresponding to log total expenditures, to be 1 . According to the Probit estimates, the probability of going on vacation is an increasirg function of family size and degree of urbanisation, although family size has an insignificant influence. The cross term between age class and education level implies that a ceterts paribus increase in age has a positive effect on the probability of going on holiday for small education levels and a negative effect for high levels of education. A similar result holds for a ceteris paribus change in the education level.

Since the Probit estimator for $\alpha_{a}$ may be inconsistent if the underlying distributional assumptions are incorrect, we performed various specification tests. Hardly any of these tests rejected the null hypothesis of no misspecification. For instance, neither normality nor homoskedasticity was rejected using the score tests given by Chesher and Irish (1987). In addition, most of the Andrew's chi-squared tests we performed, did not result in rejection of the Probit model. There was only one exception: Partitioning the support of the explanatory variable 'family size' into two classes, family size less than or equal to three, and family size larger than three, the Andrew's chi-square test statistic was 8.04 , leading to rejection of the null hypothesis of no misspecification at the $5 \%$ level ( $8.04>5.99=x_{2 ; 0.05}^{2}$ ).

The problem with the chi-squared diagnostic tests is that the results may strongly depend on the arbitrary choice of the partitions. We, therefore, also tested the Probit specification using a test proposed by Horowitz (1991). This test consists of performing a non-parametric regression of $a_{i}$ on $X_{i}^{\prime} \hat{\alpha}_{a} / \hat{\sigma}_{a}$, and computing a uniform confidence band for the regression function. If the standard normal distribution function $\Phi$ lies within this band, the null-hypothesis that the Probit specification is correct is accepted. For technical details, see proposition 1 of Horowitz (1991). The outcome of the test is presented in Figure 3.1, which includes a plot of $\Phi$, the uniform $95 \%$ confidence band, and the values of the kernel regression ${ }^{5}$ ) of $a_{i}$ on $X_{i}^{\prime} \hat{\alpha}_{a} / \hat{\sigma}_{a}$, evaluated in the points $X_{a i}^{\prime} \hat{\alpha}_{a} / \hat{\sigma}_{a}$,

Table 3.1 Estimation results of the binary choice model (Standard errors between parentheses)

| parameter | I |  | II |  |
| :--- | ---: | :--- | :--- | :--- |
| $\alpha_{a 1}$ (constant term) | -10.896 | $(0.227)$ | 0. |  |
| $\alpha_{a 2}$ (log total expenditures) | 1. |  | 1. |  |
| $\alpha_{a 3}$ (log family size) | 0.050 | $(0.083)$ | 0.052 | $(0.067)$ |
| $\alpha_{a 4}$ (age class) | 0.060 | $(0.019)$ | 0.093 | $(0.012)$ |
| $\alpha_{a 5}$ (education level) | 0.180 | $(0.063)$ | 0.236 | (0.043) |
| $\alpha_{a 6}$ (degree of urbanisation) | 0.045 | $(0.015)$ | 0.051 | $(0.012)$ |
| $\alpha_{a 7}\left(\mathrm{X}_{4 i} \mathrm{X}_{5 i}\right)$ | -0.017 | $(0.007)$ | -0.023 | $(0.005)$ |
| $\sigma_{a}^{2}$ | 0.606 | $(0.104)$ |  |  |
| log L | -1082.3 |  | -1090.4 |  |

## Explanation:

The first six regressors are the same as those in section 2; $X_{7 i}=X_{4 i} X_{5 i}$.
I : ML estimates Probit model (3.1), (3.5) (normalisation: $\alpha_{a 2}=1$ );
II : Klein-Spady estimates single index model (3.1) (normalisation: $\alpha_{a 1}=0$, $\alpha_{a 2}=1$; smoothing parameter $h_{N}=0.2$ );
$\log \mathrm{L}: \log$-likelihood value in case of Probit; quasi-log-likelihood value (without trimming) in case of the Klein-Spady estimator.

$i=1, \ldots, N$. Since the normal cumulative distribution function lies entirely within the uniform $95 \%$ confidence band, the Probit specification is accepted at the 5\% level.

A semi-parametric estimator for the single-index model (3.1) is derived by Klein and Spady (1989) . $\alpha_{a}$ can be estimated by maximising the (estimated) quasi-log-likelihood, ${ }^{6)}$ given by

$$
\begin{equation*}
Q\left(\alpha_{a} ; F_{N}\right)=\frac{1}{N} \sum_{i=1}^{N}\left\{a_{i} \log \left[F_{N}\left(\alpha_{a}^{\prime} X_{i}\right)\right]+\left(1-a_{i}\right) \log \left[1-F_{N}\left(\alpha_{a}^{\prime} X_{i}\right)\right]\right\} \tag{3.6}
\end{equation*}
$$

where $F_{N}$ represents a nonparametric estimate of $F$, specified as follows: Let $P_{N}$ denote the sample frequency estimating the unconditional probability that $a_{i}=1$, i.e.,

$$
\begin{equation*}
P_{N}=N^{-1} \Sigma_{i=1}^{N} a_{i} \tag{3.7}
\end{equation*}
$$

