Doublet-Singlet Oscillations

and

Dark Matter Neutrinos

Nobuchika Okada * †

Department of Physics, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-03, Japan

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Abstract

We examine the 'singlet majoron model' first introduced by Chikashige, Mohapatra and Peccei as a simple extension of the standard model with massive Majorana neutrinos. We can explain both the solar and the atmospheric neutrino deficits by the oscillations between electroweak doublet and singlet neutrinos without flavor mixing. Furthermore, while some light neutrinos can be the hot dark matter, tau neutrino with mass of 8.9–24MeV can be the cold dark matter through the interaction with the majoron. Thus, we can simultaneously explain the solar neutrino deficit, the atmospheric neutrino anomaly, and the cold and hot dark matters only with the Majorana neutrinos.

^{*}e-mail: n-okada@phys.metro-u.ac.jp

[†]JSPS Research Fellow

There are several observations which can be explained if the neutrinos are massive. The solar electron neutrino deficit [1] can be understood by the neutrino oscillation phenomena due to the non-zero mass difference and the flavor mixing between neutrinos. There exist two types of solutions to the solar neutrino deficit: one is the oscillation with the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [2] inside the sun, the other is the solution of the vacuum oscillation from the sun to the earth [3]. The atmospheric neutrino anomaly [4] is the observations of the deficit of muon neutrino relative to the electron neutrino both of which are produced in the atmosphere. This also can be explained by the neutrino oscillation phenomena. If the sum of the mass of all neutrinos is 5-7 eV, the neutrinos can play the role of the hot dark matter in the cold plus hot dark matter models [5], which have good agreement with the observations of the matter distribution in the universe.

The two observations of the neutrino deficits have been examined by the analysis of the neutrino oscillation with two flavor mixing or three flavor mixing scheme [6]. These analysis requires two mass squared differences: one is $\Delta m_{\odot}^2 \simeq 10^{-5} \text{eV}^2 \ (\Delta m_{\odot}^2 \simeq 10^{-10} \text{eV}^2)$ for the solar neutrino deficit with (without) the MSW mechanism, the other is $\Delta m_{\oplus}^2 \simeq 10^{-2} \text{eV}^2$ for the atmospheric neutrino anomaly. Considering all of the observations mentioned above, it is required for three neutrinos of different flavors to have nearly degenerate masses of a few eV. However, if the neutrinos are the Majorana particles, this mass spectrum is excluded by the experiments of the neutrino-less double beta decay [7], by which the effective electron neutrino mass is constrained as $\langle m_{\nu_e} \rangle < 1 \text{eV}$.

In this letter, we examine a model with the Majorana neutrino, called the 'singlet majoron model', first considered by Chikashige, Mohapatra and Peccei [8]. We show that the solar and the atmospheric neutrino deficits can be explained with oscillations between electroweak doublet and singlet neutrinos, but without flavor mixing. The existence of neutrinos as both the hot and cold dark matters is also shown.

We extend the standard model by introducing three right-handed neutrinos and one electroweak singlet scalar. Since we assume the absence of the flavor mixing in the following, we can treat each generation separately. The Yukawa interactions for one generation are described by

$$\mathcal{L}_{\text{Yukawa}} = -g_Y \overline{\nu_L} \Phi \nu_R - g_M \overline{\nu_R}^c \phi \nu_R + h.c. \quad , \tag{1}$$

where Φ is the electric-charge neutral component of the Higgs field in the standard model, and ϕ is the electroweak singlet field. The Dirac and the Majorana mass terms appear by the non-zero vacuum expectation values of these scalar fields. The mass matrix is given by

$$\begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix}, \tag{2}$$

where $m_D = g_Y \langle \Phi \rangle$ is the Dirac mass term, and $M = g_M \langle \phi \rangle = g_M v / \sqrt{2}$ is the Majorana mass term. Since the symmetry of the lepton number is spontaneously broken by $\langle \phi \rangle \neq 0$, a massless Nambu-Goldstone boson called majoron exists. For two mass eigenstates, the light one ν_ℓ and the heavy one ν_h , we obtain the couplings of the neutrinos with the majoron from eq.(1):

$$\mathcal{L}_{\chi\nu} = -\frac{i}{\sqrt{2}} \chi \left[\sin^2 \theta \,\overline{\nu_\ell} \, i\gamma_5 \,\nu_\ell - \sin \theta \cos \theta \left\{ \overline{\nu_\ell} \, i\gamma_5 \,\nu_h + h.c. \right\} + \cos^2 \theta \,\overline{\nu_h} \, i\gamma_5 \,\nu_h \right] \quad , \qquad (3)$$

where the field χ is the majoron field defined by $\chi/\sqrt{2} = \text{Im}\phi$, and θ is a mixing angle introduced by diagonalization of the mass matrix in eq.(2).

