UNIVERSITY
of
GLASGOW

Difficulties in Understanding Mathematics

An Approach Related to Working Memory and Field Dependency

by

Onyebuchi Onwumere

M.Sc.(Hons), Mathematics, University of Jos, Nigeria

A thesis submitted in fulfilment of the requirements
for the degree of
Doctor of Philosophy (PhD)

Centre for Science Education, Faculty of Education
University of Glasgow

Abstract

It is commonly agreed that learning with understanding is more desirable than learning by rote. Understanding is described in terms of the way information is represented and structured in the memory. A mathematical idea or procedure or fact is understood if it is a part of an internal network, and the degree of understanding is determined by the number and the strength of the connections between ideas. When a student learns a piece of mathematical knowledge without making connections with items in his or her existing networks of internal knowledge, he or she is learning without understanding.

Learning with understanding has progressively been elevated to one of the most important goals for all learners in all subjects. However, the realisation of this goal has been problematic, especially in the domain of mathematics where there are marked difficulties in learning and understanding. The experience of working with learners who do not do well in mathematics suggests that much of the problem is that learners are required to spend so much time in mathematics lessons engaged in tasks which seek to give them competence in mathematical procedures. This leaves inadequate time for gaining understanding or seeking how the procedures can be applied in life.

Much of the satisfaction inherent in learning is that of understanding: making connections, relating the symbols of mathematics to real situations, seeing how things fit together, and articulating the patterns and relationships which are fundamental to our number system and number operations. Other factors include attitudes towards mathematics, working memory capacity, extent of field dependency, curriculum approaches, the classroom climate and assessment. In this study, attitudes, working memory capacity and extent of field dependency will be considered. The work will be underpinned by an information processing model for learning.

A mathematics curriculum framework released by the US National Council of Teachers of Mathematics (NCTM, 2000) offers a research-based description of what is involved for students to learn mathematics *with* understanding. The approach is based on *"how learners learn,* not on *"how to teach"*, and it should enable mathematics teachers to see mathematics from the standpoint of the learner as he progresses through the various stages of cognitive development.

The focus in the present study is to try to find out what aspects of the process of teaching and learning seem to be important in enabling students to grow, develop and achieve. The attention here is on the learner and the nature of the learning process.

What is known about learning and memory is reviewed while the literature on specific areas of difficulty in learning mathematics is summarised. Some likely explanations for these difficulties are discussed. Attitudes and how they are measured are then discussed and there is a brief section of learner characteristics, with special emphasis on field dependency as this characteristic seems to be of importance in learning mathematics. The study is set in schools in Nigeria and England but the aim is not to make comparisons.

Several types of measurement are made with students: working memory capacity and extent of field dependency are measured using well-established tests (digit span backward test and the hidden figure test). Performance in mathematics is obtained from tests and examinations used in the various schools, standardised as appropriate. Surveys and interviews are also used to probe perceptions, attitudes and aspects of difficulties. Throughout, large samples were employed in the data collection with the overall aim of obtaining a clear picture about the nature and the influence of attitudes, working memory capacity and extent of field dependency in relation to learning, and to see how this was related to mathematics achievement as measured by formal examination.

The study starts by focussing on gaining an overview of the nature of the problems and relating these to student perception and attitudes as well as working memory capacity. At that stage, the focus moves more towards extent of field dependency, seen as one way by which the fixed and limited working memory capacity can be used more efficiently. Data analysis was in form of comparison and correlation although there are also much descriptive data.

Some very clear patterns and trends were observable. Students are consistently positive towards the more cognitive elements of attitude to mathematics (mathematics is important; lessons are essential). However, they are more negative towards the more affective elements like enjoyment, satisfaction and interest. Thus, they are very realistic about the value of mathematics but find their experiences of learning it much more daunting. Attitudes towards the learning of mathematics change with age. As students grow older, the belief that mathematics is interesting and relevant to them is weakened, although many still think positively about the importance of mathematics.

Loss of interest in mathematics may well be related to an inability to grasp what is required and the oft-stated problem that it is difficult trying to take in too much information and selecting what is important. These and other features probably relate to working memory overload, with field dependency skills area being important. The study identified clearly the topics which were perceived as most difficult at various ages. These topics involved ideas and concepts where many things had to be handled cognitively at the same time, thus placing high demands on the limited working memory capacity.

As expected, working memory capacity and mathematics achievement relate strongly while extent of field dependency also relates strongly to performance. Performance in mathematics is best for those who are more field-independent. It was found that extent of field dependency grew with age. Thus, as students grow older (at least between 12 and about 17), they tend to become more field-independent. It was also found that girls tend to be more field-independent than boys, perhaps reflecting maturity or their greater commitment and attention to details to undertake their work with care during the years of adolescence.

The outcomes of the findings are interpreted in terms of an information processing model. It is argued that curriculum design, teaching approaches and assessment which are consistent with the known limitations of the working memory must be considered during the learning process. There is also discussion of the importance of learning for understanding and the problem of seeking to achieve this while gaining mastery in procedural skills in the light of limited working memory capacity. It is also argued that positive attitudes towards the learning in mathematics must not only be related to the problem of limited working memory capacity but also to ways to develop increased field independence as well as seeing mathematics as a subject to be understood and capable of being applied usefully.

Acknowledgements

My deepest thanks and gratitude goes to Jehovah God for providing me the opportunity, means and perseverance for the successful completion of this study.

I am extremely grateful to my supervisor, Professor Norman Reid, for all his patience, encouragement and support sometimes at his inconvenience.

I would also like to express my thanks to Professor Rex Whitehead for carefully reading the manuscript and offering helpful suggestions; also to Dr Mark Williams for his great deal of interest and specialist advice.

I owe debts and gratitude to my mother, Mrs Oyidie Onwumere; my elder sister, Aka Ego; my brothers, Raphael, Ndubuisi and Francis Onwumere; and my brother-in-law, Ifeanyi Mbonu for their great deal of help, support and inspiration.

I must not forget my friends, John Brooks, Peter Saaremets, Mawuli Tete, Colins Ukomadu and Chief Chuka for their helpful support.

Finally, my thanks goes to my loving wife, Blessing; my daughter, Victoria; and my son, Chigozie for their wonderful support and encouragement, who kept asking "Daddy have you finished?", "When are you finishing?", "Can we play now?", especially when I tried to ignore them due to academic pressure.

Table of Contents

List of Tables

List of Figures

Chapter One

Learning through Understanding

1.1 Introduction

Every educational activity has objectives of achieving learning outcomes. More than any other species, humans are designed to be flexible learners and, from infancy, are active agents in acquiring knowledge and skills. Donovan and Branford (2005) point out that people can invent, record, accumulate, and pass on organised bodies of knowledge that helps them understand, shape, exploit, and ornament their environment. Much that each human being knows about the world is acquired informally, but mastery of the accumulated knowledge of generations requires intentional learning, often accomplished in a formal educational setting.

Petty (2004) has offered a list of objectives for formal educational learning with the hope of focussing learning goals more clearly. Indeed, what objectives teachers select during the learning process are very important because all teaching processes depend on the objectives. Based on his list, teachers and other educators want learners to:

- Acquire new knowledge and skills;
- Develop ideas;
- Increase their understanding;
- Apply intellectual, physical and creative effort to their work;
- Think and learn for themselves;
- Understand what they are doing, how well they have done and how they can improve.

These expectations are very important because they not only highlight the importance of learning processes but they also focus on the learner's strengths and weaknesses. Decades of work in the cognitive and development sciences have provided the foundation for an emerging science of learning. This foundation offers conceptions of learning processes and the development of competent performance that can help teachers and other educators support their students in the acquisition of knowledge that is the province of formal education.

Hiebert and Carpenter's (1992) definition of mathematical understanding in terms of how knowledge is structured emphasises the need for meaningful curriculum and mathematics knowledge: problem-solving, exploring patterns, making, testing, evaluating conjectures, and developing mathematically sound arguments for or against mathematics statements.

According to them:

> *"Because the goal of mathematics education should be the development of understanding by all students, the majority of the curriculum should be composed of tasks that provide students with problem situations. Two reasons support this claim. The first is the mathematics that is worth learning is most closely represented in problem solving tasks. The second is that students are more apt to engage in the mental activities required to develop understanding when they are confronted with mathematics embedded in problem situations".* (Page 187)

Mathematics education is not just the acquisition of abstract skills. Acquired mathematical skills are only meaningful if they prove to be efficient and reliable in solving problems that have been identified as being important practically (they need to be solved frequently) or theoretically (their solution allows a new understanding of the related conceptual domain). Underpinning this is the finding that learners seem naturally to be trying to make sense of what they experience. This is part of the constructivist paradigm and is the natural way by which learners relate to the world of learning (Piaget, 1973; Confrey, 1994; Gagnon and Collay, 2001). It is, therefore, important, that studies in mathematics allow the learners time and opportunity to seek to develop their understandings of the processes and procedures which are being taught.

Learning with understanding supports the creation of 'autonomous' learners (learners who can take control of their learning by defining their goals and monitoring their progress). The idea of autonomy goes back at least to the work of Piaget (1973) who proposed that the main goal of education should be the cultivation of learner autonomy. The social perspective to learning advances the idea that learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures and learn to evaluate their own thinking and that of others. Some examples of this kind of classroom environments can be found in several research reports (e.g. Lampert, 1990; Yackel and Cobb, 1996; Ball and Bass, 2003).

It is easy to state that learning mathematics with understanding is an important instructional goal for all students. It is very much harder to translate that goal into practice in the classroom. It is not easy for learners to cope with the demands of mastering mathematical procedures as well as trying to understand what is going on. It is even more difficult to add on examples to illustrate how the mathematical procedures can be applied in real-life situations. The hard-pressed teacher is also faced with the task of enabling as many students as possible to pass the examinations and gain good grades. Inevitably, all of this ensures that the emphasis is on the mastery of mathematical procedures to gain 'right' answers.

Many studies have explored the need for mathematical activity in the classroom, the decisions teachers take about what mathematics tasks to implement and how they are mediated through the curriculum materials they use (Porter, 1989; Romberg, 1992; Schmidt *et al.,* 1997; Nathan *et al.,* 2002; Zaslavsky, 2005). In all this, the aim is to emphasise meaningful learning. Such learning can be seen as reflecting the need for learners to understand sufficiently, to be able to use what they have learned, to be able to make sense of the procedures they have been taught and gain some insights into how they relate to real-life situations. Developing a suitable curriculum to achieve such goals is extremely difficult. Indeed, there needs to be research to explore the extent to which such goals are attainable at all.

Like all learning, learning in mathematics is a growth process. To facilitate the understanding of mathematical procedures in a variety of meaningful situations, it might be argued that,-

(1) Learning experiences should be purposeful and realistic.
(2) The discovery of facts, meanings and procedures should lead to insight and understanding.
(3) New material to be learned needs to be linked overtly into what is already known to give a more enriched understanding.
(4) Learning experiences and instructional materials should take account of the learner’s level of development.
(5) It is important that the learner understands the role of practice in developing competence in skills-based subject like mathematics.

Overall, learning is not just the transferring of knowledge from the teacher to the learner. It is an understanding process where relatively permanent changes are created as information is processed and experience is gained, influencing the prospects of progress and success. These changes do not solely refer to outcomes of learners’ behaviour that are manifestly observable, but also to attitudes, feelings and intellectual processes that may not be so obvious (Atkinson *et al.,* 1993; Hamachek, 1995).

1.2 Cognitive and Affective Domains of Educational Objectives

In the 1960s, there was a shift of emphasis away from subject content as a dominant factor, to consider modes of thought, and definitions of skills. This gave rise to the trend towards writing objectives (Mager, 1962). In 1956, Bloom *et al.,* published their “*taxonomy of educational objectives*” (a taxonomy comprises groups of objects of study sorted according to their similarities and differences, see: Simpson, 1966; Bowler; 1992; and Moseley *et al.,* 2005). According to them, *objectives* are “*explicit formulations of the ways in which learners are expected to be changed by the educative process*”.

This has now moved on to encompass learning outcomes (Smith, 2007). This taxonomy offered a language which was readily understood by practitioners and those designing assessments. In essence, it only involved the *cognitive*: that area of mental or intellectual activity involving remembering, thinking, problem-solving, logical argument, decision making, creativity, etc. The *affective* was largely ignored at that time although attitudinal aims started to appear in many curriculum specifications (Leder and Grootenboer, 2005).

This neglect is easy to understand, given the difficulties researchers face in this new field: theories not yet well-developed; terminology used differently and ambiguously; and varying research instruments, some untested, making the literature difficult to interpret and leaving the researchers open to criticism. Research findings also vary widely, especially with correlations between affective factors and performance (Goos *et al.*, 2008). All this makes affective assessment difficult. In addition, only what is examinable tends to be taught in the school situation.

A problem was identified in the Bloom taxonomy by Yang (2000) when she appreciated that the six cognitive skills listed by Bloom *et al.*, were almost certainly not hierarchical. She proposed an alternative model (Figure 1.1).

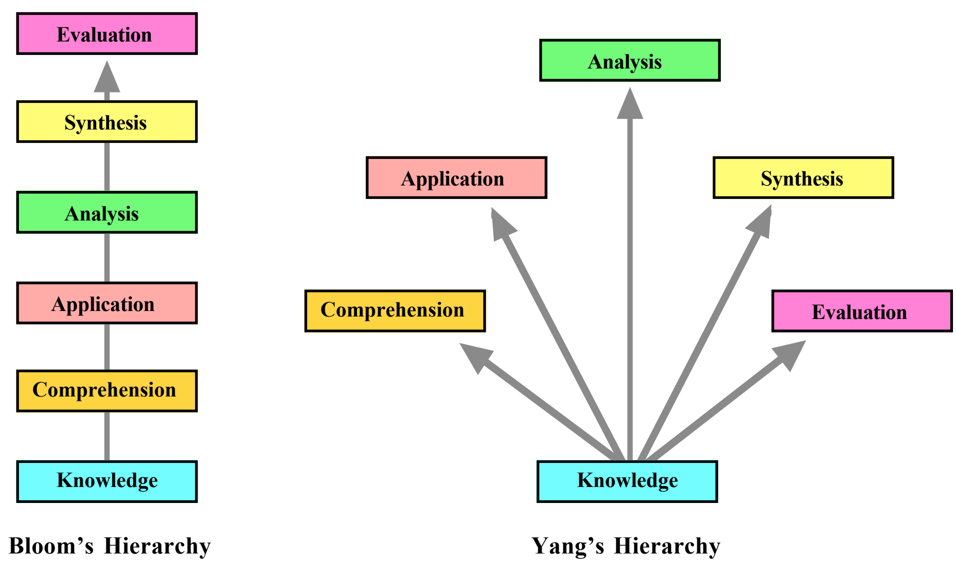


Figure 1.1 Hierarchies of Cognitive Skills

The key difference in the two hierarchies is that Yang's model does not assume that evaluation builds on an ability to synthesise and that synthesis builds on an ability to analyse and so on. She only assumes that the five skills depend on knowing something or having access to that knowledge. However, the taxonomy promotes the use of clear statements of educational objectives, even though a term like '*analysis*' may mean different things in different contexts. Potentially, it is applicable in all contexts of teaching and learning.

In mathematics education, examinations and tests check what learners have acquired in terms of knowledge, understanding and, sometimes, thinking skills. Decisions about assessments and future learning are based on cognitive skills: knowledge, comprehension, application, analysis, evaluation and synthesis. However, attitudinal objectives are general statements of syllabus philosophy that stress the importance of what students bring to the learning situation or derive from the learning situation - sets of attitudes, perspectives, values and beliefs. These elements of affective abilities are rarely measured at all; indeed, measurement may be impossible or inappropriate.

However, that does not mean that such aspects are unimportant. It is both timely and imperative to explore the potential of self-beliefs and attitudes to inform educational planning and practice. It is known that self-concepts and related attitudes are important influences in the learning choices students make, and play a role in learning behaviour and performance. For example, in a study of the factors that influenced more than 500 first-year Australian students, Cretchley *et al.,* (2000) found that self-beliefs about mathematics ability were a major influence behind their choice to study mathematics at university. Given the need to attract learners to mathematics, the legacies of low self-esteem and low interest in mathematics are serious.

In almost all countries, there is a view that mathematics is a 'difficult' subject and that students do not often have a positive attitude towards it. Educational research in a wide variety of contexts indicates considerable failure of students to perform in mathematics (e.g. Haylock, 1991; Schoenfeld 1994; Christou, 2001; Al-Enezi, 2006; Ali, 2008). The fundamental questions that arise are: why is mathematics perceived in this way? why does this subject cause so much anxiety and unease? what roles do cognitive and affective abilities play in the learning of mathematics? How can learners be helped?

The work of this thesis began with these questions and explores aspects of the learning process in an attempt to offer some useful insights. If learners do have adequate cognitive skills and positive attitudes, then it is likely that the learning of mathematics will be successful or fulfilling.

The technological and scientific revolution which is taking place today makes it imperative that the schools give added emphasis to the development of a learner's understanding and appreciation of mathematical procedures and methods of reasoning in that these depend on mathematics. The knowledge and understanding which are imparted to learners need to be carried out in such a way that the expert knowledge can be built upon to move culture forward, to solve new problems and to take meaningful decisions as citizens in a vast variety of applications.

1.3 Overview of the Study

School systems have always regarded understanding as a crucial component of classroom instruction. The problem is that learning has been determined by the subject matter to be taught and its logical ideas, neglecting the needs of the learner and the way his mind and mental processes work when trying to make sense of what is being taught. Consequently, learners often experience significant problems during learning. Johnstone (2000) considers this in relation to chemistry but the principles might widely apply in mathematics. Research indicates that there are a number of reasons for this difficulty (Reid, 1978; Schminke *et al.*, 1978; Smith, 2007; Hindal, 2007). Figure 1.2 seeks to summarise some of the main findings related to problems of understanding mathematics or possible sources of the problems.

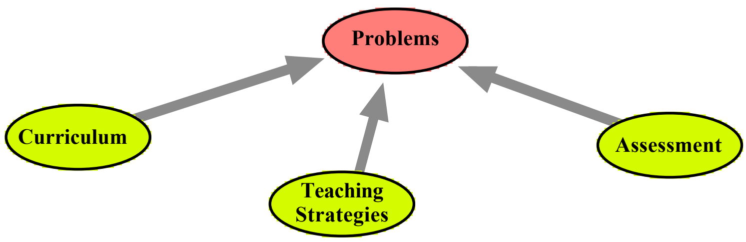


Figure 1.2 Sources of Mathematics Difficulty

Within the school curriculum (and of course the university), learning mathematics is uniquely challenging in that it is highly organised, sequential and progressive. Simpler elements must be learned successfully before moving onto others. Chin and Ashcroft (1998, page 4) note that it is a subject where one learns the parts; the parts build on each other to make a whole; knowing the whole enables one to reflect with more understanding on the parts, which in turn strengthens the whole; knowing the whole also enables an understanding of the sequences and interactions of the parts and the way they support each other so that the destination clarifies the stages of the journey. Because of the interrelating nature of the subject, "*learners who have learning problems in mathematics may sometimes appear to feel even more lost and disempowered than those who encounter problems in other subjects*" (Frederickson and Cline, 2009).

The curriculum framework and the sequence of topics to be taught may be inappropriate for a specific age and for a particular ability. Teachers are often directed in the framework to the activities and the textbooks they have to employ, and sometimes the direction may be unhelpful. All of this can result in a material being included which is likely to affect students' understanding. Mathematics may itself be intrinsically difficult and the approaches laid down in the curriculum may fail to take into account the way learners actual learn in highly conceptual areas. Thus, it is highly likely that the problem arises because of the very nature of mathematics.

