NONLINEAR CONTINUOUS-TIME GENERALISED PREDICTIVE CONTROL

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By

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To my husband, Jesús Ulises, and my daughter, Silvia.

Abstract

The development of the nonlinear version of the Continuous-time Generalised Predictive Control (NCGPC) is presented. Unlike the linear version, the nonlinear version is developed in state-space form and shown to include Nonlinear Generalised Minimum Variance (NGMV), and a new algorithm, Nonlinear Predictive Generalised Minimum Variance (NPGMV), as special cases. Through simulations, it is demonstrated that NCGPC can deal with nonlinear systems whose relative degree is not well defined and nonlinear systems with unstable zero dynamics. Geometric approaches, such as exact linearisation, are shown to be included in the NCGPC as special cases.

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Chapter 1

Introduction

A system represented as linear is to some extent a mathematical abstraction that can never be encountered in a real world. However all physical systems are nonlinear in some extent, when the effect of the nonlinearity is very small, linear controllers can be applied.

The majority of controller design techniques for process control are based on linear process models. These linear process models can be obtained by local linearisation vía Taylor series approximation. However, they are not adequate since, the linear model is valid in a small operation range, when a larger operation range is required large, the linear controller design gives poor performance or yields to unstability. If the process is highly nonlinear, these are not applicable.

The geometric approaches based directly on nonlinear process model such as (A. Isidori [44], P. Lee *et. al.* [52], C. Economu and M. Morari [25] and C. Kravaris *et al.* [48]) can deal with smooth nonlinear processes, they provide exact linearisation of the nonlinear system and are independent of the operating point. Linear controllers can then be designed for the equivalent linear system.

The main advantage of the input-output exact linearisation techniques compared to local linearisation vía Taylor series approximation is that they exactly linearise important classes of nonlinear systems, whereas methods based on local linearisation vía Taylor series approximation linearise the model only at the nominal operating point.

However, the input-output exact linearisation techniques are based on very restrictive assumptions, which are difficult to encounter in practical situations:

- state measurement
- exact model
- stable zero dynamics
- well defined relative degree

Therefore, it is necessary to design approaches with less restrictive assumptions. A perfect model is required in order to achieve linearisation of the closed loop system; thus, the robustness is compromised. A few robust controller design strategies are available for nonlinear systems. Adaptive control techniques have been designed for systems with uncertain parameters, such as (S. Sastry [72], K. Nam[64], P. Kanellakopoulus *et al.* [46], R. Marino *et. al.* [57] and [58]). If structural uncertainties are present, methods have been developed such as C. Kravaris [49]. However, these adaptive and robust controller design strategies have serious limitations.

Perfect disturbance rejection can be carried on if the model is perfect and the disturbance must satisfy a matching condition or be measured. Approaches with less restrictive assumptions have been developed such as (P. Daoutidis and C. Kravaris [17] and A. Isidori *et al.* [45]).

In all the linearisation approaches, it is assumed that all the state variables required for the linearising transformation are available. But, in general, it is often impossible to obtain measurement of all the states. Therefore, it is necessary to estimate these states through a model and output measurement.

For stable open-loop systems, the process model can be used as an open-loop observer (J. Alvarez and J. Alvarez [2] and C. Kravaris and Ch. Chang-Bock [48]). For open-loop unstable processes, an open-loop observer is inadequate. Therefore, a closed-loop observer is required. It can be constructed by feeding back the error between the actual process output and the estimated output. However, it is difficult to find an observer gain that provides a fast convergent estimate. A simple method for designing the observer gain is to linearise the process model around the steady-state conditions in conjunction with a linear observer design method (see P. J. Gawthrop [30]). However, it is very difficult in practice to determine the region around the steady operating point in which such conditions are satisfied and the convergence conditions are not available. Closed-loop observers not based on linearisation around a steady state (see B. Walcott *et al.* [75]) exist, but they are restricted to very limited classes of systems.

To summarise, design of a closed-loop observer that generates an estimate that converges at least asymptotically to the actual states is an open question.

The exact linearisation strategies are not able to control systems with time delays, few techniques have been proposed such as (C. Kravaris and A. Wright [50]).

As all input-output linearisation strategies such as (A. Isidori [44], P. Lee *et al.* [52], C. Economu and M. Morari [25] and C. Kravaris *et al.* [48]), are based on model inversion, the system is required to be a minimum phase system. The fact that placing poles at the process zeros does not destroy internal stability if the zeros are in the open left half plane, is used in nonlinear systems in order to control non-minimum phase systems. Therefore, it is required that the system is factored in minimum-phase and the nonminimum-phase parts. Then just the first part is used for the purpose of control design F. Doyle III [42]. Another technique used to control nonminimum phase systems, is based on an approximation of non-minimum phase systems by minimum phase systems, L. Benvenuti *et al.* [5]. A similar controller is applied to the inverted pendulum on a cart by R. Gurumoorthy *et al.* [37].

The applicability of the linearisation algorithm fail when the system has singular points, this happen when $L_g L_f^{r-1} h(x) \neq 0$ for $x \neq x_0$ but $L_g L_f^{r-1} h(x_0) = 0$. (See Section 2.2.2 for definition of the Lie derivative.) These can be viewed as regions where the relative degree can not be defined. A few approaches have been proposed to solve this problem such as J. Hauser [38] and D. Rangel [69].

Predictive Control Schemes such as J. Richalet [70], C. R. Cutler *et al.* [16]. R. K. Mehra and R. Rouhani [71] and D. W. Clarke *et al.* [15], have proven to be useful controller design strategies for linear process: their distinctive features are the ability to deal with:

- time-delays.
- input constraints.
- non-minimum phase system.

These predictive controllers have been successfully extended to discrete nonlinear systems (D. Q. Mayne [59], A. A. Patwardhan [68], [67], Q. M. Zhu [77], M. J. Sistu [73], R. Bars [4] and W. Wang [76]). Several control design techniques that include some of these features have been proposed in continuous time. However, these techniques are based on very restrictive assumptions. For instance, in reference M. Soroush and C. Kravaris [74] the system must be minimum phase. In references, S. Abu and M. Flies [1] and Ping Lu [55] a complete availability of the process states is assumed and the system must be minimum phase.

Emulator Based Control (EBC) provides a framework for control methods arising from Astrom's minimum-variance control; such methods include the generalised minimum-variance controller of D. W. Clarke and P. J. Gawthrop [12] and [13] and the Generalised Predictive Control of D. W. Clarke *et al* [15] (see also D. W. Clarke and C. Mohtadi [14]). EBC has been shown (P. J. Gawthrop *et al.* [35]) to have close relations with the Internal Model Control (IMC) of M. Morari and E. Zafiriou [63]; it follows that the results herein are also relevant to IMC.

Continuous – time setting is used in this thesis, for the reasons given by P. J. Gawthrop [27] [28] and [29]. In particular, we believe that a continuous-time approach exposes the fundamentals of the underlying control problem which are obscured by sampling: this is particularly the case for nonlinear systems.

Generalised Predictive Control is one of a wider set of methods called Model-based Predictive Control (D. W. Clarke [11] and M. Morari [62]), hence results of this thesis give a particular form of nonlinear Model-based Predictive Control. Such methods have achieved success in a number of applications (see, for example the survey of

D. W. Clarke [11] and the papers collected in Chapter 5 of D. W. Clarke [10]), and therefore we believe that NCGPC will also find industrial application.

There are two main approaches to Generalised Predictive Control: the transfer function approach of, D. W. Clarke *et al.* [15] and H. Demircioglu and P. J. Gawthrop [21]; and, on the other hand, the state space approach of, P. J. Gawthrop and H. Demircioglu [32], R. R. Bitmead [6], J. Lee [51] and A. W. Ordys and D. W. Clarke [66]. A. W. Ordys and D. W. Clarke [66], point out that, whereas in the context of linear systems there is no significant difference between the two approaches, in the context of nonlinear systems the state-space approach is essential. For these reasons, the state-space approach is chosen in this thesis.

A critical assessment of (discrete-time) GPC is given in R. R. Bitmead *et al.* [6], implying that it is the " thoughtless person's LQ " controller. Although we believe that their arguments can be answered in more general terms, suffice it to say here that their arguments are not directly relevant to nonlinear systems. Nevertheless the state-space observer/state feedback advocated by R. R. Bitmead *et al.* [6] is adopted here, also used by P. J. Gawthrop and D. W. Clarke [31] and by P. J. Gawthrop and H. Demircioglu [32].

In the same way that linear CGPC provides a nice way of handling systems with zeros in the right-half plane, the nonlinear version presented in this thesis, provides a nice way of handling systems with unstable zero dynamics; this is in distinction to the standard methods arising from geometric theory which can not handle such systems. The continuous-time GPC of H. Demircioglu and P. J. Gawthrop [21] provided one possible generalisation of the continuous-time GMV control in the same spirit as the generalisation of D. W. Clarke *et al.* [15] the discrete-time context. However, one particular feature of GMV (the so called P polynomial) was not used. As an intermediate step, a new algorithm - the predictive Generalised Minimum Variance controller - is derived. Like GPC, but unlike GMV, this is a *moving horizon* (see D. Q. Mayne and H. Michalska [60]) controller. This thesis provides this extension as a step towards nonlinear GPC; however we believe it can also be of general interest.

In this thesis, it is also shown that nonlinear exact linearisation by feedback approach presented by A. Isidori [44] and the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] (which is a special case of the nonlinear exact linearisation by feedback) are equivalent a special case of NCGPC - Nonlinear Generalised Minimum Variance Control (NGMV). In the same spirit as A. isidori [44], the systems considered here have smooth nonlinearities. However, certain non-differentiable actuator nonlinearities could be handled using the constraint-based approach of C. M. Chow and D. W. Clarke [9] - but this is not persuade further here.

In the same way that exact linearisation requires a state estimate, so does NCGPC. There are a number of possible approaches to nonlinear state estimation including those reviewed by B. Walcott *et al.* [75] and introduced by L. Hunt and M. S. Verma [41]. Detailed discussion of observer design is outwith the scope of this thesis.

However, in this thesis, in order to give a partial solution to this problem, assuming the system is stable, an open loop observer is used. This is essentially a model of the process that is simulated in parallel to the process. Thus, assuming that suitable observer can be found, NCGPC provides another method of providing output feedback control of nonlinear systems (see R. Marino and P. Tomei [57] and [58]).

1.1 Contribution

To summarise I would like to emphasise that the main contributions of this thesis are:

- The development of the nonlinear version of the CGPC, with the following distinctive features:
  - Provides a nice way of handling systems with unstable zero dynamics.
  - Has the ability to deal with systems which do not have a well defined relative degree.

– Considers nonlinear dynamic systems with non-affine state-space representation:

$$\begin{aligned} \dot{x}(t) &= F(x(t), u(t)) \\ y(t) &= h(x(t)). \end{aligned}$$

- The nonlinear techniques such as the nonlinear exact linearisation by feedback developed by A. Isidori [44] and the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2], as well as the linear version CGPC recast in state space form, are shown to be included in the NCGPC.

- The following equivalencies were found.

NGMV	GLC [18]
GLC [18]	exact linearisation [44]
NGMV	exact linearisation [44]
NPGMV	Model Matching Via State Feedback [44]

- The TRRMCNL developed by J. Alvarez and J. Alvarez [2] is shown to be just the Model Matching Via State Feedback [44] with an open loop observer.

- The error feedback-GLC developed by P. Dautidis and C. Kravaris [18] uses an open loop observer, which leads to an output feedback control, in this thesis a regulation model used by J. Alvarez and J. Alvarez [2] is adding to this controller in order to improve the performance.

- A design of a positioning control for an Induction Motor based on the TRRMCNL.

1.2 Outline of Thesis

The thesis is organised as follows:

Chapter 2 is a review of the Theory of Nonlinear Feedback for SISO systems, which will be used in this thesis, this material includes: the coordinates transformation, the Lie derivative, the relative degree. The nonlinear feedback linearisation, the important concept of the *zero dynamics*. The disturbance decoupling problem was mentioned and solved as well as the disturbance decoupling problem with measurement and asymptotic model matching. The summary is based on the book by A. Isidori [44]. The Theory of Nonlinear Feedback is also critically evaluated.

Chapter 3 is a review of the GLC (Globally Linearising Control) developed by C. Kravaris and C. Chung [48] and its characteristics such as the fact that it can be viewed as an output feedback control and a nonlinear analogue of placing poles at process zeros are also reviewed. The GLC is also critically evaluated.

Chapter 4 The TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2] is reviewed and critically evaluated. The position of an Induction Motor is controlled by applying this controller, this work was accomplished by the author I. Siller-Alcalá and E. Liceaga-Castro and has been reported elsewhere [54]. This work suggested the ideas investigated in Chapters 5 and 6.

Chapter 5 In this chapter, the Nonlinear Continuous-time Generalised Predictive Control (NCGPC) is recast in a state-space form and shown to include Nonlinear Generalised Minimum Variance (NGMV) and a new algorithm, Nonlinear Predictive Generalized Minimum Variance (NPGMV) and has been reported elsewhere P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33] . We found that NCGPC controller has the advantage to deal with systems which do not have a well defined degree and has the ability to deal with nonlinear non-minimum phase systems (systems with unstable zero dynamics). Simulations are presented in order to show the effectiveness of the method.

Chapter 6 Equivalencies between the exact linearisation techniques and the new algorithms are also found. It is also shown that the exact linearisation by feedback approach presented by A. Isidori [44] and the error feedback-GLC developed by P.

Dautidis and C. Kravaris [18] (which is a special case of the nonlinear exact linearisation by feedback) are equivalent to Nonlinear Minimum Variance Control (NGMV) has been reported elsewhere P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá[33] .

The fact that the TRRMCNL developed by J. Alvarez and J. Alvarez [2] is just Model Matching Via State Feedback [44] with an open loop observer is shown.

It is also shown that, the Tracking and Regulation Reference Model Control of Nonlinear Systems (TRRMCNL) developed by J. Alvarez and J. Alvarez [2] and the Model Matching Via State feedback developed by A. Isidori [44] are equivalent to NPGMV. Thus, it is concluded that the linearisation techniques are included in NCGPC.

In this chapter a regulation model used by J. Alvarez and J. Alvarez [2] is adding to the error feedback-GLC in order to improve the performance. Simulations are presented in order to show the effectiveness of the method.

Chapter 7 Conclusions are presented and possible future works suggested.

Chapter 2

Theory of Nonlinear Feedback Review

2.1 Introduction

The purpose of this chapter is to summarise some of the most important aspects of the differential geometric approach to nonlinear systems. Also, the nonlinear feedback linearisation control is critically evaluated. The results detailed here will be helpful in designing and understanding the properties of the NLCGPC (Nonlinear Continuous Time Generalised Predictive Control), the NGMV (Nonlinear Generalised Minimum Variance Predictive Control) and the NPGMV (Nonlinear Predictive Generalised Minimum Variance Control), which will be developed in the Chapters 5. This approach will be used in the Chapter 6 where will be shown that this controller is equivalent to NGMV. The review is based on the now classical book of A. Isidori [44], but for further details see H. Nijmeijer [65].

2.2 Mathematical Preliminaries

In this section, differential notations and coordinates transformations are briefly reviewed. These will be helpful in the derivation of the nonlinear control law.

The single-input single-output nonlinear systems considered here are described by the following differential equations:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)), \end{aligned} \tag{2.1}$$

where $x \in \mathbf{R}^n$ is the vector of the states variables, $u \in \mathbf{R}$ is the manipulated input and $y \in \mathbf{R}$ is the output to be controlled. f and g are smooth vector fields and h is a smooth function. Following A. Isidori [44], the function h is said to be a smooth function or C^∞ if its partial derivatives with respect to $x_1, x_2, \ldots, x_n$ exist and are continuous.

The vector field f is said to be a smooth vector, if its partial derivatives with respect to $x_1, x_2, \ldots, x_n$ exist and are continuous.

Equation (2.1) is said to be a control affine system, because the input u appears linearly.

2.2.1 Nonlinear change of coordinates in the state space

Changes of coordinates in the state space are very useful when analysing nonlinear systems. Following A. Isidori [44]," A nonlinear change of coordinates can be described in the form:

$$z = \Phi(x) \tag{2.2}$$

where $\Phi(x)$ represents a $\mathbf{R}^n$-valued function of n variables, i.e.:

$$\Phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \cdots \\ \phi_n(x) \end{bmatrix} = \begin{bmatrix} \phi_1(x_1, x_2, \ldots, x_n) \\ \phi_2(x_1, x_2, \ldots, x_n) \\ \cdots \\ \phi_n(x_1, x_2, \ldots, x_n) \end{bmatrix} \tag{2.3}$$

with the following properties:

(i) $\Phi(x)$ is invertible, i.e. there exists a function $\Phi^{-1}(z)$ such that:

$$\Phi^{-1}(\Phi(x)) = x$$

for all **x** in $\mathbf{R}^n$.

(ii) $\Phi(x)$ and $\Phi^{-1}(z)$ are both smooth mappings, i.e. have continuous partial derivatives of any order. "

The first property is needed in order to recover the original state vector. The second one guarantees that the system in the new coordinates is still smooth. This kind of transformation is called a global diffeomorphism (A. Isidori [44]). A transformation with these properties and defined for all **x** is difficult to be found; in addition these properties are difficult to check. Because of this, here a transformation defined only in a neighbourhood of a given point is used instead. Such transformations are called a local diffeomorphism.

Proposition 2.1 *(A. Isidori [44] Proposition 1.2.3) " Suppose $\Phi(x)$ is a smooth function defined on some subset U of* $\mathbf{R}^n$ *. Suppose the Jacobian matrix:*

$$\Phi(x) = \begin{bmatrix} \frac{\partial\Phi_1(x)}{\partial x_1} & \frac{\partial\Phi_1(x)}{\partial x_2} & \cdots & \frac{\partial\Phi_1(x)}{\partial x_n} \\ \frac{\partial\Phi_2(x)}{\partial x_1} & \frac{\partial\Phi_2(x)}{\partial x_2} & \cdots & \frac{\partial\Phi_2(x)}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial\Phi_n(x)}{\partial x_1} & \frac{\partial\Phi_n(x)}{\partial x_2} & \cdots & \frac{\partial\Phi_n(x)}{\partial x_n} \end{bmatrix} \tag{2.4}$$

is nonsingular at a point $x = x^0$. Then, on a suitable open subset $\mathbf{U}^0$ *of* **U***, containing x^0,* $\Phi(\mathrm{x})$ *defines a local diffeomorphism."*

2.3 Exact Linearisation via Feedback

In this section a single-input single-output nonlinear system given by equation (2.1) will be transformed into a linear and controllable system. In order to obtain this linear and controllable system, we need first to introduce a differential notation.

The Lie derivative of a scalar function $h(x)$ with respect to a vector function $f(x)$ is defined as:

$$L_f h(x) = \frac{\partial h}{\partial x} f(x) \tag{2.5}$$

As the Lie derivative of a scalar function is also a scalar function, higher order Lie derivatives can be defined recursively as:

$$L_f^k h(x) = \frac{\partial}{\partial x}(L_f^{k-1} h(x)) f(x) \tag{2.6}$$

where

$$L_f^0 h(x) = h(x) \tag{2.7}$$

An important property of a nonlinear system is its relative degree.

Definition 1 *(A. Isidori [44]) " The single-input single-output nonlinear system:*

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x) \tag{2.8}$$

has relative degree r at x^0 if:

(i) $L_g L_f^k h(x) = 0$ for all x in a neighbourhood of x^0 and all $k < r - 1$.

(ii) $L_g L_f^{r-1} h(x^0) \neq 0$ "

In essence, the relative degree is the number of times that the output has to be differentiated with respect to time for the input u to appear explicitly. If $L_g L_f^k h(x) = 0$ for all $k > 0$, then $r = \infty$. This means that the output is not affected by the input. These cases rarely are founded in practice, but the system can have singular points where the relative degree is not well defined. This happens when $L_g L_f^{r-1} h(x) \neq 0$ for $x \neq x^0$ but $L_g L_f^{r-1} h(x^0) = 0$; here point x^0 is said to be a singular point. This is an important property of nonlinear systems and will be defined as follows.

Definition 2 *The single-input single-output nonlinear system:*

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2.9}$$

has not well defined relative degree r at x^0 if:

(i) $L_g L_f^k h(x) = 0$ *for all x in a neighbourhood of x^0 and all $k < r - 1$.*

(ii) $L_g L_f^{r-1} h(x^0) = 0$

this point x^0 is said to be a singular point.

The functions $h(x)$, $L_f h(x)$, $\ldots$, $L_f^{r-1} h(x)$ can be used in order to define, at least partially, a local coordinates transformation near point x^0. This is expressed in the following statement.

Proposition 2.2 *(A. Isidori [44] Proposition 4.1.3) " Suppose the system has relative degree r at x^0. Then $r \leq n$. Set:*

$$\begin{aligned} \phi_1(x) &= h(x) \\ \phi_2(x) &= L_f h(x) \\ &\vdots \\ \phi_r(x) &= L_f^{r-1} h(x) \end{aligned} \tag{2.10}$$

If r is strictly less than n, it is always possible to find $n-r$ more functions $\phi_{r+1}(x) \ldots \phi_n(x)$ such that the mapping:

$$\Phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \cdots \\ \phi_n(x) \end{bmatrix} \tag{2.11}$$

has Jacobian matrix which is nonsingular at x^0 and therefore qualifies as a local coordinates transformation in a neighbourhood of x^0. The value at x^0 of these additional functions can be fixed arbitrarily. Moreover, it is always possible to choose $\phi_{r+1}, \ldots, \phi_n(x)$ in such a way that: $L_g\phi_i(x) = 0$ for all $r+1 \leq i \leq n$ and all x around x^0."

The system in these z-coordinates $z_i = \phi_i$ where $1 \leq i \leq n$ is represented by the so-called normal form:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(z) + a(z)u \\ \dot{z}_{r+1} &= q_{r+1}(z) \\ &\vdots \\ \dot{z}_n &= q_n(z) \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} a(z) &= L_g L_f^{r-1} h[\phi^{-1}(z)] \\ b(z) &= L_f^r h[\phi^{-1}(z)] \\ q_k(z) &= L_f \phi_{k+r}[\phi^{-1}(z)] \qquad k = 1, \ldots, n-r \end{aligned}$$

Sometimes it is difficult to construct $\phi_{r+1}, \ldots, \phi_n(x)$, such that $L_g\phi_i(x) = 0$. However, these functions can be chosen with the only property that the Jacobian matrix of $\Phi(x)$ is nonsingular at x^0, and this is sufficient to define a coordinates transformation. But it is not possible to obtain anything special for the last $n-r$ coordinates, that therefore will appear in a form like:

$$\begin{aligned} \dot{z}_{r+1} &= q_{r+1}(z) + p_{r+1}(z)u \\ &\vdots \\ \dot{z}_n &= q_n(z) + p_n(z)u \end{aligned} \tag{2.13}$$

Consider a nonlinear system transformed into its normal form given by equation (2.13) with relative degree $r = n$, *i.e.* exactly equal to the dimension of the state space, at some point $x = x^0$. If the state feedback control law is chosen as:

$$u = -\frac{b(z)}{a(z)} + \frac{v}{a(z)} \tag{2.14}$$

the closed-loop system is governed by the equations:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= v \end{aligned} \tag{2.15}$$

Thus, we infer that any nonlinear system with relative degree n at some point x^0 can be transformed into a system which, in a neighbourhood of the point $z^0 = \phi(x^0)$, is linear and controllable. It is necessary to remark that the local input-output linearisation problem is solved by the diffeomorphism in equation (2.11) and the state feedback control law in equation (2.14).

2.4 The Zero Dynamics

In this section the concept of *zero dynamics* will be introduced. Zero dynamics are analogous to the zeros of the transfer function of a linear system.

The relative degree r of a linear system is the difference between the number of poles and the number of finite zeros, i.e. when r is less than n. If $r = n$ the transfer function does not have finite zeros. Thus, in the nonlinear case, if $r = n$ the system does not have zero dynamics. The zero dynamics of nonlinear systems have the same role as the zeros have in the internal stability of the linear systems.

Consider a nonlinear system with $r < n$, z is partitioned into two column vectors:

$$\xi = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_r \end{bmatrix}, \qquad \eta = \begin{bmatrix} z_{r+1} \\ z_{r+2} \\ \vdots \\ z_n \end{bmatrix}$$

The normal form equation (2.13) can be rewritten by

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(\xi,\eta) + a(\xi,\eta)u \\ \dot{\eta} &= q(\xi,\eta) \end{aligned} \tag{2.16}$$

Suppose that we want the system output $y = 0$ for all $t \geq 0$. It is clear from this equation that $\xi(t)$ is zero, $u(t)$ has to be

$$u = \frac{-b(0,\eta(t))}{a(0,\eta(t))} \tag{2.17}$$

and $\eta(t)$ will be the solution of the following dynamics

$$\dot{\eta}(t) = q(0,\eta(t)) \tag{2.18}$$

These describe the internal behaviour of the system when the input u and the initial conditions were chosen such that the output y equal zero at all the times (A. Isidori [44]). These dynamics are very important and will be defined as follows.

Definition 3 *(A. Isidori [44])" The dynamics* $\dot{\eta} = q(0,\eta(t))$ *are called the zero dynamics of the system."*

The use of terminology *zero* is related to the zeros of the transfer function in a linear system. Consider the transfer function of a linear system with relative degree r as follows:

$$H(s) = K\frac{b_0 + b_1 s + \ldots + b_{n-r-1}s^{n-r-1} + s^{n-r}}{a_0 + a_1 s + \ldots + a_{n-1}s^{n-1} + s^n} \tag{2.19}$$

The representation in states space of $H(s)$ is given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2.20}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -a_0 & -a_1 & -a_2 & \ldots & -a_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ K \end{bmatrix}$$

$$C = [b_0\ b_1\ \ldots\ b_{n-r-1}\ 1\ 0 \ldots 0] \tag{2.21}$$

There are many ways to choose the new coordinates but the simplest one is as follows:

$$\begin{aligned} z_1 &= Cx = b_0 x_1 + b_1 x_2 + \ldots + b_{n-r-1}x_{n-r} + x_{n-r+1} \\ z_2 &= CAx = b_0 x_2 + b_1 x_3 + \ldots + b_{n-r-1}x_{n-r+1} + x_{n-r+2} \\ &\vdots \\ z_r &= CA^{r-1}x = b_0 x_r + b_1 x_{r+1} + \ldots + b_{n-r-1}x_{n-1} + x_n \\ z_{r+1} &= x_1 \\ z_{r+2} &= x_2 \\ &\vdots \\ z_n &= x_{n-r} \end{aligned} \tag{2.22}$$

In the new coordinates the normal form has the following structure

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
&\vdots \\
\dot{z}_{r-1} &= z_r \\
\dot{z}_r &= R\xi + S\eta + Ku \\
\dot{\eta} &= P\xi + Q\eta
\end{aligned} \tag{2.23}$$

where R and S are row vectors and P and Q matrices. The zero dynamics are

$$\dot{\eta} = Q\eta \tag{2.24}$$

It is easily checked that

$$\begin{aligned}
\frac{dz_{r+1}}{dt} &= \frac{dx_1}{dt} = x_2(t) = z_{r+2}(t) \\
&\vdots \\
\frac{dz_{n-1}}{dt} &= \frac{dx_{n-r-1}}{dt} = x_{n-r}(t) = z_n(t) \\
\frac{dz_n}{dt} &= \frac{dx_{n-r}}{dt} = x_{n-r+1}(t) = -b_0x_1(t) - \ldots - b_{n-r-1}x_{n-r} + z_1(t) \\
&= -b_0z_{r+1}(t) - \ldots - b_{n-r-1}z_n(t) + z_1(t)
\end{aligned} \tag{2.25}$$

Then the matrix Q is given by

$$Q = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -b_0 & -b_1 & -b_2 & \ldots & -b_{n-r-1} \end{bmatrix} \tag{2.26}$$

It is evident that the eigenvalues of this matrix have the same values as the zeros of the transfer function. Therefore we can infer that for the linear case, the zero dynamics are linear dynamics with eigenvalues coinciding with the zeros of the transfer function of the system.

Remark 1 *(A. Isidori [44] Remark 4.3.2) " In the nonlinear case the linear approximation of the zero dynamics of the system at* $\eta = 0$ *coincides with the zero dynamics of the linear approximation of the entire system at* $x = 0$.

The linear approximation is given by

$$\dot{x} = Ax + Bu \tag{2.27}$$
$$y = Cx$$

where $A = \left[\frac{\partial f}{\partial x}\right]_{x=0}$, $B = g(0)$ *and* $C = \left[\frac{\partial h}{\partial x}\right]_{x=0}$

Taking the linear approximation of the normal form, gives:

$$b(\xi, \eta) = R\xi + S\eta \tag{2.28}$$
$$a(\xi, \eta) = K \tag{2.29}$$
$$q(\xi, \eta) = P\xi + Q\eta \tag{2.30}$$

a linear system in normal form is obtained, where the Jacobian matrix

$$Q = \left[\frac{\partial q}{\partial \eta}\right]_{(\xi,\eta)=0} \tag{2.31}$$

describes the linear approximation at $\eta = 0$ *of the zero dynamics of the original nonlinear system, its eigenvalues coincide with the zeros of the transfer function of the linear approximation at* $x = 0$ *of the original nonlinear system."*

2.5 Local Asymptotic Stabilisation

The zero dynamics play an important role in the problem of asymptotically stabilising a nonlinear system at a given equilibrium point. Suppose that $f(x)$ has an equilibrium point at x^0 that, without loss of generality, we assume to be $x^0 = 0$. The local asymptotic stabilisation problem is to find a smooth state feedback $u = \alpha(x)$ defined locally around the point $x^0 = 0$ and as well it is desired to preserve the equilibrium, *i.e.* such that $\alpha(0) = 0$ (and so $u = 0$), with the property that the closed loop

$$\dot{x} = f(x) + g(x)\alpha(x) \tag{2.32}$$

is locally asymptotically stable at $x = 0$.

Proposition 2.3 *(A. Isidori [44] Proposition 4.4.1) " Suppose the linear approximation is asymptotically stabilisable, i.e. either the pair (A, B) is controllable or -in case the pair (A,B) is not controllable- the uncontrollable modes correspond to eigenvalues with negative real part. Then, any linear feedback which asymptotically stabilises the linear approximation is also able to asymptotically stabilise the original nonlinear system, at least locally. If the pair (A,B) is not controllable and there exist uncontrollable modes associated with positive real part, the original nonlinear system cannot be stabilised at all."*

However, there are cases where the pair (A,B) is not controllable and there are uncontrollable modes associated with eigenvalues on the imaginary axis (although none are in the right-half complex plane), under these conditions it is not able to know for sure that the system is asymptotically stabilisable. The zero dynamics play an important role in the solution of this problem. As mentioned before, nonlinear systems with asymptotically stable zero dynamics are analogous of linear systems with zeros in the left-half plane.

Suppose the system has the normal form as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(\xi,\eta) + a(\xi,\eta)u \\ \dot{\eta} &= q(\xi,\eta) \end{aligned} \tag{2.33}$$

Without loss of generality, assume that $(\xi, \eta) = (0, 0)$ is an equilibrium point and that input u is defined as:

$$u = \frac{1}{a(\xi,\eta)}(-b(\xi,\eta) - c_0 z_1 - c_1 z_2 - \ldots - c_{r-1} z_r + v) \tag{2.34}$$

where $c_0, c_1, \ldots, c_{r-1}$ are real numbers.

The closed loop system with this control law is given by:

$$\begin{aligned} \dot{\xi} &= A\xi + Bv \\ \dot{\eta} &= q(\xi,\eta) \end{aligned} \tag{2.35}$$

with:

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -c_0 & -c_1 & -c_2 & \ldots & -c_{r-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The matrix A has characteristic polynomial:

$$p(s) = c_0 + c_1 s + \ldots + c_{r-1} s^{r-1} + s^r \tag{2.36}$$

We can conclude that the system has a $n-r$ dimensional subsystem $\dot{\eta} = q(\xi,\eta)$ that is not observable from the output; if the zero dynamics are asymptotically stable and the coefficients of A are properly chosen, then, the closed loop system is asymptotically stable. If the zero dynamics are not asymptotically stable, the nonlinear system is called nonminimum phase. These kind of systems are not internally stable in the sense that state variables can be *unstable* despite the fact that the system is stable in an input-output sense.

Proposition 2.4 *(A. Isidori [43]) " Consider a system:*

$$\begin{aligned} \dot{\xi} &= A\xi + \gamma(\xi) \\ \dot{\eta} &= q(\xi,\eta) \end{aligned} \tag{2.37}$$

in which ξ *and* η *represent vectors (of suitable dimensions), and:*

$$\gamma(0) = 0, \qquad \frac{\partial \gamma}{\partial \xi}(0) = 0, \qquad q(0,0) = 0$$

Suppose $q(0,\eta)$ *is asymptotically stable at* $\eta = 0$. *Suppose also* A *has all the eigenvalues in the left-half complex plane and no eigenvalue coinciding with those of the matrix:*

$$Q = \left[\frac{\partial q(\xi,\eta)}{\partial \eta}\right]_{(\xi,\eta)=(0,0)}$$

Then the system is locally asymptotically stable at $(\xi,\eta) = (0,0)$*."*

Remark 2 *(A. Isidori [43]) " Note that the result stated in the Proposition 2.4 holds under the assumption that* $q(0,\eta)$ *is asymptotically stable at* $\eta = 0$. *In a nonlinear setting, this does not necessarily require that the matrix:*

$$Q = \left[\frac{\partial q(\xi,\eta)}{\partial \eta}\right]_{(\xi,\eta)=(0,0)}$$

has all the eigenvalues in the left-half complex plane (in which case all the eigenvalues of the linear approximation will have negative real part and the result of the Proposition would be a trivial consequence of the so-called principle of stability in the first approximation), but might as well be true in the presence of some eigenvalue of Q *lying on the imaginary axis. In other words, noting that the matrix* Q *is nothing else than the linear approximation of the zero dynamics at* $\eta = 0$, *we may say that the proposed stabilising feedback works not only when the linear approximation of the zero dynamics has all eigenvalues with negative real part, but in the broader situation in which the zero dynamics is simply asymptotically stable."*

Using the original coordinates the control law equation (2.34) can be rewritten as:

$$u = \frac{1}{L_g L_f^{r-1} h(x)}(-L_f^r h(x) - c_0 h(x) - c_1 L_f h(x) - \ldots - c_{r-1} L_f^{r-1} h(x) + v) \tag{2.38}$$

We can summarise all we say before in a formal statement

Proposition 2.5 *(A. Isidori [44] Proposition 4.4.2)* "*Suppose the equilibrium* $\eta = 0$ *of the zero dynamics of the system is locally asymptotically stable and all roots of the polynomial* $p(s)$ *have negative real part. Then the feedback law (2.38) locally asymptotically stabilises the equilibrium* $(\xi, \eta) = (0, 0)$."

2.6 Disturbance Decoupling and Model Matching

The process can be subjected to disturbances which enter either in the states and/or input, or at the output. In the linear case the disturbances can be treated as *output* disturbances due to the superposition principle. But, in the nonlinear case this principle no longer applies, and the disturbances must be considered separately. Consider a system with a disturbance w:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)w \\ y &= h(x) \end{aligned} \tag{2.39}$$

The local disturbance decoupling problem is to find a control law

$$u = \alpha(x) + \beta(x)v \tag{2.40}$$

and a diffeomorphism such that the system will be linear and controllable and the output y is unaffected by the disturbance w. If the disturbance has relative degree ρ at point x^0 then, $L_pL_f^kh(x) = 0$ for all x near x^0 and for all $k < \rho - 1$; and $L_pL_f^{\rho-1}h(x) \neq 0$. If the two conditions are satisfied, then $\rho \leq n$. If $L_pL_f^kh(x) = 0$ for all k then $\rho = \infty$ and w does not affect the output. If $\rho < r$ the disturbance affects the output more directly than u and w can not be decoupled from y using a control law equation (2.40). Thus, in order to solve the disturbance decoupling problem, the relative degree of the disturbance has to be $\rho > r$; this assumption is often called a disturbance matching condition. This is summarised in the following statement.

Proposition 2.6 *(A. Isidori [44] Proposition 4.6.1)* "*Suppose the system has relative degree* r *at* x^0. *The problem of finding a feedback* $u = \alpha(x) + \beta(x)v$, *defined locally* *around x^0, such that the output of the system is decoupled from the disturbance can be solved if and only if*

$L_pL_f^ih(x) = 0$ *for all* $0 \leq i \leq r-1$ *and all x near x^0.*

If this is the case, then a solution is given by

$$u = -\frac{L_f^rh(x)}{L_gL_f^{r-1}h(x)} + \frac{v}{L_gL_f^{r-1}h(x)}"$$

Sometimes, the *measurements* of the disturbance are available, and can be used to design the control law as follows:

$$u = \alpha(x) + \beta(x)v + \gamma(x)w \tag{2.41}$$

A necessary and sufficient condition for the disturbance decoupling problem with measurement is that $\rho \geq r$. The problem can solved by a feedforward-feedback control law:

$$u = -\frac{L_f^rh(x)}{L_gL_f^{r-1}h(x)} + \frac{1}{L_gL_f^{r-1}h(x)}v - \frac{L_wL_f^{r-1}h(x)}{L_gL_f^{r-1}h(x)}w \tag{2.42}$$

When the reference output is given by the output of a reference model, for instance a linear model described by:

$$\begin{aligned} \dot{\xi} &= A\xi + Bw \\ y_R &= C\xi \end{aligned} \tag{2.43}$$

the feedback control law which enables the output $y(t)$ to asymptotically converge to the output $y_R(t)$, regardless of the initial conditions of the system and of the reference model, is given by

$$\begin{aligned} u = \frac{1}{L_gL_f^{r-1}h(x)}(&-L_f^rh(x) + CA^r\xi(t) \\ &+CA^{r-1}Bw - \sum_{i=1}^{r} c_{i-1}(L_f^{(i-1)}h(x) - CA^{(i-1)}\xi)) \end{aligned} \tag{2.44}$$

This problem is known as asymptotic model matching.

2.7 Conclusions

In this chapter the nonlinear feedback linearisation control developed by A. Isidori [44] and differential notation were reviewed.

The single-input single-output nonlinear systems considered are described by equation (2.1). Note that the input u appears linearly in the equation. Nonlinear dynamic systems with the following non-affine state-space representation are not considered by nonlinear feedback linearisation control, but are considered by the proposed controllers in Chapter 5:

$$\begin{aligned} \dot{x}(t) &= F(x(t), u(t)) \\ y(t) &= h(x(t)). \end{aligned}$$

In Sections 2.2.1 and 2.2.2 the transformation of coordinates and the Lie derivative were presented in order to define the concept of relative degree and to develop the well known nonlinear feedback linearisation control given by equation (2.14), This control law is compared in Chapter 6 with the new algorithms developed in Chapter 5. Also, the relative degree and Lie derivative are used in next Chapters.

The important concept of the *zero dynamics* is given by Definition 2 in Section 2.3. It was also shown in Section 2.4 that the *zero dynamics* play an important role in the stability of the system. Proposition 2.4 assumes that *zero dynamics* are asymptotically stable in order for the feedback control given by equation (2.14) to asymptotically stabilise the system. Thus, we can see that systems with unstable *zero dynamics* can not be controlled by the nonlinear feedback linearisation control reviewed in this chapter. New algorithms will be developed in Chapter 5, which can control such systems.

It is necessary to emphasise that the relative degree r is the number of times that the output has to be differentiated with respect to time until the input u appears explicitly. Unfortunately, the system can have singular points where the relative degree is not well defined: this happens when $L_g L_f^{r-1} h(x) \neq 0$ for $x \neq x_0$ but $L_g L_f^{r-1} h(x_0) = 0$, this point

x_0 is said to be a singular point. Referring back to the nonlinear feedback linearisation control given by equation (2.38) :

$$u = \frac{1}{L_g L_f^{r-1} h(x)} (-L_f^r h(x) - c_0 h(x) - c_1 L_f h(x) - \ldots - c_{r-1} L_f^{r-1} h(x) + v)$$

we can see that this exact input-output linearisation approach is not applicable to these systems. Again new algorithms are proposed in Chapter 5 which are applicable.

If the system has a well defined relative degree r, the first r output derivatives are obtained in order to get the nonlinear feedback linearisation control given in Proposition 2.4. Thus, the relative degree has to be known. New algorithms developed in Chapter 5 remove this restrictive assumption.

In Section 2.5 the disturbance decoupling problem and disturbance decoupling problem with measurement were mentioned and solved. These were used by J. Alvarez and J. Alvarez [2] in order to develop the Tracking and Regulation Reference Model Control of Nonlinear Systems (TRRMCNL), which is reviewed in Chapter 4. In this section, asymptotic model matching is also solved, which is used in Chapter 6 in order to compare with the new algorithms developed in Chapter 5.

Chapter 3

Globally Linearising Control

3.1 Introduction

Several controllers based on feedback linearisation, such as (A. Isidori [44], P. Lee *et. al.* [52], C. Economu and M. Morari [25] and C. Kravaris and C. Chung [48]) have been proposed for non-linear processes. In this chapter one of them the GLC (Globally Linearising Control) developed by C. Kravaris and C. Chung [48] is reviewed. The most important characteristics and aspects are given, the error feedback control structure is reviewed and critically evaluated. This approach will be used in the chapter 6 where will be shown that this controller is equivalent to NGMV.

3.2 Globally Linearising Control

Consider a nonlinear system described by equation (2.1) and the problem of finding a static state feedback such that the $v - y$ input/output system is linear and of minimal order. This problem was solved by C. Kravaris and C. Chung [48]. The nonlinear state feedback law that makes this possible, is established in the following theorem.

Theorem 3.1 *(C. Kravaris and C. Chung [48]) " Consider a nonlinear system of the form of equation (2.1) and assume that it has relative degree equal to* r. *Then there is always a static state feedback of the form* $u = \alpha(x) + \beta(x)v$ *that makes the input/output* *behaviour of the closed loop system linear. In particular, the static state feedback that makes the closed-loop behaviour linear and of minimal order is of the form*

$$u = \frac{v - \sum_{i=0}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)} \tag{3.1}$$

where $\beta_1, \ldots, \beta_r$ are scalar constant parameters. The input/ouput behaviour of the closed loop system is then governed by

$$\sum_{i=0}^{r} \beta_i \frac{d^i y}{dt^i} = v(t)'' \tag{3.2}$$

The signal v is generated by an external linear control, which can be, for instance, a PI controller as follows:

$$v = K_c \left[(y_{sp} - y) + \frac{1}{\tau_I} \int_0^t (y_{sp} - y) \right] \tag{3.3}$$

The closed loop transfer function is given by

$$\frac{y(s)}{y_{sp}(s)} = \frac{K_c s + \frac{K_c}{\tau_I}}{\beta_r s^{r+1} + \beta_{r-1} s^r + \ldots + (\beta_0 + K_c)s + \frac{K_c}{\tau_I}} \tag{3.4}$$

The parameters β_i are chosen in order to the closed-loop system will be BIBO stable and the tracking error tends to zero; that is: the roots of the denominator must have negative real parts. The structure of the GLC is shown in Figure 3.1.

