

# Triangulating Unknown Environments using Robot Swarms

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## 1. INTRODUCTION

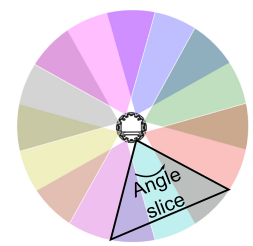
In recent years, the field of robotics has seen two diverging trends. One has been to achieve progress by increasing the capabilities of individual robots, keeping the cost of state-of-art machines relatively high. An opposite direction has been to develop simpler and cheaper platforms, at the expense of reducing the capabilities per robot. The latter raises new challenges for developing new principles and algorithms, such as coordinating many robots with limited capabilities into a swarm that can carry out difficult tasks, for example, exploration, surveillance, and guidance.

In this video, we show a recent collaboration between theory and practice of swarm robotics. We consider online problems related to exploring and surveying a region by a swarm of robots with limited communication range. The minimum relay triangulation problem (MRTP) asks for placing a minimum number of robots, such that their communication graph is a triangulated cover of the region. The maximum area triangulation problem (MATP) aims at finding a placement of  $n$  robots such that their communication graph contains a root and forms a triangulated cover of a maximum possible amount of area. We demonstrate the practical relevance of our methods by showing how they can be used on the novel real-world platform *r-one*.

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(a) r-one robot



(b) IR regions

**Figure 1:** (a) The *r-one* for multi-robot research, designed by the MRSL group at Rice University. (b) IR receiver detection regions. Each receiver detects an overlapping  $68^\circ$ , allowing to determine angles within  $22.5^\circ$ .

## 2. THEORETICAL MODEL

We are given a polygon  $P$ . Every robot in the swarm has a (circular) communication range  $r$ . Within this range, perception of and communication with other robots is possible. For ease of description, we normalize to  $r = 1$ .

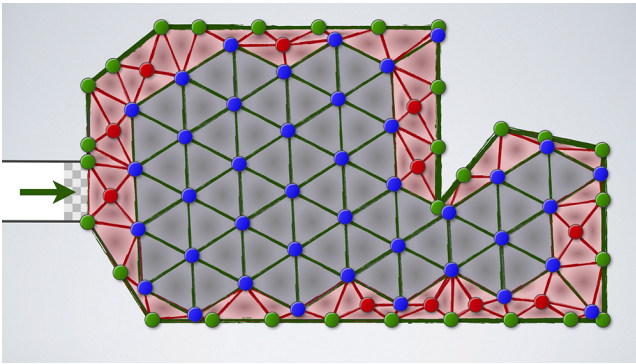
In the *Minimum Relay Triangulation Problem* (MRTP), we are given the  $n$ -gon  $P$  and a point  $z \in P$ . The goal is to compute a set  $R$  (with  $z \in R$  and  $V \subseteq R$  for the vertex set  $V$  of  $P$ ) of relays within  $P$ , such that there is a *unit triangulation* of  $P$  whose vertex set is exactly the set  $R$ , whose edges stay within  $P$  and have length at most 1. The objective is to minimize the number of relays.

In the *Maximum Area Triangulation Problem* (MATP), the number of available relays is bounded by a number  $k$ . The goal is to determine a set  $R$  of at most  $k$  relays, with a unit triangulation covering the maximum possible area.

For the online versions (OMRTP and OMATP), the polygon  $P$  is unknown. Each relay may move through the area, and has to decide on a new location for a triangulation vertex, while still being within reach of the previously placed relays. Once it has stopped, it becomes part of the static triangulation, allowing other relays to extend the exploration.

## 3. THE R-ONE ROBOT

The *r-one* [5] is an advanced, low-cost, open source robot design with an estimated unit cost of about US \$250. Mea-



**Figure 2: The 3-competitive strategy for the OM RTP.**

suring only 11 cm in diameter, it has a 32-bit ARM-based microcontroller, running at 50 MHz with no floating point unit. The local infrared (IR) communication system is used for inter-robot communication and localization. Each robot has eight IR transmitters and eight receivers. The transmitters broadcast in unison and emit a radially uniform energy pattern. The robot’s eight IR receivers are radially spaced to produce 16 distinct detection regions (shown in Figure 1b). By monitoring the overlapping regions, the bearing of neighbors can be estimated to within  $\approx \pi/8$ . Thus, it has limited capabilities for measurement, which is intertwined with local communication. The IR receivers have a maximum bit rate of 1250 bits per second. Each robot transmits  $(\Delta + 1)$  4-byte messages during each round, one being a system announce message, the others containing the bearing measurements to that robot’s neighbors. The system supports a maximum of  $\Delta = 10$ . For experimental work on coordination and navigation of r-ones, see [1].

#### 4. MRTP AND MATP

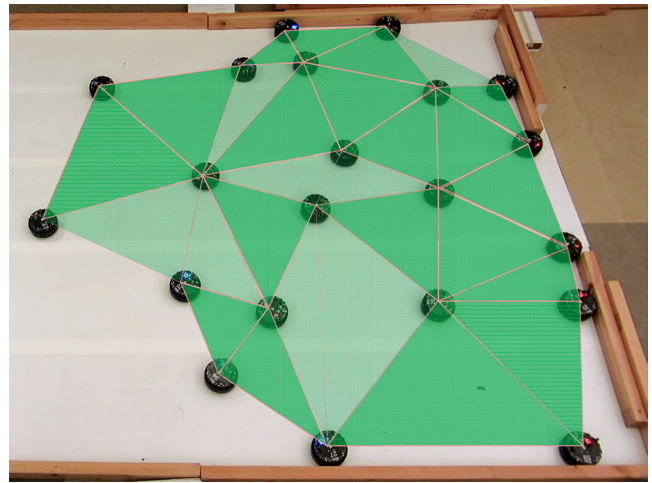
The problems MRTP and MATP were introduced in [4]; the currently best results for the online versions OM RTP and OM ATP were presented in [3, 6]. Both problems share their decision problem, which is known to be NP-hard.

For the OM RTP, there is a lower bound of  $6/5$  on the competitive factor of any deterministic strategy, as well as a 3-competitive algorithm for general polygons. This strategy is shown in Figure 2: We place robots at unit intervals along the boundary (green) and fill the interior with a regular triangular grid (blue). The space between the two is patched together using a third class (red). One can prove that the size each of the three classes is bounded by the number of robots in an optimal solution. For polyominoes, algorithms with better competitive factors exist [2].

On the other hand, the OM ATP does not admit a deterministic strategy with a constant competitive factor, if the polygon may have small corridors. If these can be excluded, greedy strategies perform well [2].

#### 5. THE VIDEO

The video starts by showing a swarm of robots. Each of them perceives only its direct environment. This leads to the question of exploring an unknown environment, and keeping it under surveillance. The next scene introduces triangulations as a way to build a covering network whose geometry describes the environment. A prototype of such



**Figure 3: A swarm of r-ones executing the greedy algorithm for the OM ATP.**

low-cost robots is the r-one platform, which is introduced. This is followed by a theoretical description of the MRTP, for which the 3-competitive algorithm is presented. Then, the MATP is introduced, and a greedy algorithm executed by r-one robots is shown—see Figure 3.

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