Further problems can occur with assessment. Recent work by Hindal (2007) shows the devastating power of assessment in reducing almost everything to a recall and recognition measurement exercise. The problem is that national assessment is determined outside the school and the realities of learning situations are often not considered. Sometimes, assessment lays emphasis on what is easy to assess thereby focusing on memorisation and recall of information or procedures.

Another potential problem arises as a result of teaching strategy or teaching style. Every individual is unique and learns in a particular way. This implies that any teaching which does not take into account student limiting factors for learning rarely succeeds. Teachers who understand the learning needs of their students are more empowered to provide the kind of instruction their students need. Knowing why a student is struggling to learn provides a basis for understanding why particular strategy or approaches are effective for him or her.

The areas of mathematics difficulty described in this section are not the central focus of this study. Instead, the study seeks to direct its focus on the learner. The aim is to throw light on aspects of how the student learns as well as student attitudes. In this respect, all of the following questions impinge on the resolution of the problem:

(1) Why is mathematics sometimes difficult to understand?
(2) What aspects of school mathematics make learning difficult or sometimes impossible?
(3) How can mathematics be presented to meet each learner's interest and ability?
(4) What roles do attitudes and memory play in the learning and application of mathematics?
(5) Do the problems of learning and understanding mathematics lie with individual's characteristics ways of learning?

The study started with these questions. Although exploratory, the approaches to the questions can serve as a motivator for the learning of mathematics.

1.4 The Aims of this Research

The explanation of human memory has been called the "*supreme intellectual puzzle of the century*" (Anderson, 1995). This may seem something of an exaggeration, but at least for educational psychologists, memory is at the core of the teaching and learning process. Of considerable importance, learners bring to the learning situation sets of attitudes, perspectives and beliefs. Again, the educational journey may well cause attitudes, perspectives and beliefs to develop, sometimes not in expected ways. Most are rarely measured at all; indeed, measurement may be impossible or inappropriate (Oraif, 2006). However, that does not imply that such aspects are unimportant in the learning process.

The mathematics teacher's job satisfaction is likely to be strongly influenced by how learners are able to use their memory in cultivating understanding and what learners may bring to the learning situation such as self-belief (or lack of it). Personal memories, from past learning experiences and, perhaps, related experiences may contribute to the development of understanding and confidence. Such memories are stored in the long-term memory where they may influence the perception of incoming information, or may influence the processing of information during the learning process.

The study starts by exploring what is meant by memory and attitude, specifically in an educational setting, and then moves on to examine how what is held in the memory is acquired, used and processed. Memory is more than the place where we retain information. Memory involves understanding and the ability to think through issues and solve problems. However, if attitudes are negative towards some aspect of the understanding process, then learning may well be seriously hindered. As Cretchley *et al.*, (2000) noted,

> *"To make the learning of mathematics less frustrating, and less fearful, and more effective, further attention by both mathematics educators and researchers should be focussed on beliefs and attitudes students bring into the mathematics classrooms or develop during their educational experiences".*

Equally, Reid (2006) has noted the powerful effect that attitudes can have on subsequent learning. Thus, achievement in learning depends heavily on attitudes while successful achievement may contribute to the development of positive attitudes.

Of course, learners show quite diverse learning characteristics (sometimes known as learning styles). It is possible that these may be related to the development of certain attitudes while some characteristics may influence successful understanding quite markedly. When learners are positive and confident, they are perhaps willing to engage in the cognitive tasks so that they can launch into new areas of thought and enquiry, making themselves better equipped to develop their potentials in mathematics. It is unlikely that future learning will be too successful or fulfilling when the opposite is the case.

The aim of this study will be:

- To observe how student attitudes towards mathematics change with age (approximately ages 11-17);
- To identify aspects of the processes of teaching and learning mathematics that seem to be important in enabling learners to grow and develop;
- To investigate the relationship between cognitive and attitudinal aspect of learning mathematics, concentrating mainly on the influence of cognitive understanding and learning difficulty on attitudes to mathematics;
- To pinpoint negative factors which were influencing learners away from mathematics; and
- To identify some individual differences and their effects on learning and achievement.

The focus is to try to find out what aspects of the process of teaching and learning seem to be important in enabling students to grow, develop and achieve. The attention here is on the learner and the nature of the learning process.

1.5 The Structure of the Research

In the early 1990's, science educators (e.g. Johnstone, 1991) attempted to take into account the psychological models of learning and the cognitive structure of the learners. These approaches look at that part of the learner's brain where information is held, organised, shaped, and worked upon before it is stored and retrieved. This model of information processing has also been studied by many (e.g. Al-Naeme, 1988; Jung, 2005; Oraif, 2006). The common theme of the model evolves through the idea of how input information is stored and processed inside the human memory and how a response to this comes into existence. It attempts to analyse cognition as a set of steps in which an abstract entity called information is processed. The investigations have identified three types of memory: the sensory memory, the short-term memory (or working memory), and the long-term memory. The procedure of the acquisition, retention and manipulation of knowledge relies mainly on the action of these memories.

The issue of attitude to learning is also of direct concern to others in the science community (Ramsden, 1998), and mathematics has drawn considerable interest as well (Leder and Grootenboer, 2005). The interest in research related to attitudes developed from the 1920s. Since then, numerous research projects, both experimental and theoretical, have been generated to investigate and explain the nature of attitudes: ways they are formed, stored and retrieved, and the way they change and influence behaviour. Interest in attitudes has continued unabated since then and this can be explained because of the important functions attitudes were thought to serve and because of the presumed ability of attitudes to direct and predict behaviour.

Chen (2004, Page 6) noted that students' perception of science, and how they store information received, are affected by the way they are taught, how motivated they are, and the way they think to develop meaningful learning. Jung and Reid (2009) also brought together evidence which explored the relationships between attitudes and working memory (Reid, 2003; Jung, 2005). They noted the intriguing possibility that limited working memory space may influence the way learning is taking place (understanding or memorisation) and how the failure to be able to understand relates to attitude deterioration which may then be a very strong factor which will influence their choice to continue with the subject. The same might be true of mathematics. Thus, mathematics, as a subject, must be taught and represented to learners in a way which is accessible in terms of working memory limitations. This might assist in developing more secure understanding and, thus, the retention or enhancement of positive attitudes.

At the tertiary as well as the secondary level of mathematics education, the methods of instruction and of organising conventional examination questions highlight some defects which require re-examination and reinterpretation in correspondence with learners' limited working memory capability. Three essential themes come together in this study: mental capacity, learner characteristics, and attitudes, all of which can affect the performance of learners in mathematics examinations or during mathematics learning.

The study intends to utilise the discipline of psychology not as an end in itself, but as a map-reading, providing a route into mathematics education itself so that points of guidance become markers, which future research may erase, shift into new positions or delineate afresh.

This study has the following structure:

- Chapter two reviews some literature on memory and learning. In particular, the information processing model was examined in order to study the learning process. It gives insight into how a learner approaches, transforms, reduces, elaborates, encodes, stores, retrieves and uses information. The overload of student's working memory space is considered as the main factor causing learning difficulty and, in consequence, potential learning failure.

- Chapter three reviews studies on learning difficulties in mathematics in an attempt to establish the reasons that students find mathematics difficult to learn, with the likely explanation for the difficulties. This describes attempts to use educational models in solving the difficulties in order to explore the knowledge of how students learn. In many topics in mathematics, grasping a concept may depend on holding many ideas cognitively at the same time. This makes considerable demand on working memory space which may lead to difficulties.
- In Chapter four, attitudes related to learning mathematics are considered. Definitions of attitudes, attitude models, the process of attitude formation and change, and especially attitude development in the context of mathematics education are considered.
- Chapter five describes approaches to attitude measurement, the relationship between attitudes and behaviour, the methods of data collection and the interaction of the cognitive and attitude models in mathematics education. Several guiding principles are suggested in relation to development of desirable and well-informed attitudes connected with the school mathematics education.
- Chapter six reviews literature on what are often known as cognitive learning styles. These are perhaps better described as learner characteristics: the field-dependent/field-independent characteristic, creativity, convergency and divergency, and the interaction between cognitive style and memory processes.
- In Chapter seven, the aim is to undertake a broad exploration of learning experiences in mathematics at certain age groups or stages (primary, secondary and university) in the attitudinal and cognitive domains. The study uses a survey of students' perceptions, with samples of students drawn from Nigerian and English schools. In addition, the working memory capacity of students aged between 16 and 18 was measured and information was gained about their performance in mathematics examinations. The data are analysed to explore their self-perceptions related to their experiences in learning mathematics and how these connect with age and gender. Relationships between these self-perceptions, mathematics examination scores and measured working memory capacity are explored.
- While working memory may influence success in mathematics examinations, the working memory of a student is fixed genetically. However, the learner may use that space more or less efficiently. Part of the efficiency lies in the concept of field dependency. Chapter eight shows the results of Experiment Two on the aspects of field dependency (field-dependent, field-independent and field-intermediate), success in mathematics, age and gender. As access to schools in Nigeria was proving impossible, the sample was drawn from schools in England.

- In Chapter nine, the results of the experiment three - further exploration of field dependency, mathematics achievement and working memory - are discussed. Much of the previous work looked at large samples and studied relationships between measurements. Here, an attempt was made to gain an insight into the actual processes students use when solving mathematics problems to see how the extent of field dependency specifically influenced the way mathematics was approached by students. Many measurements are made but a small number of more detailed interviews and surveys were also employed, in relation to specific mathematics problems which they would face at this age.
- Finally, Chapter ten presents a general summary of the outcomes of the study. It draws conclusions and makes suggestions based on evidence from this study. The limitations of the study and further work to be done are also discussed.

As noted earlier, the research measurements were made in both Nigeria and England, and the next two sections offer a brief outline of the educational systems in these two countries.

1.6 Education in Nigeria

Nigeria currently operates the 6-3-3-4 system of education and there is also a pre-primary education for children aged between three and five years prior to their entering primary school. Table 1.1 gives the summary of the formal structure of the Nigeria system of education.

School	Age	Duration *(years)*	Curriculum stage
Pre-primary	3-5	2	Nursery/Foundation stage
Primary	6-12	6	First School Leaving Certificate
Junior secondary	12-15	3	Junior secondary School Certificate
Senior secondary	15-18	3	West African School Certificate/General
Tertiary	18+	4	Degree/Diploma/National Certificate of Education

Table 1.1 Structure of Education in Nigeria

The present study focusses on primary, secondary and tertiary schools in that samples for part of the study are drawn from these schools. At the end of the six years of primary school, pupils are expected to enrol in a secondary school of their choice after they have passed the National Common Entrance Examination. Government plans that secondary education should be of six year duration and be given in two stages: the junior secondary school (JSS) and the senior secondary school (SSS). The junior secondary is made up of JSS1, JSS2, and JSS3, while the senior secondary school consists of SS1, SS2, and SS3, -

each stage being of three year duration. At the end of the three years of junior secondary school (JSS3), the junior secondary certificate examination is taken. Those who passed the examination proceed to senior secondary at the same institution or an institution of their choice, whereas those who failed, enrol on to an apprenticeship system or other scheme for out- of-school vocational training. The senior secondary school examination is sat at the end of the SS3.

The curriculum is drawn by the Ministry of Education and most of the schools are co-educational and comprehensive in nature. The Implementation Committee for the National Policy on Education (1978-79:21) identified the following weaknesses with the current education system:

(a) *Human Resources* : There are major shortages of qualified teachers, often giving oversize classes. Some teachers in the secondary education do not have any teaching training when they enter the classroom.
(b) *Material Resources*: Schools, most of the time, do not have basic teaching equipment such as computer laboratories, libraries, reading rooms, and other technological equipment such as interactive white board in the teaching and learning procedure.
(c) *Financial Constraints*: These have sometimes meant teachers not being paid and ensuing strikes, with considerable educational disruption.
(d) *The examination system*: The national examinations at the end of junior secondary and senior secondary influence how students learn. In order to achieve greater marks, questions are designed to offer high rewards for those who can recall best, and education has often been reduced to an emphasis on verbatim recall. This has led to a pattern of learning which largely ignores understanding but is only a device for examination success.

Mathematics forms the bedrock of the current education system in Nigeria. It is compulsory at both the primary and secondary levels. Teaching mathematics takes considerable time, five hours per week in the primary and secondary schools irrespective of the students' choice of studies chosen. There is a strong tendency for the choice of other subjects to depend on ability in mathematics. Thus, choice of courses in the sciences and engineering (as well as medicine and related subjects at degree level) depends critically on ability in mathematics.

1.7 Education in England

This study was aimed to focus on mathematics education in Nigeria. However, after the first stage of data gathering, it became very clear that gaining access to schools and universities in Nigeria was becoming more or less impossible. The work then continued with reference to secondary schools in the North of England where access was easier due to personal contacts.

Education in England is distinct from that of Nigeria. School education in England is divided into primary, secondary, further and higher education and overseen by the Department for Children, Schools and Families; and the Department for Innovation, Universities and Skills. At local level, the local authorities take responsibility for implementing policy for public education and state schools. Full-time education is compulsory for all children aged between 5 and 16 (inclusive).

To ensure teaching standards are consistent, pupils in compulsory education follow the English national curriculum. Children aged five to sixteen in state-maintained schools must be taught the national curriculum which sets out:

- The subjects taught;
- The knowledge, skills and understanding required in each subject;
- Standards or attainment targets in each subject;
- How a child's progress is assessed and reported.

Within the framework of the national curriculum, schools are free to plan and organise teaching and learning in the way that meets the needs of their students. Table 1.2 describes the most common patterns for schooling in the state section in England.

Age	Year Group	Stage	Schools	Statutory Test	National Examination
4-5	Reception	Foundation	Infant		
5-6	1	Key Stage 1	Primary		
6-7	2			Yes	
7-8	3	Key Stage 2			
8-9	4				
9-10	5				
10-11	6			Yes	
11-12	7	Key Stage 3			
12-13	8		Secondary		
13-14	9			Yes	
14-15	10	Key Stage 4			
15-16	11				GCSE
16-17	12	Key Stage 5			
17-18	13				'A'Levels, GNVQs

Table 1.2 Key Stages of the English National Curriculum

Again, the present study was aimed to focus on primary and secondary schools as well as university mathematics students. Difficulties in access reduced this somewhat. Although schools in Nigeria and England are involved, there is no intention to compare the two systems in any way: each has different aims, objectives and challenges. However, the one important feature in both countries is that mathematics is a core subject.

The starting point in this study is an attempt to try to describe and define what is meant by memory and, specifically, memory in a context of learning without any attempt to conceptualise it in terms of brain location or brain processes.

Chapter Two

Learning and Memory

2.1 Introduction

Many of the important insights have arisen as a result of thinking about learning in terms of the way individuals make use of their *memory* in processing incoming information. Most of the information is received in formal learning by means of sound and sight. Some incoming information is held for a very short time in the memory, while others seem to be retained permanently. Indeed, most incoming information is discarded immediately (Slavin, 2000). Wingfield and Byrnes (1981) point out that no one has ever seen a memory and no one is likely to see one. In everyday conversation, memory is used to talk about things we do. We *remember* our first day in class, or *forget* where we park the car. We *recognise* an old school friend, or *recall* a pleasant lesson in class. All these things have to do with memory.

The term memory has to do with the capacities to save or retain information, to recall it when needed, to recognise its familiarity when seen or heard again, and to process information. This study is the attempt to describe how these capacities are exercised and why, in the case of forgetting, attempts to exercise these capacities may be frustrated.

However, memory is more than just recall and recognition. If information is to be useful, it has to be capable of being understood to the extent that it can be applied. Understanding has to take place in the brain and the principles which underpin successful understanding will also be discussed.

Learning and memory are obviously very closely related to each other. Learning depends on memory for its *permanence* and memory would have no *content* without learning. Hence, Gross (2005) defined memory as the retention of learning and experience. In the broadest sense, Blakemore (1988) says that learning is the acquisition of knowledge and memory is the storage of an internal representation of that knowledge. For example, when a person learns something, what is learned is stored in the memory, and also, a good performance in a test which requires the use of memory can be seen as demonstrating that some learning has occurred. Blakemore summed up the fundamental importance of memory like this:

> *"... without the capacity to remember and learn, it is difficult to imagine what life would look like, whether it could be called living at all. Without memory, we would be servants of the moment, with nothing but our innate reflexes to help us deal with the world. There could be no language, no art, no science, no culture. Civilisation itself is the distillation of human memory ..."* (Page 46)

Both learning and memory featured prominently in the early years of psychology as a science. James (1890), one of the pioneers of psychology, was arguably the first to make a formal distinction between *primary* and *secondary memory*, which correspond to *short-term* and *long-term memory* respectively. This distinction is central to Atkinson and Shiffrin's (1968, 1971) very influential *multi-store model.*

Several major accounts of memory have emerged from criticisms of the limitations of the multi-store model. These include Craik and Lockhart's (1972) *levels-of-processing approach*, Baddeley and Hitch's (1974) *working-memory model*, attempts to identify different types of long-term memory (e.g. Tulving, 1972), and information processing models (e.g. Johnstone, 1997). These models seek to account for experimental observations and offer a description of what is happening when learning is taking place as well as accounting for the limitations of learning.

2.2 Learning as Understanding

A fundamental goal of teaching is to advance learners' understanding. We expect learners to do more than simply accumulate information; we want them to develop ideas and achieve a grasp of the subject matter. Learning as understanding is a *sense-making* activity. Understanding develops as learners use what they already know (i.e., prior knowledge) to construct meaning out of new information.

However, the word *understanding* needs some clarification. If a person understands something fully, then it is possible for the person to use that knowledge and apply it in novel situations, with some prospect of success. As learners make sense of new information, their knowledge about the topic not only increases quantitatively, but changes qualitatively by becoming more differentiated and elaborated. The result is a representation or mental model that structures the conceptual knowledge. By contrast, rote learning is a process in which the person tries to copy new information into memory. Although, the individual may be able to replicate the material, he or she does not necessarily grasp the relationship among the ideas and facts. White (1998), referred to this as 'inert knowledge', information the person can recall but cannot use productively for other thinking or problem-solving.

While learning was seen very much as memorisation and subsequent accurate recall in the earlier part of the 20th century, Piaget's (1963) work demonstrated very clearly that children naturally are seeking to make sense of what they observe. Nonetheless, the prominence of recall still exists in most examinations even today.

Ausubel (Ausubel *et al*., 1978) offered some implicit insights into understanding when he observed that:

> *"If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly."*

He proposed that understanding is actually the assimilation (rather than the formation) of concepts: "*most of what anyone really knows consists of insights discovered by others that have been communicated to him or her in a meaningful fashion*" (Ausubel, 1978, page 530). It is, therefore, important, in Ausubel's view, for teachers to present new learning in such a way that learners can relate it to their existing knowledge, taking into account the complexity of the new learning and the cognitive development of the learners. He proposed six hierarchically-ordered categories in his analysis of learning to facilitate understanding:

- Representational learning
- Concept learning
- Propositional learning
- Application
- Problem-solving
- Creativity

The key thing for understanding is that new ideas need to be linked coherently and correctly to ideas already held in the long-term memory. Johnstone (1997) went on to describe learning as follows:

> *"Learning is the reconstruction of material, provided by the teacher, in the mind of the learner. It is an idiosyncratic reconstruction of what the learner understands, or thinks she understands of the new material provided, tempered by the existing knowledge, beliefs, biases, and misunderstandings in the mind of the learner."*

He suggested the following four ways of storing information in the long-term memory:

"(1) The new knowledge finds a good fit to existing knowledge and is merged to enrich the existing knowledge and understanding (correctly filed).