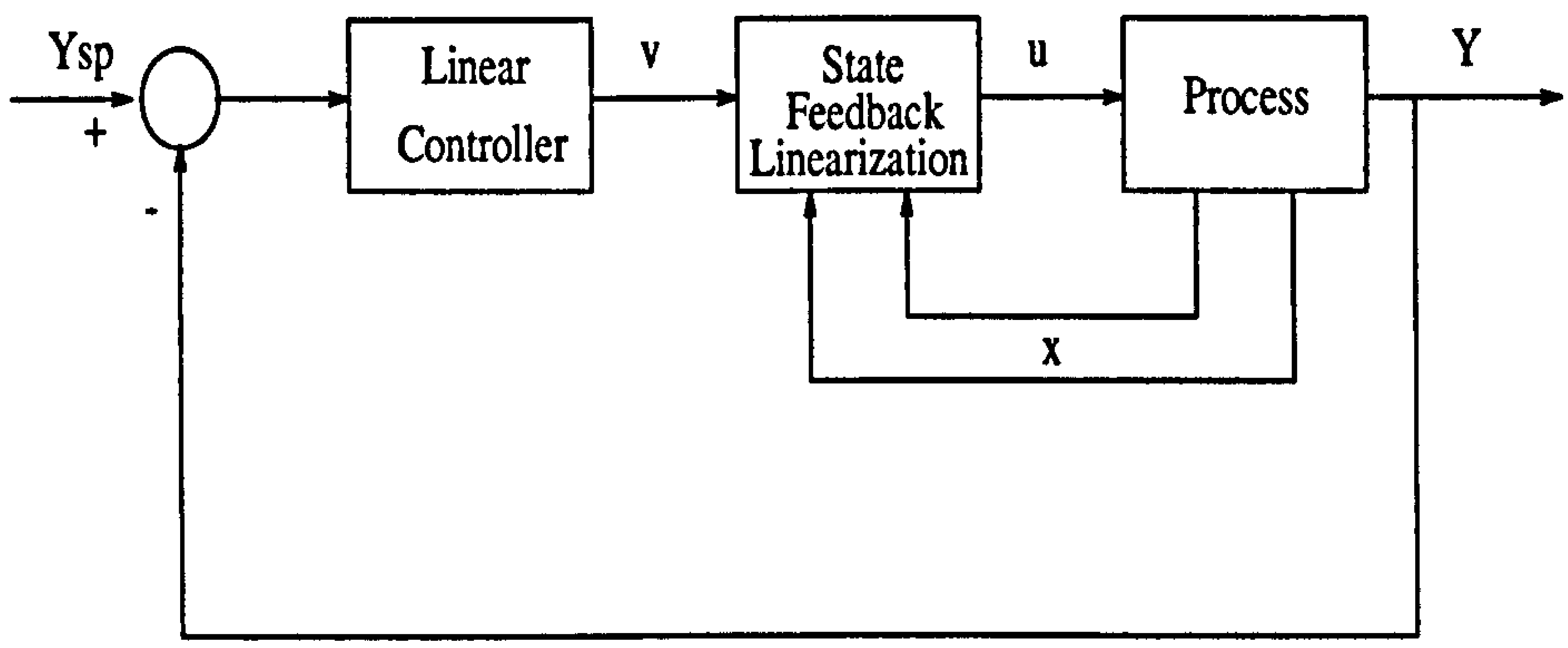

Figure 3.1: GLC structure

3.3 Error feedback GLC

In this section a dynamic output feedback controller together with a connection between the state space and input-output approaches are summarised, this section is based on

P. Daoutidis and C. Kravaris [24].

The review of key elements of the input/output approach is necessary in order to carry out the connection. First, we review the structure of the classical output feedback control, which is shown in Figure 3.2, where P and C represent nonlinear input/output operators of the nonlinear process and the classical feedback controller, respectively. Consider C as:

$$C = Q(I - PQ)^{-1} \tag{3.5}$$

where I denotes the identity operator and Q an appropriate nonlinear operator. The

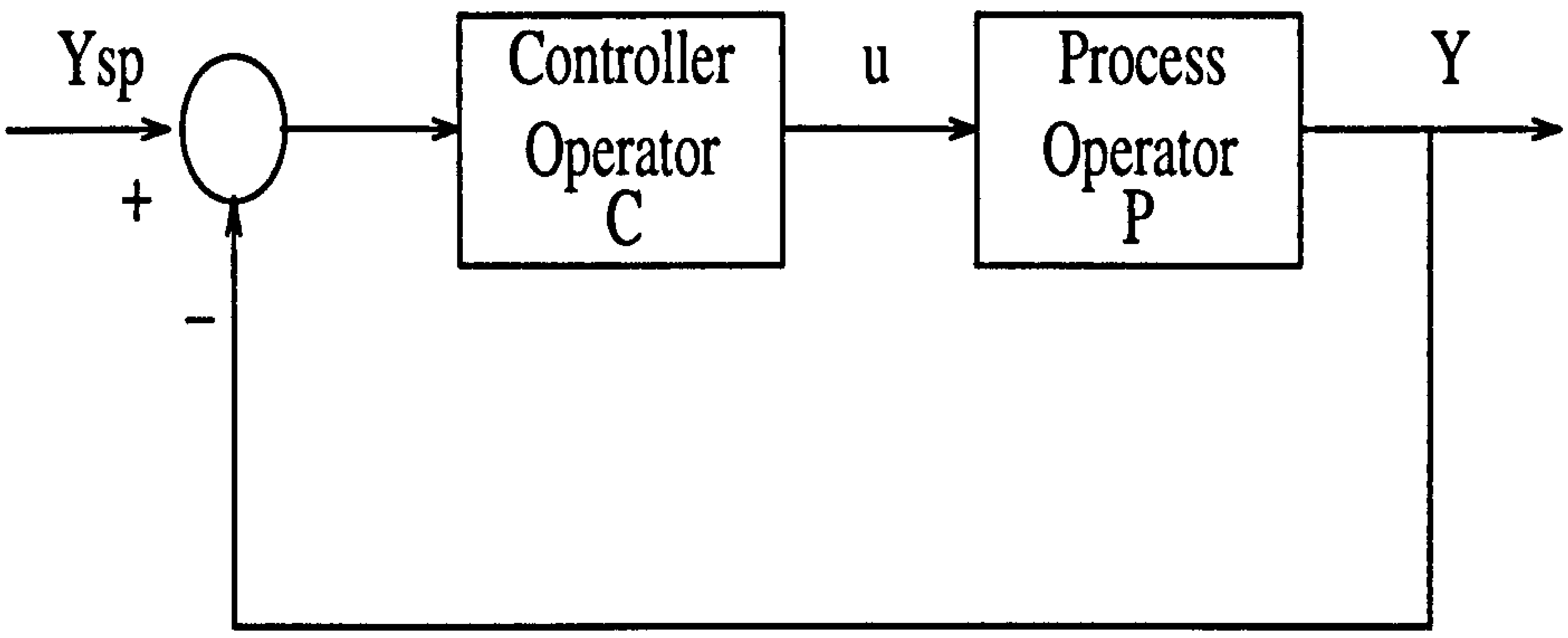

Figure 3.2: Classical output feedback control structure

equivalent control structure is shown in Figure 3.3 which is the IMC (Internal Model

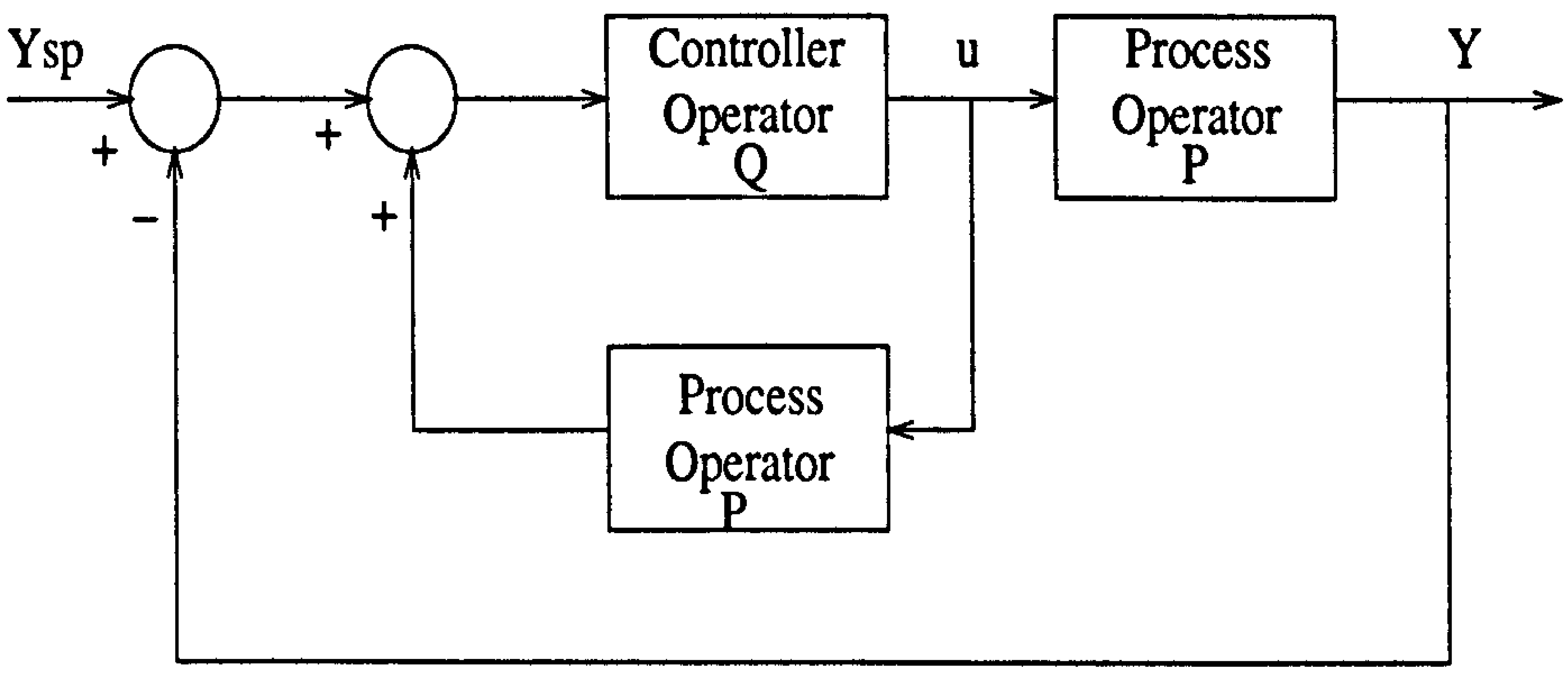

Figure 3.3: IMC structure

Control) where Q will be the IMC controller. The input-output response of the closed loop system is

$$y = PC(I + PC)^{-1} y_{sp} \tag{3.6}$$

or equivalently, by:

$$y = PQy_{sp} \tag{3.7}$$

If the choice of the controller operator is

$$C = P^{-1}R(I - R)^{-1} \tag{3.8}$$

or equivalently

$$Q = P^{-1}R \tag{3.9}$$

where R is the desired closed-loop operator, the output response will be given by

$$y = Ry_{sp}$$

The above analysis provides useful insights on the nature of the output feedback problem and illustrates clearly the importance of the inverse operator P^{-1} in the controller synthesis. On the other hand, the treatment is purely macroscopic and issues like the order of the closed-loop operator R and the implementation of the controller operator C (or Q) are obscure in the above abstract setting. This is in contrast to the linear case, where the transfer function description captures essential information (poles, zeros, relative degree, etc.) and allows an explicit solution to the controller synthesis problem.

P. Daoutidis and C. Kravaris [24] derived a dynamic output feedback controller, which induces a closed loop response between y and y_{sp} given by the following transfer function:

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{(\epsilon s + 1)^r} \tag{3.10}$$

where ϵ is an adjustable parameter. The characterisation " dynamic output feedback controller " means the controller will be a dynamic nonlinear system itself, with inputs the setpoint y_{sp}, the system output y and, possibly, some derivatives of y, and with output the manipulated input u. The output feedback controller is based on state feedback/state observer combination. P. Daoutidis and C. Kravaris [24] found an interpretation from an input/output perspective and a precise connection between the state space and input-output approaches is established.

When the system is open loop stable, the GLC is modified, the states are reconstructed through an open-loop state observer. This results in an error feedback structure shown in Figure 3.4. It is possible to see the similarity between this structure and the IMC structure. The open-loop observer is an internal model which is simulated in parallel to the process, and the input/output linearising control law of equation (3.1) can be viewed as implicitly generating an inverse of the system. P. Daoutidis and C.

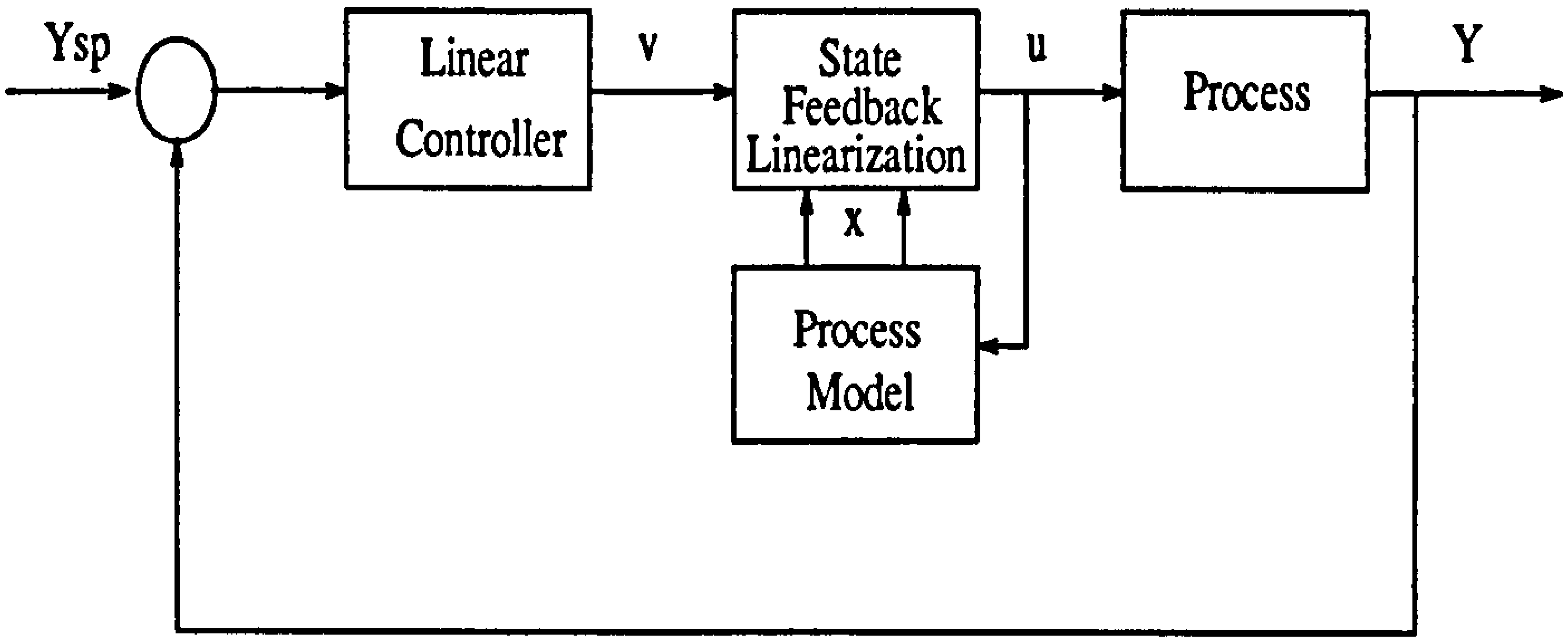

Figure 3.4: GLC structure with open loop observer

Kravaris [24] established that the GLC-open loop observer shown in Figure 3.4 can solve the output feedback control problem and also, an interpretation is found from input/output perspective in the following Theorem.

Theorem 3.2 *(P. Daoutidis and C. Kravaris [24]) " Consider a nonlinear system of the form of equation (2.1), with relative degree r. Then, the dynamic system*

$$\begin{aligned} \dot{\xi} &= A^*\xi + b^*(y_{sp} - y) \\ \dot{w} &= f(w) + g(w)u \\ u &= \frac{c^*\xi + \frac{\beta_r}{\epsilon^r}(y_{sp} - y) - \sum_{i=0}^{r}\beta_i L_f^k h(w)}{\beta_r L_g L_f^{r-1} h(w)} \end{aligned} \tag{3.11}$$

where

$$A^* = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & -\frac{r}{\epsilon^{r-1}} & \dots & -\frac{r(r-1)}{2!\epsilon^2} & -\frac{r}{\epsilon} \end{bmatrix}, \quad b^* = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and

$$c^* = \frac{1}{\epsilon^r}\left[\beta_0 \dots (\beta_{r-2} - \beta_r \frac{r(r-1)}{2!\epsilon^2}) \ (\beta_{r-1} - \beta_r \frac{r}{\epsilon})\right]$$

represents an $(n+r)-th$ order state-space realisation of a dynamic output feedback controller which, under appropriate initialisation, induces the closed-loop transfer function:

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{(\epsilon s + 1)^r} \text{"}$$

Proof: (P. Daoutidis and C. Kravaris [24]) " Define the auxiliary variable

$$v = c^*\xi + \frac{\beta_r}{\epsilon^r}(y_{sp} - y) \tag{3.12}$$

Then, the new system is

$$\begin{aligned} \dot{\xi} &= A^*\xi + b^*(y_{sp} - y) \\ v &= c^*\xi + \frac{\beta_r}{\epsilon^r}(y_{sp} - y) \end{aligned} \tag{3.13}$$

which has the following transfer function

$$\frac{v(s)}{y_{sp}(s) - y(s)} = \frac{\beta_0 + \beta_1 s + \dots + \beta_r s^r}{(\epsilon s + 1)^r - 1} \tag{3.14}$$

which can be interpreted as a choice of the linear controller in the GLC structure. The other subsystem of equation (3.12) becomes

$$\begin{aligned} \dot{w} &= f(w) + g(w)u \\ u &= \frac{v - \sum_{i=0}^{r} \beta_i L_f^k h(w)}{\beta_r L_g L_f^{r-1} h(w)} \end{aligned} \tag{3.15}$$

which is identical to the GLC control law equation (3.1) where the states are obtained from an open loop state observer. With an appropriate initialisation, it induces exactly the response of equation (3.2), where v is given by equation (3.12). The transfer function between $y(s)$ and $y_{sp}(s)$ is given by

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{(\epsilon s + 1)^r} \tag{3.16}$$

which completes the proof. We can see from an input/output perspective that the input of the controller is the tracking error $y_{sp} - y$ and its output is the value of the input u. Therefore, the control structure coincides with the classical error feedback control structure of Figure (3.2) and equation (3.12) can be interpreted as a state-space realisation of the classical controller C. From the equation (3.8), the two elements of the controller can be identified. The first element is the dynamic system

$$\begin{aligned} \dot{\xi} &= A^*\xi + b^*(y_{sp} - y) \\ y^* &= \frac{1}{\epsilon^r}\xi_1 \end{aligned} \tag{3.17}$$

which is the minimal state space realisation of the operator $R(I-R)^{-1}$, *i.e.* the transfer function

$$\frac{1}{(\epsilon s + 1)^r - 1} \tag{3.18}$$

the following expression can be easily verified

$$c^*\xi + \frac{\beta_r}{\epsilon^r}(y_{sp} - y) = \sum_{i=0}^{r} \beta_i \frac{d^i y^*}{dt^i} \tag{3.19}$$

The second element of the controller will be

$$\begin{aligned} \dot{w} &= f(w) + g(w)u \\ u &= \frac{\sum_{i=0}^{r} \beta_i \frac{d^i y^*}{dt^i} - \sum_{i=0}^{r} \beta_i L_f^k h(w)}{\beta_r L_g L_f^{r-1} h(w)} \end{aligned} \tag{3.20}$$

this element can be interpreted as a (full-order) realisation of the inverse operator P^{-1}, because under an appropriate initialisation, it can be reduced to the full-order realisation of the inverse system R. M. Hirschorn [39] given by

$$\begin{aligned} \dot{w} &= f(w) + g(w)u \\ u &= \frac{\frac{d^r y^*}{dt^r} - L_f^r h(w)}{\beta_r L_g L_f^{r-1} h(w)} \end{aligned} \tag{3.21}$$

The structure of the error feedback control is given in the Figure 3.5, where the two components of the controller identified above are given in operator form."

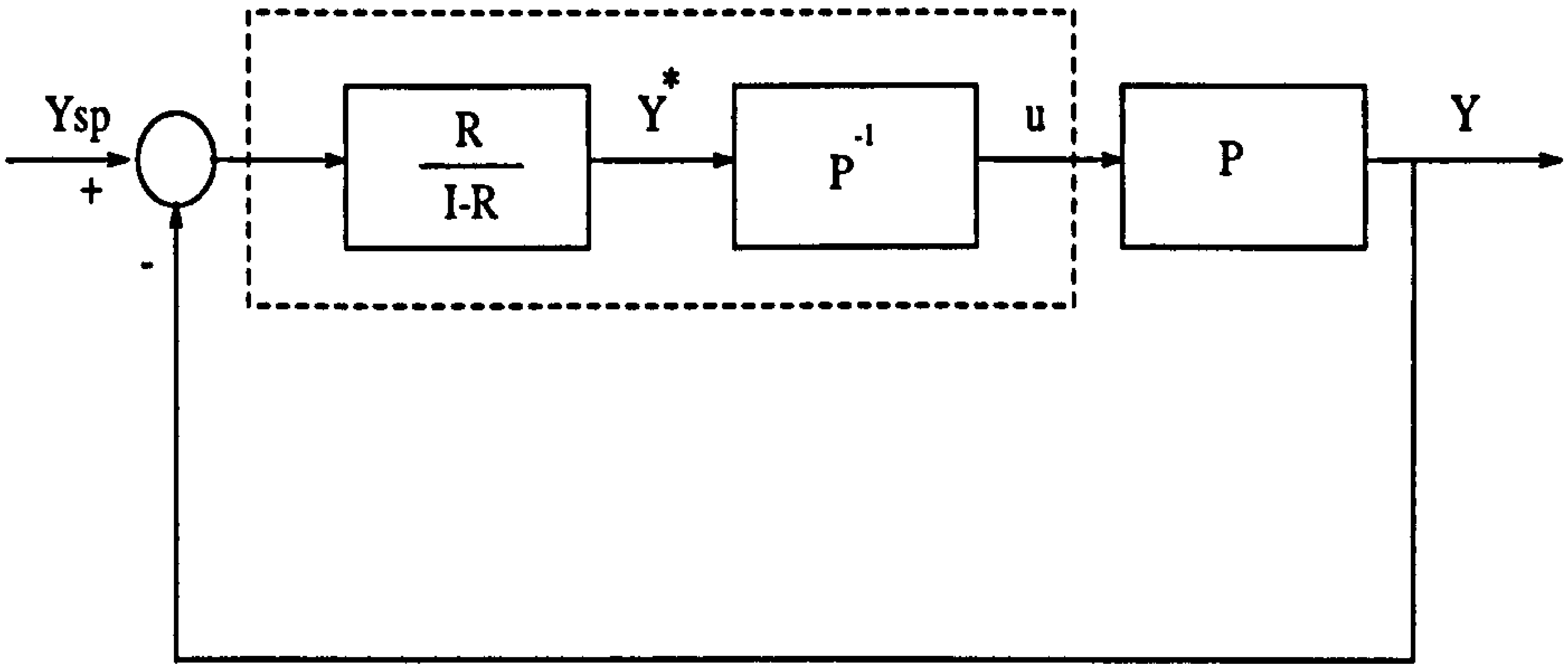

Figure 3.5: Error feedback control structure for open-loop stable processes.

3.4 A Nonlinear Analogue of Placing Poles at Process Zeros

In this section a summary based on C. Kravaris [47] is presented, where the concept of placing poles at the process zeros is extended to nonlinear systems. The concept of zero dynamics given by C. Byrnes and A. Isidori [8] and reviewed in the Chapter 2 is used for this purpose. Also, in this summary, it is shown that the GLC places poles at the process zeros in a nonlinear process. The following results will be necessary to make the analogies.

Proposition 1 *(C. Kravaris [47]) " Consider a linear system*

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx \end{aligned} \tag{3.22}$$

and assume that its relative degree is r. Then the state feedback

$$u = v - \frac{1}{cA^{r-1}b}\left(\sum_{k=0}^{r}\beta_k c A^k\right) x \tag{3.23}$$

places the poles at the roots of $(n-r)$th degree polynomial $cAdj(sI-A)b$ and at the roots of the rth degree polynomial $\sum_{k=0}^{r}\beta_k s^k$. The resulting closed-loop transfer function is of the form

$$\frac{y(s)}{v(s)} = \frac{constant}{\beta_r s^r + \beta_{r-1}s^{r-1} + \ldots + \beta_1 s + \beta_0} \tag{3.24}$$

The state feedback, equation (3.23) cancels all the zeros of the process by placing poles at them. It is clear that the closed-loop system will be internally stable if and only if all the zeros of equation (3.22) are in the left half-plane, that is, if and only if the process is minimum phase. "

Consider an invertible transformation of the system equation (2.1)

$$\xi = T(x) = \begin{bmatrix} t_1(x) \\ \vdots \\ t_{n-r}(x) \\ h(x) \\ L_f h(x) \\ \vdots \\ L_f^{r-2}h(x) \\ L_f^{r-1}h(x) \end{bmatrix}$$

that transforms the system equation (2.1) into Byrnes-Isidori canonical form C. Byrnes and A. Isidori [8]:

$$\begin{aligned} \dot{\xi}_1 &= F_1(\xi_1, \ \ldots, \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\ &\vdots \\ \dot{\xi}_{n-r} &= F_{n-r}(\xi_1, \ \ldots, \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\ \dot{\xi}_{n-r+1} &= \xi_{n-r+2} \end{aligned}$$

$$\vdots$$

$$\dot{\xi}_{n-1} = \dot{\xi}_n$$

$$\dot{\xi}_n = L_f^r h(x) + L_g L_f h(x) u(t) \tag{3.25}$$

$$y = \xi_{n-r+1} \tag{3.26}$$

The first $n-r$ equations, represent the dynamic zeros. The $y = \xi_{n-r+1}$ is affected by the zero dynamics through the right-hand side of the nth equation. Therefore, in order to cancel the zero dynamics, we need a state feedback that makes the right-hand side of the nth equation a function of $\xi_{n-r+1}, \ldots, \xi_n$ and v. It is requested to be a linear function

$$L_f^r h(x) + L_g L_f h(x) u(t) = \frac{1}{\beta_r}(v - \beta_0 \xi_{n-r+1} - \ldots - \beta_{r-1}\xi_n) \tag{3.27}$$

This leads to the closed-loop system

$$\dot{\xi}_1 = F_1(\xi_1, \ \ldots \ , \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n)$$

$$\vdots$$

$$\dot{\xi}_{n-r} = F_{n-r}(\xi_1, \ \ldots \ , \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n)$$

$$\dot{\xi}_{n-r+1} = \dot{\xi}_{n-r+2}$$

$$\vdots$$

$$\dot{\xi}_{n-1} = \dot{\xi}_n$$

$$\dot{\xi}_n = \frac{-\beta_0}{\beta_r}\xi_{n-r+1} - \ldots - \frac{\beta_{r-1}}{\beta_r}\xi_n + \frac{1}{\beta_r}v \tag{3.28}$$

It is possible to see the output is completely unaffected by the first $n-r$ equations. Equation 3.27, can be written in terms of the original variables as

$$u(t) = \frac{v - \sum_{i=0}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)} \tag{3.29}$$

We can see that this equation is identical to equation 3.1.

It is very interesting to remark here the analogies between the input/output linearising state feedback GLC and the linear state feedback equation (3.23). When

$f(x) = Ax$, $g(x) = b$ and $h(x) = cx$, equation (3.1) becomes

$$u = \frac{v}{\beta_r cA^{r-1}b} - \frac{1}{\beta_r cA^{r-1}b}\left[\sum_{k=0}^{r}\beta_k cA^k\right]x \tag{3.30}$$

which is identical to equation (3.23) under appropriate rescaling of u and v. This state feedback places poles at the zeros of equation (3.24) or equivalently cancels the zeros of equation (3.22).

In order to establish an important theorem which establish the necessary condition for internal stability, the following definitions are presented.

Definition 1 *(C. Kravaris [47]) " Consider a nonlinear system of the form of equation (2.1) whose relative degree is* r *and an invertible transformation* $\xi = T(X)$ *of the form equation (3.26) that transforms equation (2.1) into (3.28). Assume that with appropriate translation of axes the origin is an equilibrium point of equation (3.28). The (forced) zero dynamics of equation (2.1) is the* $(n-r)-$*order dynamic system*

$$\begin{aligned} \dot{z}_1 &= F_1(z_1,\ldots,z_{n-r},U_1,\ldots,U_r) \\ &\vdots \\ \dot{z}_{n-r} &= F_{n-r}(z_1,\ldots,z_{n-r},U_1,\ldots,U_r) \end{aligned} \tag{3.31}$$

(3.32)

In particular, the unforced zero dynamics is the $(n-r)-$*order unforced dynamic system*

$$\begin{aligned} \dot{z}_1 &= F_1(z_1,\ldots,z_{n-r},0,\ldots,0) \\ &\vdots \\ \dot{z}_{n-r} &= F_{n-r}(z_1,\ldots,z_{n-r},0,\ldots,0)'' \end{aligned} \tag{3.33}$$

(3.34)

Definition 2 *(C. Kravaris [47]) " Under the assumptions of definition 1, we will say that the nonlinear system of (2.1) has stable zero dynamics if for any set of initial* *conditions* $z_1(0), \ldots, z_{n-r}(0)$ *and any exponentially decaying* $U_1, \ldots, U_r$,

$$\lim_{t \longrightarrow \infty} z_i(t) = 0 \qquad i = 1, \ldots, n - r'' \tag{3.35}$$

Theorem 3.3 *(C. Kravaris [47]) " Consider the nonlinear system of equation (2.1) with the state feedback of equation (3.1) and assume that* $\beta_0, \ldots, \beta_r$ *have been chosen so that the roots of the polynomial* $\beta_r s^r + \ldots + \beta_1 s + \beta_0$ *are in the open left half-plane. The closed-loop system will be internally stable if the zero dynamics of the open-loop system equation (3.1) are stable in the sense of definition 2."*

It is well known that the closed loop system (3.28) does not guarantee internal stability; the states $\xi_1, \ldots, \xi_{n-r}$ may go unstable even if the subsystem of the last r state equations is stable. However, if the zero dynamics is stable, the input-output stability guarantees internal stability of the closed-loop system. This is analogue with the linear results: placing poles at the process zeros does not destroy internal stability if the zeros are in the open left half plane.

3.5 Conclusions

The most important characteristics and aspects of the GLC (Globally Linearising Control) developed by C. Kravaris and C. Chung [48] and the error feedback control structure developed by P. Daoutidis and C. Kravaris [24] is reviewed.

The GLC control law can be viewed as a special case of the feedback linearisation reviewed in the Chapter 2. In order to show this, it is necessary define a new input v' as follows:

$$v' = v - \sum_{i=0}^{r-1} \beta_i L_f^i h(x) \qquad (3.36)$$

If $\beta_r = 1$ and $v = v'$, the control law equation (3.1) becomes

$$u = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \qquad (3.37)$$

This control law equation is identical to the standard feedback linearisation developed by A. Isidori [44], which was reviewed in the Chapter 2. This fact is used in Chapter 6 in order to get equivalencies between the exact linearisation and the new algorithms developed in Chapter 5. Therefore, the GLC inherits the restrictive assumptions required by the controller reviewed in Chapter 2:

- Affine system
- System with stable *zero dynamics*
- Relative degree known
- Relative degree well defined

Also, the theorem 3.3 gives the necessary condition in order to guarantee internal stability, which was given in Chapter 2. This necessary condition is that the *zero dynamics* must be stable. The other two assumptions, are easy to infer from the control law given by equation (3.1). As mentioned in Chapter 2 these assumptions are removed by the new algorithms presented in Chapter 5.

Also, the following points relating to the Theorem 3.2 should be noted:

- The control law used in this theorem can be reduced to:

$$u = \frac{v - \sum_{i=0}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)}$$

where v is chosen by

$$v = y_{sp} - y$$

Choosing the β_i in order to get

$$\frac{y}{y_{sp}} = \frac{1}{(\epsilon s + 1)^r}$$

- The sentence given in Theorem 3.2 " *under an appropriate initialisation*" can be interpreted to mean the process and process model are equal. Also $y^* = y$ for all times. In Chapter 6 is considered that there are parameter uncertainties in the process model, a regulation model is added in order to improve the performance. This regulation model is used by the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2].

Finally, I would like to remark on the similarity between an error feedback structure shown in Figure 3.4 and the IMC structure, which was reviewed here. In the IMC, a " perfect" controller is designed using the inverse of the process model. The perfect controller is then augmented with a filter that can be tuned to achieve a compromise between performance and robustness. The error signal e_p used as a feedback signal to the controller provides a measure of the plant/model mismatch

$$e_p = y_{sp} - y - y_m$$

where y is the process output. The perfect controller is chosen as the right inverse of the model driven by the error signal e_p.

$$u = -\frac{L_{f_m}^r h_m(x_m)}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)} + \frac{e_p^{(r)}}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)}$$

This control law linearises the map between the error signal e_p and the model output y_m. If e_p and y_m satisfy the conditions:

$$y_m^{(k)}(0) = e_p^{(k)}(0) \qquad 0 \leq k \leq r-1$$

perfect control is achieved because:

$$y_m^{(r)}(t) = e_p^{(r)}(t) \qquad for\ all\ t \geq 0$$

therefore

$$y(t) = y_{sp}(t) \qquad for\ all\ t \geq 0$$

P. Daoutidis and C. Kravaris [24] explain that this similarity is because the open-loop observer can be viewed as an internal model and the input/output linearising control law can be viewed as implicitly generating an inverse of the system. There is a difference in the inverse: in the IMC the inverse is driven by e_p and in the GLC is driven by the output y. For this reason, I consider that similarity mentioned in this Chapter does not exist.

Chapter 4

Alvarez's Control of Nonlinear Systems

4.1 Introduction

In this Chapter, the design and properties of the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2] are reviewed and critically evaluated. In Chapter 6 it is concluded that the regulation model used by this controller is not useful. Also, a design of a positioning control for an Induction Motor based on the TRRMCNL developed by E. Liceaga and I. Siller [54] is presented. This work gave rise to development of the new controllers presented in Chapter 5.

The type of motor considered corresponds to squirrel-cage-induction machine, whose dynamics can be described by a set of highly nonlinear differential equations, called d-q model, and the control design will be obtained in terms of a simplified model. Some of the difficulties encountered in order to control this machine, are caused by severe changes of the rotor resistance during its operation, plus its inherent nonlinear behaviour.

Some simulations are presented in order to show the good results obtained when the position of the motor is controlled by the TRRMCNL.

4.2 Structure of the TRRMCNL

The TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) controller developed by J. Alvarez and J. Alvarez [2] is reviewed in this section. This controller is an extension of the scheme proposed by G. Bornard and J. P. Gauthier [7] for linear systems.

The control law is found by solving the disturbance decoupling problem with measurements J. Alvarez and J. Alvarez [2] of an extended system, composed of a internal model , a tracking (or reference model) and a regulation filter.

Its implementation only requires measurements of the output, and so an output error dynamics estimation is performed.

The scheme proposed leads to a two degrees of freedom controller, where the performance is determined by the tracking model, in the same manner as in the model matching problem. Meanwhile, the robustness properties are defined by the regulation filter.

It is assumed that the dynamics of the process to be controlled are described by the following differential equation:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{4.1}$$

Where the vector fields f and g are smooth on $\mathbf{R}^n$ and h is a smooth real valued function on $x \in \mathbf{R}^n$, $y \in \mathbf{R}$ is the process output and $u \in \mathbf{R}$ is the process control input. It also will be assumed that the relative degree of the process is r and $f(x_0) = 0$ with $x_0 = 0$.

It is very easy to obtain an output nonlinear controller for the system equation (4.1), by solving the disturbance decoupling problem with measurements (J. Alvarez and J. Alvarez [2]) of an extended system, composed of the following subsystems:

S.1 A tracking or reference asymptotically stable system with unity steady-state gain:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r z \\ y_r &= C_r x_r \end{aligned} \tag{4.2}$$

Where $x_r \in \mathbf{R}^{n_r}$, $A_r \in \mathbf{R}^{n_r \times n_r}$, $B_r \in \mathbf{R}^{n_r \times 1}$, $C_r \in \mathbf{R}^{1 \times n_r}$, $z \in \mathbf{R}$ and relative degree r_r.

S.2 An asymptotically stable regulation filter with unity steady-state gain and state space representation:

$$\begin{aligned} \dot{x}_e &= A_e x_e + B_e \varepsilon \\ y_e &= C_e x_e \end{aligned} \tag{4.3}$$

Where $x_e \in \mathbf{R}^{n_e}$, $A_e \in \mathbf{R}^{n_e \times n_e}$, $B_e \in \mathbf{R}^{n_e \times 1}$, $C_e \in \mathbf{R}^{1 \times n_e}$, $\varepsilon \in \mathbf{R}$ and relative degree r_e.

S.3 A stable minimum-phase (with stable zero dynamics) nominal representation of the process equation (4.1):

$$\begin{aligned} \dot{x}_m &= f_m(x_m) + g_m(x_m)u(t) \\ y_m &= h_m(x_m) \end{aligned} \tag{4.4}$$

Where $x_m \in \mathbf{R}^{n_m}$, f_m and g_m are smooth vector fields on $\mathbf{R}^{n_m}$, h_m is a smooth real valued function on $\mathbf{R}^{n_m}$ and with relative degree d_m. Moreover, it will be assumed that $f_m(x_{m_0}) = 0$ with $x_{m_0} = 0$.

r_r, r_t and r_m can not be chosen independently, see expression (4.9).

The resulting structure of the control system is shown in the block diagram of Figure 4.1, where:

- the block labelled Reference Model is used to obtain the performance of the closed loop system;

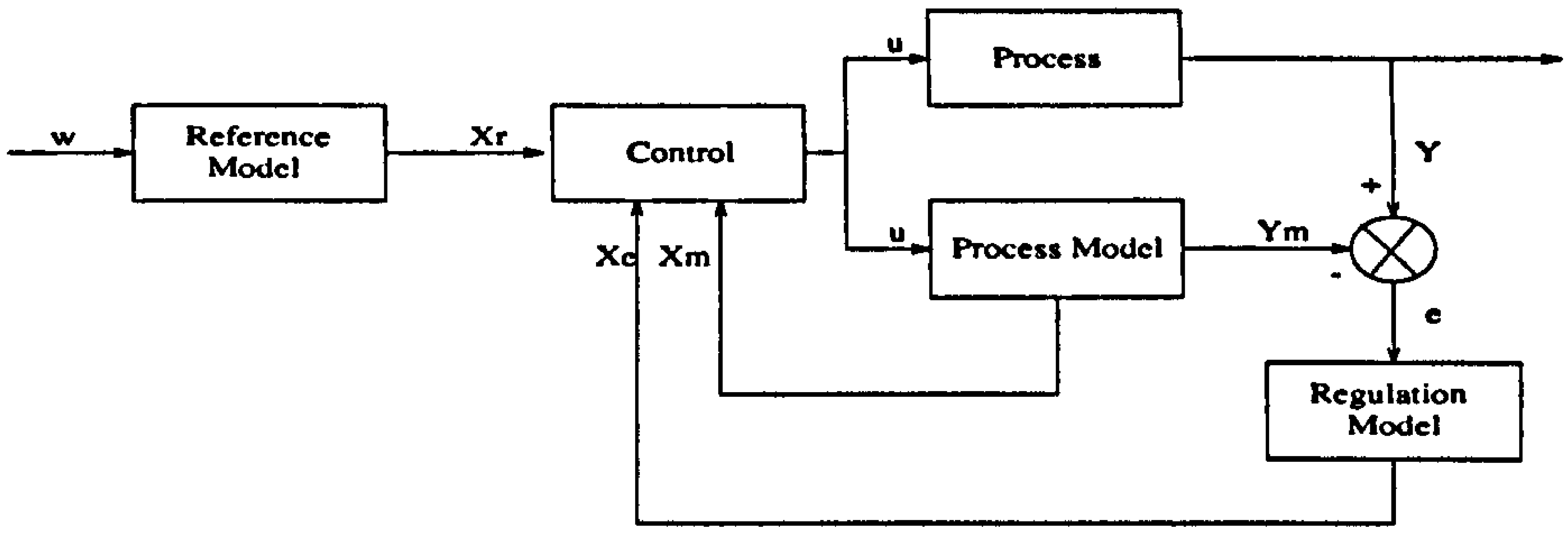

Figure 4.1: TRRMCNL Structure

- the block labelled Process Model is the open-loop observer used in order to get the model states, which are needed to emulate the output derivatives of the system;
- the block labelled Regulation Model is used to counteract the error model between output process derivatives and output process model derivatives. The input of the filter is the error between output system and output process model.
- the control input u is the same for system and model, the control system has an output feedback.

The input z, of the tracking system equation (4.2), represents the desired output of the real process. Meanwhile, the input ε, of system equation (4.3), is defined as:

$$\varepsilon = y - y_m \tag{4.5}$$

Where y is the measured output of the process and y_m is the model output, namely equation (4.5) represents the output modelling error.

Equation (4.4) is considered to be a simplified control design model of the process equation (4.1) It will be assumed that the relative degree of (4.4) and (4.1) coincide, at least inside a section of the operational envelope of the process.

The problem can be formulated to find a control law for the process so that:

The process output y tracks the reference output y_r.

The effect of disturbances on the process output is eliminated.

The use of state observers is avoided.

In order to find this control law, let us consider the following extended system which is formed by linking systems (4.2), (4.3) and (4.4) as follows:

$$\begin{aligned} \dot{x}_E &= f_E(x_E) + u g_E(x_E) + p_{E_1} y + p_{E_2} z \\ y_E &= h_E(x_E) \end{aligned} \tag{4.6}$$

Where, $x_E = (x_m, x_r, x_e)^T$, $h_E = y_r - y_e - y_m$, with

$$f_E = \begin{bmatrix} f_m(x_m) \\ A_r x_r \\ A_e x_e - h_m(x_m) \end{bmatrix}, \quad g_E = \begin{bmatrix} g_{x_m} \\ 0 \\ 0 \end{bmatrix}, \quad p_{E_1} = \begin{bmatrix} 0 \\ 0 \\ B_e \end{bmatrix} \quad \text{and} \quad p_{E_2} = \begin{bmatrix} 0 \\ B_r \\ 0 \end{bmatrix}$$

The following assumption is considered

Assumption 1 *There exists a control law such that:*

$$\lim_{t \longrightarrow \infty} \epsilon(t) = \textit{constant} \tag{4.7}$$

Then the following result is satisfied.

Lemma 4.1 *Under assumption 1, if the control law is such that the extended system output $y_E(t)$ converges to zero and if the tracking reference model input z is a constant, then the process output converges to z.*

Equation (4.6) represent a nonlinear system with input u, subject to the disturbance y and z through the vectors field $p_{E_1}(x)$ $p_{E_2}(x)$.

Therefore, the problem is to find a control law such that:

The extended system output y_E tends to zero.

The dynamics of the output y_E is decoupled from the process output y and the tracking reference model z.

Once the extended system (4.6) has been defined, the usual disturbance decoupling problem with measurements can be formulated. Namely, to design a feedback controller, such that output y_E tends to zero and is completely independent of the 'disturbances' y and z. The feedback control necessary for this has the form:

$$u(x_E) = \alpha(x_E) + \beta(x_E)v_E + \gamma_1(x_E)y + \gamma_2(x_E)z \tag{4.8}$$

The conditions required for the existence of such feedback have been established in [44]. In the present case, these conditions reduce to the selection of the regulation filter and the tracking reference model (4.3) and (4.2) with theirs relative degrees at least as great as the relative degree of the model equation (4.4):

$$r_e \geq r_m, \qquad r_r \geq r_m \tag{4.9}$$

The control law solving this problem is given by the following lemma:

Lemma 4.2 *A control law that solves the DDPM for the extended system equation (4.6) is equation (4.8), with:*

$$\begin{aligned}
\alpha(x_E) &= \frac{-L_{f_m}^{r} h_m(x_m) + C_t A_t^{r-1}(A_r x_t + b_r z) - C_e A_e^{r-1}(A_e x_e - b_e h_m(x_m))}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)} \\
\beta(x_E) &= -\frac{1}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)} \qquad (4.10) \\
\gamma_1(x_E) &= \frac{-C_e A_e^{r_m-1} b_e}{L_{g_m} L_{f_m}} \\
\gamma_2(x_E) &= \frac{-C_r A_r^{r_m-1} b_r}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)}
\end{aligned} \tag{4.11}$$

When the control law equation (4.8) is applied to the process (4.1), the extended system is linearised as follows:

$$\begin{aligned}
\xi &= h_E(x_E) \\
\dot{\xi} &= L_{f_E} h_E(x_E) \\
\xi^{(2)} &= L_{f_E}^2 h_E(x_E) \\
&\vdots \\
\xi^{(r)} &= -v_E(x_E) \\
\dot{\eta} &= q(\xi,\eta) + t(\xi,\eta)y + s(\xi,\eta)w
\end{aligned} \tag{4.12}$$

In order to place the poles of the linear part such that the controllable and observable part of the extended system is asymptotically stable, $v_E(x_E)$ is chosen as

$$v(x_E) = \sum_{i=0}^{r_m} a_i(y_r^{(i)} - y_e^{(i)} - y_m^{(i)}) \tag{4.13}$$

Where the coefficients a_i form the Hurwitz polynomial

$$p(s) = s^{r_m} + a_{d-1}s^{r_m-1} + \ldots + a_1 s + a_0 \tag{4.14}$$

and $r^{(i)}$ indicates the i-th derivative of r. It is possible to see that if $v_E(x_E)$ is substituted in the $rth-$derivative of ξ in equation (4.13), yields a linear differential equation for the expanded output.

$$\sum_{i=0}^{r_m} a_i(y_r^i - y_e^i - y_m^i) = 0 \tag{4.15}$$

Consider that the initial conditions of y_r, y_e and y_m are zero. Taking Laplace Transform of this equation

$$y_r(s) - y_e(s) - y_m(s) = 0 \tag{4.16}$$

Consider the transfer function of the regulation model given by:

$$G_e(s) = \frac{k_0}{s^{r_m} + \ldots + k_1 s^1 + k_0} \tag{4.17}$$

and defining the tracking error as $e_r(t) = y_r(t) - y(t)$ the equation (4.16) becomes

$$e_r(s) = (G_e(s) - 1)\varepsilon(s) \tag{4.18}$$

When the model and process are identical $e_r(s) = 0$. In other case, we can see that the tracking error tends to zero if ε is bounded and $G_e(s)$ has the poles with negative real part. Clearly, the right hand side of (4.18) would converge to zero with a rate determined by the regulation filter dynamics. Thus, it may be considered that the selection of the regulation filter bandwidth should be as high as possible, nevertheless, in a practical situation the measurement of the output may be contaminated with noise, for instance:

$$\varepsilon = (y + \zeta) - y_m$$

Where ζ denotes a measurement noise, thus the equation (4.16) may be

$$y_r(s) - G_e(s)((y(s) + \zeta(s)) - y_m(s)) - y_m(s) = 0 \tag{4.19}$$

rewriting this equation

$$e_r(s) = G_e(s)(\varepsilon(s) + \zeta(s)) - \varepsilon(s) \tag{4.20}$$

Therefore, the regulation filter would have to be designed with a regulation model bandwidth higher than the ε bandwidth but lower than the noise bandwidth. Making some manipulation the control law equation (4.8) becomes:

$$u(x_E) = \beta^{-1}(x_E)(-\alpha(x_E) + y_t^{(r_m)} - y_r^{(r_m)} + V(x_E)) \tag{4.21}$$

With

$$\beta_0 = L_{g_m} L_{f_m}^{r_m - 1} h_m(x_m)$$

$$\alpha_0 = L_{f_m}^{r_m} h_m(x_m)$$

$$V(x_E) = \sum_{i=0}^{r_m - 1} a_i(y_r^{(i)} - y_e^{(i)} - y_m^{(i)})$$

Before ending this section the relevant aspects of the control structure proposed can summarised as follows:

- The performance and robustness properties are determined independently. Indeed, the closed-loop system performance is defined by selecting an appropriate reference model, in the same manner as in the standard model matching structure introduced by A. Isidori [44]. Whereas, (some of) the robustness properties are adjusted by an independent regulation filter.

