# The reconstructed final state of $\mathrm{Pb}+\mathrm{Pb} 158 \mathrm{AGeV}$ reactions from spectra and correlation data of NA49, NA44 and WA98 

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The final state of $\mathrm{Pb}+\mathrm{Pb}$ reactions at CERN SPS has been reconstructed with the Buda-Lund hydro model, by performing a simultaneous fit to NA49, NA44 and WA98 data on particle correlations and spectra.

## 1. Introduction

In refs. [] , 2] it was observed, for the first time, that the parameters of particle emitting sources can be determined only if a simultaneous analysis of the momentum distributions and two-particle correlation functions is performed. Simultaneous fitting of particle correlations and spectra were reported in refs. [2] 7]. Here, we determine the reconstructed space-time picture of particle emission in $\mathrm{Pb}+\mathrm{Pb}$ collisions at CERN SPS by fitting simultaneously the NA44 and NA49 published data $[8,9]$ on two-particle correlations and single-particle spectra at $\mathrm{Pb}+\mathrm{Pb} 158 \mathrm{AGeV}$ central reactions at CERN SPS. Preliminary data [10 from WA98 experiment are also used to check the reliability and the consistency of the fit results.

## 2. Buda-Lund hydrodynamic model

The Buda-Lund hydro parameterization characterizes with means and variances the local temperature, flow and chemical potential distributions of a cylindrically symmetric, finite hydrodynamically expanding system. The four-velocity $u^{\mu}(x)$ of the expanding matter is given by a scaling longitudinal Bjorken flow appended with a linear transverse flow, characterized by its mean value, $\left\langle u_{t}\right\rangle$. A Gaussian shape of the local density distribution is assumed both in the transverse plane and in space-time rapidity. The changes of the inverse temperature are characterized with means and variances. The freeze-out hypersurface is characterized by a mean freeze-out (proper)time $\tau_{0}$ and a duration parameter $\Delta \tau$, the variance of the freeze-out propertime distribution. The following emission function $S_{c}(x, p)$ applies:

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$$
\begin{align*}
S_{c}(x, p) d^{4} x & =\frac{g}{(2 \pi)^{3}} \frac{p^{\mu} d^{4} \Sigma_{\mu}(x)}{\exp \left(\frac{u^{\mu}(x) p_{\mu}}{T(x)}-\frac{\mu(x)}{T(x)}\right)+s}  \tag{1}\\
p^{\mu} d^{4} \Sigma_{\mu}(x) & =m_{t} \cosh [\eta-y] H(\tau) d \tau \tau_{0} d \eta d r_{x} d r_{y},  \tag{2}\\
u^{\mu}(x) & =\left(\cosh [\eta] \cosh \left[\eta_{t}\right], \sinh \left[\eta_{t}\right] r_{x} / r_{t}, \sinh \left[\eta_{t}\right] r_{y} / r_{t}, \sinh [\eta] \cosh \left[\eta_{t}\right]\right),  \tag{3}\\
\frac{\mu(x)}{T(x)} & =\frac{\mu_{0}}{T_{0}}-\frac{r_{x}^{2}+r_{y}^{2}}{2 R_{G}^{2}}-\frac{\left(\eta-y_{0}\right)^{2}}{2 \Delta \eta^{2}}, \quad \frac{T_{0}-T_{r}}{T_{r}}=\left\langle\frac{\Delta T}{T}\right\rangle_{r}, \quad \frac{T_{0}-T_{t}}{T_{t}}=\left\langle\frac{\Delta T}{T}\right\rangle_{t} \tag{4}
\end{align*}
$$
\]

where $\mu(x)$ is the chemical potential and $T(x)$ is the local temperature, the subscript ${ }_{c}$ refers to the core of collision (surrounded by a halo of long lived resonances), the transverse mass is $m_{t}=\sqrt{m^{2}+p_{x}^{2}+p_{y}^{2}}$, the rapidity $y$ and the space-time rapidity $\eta$ are defined as $y=0.5 \log \left[\left(E+p_{z}\right) /\left(E-p_{z}\right)\right]$ and $\eta=0.5 \log [(t+z) /(t-z)], R_{G}$ stands for the transverse geometrical radius of the source, $\sinh \left[\eta_{t}\right]=\left\langle u_{t}\right\rangle \frac{r_{t}}{R_{G}}$ with $r_{t}=\sqrt{r_{x}^{2}+r_{y}^{2}}$. The central temperature at mean freeze-out time is denoted by $T_{0}=T\left(r_{x}=r_{y}=0 ; \tau=\tau_{0}\right)$. With the surface temperature $T_{r}=T\left(r_{x}=r_{y}=R_{G}, \tau=\tau_{0}\right)$ and the temperature after freeze-out, $T_{t}=T\left(r_{x}=r_{y}=0 ; \tau=\tau_{0}+\sqrt{2} \Delta \tau\right)$, the relative transverse and temporal temperature decrease are introduced, see refs. [1, [2, [] for further details.

### 2.1. Single particle spectra and two particle correlations

The total invariant single particle spectrum in $y$ rapidity and transverse mass $m_{t}$ is determined analytically using a saddle-point approximation,
$\frac{d^{2} n}{2 \pi m_{t} d m_{t} d y}=\frac{g}{(2 \pi)^{3}} \bar{E} \bar{V} \bar{C} \frac{1}{\exp \left(\frac{u^{\mu}(\bar{x}) p_{\mu}}{T(\bar{x})}-\frac{\mu(\bar{x})}{T(\bar{x})}\right)+s}$,
where $\bar{E}$ stands for the average energy, $\bar{V}$ for the average volume of the effective source of particles with a given momentum $p$. The correction factor $\bar{C}$ includes the effective intercept parameter $\lambda_{*}\left(y, m_{t}\right)$ of the two particle (or Bose-Einstein) correlation function, that controls the core ratio in the particle production in the core/halo picture. In Ref. [1] , a Boltzmann approximation $(s=0)$ to the above Invariant Momentum Distribution (IMD) as well as to the Bose-Einstein correlation function (BECF) was obtained in an analytic way. The analytical formulas for the BECF and IMD, as were used in the fits, have been summarized in refs. [1,2,2,4]. Bose-Einstein or Fermi-Dirac statistics $(s= \pm 1)$ was used in the analytic expressions fitted to single particle spectra.