(2) The new knowledge seems to find a good fit (or at least a reasonable fit) with existing knowledge and is attached and stored, but this may, in fact, be a misfit (a misfiling).

(3) Storage can often have a linear sequence built into it, and that may be the sequence in which things are taught.

(4) The last type of memorisation is that which occurs when the learner can find no connection on which to attach the new knowledge."

The first way reflects meaningful learning or understanding because new knowledge is linked to appropriate old knowledge and understanding; and it is very easy to retrieve and almost never lost. Conversely, the last type is labelled '*rote learning*', in which there is no interaction between new knowledge and previous knowledge. Such knowledge is very easily lost and very difficult to retrieve. When new knowledge is linked incorrectly to previously held ideas, this can lead to misconceptions which are very persistent and very difficult to change. The third type, linear learning is associated with memorising something like the alphabet and it can be accessed in only one way. According to Danili (2001), this kind of learning is useful in some cases although it is often slow and needs a lot of effort.

Gardner (1998) indicated that many students acquire little more than passing familiarity with the subjects we teach. He concluded that:

> *"An ordinary degree of understanding is routinely missing in many, if not most students. It is reasonable to expect a college student to be able to apply in a new context a law of physics, or a proof in geometry, or a concept in history of which he just demonstrated mastery in his class. If, when the circumstances of testing are slightly altered, the sought-after competence can no longer be documented, then understanding - in any reasonable sense of the term - has simply not been achieved."*

Johnstone (1997) and Reid (2008) note that deep understanding is not an automatic consequence of being taught or trying to learn. Attaining deep understanding of a subject is not simply a matter of paying attention in class and studying hard. Students can devote many hours to learning and come away with relatively little understanding of the subject.

Understanding depends heavily on experience and may well influence future experiences, success and confidence in some situations. For Gagné (1985, page 15), understanding consists of trainable intellectual skills and a strategic thinking capability that can only evolve as a function of experience and ability. Essentially, he subscribes to an information processing model of learning, emphasising the mastery that can be achieved through understanding. He believes that a better understanding of how learning operates will facilitate planning for learning, managing learning and instructing. His work has its roots in a behaviourist model, which he subsequently revised to address cognitive aspects of problem-solving. His insights are important but they were developed in the context of training rather than school or university education.

2.3 Developing Ideas

There have been many attempts to describe the human learning process. Each looks at learning from a different point of view; they supplement rather than contradict each other and often overlap in practice. The cognitivists, led by Piaget (1896-1980) looked at the connections between age and mental processes involved during learning. These mental events are concerned with how we obtain, process, and use information. The complexity of human thinking, memory, problem solving, decision making, and creativity are all cognitive activities.

Based on extensive interviewing and discussion, Piaget (1963) showed that children *construct* their own knowledge through different stages (sensorimotor, pre-operational, concrete operational and formal operational) in the same order but not at the same rate. His findings are summarised in Figure 2.1

Stages of Intellectual Development	Description
Sensorimotor *(birth to 2 years)*	Differentiates self from objects Recognises self as agent of action and begins to act Achieves object permanence, realising that things exist even when no longer present to the senses
Pre-operational *(2 to 7 years)*	Learns to represent objects by images and words Language facility and grammar expand enormously Classifies objects by a single feature eg colour or height
Concrete operational *(8-11 years)*	Can think logically about objects and events Achieves conservation of number (age 6), mass (age 7) and weight (age 9) Can classify objects according to several features and can order them in series along a single dimension
Formal Operational *(11 years onwards)*	Can think logically about abstract propositions Can test hypotheses systematically Becomes concerned with the hypothetical, the future, and ideological problems

Figure 2.1 Summary of Developmental Stages (from Piaget, 1963)

Piaget's major cognitive developmental questions were:

- What is children's thinking like at various points throughout development?
- How does this development come about?

The process of development begins at birth and culminates in adolescence. Among the four stages of cognitive development listed, the last two are significant for pupils in secondary school in that they are likely to be operating cognitively at concrete operational and formal operational levels of development. Hence, the handling of abstract ideas and the ability to establish alternative ways of looking at information may not be well developed. Indeed, there is evidence that first year university students are not necessarily operating completely at the formal stage although they might be capable of it (Herron, 1975).

While this is strongly supported as an insight into how understanding takes place, the constructivist approach has not been found to offer the key insights to aid consistent improved learning. Solomon (1994) raised some doubts many years ago while the more recent paper by Kirschner *et al.,* (2006) largely demolishes the usefulness of this (and some other) approaches.

Piaget's work has been criticised in the following areas:

- He did not use sufficiently large samples and he did not pay enough attention to statistical significance (Ausubel *et al.*, 1978).
- The boundaries of his stage development theory are too rigid (Ausubel *et al.*, 1978).
- He underestimated the significant role of social interaction and language in child development. He believed that the developmental changes in the cognitive structure of the child produce the language development (Donaldson, 1987).

There is some validity in these criticisms. However, Piaget was an acute and accurate observer of child cognitive development. His model of stages is largely supported although his boundaries may be slightly too rigid. Nonetheless, language is an important issue. Indeed, the whole question of language and social relationships was underplayed by Piaget. The work of Vygotsky (1987) made a useful contribution here, showing that the child could accelerate to a small extent through affirmative contact with those who were slightly more cognitively advanced.

Overall, Piaget's work was used to establish the basis for much modern educational thought with profound impact on educational practice and research. His work was the theoretical justification for some of the curriculum reform movements of the 1960s and especially for disciplines like science and mathematics (Novak, 1978). Piaget is important in that his ideas are still used by teachers and other educators to create specific principles for teaching and learning. The key principle was that the learners should not be seen as passive recipients of external knowledge but, rather, as active constructors of their own knowledge. The teacher's role was to provide a context whereby the learners were challenged to engage with activities requiring adaptation that are appropriate to their developmental level.

Within the framework of cognitive psychology, Ausubel (1968) placed emphasis on the learner's prior knowledge and also insisted that every learner constructs his own understandings in his own way. His major contribution to learning process is the description of conditions and factors that lead to meaningful learning. From his research, Ausubel had grasped that the key thing in meaningful learning was the correct linking together of ideas and understandings in the learner's long-term memory. If meaningful learning can be considered in terms of the extent to which the person can apply the knowledge successfully in new situations, then being able to apply ideas depends on the availability of links between ideas held in the long-term memory. Open-ended problem solving is a situation where this application of understanding in novel situations is very apparent. In very recent work, Al-Qasmi (2006) demonstrated, with first year university biology students that their success on such problems depended very much on the availability and accessibility of links between ideas held in the long-term memory.

Novak (1978) noted that Ausubel's model deals with the following two main aspects of human conceptual functions during instructional presentation:

(1) The way a learner learns different kinds of information meaningfully from the verbal/textual presentation of the material to be learned in the classroom.

(2) The importance of prior knowledge already learned by the learner during the learning process.

The first dimension looks into the different ways information is made available to the learner (*reception or discovery*). The second dimension focusses on the degree of meaningfulness (*rote or meaningful*), by which the learner assimilates the information into his existing cognitive structure. Ausubel *et al.,* (1978) argues that the two dimensions are unrelated or orthogonal (Figure 2.2).

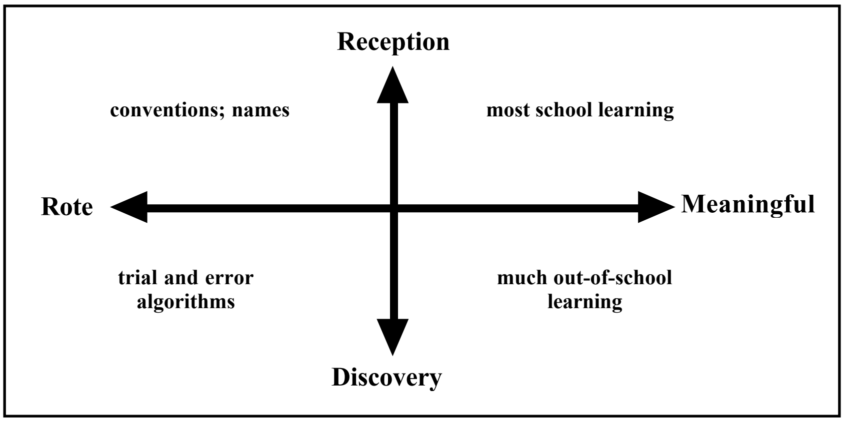


Figure 2.2 Two dimensions of Learning

The clarification of ideas illustrated in Figure 2.2 was a major contribution to the understanding of learning. For example, learning can be meaningful irrespective of whether it is teacher-directed reception learning or more learner-centred discovery learning. This important observation is often lost in debates today about student-centred learning which is often presented as some kind of major improvement on teacher-directed learning. Both have their place and both can lead to meaningful learning (Ausubel, 1968).

Ausubel argued that meaningful learning occurs when the learner makes a conscious effort to determine the key conceptual framework in knowledge which is related to existing concepts. The level to which the meaningful understanding can occur essentially depends on the quality and organisation of the existing prior knowledge. It depends also on how the new knowledge is linked to this existing prior knowledge. According to Ausubel, "*meaningful learning takes place if the learning task is related in a non arbitrary and non verbatim fashion to the learner's existing structure of knowledge." (Ausubel et al., 1968).* Various types of learning can be placed in this model (see Figure 2.3).

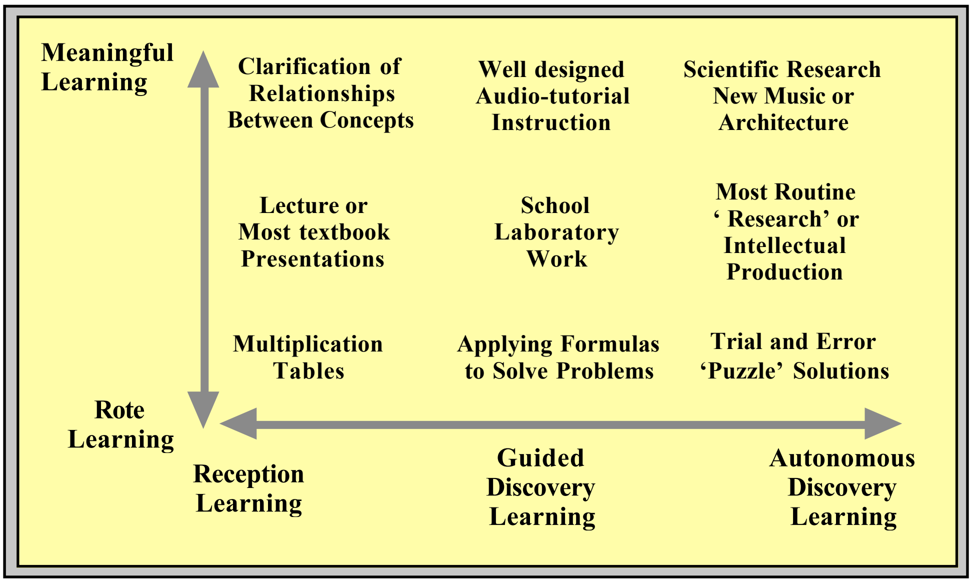


Figure 2.3 Reception Learning & Discovery Learning (Source: Novak, 1998, 2002)

Rote learning occurs when there are no relevant concepts available in the learner's cognitive structure to interact with the new knowledge or when new knowledge is not linked with previous ideas. The implication of this learning is that it results in arbitrary verbatim incorporation of new knowledge into the cognitive structure.

The distinction between meaningful and rote approaches to learning is particularly useful for teachers who want to understand their students' learning and create learning environments which encourage students to achieve desired learning outcomes. Prosser and

Trigwell (1999) offered the distinctions between the two types of learning, as follows.

When learners are engaged in meaningful learning, they:

"• *Develop understanding and make sense of what they are learning.*
- *Create meaning and make ideas their own.*
- *Focus on the meaning of what they are learning.*
- *Relate ideas together and make connections with previous experiences.*
- *Ask themselves questions about what they are learning, discuss their ideas with others and enjoy comparing different perspectives.*
- *Are likely to explore the subject beyond the immediate requirements.*
- *Are likely to have positive emotions about learning.*"

Conversely, when learners are engaged in rote learning activities they:

"• *Aim to reproduce information to meet external (assessment) demands.*
- *May aim to meet requirements minimally, and appear to be focussed on passing the assessment instead of (rather than as well as) learning.*
- *Focus on pieces of information in an atomistic way, rather than making connections between them and seeing the structure of what is being learned.*
- *Limit their study to the bare essentials.*
- *May concentrate on ways to reproduce the information."*

In general, Johnstone (1997) described meaningful learning as, "*...good, well-integrated, branched, retrievable and usable learning*." On the other hand, rote learning is "*at best, isolated and boxed learning that relates to nothing else in the mind of the learner*."

In many school (and, indeed, university) learning, the dominant feature is teacher-centred reception learning. In such learning, the teacher presents the material in a way and order that he/she thinks is suitable and the learners are encouraged to absorb and understand as much as they can. Ausubel was an advocate of this and has shown that meaningful learning can arise from this approach quiet successfully (Ausubel and Robinson, 1969).

Discovery learning tends to be a more learner-oriented process rather than a teacher based view of teaching and learning. To discover the main content, the learner is required to arrange, organise and construct the links between the new information and his existing knowledge with the help of procedural instructions provided by the teacher (Ausubel, 1968). One possible advantage of discovery learning over reception learning noted by Danili (2001) is that it motivates the learner to construct the information into a meaningful pattern rather than depending on the teacher entirely, and without the teacher being involved much in any mental activity. Langford (1989) argues that, through discovery learning, real knowledge can be acquired and such knowledge can be preserved much longer in the memory. However, discovery learning may make more demands on time. It is unlikely that learners can discover in an afternoon of activity what it took the best brains centuries to discover. There may have to be considerable teacher direction in the discovery process.

Ausubel (1968) argues that most learners learn primarily through reception learning rather than discovery learning although he does not underrate the usefulness of discovery learning. According to him, meaningful learning does not depend on the method of learning but on the way learning materials are constructed and presented. He insists that both reception and discovery learning can be classified to be either meaningful or rote learning depending on the result after the material to be learned is presented to the learner (Danili, 2001).

In general, the constructivism approach is basically a *description* of how children naturally develop mentally and learn. This approach, however, does not provide teachers with clear-cut instructions, or evidence about how teaching should be carried out. The danger inherent in the constructively theory, as argued by Danili (2001), is that it fails to come to grips with such issues as culture and power in the classroom which are thought to be very important to improve learning in the classroom because we learn by interaction, repetition, and correction of mistakes along with continuous support from adults to children. In addition, Kirschner *et al.,* (2006) argue strongly that constructivism as a model is quite inadequate in that it fails to appreciate the critical role of limited working memory capacity. This will be discussed later.

The following section will focus on the role of memory in the learning process. One major emphasis of the cognitive dimension deals with the process of knowing - how information is processed, stored and recalled. It concerns the human memory system in that what we have in the memory constitutes our knowledge. A careful investigation of how students learn, how they remember, recognise, process and recall information might provide the answer of questions such as where the limitations come from and how to help students to overcome the difficulties.

2.4 Meaning of Memory

Memory, like learning, is a hypothetical construct denoting three distinguishable but interrelated processes:

- *Registering* (or *encoding*) - the *transformation* of sensory input (such as a sound or visual image) into a form which allows it to be entered into (or registered in) memory. With a computer, for example, information can only be encoded if it is presented in a format the computer recognises.

- *Storage* - the operation of holding or retaining information in memory. Computers store information by means of changes in the system's electrical circuitry; with people, the changes occurring in the brain allow information to be stored, though exactly what these changes involve is unclear.
- *Retrieval* - the process by which stored information is *extracted* from the memory.

These three processes of memory are shown in Figure 2.4.

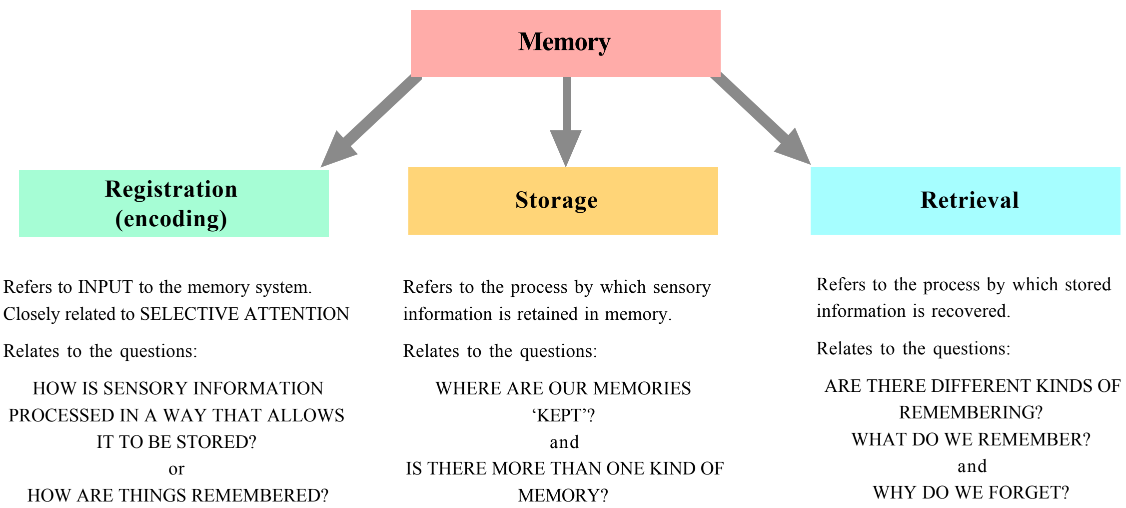


Figure 2.4 Three processes of memory (Gross, 2005)

Gross (2005) believes that registration can be a necessary condition for storage to take place, but not everything which registers on the senses is stored. The same is true of storage. It is a necessary, but not sufficient, condition for retrieval: we cannot recover information which has not been stored, but the fact that we know it is no guarantee that we will remember it on any particular occasion. Gross noted that this is a crucial distinction between *availability* (whether or not the information has been stored) and *accessibility* (whether or not it can be retrieved), which is especially relevant to the theories of forgetting.

In practice, storage is studied through testing learners' ability to retrieve. This is equivalent to the distinction between learning and performance: learning corresponds to storage, while performance corresponds to retrieval. For these reasons, it is useful to distinguish memory as *storage* and memory as *retrieval.*

Many researchers have focussed on only one of the above stages (encoding, storage and retrieval). However, it is important to note that all the three stages are closely related to, and depend on, each other. Tulving and Thomson (1973) argued, "*Only that can be retrieved that has been stored and ... how it can be retrieved depends on how it was stored*".

One way of distinguishing different kinds of questions about memory is to separate those which concern processes from questions that concern memory structures (Atkinson and Shiffrin, 1968). Questions about memory processes have to do with the mental activities that are performed in order to put information into memory and the activities that later make use of that information, namely acquisition of information and recall. Questions about structure have to do with the nature of memory storage itself, its duration and organisation - potentially subject to different causes of forgetting. Information that is to serve as the basis for remembering must first be acquired through mental representation or memory code, and retained and retrieved if it is to be used as the basis of a later act of remembering. These three logical distinct processes, acquisition, or 'encoding', retention or 'storage' and recall or 'retrieval' form the repeating themes of this study.

Understanding of the learning process and the nature and role of memory have developed enormously in the later half of the 20th century. During that time, a number of people started to see education from a learner's point of view. Among them (e.g. Atkinson and Shiffrin, 1971; Child, 1993; Johnstone, 1993) paid much attention to the learner and the process of human learning.