Next, let $g_{N}\left(\cdot \mid a_{i}\right)$ denote the density estimates for $X_{i}^{\prime} \alpha_{a}$ conditional on $a_{i}$ (for $a_{i}=1$ and $a_{i}=0$ ), given by

$$
\begin{align*}
& g_{N}\left(z \mid a_{i}=1\right)=\sum_{j=1}^{N} a_{j} K\left(\left(z-X_{j}^{\prime} \alpha_{a}\right) / h_{N}\right) /\left(h_{N} N P_{N}\right),  \tag{3.8}\\
& g_{N}\left(z \mid a_{i}=0\right)=\sum_{j=1}^{N}\left(1-a_{j}\right) \cdot K\left(\left(z-X_{j}^{\prime} \alpha_{a}\right) / h_{N}\right) /\left(h_{N} N\left(1-P_{N}\right)\right), \tag{3.9}
\end{align*}
$$

with $K$ a Kernel function, and $\left\{h_{N}\right\}$ a sequence of bandwidths which have to satisfy $N h_{N}^{6} \rightarrow \infty$ and $N h_{N}^{8} \rightarrow 0$, for $N \rightarrow \infty$. Then the estimate $F_{N}$, evaluated at $z$ $\in \mathbb{R}$, is given by

$$
\begin{equation*}
F_{N}(z)=P_{N} g_{N}\left(z \mid a_{i}=1\right) /\left[P_{N} g_{N}\left(z \mid a_{i}=1\right)+\left(1-P_{N}\right) g_{N}\left(z \mid a_{i}=0\right)\right] \tag{3.10}
\end{equation*}
$$

We made use of the kernel given by

$$
\begin{equation*}
K(z)=\left(3 / 2-(1 / 2) z^{2}\right) \varphi(z) \tag{3.11}
\end{equation*}
$$

with $\varphi$ the standard normal density function. ${ }^{71}$
Klein and Spady show that, under some regularity conditions, the estimator $\hat{\alpha}_{a}$ of $\alpha_{a}$ obtained by maximising (3.6), satisfies

$$
\begin{equation*}
\sqrt{ } N\left(\hat{\alpha}_{a}-\alpha_{a}\right) \rightarrow_{d} N\left(0, V_{a}\right) \tag{3.12}
\end{equation*}
$$

where $V_{a}$ is given by

$$
\begin{equation*}
v_{a}=\left[E\left(\partial P\left(\alpha_{a}\right) / \partial a_{a}\right)\left(\partial P\left(\alpha_{a}\right) / \partial a_{a}\right)^{\prime}\left\{1 /\left[P\left(\alpha_{a}\right)\left(1-P\left(\alpha_{a}\right)\right)\right]\right\}\right]^{-1} \text {. } \tag{3.13}
\end{equation*}
$$

with $P\left(\alpha_{a}\right)$ a shorthand notation for $F\left(X_{i}^{\prime} \alpha_{a}\right)$. The covariance matrix $V_{a}$ can be estimated consistently by using its sample analogue.

In order to guarantee identification of $\alpha_{a}$ in the single index model, one has to impose some normalisation condition. First of all, the coefficient corresponding to the constant term is set equal to zero, since the constant is absorbed in the function $F$. In addition, we have fixed the coefficient corresponding to the variable $X_{2 i}$, i.e., log expenditures, to be equal to one. The choice of this normalisation is based upon the fact that log expenditures is the only explanatory variable which can reasonably be assumed to be continuously distributed.

The estimation results for the regression coefficients of the KleinSpady estimator are presented in the right panel of Table 3.1. The function $\mathrm{F}_{\mathrm{N}}$, the estimate of F given by (3.10), is presented in Figure 3.2, together with the function $G(z)=\Phi((z-10.90) / 0.77)$, the corresponding function in case of Probit. The results are obtained using the value 0.2 for the smoothing parameter $\mathrm{h}_{\mathrm{N}}$ in (3.8) and (3.9) ${ }^{8}$ ).

The Klein-Spady estimates of the regression coefficients are quite close to those obtained using Probit: The mean function is an increasing function of family size and the degree of urbanisation; it is increasing and decreasing as a function of age class for small and large values of the education level, respectively. According to figure 3.2, the Klein-Spady estimate $\mathrm{F}_{\mathrm{N}}$ differs somewhat from the corresponding function in case of Probit. $\mathrm{F}_{\mathrm{N}}$ is initially steeper and decreases if the value of the mean function becomes high. Relatively large differences between the two functions only occur in the region where observations are sparse. The conclusions from this figure are thus well in line with those from figure 3.1.

In order to get some more insight in the performance of the Klein-Spady estimation results, we applied an informal graphical test, suggested by Horowitz (1991). This test consists of assigning observations to cells, according to the predicted probability of going on vacation. This predicted probability is given by $F_{N}\left(X_{i}^{\prime} \hat{\alpha}_{a}\right)$, with $\hat{\alpha}_{a}$ the Klein-Spady estimates of $\alpha_{a}$ and with $\mathrm{F}_{\mathrm{N}}$ given by (3.10). We used the cells [0;0.3), [0.3;0.4), $[0.4 ; 0.5), \ldots,[0.8 ; 0.9),[0.9 ; 1.0]$. The first cell is chosen larger than the others in order to obtain a comparable number of observations per cell.