- Although, the controller is based on a geometric input-output structure, no attempt is made to estimate the system states. Instead, a nominal design model (or open loop observer), together with a regulation filter, is used to estimate the modelling error dynamics. The resulting control structure is much simpler to analyse and implement than the control-observer structures.

- The system must be stable and have stable zero dynamics.

4.3 Case study

In this section, a positioning controller of an induction motor, based on structure reviewed here is presented. The type of motor considered corresponds to a squirrel-cage-induction machine, whose dynamics are described by a set of highly nonlinear differential equations J. Meisel [61]. In general, the control of this type of machines is very difficult to achieve through conventional linear control techniques. Some of the difficulties encountered are caused by the severe changes of the rotor resistance during its operation, plus its inherent nonlinear behaviour.

Several applications of the geometric approach to nonlinear control to the induction motors have been reported. Among these, are the papers of A. de Luca [19], which considered an output feedback controller; the one by R. Marino et al. [56], who considered a nonlinear adaptive control structure and Kim. Donf-Il et al. [23], who considered an output feedback linearisation approach.

The relevance of the application presented in this paper is the simplicity of the controller when compared with previous designs, moreover, it includes a mechanism which allows the designer to improve the control system robustness.

4.3.1 Induction motor dynamics

Induction motors are rugged and non-expensive devices compared with dc-motors. They are widely used in the industrial environment due to their reliability, comparative low size and low maintenance requirements. In this chapter a squirrel-cage-induction motor (NEMA D) J. Meisel [61] has been considered. This kind of machine is designed to operate under torque loads and have a high starting torque, thus, it may be used as positioning device under appropriate feedback control. In order to develop such feedback, it is necessary to determine its main dynamical characteristics, which can be represented according to J. Meisel [61], with the model referred to as 'd-q' model J. Meisel [61]:

$$\begin{bmatrix} v_d^s(t) \\ v_q^s(t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_s + L_s p) & 0 & Mp & 0 \\ (R_s + L_s p) & 0 & Mp & 0 \\ 0 & (R_s + L_s p) & 0 & Mp \\ Mp & nMw_r & (R_r + L_r p) & nL_r w_r \\ -nMw_r & Mp & -nL_r w_r & (R_r + L_r p) \end{bmatrix} \begin{bmatrix} i_d^s(t) \\ i_q^s(t) \\ i_d^r(t) \\ i_q^r(t) \end{bmatrix} \tag{4.22}$$

$$T_e = nM\left[i_d^s(t)i_q^r(t) - i_q^s i_d^r(t)(t)\right] \tag{4.23}$$

$$\begin{aligned} \dot{w}_r &= (-Dw_r + T_e)/J \\ \dot{q} &= w_r \end{aligned} \tag{4.24}$$

With

$$v_d^s = \sqrt{2}V_s cos(\omega t)$$

$$v_d^s = \sqrt{2}V_s sin(\omega t)$$

where

v_d^s, i_d^s Instantaneous stator direct axis voltage and current.

v_q^s, i_q^s Instantaneous stator quadrature axis voltage and current.

i_d^r, i_q^r Instantaneous rotor direct and quadrature-axis currents.

V_s Supply voltage amplitude.

p Operator d/dt.

w_r Rotor angular velocity.

T_e Instantaneous electromagnetic torque.

R_s, R_r Stator and rotor resistances. R_s=60 ohms, R_r=37.36 ohms.

M Peak stator-rotor mutual inductance. M=1.6 h.

L_s, L_r Stator and Rotor self-inductances. L_s=1.699 h, L_r=1.68 h.

w Excitation frequency. w=377 rad/sec.

n Number of pole-pairs. n=2.

J, D Equivalent Inertia and viscous friction. J=.0186 kg-m^2, D=.0261 newton-m-sec/rad.

l_s, l_r Stator and Rotor leakage inductances. l_s=.0991 h, l_r=.0804 h.

A much simpler representation can be derived if the average value of the electromagnetic torque is considered. In this case the motor dynamics reduce to J. Meisel [61]:

$$T_{em} = \frac{2nR_r V_s^2 / (w\phi)}{(R_s + R_r/\phi)^2 + (w\,(l_s + l_r))^2} \tag{4.25}$$

rewriting and letting $u = V_s^2$,

$$T_{em} = f(w_{r_m})u \tag{4.26}$$

where

$$f(w_{r_m}) = \frac{2nR_r / (w\phi)}{(R_s + R_r/\phi)^2 + (w\,(l_s + l_r))^2} \tag{4.27}$$

The control input is the voltage amplitude, $V_s = \sqrt{u}$. Thus the mechanical part of the motor reduces to:

$$\begin{aligned} \dot{w}_{r_m} &= (-Dw_{r_m} + f(w_{r_m})u)/J \\ \dot{q}_r &= w_{r_m} \end{aligned} \tag{4.28}$$

$$\phi = 1 - \|s - 1\| \tag{4.29}$$

Where ϕ in (4.29) represents a normalisation of the slip s, which can be written as

$$s = \frac{w_s - w_{r_m}}{w_s} \tag{4.30}$$

$$w_s = w/n \tag{4.31}$$

Where w_s is defined as the synchronous speed of the motor.

Obviously, the model described by equation (4.28) is much simpler than the original d-q model defined by equations (4.22), (4.23) and (4.24). Nevertheless, sometimes this approximation is sufficient for control design purposes, even in the presence of parametric disturbances, e. g. variations of the rotor resistance. A control system design, able to tolerate such simplifications and parametric disturbances is described next.

4.4 Control of the induction motor.

In this section, a positioning control for the d-q induction motor described by equations (4.22), (4.23) and (4.24), is presented. The control design has been obtained in terms of the simplified model equation (4.28). Thus, the relative degree of the motor and its model are the same, with the angular position as output variable and control variable Vs. Nevertheless, the torque dynamics has been neglected. The system to be controlled is shown in Figure 4.2, where the block labelled motor represents equation (4.27).

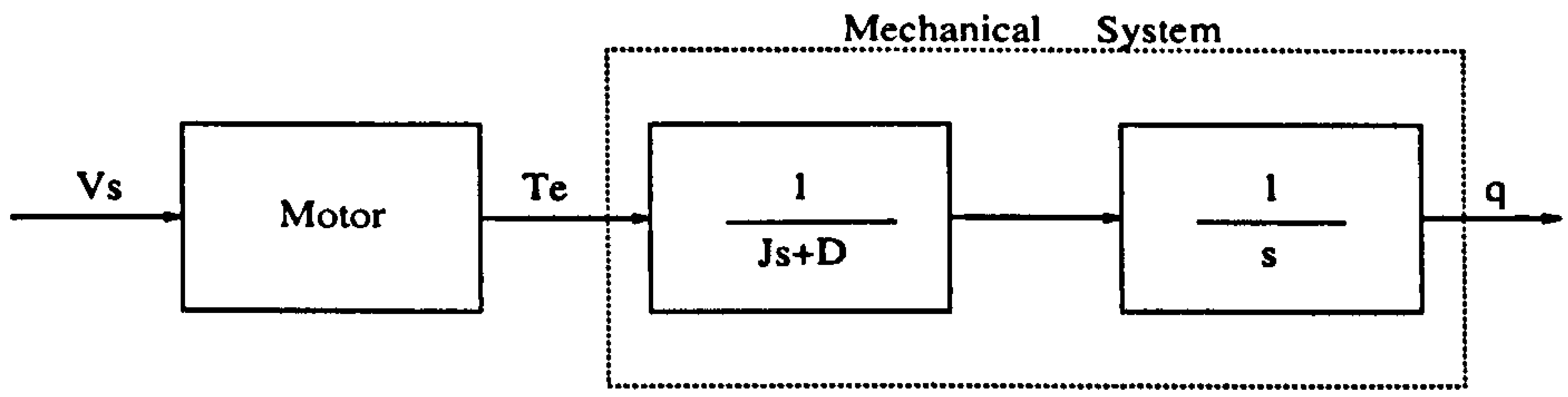

Figure 4.2: System

The control law equation (4.21) derived from the model equation (4.28) is:

$$u = \frac{J}{f(w_{r_m})}[\frac{D}{J}w_{r_m} + y_{t2} - y_{r2} + a_{t1}(y_{t1} - y_{r1} - w_m) + a_{to}(y_t - y_r - q_m)] \tag{4.32}$$

The tracking dynamics have been selected with time constants equal to those of the mechanical system, given by:

$$\dot{y_{t1}} = -a_{t1}y_{t1} - a_{to}y_t + a_{to}z, \qquad \dot{y}_t = y_{t1} \tag{4.33}$$

where $a_{to} = 4.0123$ and $a_{t1} = 4.0061$

The regulation dynamics determine the robustness of the system with respect to model uncertainty. A regulation filter with a very large bandwidth will allow large discrepancies between the motor and its model, but the influence of noise measurement may be amplified. In the present case the regulation filter is defined as:

$$\dot{y_{r1}} = -a_{r1}y_{r1} - a_{ro}y_r + a_{ro}\varepsilon, \qquad \dot{y}_r = y_{r1} \tag{4.34}$$

where $a_{ro} = 100$ and $a_{r1} = 20$

Which permitted the performance of positioning changes, with no significant effects from the non modelled dynamics, and variations up to 50% of the rotor resistance.

Some simulations results are shown in Figures 4.3-4-8.

In Figures 4.3 and 4.4 show the effect of a discrepancy in the value of the gain model. The effect of an increment (50%) in the rotor resistance value is shown in Figures 4.5 and 4.6. Meanwhile, a simulation considering a reduction of 50% of the rotor resistance is shown in Figures 4.7 and 4.8.

The simulations show the robustness of the controller.

4.5 Conclusions

A brief review of the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) controller developed by J. Alvarez and J. Alvarez [2] was presented.

The TRMCNL was used in order to develop a positioning control system for an induction motor, where the input control was the stator voltage amplitude E. Liceaga and I. Siller [54].

The characteristics of the TRRMCNL allow the performance of the closed loop system to be determined by a tracking or reference model, in the same manner as in the standard nonlinear model matching structure. Whereas, the robustness with respect to non modelling dynamics and parameter disturbances, can be counteracted through an independent regulation filter.

The simulation results show that it is possible to ignore the electrical torque dynamics from the design model without affecting considerably the positioning capabilities of the closed loop system. Moreover, the robustness of the controller was proven for changes of the rotor resistance for up to 50%.

The TRRMCNL can control systems with the same restrictive assumptions as exact linearisation feedback control reviewed in Chapter 2, which are:

- Affine system
- System with stable *zero dynamics*
- Relative degree known
- Relative degree well defined

This work suggested to me the ideas investigated in Chapters 5 and 6. In Chapter 5 new algorithms are presented, which remove the assumptions given above. Equivalencies between the TRRMCNL and the new algorithms are found as well as the fact that the regulation model is not useful.

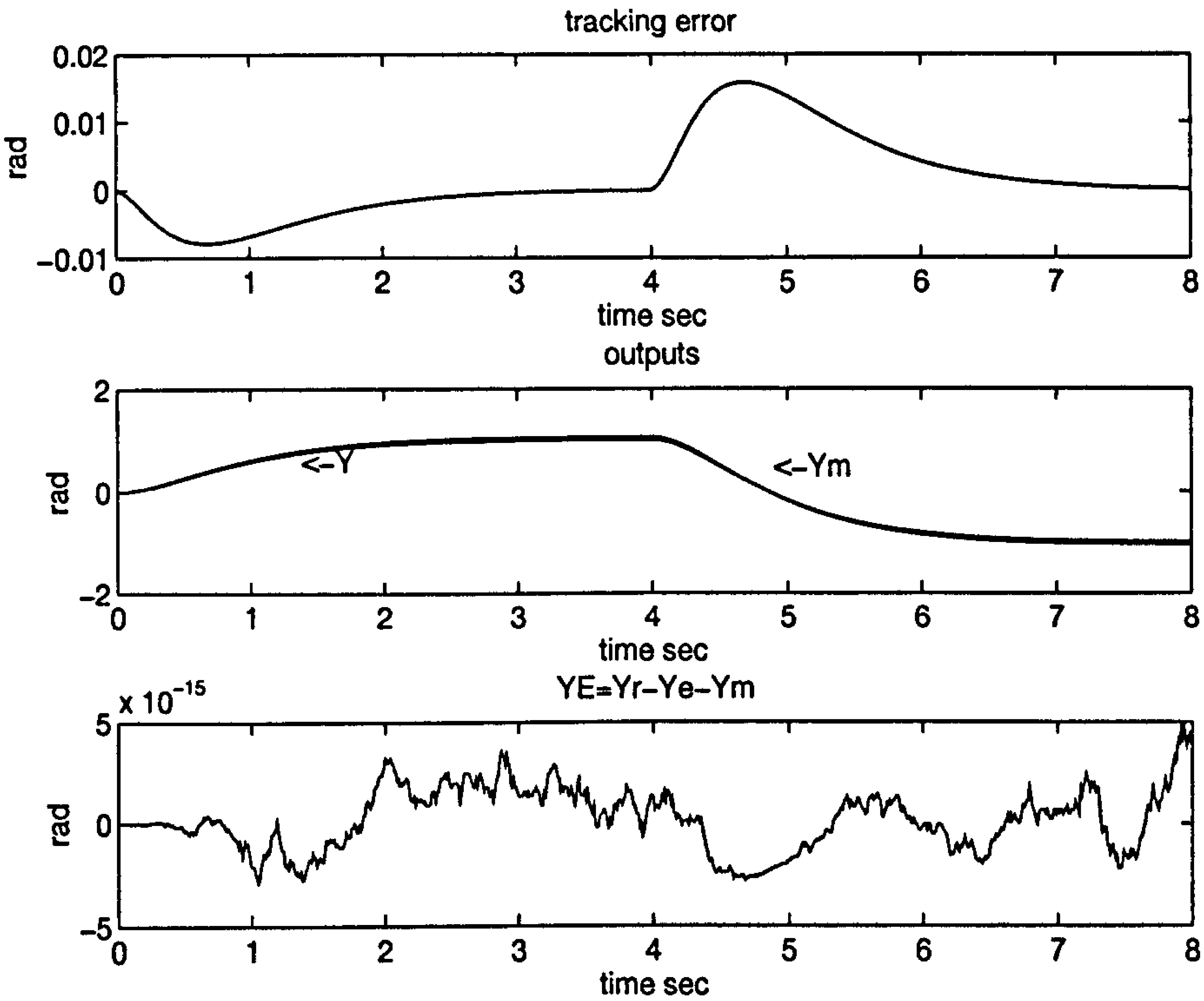

Figure 4.3: Without parametric uncertainty.

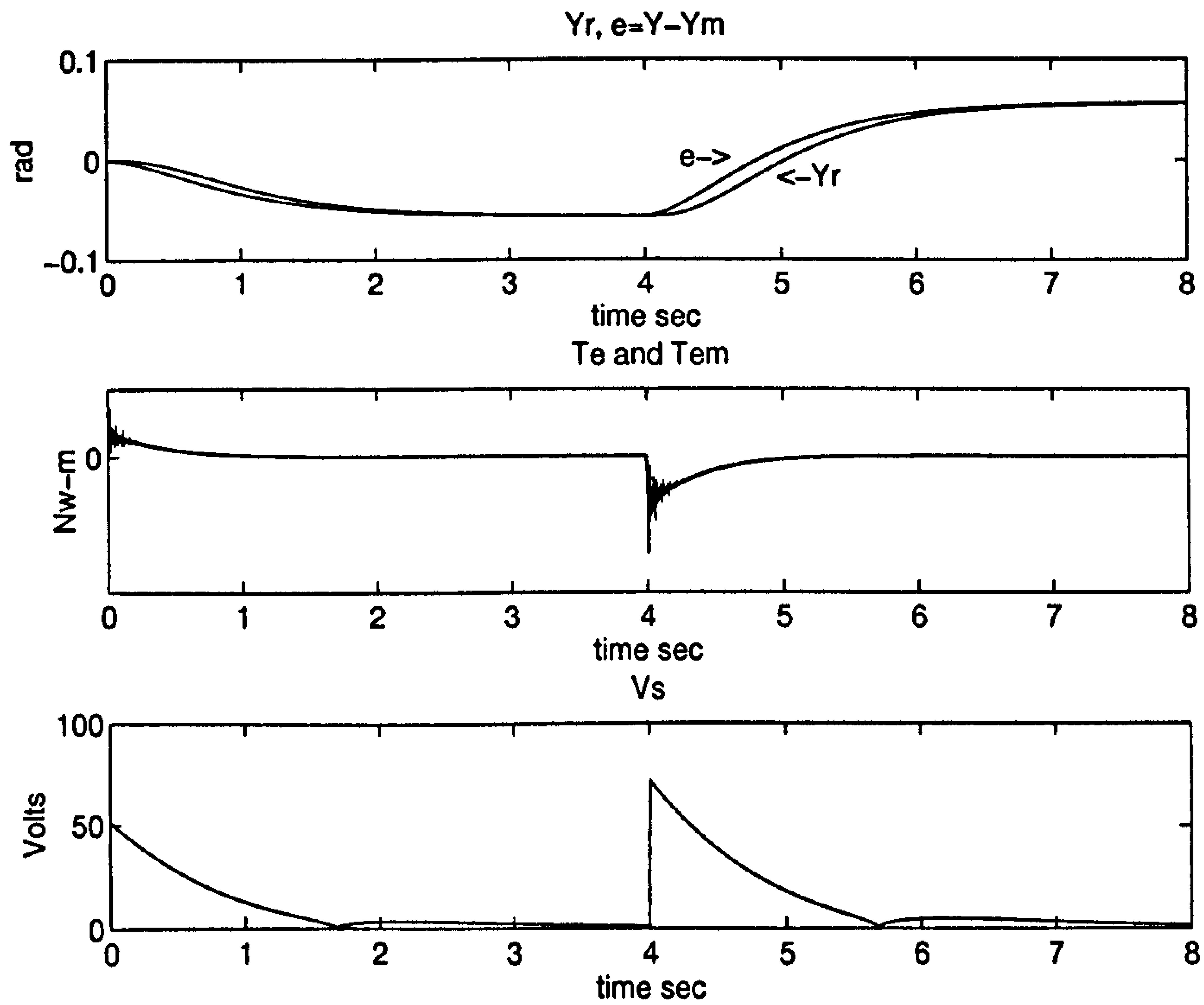

Figure 4.4: Without parametric uncertainty.

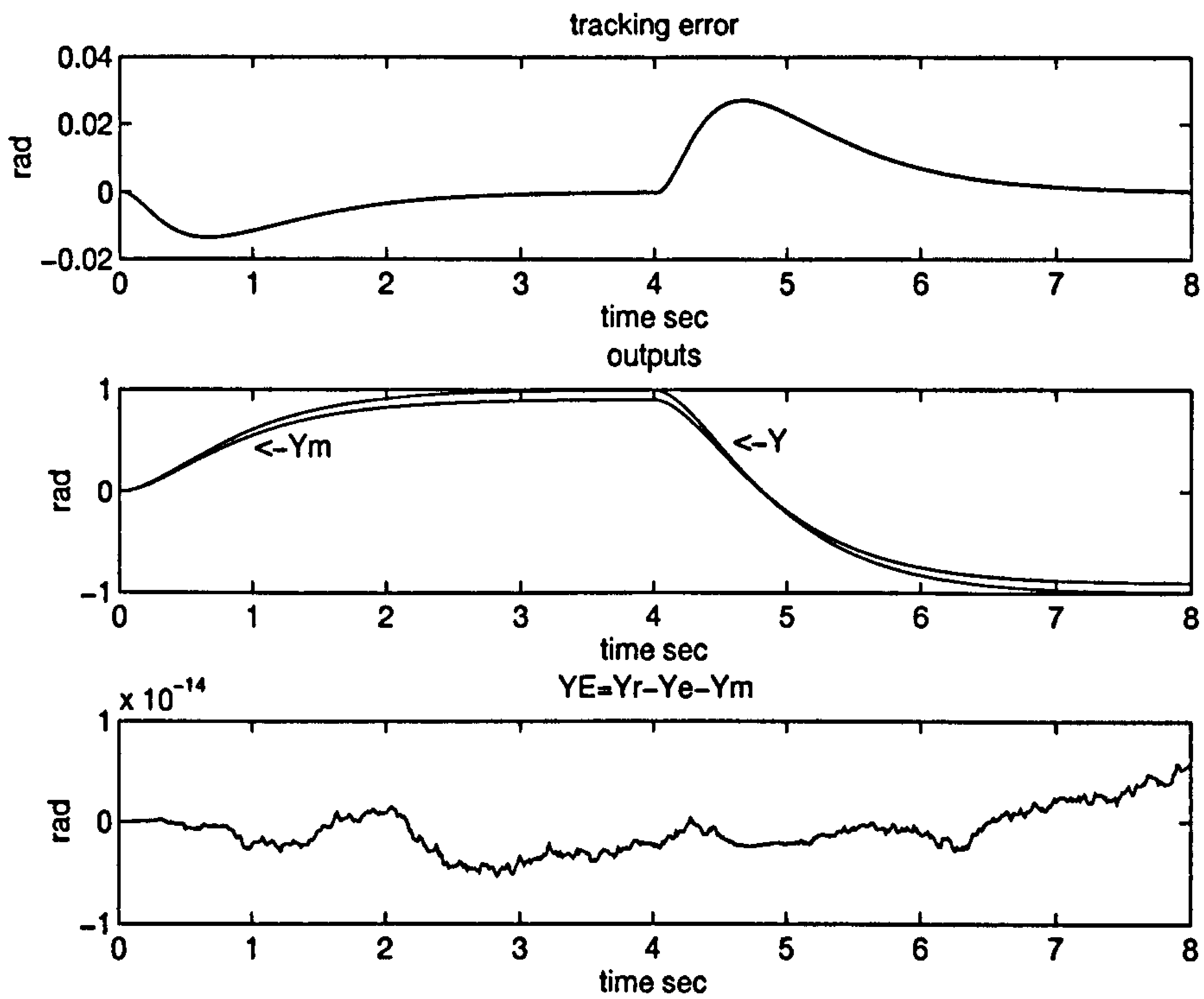

Figure 4.5: Parametric uncertainty Rr=1.5Rro.

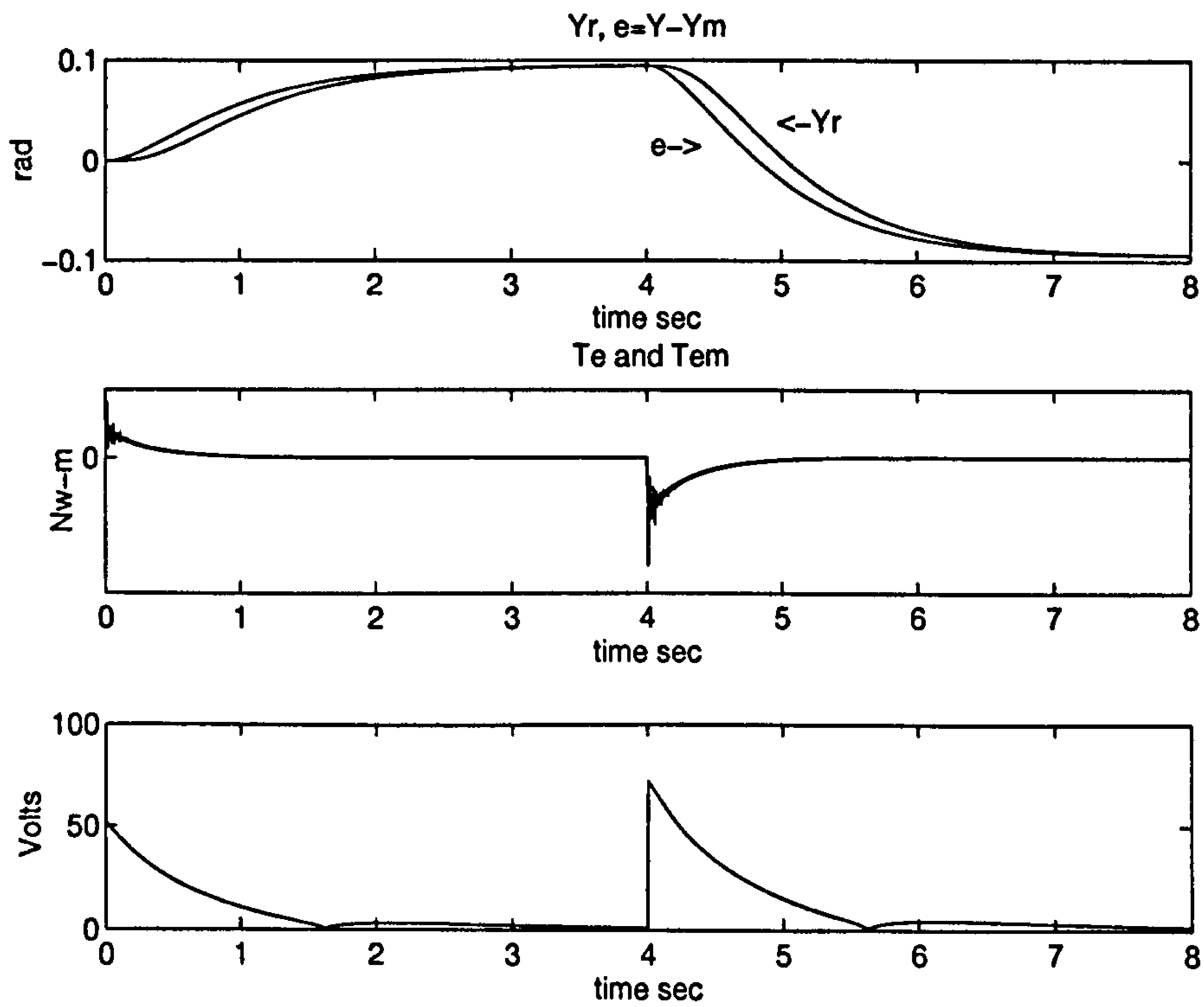

Figure 4.6: Parametric uncertainty Rr=1.5Rro.

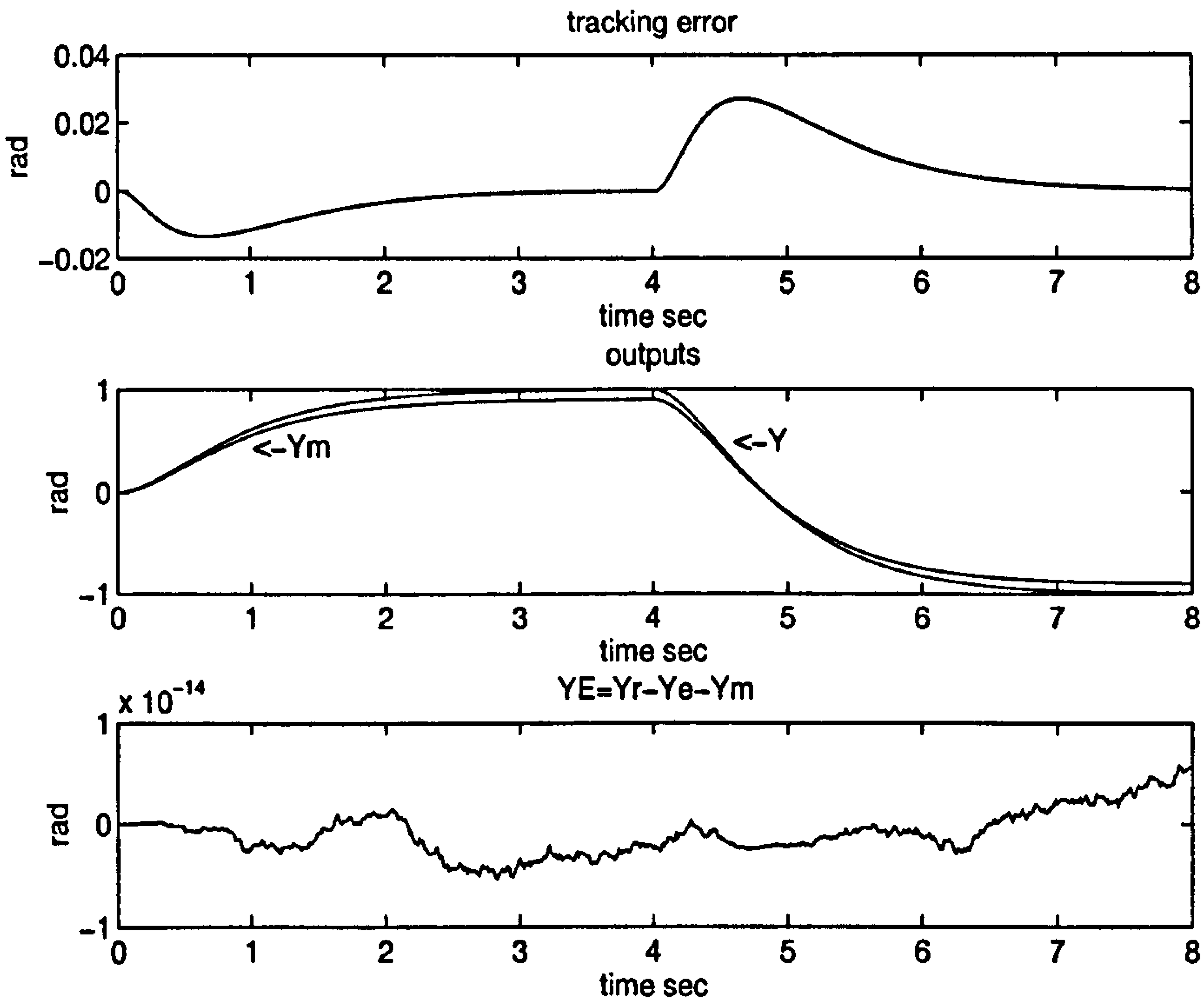

Figure 4.7: Parametric uncertainty Rr=.5Rro.

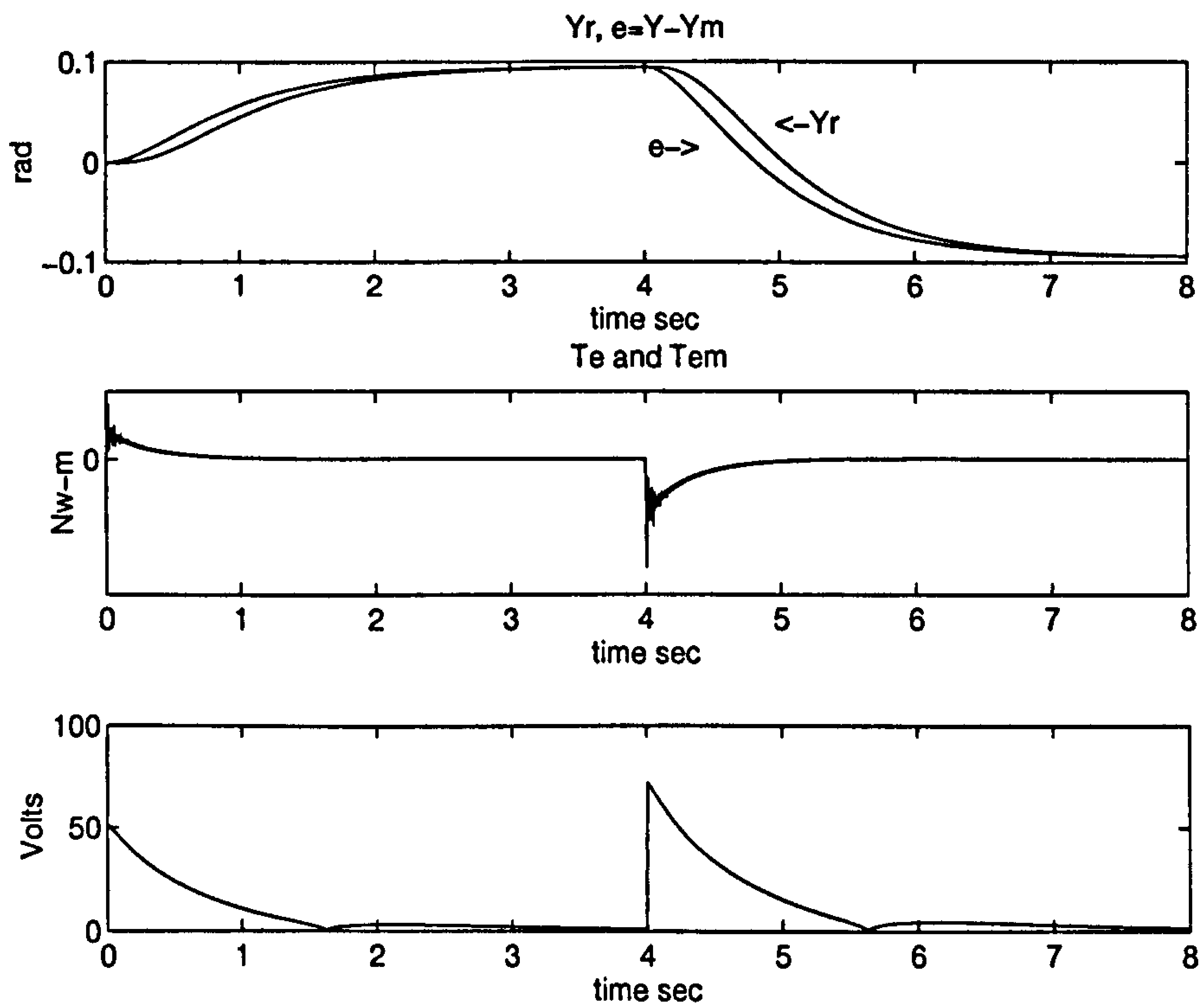

Figure 4.8: Parametric uncertainty Rr=.5Rro.

Chapter 5

Nonlinear Generalised Predictive Control

5.1 Introduction

The NGMV is an extension of the Generalised Minimum Variance Control (GMV) originally derived in discrete-time form for linear systems by Clarke and Gawthrop [12], [13] and in continuous time by Gawthrop [29]. This chapter focuses on one particular version of GMV, the model reference version (Gawthrop [28]; Gawthrop [29]) is recast in a state-space form for nonlinear systems. As an intermediate step, a new algorithm: the NPGMV (Nonlinear Predictive Generalised Minimum Variance Control) is developed making use of the concepts of *receding – horizon* control and *predictive* control, for example (Mayne and Michalska [60]; Demircioglu and Gawthrop [21]). This thesis provides this extension as a step towards NCGPC.

The chapter goes on to develop the nonlinear version of the CGPC with the following distinctive features:

- It provides a nice way of handling systems with unstable zero dynamics.
- It has the ability to deal with systems which do not have a well defined relative degree.

- It considers nonlinear dynamic systems with non-affine state-space representation:

$$\begin{aligned}\dot{x}(t) &= F(x(t), u(t)) \\ y(t) &= h(x(t)).\end{aligned}$$

The Chapter is organised as follows. First, in Section 2, system description is given. In Section 3, derivative emulation in state-space setting is developed. In Section 4, 5 and 6, development of the NGMV, NPGMV and NCGPC are presented. In Section 7, nonlinear systems with unstable zero dynamics and nonlinear systems with not well defined relative degree are treated. In Section 8 the simulation results are shown in order to illustrate the theory. Finally, in Section 9 the conclusions are presented.

5.2 System Description

This chapter considers nonlinear dynamic systems with the state-space representation:

$$\begin{aligned}\dot{x}(t) &= F(x, u) \\ y(t) &= h(x),\end{aligned} \tag{5.1}$$

where the functions F and h are smooth (to be precise differentiable N_y times with respect to each argument). $x \in \mathbf{R}^n$ is the vector of the state variables, $u \in \mathbf{R}$ is the manipulated input and $y \in \mathbf{R}$ is the output to be controlled.

A special case of equation (5.1) is one where the control enters in a linear fashion.

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)),\end{aligned} \tag{5.2}$$

where $F(x, u) = f(x(t)) + g(x(t))u(t)$. Much of geometric control theory (see, for example, Isidori [44]) is built on systems of the form of equation (5.2) rather than that of equation (5.1). However, equation (5.2) has no particular advantage for our purposes so the more general case of equation (5.1) is considered here.

5.3 Derivative Emulation

The GMV and GPC controllers for linear systems are based on taking multiple derivatives of the system output with respect to time. In principle, this procedure can be equally well applied to the outputs of nonlinear systems: this is the fundamental idea behind this thesis. It is also the basis of much of geometric theory of nonlinear systems.

The notion of an emulator was introduced in Gawthrop [27] to describe the dynamic systems which emulate unrealisable operations, for example taking derivatives of the output, which is not feasible because of noise amplification. This emulator for linear systems, is developed by making use of the fact that the derivative of a signal in the time domain corresponds to multiplication by s in the Laplace domain. Linear systems can be described by differential equations or polynomials in the Laplace operator s. Unlike the linear case, however, the nonlinear systems can not be described by polynomials in the Laplace operator s, they can just be described by differential equations, equation (5.1) and the only way to obtain the output derivatives is as follows:

$$
\begin{aligned}
\dot{y}(t) &= L_f h(x) \\
y^{(2)}(t) &= L_f^2 h(x) \\
&\vdots \\
y^{(r)}(t) &= L_f^r h(x) + L_g L_f^{r-1} h(x) u(t) \\
y^{(r+1)}(t) &= S_1(x) + J_1(x) u(t) + L_g L_f^{r-1} h(x) \dot{u}(t) \\
y^{(r+2)}(t) &= S_2(x) + J_2(x) u(t) + I_1(x) \dot{u}(t) + L_g L_f^{r-1} h(x) u^{(2)}(t) \\
&\vdots \\
y^{(N_y)}(t) &= S_{(N_y-r)}(x) + J_{(N_y-r)}(x) u(t) \\
&\quad + I_{(N_y-r)}(x) \dot{u}(t) + I_{(N_y-r+1)}(x) u^{(2)}(t) + \ldots \\
&\quad + I_{(2(N_y-r-1))}(x) u^{(N_y-r-1)}(t) + L_g L_f^{r-1} h(x) u^{(N_y-r)}(t)
\end{aligned}
\tag{5.3}
$$

where $L_f h$ represents the Lie derivative, which was defined in Section 2.3 and S_i and

J_i are some functions of x (and not u). These output derivatives are obtained from the system of equation (5.2) and N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u. In this case output and its derivatives can be rewritten by:

$$\mathbf{Y}_{N_y}(t) = \mathbf{O}(x(t)) + H(x(t))\mathbf{u} \tag{5.4}$$

where

$$\mathbf{O} = \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \\ S_1(x) \\ S_2(x) \\ \vdots \\ S_{(N_y-r)}(x) \end{bmatrix} \tag{5.5}$$

$$H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ L_g L_f^{r-1} h(x) & 0 & \dots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \dots & 0 \\ J_2(x) & I_1(x) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \dots & L_g L_f^{r-1} h(x) \end{bmatrix} \tag{5.6}$$

and

$$\mathbf{Y}_{N_y} = \left[y \; \dot{y} \; y^{(2)} \dots y^{(N_y)}\right]^T \tag{5.7}$$

$$\mathbf{u} = \left[u \; \dot{u} \; u^{(2)} \ldots u^{(N_y - r)}\right]^T \tag{5.8}$$

If N_y is not chosen as described above or the system is described by the equation (5.1), output derivatives can be written by

$$\mathbf{Y}_{N_y}(t) = \mathbf{O}_{\mathbf{Gen}}(x(t), u(t)) \tag{5.9}$$

where, $\mathbf{O}_{\mathbf{Gen}}$ is a more general form of the function $\mathbf{Y}_{N_y}(t)$. It is possible to see from equations (5.4) and (5.9) that the states are required, which are not always available. For this reason a state observer is needed. Unlike the linear case, however, there is no general theory of state estimation for non-linear systems. For the purposes of this thesis, observers are taken to be of the form.

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t)) + g(\hat{x}(t))u + L(\hat{x}(t))e(t) \\ \hat{y}(t) &= h(\hat{x}(t)) \\ e(t) &= \hat{y}(t) - y(t) \end{aligned} \tag{5.10}$$

Unlike the linear case, the stability of such an observer is not guaranteed in general and its design is non trivial (Walcott,Corless and Zak [75]; Hunt and Verma [41]). Although observer design is an important issue for the algorithms developed in this thesis it is beyond the scope of this thesis. It is an area of continuing research by the authors; in particular we are looking at *physical model – based* observers (Gawthrop, Jones and MacKenzie [34]; Gawthrop and Smith [36]). The corresponding *emulated* $\mathbf{Y}(t)$, $\hat{\mathbf{Y}}(t)$, is defined as:

$$\hat{\mathbf{Y}}_{N_y}(t) = \mathbf{O}(\hat{x}(t)) + H(\hat{x}(t))\mathbf{u} \tag{5.11}$$

or

$$\hat{\mathbf{Y}}_{N_y}(t) = \mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), u(t)) \tag{5.12}$$

where $\hat{x}(t)$ is an estimate of the state $x(t)$. An open loop observer can be used when the system is stable. The open-loop observer is a process model that is simulated in parallel to the process, the process model is given by:

$$\begin{aligned} \dot{x}_m(t) &= F_m(x_m(t), u(t)) \\ y_m(t) &= H_m(x_m(t)), \end{aligned} \tag{5.13}$$

where the function F and H are smooth (to be precise differentiable N_y times with respect to each argument). $x \in \mathbf{R}^n$ is the vector of the state variables, $u \in \mathbf{R}$ is the manipulated input and $y \in \mathbf{R}$ is the output to be controlled.

Or the special case

$$\begin{aligned} \dot{x}_m(t) &= f_m(x_m(t)) + g_m(x_m(t))u(t) \\ y_m(t) &= h_m(x_m(t)), \end{aligned} \tag{5.14}$$

where $x_m \in \mathbf{R}^n$ is the vector of the state variables, $u \in \mathbf{R}$ is the manipulated input and $y_m \in \mathbf{R}$ is the output to be controlled. f_m and g_m are smooth vector fields and h is a smooth function.

5.4 Nonlinear Generalised Minimum-Variance Control (NGMV)

Generalised Minimum Variance Control (GMV) was originally derived in discrete time form by Clarke and Gawthrop [12] (see also (Gawthrop [26]; Clarke and Gawthrop [13])) (based on the Self-tuning Regulator of Aström and Wittenmark [3] and more recently in continuous time form (Gawthrop [28], [27] and [29]). In each case, GMV had a transfer function formulation. As discussed by Ordys and Clarke [66] in the context of nonlinear systems the state space approach is essential.

5.4.1 Development of NGMV

This section focuses on one particular version of GMV, the model-reference version of GMV (Gawthrop [26] and [29]). In the state space formulation, this is equivalent to defining the *unrealisable* vector $\phi(t)$ given by

$$\phi(t) = \mathbf{P}Y_{N_y}(t) \tag{5.15}$$

and its realisable emulated version $\phi(t)$

$$\hat{\phi}(t) = \mathbf{P}\hat{Y}_{N_y}(t) \tag{5.16}$$

where

$$\mathbf{P} = \begin{bmatrix} p_0 \, p_1 \, \dots \, p_{N_p} \end{bmatrix} \tag{5.17}$$

In the case of GMV, $N_y = N_p$. A corresponding polynomial can be defined as:

$$p(s) = \sum_{i=0}^{N_p} p_i s^i \tag{5.18}$$

Because, in the context of GMV, $N_y = r$ (the relative degree of the system) then only $u(t)$ (not its derivatives) will appear in equation (5.16). Following Gawthrop [29], the corresponding NGMV control is thus implicit defined (at each time t) by:

$$\hat{\phi}(t) = \mathbf{PO}_{\mathbf{Gen}}(\hat{x}(t), u(t)) = w \tag{5.19}$$

where w is the set-point.

This equation may have none, one or many solutions depending on the form of $PO_{\mathbf{Gen}}(\hat{x}(t), u(t))$ and the current value $\hat{x}(t)$. In general, the equation must be solved numerically online. The precise conditions for solution are not of concern here; we merely note that NGMV is not a practically useful controller, because, exact model-matching is the fundamental problem associated with NGMV. It is well known that exact model-matching implies severe restriction on the process.

However, if the system of equation (5.1) has the special form of equation (5.2) then (and noting that $N_y = r$) the implicit control of equation (5.16) becomes

$$\mathbf{PO}(\hat{x}(t)) + \mathbf{P}H(\hat{x}(t))u(t)) = w \tag{5.20}$$

This has the obvious solution

$$u = \mathbf{P}H^{-1}(\hat{x}(t))[w - \mathbf{PO}(\hat{x}(t))] \tag{5.21}$$

if $\mathbf{P}H^{-1}(\hat{x}(t)) \neq 0$.

5.5 Nonlinear Predictive Generalised Minimum Variance Control (NPGMV)

5.5.1 Development of NPGMV

The fundamental problems with NGMV are:

- the need to know the system relative degree r precisely and
- the fact that it cancels system zeros.

To some extent, this can be overcome using *control weighting* (Clarke and Gawthrop [12]; Gawthrop [26]; Clarke and Gawthrop [13]; Gawthrop [29]); however, this requires detailed design in its own right and is not considered further here.

The approach used here is to combine the twin concepts of *receding – horizon* control and *predictive* control in, for example (the Generalised Predictive Control of Clarke, Mohtadi, and Tuffs [15] and the corresponding continuous time version of Demircioglu and Gawthrop [21]; Demircioglu and Gawthrop [22]). This section presents a new algorithm based on these ideas which is developed in a state-space context.

There are five distinct, but related concepts associated with the predictive control considered here:

- prediction,
- Taylor series expansion,
- moving-horizon control,
- control constraints (within the moving horizon time-frame),
- optimisation.