As was noted earlier, it was James who first distinguish between primary and secondary memory. Many psychologists since James have also made the distinction, including Hebb (1949); Broadbent (1958), and Waugh and Norman (1965). In Atkinson and Shiffrin's (1968, 1971) multi-store model, they are called short-term memory (STM) and long-term memory (LTM) respectively. Strictly, the STM and the LTM refer to the experimental procedures for investigating short-term and long-term storage respectively. These storage systems will be discussed in this chapter.

2.5 The Multi-Store Model

Atkinson and Shiffrin's (1968, 1971) *multi-store model* (sometimes called the *dual-memory model* because of the emphasis on STM and LTM) was an attempt to explain how information flows from one storage system to another. The model sees sensory memory, short-term memory and long-term memory as permanent structural components of the memory system (built-in features of the human information-processing system). In addition to this structural components, the memory system comprises more *transient control processes*. *Rehearsal* is a key control process, serving two main functions:

(a) To act as a buffer between sensory memory and long-term memory by maintaining incoming information within the short-term memory.

(b) To transfer information to the long-term memory.

Figure 2.5 shows the multi-store/dual-memory model of memory proposed by Atkinson and Shiffrin.

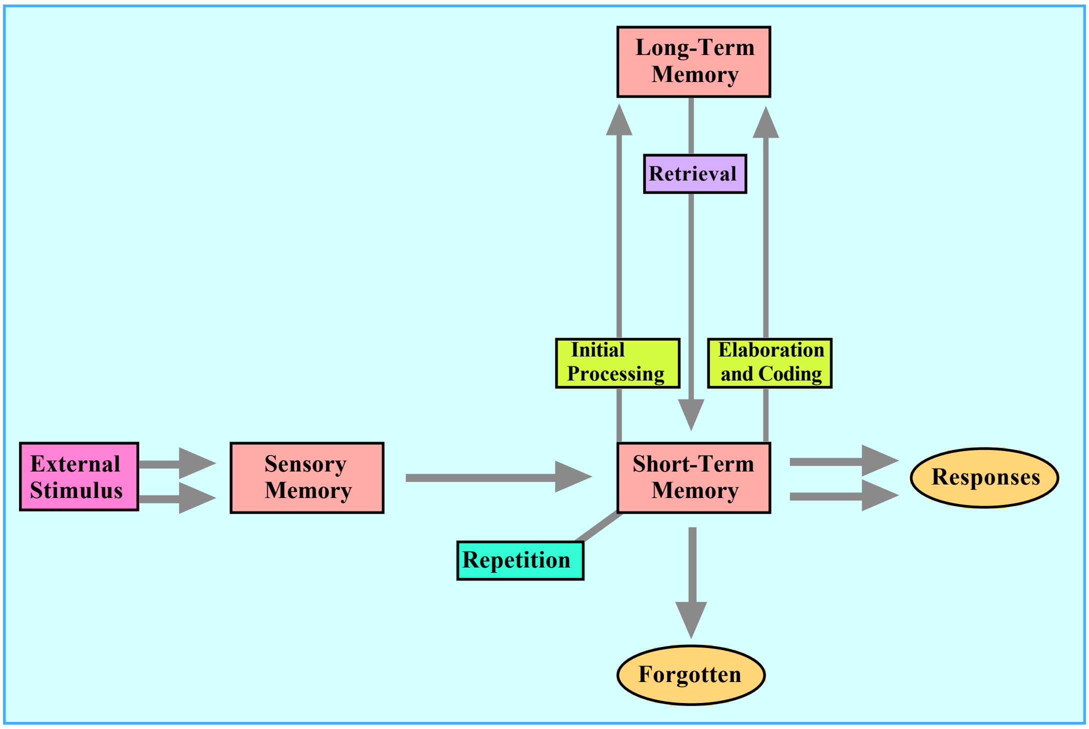


Figure 2.5 The Memory Model of Atkinson and Shiffrin (1968)

Information from the sensory memory is scanned and matched with the information in the long-term memory, and if matched (that is, pattern recognition) occurs, then it might be fed into the short-term memory along with a verbal label from the long-term memory.

The multi-store model sees rehearsal as a key control process which helps to transfer information from STM to LTM. There is also only one type of rehearsal as far as the model is concerned, what Craik and Watkins (1973) call maintenance rehearsal. This means that what matters is how much rehearsal occurs. But maintenance rehearsal may not even be necessary for storage. Jenkin (1974) found that participants could remember material even though they were not expecting to be tested - and so were unlikely to have rehearsed the material. This is called *incidental learning.* According to them, this is a kind of rehearsal that is important. They also considered that the multi-store model's view of the relationship between structural components and the control processes was, essentially, the wrong way round.

According to the multi-store model, the structural components (sensory memory, STM, LTM) are fixed, while the control processes (such as rehearsal) are less permanent. Crait and Lockhart's *levels-of-processing* (LOP) model begins with the proposed control processes. The structural components (the memory system) are what results from the operation of these processes. In other words, memory is a *by-product of perceptual analysis.* This is controlled by the central processor, which can analyse a stimulus (such as a word) on various levels:

- At a superficial (or *shallow*) level, the surface features of a stimulus are processed.
- At an intermediate (*phonemic or phonetic*) level, the word is analysed for its sound.
- At a deep (or *semantic*) level, the word's meaning is analysed.

The level at which a stimulus or information is processed depends on both its nature and the processing time available. The more deeply information is processed, the more likely it is to be retained.

2.5.1 Sensory Memory

Many of the important insights have developed as a result of thinking about learning in terms of the way the individual processes incoming information. Information is received by the senses and, in an educational setting, sound and sight are the most important routes. The sensory memory receives information from the senses, selecting what is important, and passing it on to the short-term memory before it is processed. However, according to Johnstone (1991), sensory memory is known as perception filter; or sensory register (Atkinson and Shiffrin, 1968). Thus, the learner selects information which is important, interesting and understandable through this filtering process, and only those pieces of information which are selected can be processed further in the learner's brain.

The most explored sensory stores are the *iconic store* and the *echoic store*. The iconic store is a memory store which holds visual information for about half a second. Sperling (1960) found that there was a very rapid decay of information for the iconic store. The length of time information can be held in the iconic registers is about 0.3 seconds after the visual display.

The echoic store is a memory store which holds auditory information for a short period of time. The findings of the study by Treisman (1964) provide an estimate of the duration of information of approximately two seconds in the echoic store, but other researchers have argued that this is an underestimate (e.g. Darwin *et al.*, 1972). This ability to retain sensory information, even for a very brief duration, gives us additional time to process this information into some more enduring form.

Human perception is directly linked with what people already have stored in the long-term memory as shown in Figure 2.2. The sensory memory must be driven by what people already have perceived, know, and understand. This means that the learner is influenced by his previous knowledge, attitudes, abilities, preferences, prejudices and experiences which is based on Ausubel's models for meaningful learning discussed in the earlier section.

Thus, what a learner already knows will influence the selection process for new incoming information. This is very important in all learning and explains why learners respond and pay attention to certain stimuli.

2.5.2 Short-Term Memory

Once the sensory memory receives new information from the senses, it goes to the short-term memory where the manipulative activities take place. Clearly, if we possessed only the sensory memory, our capacity for retaining information about the world would be extremely limited (Gross, 2005). However, according to models of memory such as Atkinson and Shiffrin's multi-store memory (1968, 1971), some information from the sensory memory is successfully passed on to the short-term memory. It stores what we are thinking about at the time, along with information that has come from our senses.

Short-term memory (and long-term memory) can be analysed in terms of:

- *Capacity* - how much information can be stored.
- *Duration* - how long the information can be held in storage.
- *Coding* - how sensory input is represented by the memory system.

In their multi-store model, Atkinson and Shiffrin saw the short-term memory as a system for temporarily holding and manipulating information.

However, Baddeley and Hitch (1974) criticised the model's concept of a *unitary* short-term memory (meaning that it operates in a single, uniform fashion). Instead of a single, simple short-term memory, they proposed a more complex, multi-component working memory. This comprises a *central executive*, which is in overall charge (involved in many higher mental processes, such as decision-making, problem-solving and making plans) plus sub - or *slave systems,* whose activities are controlled by the central executive. These are the articulatory (or phonological) loop and the visuo-spatial scratch (or sketch) pad. Much evidence supports this analysis (see Baddeley, 1997).

While accepting that the short-term memory rehearses incoming information for transfer to the long-term memory, they argued that the short-term memory was much more complex and versatile than a mere '*stopping-off station*' for information. According to them, the short-term memory should be replaced by the concept of a working memory system consisting of the following three components:

- *Central executive*: this is an attention-like system of limited capacity. Its functions include the regulation of information flow within the working memory, the retrieval of information from other memory system such as the sensory and long-term memory. The processing resources used by the central executive to perform these various functions are limited in capacity. The greater the composition for the limited resources of the executive, the more its efficiency at fulfilling particular functions will be reduced (Mahdi, 1995).
- *Articulatory loop*: this is a limited capacity system which contains information in a phonological (speech-based) form; it is used for verbal rehearsal; it is also defined in terms of time, and known as the phonological loop. The number of items that can be fitted on the articulatory loop (words, digits, etc.) depends on the time taken to articulate them. Baddeley *et al.,* (1975) found a difference in immediate memory (serial recall) for words of different length. This word-length effect can be explained by the fact that longer words take longer to articulate than shorter words and, therefore, take up more of the available space of the tape loop. Further investigations of reading rates and recall for different words showed a consistent relationship between word length, recall, and reading rate (Baddeley, 2000). As word length increases, memory span and reading rate both fall. Thus, the capacity of the articulatory loop can be best expressed, not as a number of items but rather as the time taken to articulate a sequence of items.
- *Visuo-spatial*: this is a limited capacity system which stores visual and/or spatial information.

The working memory holds the information for a few seconds. This is the place where interpreting, rearranging, understanding and problem-solving may take place. The information can be passed to the long-term memory for permanent storage and the working memory is then free to take in more information.

The two phrases, 'short-term memory' and 'working memory' are now being used somewhat interchangeably. Originally, this part of memory was conceptualised as a space for simply holding information. Later, it was appreciated that the space is where thinking, understanding and problem solving activity take place (Eysenck, 1998). The long-term memory is a place where knowledge and understandings are stored. Indeed, the phrase 'working memory' is more appropriate and will now be used.

The important point to note is that working memory is where the person thinks, holds information and solves problems. It is connected not only to the long-term memory but also to the sensory memory by a way of receiving and responding to incoming information. It refers to the use of temporary storage mechanisms in the performance of more complex tasks (Baddeley and Hitch, 1977). Gross (2005) states two significant important functions of working memory:

- Helps us to keep track of what we are doing or where we are from moment to moment.
- Holds information long enough to allow us to make a decision, dial a telephone number, or repeat a strange foreign word that we have just heard.

These features have important implication in all learning, most especially, for the way teachers should plan and deliver their lessons.

In some brilliant work, Miller (1956) found ways to measure the capacity of what he called the '*short-term memory*' of individuals. The problem is how to express this capacity: what units are to be used. He developed the concept of '*chunks*'. A chunk is a parcel of information, the size of which is in the control of the individual learner. It might be a single number or a single letter, or many pieces of information grouped together.

Miller (1956) demonstrated that the average adult capacity of the working memory is 7 chunks. He also showed that almost all adults have capacities lying between 5 and 9. The capacity of working memory grows with age until about age 16. This corresponds to the age when Piaget found that formal operational thought was fully available. At age 14, the average capacity is nearer 6, and, at 12, it is nearer 5. Working memory capacity cannot be expanded. However, it can be used more efficiently. One way was what Miller called '*chunking*'.

Miller showed how chunking can be used to expand the limited capacity of the working memory by using already established memory stores to categorise or encode new information. If we think of working memory capacity as seven 'slots', with each slot being able to accommodate one bit or unit of information, then seven individual letters would each fill a slot and there'd be no 'room' left for any additional letters. But if the letters are chunked into a word, then the word would constitute one unit of information, leaving six free slots.

In the example below (from Gross, 2005, page 284), the 25 bits of information can be chunked into (or reduced to) six words, which could quite easily be reduced further to one 'bit' (or chunk) based on prior familiarity with the words:

S	A	V	A	O
R	E	E	E	G
U	R	S	Y	A
O	O	D	N	S
F	C	N	E	R

To be able to chunk, you have to know the 'rule' or the code, which in this case is: starting with F (bottom left-hand corner) read upwards until you get to S and then drop down to C and read upwards until you get to A, then go to N and read upwards and so on. This should give you 'four score and seven years ago'.

Thus, chunking is involved whenever we reduce a larger amount of information to a smaller amount. Gross (2005) noted that this appears to increase the capacity of the working memory (although, in fact, all it does is use the fixed space more efficiently) and offers a form of encoding information, by imposing a meaning on otherwise meaningless material. For example:

- Arranging letters into words, words into phrases, phrases into sentences.
- Converting 1066 (four bits of information) into a date (one chunk).
- Using a rule to organise information: the series 1492536496481100121 (21 bits) is generated by the rule by which number = 1^2, 2^2, 3^3 and so on. The rule represents a single chunk, and that is all that has to be remembered.

The experienced learner knows enough how to be able to group ideas together powerfully and meaningfully. The novice learner lacks this experience and cannot chunk efficiently. However, chunking skills cannot be taught easily in that they are, to some extent, idiosyncratic.

It is important to note that, if the working memory has to hold many ideas at once, there is very little space left for thinking about these ideas. Equally, if much thinking is going on, then there is little space for holding information. The working memory is, therefore, a shared thinking-holding space.

Individuals will group information into a recognisable pattern in a variety of ways, largely dependent on experience. However, if the working memory is overloaded, learning more or less ceases, understanding is highly unlikely and problem solving will not take place. This is the origin of most common learning difficulties in highly conceptual subjects like mathematics and the sciences. Concepts, by their nature demand many ideas to be held and brought together, at the same time. In mathematics education, the working memory space is easy to overload and the working memory demand of much learning strongly influences success (Al-Enezi, 2006; Ali, 2008). This may be related to the nature of mathematics itself and the method by which mathematics is taught in schools. Some learning difficulties will be discussed in the next chapter.

It is important to recognise that working memory capacity grows with age. This explains why certain skills are extremely difficult (sometimes impossible) for younger learners but become much more accessible with age. This is critically important in mathematics where some ideas make heavy demands on limited working memory space while, at the same time, ideas tend to build sequentially and logically one on the other. A mathematics curriculum has to be constructed with great care to ensure accessibility of ideas in terms of working memory capacity as well as generating a logical sequence of understandings.

Another problem, may arise when strategies and memory aids (e.g. BIDMAS) are invoked. It is essential that these strategies do not exceed the working memory capacity. They may actually end up making understanding even less likely.

Many researchers (e.g. Baddeley and Hitch, 1974; Johnstone, 1984) have measured working memory capacity. However, the following two ways are most commonly used:

(a) *Figure Intersection Test* (developed by Pascual-Leone, 1970). In this test, there are two sets of simple geometric shapes, the set of shapes on the right (called the presentation set), and the other set of overlapping shapes on the left (called the test set). The presentation set consists of a number of shapes separated from each other. The test set consists of the shapes but overlapping, so that there exists a common area which is inside all the shapes of the presentation set. What the respondents have to do is to look for and shade-in the common area of intersection in the test set. There are usually 20 items in the test, sometimes in some items, a misleading irrelevant shape (not present in the presentation set) is included in the test test. The number of shapes varies from 2 to 8, and the number of shapes in the test set is equal to the score given if the item is marked correctly. Usually, as the number of shapes increase, the tasks become more complex.

(b) *Digit Span Test* (developed by Miller, 1956). In Chapter 7 (and, more detailed in Appendix A), the procedure for using the latter will be discussed in that it is employed in the present study to obtain experimental data. In essence, it involves recalling numbers in reverse order. The two tests give highly consistent outcomes (El-Banna, 1987).

2.5.3 Long-Term Memory

Once the working memory has 'made sense' of the information, it is passed into the long-term memory where, unless it is subsequently used or recalled in some other way, it is again eventually forgotten. The capacity of long-term memory seems to be limitless and its duration virtually endless (Solso, 1995). It is a memory where everything is relatively permanently stored: facts, skills, abilities, experiences, understandings, concepts, emotions, attitudes, memories and prejudices, but which are potentially retrievable.

According to Bower (1975), some of the kinds of information contained in the long-term memory include:

- A spatial model of the world around us.
- Knowledge of the physical world, physical laws and properties of objects.
- Beliefs about people, ourselves, social norms, values and goals.
- Motor skills, problem-solving, skills, and plans for achieving various things.
- Perceptual skills in understanding language, interpreting music, and so on.

Many of these are included in what Tulving (1972) calls *semantic memory* (memory for meaning). Indeed, long-term memory is the store for everything learned that can ever be used later. It is thought that anything stored is present for life (excluding later brain damage). However, that does not guarantee that the information can be accessed later. This seems to depend very much on the extent of inter linking between ideas. The more inter linking, the more potential 'routes' for access my be available (Al-Qasmi, 2006).

One significant feature of long-term memory in terms of learning is its linkage with other memories to give a complex matrix of memory. What is available in the long-term memory is very important because it may distort the selection process and provide, for the working memory, information which is incompatible with what is coming in from outside (Driver *et al.,* 1985). Thus, what is available in the long-term memory has a direct influence with learning, perception and understanding.

One of the major distinctions between the working memory and the long-term memory stores is in terms of the forgetting mechanism involved. Forgetting from the working memory store seems to occur because of diversion of attention away from the information within the store, and because of interference from other incoming information (Reitman, 1974). Forgetting, however, is usually seen as the loss of access to information stored in long-term memory. Both forgetting and remembering, are not under direct conscious control; they are automatic. Petty (2004) argues that repetition is the only one way to ensure that something is remembered. This means that learners can learn and remember if the knowledge is recalled and used frequently. However, this looks at learning in a somewhat limited way. If a concept is genuinely understood, it tends to be remembered. The overall idea makes sense while the many ideas implicit in the concept are inter linked. This involves more than what is conventionally understood as repetition.

Of course, there are exceptions to this rule; sometimes a one-off experience will be remembered for a lifetime - for example, an event with great emotional significant. However, in a teaching-learning situation, we are at the mercy of the brain’s automatic mechanisms, and repetition may be important.

The Table 2.1 shows the summary of the main difference between the working memory and the long-term memory.

	Capacity	Duration	Coding
Working Memory	An average of 7 'chunks' of information. Much less will be held if there is processing	15-30 seconds; can be increased by repetition or rehearsal	Mainly acoustic. Some semantic. Visual is also possible
Long-term Memory	Unlimited	From a few seconds to a lifetime	Semantic, visual, acoustic; and also by smell, taste. Overall very flexible

Table 2.1 Working and Long-term Memories

A key question which arise is: how is information retrieved or recovered? Almost all educational assessment relies on this recovery process. The systematic scientific investigation of memory began with Ebbinghaus (1885) where he showed the innate human characteristic - the seeking of meaning. Other techniques for measuring memory include the following:

(a) *Recognition:* This involves deciding whether or not a particular piece of information has been encountered before (as in a multiple-choice test, where the correct answer is presented along with incorrect ones). The *sensitivity* of recognition as a form of retrieval is demonstrated by Standing (1973).

(b) *Recall:* This involves participants actively searching their memory stores in order to retrieve particular information (as in timed essays). Retrieval cues are missing or very sparse. The material can be recalled either in the order in which it was presented (serial recall) or in any order at all (free recall).

(c) *Memory-span procedure:* This is a version of serial recall, in which a person is given a list of unrelated digits or letters, and then required to repeat them back immediately in the order in which they were heard. The number of items on the list is successively increased until an error is made. The maximum number of items that can consistently be recalled correctly is a measure of *immediate memory span* (Gross, 2005).