Note that the oscillation between the electroweak doublet and singlet neutrinos is possible [9], since the mass matrix is not diagonal. In the following discussion, this type of oscillation is called the 'doublet-singlet oscillation'. The information for the mass squared difference and the mixing angle is related to the values of the matrix elements in eq.(2). The small mixing angle ($\sin \theta \ll 1$) requires $m_D \ll M$, called the see-saw type mass matrix [10], and M is fixed by the value of the mass squared difference, $M \simeq \sqrt{\Delta m^2}$. The almost pseudo-Dirac type mass matrix [11], $m_D \gg M$, is required by the large mixing angle ($\sin \theta \sim 1$), and the relation, $\Delta m^2 \simeq 2m_D M$, is obtained.

It is clear that the solar electron neutrino deficit and the atmospheric muon neutrino deficit can be explained by the 'doublet-singlet oscillation', since experiments observe only the deficits, but not appearance of the converted partner through the oscillation. The solar neutrino deficit can be interpreted by the 'doublet-singlet oscillation' in the first generation, and the atmospheric one can be interpreted by the same in the second generation. This type of the model of the neutrino oscillation is a kind of the model including the oscillation between the electroweak doublet neutrino and a 'sterile' neutrino [12]. In our model, the physical meaning of the 'sterile' neutrino is clear: it is the right-handed neutrino, which is introduced to generate the Majorana mass. Since we have little information for tau neutrino except for its existence and the upper bound on the mass $m_{\nu_{\tau}} < 24$ MeV [13], it is not needed to consider the oscillation in the third generation.

However, since we have six neutrinos, we should consider the constraint on the number of neutrino species from the big bang nucleosynthesis (BBN) [14]: $N_{\nu} \simeq 3$, where N_{ν} is the number of neutrino species which are in thermal equilibrium at the BBN era (temperature of the universe $\simeq 1 \text{MeV}$). It is known that, in the first generation, only the electroweak doublet neutrino contributes at the BBN era (see ref.[15] for brief discussion), if we take the small-angle MSW solution $(\Delta m_{\odot}^2 \simeq 10^{-5} \text{ and } \sin^2 2\theta_{\odot} \simeq 10^{-2})$ or the vacuum oscillation solution $(\Delta m_{\odot}^2 \simeq 10^{-10} \text{ and } \sin^2 2\theta_{\odot} \simeq 1)$ to the solar neutrino deficit ¹. Then we take these solutions in the first generation. However, this is not the case in the second generation, since $\Delta m_{\oplus}^2 \simeq 10^{-2} \text{eV}$ and the large mixing angle, $\sin^2 2\theta_{\oplus} > 0.6$, are required to explain the atmospheric neutrino anomaly (also see ref.[15]). Both two neutrinos in the second generation contribute to N_{ν} . Then, there already exist three species of neutrinos which is in thermal equilibrium at the BBN era: the doublet neutrino in the first generation, and two neutrinos in the second generation. Thus, the energy density of the remaining two neutrinos in the third generation should be small at the BBN era.

This situation can be realized in two ways. One is that neutrinos decay rapidly, and

¹ Considering the matter effect on the earth, the large-angle MSW solution ($\Delta m_{\odot} \simeq 10^{-5}$ and $\sin^2 2\theta_{\odot} \simeq 0.6$) with $\nu_e \to \nu_s$ (the 'sterile' neutrino) oscillation is disfavored without cosmological discussion. This fact is pointed out by Hata and Langacker in ref.[6].

disappear until the time of the BBN era ($\simeq 1$ s). This can be applied to the heavy neutrino in the third generation, since it decays into the light neutrino and the majoron through the interaction in eq.(3). The other way is that neutrinos decouple from other particles in nonrelativistic regime. This way should be applied to the light neutrino in the third generation, since it is stable.