Prediction of $\phi(t)$ by Taylor series expansion

In a continuous-time formulation (Demircioglu and Gawthrop [21]) prediction is approximated by a Taylor series expansion of the system output y; here this concept is extended slightly to use a corresponding expansion of ϕ. The predictor is a function of T into the future and is approximated by a Taylor series truncated as follows.

$$\phi(t,T) = \phi(t) + \dot{\phi}(t)T + \phi^{(2)}(t)\frac{T^2}{2!} + ... + \phi^{(N_\phi)}(t)\frac{T^{N_\phi}}{N_\phi!}. \tag{5.22}$$

or

$$\phi(t,T) = \mathbf{T}(T)\mathbf{\Phi}(t) \tag{5.23}$$

where

$$\mathbf{T} = \left[1 \; T \; \frac{T^2}{2!} \; \cdots \frac{T^{N_\phi}}{N_\phi!}\right] \tag{5.24}$$

and

$$\mathbf{\Phi}(t) = \left[\phi \; \dot{\phi} \; \phi^{(2)} \ldots \phi^{(N_\phi)}\right]^T \tag{5.25}$$

Following Demircioglu and Gawthrop [21], the unrealisable derivatives in equation (5.23) are replaced by the emulated versions to give:

$$\hat{\phi}(t,T) = \mathbf{T}(T)\hat{\mathbf{\Phi}}(t) \tag{5.26}$$

where $\hat{\Phi}(t)$ is given in terms of the state estimate of equation (5.10) by:

$$\hat{\Phi}(t) = \Pi\hat{\mathbf{Y}} \tag{5.27}$$

where $N_y = N_\phi + N_p$ and Π is the $N_\phi + 1 \times N_y + 1$ matrix given by:

$$\Pi = \begin{bmatrix} p_0 & p_1 & p_2 & \dots & p_n & 0 & 0 & 0 & \dots & 0 \\ 0 & p_0 & p_1 & \dots & p_{n-1} & p_n & 0 & 0 & \dots & 0 \\ 0 & 0 & p_0 & \dots & p_{n-2} & p_{n-1} & p_n & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & p_0 & \dots & p_n \end{bmatrix} \tag{5.28}$$

Prediction of Reference Trajectory by Taylor series expansion

The objective of the control is to drive the predicted output $y(t)$ along a desired smooth path to a setpoint w. Such a path is called a reference trajectory. The reference trajectory $y_r(t)$ is the output of the reference model represented by

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r w \\ y_r &= C_r x_r, \end{aligned} \tag{5.29}$$

where $x_r \in \mathbf{R}^{n_r}$, $A_r \in \mathbf{R}^{n_r \times n_r}$, $B_r \in \mathbf{R}^{n_r \times 1}$, $C_r \in \mathbf{R}^{1 \times n_r}$, $w \in \mathbf{R}$.

In order to define the predicted output of the reference trajectory $y_r(t, T)$ a truncated Taylor series is used, obtaining:

$$y_r(t,T) = y_r(t) + \dot{y_r}T + y_r^{(2)}\frac{T^2}{2!} + ... + y_r^{(N_y)}\frac{T^{N_y}}{N_y!}, \tag{5.30}$$

where the derivatives are easy to obtain from the reference model simulation. Rewriting this equation

$$w_r^*(t,T) = \mathbf{T}(T)\mathbf{w}_r(t) \tag{5.31}$$

where

$$\mathbf{w}_r(t) = \left[y_r\ \dot{y}_r\ y_r^{(2)} \ldots y_r^{(N_y)}\right]^T \tag{5.32}$$

Another possibility is to choose w_r as Demircioglu and Gawthrop [21] did it

$$\mathbf{w}_r(t) = \mathbf{R}_0 y(t) + \mathbf{R}(w - y(t)) \tag{5.33}$$

where $\mathbf{R}$ is a column vector containing the Markov parameters of a reference dynamic system and $\mathbf{R}_0$ has first element 1 and the rest 0. Both $\mathbf{R}$ and $\mathbf{R}_0$ have the same dimension $N_y + 1$. A particularly simple case arises when the reference is just a unit gain and so $\mathbf{R} = \mathbf{R}_0$:

$$\mathbf{w}_r(t) = \mathbf{R}_0 w \tag{5.34}$$

Moving Horizon Control and Cost Function Optimisation with constraints

The concept of *moving−horizon* control has been discussed in detail elsewhere (Mayne and Michalska [60]). The basic idea is to design with a moving time frame located at time t regarding $\hat{x}(t)$ as the initial condition of a state trajectory $x^*(t,T)$ driven by an input $u^*(t,T)$ together with associated predicted outputs $y^*(t,T)$ and $\phi^*(t,T)$. None of the starred variables have a direct relationship with the actual variables, in particular $u(t+T) \neq u^*(t,T)$ except when $T = 0$.

Within this moving time frame, the predicted value of ϕ at time T is given by an equation of the same form as equation (5.26)

$$\phi^*(t,T) = \mathbf{T}(T)\hat{\mathbf{\Phi}}(t) = \mathbf{T}(T)\Pi\mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}^*(t,0)) \tag{5.35}$$

Following Demircioglu and Gawthrop [21], the control $u^*(t,T)$ within the moving frame is *constrained to be a polynomial of order* N_u function of time. The optimisation problem can now be formulated as the *minimisation with respect to* $\mathbf{u}^*_{N_u}(t,0)$ of the *non − dynamic* cost function:

$$
\begin{aligned}
J_{NPGMV}(\mathbf{u}^*_{N_u}(t,0)) &= \frac{1}{2}(\phi^*(t,T) - w^*(t,T))^2 \\
&= \frac{1}{2}[\Pi\mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}^*_{N_u}(t,0)) - \mathbf{w}_r(t)]^T \\
&\mathbf{T}^T\mathbf{T}[\Pi\mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}^*_{N_u}(t,0)) - \mathbf{w}_r(t)]
\end{aligned} \tag{5.36}
$$

The control is calculated by setting $u(t) = u^*(t,0)$, the first element of $\mathbf{u}^*_{N_u}(t,0)$ obtained by the minimisation of the cost function.

This is discussed further (and an example given) by P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33].

5.6 Nonlinear Version of CGPC

5.6.1 Development of the NCGPC

The development of the NCGPC will be carried out following the receding horizon strategy of its linear counterpart, which principles can be summarised as follows:

1 Predict ϕ over a range of future times.

2 Assuming that the future setpoint is known, choose a set of future controls which minimise the future errors between the predicted future output and the future setpoint.

3 Use the first element $u(t)$ as a current input and repeat the whole procedure at the next time instant; that is, use a receding horizon strategy.

Cost Function Minimisation

Given a predicted output over a time frame the CGPC calculates the future controls. The first element u(t) of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. This makes the predicted output depend on the input $u(t)$ and its derivatives, and the future controls being function of

$u(t)$ and its N_u-derivatives. The CGPC of Demircioglu and Gawthrop [21] does not use the polynomial P. This chapter provides the derivation of the more general case based on NPGMV. The NCGPC will be a direct generalisation of the NPGMV of the previous section.

The NPGMV cost function focuses on a single time instant T. In contrast, the NCGPC cost averages the error over time intervals

$$T_1 < T < T_2 \tag{5.37}$$

The cost function is:

$$J = \int_{T_1}^{T_2} [\phi^*(t,T) - w_r^*(t,T)]^2 dT \tag{5.38}$$

where T =prediction horizon.

With the substitution of equations (5.35) and (5.31) the cost function becomes

$$J = \int_{T_1}^{T_2} [\Pi \mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}_{N_u}^*(t,0)) - \mathbf{w}_r]^T \mathbf{T}^T \mathbf{T} [\Pi \mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}_{N_u}^*(t,0)) - \mathbf{w}_r] dT$$

or

$$J = [\Pi \mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}_{N_u}^*(t,0)) - \mathbf{w}_r]^T T_y [\Pi \mathbf{O}_{\mathbf{Gen}}(\hat{x}(t), \mathbf{u}_{N_u}^*(t,0)) - \mathbf{w}_r] dT \tag{5.39}$$

where

$$T_y = \int_{T_1}^{T_2} \mathbf{T}^T \mathbf{T} dT \tag{5.40}$$

the $ijth$ element of T_y is:

$$T_{y_{ij}} = \frac{T_2^{i+j-1} - T_1^{i+j-1}}{(i-1)!(j-1)!(i+j-1)!} \tag{5.41}$$

This non-dynamic optimisation problem equation must be solved numerically for $\mathbf{u}_{N_u}^*(t,0)$ at each time t. The implications of this are discussed further (and an example given) by P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33].

5.7 Nonlinear Systems with Unstable Zero Dynamics

All input-output linearisation strategies are restricted to non-linear systems with stable zero dynamics. The concept of nonlinear zeros presented by Byrnes and Isidori [8] is used in Kravaris [47] to interpret input-output linearization state feedback as a nonlinear analogue of placing poles at the process zeros, this was reviewed in Chapter 3. Consider an invertible transformation

$$\xi = T(x) = \begin{bmatrix} t_1(x) \\ \vdots \\ t_{n-r}(x) \\ h(x) \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{bmatrix} \tag{5.42}$$

that transforms the equation (5.2) into Byrnes-Isidori canonical for

$$\begin{aligned} \dot{\xi}_1 &= F_1(\xi_1, \ \ldots, \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\ &\vdots \\ \dot{\xi}_{n-r} &= F_{n-r}(\xi_1, \ \ldots, \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\ \dot{\xi}_{n-r+1} &= \xi_{n-r+2} \\ &\vdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= L_f^r h(x) + L_g L_f h(x) u(t) && (5.43) \\ y &= \xi_{n-r+1} && (5.44) \end{aligned}$$

The first $n-r$ equations, represent the dynamic zeros. The $y = \xi_{n-r+1}$ is affected by the zero dynamics through the right-hand side of the nth equation. Therefore, in order to cancel the zero dynamics, we need a state feedback that makes the right-hand side of the nth equation a function of $\xi_{n-r+1}, \ldots, \xi_n$ and v. It is requested to be a linear function

$$L_f^r h(x) + L_g L_f h(x) u(t) = \frac{1}{\beta_r}(v - \beta_0 \xi_{n-r+1} - \ldots - \beta_{r-1}\xi_n) \tag{5.45}$$

This leads to the closed-loop system

$$\begin{aligned}
\dot{\xi}_1 &= F_1(\xi_1, \ \ldots \ , \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\
&\vdots \\
\dot{\xi}_{n-r} &= F_{n-r}(\xi_1, \ \ldots \ , \xi_{n-r}, \xi_{n-r+1}, \ \ldots \ \xi_{n-1}, \xi_n) \\
\dot{\xi}_{n-r+1} &= \xi_{n-r+2} \\
&\vdots \\
\dot{\xi}_{n-1} &= \xi_n \\
\dot{\xi}_n &= -\frac{\beta_0}{\beta_r}\xi_{n-r+1} - \ldots - \frac{\beta_{r-1}}{\beta_r}\xi_n + \frac{1}{\beta_r}v
\end{aligned} \tag{5.46}$$

It is possible to see the output is completely unaffected by the first $n-r$ equations. Equation 5.45, can be written in terms of the original variables as

$$u(t) = \frac{v - \sum_{i=1}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)}, \tag{5.47}$$

This is the control law required to cancel the zero dynamics. The NCGPC leads to this control law, when the following assumptions are considered.

Assumptions (NCGPC)

- The system is described by equation (5.2).
- $P = 1$.
- $N_u = N_y - r$.
- State feedback is considered so that $\hat{x} = x$.

Under these assumptions the cost function equation (5.39) becomes:

$$\begin{aligned}
J(\mathbf{u}^*(t,0)) = [\mathrm{O}(x(t)) + H(x(t))\mathbf{u} - \mathbf{w}_r(t)]^T \\
T_y[\mathrm{O}(x(t)) + H(x(t))\mathbf{u} - \mathbf{w}_r(t)]
\end{aligned} \tag{5.48}$$

and the minimisation results in

$$\mathbf{u}^*_{N_u}(t,0) = [H^T T_y H]^{-1} H^T T_y [\mathbf{w}_r - \mathbf{O}] \tag{5.49}$$

Defining

$$\mathbf{K} = [H^T \mathbf{T}^T \mathbf{T} H]^{-1} H^T \mathbf{T}^T \mathbf{T} \tag{5.50}$$

As explained above, just the first element of $\mathbf{u}^*(t,0)$ is applied. Then, the first row of $\mathbf{K}$, which will be called $\mathbf{k}$, the control law is given by

$$\mathbf{u}^*_{N_u}(t,0) = \mathbf{k}[\mathbf{w}_r - \mathbf{O}] \tag{5.51}$$

The matrix H can be decomposed as

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{5.52}$$

H_1 is a zero matrix with dimension $r \times (N_y - r + 1)$, and H_2 is a lower triangular matrix with dimension $(N_y - r + 1) \times (N_y - r + 1)$, of the following form.

$$H_2 = \begin{bmatrix} L_g L_f^{r-1} h(x) & 0 & \dots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \dots & 0 \\ J_2(x) & I_1(x) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \dots & L_g L_f^{r-1} h(x) \end{bmatrix} \tag{5.53}$$

The matrix T_y is decomposed as

$$T_y = \begin{bmatrix} T_{y_{11}} & T_{y_{12}} \\ T_{y_{21}} & T_{y_{22}} \end{bmatrix} \tag{5.54}$$

where

$T_{y_{11}}$ is $r \times r$

$T_{y_{12}}$ is $r \times (N_y - r + 1)$

$T_{y_{21}}$ is $(N_y - r + 1) \times r$

$T_{y_{22}}$ is $(N_y - r + 1) \times (N_y - r + 1)$

Equation (5.50) can now be written as

$$\mathbf{K} = H_2^{-1}[T_{y_{22}}^{-1} T_{y_{21}} I] \tag{5.55}$$

I is the unitary matrix with dimension $(N_y - r + 1) \times (N_y - r + 1)$. The first row of the inverse of H_2 will be as follows

$$h_2 = [1/L_g L_f^{r-1} h(x) \; 0 \; \dots \; 0] \tag{5.56}$$

As explained above, just the first element of $\mathbf{u}^*(\mathbf{t}, \mathbf{0})$ is applied. Then, the first row of $\mathbf{K}$ will be needed, which will be called $\mathbf{k}$ is given as

$$\mathbf{k} = \frac{1}{L_g L_f^{r-1} h(\hat{x})}[t_1 \; t_2 \; \dots \; t_r \; 1 \; 0 \; \dots \; 0] \tag{5.57}$$

where $t_1 \; t_2 \; \dots \; t_r$ are the elements of the first row of $T_{y_{22}}^{-1} T_{y_{21}}$. They are nonlinear functions of T and the row vector has dimension $1 \times r$.

Thus, the control law is given by

$$u(t) = \frac{(t_1 p_0 \dots t_r p_{r-1})(w - y) - \sum_{i=1}^{r-1} t_{i+1} L_f^i h(x) - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \tag{5.58}$$

which can be rewritten as,

$$u(t) = \frac{(w - y(t)) - \sum_{i=1}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)}, \tag{5.59}$$

where

$$\beta_r = 1/(t_1 p_0 + t_2 p_1 \dots t_r p_{r-1}) \tag{5.60}$$

$$\beta_i = t_{i+1}/(t_1 p_0 + t_2 p_1 \dots t_r p_{r-1}) \qquad i = 1, \dots, r-1 \tag{5.61}$$

We can notice, that incredible as it may seem, large N_y does not require a bigger computational effort, because as we can see from (5.59) , the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of t_i, which just depends on T. Moreover, N_y can be chosen as the smallest predictor order. Which is such that the predicted output depends on $u(t)$. The relative degree r of the system is exactly equal to the number of times the output has to be differentiated in order for the input to explicitly appear. Because of this, the relative degree r will be the smallest predictor order N_y.

We can see that choosing $v = w - y$ the equation 5.59 becomes into equation (5.47). Therefore, when $N_u = N_y - r$ the NCGPC leads to a linearisation state feedback, which cancels the zero dynamics.

It is well known that the closed loop system (5.46) does not guarantee internal stability; the states $\xi_1, \ \ldots \ , \xi_{n-r}$ may go unstable even if the subsystem of the last r state equations is stable. However, if the zero dynamics are stable, the input-output stability guarantees internal stability of the closed-loop system (See Sections 2.4 and 3.4. This is analogous with the linear results: placing poles at the process zeros does not destroy internal stability if the zeros are in the open left half plane. This fact is used in nonlinear systems in order to control non-minimum phase systems. Therefore, the system is required to be factored in minimum-phase and the nonminimum-phase parts and just the first one is used for purpose of control design. F. III Doyle [42], developed two approaches to control nonminimum-phase system, in the first approach, a partial linearisation is made which preserves stability by using an approximate stable/anti-stable factorisation. The second one is an inner-outer factorisation approach, which derives a minimum-phase nonlinear system P_m with the following characteristics:

i Poles of the linearisation of the original system around a given equilibrium point =Poles of the linearisation of P_m around the same point (along the whole equilibrium manifold),

ii Zeros of the linearisation of the original system around a given equilibrium point = " reflection " of the zeros of the linearisation of P_m around the same point (along the whole equilibrium manifold), iii Static gain of the original system = Static gain of P_m.

This approach is just applied on maximally nonminimum-phase systems (i.e., systems with all zeros of the linearisation of P around the equilibrium point to be in the right-hand plane), due to (ii).

Another technique used to control nonminimum phase systems, is based on an approximation of non-minimum phase systems by minimum phase systems, L. Benvenuti *et al* [5]. A modification of the output of nonlinear system is made by using a transformation performed on the Jacobian linearisation of the system. This transformation removes the right-half plane zeros while left-half plane zeros remain in their original positions. This approach can deal with a nonlinear system whose linearisation possesses real right-half plane zeros. They cannot deal with system whose linearisation possesses complex zeros.

NCGPC has two main advantages. The first advantage is that it can constrain the predicted control through N_u, when $N_u = 0$ the predicted input is constrained to be constant in the future. It is possible to infer that $u(t)$ is indirectly constrained by N_u. Additionally, the response becomes slow and the control is not very active, this fact will be illustrated by simulations. The second advantage is, when $N_u < N_y - r$ the cancellation of the zero dynamics does not occur with the NCGPC. Therefore, the internal stability is preserved.

Here is presented, for the first time, a nonlinear controller derived on the basis of the predictive control, which was not derived with the objective to cancel the nonlinearities as the feedback linearisation techniques. Therefore, it can control systems with unstable zero dynamics as mentioned above.

5.8 Systems with not Well Defined Relative Degree

The application of geometric linearisation theory has permitted the extension of the applications of linear control algorithms to nonlinear systems. It is done by using the feedback linearisation techniques through differential geometry approach, to transform a nonlinear input-output system into a linear input-output system; then a linear control design algorithm is applied to the linearised model. However, the applicability of the linearisation algorithm fails when the system has singular points, this happens when $L_gL_f^{r-1}h(x) \neq 0$ for $x \neq x_0$ but $L_gL_f^{r-1}h(x_0) = 0$. These can be viewed as regions where the relative degree can not be defined (See Section 2.3). This problem can be overcome using control weighting (Clarke and Gawthrop [12]; Gawthrop [26]; Clarke and Gawthrop [13]; Gawthrop [29]). The control weight λ in the cost function J plays a very important role because it can stop the singularity condition in the control law, but it does not ensure stability.

The following assumptions needs to be made in order to get the above objectives:

Assumptions

- The system is described by equation (5.2).
- stable zero dynamics.
- $N_u = N_y - r$.
- $P = 1$.
- The trajectory reference is given by equation (5.32).
- $N_y = r$.
- $J_{NPGMV}(u^*(t,0)) = [y_r^*(t,T) - y^*(t,T)]^2 + \lambda[u^*(t,0)]^2$

Under these assumptions the cost function becomes:

$$\begin{aligned} J_{NPGMV}(u^*(t,0)) = [\mathbf{y}_r(t) - \mathbf{O}(x(t)) + H(x(t))u^*(t,0)]^T \\ \mathbf{T}^T\mathbf{T}[\mathbf{y}_r(t) - \mathbf{O}(x(t)) + H(x(t))u^*(t,0)] + \lambda[u^*(t,0)]^2 \end{aligned} \tag{5.62}$$

and the minimisation results in the following control law:

$$u(t) = \frac{[(y_r - y) + (\dot{y}_r - L_fh)T + \ldots + (y_r^{(r)} - L_f^rh)\frac{T^r}{r!}]L_gL_f^{r-1}h\frac{T^r}{r!}}{\lambda + [L_gL_f^{r-1}h\frac{T^r}{r!}]^2} \tag{5.63}$$

In order to see, the role of the λ, some approximations made by [69] are recalled:

- As $\mid \frac{T^r}{r!} L_g L_f^{r-1} h(x) \mid \gg \lambda$, when the system is relatively far from any singular point, then,

$$u(t) \approx \frac{(y_r - y) + (\dot{y}_r - L_f h)T + \ldots + (y_r^{(r)} - L_f^r h)\frac{T^r}{r!}}{L_g L_f^{r-1} h \frac{T^r}{r!}} \tag{5.64}$$

- But, when $\mid \frac{T^r}{r!} L_g L_f^{r-1} h(x) \mid \ll \lambda$, when the system is very close to any singular point, we have

$$u(t) \approx 0 \tag{5.65}$$

This case is related directly to the control introduced by D. Rangel [69], who has made a modification into the nonlinear state feedback control adding the control weight. Nevertheless, it is sometimes difficult to find λ. Making use of N_u, the NCGPC allows another solution, if $N_u < N_y - r$, the problem of singularity points is also removed.

5.9 Simulation Study

In order to show the effectiveness of the proposed controller and to study the effects of the NCGPC design parameters (N_y, N_u, T_1, T_2, R_n/R_d) simulations will be presented where polynomial P is chosen equal 1. The examples used in the simulation are as follows:

Example 1:

$$\begin{aligned} \dot{x}_1 &= -x_1 - x_2 \\ \dot{x}_2 &= exp(-x_2) - 1 - u \\ y &= x_1 \end{aligned}$$

Example 2:

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 \\
\dot{x}_2 &= -3x_2 + x_1^2 + 2u \\
\dot{x}_3 &= x_1 - 2x_3 \\
\dot{x}_4 &= -x_4 + x_3^2 \\
y &= x_1 - 3x_3
\end{aligned}
$$

Example 3:

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 \\
\dot{x}_2 &= -3x_2 + x_1^2 + (2 + sin^2(x_4))u \\
\dot{x}_3 &= x_1 - 2x_3 \\
\dot{x}_4 &= -x_4 + x_3^2 \\
y &= x_1 - 3x_3
\end{aligned}
$$

Example 4:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - 2x_2 + x_3 + 10x_4 + x_3x_4 + u \\
\dot{x}_3 &= -2x_1 - 3x_2 + x_3 \\
\dot{x}_4 &= -x_4 + .3x_2^2 \\
y &= x_1
\end{aligned}
$$

Example 5:

$$
\begin{aligned}
\dot{x}_1 &= x_3 - x_2^2 \\
\dot{x}_2 &= -x_2 - u \\
\dot{x}_3 &= x_1^2 - x_3 + u
\end{aligned}
$$

5.9.1 The predictor order N_y

As we explained before, the future output is approximated by an N_y^{th} order truncated Maclaurin series. It is clear that N_y needs to be chosen such that a good approximation can be obtained over the range in which T varies. We will chose N_y so that a good approximation of the open-loop system step response over the range $0 < T < T_2$ is obtained.

The system considered in order to illustrate the variations of N_y is example 1. The derivatives of the output are given by

$$\begin{aligned}
\dot{y} &= -x_1 - x_2 \\
y^{(2)} &= x_1 + x_2 - exp(-x_2) + 1 + u \\
y^{(3)} &= -x_1 - x_2 + (exp(-x_2) - 1 - u)(1 + exp(-x_2)) + \dot{u}
\end{aligned}$$

We can see from the figure 5.1, that if $N_y = 3$, the approximation will be poor if $T > 1$ and so $N_y > 3$ if T is to be chosen greater than 1. As we concluded before if $N_u = N_y - r$, the control law is independent of the last $N_y - r$ derivatives. Then it is possible to calculate the parameters β_i considering the largest N_y, without the use of the remaining derivatives. We will consider this case, in all the processes, except in the nonminimum phase systems.

5.9.2 The maximum prediction horizon T_2

Example 1 is chosen to illustrate the effects of T_2. In this simulation $N_y = 3$, $N_u = N_y - r = 1$, $R_n/R_d = 1/(s+1)$ and $T_1 = 0$. Variations of T_2 are chosen to vary from .5, 1, 2 and 3. Figures (5.2), (5.3) and (5.4) illustrate the effects due to these variations. We can infer from the simulations that the small value T_2 corresponds the fastest step response. The figures show as well that the response becomes slower and poles move towards the origin as T_2 increases. In this simulation, a perfect model is considered, then $y(s) = G(s)w$.

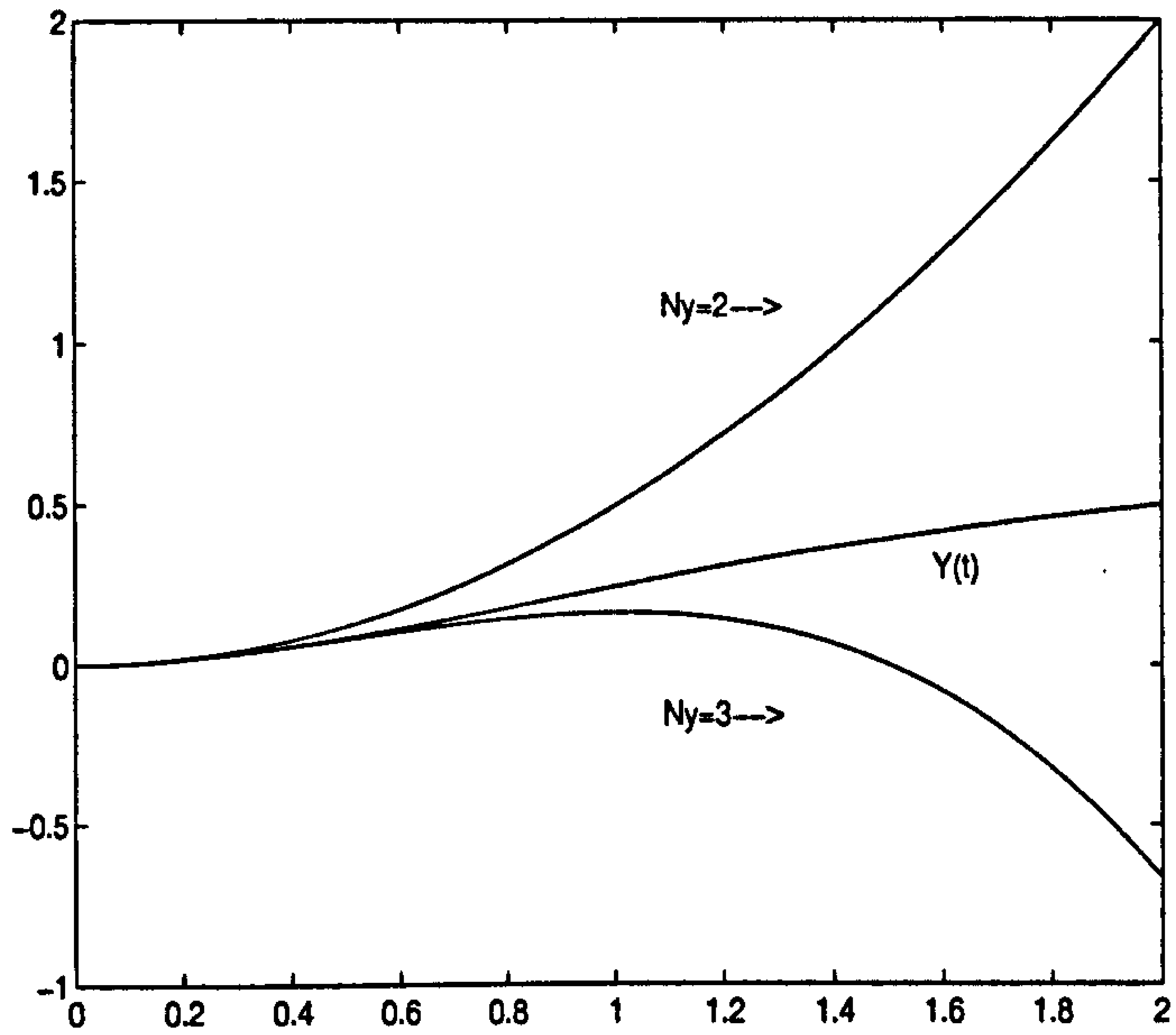

Figure 5.1: Example1: Illustration of N_y

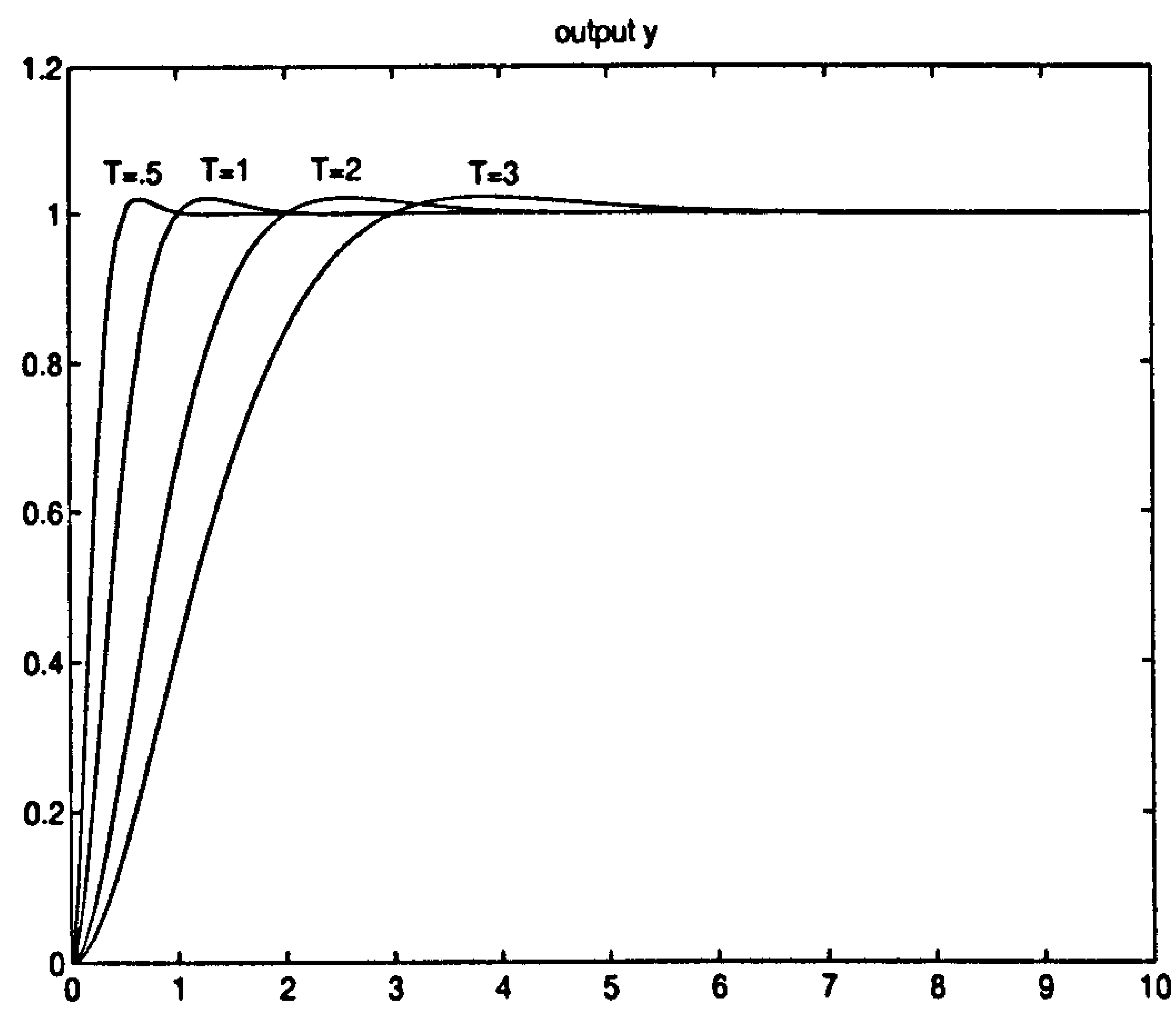

Figure 5.2: Example 1: The effects on y when T_2 is varied

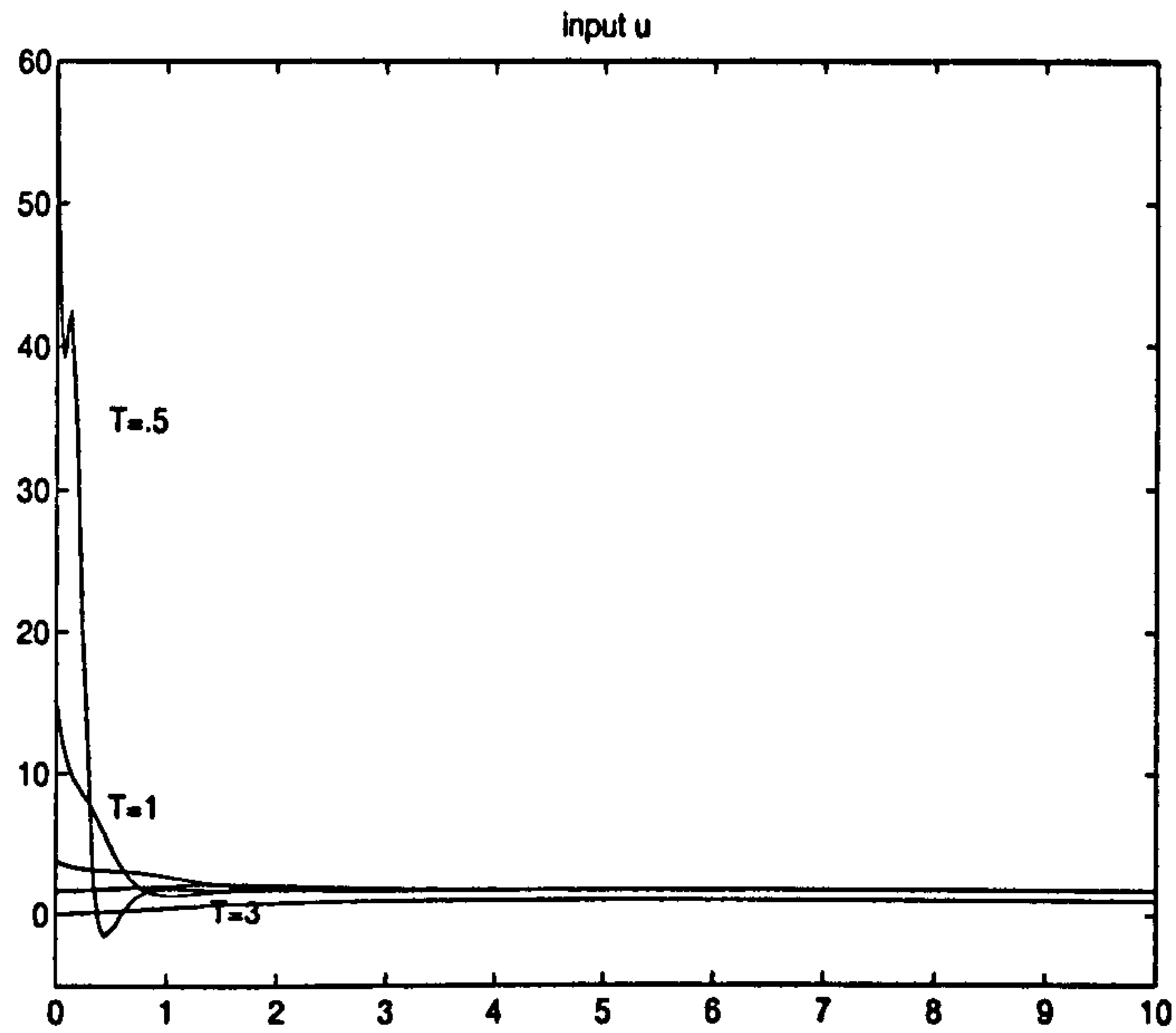

Figure 5.3: Example 1: The effects on u when T_2 is varied

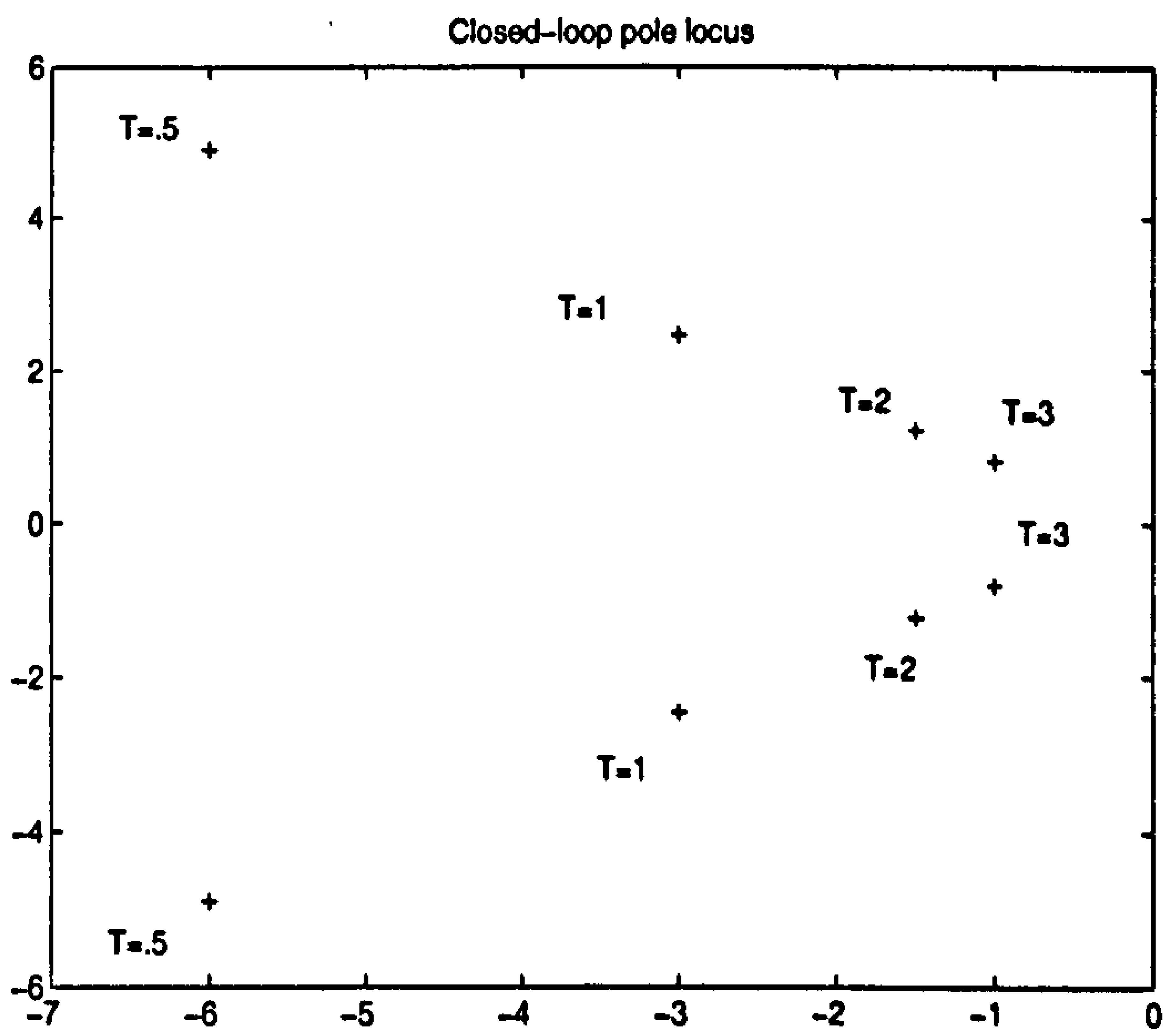

Figure 5.4: Example 1: The effects on poles when T_2 is varied

5.9.3 The Control order N_u

The control order has the function to constrain the predicted input $u_r^*(t,T)$. When $N_u = 0$ the predicted input is constrained to be constant in the future, when $N_u = 1$ the predicted input will be a ramp. We can say that u(t) is indirectly constrained by N_u. As in CGPC, a small value of N_u gives less active control $u(t)$ and slow output response. We will use the example 1 to illustrate this, the design parameters are $N_y = 3$, $N_u = 0$ and $N_u = 1$ $R_n/R_d = 1/(s+1)$. From Figures (5.5) and (5.6) that smaller value of N_u gives less active control $u(t)$ and slow output response. We are going to analyse in detail, the effects when $N_u < N_y - r$.

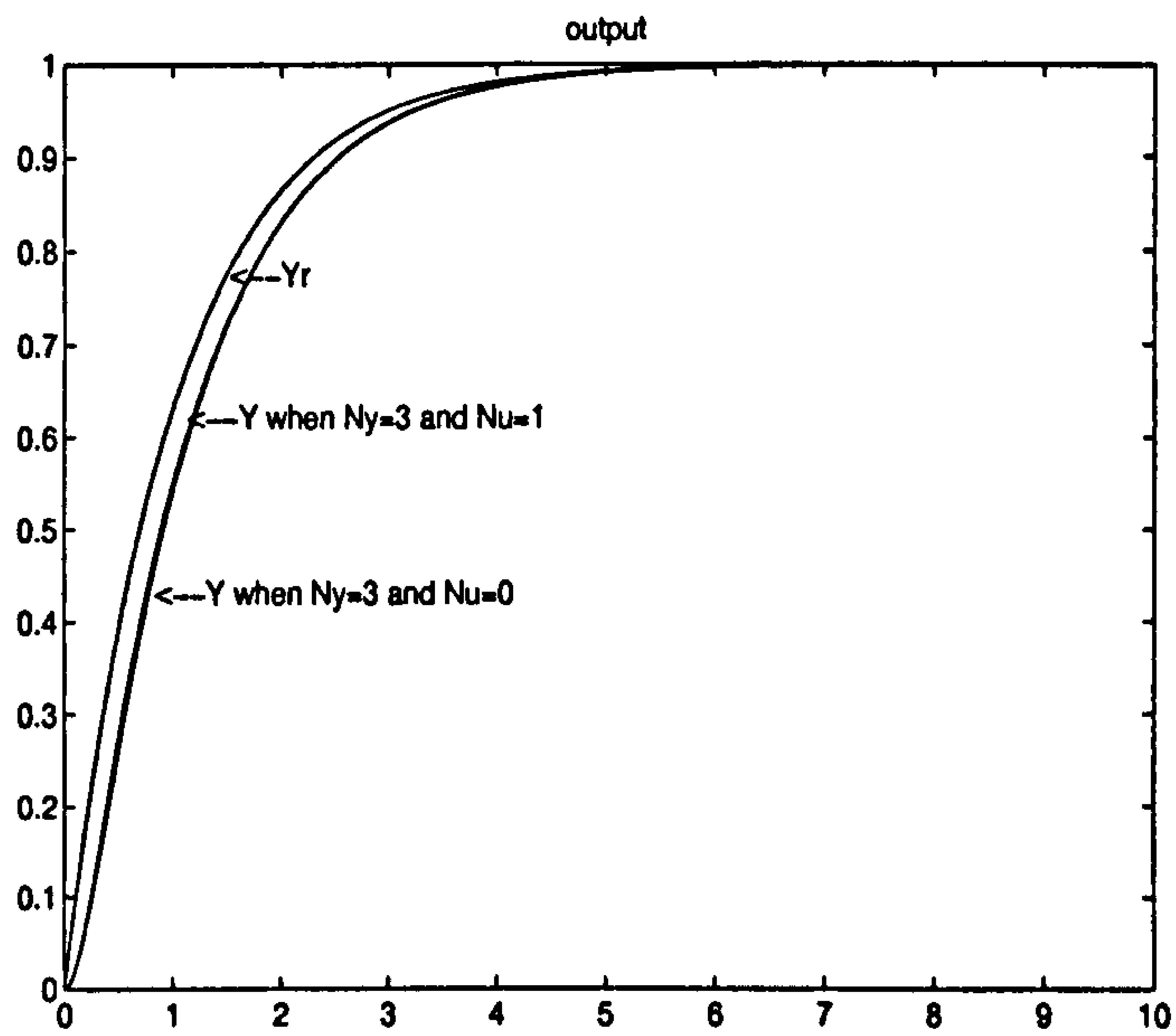

Figure 5.5: Example 1: The effects on y when N_u is varied

5.9.4 The Reference Model R_n/R_d

The Reference Model R_n/R_d can either be used to penalise the overshoot or as an approximate model. The NCGPC, as CGPC control law, tries to match the system output to the reference model output. But, it is not possible to get exact model-following, since the NCGPC only changes the closed-loop pole locations. A good choice for R_n/R_d, is a first order. When the parameters are chosen as $N_u = N_y - r$ and R_n/R_d as a first order, the NCGPC places one of the closed-loop poles at the pole of

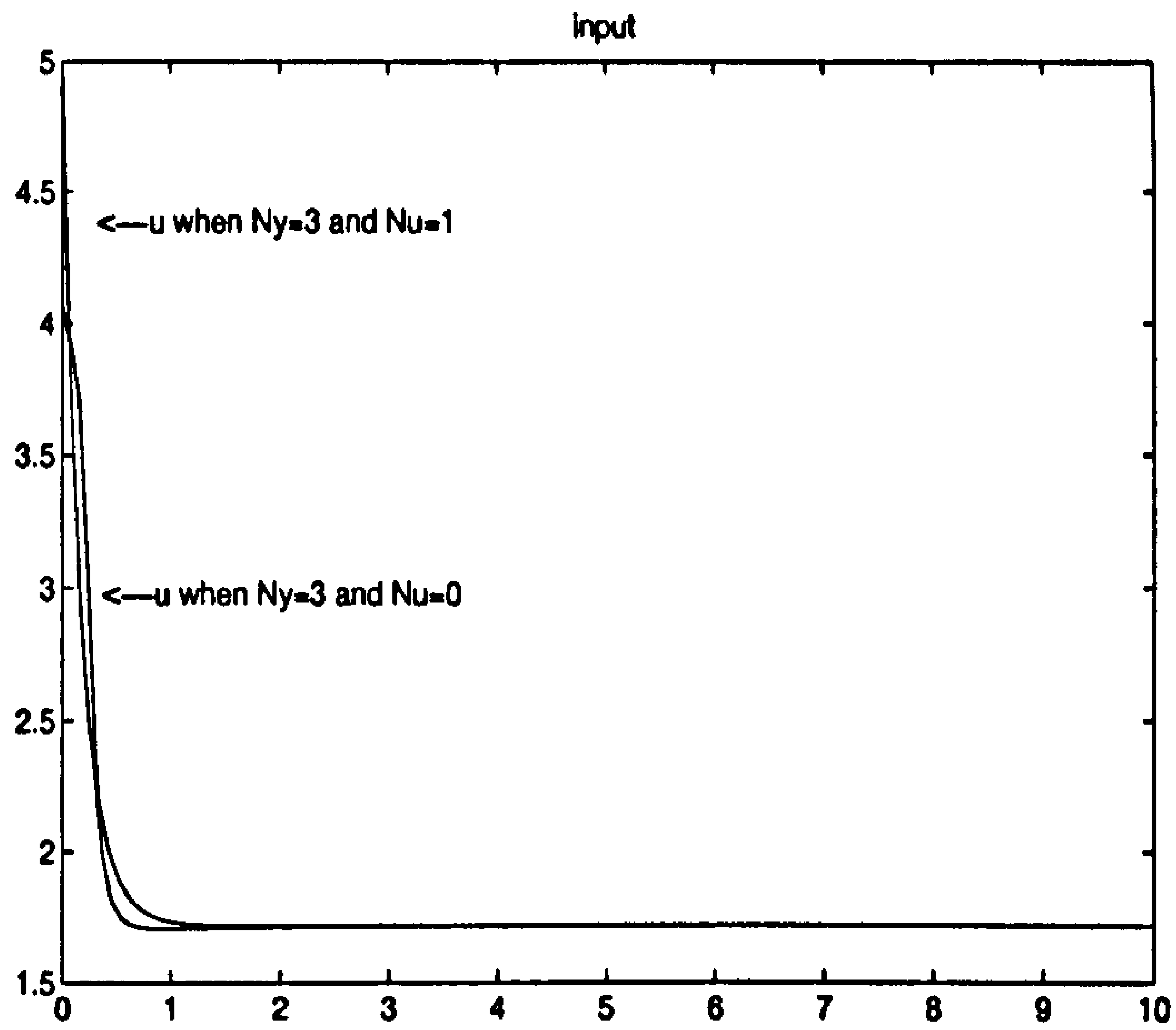

Figure 5.6: Example 1: The effects on u when N_u is varied

R_d, the rest away from the imaginary axis, (T_2 gives the distance). It is possible to get a very close model-following by choosing the right T_2. Figures (5.7) and (5.8) show the simulation results , where the example 1 is used, $N_u = 1$, $N_y = 3$, $R_n/R_d = 1/(s+1)$ and $T_2 = 1$. We can see that the one of the closed-loop poles is placed at -1 and the overshoot is removed. When $N_u < N_y - r$ it is not possible to get a close following-model, instead of this, R_n/R_d just penalises the overshoot. This case will be studied later.