(d) *Paired-associate recall:* Participants are required to learn a list of paired items (such as 'chair' and 'elephant'). When one of the words (e.g. chair) is represented, the participants must recall the paired word (elephant).

2.6 Information Processing Models of Learning

From his research, Ashcraft (1994) maintained that a standard information processing model should contain three major components:

- Sensory memory (sensory register or perception filter)
- Short-term memory (working memory or working memory space)
- Long-term memory

The differences between these three types of memory have been discussed previously in this chapter in the ways of their information processing functions and capacity. Overall, the information we receive from the external environment is mediated by sensory memory, processed in the working memory and stored in the long-term memory.

For many years, Johnstone (1993, 1997) had explored the difficulties which students found in chemistry, physics, biology and mathematics. In one study, it became clear that the likely source of the difficulties lay in the overload of what became known as working memory (Johnstone and Kellett, 1980). He then tested this hypothesis, with quite remarkable outcomes (Johnstone and El-Banna, 1986, 1989).

Johnstone and El-Banna took many questions in assessments in chemistry and found the working memory load which was demanded for any chance of success. They expected to find that, as the question load increased, the performance would fall (Figure 2.6)

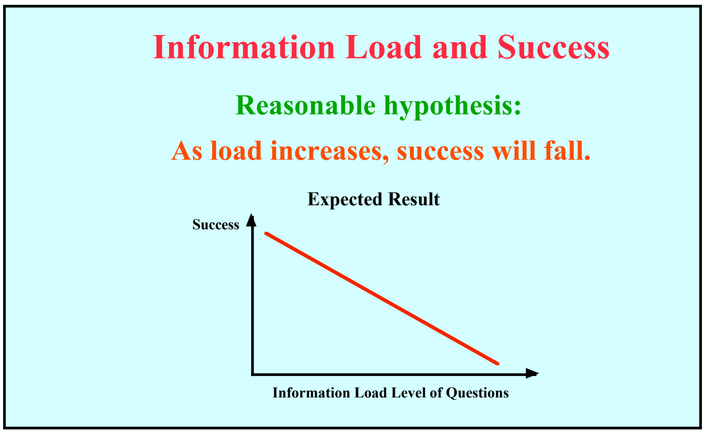


Figure 2.6 Predicted Graph (Johnstone and El-Banna, 1986, 1989)

However, what they obtained was (Figure 2.7):

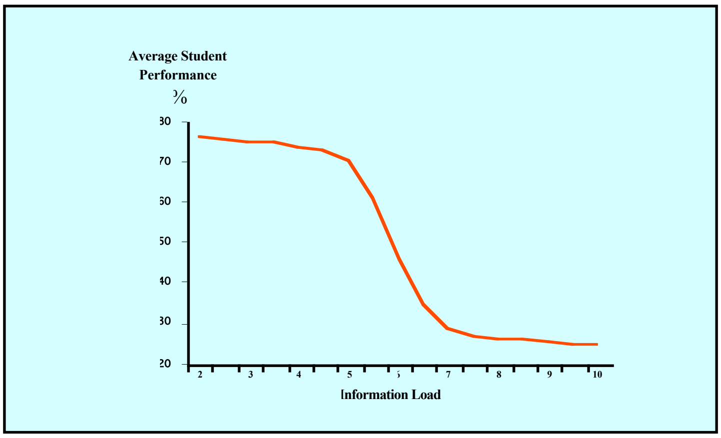


Figure 2.7 Graph Obtained (Johnstone and El-Banna, 1986, 1989)

There was a sudden fall in performance when the working memory load of the question reached about 6. To make the matter even more clear, they had measured the working memory capacity of the students (aged about 18-19) using both the figural intersection test and the digit span backwards test.

These two tests are standard tests for the measurement of working memory capacity. The former is based on inversion and recall of sets of numbers of increasing length, while the latter is based upon detecting the common overlap between increasing numbers of geometrical shapes. Johnstone and El-Banna found that the two tests gave the identical outcomes for the vast majority of the students. They divided the student group into three: those with above average working memory capacities (greater than 7); those with average working memory capacities (7); and those with below average working memory capacities (less than 7). They then re-plotted the data (Figure 2.7) to obtain Figure 2.8.

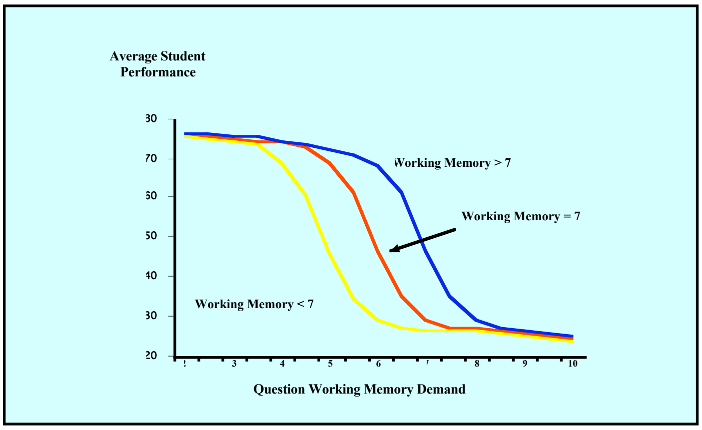


Figure 2.8 Graph with three groups (Johnstone and El-Banna, 1986, 1989)

This showed clearly that it was the limited working memory capacity which was influencing success in the questions.

Later, Johnstone brought all the findings together to develop an information processing model. Johnstone's model includes all three components of Ashcraft's (1994) model: perception filter, working memory, and long-term memory. Johnstone showed how the model brought in the earlier insights from Ausubel (1968), Pascal-Leone's (1970) ideas of limited working memory, the constructivist view of learners' cognitive development, knowledge construction, and building knowledge (Piaget, 1973; Miller, 1993; Atkinson and Shiffrin, 1971). Thus, this model suggests explanations of how learning has occurred, and why learning is sometimes difficult or impossible. In other words, it suggests a simplified mechanism of the learning process and enables us to understand the limitations of learning.

The flow of information during learning by Johnstone (1994) is shown in Figure 2.9 overleaf.

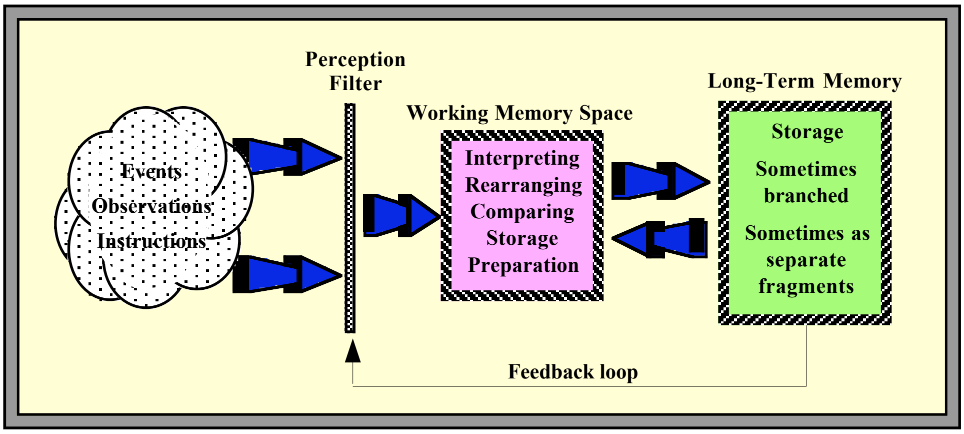


Figure 2.9 A model of learning of information processing. (Johnstone, *et al.*, 1994, p.78)

His model has been used predictively to underpin numerous studies which have explored learning and found to offer very useful insights (Johnstone *et al.*, 1998; Sirhan and Reid, 2001; Danili and Reid, 2004; Hussein and Reid, 2009). According to Reid (2008), many of these insights have been tested and, in every case so far, the model has been found to predict successfully. A careful study of the Johnstone's model might offer insights as to why students underachieve (working memory overload) and how to help students overcome their learning difficulties.

2.7 Causes of Forgetting After Learning

To understand why we forget, we must recall the distinction between *availability* (whether or not the material has been stored) and *accessibility* (being able to retrieve what has been stored). In terms of the multi-store memory, since information must be transferred from the working memory to the long-term memory for permanent storage:

- Availability mainly concerns working memory and the transfer of information from the working memory to the long-term memory.
- Accessibility has to do mainly with the long-term memory.

Forgetting can occur at the encoding, storage, or retrieval stages. One way of looking at forgetting is to ask what prevents information staying in the working memory long enough to be transferred to the long-term memory. Some answers are provided by *decay* and *displacement theories.* Some answers to the question about what prevents us from locating the information that is already in the long-term memory include those offered by *interference theory*, *cue-dependent forgetting* and *motivated forgetting.*

Decay (or trace decay) theory tries to explain why forgetting increases with time. Clearly, memories must be stored somewhere in the brain. According to Gross (2005), presumably, some sort of structural change (the *engram*) occurs when learning takes place. According to decay theory, metabolic processes occur over time which cause the engram to degrade/break down, unless it is maintained by repetition and rehearsal. This results in the memory contained within it becoming unavailable. Hebb (1949) argued that, while learning is taking place, the engram which will eventually be formed is very delicate and liable to disruption (the *active trace*). With learning, it grows stronger until a permanent engram is formed (the *structural trace*) through neurochemical and neuroanatomical changes.

In the limited-capacity working memory system, forgetting might occur through displacement. When the system is 'full', the oldest material in it would be displaced ('pushed out') by incoming new material. This possibility was explored by Waugh and Norman (1965) using the *serial probe task*. Participants were presented with 16 digits at the rate of either one or four seconds. One of the digits (the probe) was then repeated, and participants had to say which digit followed the probe. Presumably:

- If the probe was one of the digits at the beginning of the list, the probability of recalling the digit that followed would be small, because later digits would have displaced earlier ones from the system.
- If the probe was presented towards the end of the list, the probability of recalling the digit that followed would be high, since the last digits to be presented would still be available in the working memory.

From Figure 2.10, when the number of digits following the probe was small, recall was good, but when it was large, recall was poor. This is consistent with the idea that the earlier digits are replaced by later ones.

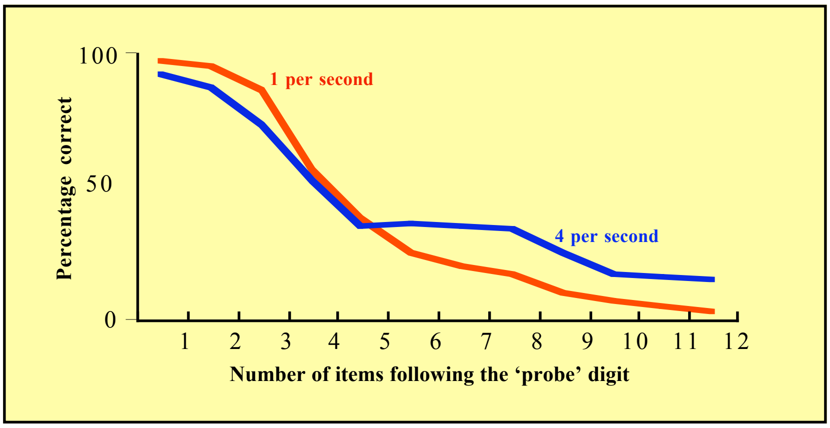


Figure 2.10 Waugh and Norman's (1965) Serial Probe Experiment. (Derived from: Gross, 2005, page 301)

Since less time had elapsed between presentation of the digits and the probe in the four-per-second condition, there would have been less opportunity for those digits to be decayed away. This make it unclear whether displacement is a process distinct from decay.

According to *retrieval-failure theory*, memories cannot be recalled because the correct retrieval cues are not being used. The role of retrieval cues is demonstrated by the *tip-of-the tongue phenomenon*, in which we know that we know something but cannot retrieve it at that particular moment in time (Brown and McNeill, 1966).

Interference theory is another cause of forgetting. According to this theory, forgetting is influenced more by what we do before or after learning than by the mere passage of time. In retroactive interference or inhibition, later learning interferes with the recall of earlier learning. For example, if you originally learned to drive in a manual car, then learn to drive an automatic car, when returning to a manual, you might try to drive it as though it was an automatic. On the other hand, in proactive interference, earlier learning interferes with the recall of later learning.

To summarise this section, working memory has a limited capacity. Forgetting may be as a result of information overload. As a result of this forgetting mechanism, we only remember:

- That which have been recalled frequently, or
- That which we have heard recently

Of course, forgetting may simply be because the information never reached the long-term memory. This can occur for many reasons. One of them is that, during the learning process, overloading of the working memory can occur if the learning task is beyond the working memory capacity of the learner. Learning more or less ceases and the information may well never be stored at all. Thus, Barber (1988) argued that, if the information to be used is beyond the limit of the learner's working memory capacity, then an overload may occur and loss in productivity or efficiency may arise.

This raises the question about how to improve the performance of the working memory. Can strategies be taught by restructuring teaching and learning to improve understanding in such a way that working memory overload is reduced? Three studies have shown this to be true (see Hussein and Reid, 2009). The aim of examination is to test mathematics in some forms; it is possible that the question does not so much test the mathematics but the capacity of the working memory or field dependence (the skill of being able to see what is important for a particular task and disembedding it from the surrounding information). The early work of field dependency was developed by Witkin *et al.,* (1974) who emphasised that some learners have more difficulty than others in separating '*signal*' from '*noise*'. Research has not yet shown clearly if and how this skill can be developed (Danili, 2004).

Johnstone and El-Banna (1989) noted that a necessary (but not sufficient) condition for students to be successful in mathematics or in examination question is that the demand of the tasks should not exceed the working memory capacity of the students. If the capacity is exceeded, the students' performance will fall unless they have a kind of strategy which enable them to structure the question and bring it within their capacity.

The use of group work has been suggested by Reid and Yang (2002b) as one way to reduce working memory problems. Working in small group offers the capacity of several working memories when facing some problem-solving activities. Although the working memory is the property of an individual, it is possible for several learners to work together thereby reducing the potential overload. Malacinski (1994) pointed out that collaborative learning groups provide an ideal structure for students to 'unwrap' new information, construct understanding, and develop critical thinking skills

Thus, learning difficulties can be seen as arising from information overload. Learning mathematics is very likely to produce this kind of overload. This is where practice with mathematical exercises can be helpful. For example, after a student has completed ten, say, exercises in the solving in straightforward simultaneous equations, the procedures used are cognitively automated and thus the whole sequence of the procedures occupies, perhaps, only one space in the working memory, leaving space for thought and understanding.

2.7 Chapter Summary

The fundamental assumption of the multi-store model is that there are three different types of memory store: the sensory memory, the short-term stores, also known as the working memory, and the long-term memory.

This assumption has stood the test of time. There is a strong evidence that the three memory stores differ from each other in a number of important ways. Eysenck (1998) identified four of the major differences:

- *Temporary Duration*: information stays in the sensory memory for a fraction of a second (iconic store) or two to three seconds (echoic store); it stays in the short-term memory for a few seconds; and it stays in the long-term memory for months or years.

- *Storage Capacity*: the sensory stores have rather limited capacity; the working memory has a capacity of approximately seven items; and the long-term memory has essentially unlimited capacity.
- *Entering Process*: information enters the sensory memory without the individual engaging in any active processing; it enters the working memory as a result of attention; and it enters the long-term memory as a result of rehearsal.
- *Forgetting Mechanism*: information is lost from the sensory memory through decay; it is lost from the working memory via diversion of attention and interference; and it is forgotten from the long-term memory mainly through in accessibility.

Learning difficulties can be interpreted in terms of information flow. Thus, the selection process may be flawed, the working memory may be overloaded, saving to the long-term memory may involve poor linkage to previous ideas. The next chapter will consider such difficulties in more detail.

Learning is not the same as remembering; it is an active ‘meaning-making’ process. Only information that has been structured and organised by the learner can pass into the long-term memory and can be used in real life. This organisation process is helped by doing rather than by listening.

Chapter Three

Learning Difficulties in Mathematics

3.1 Introduction

The Cockcroft report (1981) on the effects of teaching and learning for meaning and understanding offers some reasons why people should learn mathematics: it is useful for everyday life, for science, for commerce and for industry, it provides a powerful, concise and unambiguous means of communication, and it provides means to explain and predict. Also, the report claims that mathematics develops logical thinking, and it has aesthetic appeal.

Looking at the Cockcroft suggestions, some of them need challenged. For example, most people only need basic arithmetic and perhaps a rudimentary understanding of probability for ordinary life. For most of the population, the power of mathematics in giving a powerful, concise and unambiguous means of communication is largely irrelevant while the argument that it develops logical thinking needs supporting evidence. It is far more likely that those who can think logically find, in mathematics, a way to express and use their ability.

It is paradoxical that, although mathematics has enormous power to solve practical problems, it is yet regarded justifiably as an 'abstract' subject, and "*for many people, free association with the word 'mathematics' would produce strong negative images*" (Donovan and Bransford, 2005). A mathematical calculation, or a formula such as:

$$e^{i\pi} = -1$$

does not of itself demonstrate any practical relevance. Yet the most complex mathematics has its feet firmly planted in the real world. It is rightly called an abstraction from the real world (Liebeck, 1984). Even 'two' is an abstract concept. We cannot understand 'two' until we have met many pairs (for examples, a pair of eyes, a pair of shoes, a pair of wings), and abstracted what all pairs have in common.

We cannot understand what is meant by 'number' until we have understood 'two', 'three', 'four' and other similar concepts. 'Number' is an abstraction from a set of abstractions. The concept of 'addition of numbers' is an even higher abstraction than 'number'. Mathematics involves a hierarchy of abstractions, and we cannot understand any mathematical concept without also understanding the concepts on which it depends lower in the hierarchy.

Of course, language itself is abstract, and we communicate mathematics through language. But language in general does not involve this hierarchical structure to the degree that mathematics does. The teacher's task is to lead learners through this hierarchy without losing the chain of connections with the real world. To do this, it is essential that the teacher understands how the learner grows in ability to encounter and understand mathematics, and how the educational and environmental climate affects mathematical growth. More explicitly, the teacher must decide what learners are able to learn, what learners should learn, and what techniques best bring about learning. This means that there needs to be a sound understanding of underpinning educational principles which govern effective and efficient learning of mathematical skills and ideas.

In this chapter, the complexity of mathematics concepts and the identification of instructional barriers are discussed in terms of current understandings of how learning occurs. Consequently, the focus will be on the cognitive quantitative growth of the learner from developmental and intellectual points of view, along with selected psychological considerations which affect the learning of mathematics. This chapter also seeks to summarise some of the main findings related to areas, themes or topics which are causing difficulties for learners in learning mathematics with special emphasis on secondary school students.

3.2 Areas of Mathematics Difficulties

In the 1970s, education reformers laid great stress on the necessity for children to understand mathematical structure rather than become competent at routine calculations. The standard joke against these pioneers was that they insisted that children should understand that 5 x 3 = 3 x 5, but did not care whether they also knew that five threes are fifteen. The emphasis was certainly on mathematical structure rather than calculating techniques. The following little excursion explains the trend:

The numbers 1, 3, 5, 7 and so on are called odd numbers. Let us pose the problem of adding up the *first hundred odd numbers*. Even with modern calculator, the task could prove very long and boring. Let us add the first two odd numbers:

$$1 + 3 = 4$$

Next, the additions of first three, four and five odd numbers give:

$$1 + 3 + 5 = 9$$
$$1 + 3 + 5 + 7 = 16$$
$$1 + 3 + 5 + 7 + 9 = 25$$

The patterns are:

The first two odd numbers add to 2×2,
The first three odd numbers add to 3×3,
The first four odd numbers add to 4×4,
The first five odd numbers add to 5×5.