First, we discuss the case of the light neutrino. If it decouples in non-relativistic regime, its energy density is suppressed by the Boltzmann factor $e^{-m/T}$ (m > T), and becomes negligible, where m and T are the mass of the neutrino and the decoupling temperature, respectively. Since this suppression should works at the BBN era (1 MeV), m > 1MeV is required. However, this region of the mass of tau neutrino is cosmologically excluded [16], since the density parameter Ω in the present universe becomes too large, $\Omega \gg 1$. This is true, if we consider only the electroweak interaction. However, note that there is the interaction between neutrinos and the majoron. Carlson and Hall, and Kitazawa et al. [17] pointed out that neutrinos can be the cold dark matter through the interaction. We investigate that the light neutrino in the third generation can really be the cold dark matter in the following.

Let us consider the interaction between neutrinos in the third generation and the majoron. We assume that the mass matrix in the third generation is the see-saw type: $m_D \ll M$ in eq.(2). Then, the light mass eigenstate (ν_ℓ) and the heavy one (ν_h) have masses $m_\ell \simeq m_D^2/M$ and $m_h \simeq M$, respectively. The light neutrino is almost electroweak doublet state, or tau neutrino, and the heavy one is almost electroweak singlet state by the see-saw mechanism. Using m_ℓ and m_h , the couplings of the neutrinos with the majoron in eq.(3) are rewritten by

$$\mathcal{L}_{\chi\nu} \simeq -\chi \left[\left(\frac{m_{\ell}}{v} \right) \overline{\nu_{\ell}} \, i\gamma_5 \, \nu_{\ell} - \sqrt{\frac{m_{\ell}m_h}{v^2}} \left\{ \overline{\nu_{\ell}} \, i\gamma_5 \, \nu_h + h.c. \right\} + \left(\frac{m_h}{v} \right) \overline{\nu_h} \, i\gamma_5 \, \nu_h \right] \quad , \tag{4}$$

where the relation $m_h \simeq g_M v / \sqrt{2}$ is used.

The energy density of the cold dark matter in the present universe is given by

$$\rho_{CDM} \simeq 2.0 \times 10^{-6} \text{ GeV/cm}^3 \tag{5}$$

in the cold plus hot dark matter models [5]. These cosmological models agree very well with the observations of the matter distribution in the universe with the total density parameter $\Omega = 1$ and the Hubble constant $h \equiv H_0/100$ km s⁻¹ Mpc⁻¹ = 0.5.

The relation between the mass $m_{\ell}(=m_{\nu_{\tau}})$ and v is obtained by using the value of ρ_{CDM} . The decoupling temperature T_D is defined by [18]

$$n(T_D) \left\langle \sigma | v | \right\rangle_{T_D} = H(T_D) , \qquad (6)$$

where $n(T_D)$ is the number density of the tau neutrino at the decoupling temperature, $\langle \sigma | v | \rangle_{T_D}$ is the average value of the annihilation cross section of tau neutrino times relative velocity, and H is the Hubble parameter. For the non-relativistic tau neutrino, $n(T_D)$ is approximately given by

$$n(T_D) \simeq \frac{1}{\sqrt{2\pi^3}} x^{-\frac{3}{2}} e^{\frac{1}{x}} T_D^3 \quad , \tag{7}$$

where $x = T_D/m_{\nu_{\tau}}$. Considering the non-relativistic annihilation process of the tau neutrino, $\nu_{\tau}\nu_{\tau} \rightarrow \chi\chi$, we obtain

$$\langle \sigma | v | \rangle_{T_D} \simeq \frac{1}{32\pi} \, \frac{m_{\nu_\tau} T_D}{v^4} \tag{8}$$

from eq.(4). The Hubble parameter H is given by

$$H(T_D) = \left(\frac{8\pi^3 g_*}{90}\right)^{\frac{1}{2}} \frac{T_D^2}{M_P} \quad , \tag{9}$$

where $M_P \simeq 1.2 \times 10^{19}$ GeV is the Planck mass, and g_* is the total degrees of freedom of all particles in thermal equilibrium (we set $g_* = 43/4 + 1$). The energy density of the tau neutrino in the present universe is given by

$$\rho_{\nu_{\tau}} = m_{\nu_{\tau}} n(T_D) \left(\frac{T_0}{T_D}\right)^3 \quad , \tag{10}$$

where $T_0 \simeq 1.9$ K is the temperature of the tau neutrino at present. From eqs.(6)-(10), and the condition $\rho_{\nu_{\tau}} = \rho_{CDM}$, we can obtain the relation between $m_{\nu_{\tau}}$ and v. This relation is shown in Table I together with the contribution of the tau neutrino at the BBN era as another species, $\Delta N_{\nu} (= N_{\nu} - 3)$. Considering the experimental upper bound on the mass of tau neutrino, $m_{\nu_{\tau}} < 24$ MeV, and the BBN constraint (we take $\Delta N_{\nu} \le 0.01$), the region, $m_{\nu_{\tau}} \simeq 8.9-24$ MeV and $v \simeq 2.7-4.3$ GeV is allowed as the cold dark matter.