5.9.5 Simulation Results when $N_u < N_y - r$

Example 1 is again chosen as before. The design parameters are $N_y = 3$, $N_u = 0$, $T_2 = 1$ and $R_n/R_d = 1/(s+1)$.

$$
\begin{aligned}
y(t) &= x_1 \\
\dot{y}(t) &= y_1 \\
y^{(2)}(t) &= y_2 + u \\
y^{(3)}(t) &= y_3 - u(1 + exp(-x_2)) + \dot{u}
\end{aligned}
$$

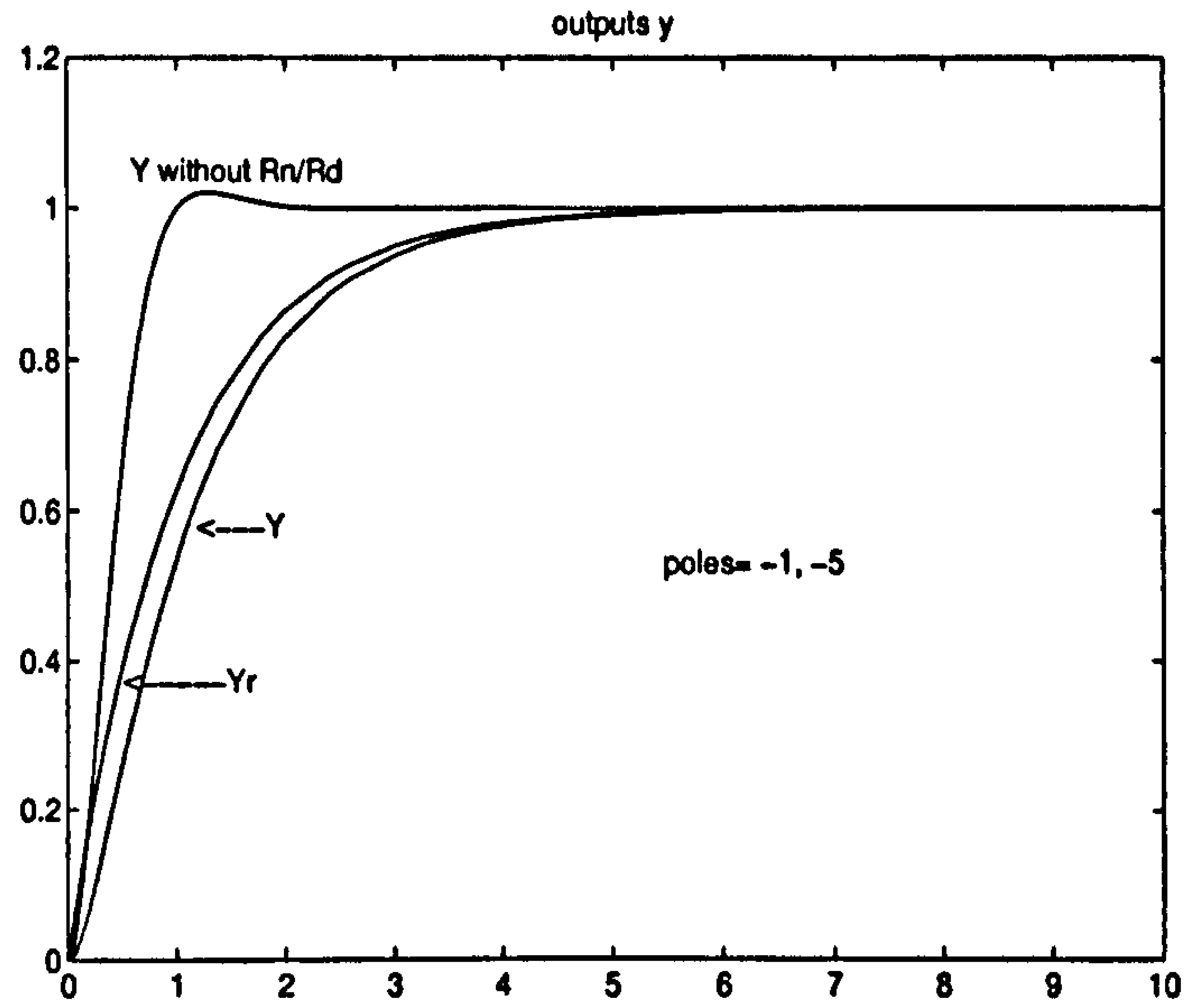

Figure 5.7: Example 1: The effects on y when R_n/R_d is used

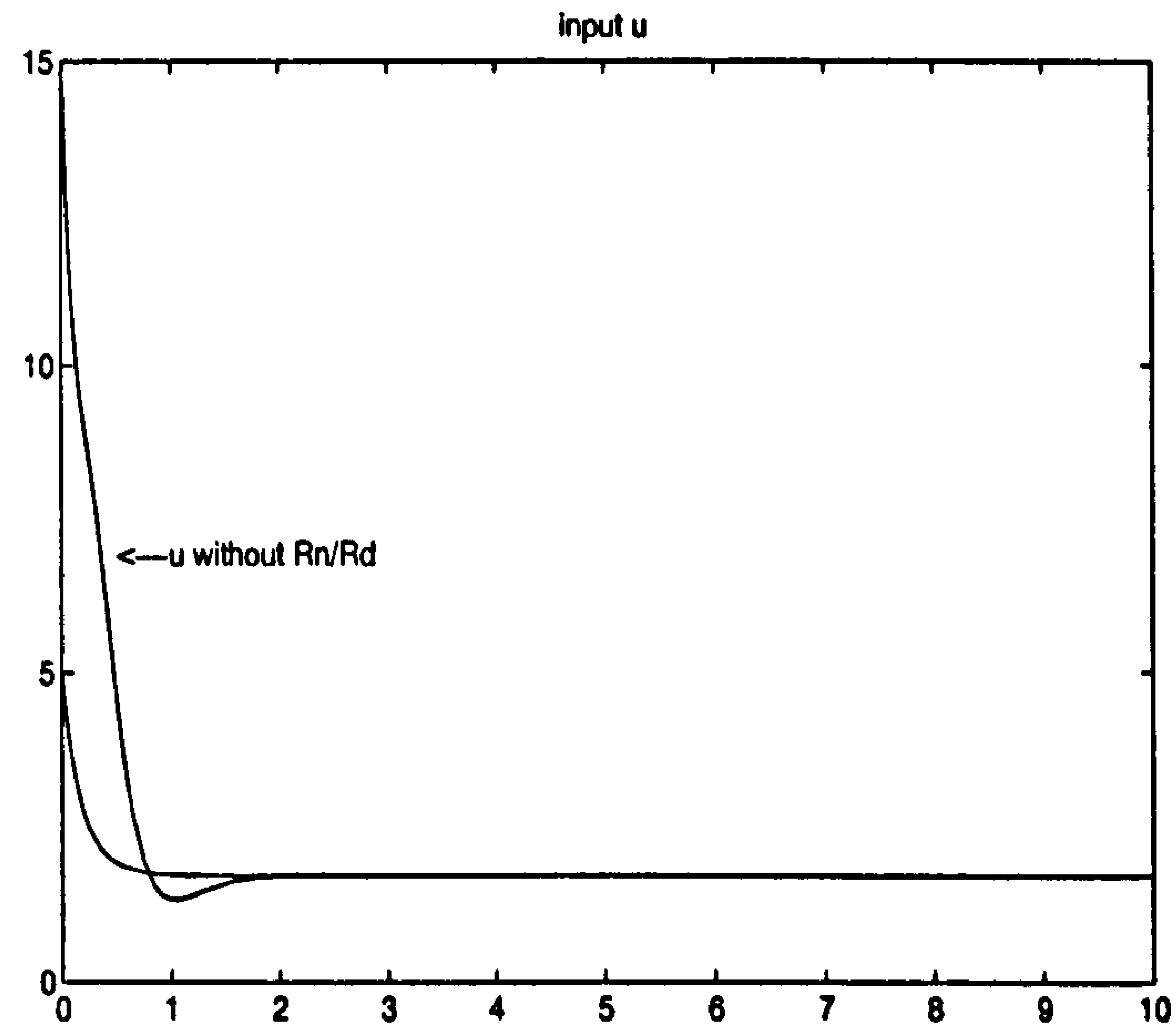

Figure 5.8: Example 1: The effects on u when R_n/R_d is used

$$
\begin{aligned}
y_1 &= -x_1 - x_2 \\
y_2 &= x_1 + x_2 - exp(-x_2) \\
y_3 &= -x_1 - x_2 + (exp(-x_2) - 1)(1 + exp(-x_2))
\end{aligned}
$$

The control law is given by

$$u(t) = (k_2(x) - k_3(x) + k_4(x))(z - y) - k_2(x)y_1 - k_3(x)y_2 - k_4(x)y_3$$

where, k_2, k_3 and k_4 are function of x due to $H = [1, -(1+exp(-x_2)]^T$. The simulation result is shown in Figures (5.9) and (5.10)

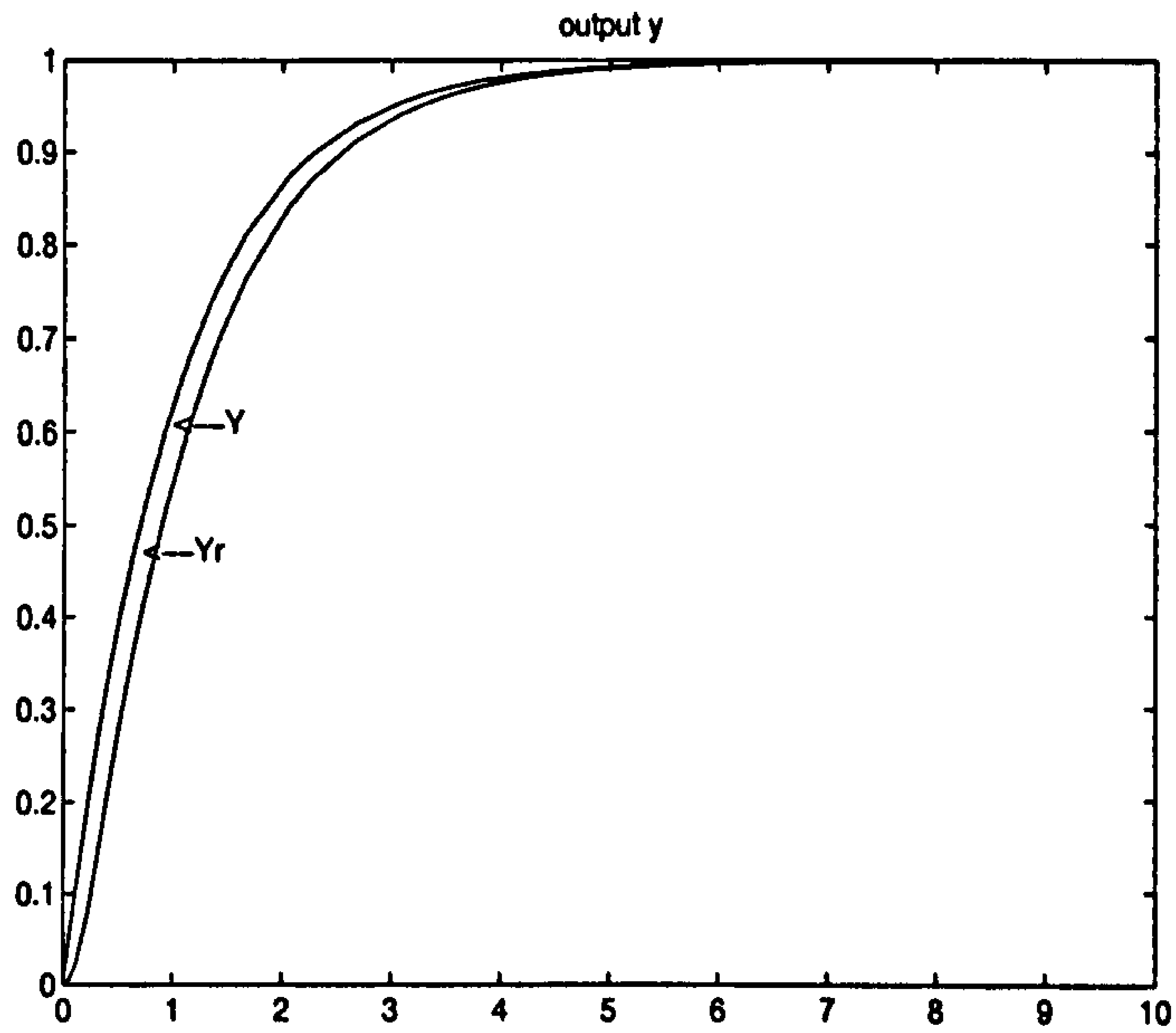

Figure 5.9: Example 1: output when $N_y = 3$ and $N_u = 0$

5.9.6 Nonlinear Systems with Unstable Zero Dynamics

Example 2 was treated by [40], it has a well defined relative degree $r = 2$ and unstable zero dynamics given by

$$
\begin{aligned}
\dot{x}_3 &= x_1 - 2x_3 \\
\dot{x}_4 &= -x_4 + x_3^2
\end{aligned}
$$

The output derivatives are given by

$$y(t) = x_1 - 3x_3$$

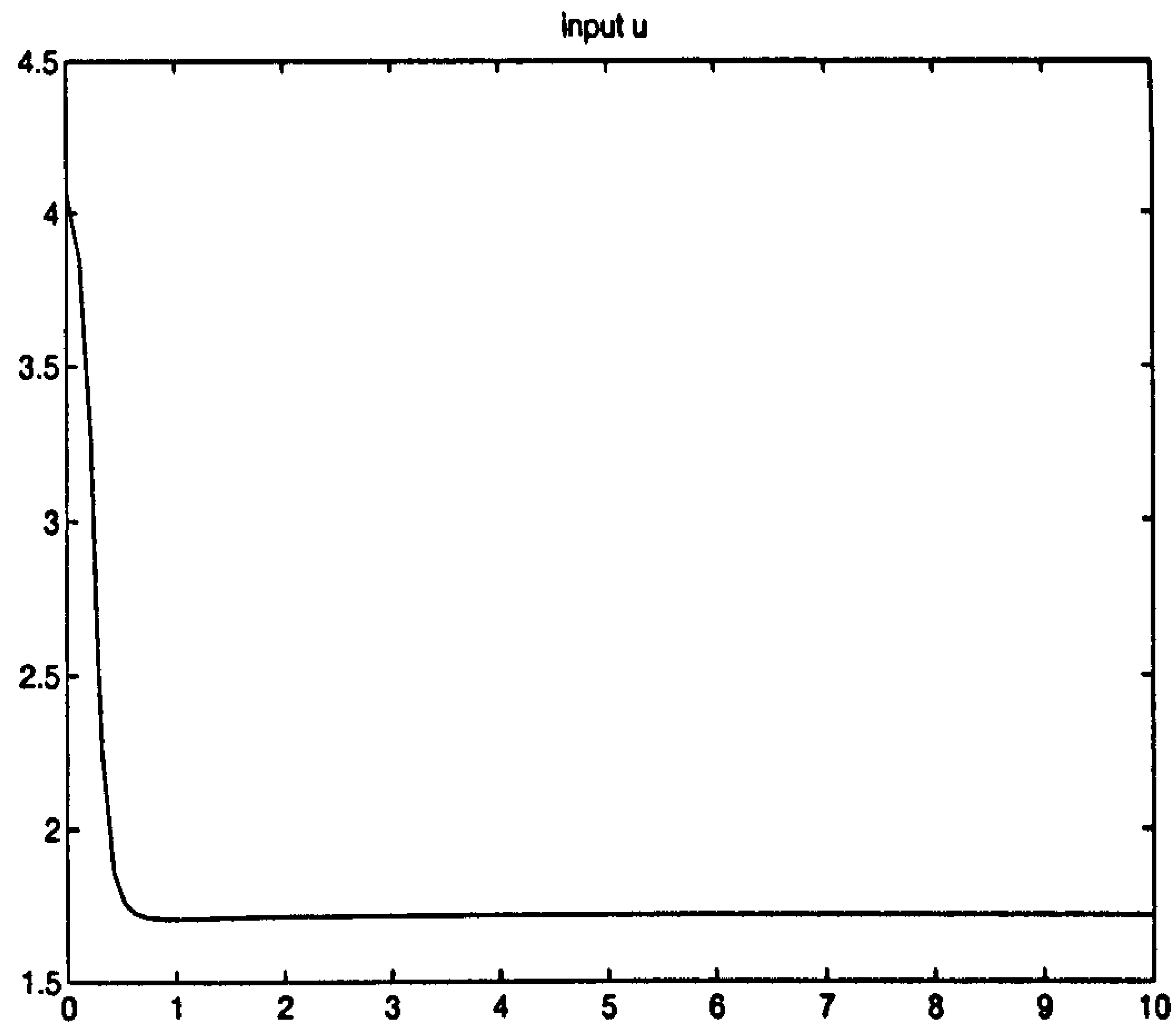

Figure 5.10: Example 1: input u when $N_y = 3$ and $N_u = 0$

$$\begin{aligned} \dot{y}(t) &= y_1 \\ y^{(2)}(t) &= y_2 + 2u \\ y^{(3)}(t) &= y_3 - 14u + 2\dot{u} \end{aligned}$$

where

$$\begin{aligned} y_1 &= -4x_1 + x_2 + 6x_3 \\ y_2 &= 10x_1 - 7x_2 + x_1^2 - 12x_3 \\ y_3 &= -22x_1 + 31x_2 - 9x_1^2 + 2x_1x_2 + 24x_3 \end{aligned}$$

The control law is given by

$$u(t) = (f_1 - f_2 + f_3)(z - y(t)) - f_1y_1 - f_2y_2 - f_3y_3$$

where, f_1, f_2 and f_3 are constants due to $H = [2 - 14]^T$. We can see that this control law is different to the control law required to cancel the zero dynamics equation (5.47).

It was simulated to show the control of non-minimum phase nonlinear systems. The design parameters are $N_u = 0$, $N_y = 3$, $T_1 = 0$, $T_2 = 1$ and $R_n/R_d = 1/(s+1)$. As in CGPC for a non-minimum phase system N_u must be $N_u < N_y - r$. The simulation result is shown in Figures (5.11) , (5.12) and (5.13). We can see that the output response is good, the input and states tend to a constant value. Figure (5.14) shows the output responses, when the setpoints w are 1, 5 and 10. We can see that the output has not the same response, when $w = 10$ the response becomes slower. These differences are because the non-linear system was not linearised. The responses can be improved by choosing appropriate R_n/R_d.

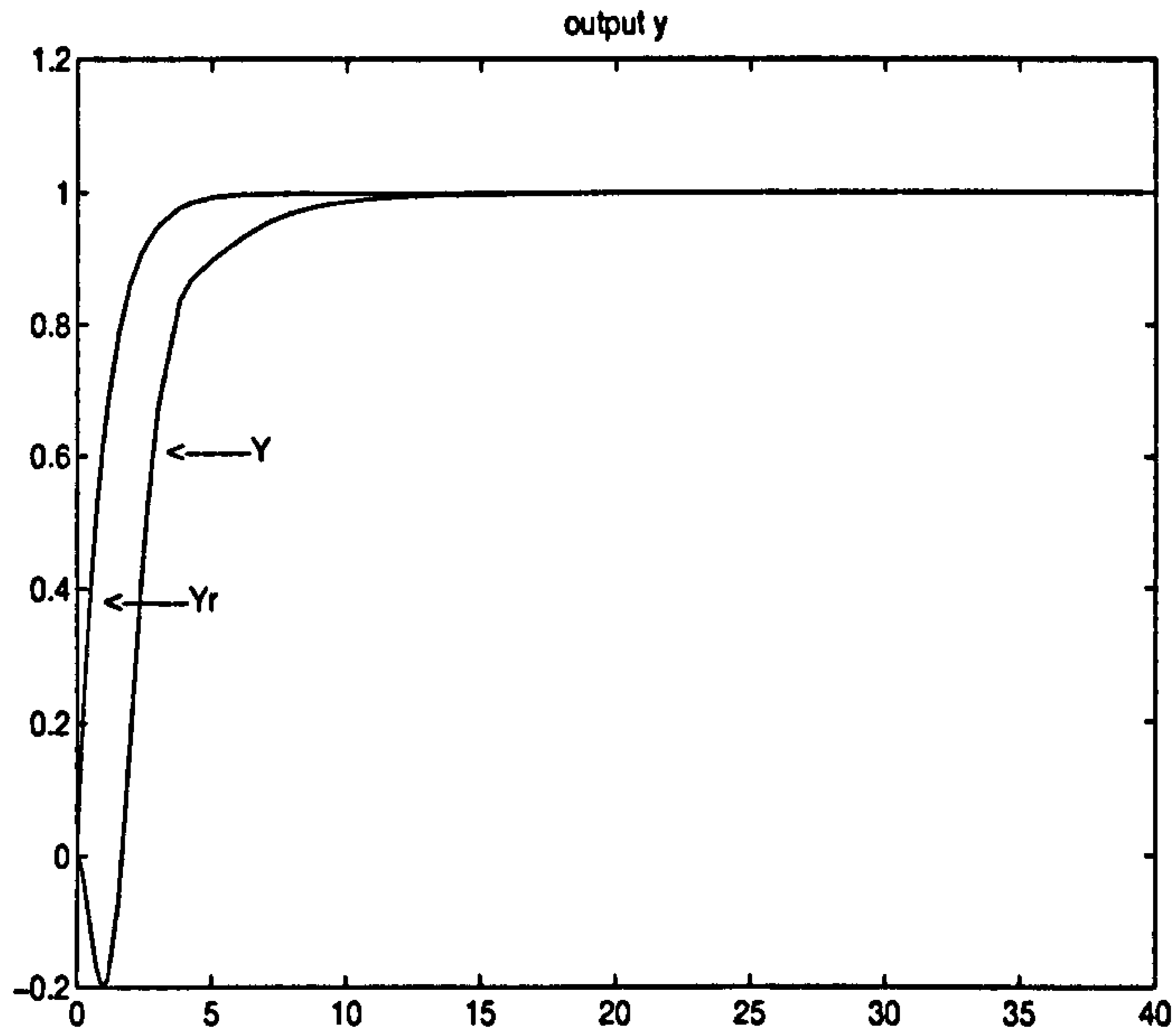

Figure 5.11: Example 2: Output y of the Non-minimum phase system

We will consider the process as the following differential equations

$$\dot{x}_1 = -1.1x_1 + .9x_2$$

$$\dot{x}_2 = -3x_2 + .8x_1^2 + 2u$$

$$\dot{x}_3 = 1.1x_1 - 2x_3$$

$$\dot{x}_4 = -.9x_4 + x_3^2$$

The system output is $y = x_1 - 3x_3$, initial conditions will be $x_1 = .1$, $x_2 = -.2$, $x_3 = .2$ and $x_4 = -.1$ The process model is chosen with the same parameters as example 2. Figures (5.15), (5.17) and (5.16) show good output response when the model is not perfect and the initial conditions are not known. The design parameters were chosen as before.

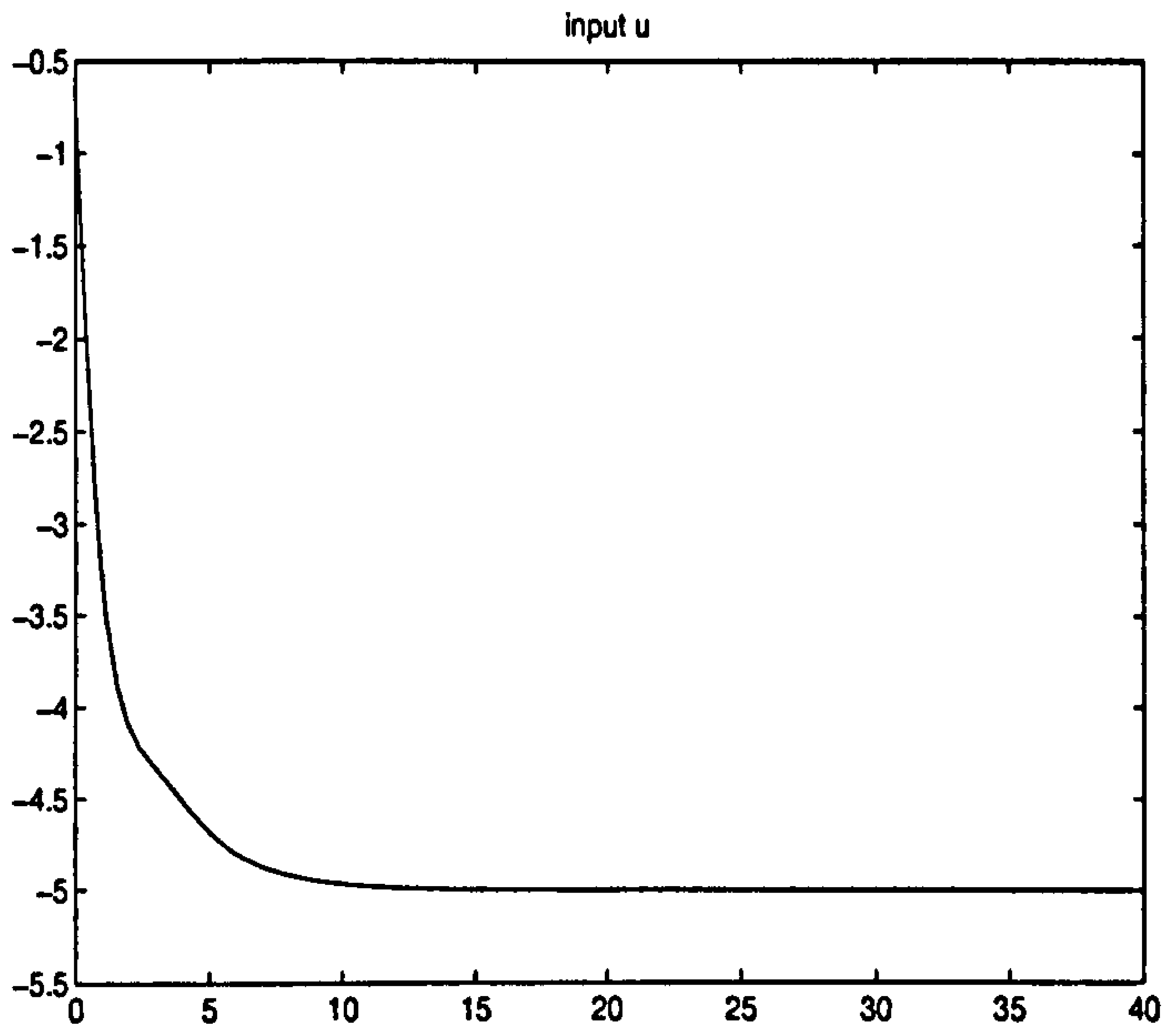

Figure 5.12: Example 2: Input u of the Non-minimum phase system

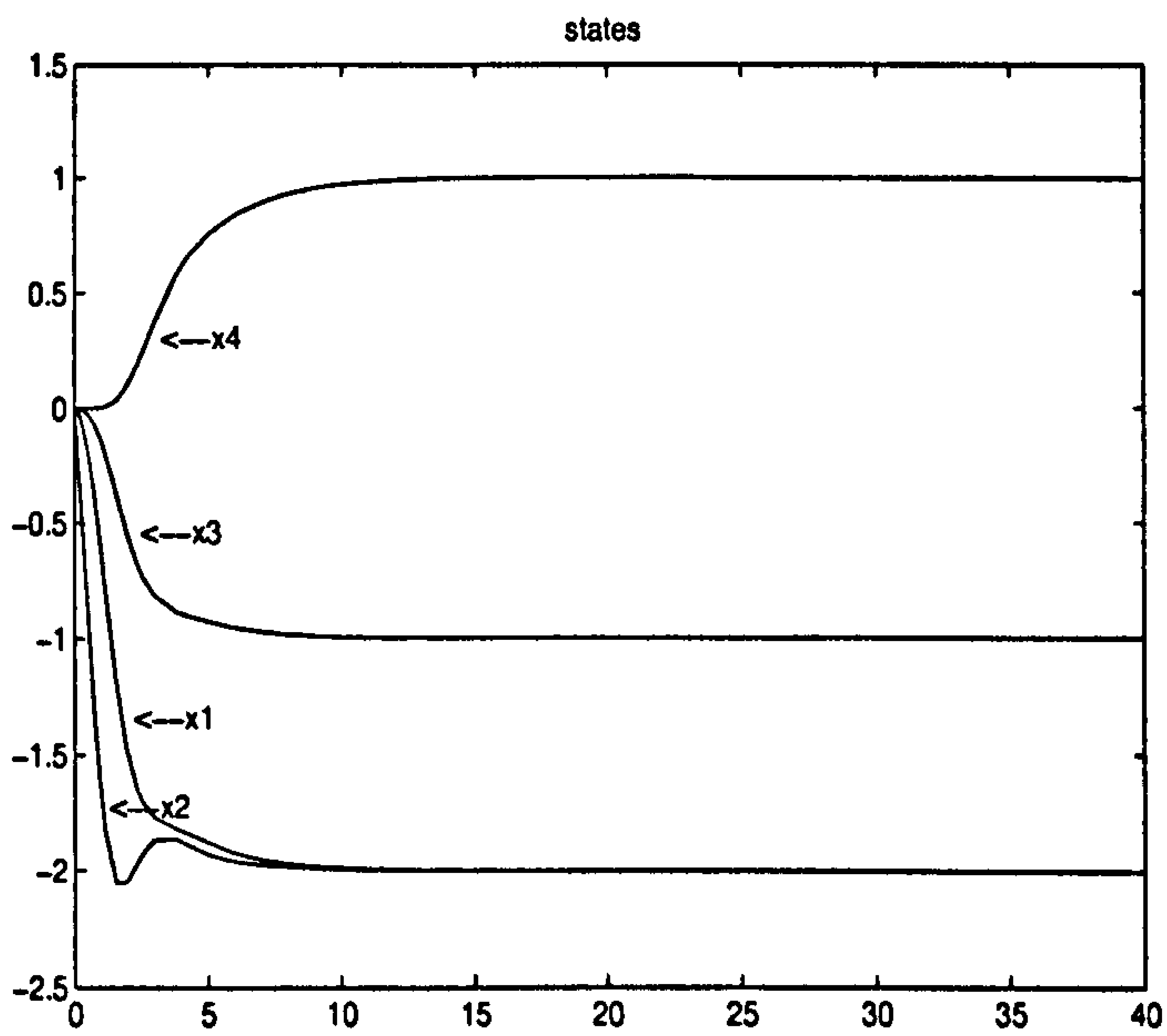

Figure 5.13: Example 2: States of the Non-minimum phase system

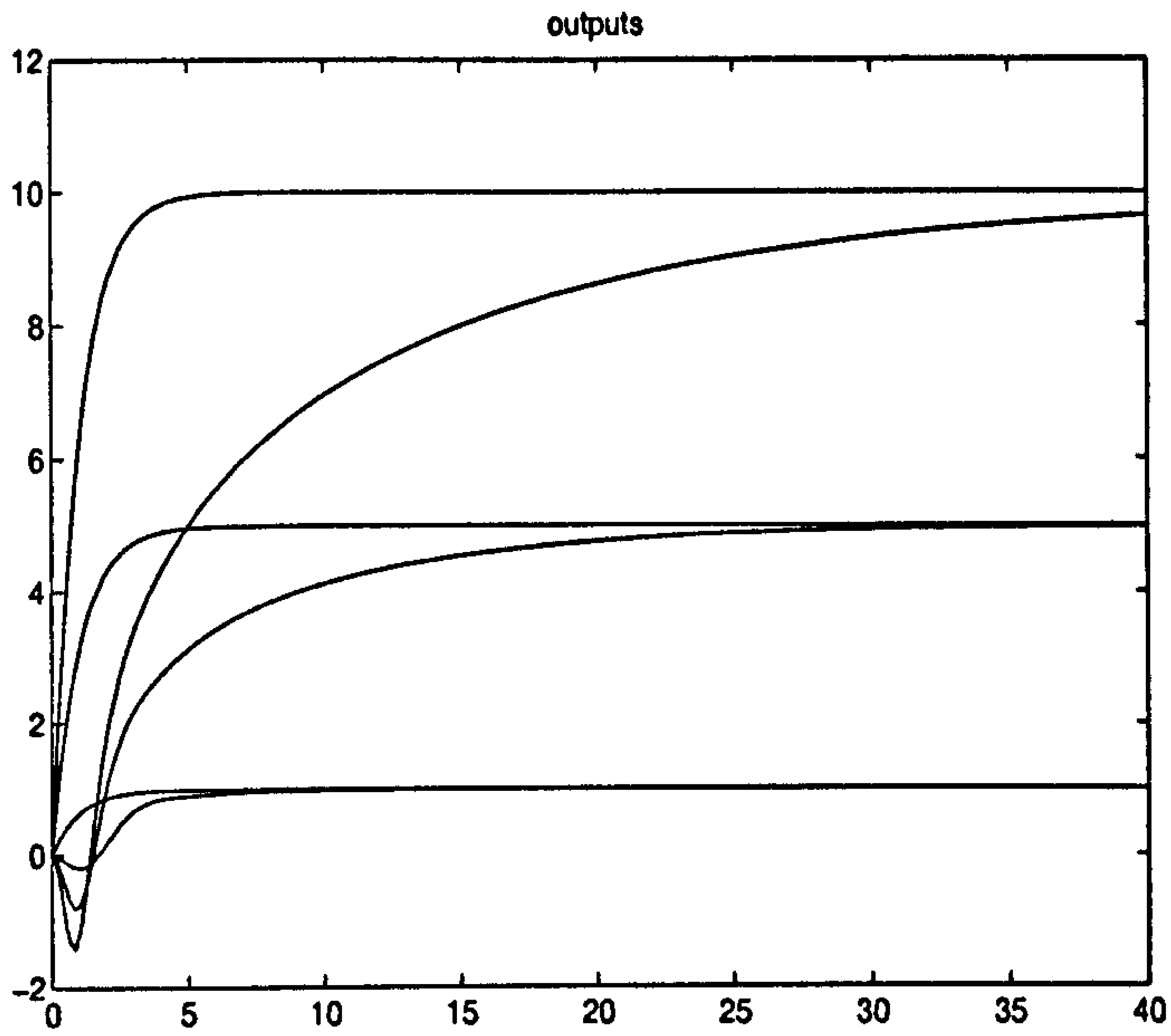

Figure 5.14: Example 2: Output responses of the Non-minimum phase system

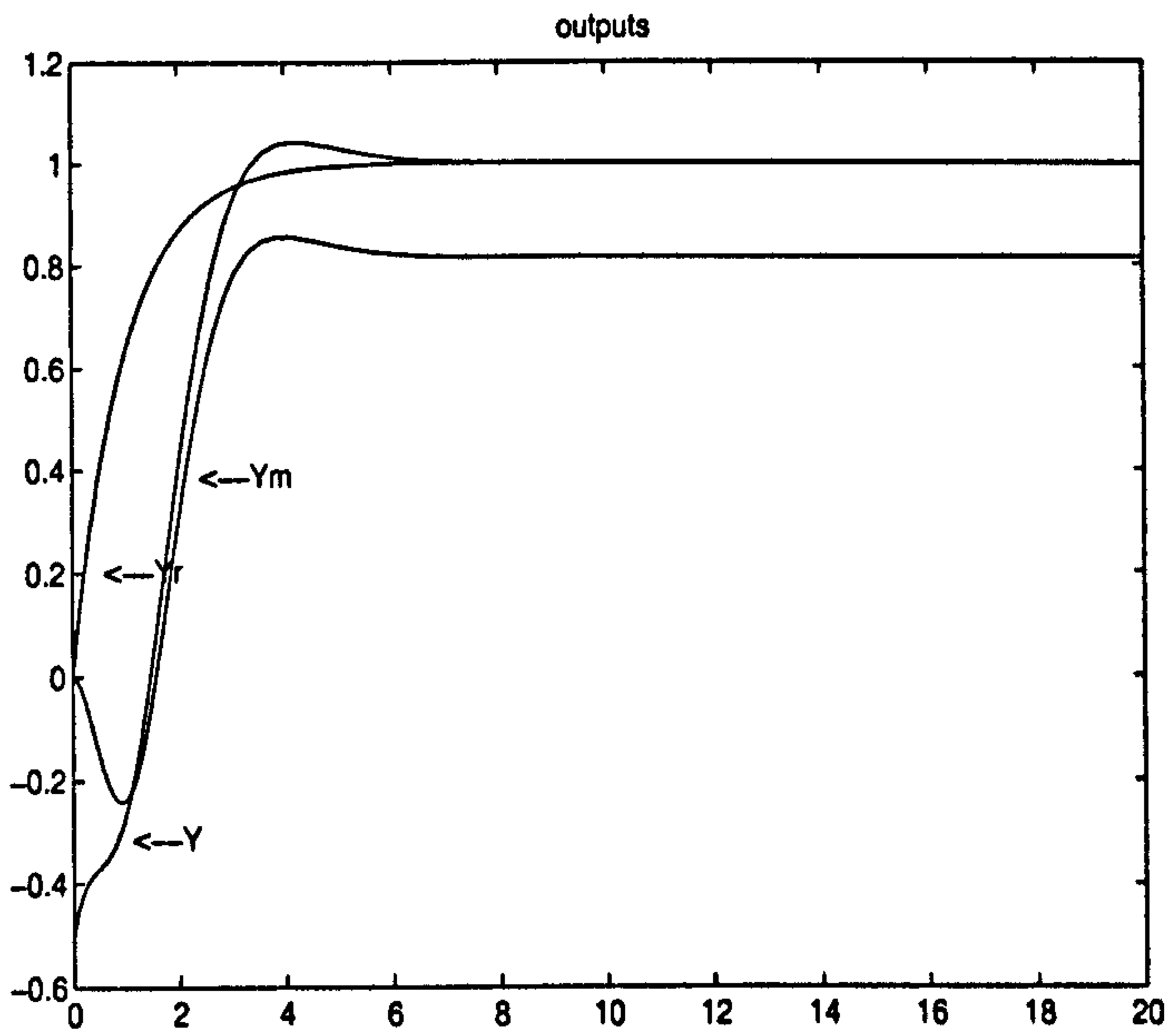

Figure 5.15: Example 2: y of the system with parameter variations

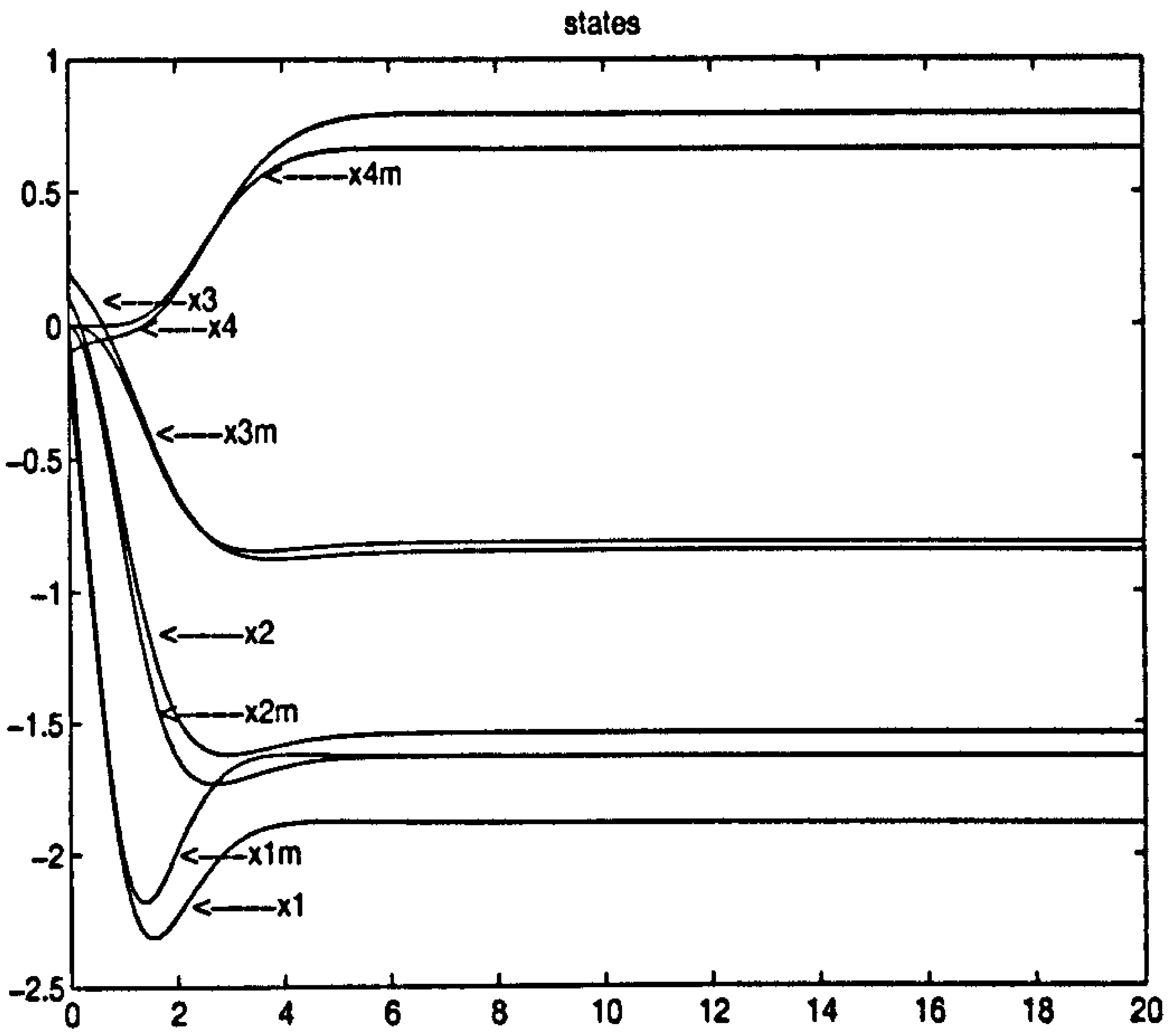

Figure 5.16: Example 2: States of the system with parameter variations

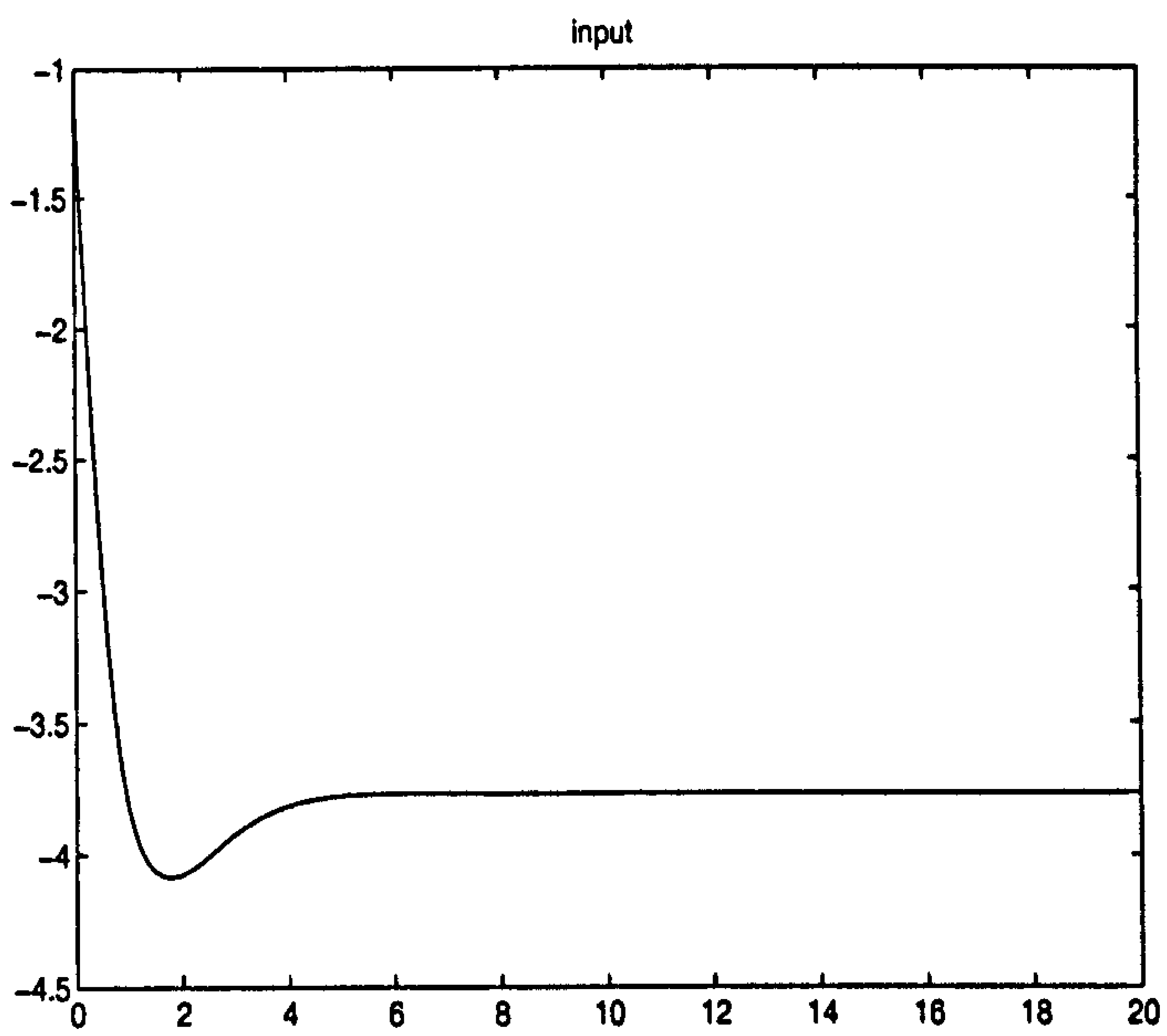

Figure 5.17: Example 2: u of the system with parameter variations

Example 3 was treated by [20] has $g(x) = 2 + sin^2(x_4)$. This example will be used to show that the NCGPC can control non-minimum phase systems with $g(x)$ being a nonlinear function of the states. The design parameters are chosen to be $T_1 = 0$, $T_2 = 1$, $N_y = 3$, $N_u = 0$, $R_n/R_d = 1/(s+1)$ and the setpoint $w = 1$. The Figures (5.18), (5.19) and (5.20) show the good response of the output and the states and input converge asymptotically to constant values.