## Figure 3.2: Estimated distribution functions



For each cell, one can compute the quantities

$$
\begin{align*}
& Q_{\text {pred }}=\sum_{i E_{c e l l}} F_{N}\left(X_{i}^{\prime} \hat{\alpha}_{a}\right),  \tag{3.14}\\
& Q_{\text {obs }}=\Sigma_{i E_{c e l l}} a_{i} / N_{c e l l}, \tag{3.15}
\end{align*}
$$

with $N_{\text {cell }}$ the number of observations in the cell. If the single-index model is correctly specified, $Q_{\text {obs }}$ and $Q_{p r e d}$ will be close. The same test can also be applied to the Probit model.

Figures 3.3 and 3.4 represent graphs of $Q_{o b s}$ against $Q_{\text {pred }}$ for the Probit and the Klein-Spady specification, respectively. In both cases, the outcomes are quite close to the $45^{\circ}$-line, suggesting that the model fits the data well.

In conclusion, formal and informal tests suggest that the Probit specification works quite well for the data at hand. Not surprisingly, the single index model, of which Probit is a special case, yields quite similar results and also works quite well.

Figure 3.3: Obs. and pred. probabilities for Probit model


Figure 3.4: Obs. and pred. probabilities for Kl ./Sp.-model


The Regression part of the model

Once the decision to go on holiday has been made, one has to decide how much to spend on it. We model this decision by means of the following conditional regression equation for those families with $y_{i}>0$ :

$$
\begin{equation*}
E\left(y_{i} \mid a_{i}=1, x_{i}\right)=X_{i}^{\prime} \alpha_{b} . \tag{3.16}
\end{equation*}
$$

In terms of the error term $\varepsilon_{b i}=y_{i}-X_{i}^{\prime} \alpha_{b},(3.16)$ can also be written as

$$
\begin{equation*}
y_{i}=X_{i}^{\prime} \alpha_{b}+\varepsilon_{b i} ; E\left(\varepsilon_{b i} \mid a_{i}=1, X_{i}\right)=0 \tag{3.17}
\end{equation*}
$$

As in section 2, $y_{i}$ is the budget share of vacation expenditures (in \%). In the vector of covariates $X_{i}$ we included $X_{1 i}, \ldots, X_{6 i}$, described in section 2 . On the basis of preliminary estimation results with squares and cross products of the regressors $X_{1 i}, \ldots, X_{6 i}$, we also included the cross product $X_{7 i}=X_{2 i} X_{6 i}$.

The parameter $\alpha_{b}$ can simply be estimated by Ordinary Least Squares of $y_{i}$ on $X_{i}$, using the subsample for which $y_{i}>0$ only. The estimation results are presented in the left panels of table 3.2. The first set of standard errors is computed in the standard way, assuming the error terms to be homoskedastic and independent of the covariates. The second set of standard errors is based upon an estimator of the covariance matrix of the OLSestimator which, under weak regularity conditions, remains consistent in case of heteroskedasticity. This estimator will be discussed below (equation (3.27)). We also present the estimation results using the mean function without cross term. According to the OLS-estimation results without cross term, only the variable log family size has a negative influence on the budget share. The other variables have a positive influence, although the coefficients corresponding to $10 g$ expenditures and education level are insignificant. The results with the cross term $X_{2 i} X_{61}$ included imply that the ceteris paribus effect of log expenditures is large and positive for people living in the country, but small or even negative for people living in cities.

The OLS estimator is asymptotically efficient if the errors are independent of the covariates and normally distributed. In order to test these distributional assumptions, we performed various specification tests. Using the same score test as in section 2 , normality was strongly rejected.

To test against the alternative of heteroskedastic error terms, we used the same two parametric forms of heteroskedasticity as in section 2 and again applied similar score tests. In each case, the hypothesis of homoskedastic error terms was strongly rejected.

The outcomes of these tests suggest that either the normality or the homoskedasticity assumption or both do not hold. In this case, given (3.16), the OLS estimator is consistent, but not asymptotically efficient. A semiparametric asymptotically efficient estimator for $\alpha_{b}$ is presented by Robinson (1987). It is given by

$$
\begin{equation*}
\hat{\alpha}_{b}=\left[\sum_{i=1}^{N} X_{b i} X_{b i}^{\prime} \hat{\sigma}_{i}^{-2}\right]^{-1}\left[\sum_{i=1}^{N} X_{b i} y_{i} \hat{\sigma}_{i}^{-2}\right], \tag{3.18}
\end{equation*}
$$

where now $N=1143$, the number of observations used in the regression, and $\hat{\sigma}_{i}^{2}$ is the so-called uniform $k-N N$ estimator of $\sigma_{i}^{2}$, given by

$$
\begin{equation*}
\hat{\sigma}_{i}^{2}=\sum_{j=1}^{N} \hat{\varepsilon}_{b j}^{2} W_{i j} \tag{3.19}
\end{equation*}
$$