It is possible to show using the power of the pattern that $10^2 \times 10^2 = 10000$ is the correct result by using algebra or by resorting to calculation. However, the result is predicted without either. The difficulty here lies not only on mindless calculating techniques, but also competence at using and understanding relevant mathematics structure. It is through both of them that learners are equipped to solve real problems (Liebeck, 1984). To solve real problems, we need to understand mathematics. Paradoxically, to understand mathematics, we need to explore real problems. When learners learn mathematics, they need to play with real objects and explore real problems that interest them.

The 1988 Education Reform Act in England introduced a National Curriculum for schools. As far as mathematics is concerned, the National Curriculum sets out to stabilise rather than reform current teaching. It lays down core content of school mathematics, but refrains from prescribing *how* it should be taught for learners to understand and become competent.

Within the school curriculum, learning mathematics is uniquely challenging in that it is highly organised, sequential and progressive. Simpler elements or concepts must be learned successfully before moving on to others. Because of the interrelating nature of the subject, learners who have learning difficulty in mathematics may sometimes appear to feel even lost in trying to 'make sense' of what is required in mastering the concepts, processes and symbolism, which is a natural way of learning with understanding.

Frederickson and Cline (2009) identified three major sources of difficulty:

(1) Confusion between trying to achieve mathematical understanding ('knowing both what to do and why') and trying to learn mathematical procedures ('knowing rules and routines without appreciating the reasons for them').

(2) Increased anxiety, relating particularly to problems of miscommunication. Barwell (2002) and Christou (2001) have shown how 'real life mathematics problems' and 'word problems' create additional challenges.

(3) Reading mathematics and understanding the language of mathematics is challenging. Some words are used only in mathematical English, and are therefore unfamiliar until learners have been taught them (e.g., hypotenuse, parallel), while some other words are used confusingly with different meanings in mathematical English and ordinary English (e.g., mean, product, odd).

Several studies have identified topics and themes that are causing greatest difficulty for the learners in learning mathematics. For example, Human Sciences Research Council (HSRC, 2008) investigated the crisis in school-level mathematics education curriculum across all schools in South Africa. Their findings confirm that problems often observed in students are linked with aspects such as:

- Algebraic manipulation: simplifications, formula, equations, etc.
- Numeracy: basic number relationships, place value, decimal, measurement, etc.
- Graphs of functions.
- Limited insight into the nature of concepts such as:
    - (a) functions
    - (b) inverses
    - (c) zero product property
    - (d) number properties - Natural, integer, rational and complex numbers
- Limited ability to translate between language and mathematics.
- Computational skills - many students can only calculate if they have a calculator, and have no sense of order of magnitude as a result of this calculator dependence.
- Space and Measure - confusion exists with regard to calculations of areas, perimeters, volume, etc., which undermines study in other areas.
- Trigonometry - trigonometric ratios, angles, bearings, etc.

In implementing this curriculum, teachers were expected to make significant changes to their more familiar ways of working. However, almost all teachers received very limited support in terms of training and there also had been constant complaints that the limited short-term training provided for teachers to enable them teach the new curriculum had not been effective.

Instead of spreading fear of mathematics, the study suggested that teaching and learning should promote general appreciation of mathematics in terms of its:

- Philosophical ideas
- Cognitive aspects
- Logic
- Principles of construction of mathematical objects and theories

They also suggested that teacher education programmes should attempt to familiarise prospective teachers with common, sometimes erroneous, cognitive processes used by students. While their study seemed to be suggesting that much mathematics produced problems, it did identify the teacher perceptions that they were being asked to do something for which they had no experience or training. However, it assumes that training would assist. Perhaps the teachers were being asked to do the impossible in terms of the cognitive demand on the learners.

According to Donovan and Bransford (2005), students mostly display the following four deficiencies when solving problems related to equations and formulae. They argued that an attempt to correct and promote the development of these areas may lead to mathematical proficiency:

(1) Conceptual based mistake.
(2) Procedural fluency mistake - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
(3) Mistakes based on strategic competence - ability to formulate, represent, and solve mathematical problems.
(4) Adaptive reasoning based mistake - capacity for logical thought, reflection, explanation, and justification.

The study concluded that these are widespread and are very prevalent with students even after they had graduated from school. They affect not only their conceptual understanding and retrieval but also their problem solving ability (strategic competence). The study showed that the preconceptions students bring to the study of mathematics affect more often their understanding and problem solving; those perceptions also play a major role in whether students have a productive disposition toward mathematics, as do, of course, their experiences in learning mathematics.

In looking at *shape*, *space* and *measure*, Clements *et al.,* (1999) surveyed criteria pre-school children use to recognise members of a class of shapes from other figures. Their study was based on three dominant lines of inquiry presented by the theories of Piaget, van Hieles, and other cognitive psychologists (Clements and Battista, 1992b). Ninety seven (97) children were involved in the survey which sought to probe how they identify and describe shapes, and reasons for these identifications. The study concluded that, *"Young children initially form schemas on the basis of feature analysis of visual forms. While these schemas are developing, children continue to rely primarily on visual matching to distinguish shapes."*

The study also found that these children were capable of recognising components and simple properties of familiar shapes. They concluded that abstraction and representation are closely related and that children can represent at or below their level of abstraction but not above this level. The findings also supported the views of Kato *et al.,* (2002) that learning should build on a child's existing ideas and knowledge. Although the study involved pre-school children, the same is likely to be true for other learners.

Donovan and Bransford (2005) found that Brazilian street children could perform mathematics operations about numbers: addition, subtraction, multiplication and division when making sales in the street, but were unable to answer similar problems presented in a school context. Likewise, a study of housewives in California, USA, uncovered an ability to solve mathematical problems when they are engaged in shopping, even though the women could not solve problems presented abstractly in a classroom that required the same mathematics.

Teaching and learning of the fundamental concepts of transformation and symmetry usually have been a problem at school level. Mostly, student understanding is not only not more than the knowledge of a procedure or formula application to solve mathematics exercises but also it is not rich in making connections about models and pictures. Haylock (2006) found that difficulties in understanding transformation and symmetry are likely related to teaching methodologies that focus on rules rather than on the fundamental ideas of shapes. To help the learner to recognise the equivalencies that exist within various transformations he suggested that learners should be taught to: recognise when shapes are identical; visualise and describe movements of shapes using appropriate language; transform objects and images in practical situations using ICT; visualise and predict the position of a shape following a rotation, reflection or translation; identify and draw two-dimensional shapes in different orientations on grid; and recognise reflective symmetry in regular polygons.

These examples are not exhaustive. However, they suggest that people possess resources in the form of informal strategy development and mathematical reasoning that can serve as a foundation for learning more abstract mathematics. They also suggest that the link is not automatic. If there is no bridge between informal and formal mathematics, the two often remain disconnected. Overall, the study emphasises the need to determine the mental ability level at which learners are expected to understand, the need to build on existing knowledge, and the need to engage students' preconceptions.

3.3 The Origin of Mathematics Difficulties

In Chapter two, the information processing model was discussed. Memory processes can be broken down into three stages:

- *Input* (attending, using the senses, taking in information);
- *Storage* (working and long-term memory: what you do with the information to remember it);
- *Output* (recalling the information).

Learners can experience difficulties at any or all of these stages and difficulty at one stage can affect the others (Christina *et al.,* 2000). For example, in mathematics, the learner's working memory is easily overloaded in a learning situation because the content of the working memory is severely limited in capacity and duration (time it can hold items of information). Much research has revealed the effects of capacity limitations. Little work has been carried out to explore the duration implications.

To learn a concept means to hold a number of ideas at once, and to play with these ideas mentally and restructure them into a meaningful whole (Al-Enezi, 2006). This makes huge demands on the size of the working memory. The capacity of working memory is known to grow on average by 1 unit for each two years until about age 16. Thus, pupils at age 10, for example, will only have a mean working memory capacity of 4.

This limiting factor which influences success in learning arises because of the:

- Nature of mathematics itself;
- Way in which mathematics is taught in schools; and/or
- Way in which mathematics curriculum is designed.

Petty (2004) states that learners have preferences in the way they like to learn, especially when facing new ideas. However, most learners can cope to varying extents when faced with a style of teaching that is not their first preference. He suggests that teachers ensure that learners are aware of and encouraged to make best use of their strengths and that teaching includes opportunities to use learners' strength.

This is not as straightforward as suggested. Evidence suggests that school pupils aged about 13 are not very aware of their preferred learning characteristics (Hindal, 2007). In addition, it is not easy to see how teachers can be made aware of the specific learning characteristics of individual pupils without using a battery of tests. Indeed, it is asking the impossible of any teacher to be able to fashion their teaching style to suit the learning characteristics of each member of a class of, say, thirty.

The reality is that mathematics is a very complex subject dealing with basic features of the natural world and is often regarded as a difficult subject for many students. The possible factors are considered in the following section.

3.3.1 The Nature of Mathematics and Learning

Thinking of chemistry, Johnstone (2000) noted that a most important factor causing difficulty might be the '*intrinsic nature of the subject.*' The same is likely to be true for mathematics. The psychology of the formation of most of the concepts in mathematics is quite different from that of the everyday world. He suggests that we need three levels of thought when thinking within the discipline of chemistry:

- The macro or tangible: what can be seen, touched or smelled;
- The sub-micro: structures, and interpretation; and
- The representation: use of symbols, formulae, equations, manipulations graphs.

Johnstone argued that the learning process starts with the things that can be seen, touched and smelled. Examples of everyday experience help us to understand more abstract concepts. Learners need to experience all the three levels to understand chemistry. However, information processing models suggests that this is impossible because our working memory has a limited capacity. It can process only a few pieces of information at a time. The situation in mathematics may well be very similar.

A simple model which enables us to talk about understanding in mathematics is to view the growth of understanding as the building up of (cognitive) connections. More specifically, when we encounter some new experience, there is a sense in which we understand it if we can connect it to previous experiences, or, better, to a network of previously connected experiences. The more strongly connected the experience the more we understand it. Learning without making connections is what we would call learning by *rote.* Trying to master the processes, symbolisms, and making connections may well create pressure on limited working memory capacity. The learner cannot cope with concepts (understanding), procedure, symbolisms and problem solving at the same time.

Ausubel (1968) described this meaningful learning in terms of 'internalisation'. This seems to involve taking the new material and linking it in multiple of ways to previously learned knowledge and to develop an enriched and enhanced network of ideas and understandings. The understanding becomes one's own. However, the limitations in working memory capacity may make such understanding, connection, and internalisation very difficult. Part of the teacher's role in developing understanding is, then, to help the child to build up connections between new experiences and previous learning.

It is very helpful to think of understanding the concepts of numbers and number-operations (i.e., number, place value, addition, subtraction, multiplication, division, equals, number patterns, and so on) as involving the building up of a network of cognitive connections between the four types of experience of mathematics identified in Figure 3.1 (concepts, procedures, symbolic representations and applications).

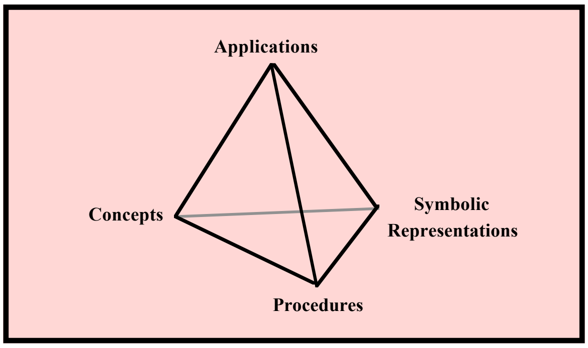


Figure 3.1 Some significant connections in understanding mathematics (The Mathematics Tetrahedron: source: Ali, 2008)

The tetrahedron model presented in Figure 3.1 reflects the kind of thinking shown by Johnstone (2000) in the development of his triangle. Understanding can be thought of as building up cognitive connections between these four components (applications, concepts, procedures and symbols). The key point is that, if the learner is required to think of ideas drawn from all four (or even three) vertices of the tetrahedron, then working memory overload is more or less guaranteed. There is no easy answer to this but the general principle is, especially with young learners, to work at as few vertices as possible at the same time.

Different mathematical topics may require different approaches. Thus, for example, in approaching measurement, starting with physical objects and an appropriate measuring device (a ruler or tape), dimensions of physical reality can be 'translated' to numbers and units. With older students, when introducing linear equations, establishing the skills of the correct conduct of procedures to obtain right answers may be the starting point. Once this is established and become more or automated (thus, using little working memory space), the underpinning concepts can be explained. Only when the students have considerable experience, as well as the confidence coming from the expectation of success, can applications based on real-life or novel situations may be considered.

The key point is not to work at too many vertices at the same time. The expert mathematician can work comfortably in the space inside the tetrahedron, moving effortlessly from vertex to vertex. The 'novice' learner has insufficient experience to be able to chunk any of the skills, and working memory overload is more or less inevitable.

3.3.2 Difficulties in Understanding Concepts

As already noted in this chapter, many of the concepts and procedures studied in mathematics are abstract because rules and algorithms dominate them, and, therefore, inherently difficult to learn. According to Sawer (1959), abstraction is a process of forgetting unimportant details, and without abstraction thought will be impossible. This gives rise to many difficulties in teaching some mathematical ideas without appropriate use of analogies or models. Learners tend to be good at mastering facts but they are not so good at grasping the underlying concepts. This is because the way we build up concepts depends on the content which we can see, sense and experience (Chen, 2004). The investigations carried out by Jung (2005) and Al-Enezi (2006) showed similar results which indicated major problems in learning mathematics.

The work of Piaget (1963) has established the learner as a person who is trying to make sense of what is experienced. In mathematics, trying to recognise or master the concepts without appropriate use of models may well create enough pressure on limited working memory capacity. When the teacher teaches the concept of 'directed numbers', he or she can write a pile of numbers, some negative and some positive and says 'these are all directed numbers'. However, there is nothing that appeals to the senses that helps to set out the concept of directed numbers. Moreover, the statement about 'number line' is not available to the senses either and students in general have no anchoring idea in the long-term memory.

Russell's (1921) characterised pure mathematics as '*the subject in which we never know what we are talking about, nor whether what we are saying is true.*' Thus, the difficulties may arise because these concepts do not exist in the mind. Learners can learn where concepts are seen, sensed and experienced.

In looking at the phrase 'making sense of', dimension will almost certainly generate overload and yet it is this dimension which is the natural way of learning and understanding. Ausubel (1968) talks of meaningful learning where what is learned is 'internalised'. This is critical but the limitations of working memory may make such learning and internalisation very difficult. There are good arguments, therefore, for making the mathematics taught meaningful (Ali, 2008).

3.3.3 The Language Barrier and Working Memory

Language is the most important tool of communication. Teaching and learning depends heavily on language. Learning is not a passive absorbing of knowledge but an active process in which the learner is engaged in constructing meanings from text, dialogue and physical experience. However, the use of language could cause overloading in the working memory. Lack of clarity often comes in the form of language ambiguity. The work of Johnstone and Selepeng (2001) demonstrated that less familiar language places a burden on limited working memory resources while the later work of Durkin (1991) showed the specific problems language creates in mathematics with younger students and how this can be directly related to achievement.

Mathematics is full of words such as formula, index, limits, planes, inequality, roots, set, identities, etc. These words have been given precise technical meanings which are often closely related to but not identical to their everyday meanings. Words, whose meaning in everyday life might not be the same as the mathematical ones, create confusion in the learners' mind. For example, the word '*function*' is activity in everyday usage. However, in mathematics, it is used to denote a rule which 'relates' or 'maps' an input onto one output (written as $\mathbf{f: x \rightarrow 2x - 1}$**; or** $\mathbf{f(x) = 2x - 1}$**).** Again, in the ordinary usage, the word 'set' means, a set of shoes, a set of lines, a set of boxes, a set of oranges, and so on. However, in mathematics (in particular, algebra), a set is a collection of objects/numbers with a common defined property.

Learners use their previous language of concepts to interpret the new information which they receive. Familiar vocabulary changing its meaning as it moves into mathematics might cause confusion and lead to a misunderstanding of mathematical concepts. Cassels and Johnstone (1982) found that unfamiliar vocabulary, familiar vocabulary changing its meanings, pompous language where plain language would do, use of unfamiliar constructions all make learning difficult.

3.3.4 Mathematical Notations

Symbols and formal notations are an essential and fundamental ingredients of mathematics. They condense a hierarchy of concepts into 'manageable' form. However, this notation is sometimes a cause of confusion in the minds of learners. Consider the following notations:

$=$ equal to
$\approx$ approximately equal to
$\leq$ less or equal to

$\neq$ not equal to
$\sum$ sum of
∞ infinity
π pi
∂ an infinitesimally small part of ...
S_∞ sum to infinity, and many others.

These representations emphasise that the visible appearance of mathematics is determined by notations - often to be remembered cognitively at the same time. However, information processing models suggest this will pose problems because working memory has a limited capacity. Only teachers who have acquired efficient chunking strategies can cope. The learner may have to cope with grasping the meaning of the notation, understanding what is required and remembering the procedure to be adopted - all at the same time. This may prove to be cumbersome and, working memory will almost certainly overload. In addition, a key problem for the learner is the ability to distinguish whether a particular notation is appropriate or not to a particular mathematics problem.

3.3.5 The Complex Nature of Mathematics

From the earliest stage of learning mathematics, mathematical concepts are built in a hierarchical structure, and unless the earlier ideas of mathematics are mastered, new ideas cannot be (or are rarely) understood. A concept can be defined as a generalisation about related data, objects, or ideas. To give a precise definition is difficult because there are different kinds of concept and many of these have intrinsic characteristics. Among the first mathematical concepts developed by learners are the concept of ‘number’. Lumb and Papendick (1978) noted that the formation of a concept depends upon the process of abstraction (that is, the drawing away from the elements of the learners’ experience and discovering what they have in common) and the process of discrimination.

Thus, the learners when developing the concept of number must identify the common property in the various sets of objects and then be able to discriminate between sets containing other numbers of objects. Obviously, concepts are not inherited: they have to be gradually learned. In a mathematics lesson, there are often too many things to be manipulated cognitively at the same time - including previous learning. There are rules and formulas to be applied, skills to be recalled and verbal instructions to be used. In this situation, learners cannot discriminate what is important the ‘signal’ from what is peripheral the ‘noise’ (Jung, 2005).

Research by the National Training Laboratories (2005) in Bethel, Maine, USA on students memory shows the percentage of material that students can recall after engaging in different learning activities (see Figure 3.2).

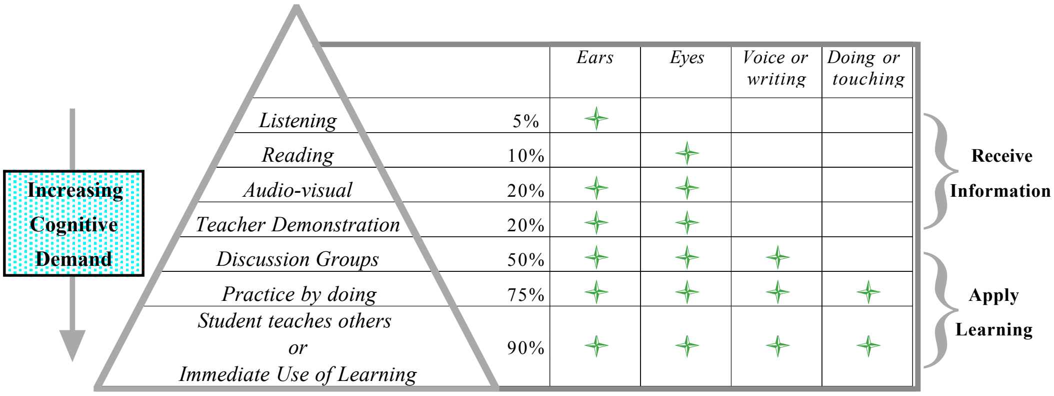


Figure 3.2: The Learning Pyramid
Source: National Training Laboratories (2005)

Their studies indicated that the activities with best recall rates:

- Require learners to actively practice and apply their learning.
- Require the learner to construct their own learning.
- Involve tasks that are higher on cognitive demand.
- Require the learner to process information in more parts of their brain, and to make use of more senses or learning styles (multi-sensory learning).