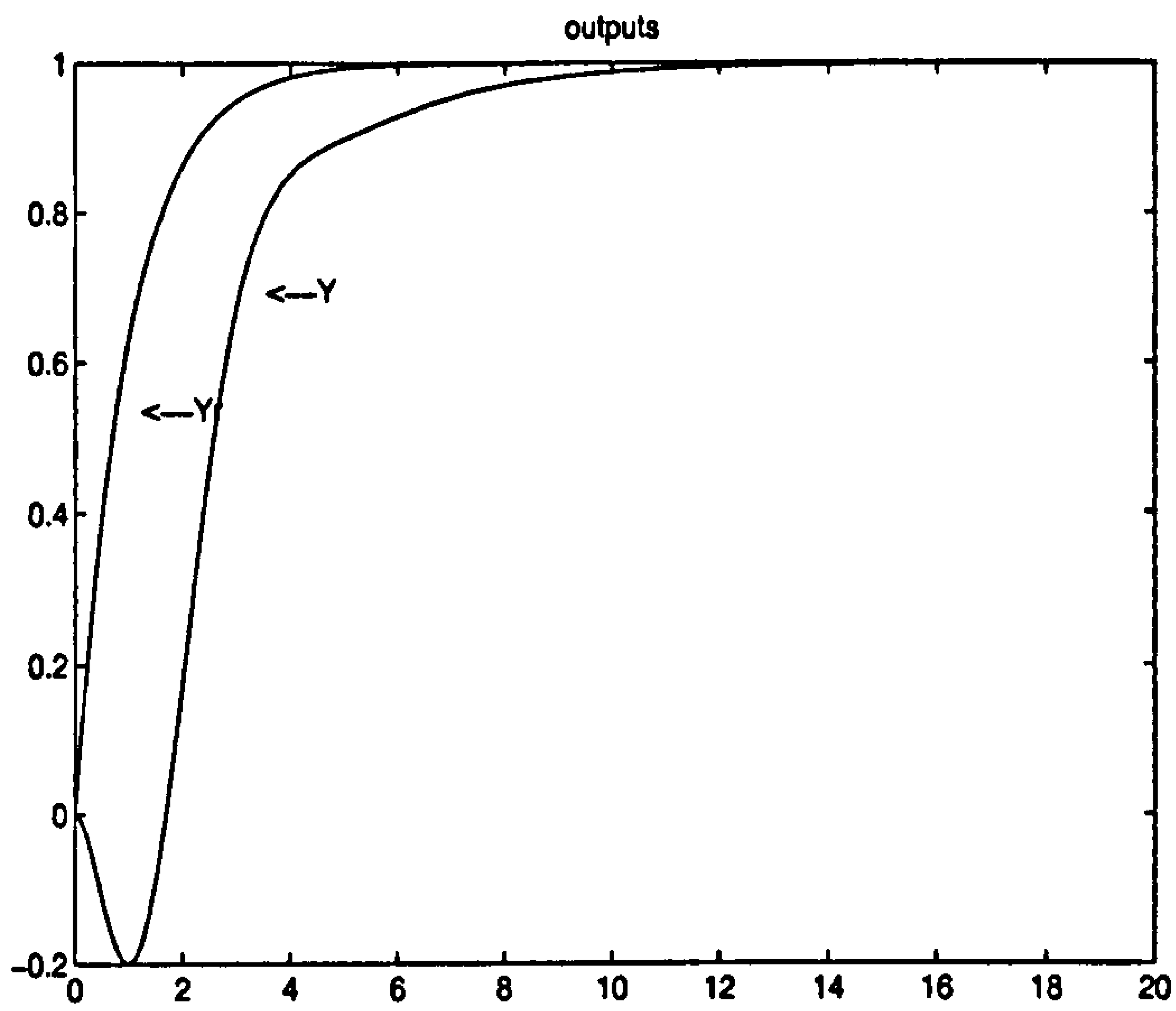

Figure 5.18: Example 3: Output y of the Non-minimum phase system

Figures (5.21), (5.22) and (5.23) show the simulation results, when the reference is as $w = 10$, design parameters were chosen the same, just the previous reference model was changed to $R_n/R_d = 1/(s+3)$ since $R_n/R_d = 1/(s+1)$ made the output response very slow.

Example 4 was treated by [42], is a nonminimum phase system, Figures (5.24) (5.26) and (5.25) show the good response of the output, the states and control input converge asymptotically to constant values respectively. In this example the parameters were chosen as $N_u = 0$, $N_y = 3$, $T_1 = 0$, $T_2 = 2$, $R_n/R_d = 1/(s+.8)$ and $w = 1$.

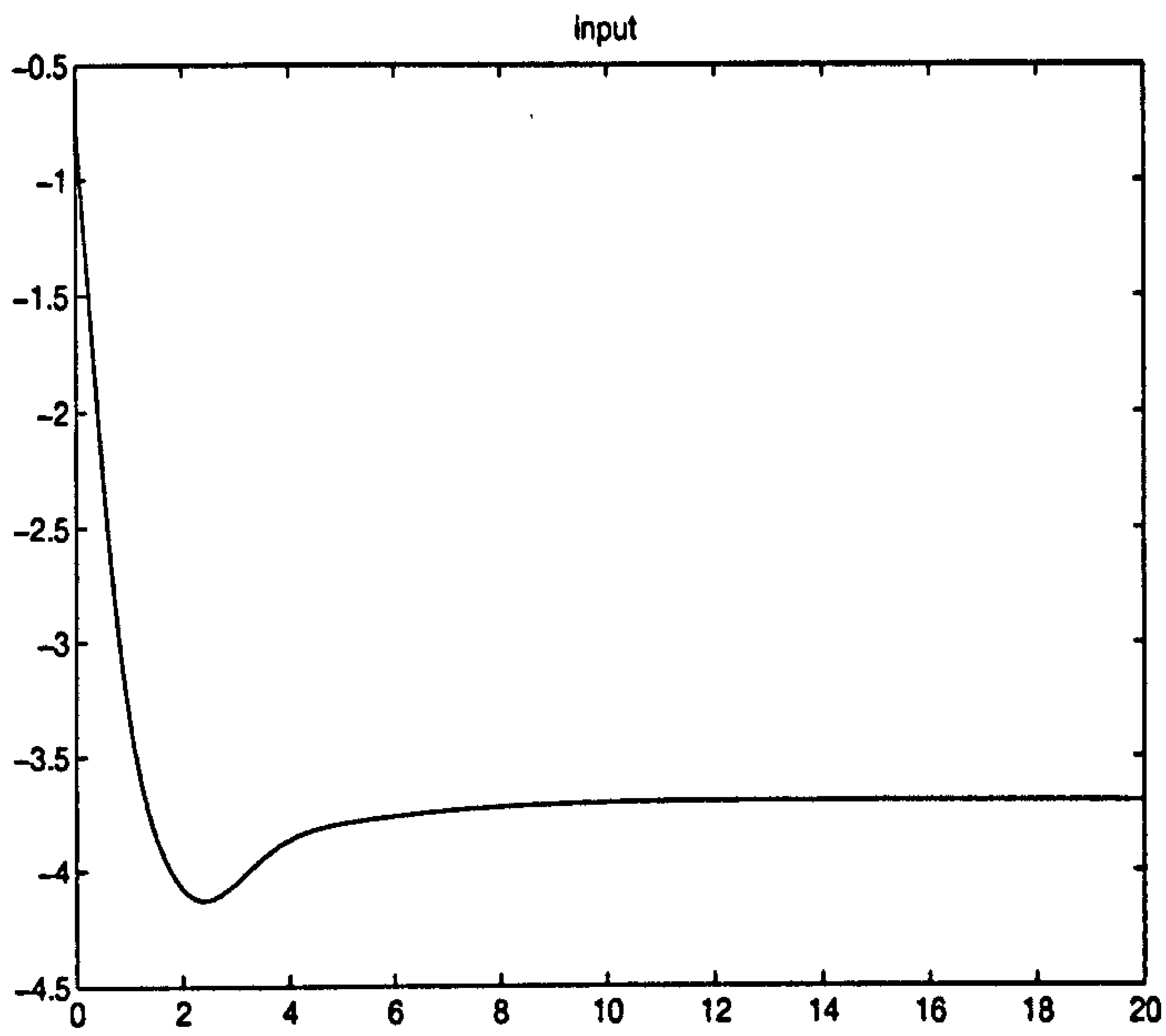

Figure 5.19: Example 3: States of the Non-minimum phase system

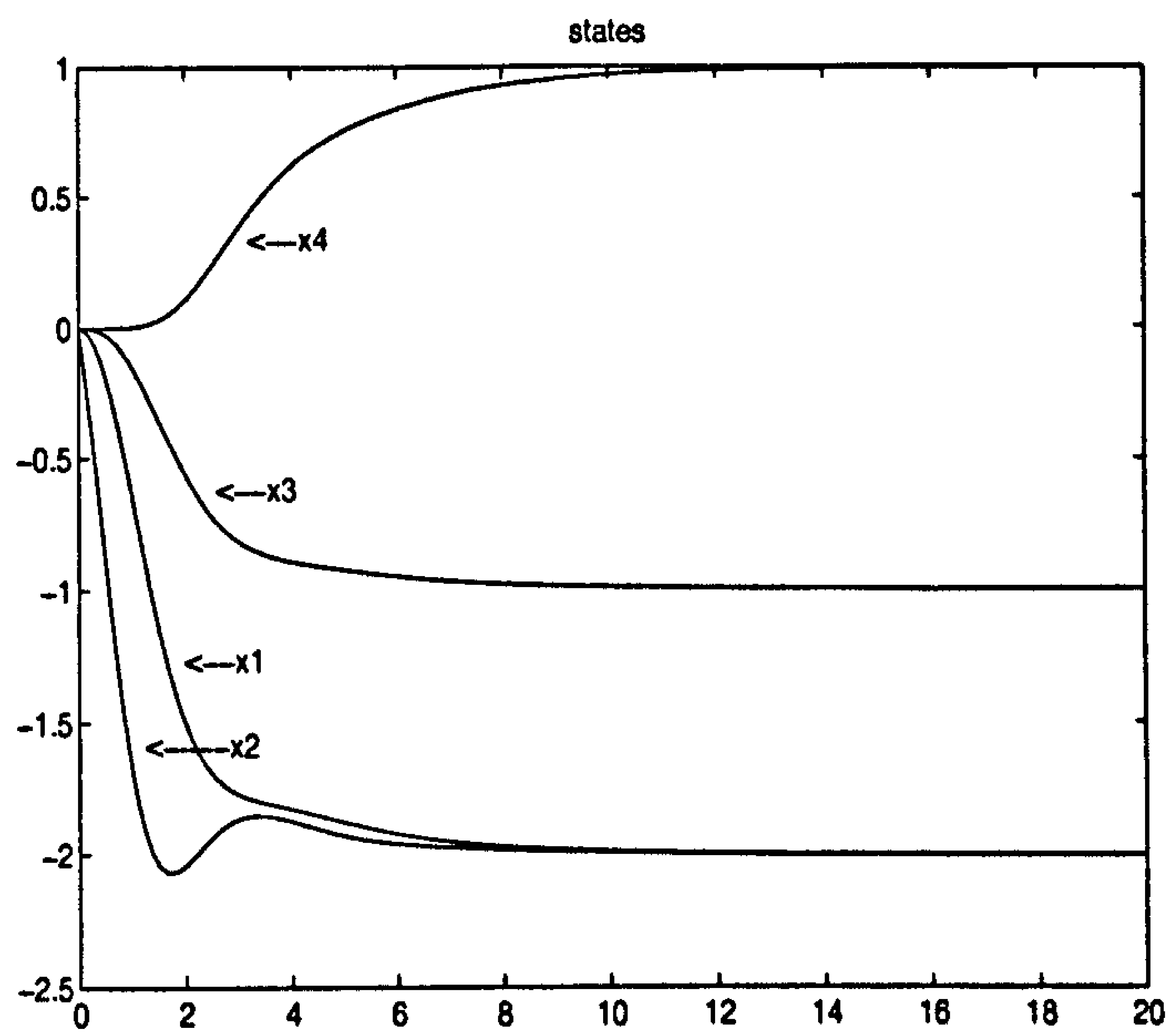

Figure 5.20: Example 3: Input u of the Non-minimum phase system

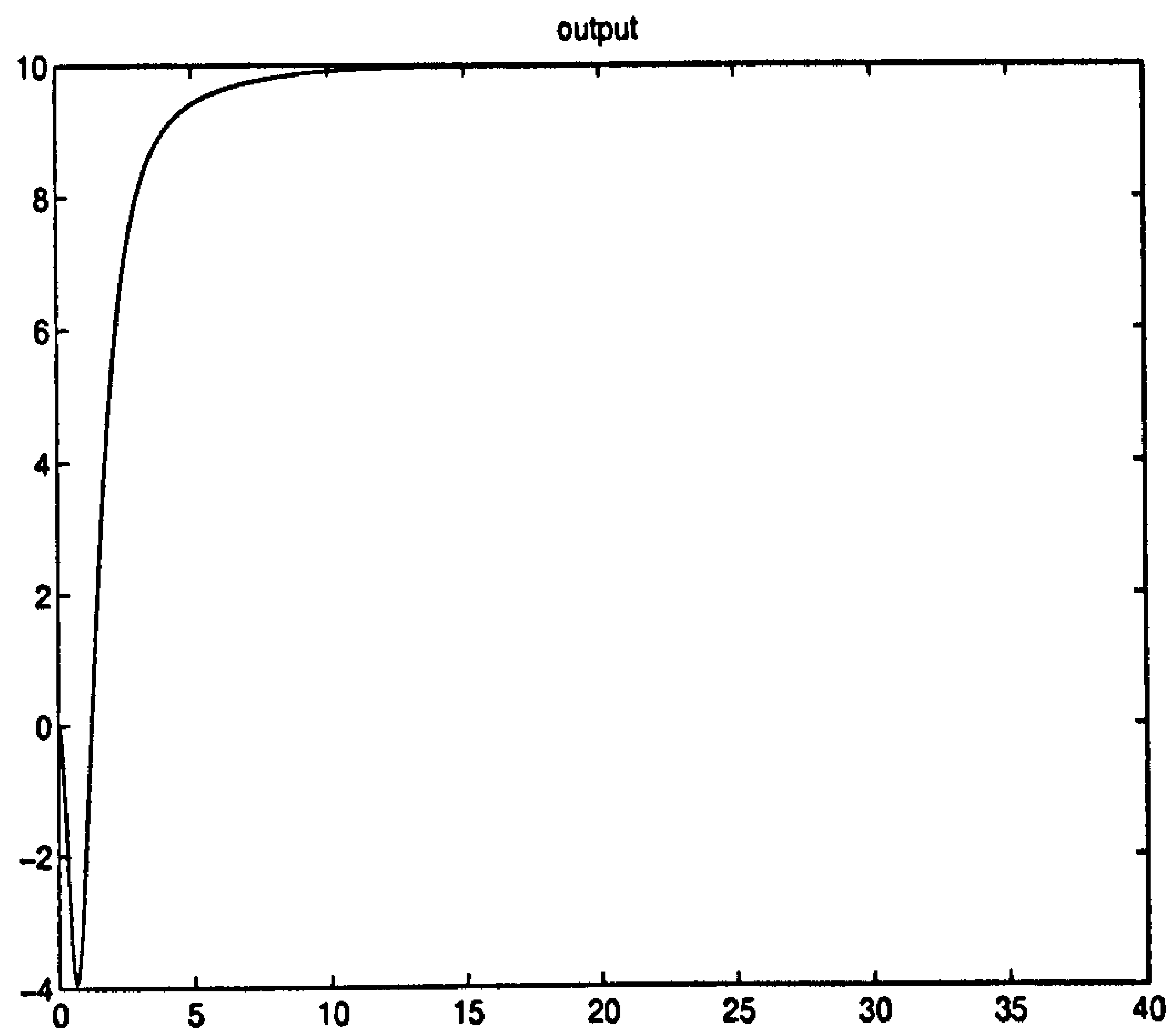

Figure 5.21: Example 3: Output y of the Non-minimum phase system

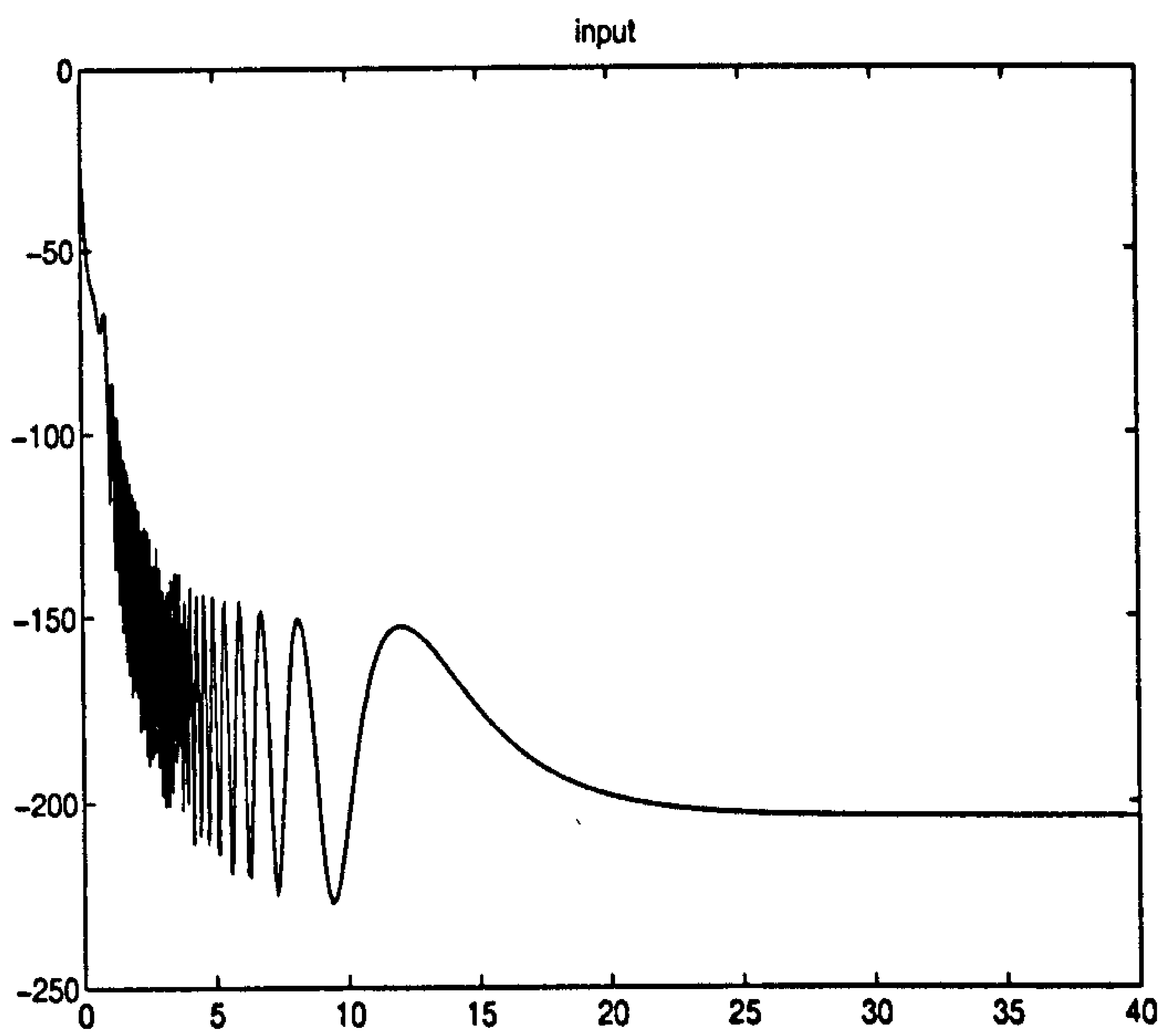

Figure 5.22: Example 3: Input u of the Non-minimum phase system

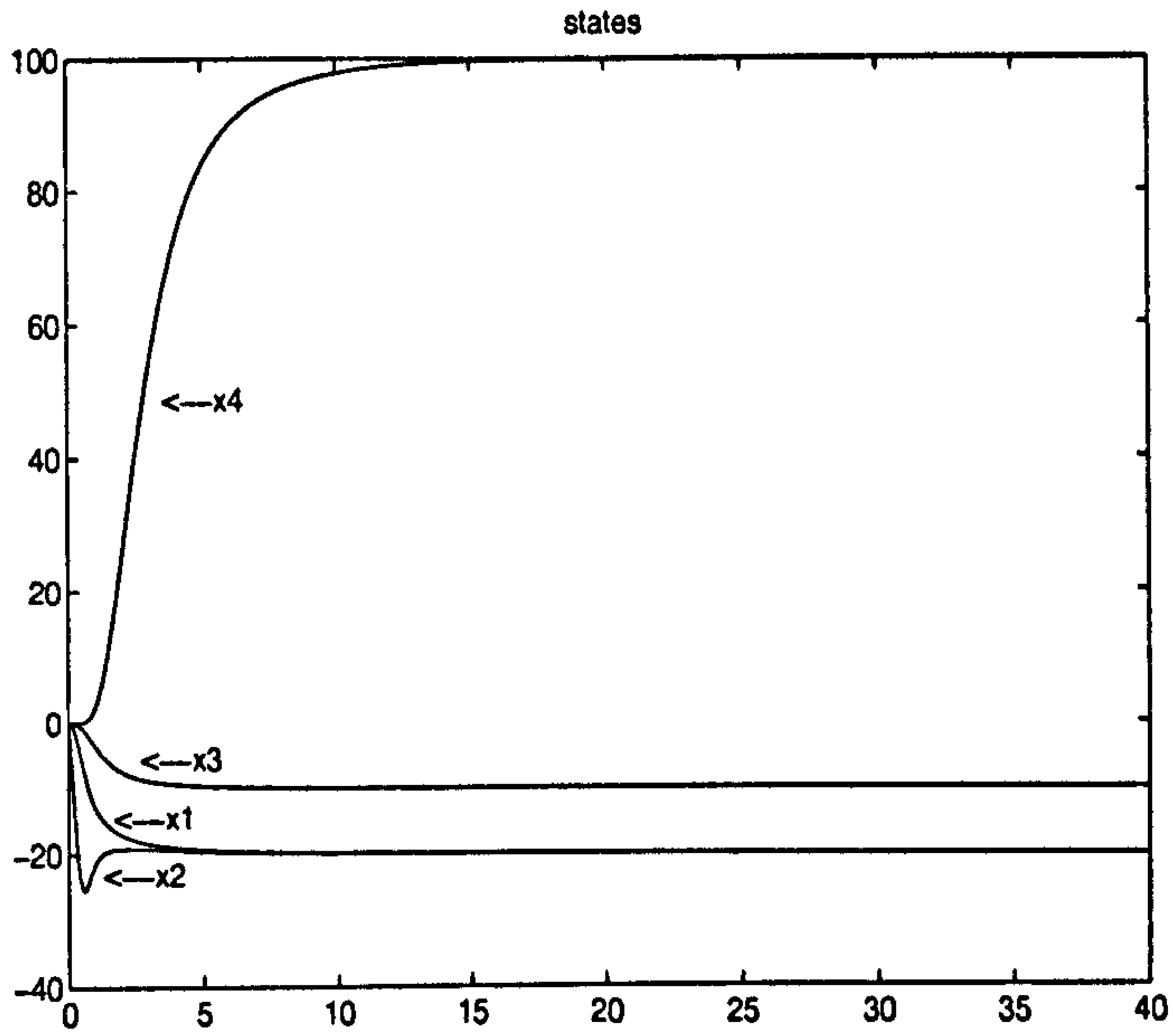

Figure 5.23: Example 3: States of the Non-minimum phase system

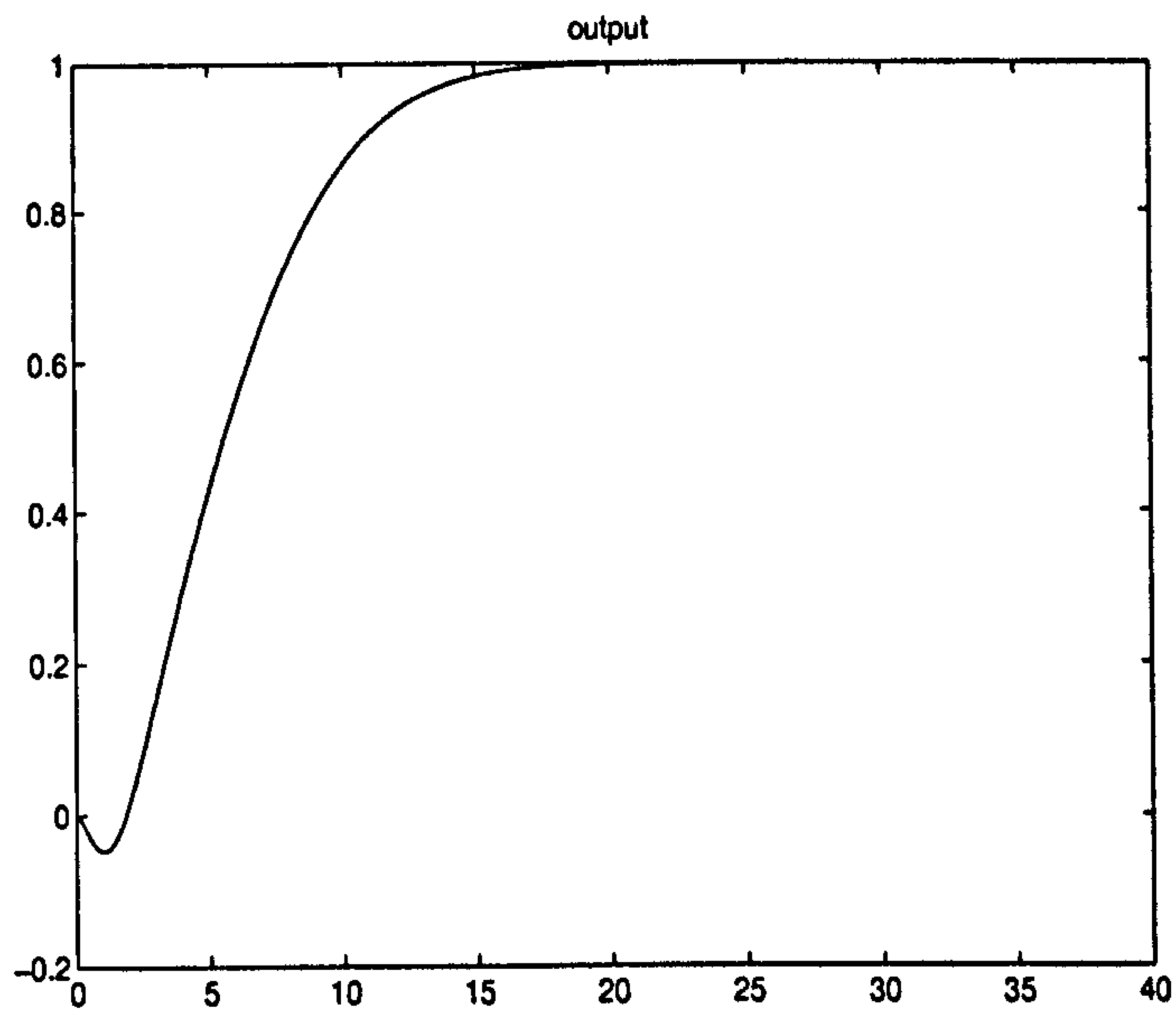

Figure 5.24: Example 4: Output y of the Non-minimum phase system

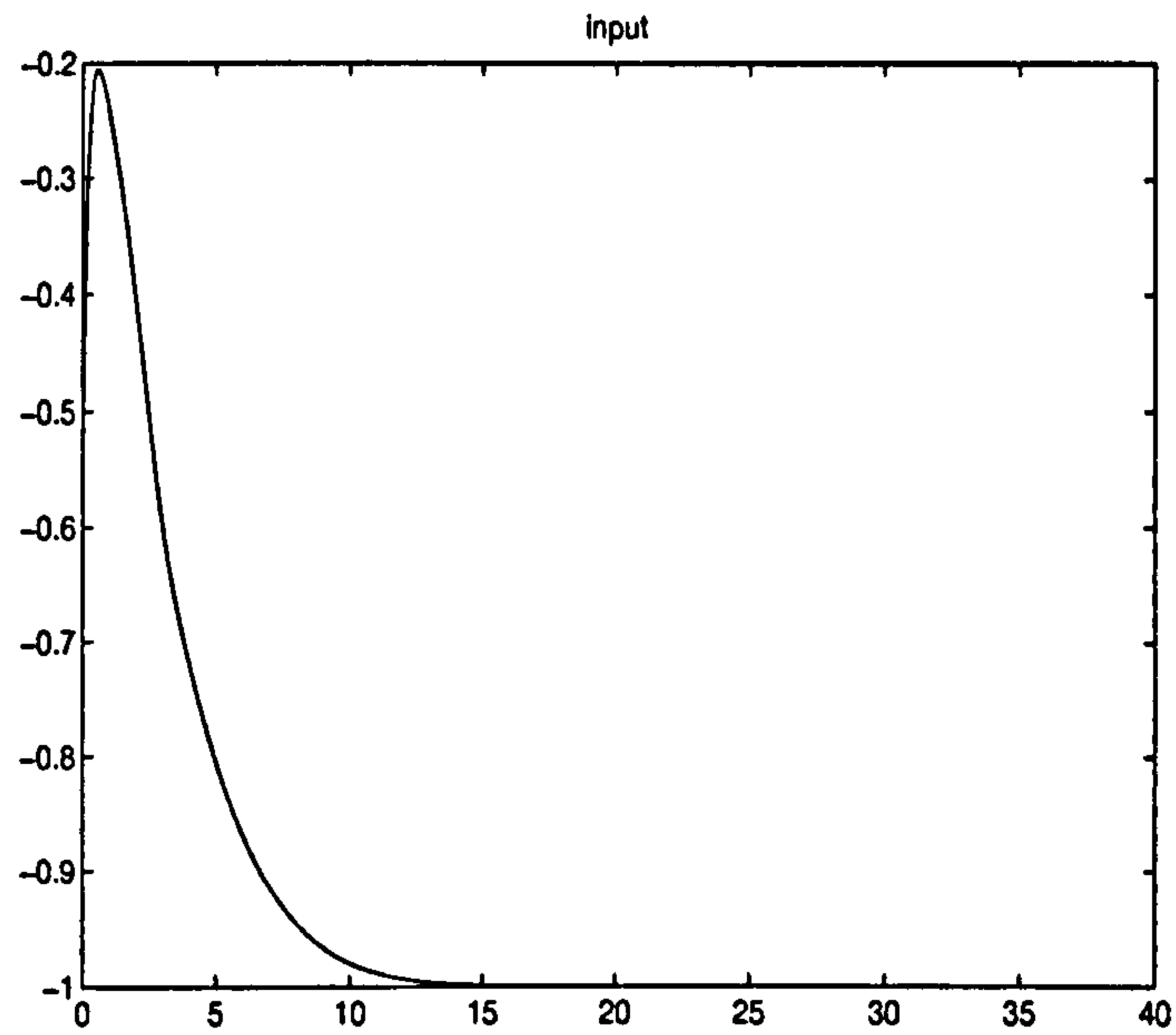

Figure 5.25: Example 4: Input u of the Non-minimum phase system

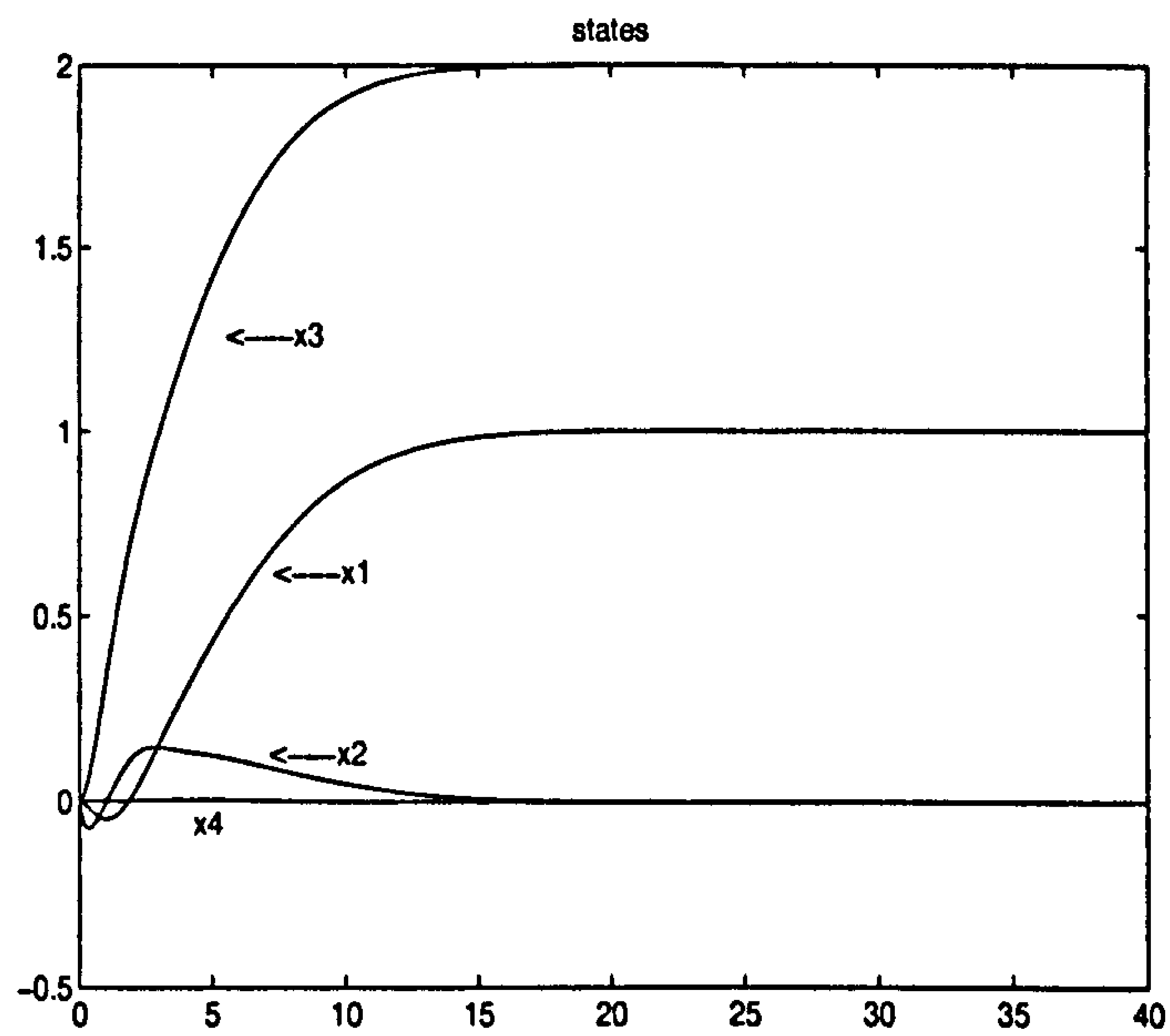

Figure 5.26: Example 4: States of the Non-minimum phase system

5.9.7 System with Not Well Defined Relative Degree

The following example has a singular point when $x2 = -.5$

$$
\begin{aligned}
\dot{x}_1 &= x_3 - x_2^2 \\
\dot{x}_2 &= -x_2 - u \\
\dot{x}_3 &= x_1^2 - x_3 + u \\
y &= x_1
\end{aligned}
$$

Figures (5.27) and (5.28) show that the output tracks the reference. Figures (5.29) and (5.30) show the states and the control input. It is possible to see that when the system is very close to the singular point $x_2 = -.5$, the control input is approximately zero, and when is far from this point the input becomes:

$$u(t) \approx \frac{(y_r - y) + (\dot{y}_r - L_f h)T + \ldots + (y_r^{(r)} - L_f^r h)\frac{T^r}{r!}}{L_g L_f^{r-1} h \frac{T^r}{r!}}$$