Here $\hat{\varepsilon}_{b i}$ is the OLS-residual, and where $W_{i j}$ are weights, given by (if there are no ties):

$$
\begin{align*}
& W_{i j}=k_{N}^{-1} \text {, if } x_{j} \text { is one of the } k_{N} \text { nearest neighbors of } x_{i}, \\
& W_{i j}=0, \quad \text { otherwise. } \tag{3.20}
\end{align*}
$$

The distance between $X_{j}$ and $X_{i}$ is deterained on the basis of the distance function $P$, given by

$$
\begin{equation*}
p\left(X_{i}, X_{j}\right)=\sum_{\ell=2}^{L}\left(X_{\ell i}-X_{\ell j}\right)^{2} / s_{\ell} \tag{3.21}
\end{equation*}
$$

where $s_{\ell}$ is the (univariate) sample variance of $X_{\ell_{i}}$, which is included to correct for differences in measurement units of the $X_{\ell_{i}}{ }^{\prime} s$, and where $L$ denotes the number of covariates.

Table 3.2 Estimation results of the regression model

| Parameter | I |  | II |  | III | IV | v |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{b 1}$ (constant) | $\begin{aligned} & -2.16 \\ & (4.34) \end{aligned}$ | [4.37] | $\begin{aligned} & -22.00 \\ & (10.04) \end{aligned}$ | [9.21] | $\begin{aligned} & -3.78 \\ & (4.01) \end{aligned}$ | $\begin{gathered} -18.67 \\ (8.54) \end{gathered}$ | $\begin{gathered} -6.39 \\ (18.29) \end{gathered}$ |
| $\alpha_{b 2}\left(\log\right.$ total $\left.\exp ^{\prime} \mathrm{s}\right)$ | $\begin{gathered} 0.67 \\ (0.44) \end{gathered}$ | [0.44] | $\begin{gathered} 2.57 \\ (0.97) \end{gathered}$ | [0.09] | $\begin{gathered} 0.87 \\ (0.40) \end{gathered}$ | $\begin{gathered} 2.28 \\ (0.83) \end{gathered}$ | $\begin{gathered} 1.06 \\ (1.56) \end{gathered}$ |
| $\alpha_{b 3}(\log$ family size) | $\begin{aligned} & -1.42 \\ & (0.42) \end{aligned}$ | $[0.40]$ | $\begin{aligned} & -1.46 \\ & (0.42) \end{aligned}$ | [0.39] | $\begin{aligned} & -1.60 \\ & (0.38) \end{aligned}$ | $\begin{aligned} & -1.56 \\ & (0.37) \end{aligned}$ | $\begin{aligned} & -1.47 \\ & (0.39) \end{aligned}$ |
| $\alpha_{b 4}$ (age class) | $\begin{gathered} 0.16 \\ (0.05) \end{gathered}$ | [0.05] | $\begin{gathered} 0.15 \\ (0.05) \end{gathered}$ | [0.05] | $\begin{gathered} 0.15 \\ (0.05) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.16 \\ (0.05) \end{gathered}$ |
| $\alpha_{b 5}$ (education) | $\begin{aligned} & 0.005 \\ & (0.13) \end{aligned}$ | [0.12] | $\begin{array}{r} 0.002 \\ (0.13) \end{array}$ | [0.12] | $\begin{aligned} & -0.007 \\ & (0.11) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.11) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.15) \end{gathered}$ |
| $\alpha_{b 6}$ (deg. of urban.) | $\begin{gathered} 0.45 \\ (0.08) \end{gathered}$ | [0.07] | $\begin{gathered} 5.37 \\ (2.25) \end{gathered}$ | $[2.28]$ | $\begin{gathered} 0.40 \\ (0.07) \end{gathered}$ | $\begin{gathered} 4.28 \\ (2.12) \end{gathered}$ | $\begin{gathered} 0.40 \\ (0.10) \end{gathered}$ |
| $\alpha_{b 7}\left(\mathrm{X}_{2 i} \mathrm{X}_{6 i}\right)$ | - |  | $\begin{aligned} & -0.47 \\ & (0.22) \end{aligned}$ | $[0.22]$ | - | $\begin{aligned} & -0.37 \\ & (0.20) \end{aligned}$ | - |
| $\lambda$ | - |  | - |  | - | - | $\begin{gathered} 0.48 \\ (2.39) \end{gathered}$ |

Explanation:
The model specification is given in (3.16). The first six regressors are the same as in section $2 ; X_{7 i}=X_{2 i} X_{6 i}$; $\lambda$ is the coefficient corresponding to the inverse of Mill's ratio? ${ }^{i}$
I, II: Ordinary Least Squares estimates;
(.): standard errors, computed in the standard way,
[.]: heteroskedasticity corrected standard errors, based upon (3.27). III, IV: Robinson-estimates; smoothing parameter $\mathrm{k}_{\mathrm{N}}=150$,
(.): standard errors, computed using (3.24) ${ }^{\text {N }}$

V : Robinson-estimates with the inverse of Mill's ratio included (without cross term),
(.): standard errors, computed using (3.24).