The heavy neutrino ν_h rapidly decay into the light neutrino (the tau neutrino) and the majoron. From eq.(4), the life time of ν_h is described by

$$\tau \simeq \frac{32\pi}{g_M^2 m_{\nu\tau}} \ . \tag{11}$$

Substituting our result $m_{\nu_{\tau}} = 8.9-24$ MeV into above equation, $\tau < 7.4 \times 10^{-21}/g_M^2$ s. The life time is far shorter than the age of the universe at the BBN era ($\simeq 1$ s), unless g_M is extremely small. Therefore, the heavy neutrino disappears until the time at the BBN era.

The neutrinos in the first and second generation can be the hot dark matter. Since the mass squared differences required the solar and the atmospheric neutrino deficits are far smaller than the mass scale of the hot dark matter, neutrinos as the hot dark matter have nearly degenerate masses. Two cases can be considered. One is that the two neutrinos in the second generation are the hot dark matter with mass $\simeq 3 \text{eV}$, if the small-angle MSW solution is taken in the first generation. In this case, the two neutrinos in the first generation have masses, $\sin^2 \theta \sqrt{\Delta m_{\odot}^2}$ and $\sqrt{\Delta m_{\odot}^2}$, respectively. The other case is that the neutrinos in both the first and second generations are the hot dark matter with mass $\simeq 2 \text{eV}$, if the solution of the vacuum oscillation is taken. These mass spectra in two generations are shown in Table II.

There is no conflict with the experiments of the neutrino-less double beta decay, even if we take the vacuum oscillation solution and the mass $\simeq 2 \text{eV}$ in the first generation. Note that the mass matrix in eq.(2) is almost pseudo-Dirac type, $m_D \gg M$. Since we ignore the CP violating phase, the two mass eigenstates in the first generation have opposite CP eigenvalues: $\eta_{\pm} = \pm 1$. Considering that the solution of the vacuum oscillation requires the large mixing angle ($\theta_{\odot} \simeq \pi/4$), the effective electron neutrino mass is described by

$$\langle m_{\nu_e} \rangle \simeq \frac{1}{2} |m_+ + \eta_+ \eta_- m_-| \quad ,$$
 (12)

where $\eta_{\pm} = \pm 1$ are the CP eigenvalues, and $m_{\pm} \simeq m_D \pm M/2$ are the mass eigenvalues. Then, we obtain

$$\langle m_{\nu_e} \rangle \simeq \frac{M}{2} \simeq \frac{\Delta m_{\odot}^2}{4m_D} \simeq \frac{10^{-10} (\text{eV}^2)}{4 \times 3 (\text{eV})} \ll 1 \text{eV} \quad , \tag{13}$$

where the relation $M\simeq \Delta m_\odot^2/2m_{\scriptscriptstyle D}$ is used.

Here, we must discuss the phenomena caused by the existence of the majoron. Although we showed that the number of neutrinos which exist at the BBN era is three, the majoron is in the thermal equilibrium at the BBN era and contribute $\Delta N_{\nu} = 0.57$ as the additional species. Thus, our model results $N_{\nu} = 3.57$. There are diverge BBN constraints obtained by many authors [14]: $N_{\nu} < 2.6-3.9$. Our result $N_{\nu} = 3.57$ lies in this region, and is cosmologically allowed.