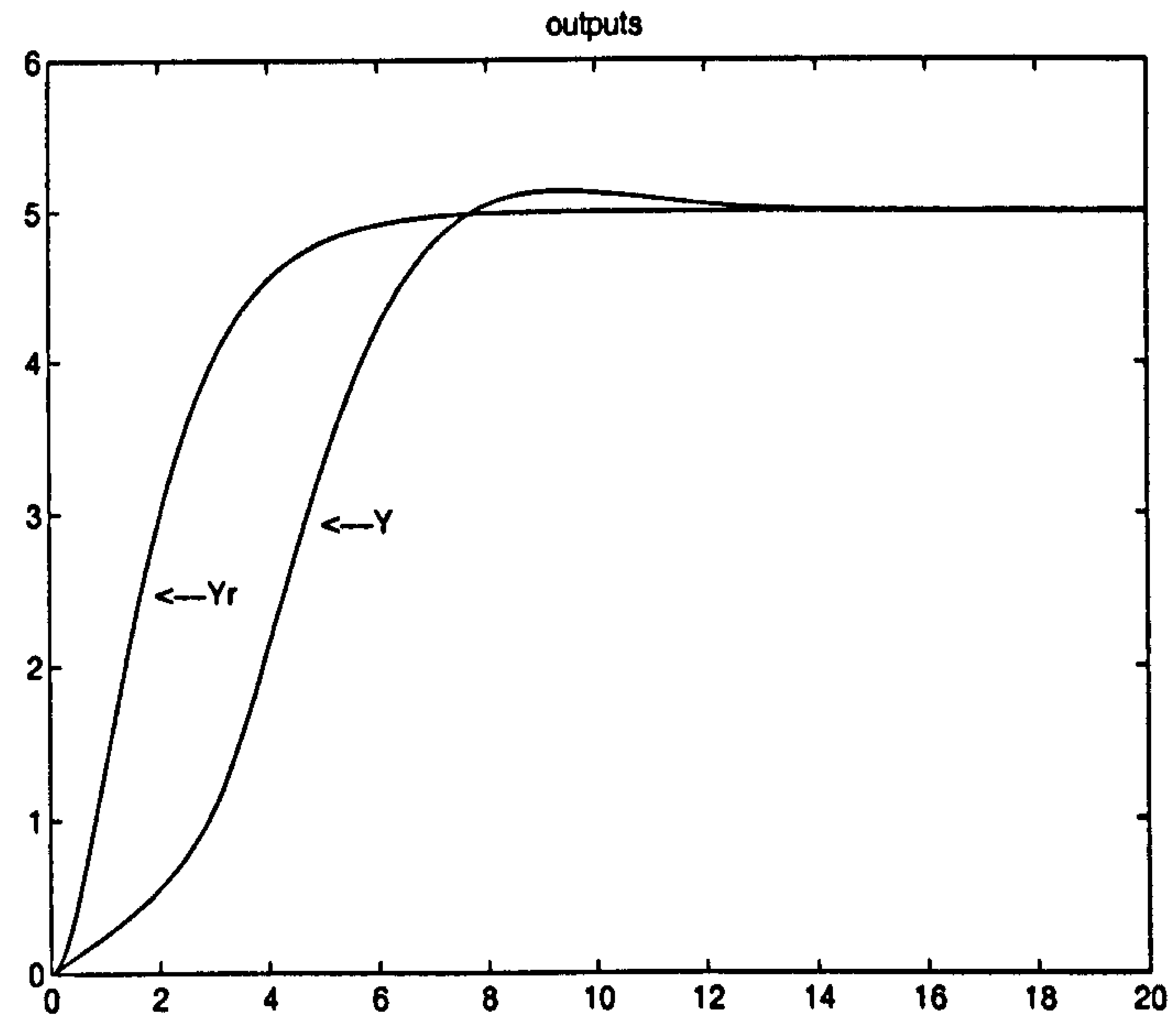

Figure 5.27: Example 5: system output and reference

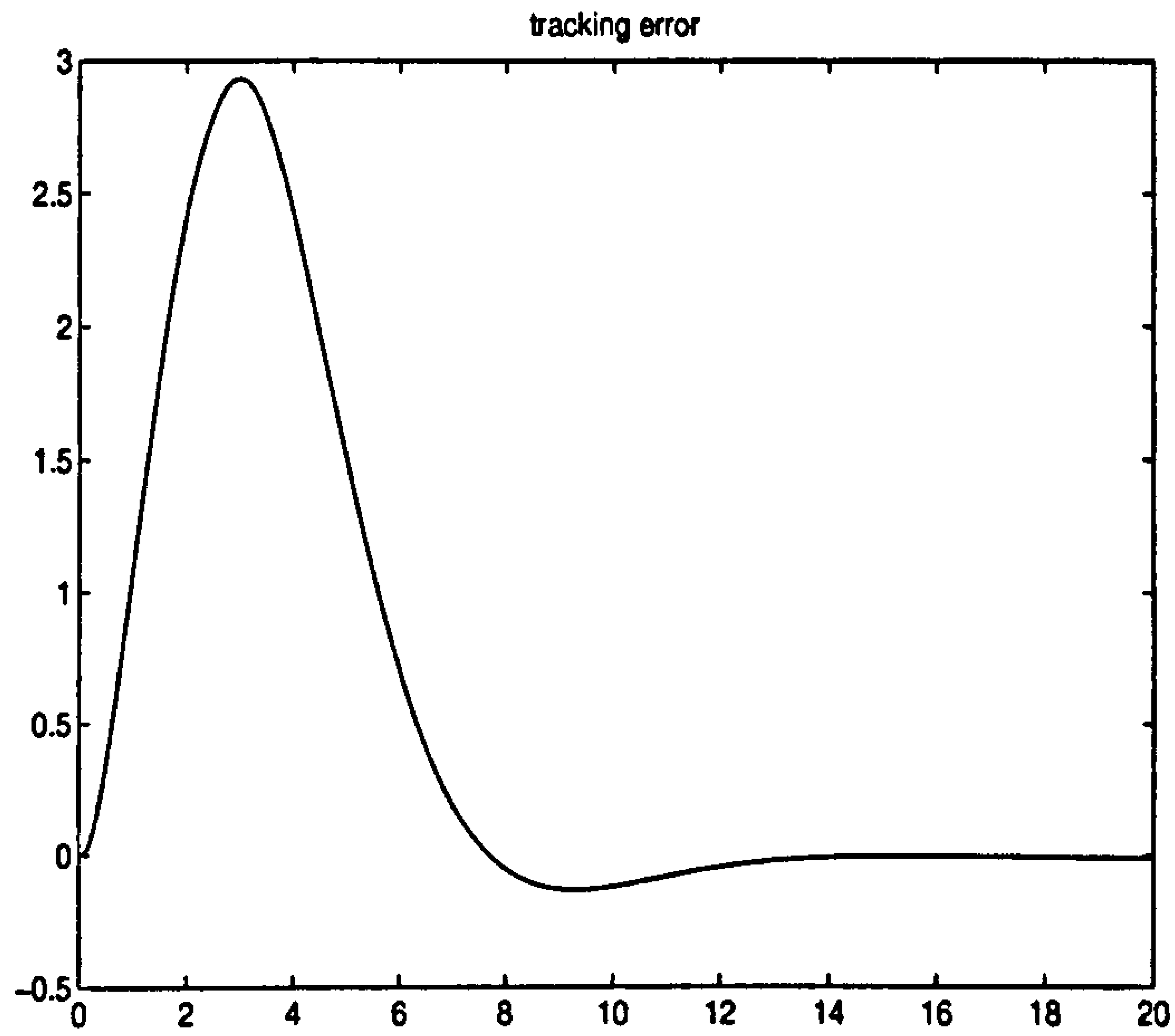

Figure 5.28: Example 5: tracking error

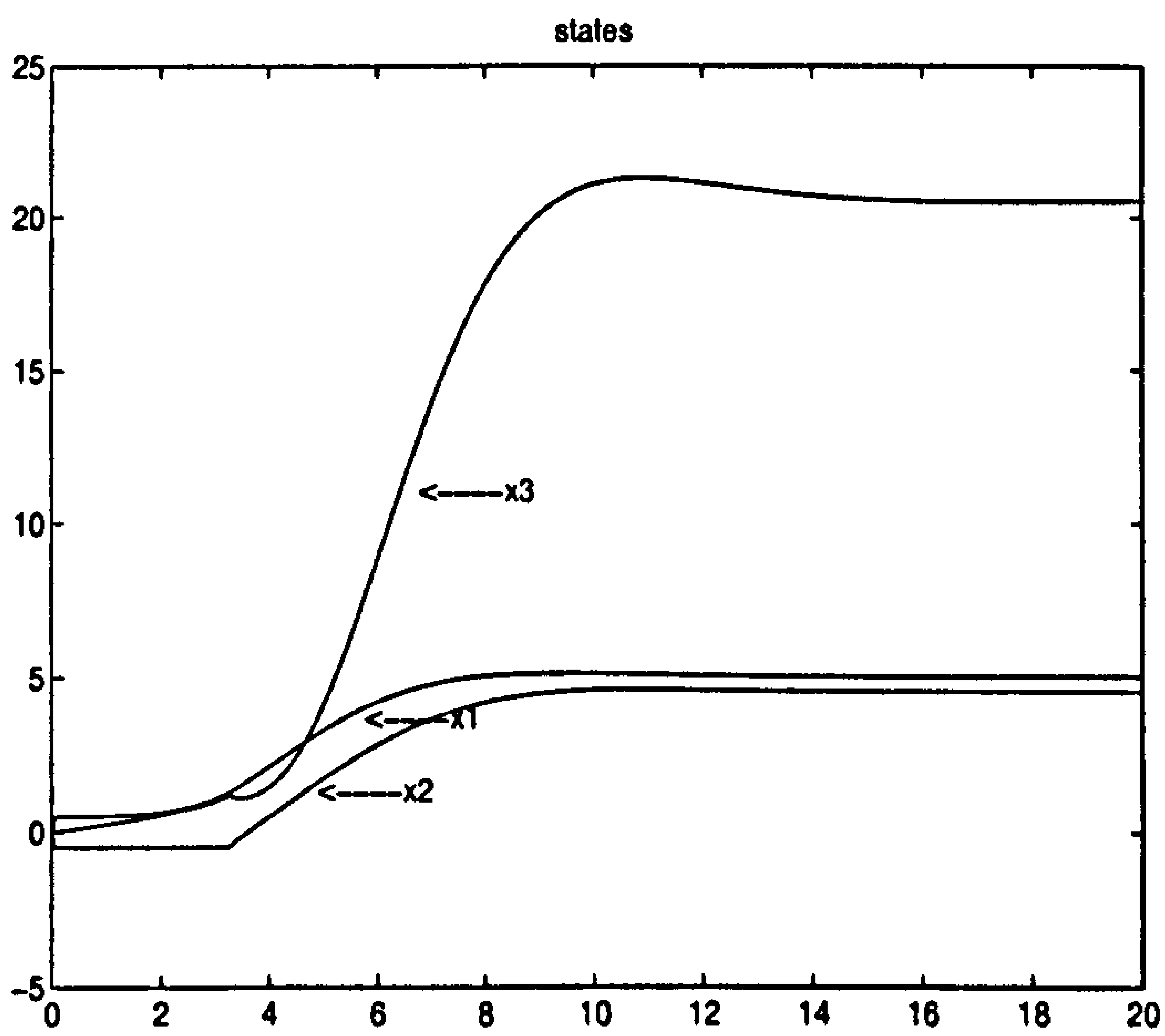

Figure 5.29: Example 5: system states

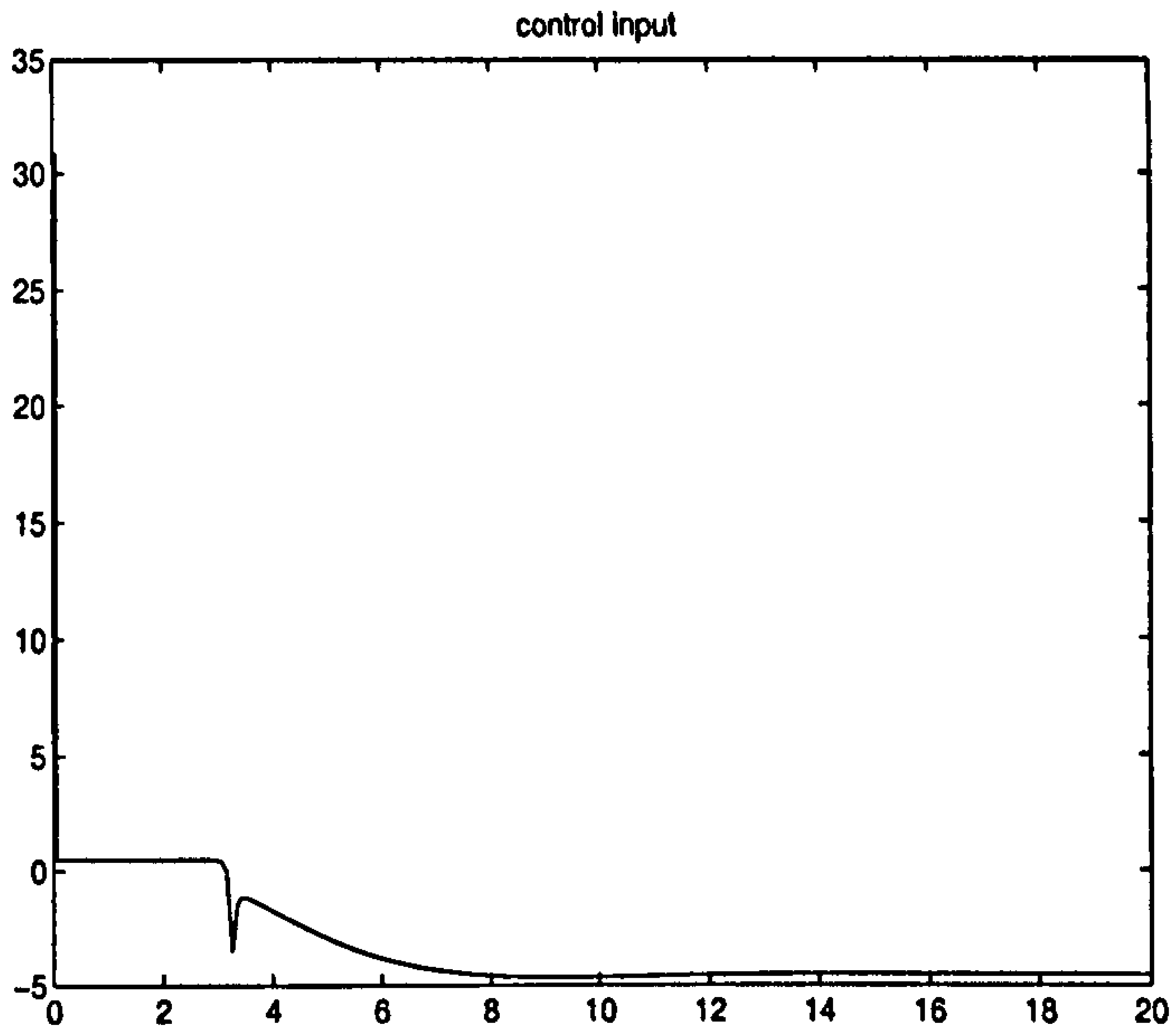

Figure 5.30: Example 5: u(t)-note notch when x_2 is close to 0.5

5.10 Conclusion

In this Chapter Continuous-time Nonlinear Generalised Minimum Variance Control (NGMV), Nonlinear Predictive Generalised Minimum Variance Control (NPGMV) and Nonlinear Continuous-time Generalised Predictive Control (NCGPC) are developed. The development of these algorithms has been reported in Section 6 of the paper P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33]

It was found that the NCGPC does not cancel the dynamics zero when $N_u < N_y - r$, therefore, this new controller can control systems with unstable zero dynamics. The NCGPC can also control systems with not well defined relative degree using the control weight λ or choosing $N_u < N_y - r$.

When NCGPC is applied on systems of the form

$$\begin{aligned} \dot{x}(t) &= F(x,u) \\ y(t) &= h(x), \end{aligned}$$

or when N_y is not chosen less than the number of the times that the output has to be differentiated in order to obtain nonlinear terms in u. As explained in Section 5.5.1, these cases give rise to a mathematical structure which is akin to a Differential

Algebraic Equation replaced by a nondynamic optimisation, implications of this are discussed further (and an example given) by P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33].

Finally, simulations are presented in order to show the effectiveness of the method.

Chapter 6

Geometric Interpretation

6.1 Introduction

In this Chapter, the nonlinear exact linearisation approaches reviewed in Chapters 2, 3 and 4 are shown to be included in NCGPC developed in Chapter 5; equivalencies between the nonlinear exact linearisation approaches and the new algorithms presented in Chapter 5 are shown; and the TRRMCNL developed by J. Alvarez and J. Alvarez [2] is shown to be just the Model Matching Via State Feedback [44] with an open loop observer. Finally, in order to improve the performance, a regulation model used by J. Alvarez and J. Alvarez [2] is added to the error feedback-GLC developed by P. Dautidis and C. Kravaris [18]. This uses an open loop observer to lead to output feedback control.

The outline of this Chapter is as follows. First in Section 2 the linear version CGPC is recast in state space form and shown to be included in NCGPC. In Section 3 the exact linearisation by feedback approach presented by A. Isidori [44] and the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] (which is a special case of the nonlinear exact linearisation by feedback) are shown to be equivalent to Nonlinear Generalised Minimum Variance Control (NGMV). In Section 4 the Tracking and Regulation Reference Model Control of Nonlinear Systems (TRRMCNL) developed by J. Alvarez and J. Alvarez [2] and the Model Matching Via State feedback developed by A. Isidori [44], are shown to be equivalent to NPGMV. In Section 5 the TRRMCNL developed by J. Alvarez and J. Alvarez [2] is shown to be just Model Matching Via State Feedback [44] with an open loop observer. In Section 6 a regulation model used by J. Alvarez and J. Alvarez [2] is added to the error feedback-GLC in order to improve the performance. Finally, in Section 7 Conclusions are presented.

6.2 Linear Systems

In this section, the linear version of CGPC is recast in state space form following the same steps as the NCGPC presented in Chapter 5.

Consider a linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= cx(t) \end{aligned} \tag{6.1}$$

where $x(t) \in \mathbf{R}^n$, $y \in \mathbf{R}$ and A, b, c are matrices of appropriate dimensions. As in the nonlinear case the process output derivatives are emulated by

$$\begin{aligned} \dot{y}(t) &= cAx(t) \\ y^{(2)}(t) &= cA^2x(t) \\ &\vdots \\ y^{(r)}(t) &= cA^r x(t) + cA^{r-1}bu(t) \\ y^{(r+1)}(t) &= cA^{r+1}x(t) + cA^r bu(t) + cA^{r-1}b\dot{u}(t) \\ &\vdots \\ y^{(N_y)}(t) &= cA^{N_y}x(t) + \ldots + cA^{r-1}bu^{N_y - r}(t) \end{aligned} \tag{6.2}$$

The predicted output is given by

$$y(t,T) \approx \mathbf{T}_{N_y} H\mathbf{u} + \mathbf{T}_{N_y}\mathbf{Y}^0 \tag{6.3}$$

where

$$\mathbf{Y}^0 = \begin{bmatrix} 0 \\ cAx(t) \\ cA^2x(t) \\ \vdots \\ cA^rx(t) \\ cA^{r+1}x(t) \\ \vdots \\ cA^{N_y}x(t) \end{bmatrix} \tag{6.4}$$

$$H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ cA^{r-1}b & 0 & \dots & 0 \\ cA^rb & cA^{r-1}b & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ cA^{N_y-1}b & cA^{N_y-2} & \dots & cA^{r-1}b \end{bmatrix} \tag{6.5}$$

Considering that $N_u = N_y - r$, it is easy to show that the control law is given by

$$u(t) = \frac{1/\beta_r(w - y(t)) - \sum_{i=1}^{r-1} \beta_i/\beta_r cA^ix(t) - cA^rx(t)}{cA^{r-1}b} \tag{6.6}$$

If the model is considered as a perfect model, this state feedback places the poles at the roots of the $(n-r)th$ degree polynomial $cAdj(sI - A)b$ and at the roots of the rth degree polynomial $\sum_{k=0}^{r} \beta_k s^k$ The resulting closed-loop transfer function is of the form

$$G(s) = \frac{1}{\beta_r s^r + \beta_{r-1}s^{r-1} + \ldots + \beta_1 s + 1} \tag{6.7}$$

The state feedback of equation (6.6) cancels all the zeros of the process by placing poles at the same values. This fact is analogous with NCGPC when $N_u = N_y - r$, the control feedback cancels the zero dynamics. Therefore, as with NCGPC, the process has to be minimum phase in order to preserve internal stability, unless $N_u < N_y - r$, in which case the zero cancellation is not carried out.

6.3 Geometric Interpretation

6.3.1 Nonlinear Generalised Minimum Variance Control (NGMV)

There has been a recent resurgence of interest in non-linear control driven by Geometrical Control Theory; to avoid proliferation of references the book of Isidori [44] is used as a summary of such results. The purpose of this section is to recast our results in a geometric setting. In particular, the construction of the NGMV is shown to follow the same steps as the development of the *Exact Linearisation via Feedback* given in Section 4.2 of Isidori [44].

Following Isidori [44], state feedback is considered so that $\hat{x} = x$ and the special (linear in the control) system of equation (5.2) is considered. Differentiating the output y with respect to time, until $u(t)$ appears, (which is the definition of relative degree r in the nonlinear context [44] and in Section 2.3), equation (5.4) gives the output derivatives, and thus

$$Y_r(t) \quad = \quad \mathbf{O}_r(x(t)) + \mathbf{H}_r(x(t))u \tag{6.8}$$

where

$$\mathbf{O}_r \quad = \quad \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \end{bmatrix} \tag{6.9}$$

$$\mathbf{H}_r \quad = \quad \begin{bmatrix} 0 \\ 0 \\ \vdots \\ L_g L_f^{r-1} h(x) \end{bmatrix} \tag{6.10}$$

and

$$\mathbf{Y}_r = \left[y\ \dot{y}\ y^{(2)} \ldots y^{(r)}\right]^T \tag{6.11}$$

The NGMV controller of equation (5.21) then becomes:

$$u = \frac{v_1 - \sum_{i=0}^{r} p_i L_f^i h(x)}{p_r L_g L_f^{r-1} h(x)} \tag{6.12}$$

where $v_1 = w$. Externally, the closed-loop system is defined by:

$$\sum_{i=1}^{r} p_i y_i = v_1 \tag{6.13}$$

We can see that equation (6.12) is the GLC (Globally Linearising Control) developed by Dautidis and Kravaris [18]. On the other hand, the GLC control law can be regarded as a special case of the state feedback control law. If $p_r = 1$ and defining a new input v as follows:

$$v = v_1 - \sum_{i=0}^{r-1} p_i L_f^i h(x) \tag{6.14}$$

the control law equation (6.12) becomes

$$u = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \tag{6.15}$$

This is the state feedback control law developed by Isidori [44]. Thus, the GLC control law developed by Dautidis and Kravaris [18] and the exact linearisation by state feedback described by Isidori [44] are equivalent to NGMV.

6.3.2 Nonlinear version of NPGMV

The following assumptions must be made in order to establish the geometric interpretation.

Assumptions (NPGMV)

- The system is described by equation (5.2).
- It has stable zero dynamics.
- $P = 1$.
- $N_u = N_y - r$.
- The trajectory reference is given by equation (5.30).
- State feedback is considered so that $\hat{x} = x$.

Under these assumptions the cost function equation (5.36) becomes:

$$J_{NPGMV}(\mathbf{u}^*(t,0)) = \frac{1}{2}[\mathbf{O}(x(t)) + H(x(t))\mathbf{u} - \mathbf{y}_r(t)]^T \mathbf{T}^T\mathbf{T}[\mathbf{O}(x(t)) + H(x(t))\mathbf{u} - \mathbf{y}_r(t)] \tag{6.16}$$

and the minimisation results in

$$\mathbf{u}^*_{N_u}(t,0) = [H^T\mathbf{T}^T\mathbf{T}H]^{-1}H^T\mathbf{T}^T\mathbf{T}H[\mathbf{y}_r - \mathbf{O}] \tag{6.17}$$

Defining

$$\mathbf{K} = [H^T\mathbf{T}^T\mathbf{T}H]^{-1}H^T\mathbf{T}^T\mathbf{T}H \tag{6.18}$$

(as explained above, just the first element of $\mathbf{u}^*(t,0)$ is applied) then, the first row of $\mathbf{K}$, which will be called $\mathbf{k}$, the control law, is given by

$$\mathbf{u}^*_{N_u}(t,0) = \mathbf{k}[\mathbf{y}_r - \mathbf{O}] \tag{6.19}$$

Following the same steps as in Section 5.7, $\mathbf{k}$ is given as

$$\mathbf{k} = \frac{1}{L_g L_f^{r-1} h(\hat{x})}[t_1 \; t_2 \; \dots \; t_r \; 1 \; 0 \; \dots \; 0] \tag{6.20}$$

where $t_1 \; t_2 \; \ldots \; t_r$ are the elements of the first row of $T_{22}^{-1}T_{21}$. They are nonlinear functions of T and the row vector has dimension $1 \times r$.

Substituting the equations (5.4) and (5.32) into (6.19) the control law becomes

$$u(t) = \frac{\sum_{i=0}^{r-1} t_{i+1}[y_r^i - L_f^i h(x)] - L_f^r h(x) + y_r^r}{L_g L_f^{r-1} h(x)} \tag{6.21}$$

We can see that the control law is identical to the control law presented by Isidori [44], which solves the problem known as asymptotic model matching. Thus, it is shown that Isidori controller is equivalent to the NPGMV.

6.3.3 A Special Case: TRRMCNL

Three objectives can be achieved by analysing of the closed loop system: first, it will be shown that the output system matches the reference trajectory y_r. Secondly, that the regulation model used by the controller TRRMCNL [2], is unnecessary and finally, that the TRRMCNL is just the PNGMV with an open loop observer. The following assumptions need to be made in order to proceed:

Assumptions

- The system is described by equation (5.2).
- The system has stable zero dynamics.
- $N_u = N_y - r$.
- $P = 1$
- $y(t) - y_m(t) \longrightarrow constant$ as $t \longrightarrow \infty$.
- The trajectory reference is given by equation (5.32)
- An open loop observer is used in order to estimate the states.
- $N_y = r$.

The emulated output derivatives are given as:

$$\begin{aligned}
\dot{y}_m(t) &= L_{f_m} h_m(x_m) \\
y_m^{(2)}(t) &= L_{f_m}^2 h_m(x_m) \\
&\vdots \\
y_m^{(r)}(t) &= L_{f_m}^r h_m(x_m) + L_{g_m} L_{f_m}^{r-1} h_m(x_m) u(t)
\end{aligned} \tag{6.22}$$

and the control law as:

$$u(t) = \frac{t_1[y_r - y] + \sum_{i=1}^{r-1} t_{i+1}[y_r^i - L_{f_m}^i h_m(x_m)] - L_{f_m}^r h_m(x_m) + y_r^r}{L_{g_m} L_{f_m}^{r-1} h_m(x_m)} \tag{6.23}$$

where

$$t_i = \frac{r!}{T^{r-i} i!}$$

Substituting the control law $u(t)$ given by equation (6.23) into the last equation of (6.22) leads to:

$$y_m^{(r)}(t) = v(t) \tag{6.24}$$

where

$$v(t) = \frac{r!}{T^r}(y_r - y) + \sum_{i=1}^{r-1} \frac{r!}{T^{r-i} i!}(y_r^{(i)} - y_m^{(i)}) + y_r^{(r)}.$$

By making some manipulations, equation (6.24) reduces to:

$$\frac{r!}{T^r}(y - y_m) = \sum_{i=0}^{r} \frac{r!}{T^{r-i} i!}(y_r^{(i)} - y_m^{(i)}). \tag{6.25}$$

Taking Laplace transforms (with zero initial conditions, because the initial conditions of y_r and y_m always are chosen zero):

$$\frac{r!}{T^r}(y(s) - y_m(s)) = \left(\frac{r!}{T^r} + \frac{r!}{T^{r-1}} s + \ldots + s^r\right)(y_r(s) - y_m(s)). \tag{6.26}$$

It is better to rewrite this as:

$$y_r(s) - y(s) = (G(s) - 1)(y(s) - y_m(s)), \tag{6.27}$$

where

$$G(s) = \frac{\frac{r!}{T^r}}{s^r + ... + \frac{r!}{T^{r-1}}s + \frac{r!}{T^r}}.$$

Note that $G(s)$ has unity steady-state gain so that the assumption

$$y(t) - y_m(t) \longrightarrow constant \qquad \text{as} \qquad t \longrightarrow \infty.$$

implies, that

$$y(t) \longrightarrow y_r(t) \qquad \text{as} \qquad t \longrightarrow \infty.$$

and that small values of T correspond to fast convergence of y to the reference y_r and high bandwidth. Recall that in Chapter 4 it was established that when the controller TRRMCNL is applied, the tracking error is given by equation (4.18), i.e.:

$$y_r(s) - y(s) = (G_e(s) - 1)(y(s) - y_m(s)) \tag{6.28}$$

where, $G_e(s)$ is the transfer function of the regulation model given by:

$$G_e(s) = \frac{k_0}{s^r + \ldots + k_1 s + k_0} \tag{6.29}$$

The TRRMCNL controller has to use the regulation model in order to guarantee that the process output matches the reference when a perfect nonlinear model is not available. In other words the regulation filter is intended to provide the robustness of the controller, (E. Liceaga [53]). But, it is possible to show that the regulation model is unnecessary. If the filters are chosen such that $G_e(s) = G(s)$. Equations (6.27) and (6.28) are identical, implying that, the tracking errors are exactly the same. We can deduce that the regulation model is unnecessary, because the same tracking error results when the NPGMV with just an open loop observer is considered. Thus the TRRMCNL is equivalent to NPGMV with an open loop observer or the controller developed by Isidori with an open loop observer, which solve the problem known as asymptotic model matching.

6.4 Performance Improvement

Under the assumptions given in Section 5.7

Assumptions (NCGPC)

- The system is described by equation (5.2).
- $P = 1$.
- $N_u = N_y - r$.
- State feedback is considered so that $\hat{x} = x$.

the control law is given by equation (5.59), i.e.:

$$u(t) = \frac{(w - y(t)) - \sum_{i=1}^{r} \beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1} h(x)}, \tag{6.30}$$

where

$$\beta_r = 1/(t_1 p_0 + t_2 p_1 \dots t_r p_{r-1}) \tag{6.31}$$

$$\beta_i = t_{i+1}/(t_1 p_0 + t_2 p_1 \dots t_r p_{r-1}) \qquad i = 1, \dots, r-1 \tag{6.32}$$

This control law is identical to the error feedback-GLC, developed by P. Dautidis and C. Kravaris [18] and reviewed in Chapter 3, which uses an open loop observer in order to lead to an output feedback control. It is also identical to NGMV, because $N_u = N_y - r$.

The resulting closed-loop transfer function is given by:

$$G(s) = \frac{1}{\beta_r s^r + \beta_{r-1} s^{r-1} + \dots + \beta_1 s + 1} \tag{6.33}$$

In order to improve the performance, a correction must be applied to the model output derivatives to account for plant/model mismatch. The correction will be achieved by applying the regulation filter developed by J.Alvarez *et al* [2] and reviewed in Chapter 4:

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + B_e e(t) \\ y_e(t) &= C_e x_e(t), \end{aligned} \tag{6.34}$$

where $x_e \in \mathbf{R}^n$, $A_e \in \mathbf{R}^{n\times n}$, $B_e \in \mathbf{R}^{n\times 1}$, $C_e \in \mathbf{R}^{1\times n}$, $e \in \mathbf{R}$.

The filter has an unity steady-state gain, the matrix A_e has eigenvalues with negative real parts and its input is the difference between process output and model output, $e(t) = y(t) - y_m(t)$.

The derivatives of the process output are emulated by the model output derivatives plus regulation filter output derivatives that are used to counteract the derivative errors. Thus, the emulated output derivatives of the process are given by:

$$\begin{aligned} \dot{y}(t) &\approx \dot{y}_e(t) + L_{f_m} h_m(x_m(t)) \\ y^{(2)}(t) &\approx y_e^{(2)}(t) + L_{f_m}^2 h_m(x_m(t)) \\ &\vdots \\ y^{(r)}(t) &\approx y_e^{(r)}(t) + L_{f_m}^r h_m(x_m(t)) + L_{g_m} L_{f_m}^{r-1} h_m(x_m(t)) u(t). \end{aligned} \tag{6.35}$$

Considering this modification in the emulation of the derivatives, it is easy to show that the control law is given by

$$u(t) = \frac{(w - y(t)) - \sum_{i=1}^{r-1} \beta_i (L_{f_m}^i h_m(x_m) + y_e^{(i)}) - L_{f_m}^r h_m(x_m) - y_e^{(r)}}{\beta_r L_{g_m} L_{f_m}^{r-1} h(x_m)}, \tag{6.36}$$

The structure of the control will be as in Figure 6.1

- the block labelled Process Model is the open-loop observer used to get the model states; these are needed to emulate the output derivatives of the system;
- the block labelled Regulation Filter is used to counteract the error between output process derivatives and output model derivatives. The input of the filter is the error between output system and output model;
- the control input u is the same for system and model, the control system has an output feedback.

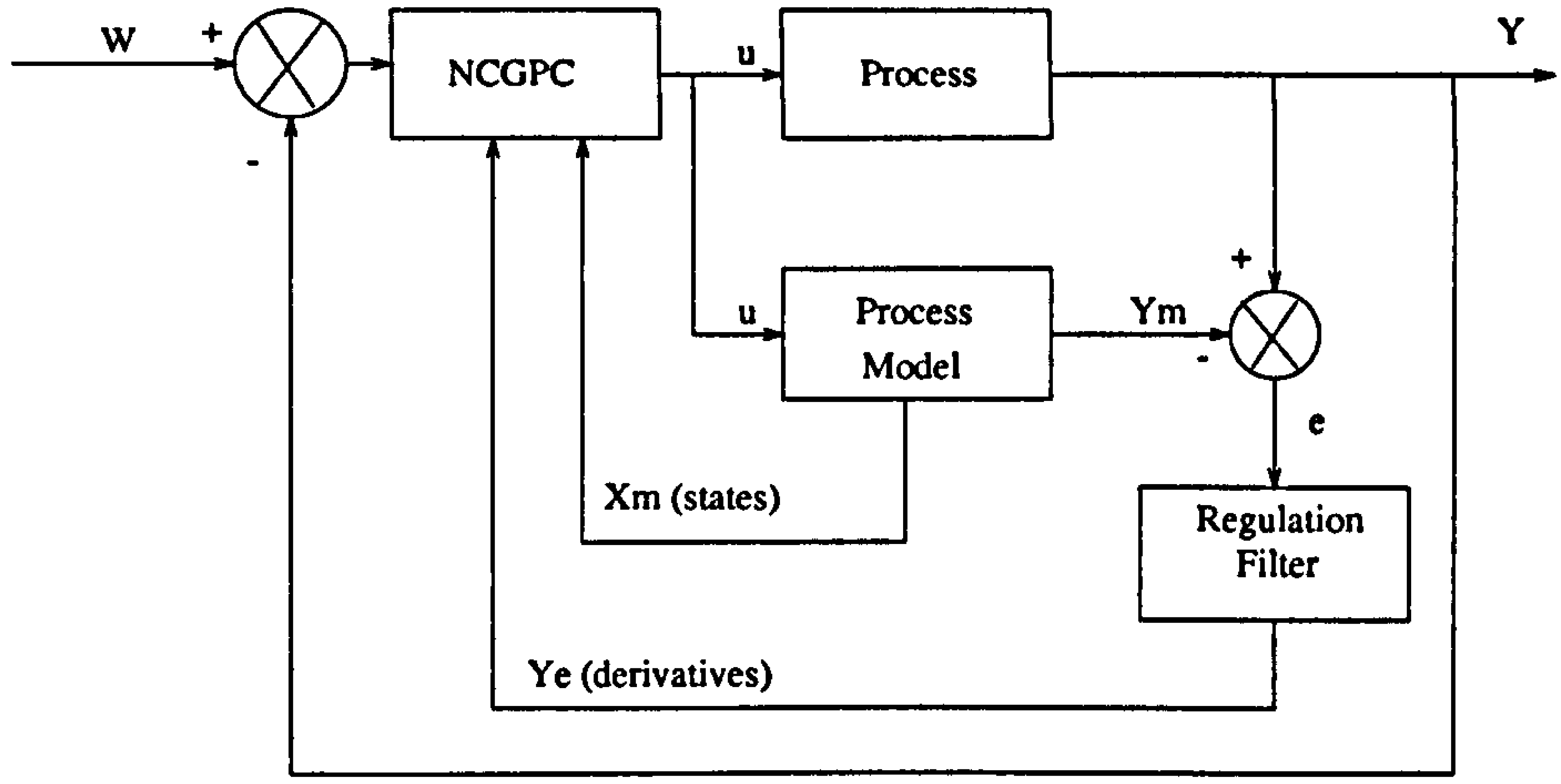

Figure 6.1: NCGPC with a regulation filter Structure

Substituting equation (6.36) into the rth derivative given by equation (6.35) leads to:

$$y_m^{(r)}(t) = 1/\beta_r(w - y(t)) - \sum_{i=1}^{r-1} \beta_i(y_m^{(i)} + y_e^{(i)}) - y_e^{(r)}(t) \quad (6.37)$$

From the definition of the regulation filter output, we can see that

$$y^{(r)}(t) = e^{(r)}(t) + y_m^{(r)}(t) \quad (6.38)$$

Substituting equation (6.37) into this equation, gives:

$$y^{(r)}(t) = e^{(r)}(t) + 1/\beta_r(w - y(t)) - \sum_{i=1}^{r-1} \frac{\beta_i}{\beta_r}(y_m^{(i)} + y_e^{(i)}) - y_e^{(r)}(t) \quad (6.39)$$

Rearranging and taking Laplace transforms (with zero initial conditions, because the initial conditions of y_e and y_m always are chosen zero) gives:

$$y_m(s) + y_e(s) = G(s)w(s) - G(s)(y(s) - y_m(s) - y_e(s)) \quad (6.40)$$

where, $G(s)$ is given by equation (6.33)

Adding $y(s)$ to both sides of this equation and rewriting:

$$y(s) = G(s)w(s) + (1 - G(s))(y(s) - y_m(s) - y_e(s)) \quad (6.41)$$

From equation (6.34) $y_e(s)$ can be obtained by the following transfer function

$$y_e(s) = G_e(s)(y(s) - y_m(s)). \tag{6.42}$$

Substituting $y_e(s)$ into equation (6.41), gives

$$y(s) = G(s)w(s) + (1 - G(s))(1 - G_e(s))(y(s) - y_m(s)) \tag{6.43}$$

When the process model is perfect, the response is given as:

$$y(s) = G(s)w(s) \tag{6.44}$$

Whereas, if the process model is not perfect the response is given by equation (6.43). From this equation, it can be deduced that

$$y(t) \longrightarrow w \qquad \text{as} \qquad t \longrightarrow \infty,$$

We can also see that the second term on the right hand side of equation (6.43) will tend to zero, if $y(t) - y_m(t)$ is a constant.

When the regulation filter is not considered, the control law becomes the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] and reviewed in Chapter 3; this uses an open loop observer. Its response will be given by

$$y(s) = G(s)w(s) + (1 - G(s))(y(s) - y_m(s))) \tag{6.45}$$

As explained before, in order to reduce the effect of the mismatch between plant and model, which is present in the second term of the right hand side of equation (6.45), it is necessary to increase the bandwidth of $G(s)$. However this may cause an overshoot in the response and an excessive control signal. In other words $G(s)$ has influence on the performance and on the robustness. While, when the regulation filter is added, we can see from the following equation

$$y(s) = G(s)w(s) + (1 - G(s))(1 - G_e(s))(y(s) - y_m(s)) \tag{6.46}$$

that we can increase the bandwidth of $(1-G(s))(1-G_e(s))$ by increasing the bandwidth of $G_e(s)$. Thus, the performance is determined by $G(s)$ and the robustness by $G_e(s)$.

Therefore, the performance of the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] and reviewed in Chapter 3 can be improved by adding the regulation model.

6.5 Simulation Results

In this section simulations are presented in order to illustrate the effects of the parameter T and of the regulation model and to make comparison between the controllers. The polynomial P is chosen equal 1. The system is given by

$$\begin{aligned} \dot{x}_1 &= -x_1 - a_1 x_2 \\ \dot{x}_2 &= exp(-a_2 x_2) - 1 - a_3 u \end{aligned} \tag{6.47}$$

The system output is $y = x_1$.

The process model is

$$\begin{aligned} \dot{x}_{1m} &= -x_{1m} - x_{2m} \\ \dot{x}_{2m} &= exp(-x_{2m}) - 1 - u \end{aligned} \tag{6.48}$$

The model output is $y_m = x_{1m}$

The regulation filter is

$$\begin{aligned} \dot{y}_e &= y_{1e} \\ \dot{y}_{1e} &= -a_{0e} y_e - a_{1e} y_{1e} + a_{0e} e \end{aligned} \tag{6.49}$$

where $e = y - y_m$

The tracking model is

$$\begin{aligned} \dot{y}_r &= y_{1r} \\ \dot{y}_{1r} &= -a_{0r} y_r - a_{1r} y_{1r} + a_{0r} z \end{aligned} \tag{6.50}$$

where z is the desired output.

If the process model is perfect $a_1 = 1$, $a_2 = 1$ and $a_3 = 1$, but to reflect parametric uncertainties, these parameters are chosen as $a_1 = 1.4$, $a_2 = .7$ and $a_3 = 1.1$.

6.5.1 The effects of T

The initial conditions of the system and process model are zero. Parameter T is varied from 0.01, 0.1, 0.5, to 1.0, in order to show that the TRRMCNL controller is just the NPGMV controller with an open loop observer. The regulation model parameters were chosen as follows: $a_{0r} = \frac{2}{T^2}$ and $a_{1r} = \frac{2}{T}$, in order to make $G_e(s) = G(s)$ (as defined in Section 6.4). Figures (6.2), (6.3) and (6.4) illustrate the effects due to these variations. We can infer from the simulations that the small value T corresponds to small tracking error. Also, it is possible to see that the simulation results for the TRRMCNL and for the NPGMV with an open loop observer, are exactly the same.

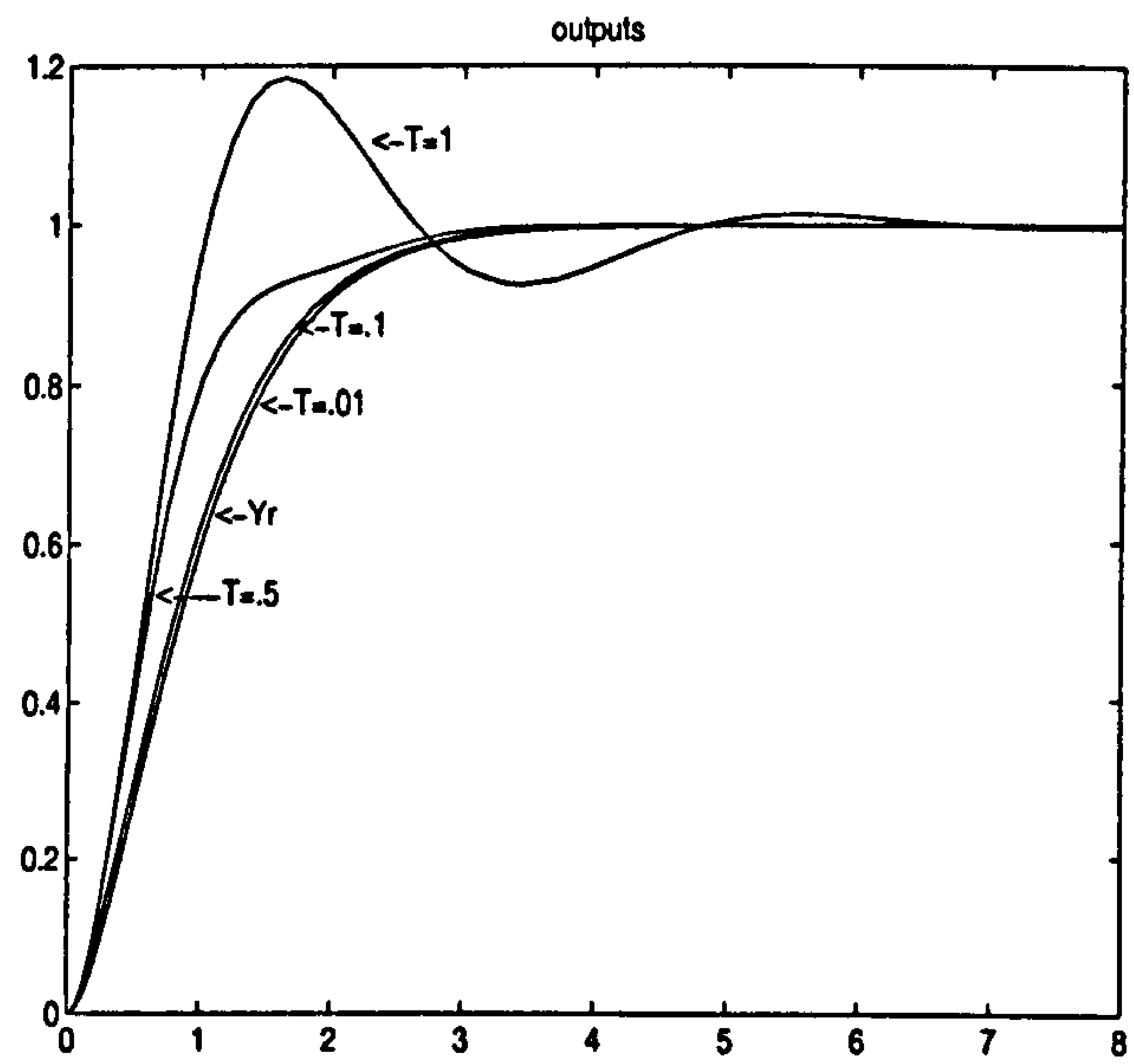

Figure 6.2: The effects on y when T is varied

6.5.2 Initial Conditions

The initial conditions considered in these simulations are $x_1(0) = -0.1$ and $x_2(0) = -0.2$, with $T = .1$ and with the same parametric uncertainties. Figure (6.5) represents the output of the system when the NPGMV with an open loop observer is applied. Similarly in Figures (6.6) and (6.7) the tracking errors $e = y_r - y$ and the control signals u are shown.