It is shown by Robinson that under some regularity conditions the limiting distribution of the estimator $\hat{\alpha}_{b}$ satisfies

$$
\begin{equation*}
\sqrt{N}\left(\dot{\alpha}_{b}-\alpha_{b}\right) \rightarrow_{d} N\left(0, v_{b}\right), \tag{3.22}
\end{equation*}
$$

where $V_{b}$ is given by

$$
\begin{equation*}
V_{b}=\left[E\left(X_{i} X_{i}^{\prime} \sigma^{-2}\left(X_{i}\right)\right)\right]^{-1} . \tag{3.23}
\end{equation*}
$$

Here $\sigma^{2}\left(X_{i}\right)=E\left(\varepsilon_{b i}^{2} \mid X_{i}\right) \cdot V_{b}$ can be consistently estimated by

$$
\begin{equation*}
\hat{V}_{b}=\left[N^{-1} \Sigma_{i=1}^{N} X_{i} X_{i}^{\prime} \hat{\sigma}_{i}^{-2}\right]^{-1} \tag{3.24}
\end{equation*}
$$

Estimation results using the Robinson estimator are mentioned in the third and fourth panel of table 3.2. The smoothing parameter $k_{N}$ (c.f. equation (3.20)), is set equal to 150. Changing $k_{N}$ hardly affected the outcomes. Because the coefficient of the cross term turned out to be insignificant on the $5 \%$ level, we also estimated the coefficients of the mean function without the cross term. ${ }^{9)}$ The resulting estimates are quite close to those obtained using OLS. The coefficient of the education level changes sign but remains quite insignificant. Estimated standard errors of the Robinson-estimator are smaller than (heteroskedasticity corrected) OLS standard errors. Surprisingly, however, the differences are quite small.

Comparing the asymptotically efficient Robinson estimates and the consistent OLS-estimates, a Hausman specification test for (3.16) can be performed. Under the assumptions given by Robinson (1987), it follows that the OLS-estimator of the regression coefficients, $\hat{\alpha}_{b, O L S}$, satisfies,

$$
\begin{equation*}
\sqrt{N}\left(\dot{\alpha}_{b}, \mathrm{OLS}^{-\alpha}\right) \rightarrow_{\mathrm{d}} \mathrm{~N}\left(0, \mathrm{v}_{\mathrm{b}, \mathrm{OLS}}\right), \tag{3.25}
\end{equation*}
$$

with

$$
\begin{equation*}
V_{b, O L S}=\left[E X_{i} X_{i}^{\prime}\right]^{-1}\left[E X_{i} X_{i}^{\prime} \sigma^{2}\left(X_{i}\right)\right]\left[E X_{i} X_{i}^{\prime}\right]^{-1} \tag{3.26}
\end{equation*}
$$

Using the proofs of Robinson, it is easy to show that a consistent estimate of $\mathrm{V}_{\mathrm{b}, \text { OLS }}$ is given by

$$
\begin{equation*}
\hat{V}_{b, 0 L S}=\left[N^{-1} \sum_{i=1}^{N} X_{i} X_{i}^{\prime}\right]^{-1}\left[N^{-1} \sum_{i=1}^{N} X_{i} x_{i}^{\prime} \hat{\sigma}_{i}^{2}\right]\left[N^{-1} \sum_{i=1}^{N} x_{i} x_{i}^{\prime}\right]^{-1} \tag{3.27}
\end{equation*}
$$

where $\hat{\sigma}_{i}^{2}$ is the estimator of $\sigma^{2}\left(X_{i}\right)$ given in (3.19). The Hausman test statistic can thus easily be calculated. The outcome (in the case without cross term) is 7.1 , which is less than $x_{6}^{2} ; 0.05=12.6$. Thus, on the basis of the Hausman test we cannot reject specification (3.16). 10)

Finally, we tested the assumption that the error term $\varepsilon_{b i}$ of the regression equation (3.17) is independent of the error term occurring in the latent equation underlying the binary choice equation, generalising the Heckman (1979) test for selectivity bias. If the function $F$ in (3.1) is a distribution function, then (3.1) can be rewritten as

$$
\begin{equation*}
a_{i}^{*}=X_{i}^{\prime} \alpha_{a}+\varepsilon_{a i}, a_{i}=0 \text { if } a_{i}^{*} \leq 0, a_{i}=1 \text { if } a_{i}^{*}>0 \tag{3.28}
\end{equation*}
$$

where the error term $\varepsilon$ ai has distribution function $F$. The regression model (3.17) can then be embedded in the following, more general, setting, which allows for dependence between $\epsilon_{\text {ai }}$ and $\varepsilon_{b i}$,

$$
\begin{equation*}
E\left(\varepsilon_{b i} \mid X_{i}, \varepsilon_{a i}\right)=\lambda \varepsilon_{a i} . \tag{3.29}
\end{equation*}
$$

Notice that (3.29) is the usual assumption imposed in the standard Heckman two-step procedure. Obviously, (3.29) implies

$$
\begin{equation*}
E\left(\varepsilon_{b i} \mid x_{i}, a_{i}=1\right)=\lambda E\left(\varepsilon_{a i}\left|\varepsilon_{a i}\right\rangle-x_{i}^{\prime} \alpha_{a}\right) . \tag{3.30}
\end{equation*}
$$

In case of Probit, the expectation on the righthand side is the inverse of Mill's ratio. Since Probit appears to describe the binary choice problem quite well, the hypothesis $H_{0}: \lambda=0$, which corresponds to model (3.17), can easily be tested by including the inverse of Mill's ratio (with $\alpha_{a}$ replaced by its Probit ML estimate) as an extra regressor in the regression equation. We estimated the resulting regression equation (without cross-term) using the Robinson-estimator. The estimates are presented in the righthand panel of Table 3.2. It follows that the null hypothesis $\lambda=0$ is not rejected.