Much of this can be interpreted in terms of information processing (Reid, 2008). Indeed, understanding means that ideas have to be linked in the long-term memory. These links can only be formed securely when there is time and opportunity for the ideas and concepts to be applied in various relevant situations.

3.3.6 Level of Work

It is crucial for motivation that the level of work is such that each learner feels he or she has accomplished something of value in the lesson, and gets recognition for these accomplishments. Clearly, this can only be achieved if the level of work is matched to the ability of the memory capacity of each learner, and if the speed of the teaching matches the speed of the learning.

The whole area of how attitudes relate to learning is complex. There are attitudes related to mathematics as a subject, attitudes related to the way mathematical skills is taught and assessed, attitudes related to the teacher and the classroom climate.

Previous work has shown clearly that a key to the development of positive attitudes is that the learners perceive what they are doing as related in some real way to their lives in the context of their social and developmental settings (Reid and Skryabina, 2002a). This led Reid to suggest the idea of the '*applications-led*' curriculum (Reid, 1999, 2000). In such a curriculum, students are introduced to the mathematics that is needed to make sense of the world around as they know it, giving insights into the perspectives and methods of mathematical enquiry as well as its outcomes. *The key point is that the actual mathematics to be taught is determined by the applications used.* However, while such an approach has been shown to develop very positive attitudes with learners in the sciences, there is no certainty at all that this approach could be developed for mathematics. The working memory limitations make the introduction of applications at an early stage very difficult.

Satisfaction in learning comes when the learner can make sense of what is taught. In other words, understanding, as the natural goal for the learner, must be possible. In some very recent work in the sciences, Jung (2009) shows this very clearly. This was confirmed in a very different context. Oraif (2006) was exploring confidence in learning, mainly with senior school and university students. She was able to show that academic confidence was related to almost nothing other than perceived previous examination success. This developed very positive attitudes with the learners. The real question is how to make success possible for all. Not all will pass mathematics examinations, as currently practised. Indeed, the very 'black and white' nature of mathematics where answers are often seen as 'right' or 'wrong', with more or less no room for opinion, makes lack of success very apparent to the young learner. Learners tend not to develop very positive attitudes relating to a subject where success is persistently elusive.

Clearly the classroom climate can be a very rich experience. This will depend on a good teacher control, a teacher respect for the learners, an atmosphere of encouragement and expectation of success, and an atmosphere where learner difficulties are handled affirmatively. Thus, Schminke *et al.,* (1978) describe the need for empathy for the struggling learner and the need to encourage, support and affirm. Part of the art of teaching is knowing how to develop the potential of the learners to the maximum without generating a feeling of frustration and failure.

There is, of course, the whole problem of mixed ability. In all subjects at all levels, some students will progress at a greater speed with greater success. This is especially marked in mathematics. There are endless arguments about the need to keep the learners as one learning group or whether to form some kind of arrangement by which the learners are grouped by ability. Strong views are held and it has been noted that many educational thinkers are adept at gathering the evidence to support their view, sidelining the evidence to the contrary (Christou, 2001).

Much of the argument revolves around the balance between the generally accepted evidence that setting by mathematical ability seems to generate better mathematical performance against the social downside by which some learners tend to see themselves as failures and there is a lack of a social mix in the mathematics classroom. Perhaps, provided the social downside can be minimised, the gains in performance suggest that some form of setting by ability is helpful.

Some important pupils actions are identified in Table 3.1.

Lower Level	Intermediate Level	Higher Level
Memorising-recalling	Translating	Interpreting
Practicing	Comparing	Applying
Listening	Constrasting	Analysing
Watching	Proving	Generalising
Manipulating	Verifying	Synthesising
Drawing	Categorising	Conceptualising
	Classifying	Labelling
	Organising	Evaluating

Table 3.1 Learners-Actions or Learning Activities

Table 3.1 identified ways children tend to organise their thinking and learning in a fashion similar to the way they are taught, although this may not be general for all children. Relationships which are apparent are most easily accommodated into cognitive structure. From observations of various relationships within a discipline, children organise sets of principles with which they construct what Gagné (1974) terms the "*structure of organised knowledge about a topic.*"

To test this assumption, Gagné isolated a mathematical principle to be learned and identified the subordinate knowledge he believed necessary to the acquisition of that principle. Through testing, he found that learners without a grasp of the necessary subordinate skills were unsuccessful in demonstrating competencies at higher levels. His study affirms the need for carefully structured and sequential mathematical experiences.

3.3.7 Curriculum Structure

Mathematics is, of course, a discipline in its own right with its own ways of thinking, methodologies and intellectual rigour. By its very nature, mathematics underpins studies in many other subject areas including the sciences, engineering, medicine, as well as many areas in the social sciences and many careers in the world of finance. The service role of mathematics is quite enormous and this is an important factor in curriculum design.

The mathematics curricula for schools are often constructed by those who have left the classroom or may have limited classroom experience. Mathematics curricula are rarely constructed by those who are still practising teachers who have been successful in the study of mathematics, and committed to the teaching and learning of the subject to serve the need of the whole population, including those for whom mathematics may be neither attractive nor easy.

The curriculum designers are almost always drawn from those who are not involved in the actual teaching of mathematics who themselves may have demonstrated considerable skill and commitment in the field. The implication is that such curricula are usually designed around the logic of mathematics without an adequate consideration of the needs of those who will use or depend on mathematics later in life or, indeed, often with inadequate regard to the developmental needs of the learners. Al-Enezi (2006) observes that this can result in material being included in the curricula which poses problems for learners, and the teachers are then faced with developing ways to teach material which is not readily accessible to the learners at that age.

Ali (2008) identified another problem, this time in the area of assessment. Teachers do not decide national certification, and yet teachers may be blamed if their pupils are not successful enough. Ali then notes that "*this can lead to a dependence on the memorisation of procedures, understanding being a casualty.*"

In general, the difficulties in some aspects of mathematics are so widespread and occur across curricula, cultures and methodologies. It is therefore, unlikely that it is caused by teachers. This is because the teachers do not decide the curriculum. They are supposed to follow what has been designed and provided (textbooks, syllabus, etc.) by the authorities and they have little freedom to make many adjustments. The situation in many countries is that teachers have been reduced to technicians, charged with delivering a curriculum over which they have little influence and assessed by approaches which they cannot alter.

Figure 3.3 below portrays a hierarchy that represents the general order of development and presentation of concepts in school mathematics programs. The extent to which topics appear and the specific level at which they appear varies considerably from programme to programme. Nonetheless, threads of continuity in the areas of structure and proof tie the different programmes together, which include topics such as set theory, logic, functions, probability, co-ordinate systems, graphing, and number theory.

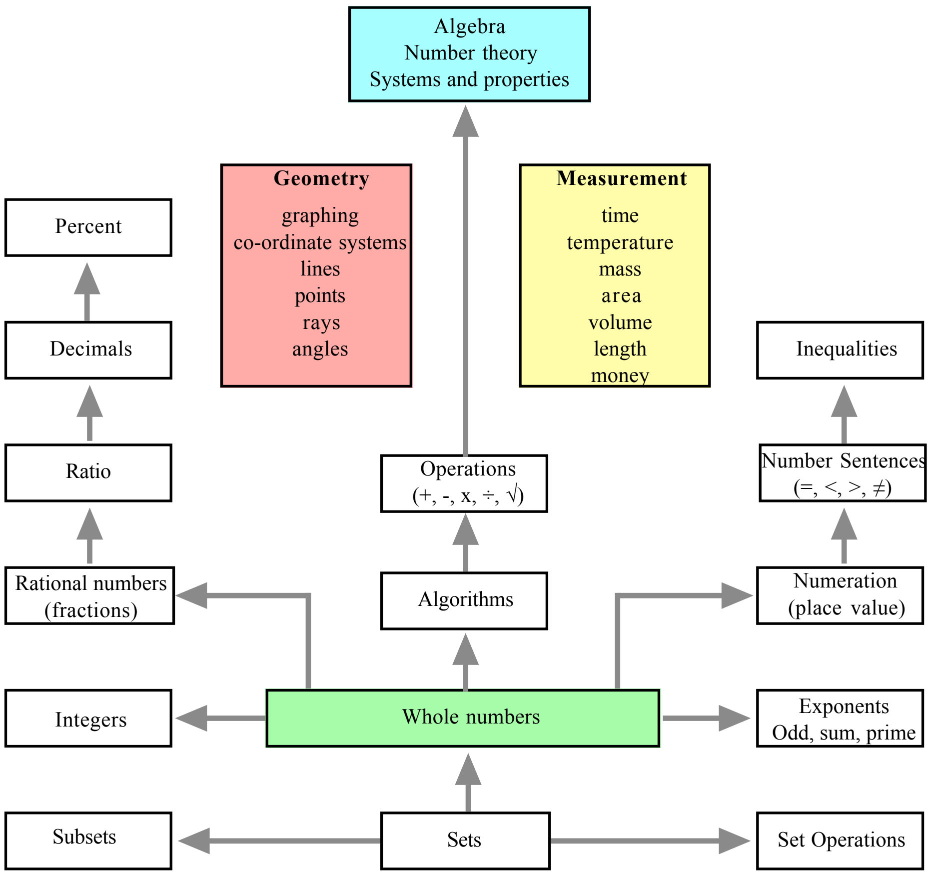


Figure 3.3 Hierarchy of School Mathematics Programme (source: Schminke, *et al.*, 1978)

3.3.8 Difficulty in Associating Meaning and Symbols

When a mathematical word problem is given to students, an amount of information is given to them. This information is received by their senses: they seem to read it fluently and yet it does not seem that they understand what they have read, and they often fail to establish an association between the meaning of a word and its graphic representation. In this case, the students have no necessary decoding skills. According to Information Processing Model already discussed in Chapter 2, working memory space is of major importance in mathematical word and applied problem procedures, and is possibly responsible for differences in students' problem solving performance (Hitch, 1978; Brainerd and Reyna, 1988).

Most of the study that has taken place in this field has demonstrated a high correlation between working memory space and performance in decoding and representing word problems. Just and Carpenter *et al.,* (1994) argued that many processes operate in a parallel way and that working memory is used as a common work place, where processes can place partial and final results. In like manner, Kintsch and Greeno (1985), refer to working memory as an important variable affecting comprehension of text propositions (like relational, assignment and question propositions), and succeeding in problem representation. In a very recent study (Akhtar, 2008), the problem of language and the overload of working memory were studied in relation to mathematics performance, showing some quite clear connections.

In the learning situation, the teacher's task is to help learners to extract meaning from their reading. First, it is necessary to find out if they grasp the meaning better when they read aloud or when they read silently; the more difficult mode (aloud or silent) is then dropped. A child who has difficulty in associating symbols with their meaning should read only a sentence or two at a time, and he should be told to try to make a mental picture of what he reads. However, in these processes many things are likely to produce overload of working memory space: difficult vocabulary, different kind of propositions, extraneous information or negative questions. At the same time, a problematic storing or retrieving from a long-term memory could lead to incorrect outcomes which could likely be caused by previous incorrect storing or retrieving of the wrong structure.

3.4 Improving Learners Achievement in Mathematics

The number of research studies conducted in mathematics education over the past three decades has increased dramatically (Kilpatrick, 1992). Some of the findings are discussed critically. He makes the point that the more opportunities students have to do mathematics, the better they tend to be in doing mathematics (McKnight 1987; Schmidt et al., 1997). This is utterly what might be expected.

There is a long history of research, going back to the work of Brownell (1945, 1947), on the effects of teaching for meaning and understanding. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on students' learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations. This is inevitable in that when a student genuinely understands, there is a reasonable prospect of applying knowledge in a new situation. Lack of understanding will make such application more or less impossible.

Students can learn both concepts and skills by solving problems. Research suggests that students who develop understanding early perform best on procedural knowledge later. The problem is that their acquisition of understanding may prove impossible as procedural skills are being mastered, simply because of working memory overload.

3.5 Chapter Summary

Mathematics learning difficulties are common, significant across cultures, curricula and methodologies, and worthy of serious instructional attention. Thus, there are not caused primarily by specific curricula, specific ways of teaching or even by the teachers themselves. There must be something intrinsic in mathematics and the way young learners approach it which underpins the widespread problems observed.

Mathematical concepts are hierarchical by nature. Before a learner can understand a higher-order concept, he must have a firm understanding of basic concepts as well as a grasp of a sufficient number of concepts of a similar order. Mathematics has its own peculiar language patterns and vocabulary, and a major part of the development of understanding of mathematics must focus on building up confidence in handling these and in connecting them with the corresponding mathematical symbols and manipulation of concrete materials.

The highly conceptual nature of mathematics places considerable demands on working memory. In order to 'make sense' of concepts, much information and experience must be held and manipulated cognitively at the same time. Working memory limitations may make this very difficult or almost impossible.

Levels of learning and objectives must be realistic and relevant to the actual needs of the learners. Merely presenting mathematical content to learners is not sufficient to ensure good teaching. Learners need a meaningful model to see and work with. Mathematical models take many forms. For example, multiplication may be shown as constructions to form arrays, equidistant moves on a number line, or as Cartesian products of even repeated addition with equal addends. Wisely selected, mathematical models help children to understand the mathematical notion being taught, how the model relates to real-life situations, and how it can be used to advance their knowledge of other mathematical topics.

The work of Piaget is particularly useful as he has put forward a comprehensive model of intellectual development describing the stages through which a learner progresses. This model has had a significant influence on the evolution of modern mathematics learning.

Learners' mathematical experience must progress through a sequence of abstraction:

- *Experience* with physical object;
- Spoken *language* that describes that experience;
- *Pictures* that represent the experience; and
- Written *symbols* that generalise the experience.

Very perceptively, Johnstone (1991) observes that difficulties can lie, "*in the transmission system itself, the methods used and the facilities available or the learners and the nature of their learning or even with the nature of the message itself*".

So far, we have examined why learning mathematics is sometimes difficult in the light of the information processing model for learning, it is also essential to look at attitudes as a possible factor that affects achievement in mathematics. Attitudes of teachers in encouraging learners; and attitudes of students which is thought to be critical in enabling them to be successful (Reid and Skryabina, 2002a) will be considered in the next chapter.

Chapter Four

Attitudes Related to Learning Mathematics

4.1 Introduction

About 40 years ago, the major focus of research in education was on educational objectives in the cognitive domain (human thinking or mental activities). Since 1975, interest in attitudinal studies in education has grown rapidly, and has become the focus of considerable research. "*It is almost universally acknowledged that educational objectives in the affective domain - those relating with attitudes, interests and values - are of great importance*" (Choppin and Frankel, 1976). However, the study of attitudes has undergone many important changes during that time, with different questions becoming the focus of theory and research.

Memory and information processing were the central themes in the previous chapters. Following the earlier work of Katz and Stotland (1959), Reid (2003) suggested that there are good reasons why, in general terms, people develop attitudes:

> *"Attitudes in life help us to:*
> - *Make sense of ourselves;*
> - *Make sense of the world around us;*
> - *Make sense of relationships."*

Attitudes generate the readiness to behave in specific ways, and the way we behave is controlled by the way we receive and process information. People appreciate the world in terms of knowledge, feelings and behaviour with a logical, rational wholeness of meaning, and determined acceptable patterns of social interaction.

Stainton and Rodgers *et al.,* (1995) have noted that psychologists have tried to answer four fundamental questions over the last 70 years:

"(1) Where do attitudes come from? How are they moulded and formed in the first place?
(2) How and why do attitudes change? What forces are involved and what mechanisms operate when people shift in their opinions about particular 'attitude object'?
(3) How can attitudes be measured?
(4) How do attitudes relate to behaviour? What is it that links the way people think and feel about an attitude object, and what they do about it?"

These are also the main focus of this study. In this chapter, the emphasis is on the first two questions, with special reference to learning in mathematics. In the following chapter, the emphasis is on some of the answers that have been offered to questions 3 and 4.

To learn a skill, learners must know what they are expected to be able to do, and how it is best done; they must know why it is best done that way along with relevant background information or explanation. They must have had the opportunity to practise; and have had the practice checked and connected. Each of these needs can be met by a multitude of learning experiences. This explains why the importance of creating the learning atmosphere which can generate students' positive attitudes toward learning was widely emphasised in the work of Ramsay and Howe (1969): "*A student's attitudes towards science may well be more important than his understanding of science, since his attitude will determine how he will use his knowledge.*" The same might be said of mathematics.

Information about students' attitudes toward mathematics can be considered as a necessary one to predict mathematics - related behaviours such as having interest or lack of it - that is, whether or not a student will have the desire to study the subject further and even in taking it for a career. The present chapter provides a general review of attitude literature: the definitions of attitudes, the functions of attitude, the process of attitude formation and change. However, before discussing attitudes, it is relevant to look at why attitudes are important.

4.2 Why Look at Attitudes in Mathematics

Durrani (1998), referring to England, stated that, "*The declining popularity of mathematics is a well known fact. The number of students studying mathematics at A-Level and in the University fell from 42% in 1963 to just 16% in 1993*". Simon (1995) suggested that "*Physics and Mathematics (at school) are only taken by students who do well and not as incidental or additional subjects*". The investigations carried out in many countries, for example, in Portugal, which have different curricula showed similar results which indicated major problems in mathematics (Borasi, 1990; Shoenfeld, 1985).

There is a tendency to think that the lack of positive attitudes towards mathematics and the low numbers opting for courses is common to all countries. However, in Scotland, mathematics is increasingly popular, as judged by uptakes at the Higher Grade (the university entry course). Scotland has had a long history of universal school education and had four ancient universities, open to all who showed the ability to gain benefit. This existed by the early 17th century with mathematics regarded as an integral part of this. The popularity of mathematics can be seen in Figure 4.1.

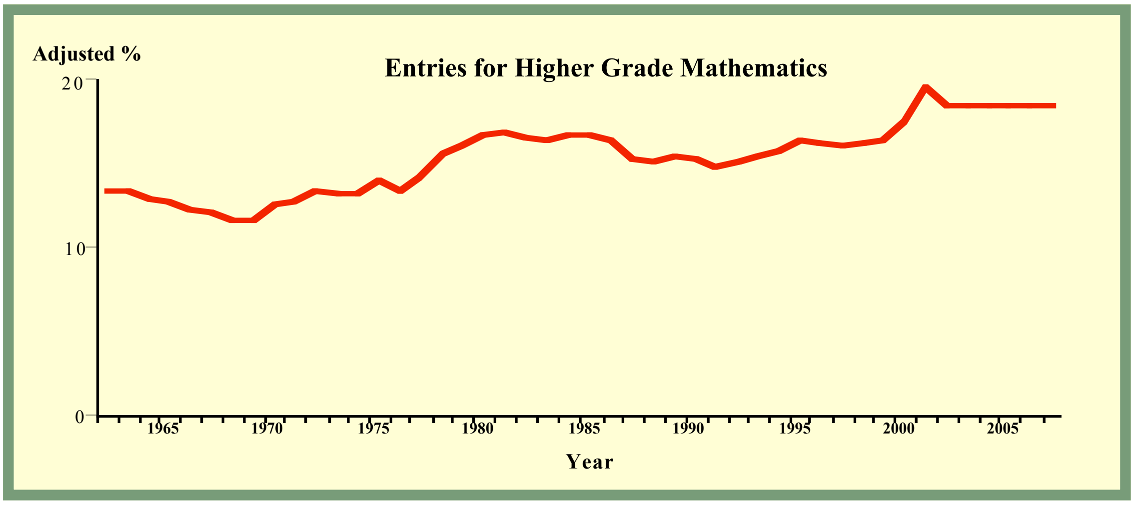


Figure 4.1 Entries for Higher Grade Mathematics in Scotland (Source: Scottish Qualifications Authority http://www.sqa.org.uk/sqa/24759.html)

It is interesting to speculate on what has caused this positive attitude. There is a full, well qualified, secondary school teaching force in mathematics. University uptakes are high, as are uptakes in subjects highly dependent on mathematics. The curricula at school are designed to educate in mathematics rather than just produce mathematicians. This has generated a population where mathematics is taken by large numbers to the final stages of schooling, where there is no great population of mathophobes, and where large numbers choose to go on to take degrees very successfully in mathematics and related areas.