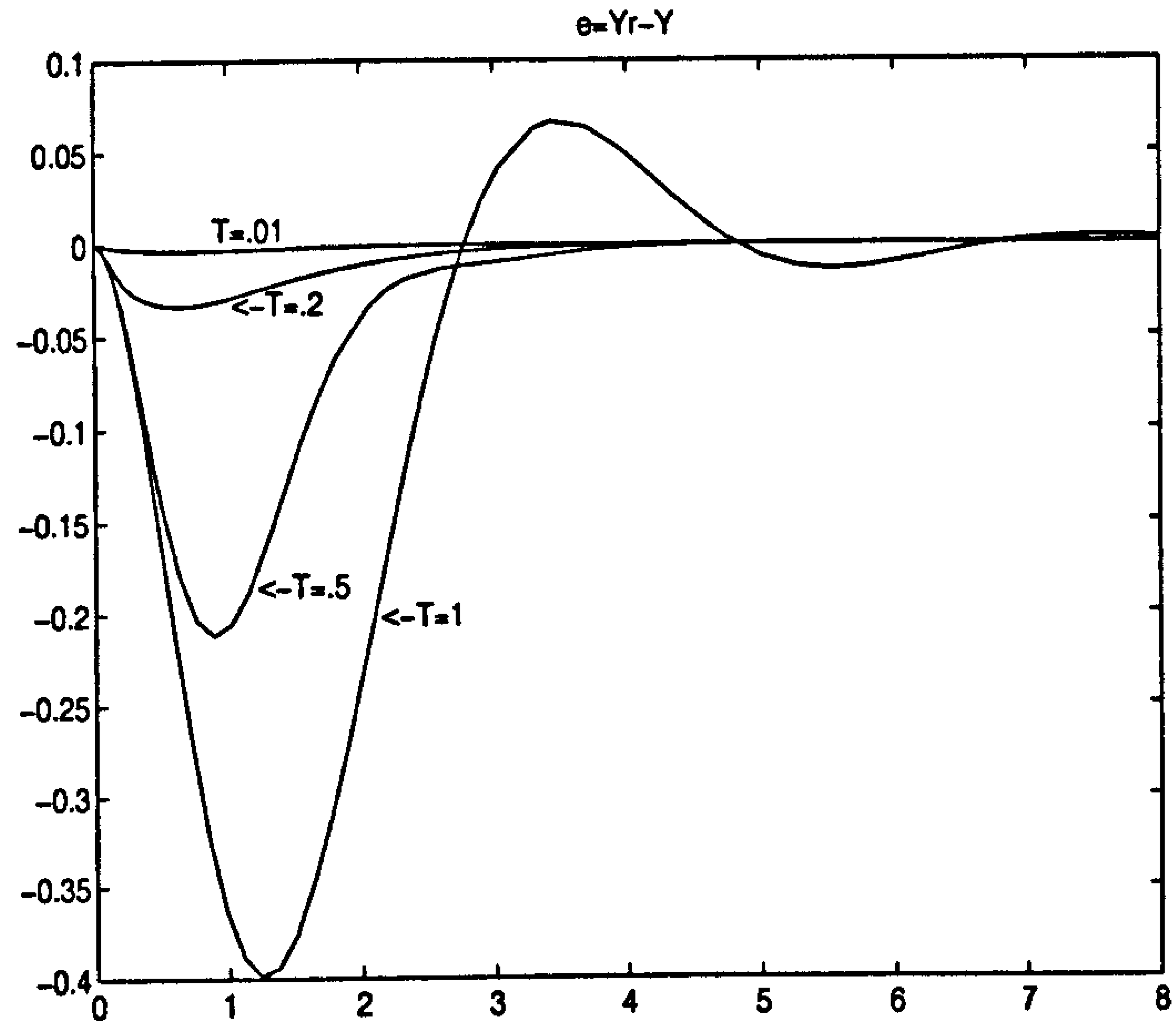

Figure 6.3: The effects on e when T is varied

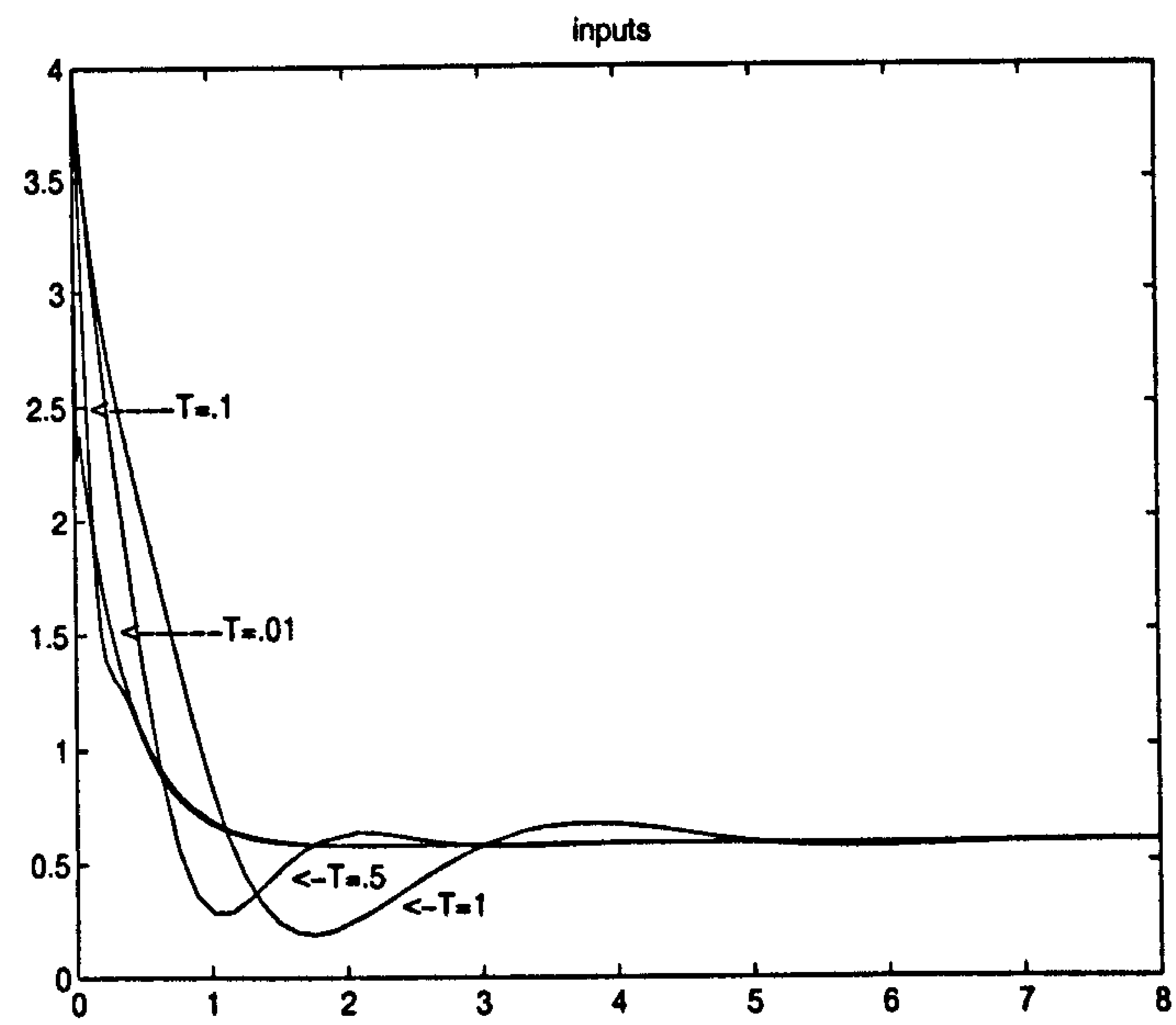

Figure 6.4: The effects on u when T is varied

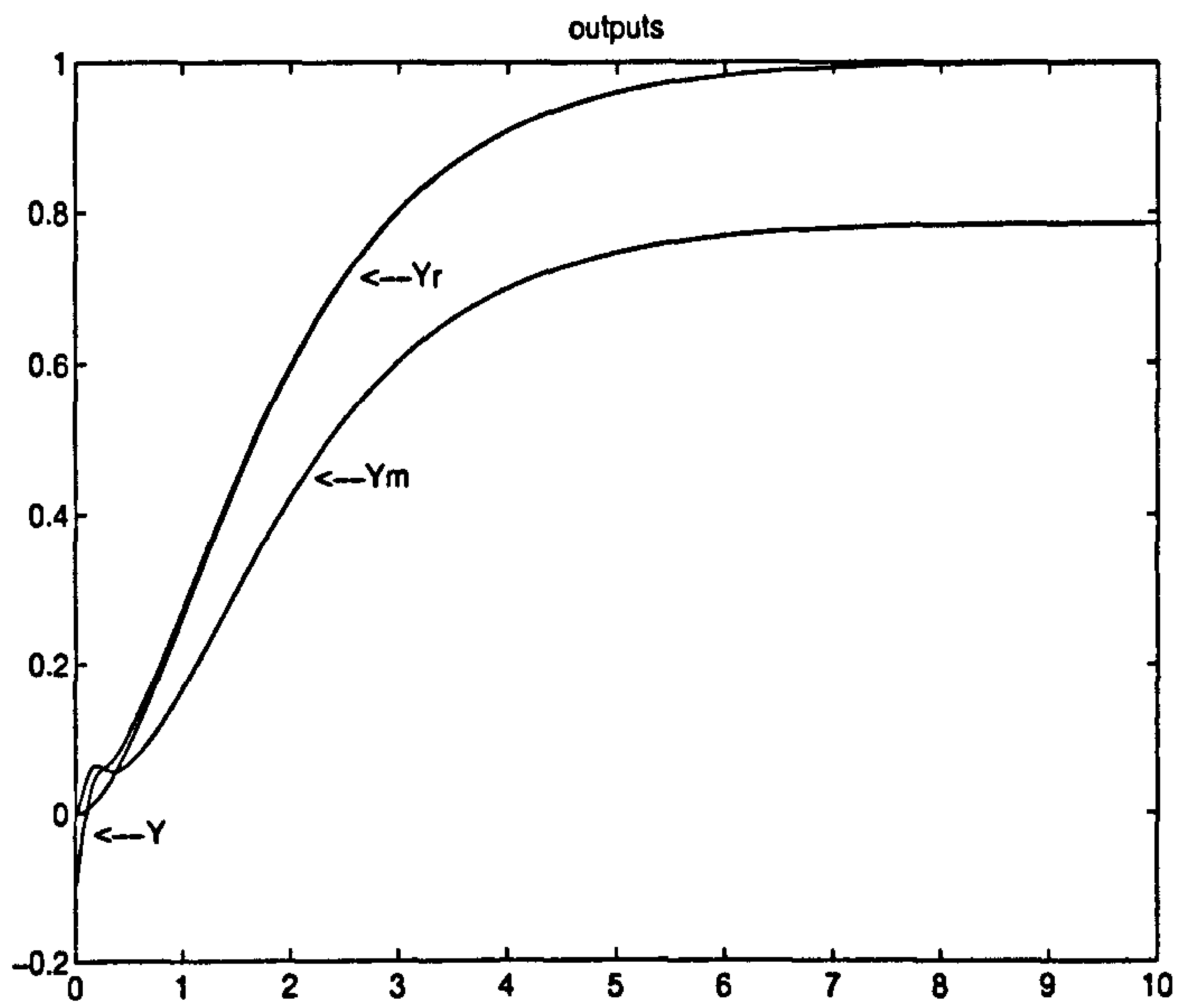

Figure 6.5: system output and reference

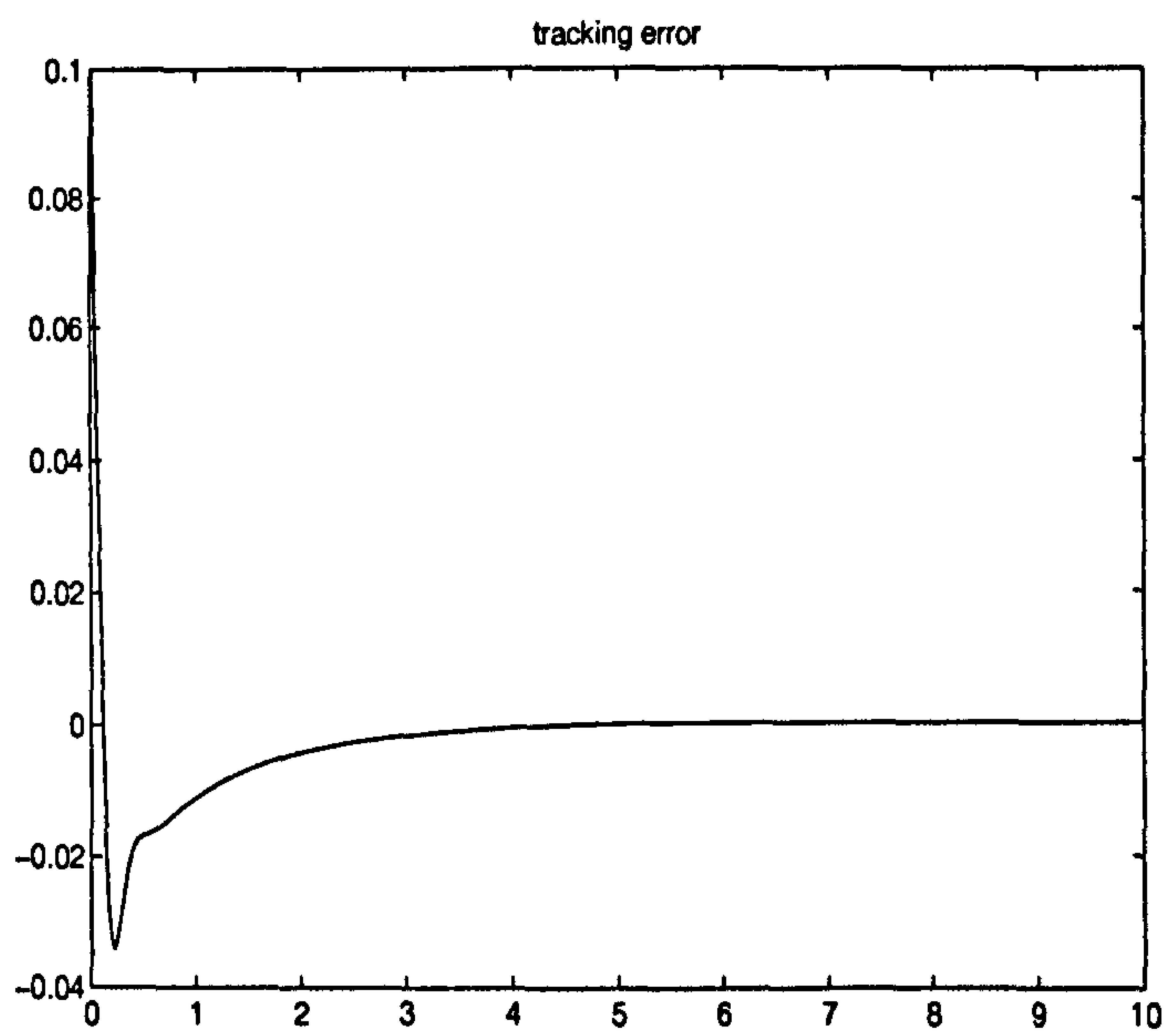

Figure 6.6: tracking error

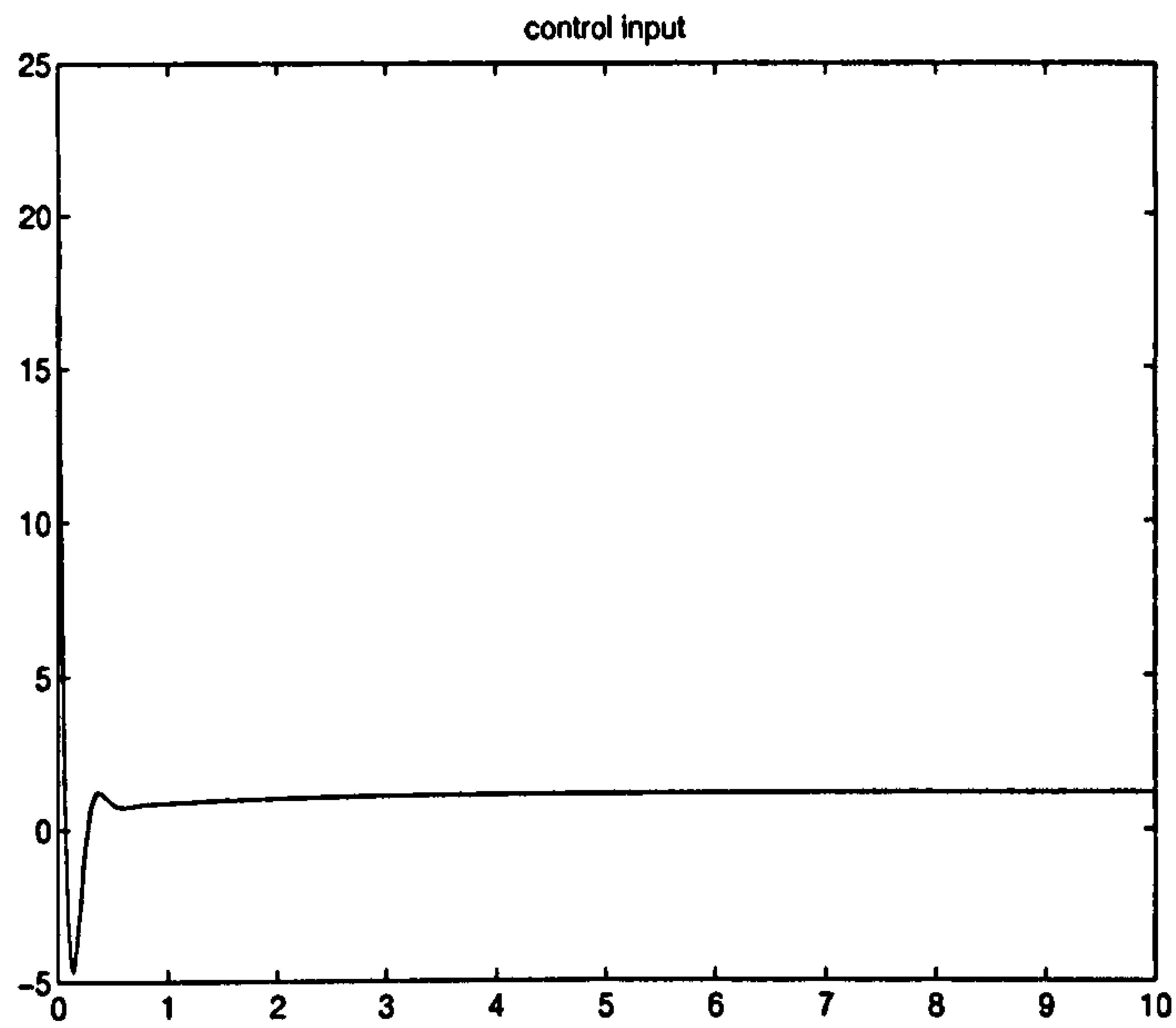

Figure 6.7: input signal

6.5.3 The effects of regulation filter

The system is given by

$$\dot{x}_1 = -x_1 - a_1 x_2$$
$$\dot{x}_2 = exp(-a_2 x_2) - 1 - a_3 u \qquad (6.51)$$

The system output is $y = x_1$.

The process model is

$$\dot{x}_{1m} = -x_{1m} - x_{2m}$$
$$\dot{x}_{2m} = exp(-x_{2m}) - 1 - u \qquad (6.52)$$

The model output is $y_m = x_{1m}$

The regulation filter is

$$\dot{y}_r = y_{1r}$$
$$\dot{y}_{1r} = -a_{0r} y_r - a_{1r} y_{1r} + a_{0r} e \qquad (6.53)$$

where $e = y - y_m$

For this study T is chosen as $T = 0.1$ and the initial conditions are $x_1(0) = -0.1$ and $x_2(0) = -0.2$. If the process model is perfect, $a_1 = 1$, $a_2 = 1$ and $a_3 = 1$. Once again parameter uncertainties, are modelled by setting , these parameters as $a_1 = 1.4$, $a_2 = .7$ and $a_3 = 1.1$.

The variations of the regulation filter bandwidth are chosen as 50 rad/sec and 5 rad/sec and one case without this filter.

Figures (6.8), (6.9) and (6.10) illustrate the effects due to the variations of regulation filter. It is possible to see that the reference is followed satisfactorily when a regulation filter is used. In fact it can be concluded from these figures that in order to improve the performance this filter is useful.

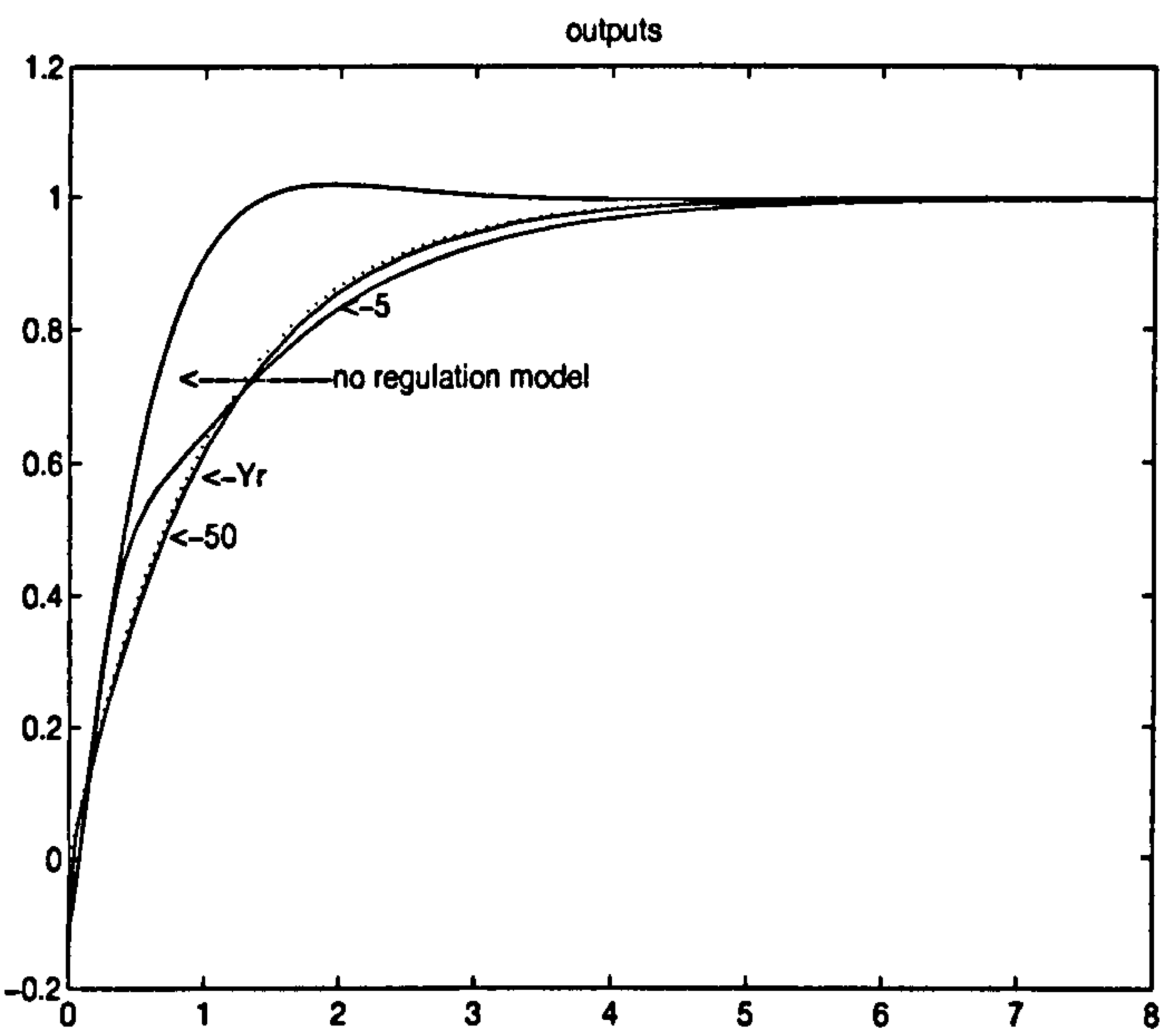

Figure 6.8: The effects on y when the regulation filter is varied

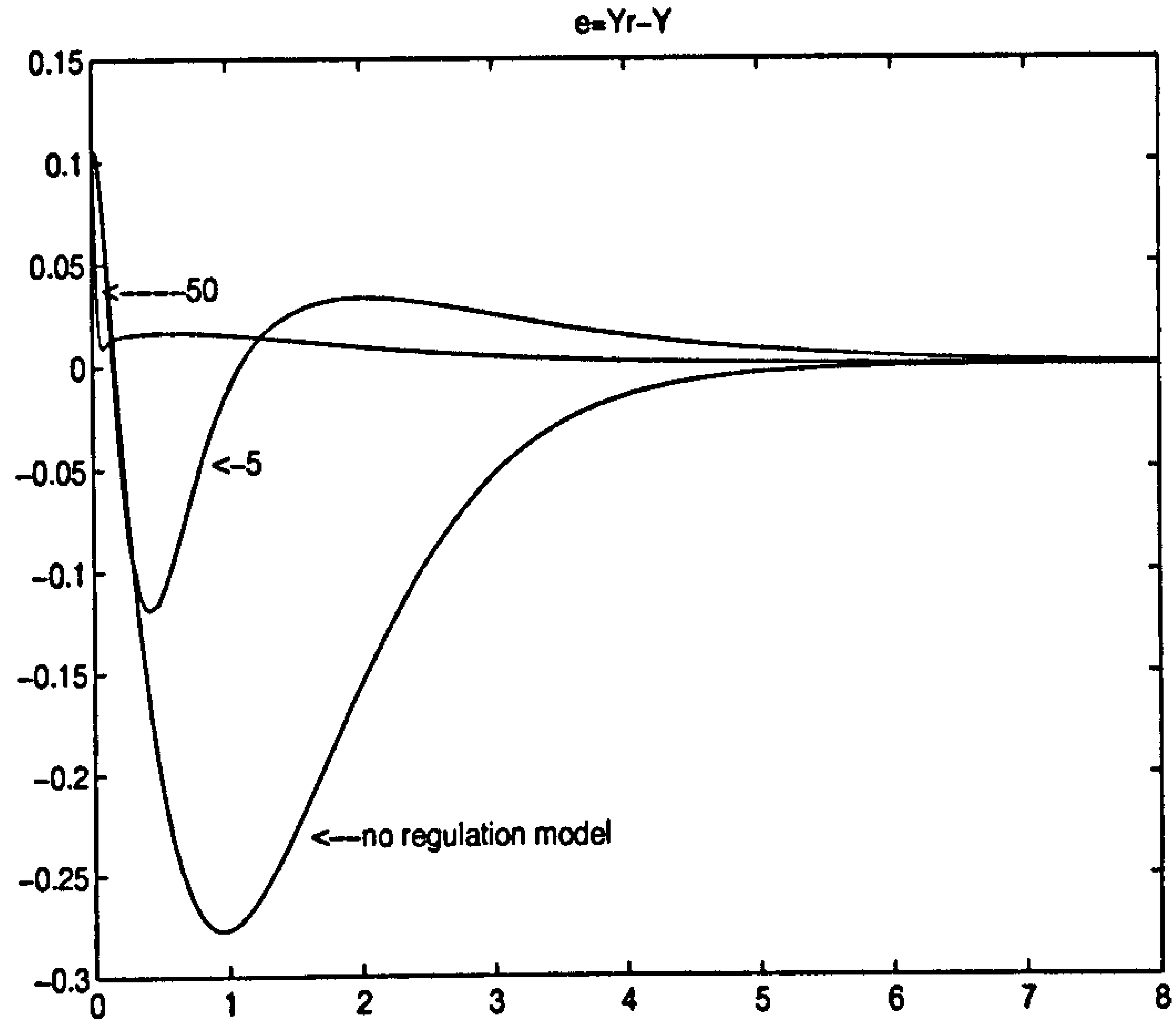

Figure 6.9: The effects on $er = y_r - y$ when the regulation filter is varied

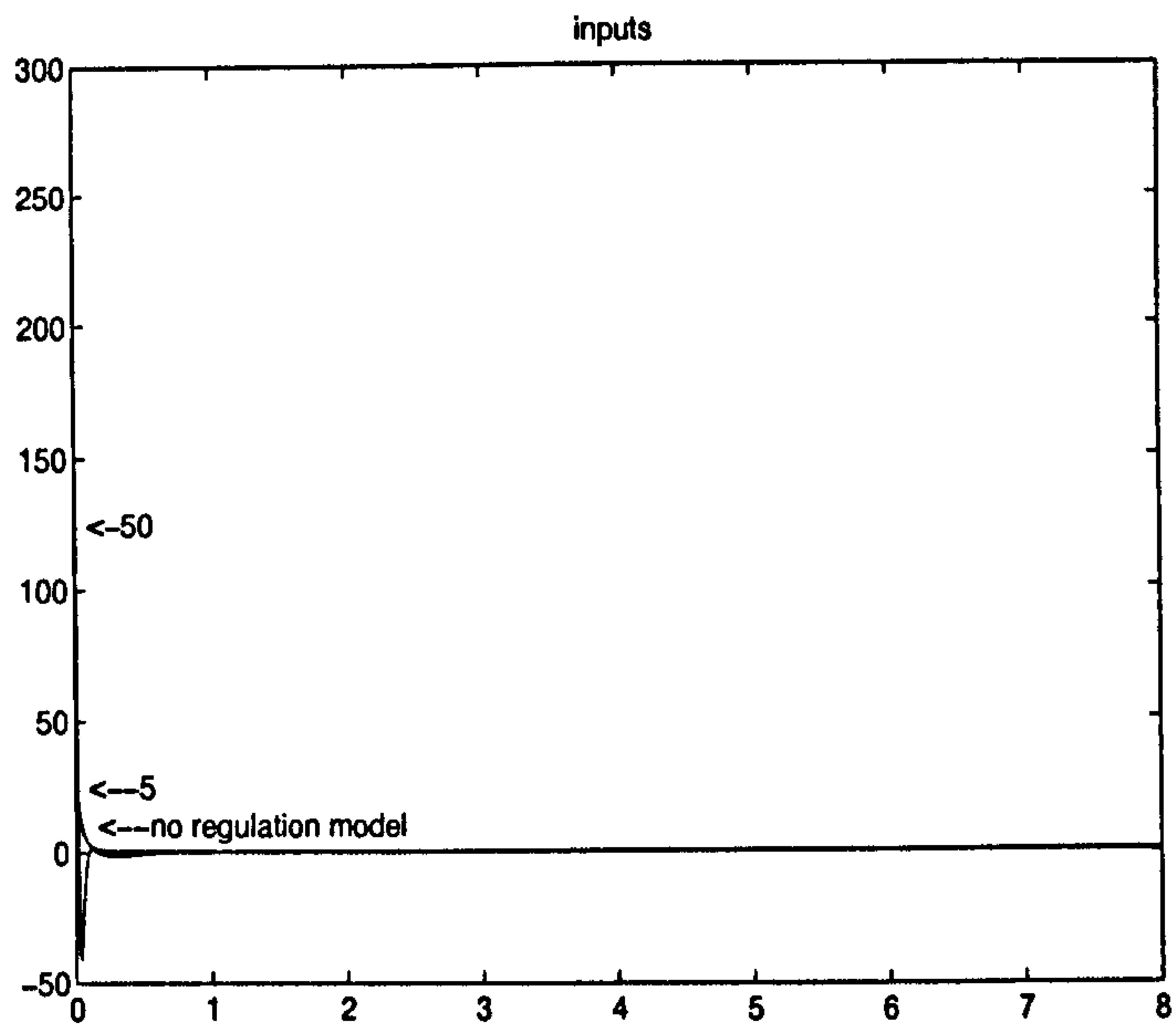

Figure 6.10: The effects on u when the regulation filter is varied

6.6 Conclusion

In this Chapter the following equivalencies are established.

NGMV	GLC [18]
GLC [18]	exact linearisation [44]
NGMV	exact linearisation [44]
NPGMV	Model Matching Via State Feedback [44]

Also, the TRRMCNL developed by J. Alvarez and J. Alvarez [2] is shown to be just Model Matching Via State Feedback [44] with an open loop observer. As a consequence, NGMV, NPGMV are included in NCGPC. Therefore, the nonlinear exact linearisation by feedback developed by A. Isidori [44], the error feedback-GLC developed by P. Dautidis and C. Kravaris [18], the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2], as well as the linear version CGPC recast in state space form, are included in the NCGPC. The TRRMCNL controller has to use the regulation model in order to guarantee that the process output matches the reference when a perfect nonlinear model is not available. In other words the regulation filter is intended to provide the robustness of the controller, E. Liceaga [53]. But, it was shown that the regulation model is unnecessary.

When the system is stable, in order to improve the performance, an open loop observer and a regulation model used by J. Alvarez and J. Alvarez [2] are added to the GMV. The open loop observer is used to get the states and the regulation model is used to counteract the error between output process derivatives and output model derivatives. The GMV then becomes an output feedback control. The error feedback-GLC developed by P. Dautidis and C. Kravaris [18] is equivalent to GMV. Thus, this controller can also improve its performance using this regulation model.

Chapter 7

Conclusions

The development of the nonlinear version of the Continuous-time Generalised Predictive Control (NCGPC) is presented. Because transfer function models are no longer appropriate, the nonlinear version of CGPC is developed in state-space form and shown to include Nonlinear Generalised Minimum Variance (NGMV), and a new algorithm, Nonlinear Predictive Generalised Minimum Variance (NPGMV), as special cases. The development of these algorithms have been reported in Section 6 of the paper by P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33]).

In Chapter 5 the nonlinear version of the CGPC of H. Demircioglu and P. J. Gawthrop [21] is developed in a state space setting. The NCGPC provides a nice way of handling systems with unstable zero dynamics, which is one of the most important objectives of this thesis. The NCGPC has two main advantages, which are exploited for this objective. The first advantage is that it can constrain the predicted control through N_u. It is possible to infer that $u(t)$ is indirectly constrained by N_u. Additionally, the response becomes slow and the control is not very active, this fact is illustrated by simulations. The second advantage is, when $N_u < N_y - r$ the cancellation of the *zero dynamics* does not occur with the NCGPC. Therefore, the internal stability is preserved.

Because NCGPC does not cancel the nonlinearities (as in the geometric approaches), this controller can deal with systems whose relative degree is not well defined and unstable zero dynamics systems by choosing $N_u < N_y - r$. In addition, it can control systems of the form $\dot{x} = F(x, u)$, $y = h(x)$. As it well known and reviewed in Chapters 2 and 3 the geometric approaches can not deal with these kind of systems.

When NCGPC is applied on systems of the form

$$\begin{aligned} \dot{x}(t) &= F(x,u) \\ y(t) &= h(x), \end{aligned}$$

or when N_y is not chosen less than the number of the times that the output has to be differentiated in order to obtain nonlinear terms in u. As explained in Section 5.5.1, these cases give rise to a mathematical structure which is akin to a Differential Algebraic Equation replaced by a nondynamic optimisation. The precise mathematical description and investigation of such *differential–optimisation equations* remains an open question. However, from a practical point of view the corresponding differential equations may be discretised in time and the optimisation performed at each time step (see Section 8 P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33]).

In Chapter 5 the following equivalencies are established.

NGMV	GLC [18]
GLC [18]	exact linearisation [44]
NGMV	exact linearisation [44]
NPGMV	Model Matching Via State Feedback [44]

Also, the TRRMCNL developed by J. Alvarez and J. Alvarez [2] is shown to be just the Model Matching Via State Feedback [44] with an open loop observer. And because NGMV, NPGMV are included in NCGPC. Therefore, the nonlinear exact linearisation by feedback developed by A. Isidori [44] and the error feedback-GLC developed by P. Dautidis and C. Kravaris [18] the TRRMCNL (Tracking and Regulation Reference Model Control of Nonlinear Systems) developed by J. Alvarez and J. Alvarez [2], as well as the linear version CGPC recast in state space form, are included in the NCGPC. The fact that the GLC control law developed by Dautidis and Kravaris [18] and the exact linearisation by state feedback described by Isidori [44] are equivalent to NGMV has been reported in Section 7.2 P.J. Gawthrop, H. Demircioglu and I.I. Siller-Alcalá [33]).

When the system is stable, in order to improve the performance, an open loop observer and a regulation model used by J. Alvarez and J. Alvarez [2] are added to the GMV. The open loop observer is used to get the states and the regulation model is used to counteract the error between output process derivatives and output model derivatives. The GMV then becomes an output feedback control. The error feedback-GLC developed by P. Dautidis and C. Kravaris [18] is equivalent to GMV. Thus, this controller can also improve its performance using this regulation model.

Finally, in Chapter 4 a design of a positioning control for an Induction Motor based on the TRRMCNL is developed. Where, Induction Motor dynamics are represented by $d-q$ model and the Induction Motor model dynamics are represented by a simplified model obtained when the average value of the electromagnetic torque is considered.

The simplified model is used as process model in order to estimate the states. The simulation results shown the good results when there are parametric uncertainties. The work presented in Chapter 4, gave to me the ideas investigated in Chapters 5 and 6.

Suggestions for further work

We will end this thesis by suggesting some possible further works. These are:

1 In order to control an open loop unstable process, the proposed controllers are , of course, inappropriate, since estimation of the states is necessary. The future work will be to control the unstable plants, the state estimation will be carried out by using MBO (Model Based Observer) developed by P.J. Gawthrop [30]. Unlike the linear case, the stability of such an observer is not guaranteed in general and its design is non trivial (Walcott *et al.* [75], Hunt and Verma [41]).

2 Systems with time delay is an area of continuing research.

3 The relationship between the NCGPC when $N_u < N_y - r$ with output regulation

of nonlinear systems.

4 The application to the ball and beam example, which is treated by J. Hauser, S. Sastry and P. Kokotovic [38]. In order to see the effectiveness of the NCGPC.

5 The application of the NCGPC to the inverted pendulum.

Bibliography

[1] S. Abu el Ata-Doss and M. Fliess. Nonlinear predictive control by inversion . *IFAC Symposium on Nonlinear Control Systems Design, Capry Italy*, pages 119–124, 1989.

[2] J. Alvarez and J. Alvarez. Tracking and regulation reference model control of nonlinear systems. *IFAC Symposium on Nonlinear Control Systems Design, Capri Italy*, pages 466–471, 1989.

[3] K.J. Astrom. *Introduction to Stochastic Control Theory.* Academic Press, New York, USA, 1970.

[4] R. Bars and R. Haber. One-step ahead predictive control algorithms for nonlinear nonparametric models. *European Control Conference, Groningen, The Netherlands*, 2:941–945, 1993.

[5] L. Benvenuti, M.D. Di Benedetto, and J.W. Grizzle. Approximate output tracking for nonlinear non-minimum phase systems with an application to flight control. *International Journal Control of Robust and Nonlinear Control*, 4:397–414, 1994.

[6] R.R. Bitmead, M. Gevers, and V. Wertz. *Adaptive Optimal Control. The Thinking Man's GPC.* New York: Prentice-Hall, 1990.

[7] G. Bornard and J.P. Gauthier. Commande dynamique multivariable des systemes industriales de production. *Note Interne No. 77-29. Automatic Control Lab. Grenoble.*, 1977.

[8] C.I. Byrnes and I. Isidori. Global feedback stabilization of nonlinear systems. *Proc. 24th IEEE CDC Pt. Lauderdale*, pages 1031–1037, 1985.

[9] C. M. Chow and D.W. Clarke. *Actuator nonlinearities in predictive control. In D.Clarke(Ed.), Advances in Model-based predictive control.* Oxford University Press, 1994.

[10] D.W. Clarke. *Advances in Model-based predictive control.* Oxford University Press, 1994.

[11] D.W. Clarke. *Advances in Model-based predictive control. In D.Clarke(Ed.), Advances in Model-based predictive control.* Oxford University Press, 1994.

[12] D.W. Clarke and P.J. Gawthrop. Self-tuning controller. *Proceedings IEE*, 122:929–934, 1975.

[13] D.W. Clarke and P.J. Gawthrop. Self-tuning control. *Proceedings IEE*, 126:633–640, 1979.

[14] D.W. Clarke and C. Mohtadi. Properties of generalized predictive control. *Automatica*, 25:859–875, 1989.

[15] D.W. Clarke, C. Mohtadi, and P. S. Tuffs. Generalized predictive control-parts i and ii. *Automatica*, 23:137–160, 1987.

[16] C.R. Cutler and B.L. Ramaker. Dynamic matrix control - a computer control algorithm . *Proceedings JACC San Francisco, USA.*, 1980.

[17] P. Daoutidis and C. Kravaris. Synthesis of feedforward/state feedback controllers for nonlinear systems. *AIChE J.*, 35:1602–1616, 1989.

[18] P. Daoutidis and C. Kravaris. Dynamic output feedback control of minimum-phase nonlinear process. *Chem. Eng. Sci.*, 47:837, 1992.

[19] A. de Luca and G. Ulivi. Design of an exact non-linear controller for induction motors. *IEEE Transactions on Automatic Control*, 34:1304–1307, 1989.

[20] Ch. Degang. An iterative solution to stable inversion of non-minimum phase systems. *American Control Conference*, pages 2960–2964, 1993.

[21] H. Demircioglu and P.J. Gawthrop. Continuous-time generalized predictive control (cgpc). *Automatica*, 27:55–74, 1991.

[22] H. Demircioglu and P.J. Gawthrop. Multivariable continuous-time generalized predictive control (cgpc). *Automatica*, 28:697–713, 1992.

[23] Kim. Donf-Il, Ha. In-Joong, and Ko. Myoung-Sam. Control of induction motors via feedback linearization with input-output decoupling. *Int. Journal Control*, 51:863–883, 1990.

[24] P. Doutidis and C. Kravaris. Dynamic output feedback control of minimum-phase nonlinear systems. *Proc. American Control Conference, Boston Mass., USA*, pages 1806–1811, 1991.

[25] C.G. Economou and M. Morari. Internal model control. 5. extension to nonlinear systems. *Ind. Eng. Chem. Proces Des. Dev.*, 25:403–419, 1986.

[26] P.J. Gawthrop. Some interpretation of the self-tuning controller. *Proceedings IEE*, 124:889–894, 1977.

[27] P.J. Gawthrop. Continuous-time self-tuning control - a unified approach. *Preprints 2nd. IFAC Workshop on Adaptive Systems in Control and Signal Processing, Lund Sweden*, pages 19–24, 1986.

[28] P.J. Gawthrop. Self-tuning pid controllers: Algorithms and implementation. *IEEE Transactions on Automatic Control*, 31:201–209, 1986.

[29] P.J. Gawthrop. *Continuous-time Self-tuning Control. Vol 1 Design.* Lechworth, England: Research Studies Press, Engineering control series., 1989.

[30] P.J. Gawthrop. Model-based observer control: An introduction. *tech. rep., University of Glasgow, Department of Mechanical Engineering, Control Group Report R-91/9*, 1991.

[31] P.J. Gawthrop and D.W. Clarke. Hibrid self-tuning control and its interpretations. *In proceedings of 3rd IMA Conference Control Theory. Academic Press*, 1980.

[32] P.J. Gawthrop and H. Demircioglu. Continuous-time generalised predictive control. *In Preprinters of the 3rd IFAC workshop on Adaptive Systems in Control and Signal Processing, Glasgow, U.K.*, 1989.

[33] P.J. Gawthrop, H. Demircioglu, and I. Siller-Alcala. Multivariable continuous-time generalised predictive control: A state-space approach to linear and nonlinear systems. *IEE Proc.-Control Theory Appl.*, 145(3):241–250, 1998.

[34] P.J. Gawthrop, R.W. Jones, and S.A. MacKenzie. Bond graph based control:a process engineering example. *American Control Conference*, 1992.

[35] P.J. Gawthrop, R.W. Jones, and D.G. Sbarbaro. Emulator-based control and internal model control: Complementary approaches to robust control design. *Automatica,(Submitted)*, 1995.

[36] P.J. Gawthrop and L.P.S. Smith. *Metamodelling: Bond Graphs and Dynamic Systems.* Hemel Hempstead, Herts, England.:Prentice Hall., 1996.

[37] R. Gurumoorthy and R. Sanders. Controlling non-minimum phase nonlinear systems- the inverted pendulum on a cart example. *American Control Conference*, 1:680–685, 1993.

[38] J. Hauser, S. Sastry, and P. Kokotovic. Nonlinear control via approximate input-output linearization: The ball and beam example. *IEEE Transaction on Automatic Control*, 37:392–398, 1992.

[39] R.M. Hirschorn. Invertibility of nonlinear control systems. *SIAM J. Contr. Optim.*, 17:289, 1979.

[40] Z. Hongchao and Ch. Degang. Minimum-energy approach to stable inversion of non-minimum phase systems. *American Control Conference*, pages 2705–2709, 1994.

[41] L. Hunt and M.S. Verma. Observers for nonlinear systems in steady state. *IEEE Transaction on Automatic Control*, 39:2113–2118, 1994.

[42] F. III, Doyle, F. Allgower, S. Oliveira, E. Gilles, and M. Morari. On nonlinear systems with poorly behaved zero dynamics. *American Control Conference*, pages 2571–2575, 1992.

[43] A. Isidori. *Lecture Notes in Nonlinear Control Systems.* in Rome, Italy, 1985.

[44] A. Isidori. *Nonlinear Control Systems Third Edition.* Springer, 1995.

[45] A. Isidori and C.I. Byrnes. Output regulation of nonlinear systems. *IEEE Transaction on Automatic Control*, 35:131–140, 1990.

[46] I. Kanellakopoulus, P.V. Kokotovic, and A. Morse. Adaptive output-feedback control of systems with output nonlinearities. *P.V. Kokotovic, Ed., Foundations of Adaptive Control, Berlin: Springer Verlag*, 1991.

[47] C. Kravaris. Input/output linearization: A nonlinear analog of placing poles at process zeros. *AIChE Journal*, 34:1803–1812, 1988.

[48] C. Kravaris and Ch. Chang-Bock. Nonlinear state feedback synthesis by global input/output linearization. *AIChE Journal*, 33:592–603, 1987.

[49] C. Kravaris and S. Palanki. Robust nonlinear state feedback under structured uncertainty. *AIChE J.*, 34:1119–1127, 1988.

[50] C. Kravaris and R.A. Wright. Robust deadtime compensation for nonlinear processes. *American Control Conference*, pages 1565–1571, 1988.

[51] J. Lee, M. Morari, and C. Garcia. State-space interpretation of model predictive control. *Automatica*, 30(4):707–717, 1994.

[52] P.L. Lee and G.R. Sullivan. Generic model control (gmc). *Computers and Chem. Eng.*, 12:573–580, 1988.

[53] E. Liceaga-Castro. A two degrees of freedom nonlinear controller. *Industrial Control Centre, University of Strathclyde, ICC/8/June*, 1992.

[54] E. Liceaga-Castro and I. Siller-Alcala. A nonlinear positioning controller for an induction motor. *The third IEEE Conference on Control Applications, Strathclyde Uni., Glasgow.*, 1:447–452, 1994.

[55] Ping Lu. Nonlinear predictive controllers for continuous systems. *Journal of Guidance, Control and Dynamics*, 17:553–560, 1994.

[56] R. Marino, S. Peresada, and P. Valigi. Adaptive input-output linearizing control of induction motors. *IEEE Transactions on Automatic Control*, 38:208–221, 1993.

[57] R. Marino and P. Tomei. Global adaptive output-feedback of nonlinear systems, part i: Linear parameterization. *IEEE Transactions on Automatic Control*, 30:17–32, 1993.

[58] R. Marino and P. Tomei. Global adaptive output-feedback of nonlinear systems, part ii: Nonlinear parameterization. *IEEE Transactions on Automatic Control*, 30:33–48, 1993.

[59] D.Q. Mayne and Michalska H. Model predictive control of nonlinear systems. *American Control Conference, Boston, Massachussets*, 3:2343–2348, 1991.

[60] D.Q. Mayne and H. Michalska. Receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, 35:814–824, 1990.

[61] J. Meisel. *Principles of Electromechanical Energy Conversion.* Mc Graw Hill Bo. Co., 1966.

[62] M. Morari. *Model Predictive Control: Multivariable control technique of choice in the 1990s? In Advances in Model-based Predictive Control, pp. 22-37.* Oxford University Press, 1994.

[63] M. Morari and E. Zafiriou. *Robust Process Control.* Englewood Cliffs:Prentice-Hall, 1989.

[64] K. Nam and A. Arapostathis. A model reference adaptive control scheme for pure-feedback nonlinear systems. *IEEE Transactions on Automatic Control,* 33:803–811, 1988.

[65] H. Nijmeijer and A.J. van der Schaft. *Nonlinear Dynamical Control Systems.* Springer-Verlag, 1990.

[66] A.W. Ordys and D. W. Clarke. State-space description for gpc controllers. *International Journal of Systems Science,* 29(9):1727–1744, 1993.

[67] A.A. Patwardhan, J.B. Rawlings, and T.F. Edgar. Model predictive control of nonlinear process in the presence of constraints. *IFAC Symposium on Nonlinear Control Systems Design, Capri Italy,* pages 345–349, 1989.

[68] A.A. Patwardhan, J.B. Rawlings, and T.F. Edgar. Nonlinear model predictive control. *Chem. Eng. Comm.,* 87:123–141, 1990.

[69] D. Rangel, L. Chun-Yea, and W. Tse-Wei. Application of feedback linearization to the control of nonlinear chemical reactors. *AICHE Annual meeting 1990.,* 1990.

[70] J. Richalet, A. Rault, J.L. Testud, and J. Papon. Model predictive heuristic control: applications to industrial processes. *Automatica,* 14:413–428, 1978.

[71] R. Rouhani and R.K. Mehra. Model algorithmic control (mac); basic theoretical properties* . *Automatica,* 18:401–404, 1982.

[72] S. Sastry and A. Isidori. Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control,* 11:1123–1131, 1989.

[73] P.B. Sistu, R.S. Gopinath, and B.W. Bequette. Implementation issues in nonlinear predictive control. *American Control Conference, Boston, Massachussets,* 3:2361–2366, 1991.

[74] M. Soroush and C. Kravaris. A continuous-time formulation of nonlinear model predictive control. *Proc. American Control Conference, Chicago*, pages 1561–1567, 1992.

[75] B. Walcott, M.J. Corless, and S.H. Zak. Comparative study of non-linear state-observation techniques. *International Journal of Systems Science*, 29:1727–1744, 1987.

[76] W. Wang and R. Henriksen. Generalized predictive control of nonlinear systems of the hammerstein form. *European Control Conference, Groningen, The Netherlands*, 2:926–931, 1993.

[77] Q.M. Zhu, K. Warwick, and J.L. Douce. Adaptive general predictive controller for nonlinear systems. *IEE Proceedings-D*, 138:33–40, 1991.