## Simulations

Following Van Soest and Kooreman (1987), we performed various simulations on the same sample of 1815 households used for estimation and testing. According to the two-equation model, the expected value of $y_{b i}$, the budget share spent on vacation expenditures, conditional on the covariates, is given by,

$$
\begin{equation*}
E\left(y_{i} \mid x_{i}\right)=E\left(y_{i} \mid a_{i}=1, x_{i}\right) P\left(a_{i}=1 \mid x_{i}\right) \tag{3.30}
\end{equation*}
$$

$P\left(a_{i}=1 \mid X_{i}\right)$ can be estimated using the Probit estimates or the Klein-Spady estimates of $\alpha_{a}$, together with the non-parametric estimate $F_{N}$ drawn in figure 3.2. According to (3.17), $E\left(y_{i} \mid a_{i}=1, X_{i}\right)$ is equal to $X_{i}^{\prime} \alpha_{b}$ and can be estimated by replacing $\alpha_{b}$ by its OLS or its Robinson estimate. We estimated $E\left(y_{i} \mid a_{i}=1, X_{i}\right)$ using the estimation results without cross term.

Table 3.3 . Effects of a change of all total expenditures by the same factor

| factor | I | II | III | IV | V | VI |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.90 | 0.583 | 0.586 | 1841.2 | 1830.4 | 1188.0 | 1185.6 |
| 0.99 | 0.625 | 0.627 | 2046.9 | 2041.4 | 1402.4 | 1399.6 |
| 1.00 | 0.630 | 0.631 | 2069.8 | 2065.0 | 1426.6 | 1423.9 |
| 1.01 | 0.634 | 0.635 | 2092.8 | 2088.7 | 1450.9 | 1448.2 |
| 1.10 | 0.671 | 0.670 | 2300.8 | 2302.7 | 1672.4 | 1669.9 |

## Explanation

$I: P\left(y_{i}>0 \mid X_{i}\right)$ according to Probit;
II : $P\left(y_{i}>0 \mid X_{i}\right)$ according to Klein-Spady;
III: $E\left(y_{i} T E_{i} \mid y_{i}>0, X_{i}\right)$ according to OLS;
IV : $E\left(y_{i} \mathrm{TE}_{i} \mid y_{i}>0, X_{i}\right)$ according to Robinson-estimator;
$V: E\left(y_{i} T E_{i} \mid X_{i}\right)$ according to Probit and OLS;
VI : $E\left(y_{i} \mathrm{TE}_{i} \mid \mathrm{X}_{i}\right)$ according to Klein-Spady and Robinson-estimator.

The first simulations concern changes of all family incomes by the same factor. Table 3.3 presents the consequences of different overall changes, ranging from $-10 \%$ to $+10 \%$. 11) Comparing the various estimation results on the basis of the outcomes of these macro-economic simulation results, we hardly find any difference between Probit in combination with OLS and the Klein-Spady estimator in combination with the Robinson estimator. According to the last two columns in the table, the elasticity of aggregate vacation expenditures with respect to total expenditures is 1.70 . This outcome is much smaller that the one in table 2.3. The main reason is that in table 2.3 the truncation was not taken into account.

In addition to overall changes in household incomes, we also consider the impact of a redistribution of incomes in the sample. Following Van Soest and Kooreman (1987), we used a redistribution of incomes such that the sample standard deviation of the logarithm of incomes decreases with $10 \%$, while the average income remains constant. ${ }^{12)}$ As a result, the ratio of the maximum and minimum income in the sample falls from 8.4 to 6.8 . According to the Probit estimates, the participation probability rises from 0.630 to 0.637. Using the Klein-Spady estimator, we find a rise from 0.631 to 0.637 . On the other hand, expected expenditures would slightly fall (with Dfl 4.6 or Dfl 5.1, according to the parametric and semi-parametric estimates,
respectively). Thus, again, we hardly find any differences between parametric and the semi-parametric outcomes.

Thus, although, for example, figure 3.2 suggests that for at least some families, substantial differences between Probit and Klein-Spady estimates of participation probabilities exist, these are not reflected in the particular macro-economic simulations we consider.

## 4. Evaluation and conclusions

In recent years, the use of limited dependent variable models in empirical micro-econometrics has become quite popular. In the literature on structural labour supply models, for example, more and more complicated models are now being used, with various error terms reflecting different sources of random variation. Most of these models are estimated by maximum likelihood under the assumptions of normality and homoskedasticity. People have by now started to realise the consequences of the fact that this estimation procedure may yield inconsistent estimates if the rather strong assumptions are violated.