## 3. Fitting NA49, NA44 and WA98 $\mathrm{Pb}+\mathrm{Pb}$ data

The kinematic parameters of the Buda-Lund model are fitted simultaneously to IMD and HBT radii measured by the CERN NA49, NA44 and WA98 experiments in central $\mathrm{Pb}+\mathrm{Pb}$ collisions at 158 AGeV . Core/halo correction $\propto 1 / \sqrt{\lambda_{*}}$ is applied and the corresponding errors are propagated properly. Due to the these conditions unique minima are
found, a good $\chi^{2} / N D F$ is obtained for all reactions, and the strongly coupled, normalization sensitive $\left\langle\frac{\Delta T}{T}\right\rangle_{t}$ and $\Delta \tau$ parameters are determined. On Figure 1 and 2, the fits to measured data are shown together with the published data. Note that the first 5 points of the NA44 pion spectrum were contaminated (9) and were not included in the fit.


Figure 1. Simultaneous fits to NA49 particle spectra and HBT radius parameters.

Fits to preliminary data of WA98 experiment provide source parameter values and errors similar to those obtained by NA49 and NA44. The normalizations of the NA44 pion spectrum and the WA98 $h^{-}$spectrum had to be fixed manually to that of NA49.

The hypothesis that pions, kaons and protons are emitted from the same hydrodynamical source is in a good agreement with all the fitted data. The fit parameters are summarized in Table 1, shown with statistical errors, only. The reconsructed space-time emission function $S(x)$ (which is the source function $S(x, p)$ integrated over the momentum $p$ ) is shown on Figure 3 and 4.

## 4. Conclusions

We find that the NA49, NA44 and WA98 data on single particle spectra of $h^{-}$, identified $\pi, K$ and $p$ as well as detailed rapidity and $m_{t}$ dependent HBT radius parameters are consistent with each other. The final state of central $\mathrm{Pb}+\mathrm{Pb}$ collisions at CERN SPS corresponds to a cylindrically symmetric, large ( $R_{G}=7.1 \pm 0.2 \mathrm{fm}$ ) and homogenous ( $T_{0}=139 \pm 6 \mathrm{MeV}$ ) fireball, expanding three-dimensionally with $\left\langle u_{t}\right\rangle=0.55 \pm 0.06$. A large mean freeze-out time, $\tau_{0}=5.9 \pm 0.6$ is found with a short duration of emission.

Table 1
Source parameters from simultaneous fitting of NA49, NA44 and preliminary WA98 particle spectra and HBT radius parameters with the Buda - Lund hydrodynamical model.

|  | NA49 |  | NA44 |  | WA98 |  | Averaged |  |
| :--- | ---: | :--- | ---: | :--- | ---: | :--- | ---: | :--- |
| Parameter | Value | Error | Value | Error | Value | Error | Value | Error |
| $T_{0}[\mathrm{MeV}]$ | 134 | $\pm 3$ | 145 | $\pm 3$ | 139 | $\pm 5$ | 139 | $\pm 6$ |
| $\left\langle u_{t}\right\rangle$ | 0.61 | $\pm 0.05$ | 0.57 | $\pm 0.12$ | 0.50 | $\pm 0.09$ | 0.55 | $\pm 0.06$ |
| $R_{G}[\mathrm{fm}]$ | 7.3 | $\pm 0.3$ | 6.9 | $\pm 1.1$ | 6.9 | $\pm 0.4$ | 7.1 | $\pm 0.2$ |
| $\tau_{0}[\mathrm{fm} / \mathrm{c}]$ | 6.1 | $\pm 0.2$ | 6.1 | $\pm 0.9$ | 5.2 | $\pm 0.3$ | 5.9 | $\pm 0.6$ |
| $\Delta \tau[\mathrm{fm} / \mathrm{c}]$ | 2.8 | $\pm 0.4$ | 0.01 | $\pm 2.2$ | 2.0 | $\pm 1.9$ | 1.6 | $\pm 1.5$ |
| $\Delta \eta$ | 2.1 | $\pm 0.2$ | 2.4 | $\pm 1.6$ | 1.7 | $\pm 0.1$ | 2.1 | $\pm 0.4$ |
| $\left\langle\frac{\Delta T}{T}\right\rangle_{r}$ | 0.07 | $\pm 0.02$ | 0.08 | $\pm 0.08$ | 0.01 | $\pm 0.02$ | 0.06 | $\pm 0.05$ |
| $\left\langle\frac{\Delta T}{T}\right\rangle_{t}$ | 0.16 | $\pm 0.05$ | 0.87 | $\pm 0.72$ | 0.74 | $\pm 0.08$ | 0.59 | $\pm 0.38$ |
| $\chi^{2} / N D F$ | $\mid 63 / 98=1.66$ |  | $63 / 71=0.89$ | $115 / 108=1.06$ | 1.20 |  |  |  |



Figure 3. The reconstructed source function $S(t, z, x=0, y=0)$ in the $(t, z)$ plane.


Figure 4. The reconstructed source function $S(x, y)$ at the mean freeze-out time.

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