The astrophysical bounds on the 'singlet majoron model' should also be considered. The most restrictive constraint is obtained by the observations of neutrinos from the supernova 1987A [19]. These observations conclude that the gravitational binding energy is released almost by the emission of neutrinos. Thus, the energy release by other exotic particles is constrained smaller than that by neutrinos. Considering the majoron emission from the supernova, the constraint on the parameters in our model is obtained in two cases. One is the case in which the electroweak singlet Higgs boson, defined by $\sqrt{2}\text{Re}\phi$, have mass less than the temperature of the core of the supernova ($T_{\text{core}} = 30-70\text{MeV}$). The forbidden region is given by [20]

$$2 \times 10^{-8} < \frac{m_{\nu_{\tau}}(\text{MeV})}{v(\text{GeV})} < 3 \times 10^{-7}$$
 (14)

On the other hand, if the mass of the singlet Higgs boson is larger than the temperature of the core, the forbidden range is given by [21]

$$2.3 \times 10^{-5} < \left(\frac{m_{\nu_{\tau}}}{\text{MeV}}\right) \left(\frac{\text{GeV}}{v}\right)^2 < 3.3 \times 10^{-3}$$
 (15)

The region shown in Table I is outside these forbidden regions. Therefore, our model is astrophysically allowed.

Next we consider the effect due to the interactions of the singlet neutrinos in the first and second generations with the majoron. Since the values of M in the first and second generations are given by $M \simeq \sqrt{\Delta m_{\odot}^2}$ and $\sqrt{\Delta m_{\oplus}^2}$, respectively, the coupling constants of these neutrinos with the majoron is extremely small: $g_M \simeq \sqrt{\Delta m_{\odot}^2}/v$ in the first generation, and $g_M \simeq \sqrt{\Delta m_{\oplus}^2}/v$ in the second generation. Such an extremely weak interaction cannot affect in any cosmological or astrophysical observation.

We would like to comment on the LSND experiment [22]. This experiment may be the first direct observation of the neutrino oscillation with $\overline{\nu_{\mu}} \rightarrow \overline{\nu_{e}}$. Since the LSND experiment is the type of the 'appearance' experiment, it is clear that our model cannot explain this experiment. However, the explanation of the LSND result can be included, if we extend our model, and introduce flavor mixing between the first and the second generations. This extended model is the same, in form, as the models of ref. [12], in which the 'sterile' neutrino is introduced, and the mixings among three flavor neutrinos and the 'sterile' neutrino are investigated. However, note that our model is very restrictive, since there are only three mixing angles: one angle related to flavor mixing, and two angles corresponding to the mixing between the doublet and singlet neutrinos in the first and the second generations.

Finally, we would like to mention the future solar neutrino experiments. The presence of the 'doublet-singlet oscillation' in the first generation will be revealed in the future SNO [23] and Super-Kamiokande [24] experiments as is pointed out by Bilenky and Giunti [25]. If the electroweak doublet neutrino converts to singlet one, the deficit of total flux of the solar neutrino is observed. The discovery of the 'doublet-singlet oscillation' is a direct evidence of new physics beyond the standard model.

In conclusion, we examine the 'singlet majoron model' first introduced by Chikashige, Mohapatra and Peccei as a simple extension of the standard model with massive Majorana neutrinos. In this model, we can explain the solar and the atmospheric neutrino deficits by the 'doublet-singlet oscillations' without flavor mixing. Furthermore, while some light neutrinos can be the hot dark matter, tau neutrino with mass of 8.9–24MeV can be the cold dark matter through the interaction with the majoron. Thus, we can simultaneously explain the solar neutrino deficit, the atmospheric neutrino anomaly, and the cold and hot dark matters only with the Majorana neutrinos. The presence of the 'doublet-singlet oscillation' (in the first generation) will be revealed in future SNO and Super-Kamiokande solar neutrino experiments.

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TABLES

v (GeV)	$m_{\nu_{\tau}}$ (MeV)	$\Delta N_{ u}$
7	67	1.6×10^{-5}
6	48	1.6×10^{-5}
5	33	1.6×10^{-5}
4	20	1.7×10^{-5}
3	11	2.2×10^{-3}
2	4.6	0.21
1	1.0	0.92

TABLE I. The relations among $v,\,m_{\nu\tau},\,{\rm and}\,\,\Delta N_{\nu}$

TABLE II. The mass spectra of neutrinos in the first and second generations

solution	first generation	second generation
small-angle MSW	$m_\ell\simeq 8\times 10^{-6}~{\rm eV}$	$m_\ell \simeq m_h \simeq 3~{\rm eV}$
	$m_h \simeq 3 \times 10^{-3} \text{ eV}$	
vacuum oscillation	$m_\ell \simeq m_h \simeq 2 \ {\rm eV}$	$m_\ell \simeq m_h \simeq 2 \ {\rm eV}$