One implication of this is the need to test model assumptions thoroughly. A number of specification tests is available in a rather general framework. See, e.g., the studies in Blundell (1987). If the model then appears to be misspecified, the next step may be to relax the model assumptions and find estimators which remain consistent under these more general assumptions. Although various semi-parametric models and estimation techniques have been developed for this goal in the recent literature (cf. Robinson (1988) for a survey), applications are still sparse. The aim of this paper has been to analyse models explaining vacation expenditures, with emphasis on specification testing and comparing parametric and semiparametric techniques.

We have considered two types of models. In section 2, we considered various specifications of the single-equation censored regression model. On the basis of specification tests, such as chi-squared diagnostics, all parametric specifications we considered are rejected. We estimated a semiparametric specification allowing for heteroskedasticity, using two different estimators. One of the drawbacks of this approach is that the distribution function of the error terms cannot be estimated. Thus, we were not able to compute, for example, the income elasticity of vacation expenditures. This elasticity seems, from a practical point of view, much more interesting than the elasticity of the underlying latent variable which
we did compute. A second drawback is that hardly any methods are available to test whether this semi-parametric model fits the data.

In section 3, we considered two-equation models, in which the decisions whether or not to go on holiday and how much to spend are modelled recursively. The participation decision was modelled by a binary choice model. The semi-parametric single index specification, and even the Probit model, a special case of the single index model, appeared to fit the data quite well. Predicted probabilities according to these models differ for sparse values of the covariates only.

Conditional on the decision to participate, the decision on how much to spend was modelled by a regression equation. This equation was estimated using both OLS and a semi-parametrically efficient estimator, with nearly identical results. The assumption of independence between the error in the regression equation and the error in the Probit model (written as a latent variable model) was tested and not rejected.

The two-equations model has the advantage that, even in case of the semi-parametric specification, it can be used to compute expected vacation expenditures for each family. It thus also allows for the computation of , for example, the elasticity of vacation expenditures with respect to total expenditures. The reason is that the probabilities in the binary choice model can be estimated satisfactorily. Problems would arise if we would try to estimate the distribution of the error term in the regression equation, but because of the linearity of this equation, this distribution is not needed. We therefore think that, at least in our case, the two-equations model is more appropriate than the single equation censored regression model.

If we repeat the simulations discussed in section 3 with some of the parametric models in section 2, we find similar income elasticities (cf. appendix 2). On the other hand, it appears that the models in section 3 capture the average value of vacation expenditures in the data much better than those in section 2. This seems one more reason to prefer the twoequations model.

Semi-parametric estimators are at this moment only available for some specific, relatively simple, univariate models. The recursive model in section 3 serves to illustrate that such simple models can be combined into a model which in some sense captures the complicated economic phenomena of interest.

## Appendix 1: The Data

The data stem from the Consumer Expenditure Survey drawn in 1981 by the Netherlands Central Bureau of Statistics. Sample statistics for the exogenous variables are given in table A. 1.

Table A.1. Sample statistics

|  | All observations | Observations with zero <br> vacation expenditures |  | Observations with non-zero <br> vacation expenditures |  |  |
| :--- | ---: | :--- | ---: | :--- | ---: | :--- |
|  | Mean | Std. dev. | Mean | Std. dev. | Mean | Std. dev. |
| $\mathrm{X}_{2 i}$ | 10.37 | 0.36 | 10.22 | 0.34 | 10.46 | 0.35 |
| $\mathrm{X}_{3 i}$ | 1.10 | 0.37 | 1.05 | 0.36 | 1.13 | 0.37 |
| $\mathrm{X}_{4 i}$ | 7.09 | 3.18 | 7.32 | 3.44 | 6.96 | 3.01 |
| $\mathrm{X}_{5 i}$ | 2.31 | 1.06 | 2.06 | 0.96 | 2.45 | 1.09 |
| $\mathrm{X}_{6 i}$ | 3.86 | 1.74 | 3.76 | 1.77 | 3.91 | 1.72 |

Explanation:
$X_{2 i}$ : logarithm of total family expenditures in 1981 (in Dfl)
$\mathrm{X}_{3 i}$ : logarithm of family size ( $2 \leq$ family size $\leq 7$ )
$X_{4 i}$ : age class family head; $X_{4 i}=1:<20$ years old; $X_{4 i}=2: 20-24$ years old;
$x_{4 i}=3: 25-29$ years old; ...; $X_{4 i}=13:>74$ years old
$\mathrm{X}_{5 i}$ : education level family head, ranging from 1 (low) to 5 (high)
$X_{6 i}$ : degree of urbanisation, ranging from 1 (country village) to 6 (big city)

Vacation expenditures are defined as expenditures of any member of the family on a vacation, which is defined as a 'stay away from home for recreation purposes for at least four successive nights'. The average annual amount spent on vacations per family is Df1 1415.4 , zeroes included, and Dfl 2247.5 if zero expenditures are not included.

The distribution of positive budget shares of vacation expenditures is presented in Figure A. 1.

Appendix 2: Table 3.3 for two censored regression specifications

The corresponding outcomes of table 3.3 for the Tobit specification and the censored regression specification assuming normality and exponential heteroskedasticity are represented in table A. 2.

zero shares excluded; 1143 observations
Figure A.1: Distribution of budget shares spent on vacations (in \%)
Table A. 2 Effects of a change of all total expenditures by the same factor

| factor | I | II | III | IV | V | VI |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.90 | 0.562 | 0.590 | 1764.3 | 1922.2 | 1100.8 | 1244.6 |
| 0.99 | 0.595 | 0.627 | 2013.2 | 2056.2 | 1311.3 | 1468.6 |
| 1.00 | 0.599 | 0.631 | 2041.1 | 2182.3 | 1335.1 | 1493.6 |
| 1.01 | 0.602 | 0.635 | 2069.1 | 2208.5 | 1359.0 | 1518.7 |
| 1.10 | 0.629 | 0.666 | 2322.5 | 2444.2 | 1576.3 | 1744.7 |

## Explanation

I : $\mathrm{P}\left(\mathrm{y}_{\mathrm{i}}>0 \mid \mathrm{x}_{\mathrm{i}}\right)$ according to Tobit;
II : $\mathrm{P}\left(\mathrm{y}_{\mathrm{i}}>0 \mid \mathrm{X}_{\mathrm{i}}\right)$ according to censored regression model with exponential heteroskedasticity and normality;
III: $E\left(y_{i} \mathrm{TE}_{i} \mid y_{i}>0, X_{i}\right)$ according to Tobit;
IV : $E\left(y_{i} T E_{i} \mid y_{i}>0, x_{i}\right)$ according to censored regression model with exponential heteroskedasticity and normality;
$\mathrm{V}: \mathrm{E}\left(\mathrm{y}_{\mathrm{i}} \mathrm{TE}_{\mathrm{i}} \mid \mathrm{X}_{\mathrm{i}}\right)$ according to Tobit;
VI : $\mathrm{E}\left(\mathrm{y}_{\mathrm{i}} \mathrm{TE}_{\mathrm{i}} \mid \mathrm{X}_{\mathrm{i}}\right)$ according to censored regression model with exponential heteroskedasticity and normality.

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## Notes

1) 'To go on holiday', 'to participate' and 'positive budget share of vacation expenditures' are used as synonyms. Vacation expenditures include expenditures of all members of the family. Thus a family is said to participate or to go on holiday if at least one member of the family goes on holiday.
2) Preliminary estimation results with squares and cross products of the regressors suggested that only $x_{2 i}^{2}$ should be incorporated.
3) Moon (1989) presents a Monte-Carlo comparison of the finite sample performance of the CLAD-estimator and several other semi-parametric estimators.
4) A test for (3.4) which generalises the usual test for selection bias in this type of two equation models (cf. Heckman, 1979), will be discussed below.
5) Following Horowitz we used the normal density. The smoothing parameter $\delta$ was set equal to $4 / 3$, and the smoothing parameter $h_{N}$ was chosen such that $\omega_{N}$, the kernel-bandwidth, was equal to the standard deviation of $X_{a i}^{\prime} \hat{\alpha}_{a} / \hat{\sigma}$. Other choices of $h_{N}$ yield slightly different curves, but the conclusion that the Probit specification is accepted, remains.
6) Klein and Spady (1989) suggest that the finite sample performance of their estimator can be improved by multiplying the quasi-log-likelihood contributions with a trimming function $\tau_{\mathrm{Ni}}$, which is used to downweight observations with $X_{i}^{\prime} \hat{\alpha}_{a}$ near the support boundary, where density estimates may not be reliable. The parameter estimation results we obtained using such trimming functions were virtually identical to those presented in table 3.1. Only the value of the quasi-loglikelihood increased, as might be expected.
7) An alternative kernel, which we used for comparison, can be found in Horowitz (1991).
8) Almost identical results were obtained using the kernel used by Horowitz (1991) (with correspondig smoothness parameter $h_{N}=0.1$ ). Other values of the smoothing parameters resulted in slightly different outcomes. The (informal) graphical test to be presented later suggests that the present choice fits the data reasonably well.
9) In addition, we tried other cross products and squares of $x_{2 i}, \ldots, x_{6 i}$, using the Robinson-estimator. None of these turned out to be significant on the 5\% level.
10) The Hausman test may not be very powerful, since, for example, it is not easy to think of alternative specifications under which the Robinson estimator is not consistent while OLS remains consistent.
11) For comparison, we present in appendix 2 the corresponding results for the Tobit model as well as the censored regression model with normality and exponential heteroskedasticity.
12) For details we refer to Van Soest and Kooreman (1987), footnote p. 224.
(For previous papers please consult previous discussion papers.)

| No. | Author(s) | Title |
| :---: | :---: | :---: |
| 9025 | K. Kamiya and <br> D. Talman | Variable Dimension Simpliciel Algorithm for Balanced Games |
| 9026 | P. Skott | Efficiency Wages, Mark-Up Pricing and Effective Demand |
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