Perhaps, this offers one key. Mathematics syllabuses are designed, inevitably, by mathematicians and they are committed to mathematics. It takes a major shift in thought to develop curricula which are specifically designed to educate *all* rather than focus only on the future potential mathematicians. This need not sacrifice rigour as the Scottish situation illustrates.

Nonetheless, the Scottish experience is not typical and, despite the relevance of mathematics in the school curriculum and the overwhelming interest in scientific and technological development for modern living, students' attitudes are often negative and may indeed be deteriorating in many countries. For these reasons, attitude development is fairly important in the learning situation: "*Attitudes of students regarding mathematics and mathematics teaching will be a significant factor underlying their school experience and achievement where the individual has neither the motivation nor the opportunity to engage in reasonable decisions.*" (Fazio, 1990).

Speaking of the physical sciences, Reid (2003) notes, "*The first thing to recognise is that students will develop their own attitudes - it's going to happen anyway! If we ignore attitudes* *in our thinking about teaching and learning, that will not stop the students developing attitudes ... If we think that our task is to communicate astronomy, chemistry, and physics ideas and nothing more, that will not stop attitudes developing. Thus, attitudes are crucial because they cannot be separated from study."*

Some studies conducted in the 1970s (e.g. Khan and Weiss, 1973) and in the 1990s (e.g. Oskamps, 1991) on possible factors which cause a person to acquire a particular attitude toward an object found two determining factors.

(1) Internal and personal determinants:
- Genetics and physiological factors
- Direct experience
- Element of choice

(2) External Influence:
- Parental influence
- School teaching
- Peer groups
- Mass media

According to Chaiken and Eagly (1993), these two groups correlate with each other and affect each other. Some internal variables (e.g. personal experience) may be the result of external variables, and some internal variables may interact with external variables in bringing about their effects, like certain behaviour of parents may exert on children of differing personalities. Thus, a person's attitudes towards mathematics can be seen as a learned disposition to evaluate in certain ways based on the manner in which the subject is taught, in which the curriculum is presented, and in which the instructions are presented.

Another study in Scotland conducted by Skryabina (2000) explored possible factors which can have direct or indirect influence on attitude formation towards physics: the teacher, classroom environment, subject instructions, pupils' socio-economic status and their religious background, pupils' gender and age, their achievement and personality, and curriculum (see Figure 4.2 overleaf).

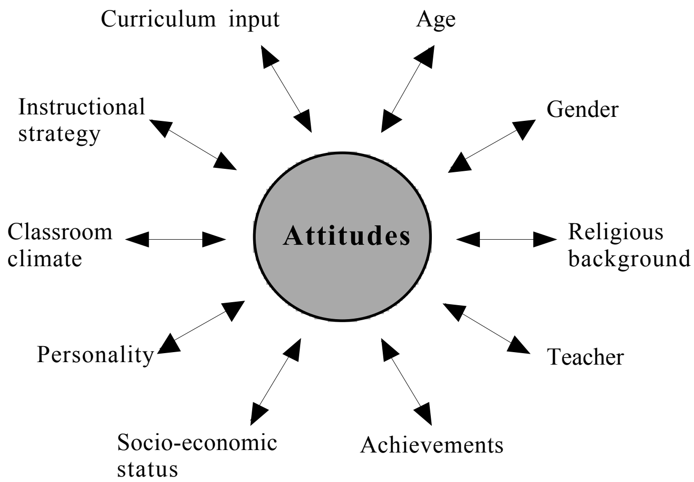


Figure 4.2 Factors Influencing Attitudes (source: Khan and Weiss, 1973)

Some of these factors may have a stronger or more direct effect on attitude towards mathematics, while some may be less influential, and this may vary from person to person. However, it is important to recognise that many of these factors are not easily modified in any way.

Thus, it may be possible to change the instructional strategy and the classroom climate. Most of the others are not open to change. For example, the teacher or researcher has very limited opportunity to change the curriculum in any way. This is usually determined outside the school. However, its delivery may be open to some change. Thus, it is important for mathematics teachers to provide learners with opportunities to develop attitudes in a more positive and structured way. Many researchers (e.g. Germann, 1988; Reid and Skryabina, 2002a; Borich, 2004; Jung 2005) found that teachers and their instructional methods play a very important role in forming learners' attitudes toward their studies. In particular, Smith (2007) suggested the characteristics of teachers who are able to foster gains in achievement and stimulate positive attitude to learning must aim to achieve many things, including:

- Beginning from where the learner is and lead him into the subject.
- Encourage learners to understand mathematics, rather than to memorise it, by drawing the link between concrete and abstracts.
- Using different methods and resources to motivate learning.
- Preparing learners to be informed citizens, able to make informed and rational decisions.
- Encouraging learners active involvement, experimentation and the use of visual aids.

This list is interesting but does not take us forward very much. It would appear that such teacher characteristics are those which describe the effective teacher in any subject area and they do not offer a clear way forward which is specific to mathematics.

As children grow up, not only content but also the way mathematical knowledge to be used become significant for many learners (Gardner, 1983). Thus, teenagers begin to be concerned about social issues. Several researchers (e.g. Donovan and Bransford, 2005) argue for the benefits of teaching mathematics in ways which permit the social and environmental consequences of mathematics to be considered. However, this will not be easy. The limitations of working memory capacity may make it very difficult to introduce consequences of mathematics while they are trying to master the skills and grasp the concepts.

A further problem which makes students give up interest in studying mathematics are its perceived difficulties. Many students state that they do not want to continue with mathematics because, "*it is too abstract, and too difficult. Consequently, mathematics is only taken by students who do well and this reinforces the notion that mathematics is difficult and is, therefore, only for the intelligent*" (Al-Enezi 2006).

Finally, the danger is to put all the blame on mathematics teachers. As noted earlier, mathematics curricula are rarely designed by practising teachers. Teachers are supposed to follow the syllabus, textbooks, and other resources provided by the authorities and they also do not have much freedom to make any amendments or changes in the provided curriculum.

Many years ago, Katz and Stotland (1959) argued that attitudes serve functions for individuals. By implication, attitude development will take place as the individual perceives, consciously or subconsciously, that the new attitude position has an advantage. They discussed in some detail the main types of functions which attitudes serve and Reid (2003) simplified this. A possible way is to consider that attitudes,

(a) Help us to understand ourselves - our goals and values;
(b) Help us to understand the social world we live in by structuring or organising information we encounter. When well established, they are capable of guiding information processing;
(c) Allows people to express their fundamental values. By expressing attitudes towards issues or people one feels about, one validates their own self-concept;
(d) Help us to adjust in a complex world by making it easier to get along with people who have similar attitudes.

These features have implications for mathematics. The aim of school courses is not only to produce academic mathematicians, for few will pursue such careers, but also to educate people in mathematics, its ways of thought and enquiry, its applications in many areas of life, and its huge contributions in science, engineering and technology to human understanding of the world.

Considerable research has been carried out in the areas of attitudes towards subjects like chemistry and physics (see Reid and Skryabina, 2002a and 2002b), and attitudes towards the study itself (e.g. Perry, 1999). In these areas, the major concern among educators is how to encourage positive attitudes because, without interest in the subject being studied, it is likely to be difficult for the learner to be motivated to learn. What is to be taught and how it is taught might create major influences on learners' attitudes. In looking at the attitudes towards the study itself (process of learning), there are skills for effective learning and it is essential to look at attitudes towards learning these skills and as well as using them. Learners need to develop critical understanding about the nature of knowledge and how it is gained, about approaches to successful study, and about the nature of learning as a lifelong process.

For these reasons, attitude development is fairly important in mathematics education. Moreover, mathematics teachers should consider that learners will develop attitude no matter what they do. It is therefore, important for the teacher to provide learners with opportunities to develop attitude in a more structured and positive way. Mathematics is sometimes taught in a way that is fairly cognitive and theoretical. There are high tendencies of misunderstanding about mathematical ideas and learners' attitudes could be based on incorrect or mere partial knowledge. Every mathematics teacher wants learners to make intellectual sense of the world around them. It is the very nature of the work of mathematics. Without careful attention to attitudes, the teacher might encourage the development of attitudes that is undesirable or unfulfilling.

4.3 Definitions of Attitudes

The concept of attitudes has a long history and has had a central place in social research. Allport (1935) noted that, "*Attitude is the most distinctive and indispensable concept in social psychology*", and Eagly and Chaiken's (1993) review of attitude research showed that this held true up to now.

Although, there are numerous definitions of the concept, most researchers would agree that attitudes refer to a general or rather stable orientation towards an object. An 'attitude object' refers to an entity to which one can respond positively or negatively: for example, social groups, individuals, inanimate objects, social and individual actions, concepts, social issues and so on.

Numerous descriptions have been offered:

> *"The affect for or against a psychological object."* (Thurstone 1929)
>
> *"Certain range within responses move."* (Likert, 1932)
>
> *"A mental and and neural state of readiness, organised through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related."* (Allport, 1935)
>
> *"A stable or fairly stable organisation of cognitive and affective processes."* (Katz and Sarnoff, 1954)
>
> *"A concept with an evaluative dimension."* (Rhine, 1958)
>
> *"A learned orientation, or disposition, toward an object, which provides a tendency to respond favourably or unfavourably to the object or situation ..."* (Rokeach, 1968)
>
> *"A state of readiness or predisposition to respond in a certain manner when confronted with certain stimulus ... attitudes are reinforced by beliefs (the cognitive component), often attract strong feelings (the emotional component) which may lead to particular behavioural intents (the action tendency component)."* (Oppenheim, 1992)
>
> *"Tendency to evaluate an entity into some degree of favour or disfavour, ordinarily expressed in cognitive , affective, and behavioural responses."* (Eagly and Chaiken, 1993)
>
> *"...express our evaluation of something or someone. They may be based on our knowledge, our feelings and our behaviour and they may influence future behaviour."* (Reid, 2003)
>
> "*A core human individuality.*" (Chu, 2008)

The common element that runs through most definitions, however, is 'the readiness to respond' to a situation. This readiness can refer to '*mental attitudes*' (Spencer, 1862) and the ability to interpret correctly what is being said, as a result of holding those attitudes.

Another key element noted by Skryabina (2000) is that a person who has certain knowledge about an attitude object will not have an attitude towards it until the evaluative response about this object occurs. The evaluation of an attitude object can done on a cognitive, affective or behavioural basis or any combination of them.

It is impossible to make a list of all definitions and descriptions of attitudes in the literature as there has been a tendency for each researcher to emphasise different aspects according to the context in which attitudes are considered and the point of view of a person defining it. For the basis of the present study, the definition given by Chaiken and Eagly (1993) and that stressed by Reid (2003) above were adopted. These focus on evaluation and the nature of attitudes as involving knowledge, feeling and experience as well as the power of attitudes to influence future behaviour. However, it is worth discussing attitude models which are relevant in constructing attitude theory.

4.4 Attitude Models

There are two main models of attitudes - the *tripartite model* and the *unidimensional model.* The first is a three-component model. According to this, an attitude is a theoretical construct which can be accessed only by means of three distinguishable reactions to an object. These classes of reactions are: cognitive, affective and behavioural responses.

Many (e.g. Rosenberg and Hovland, 1960; Eagly and Chaiken, 1993; Bagozzi and Burnkrant, 1979; McGuire, 1969) have also noted that attitudes have three components. These components may be present in varying degrees and proportions and can be described (Reid, 2006):

"• *A knowledge about the object, the beliefs, ideas opinion components (cognitive);*
• *A feeling about the object emotions like or dislike components (affective); and*
• *A tendency towards actions intentions, the object component (behavioural)."*

This three-component model, which is much more a model of attitude structure and development than a simple definition, can be illustrated as in Figure 4.3.

An attitude is often considered as an intervening variable between observable stimuli and responses. This reflects the influence that behaviourism was still having, even in social psychology, at the start of the 1960s (Gross, 2005). However, the model is still valid today.

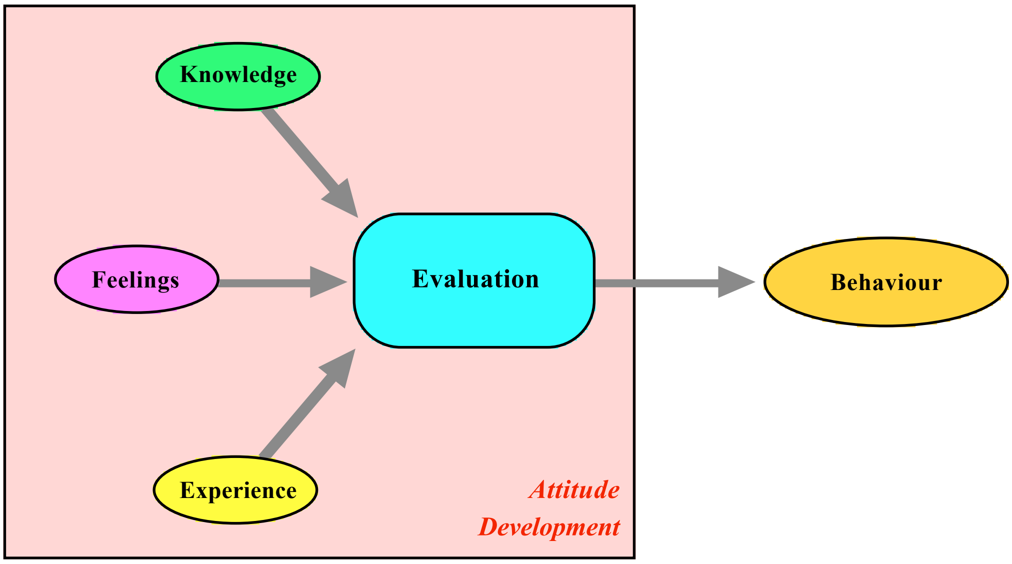


Figure 4.3 A Three-component view of attitudes

Unidimensional models of attitude stem from the recognition that the three components of attitudes described above are not always highly correlated, if at all. Clearly, individuals do not always act in ways consistent either with their beliefs or their emotions.

Unidimensional models regard the affective component as the only reliable indicator of the orientation towards the attitude object and use the terms emotions and evaluation interchangeably. Thus, Petty and Cacioppo (1981) argued that "*the term attitude should be used to refer to a general, enduring positive or negative feeling about some person, object or issue*".

However, what does *correlation* mean? Things do not have to be correlated to all been involved. A little thought shows that we cannot develop an evaluation of anything without some knowledge. An attitude founded on inadequate knowledge might be regarded as a prejudice but even prejudices are based on some knowledge. An attitude, being evaluative, must involve some element of like-dislike, approval-disapproval. This has some affective character. An attitude which is purely emotional, can scarcely be called evaluative while the total absence of any affect reduces an attitude to something akin to evaluating knowledge (rather like what a student might do at the end of a mathematical calculation: is the answer reasonable?) and this is not an attitude but closer to the evaluation described by Bloom *et al.,* (1956).

One of the most powerful influences on attitude development is experience: the behaviour of others in the past and present as well as the behaviour of the person involved. Overall, it is difficult to argue against the involvement of knowledge, affect and experience in attitude formation, development and change. These three may inter-correlate or they may not. McGuire (1969) regards attitudes as responses that locate '*objects of thought*' on '*dimensions of judgement*', while Zanna and Rempel (1988) define attitudes as the categorisation of the attitude object along an evaluative dimension. However, neither of these presume a unidimensional or multidimensional model. Reviews of empirical studies carried out by Chaiken and Stangor (1987) and Eagly and Chaiken (1993) showed that there is no conclusive evidence to support either of the two models.

Factor analytic studies (a complex statistical technique based on correlation used to calculate the number and nature of factors within a test) showed that it is difficult to distinguish between the three components of attitudes (McGuire, 1969) whereas other studies (e.g. Gross, 2005) claimed with certainty that the three components are strongly interconnected.

Ultimately, the experiences of people determine their attitudes. As attitudes develop, cognitions become more differentiated, integrated, and organised, and affect and behavioural intentions become associated with these conditions. Thus, it can be said that an attitude can be formed through cognitive, affective or behavioural processes exclusively or through different combinations of them (Zanna and Rempe, 1988).

4.5 Theories of Attitude Change

In reviewing attitude change literature, it soon becomes clear that there are theoretical dimensions of attitude change. According to Skryabina (2000), two extreme dimensions of this process can be defined:

"• *The internal dimension - where attitude is changed mostly due to motivation, desire and control of an individual.*
• *The external dimension - where attitude is changed mostly due to pressure from outside (e.g. new information) and which forces a change in attitude. This type of attitude change is not always under the control of the individual."*

There are many examples and models explaining attitude change in terms of the internal dimension in the literature, but only one of them (*cognitive dissonance theory*) will be discussed in that it has possible relevance to education. The idea of dissonance was developed by Festinger (1957). He described the situation where behaviour was inconsistent with attitude as *cognitive dissonance,* and the situation where behaviour and attitude were consistent as *consonance*.

According to cognitive dissonance theory, whenever we simultaneously hold two cognitions which are psychologically inconsistent, we experience dissonance. This is a negative drive state, a state of "*psychological discomfort or tension*", which motivates us to reduce it by achieving consonance (Gross, 2005). Attitude change is a major way of reducing dissonance. Cognitions are "*the things a person knows about himself, about his behaviour and about his surroundings*" (Festinger, 1957) and any two cognitions can be consonant (A implies B), dissonant (A implies not-B) or irrelevant to each other. For example, the cognition 'I smoke' is psychologically inconsistent with the cognition 'smoking causes cancer' (assuming that we do not wish to get cancer).

Cognitive dissonance theory was developed on the basis of the Heider's balance theory (1958). Balance theory propose that internal inconsistencies tend to lead to internal instability (a very uncomfortable state for a person), and this instability can be observed through overt behaviour (Skryabina, 2000). She noted that attitude change can be considered as one of the outcomes of reducing this instability.

In general, Festinger considered dissonance as a psychological state that leads to arousal (arousal is observable) and stressed the importance of dissonance, "*as essentially a motivational state that energises and directs behaviour... Just as hunger is motivating, cognitive dissonance is motivating. Cognitive dissonance will give rise to activity oriented towards reducing or eliminating the dissonance. Successful reduction of dissonance is rewarding in the sense that eating when one is hungry is rewarding"* (Festinger, 1957).

(1) *Dissonance following a decision* - If we have to choose between two equally attractive objects or activities, then one way of reducing the resulting dissonance is to emphasise the undesirable features of the one we have rejected. This adds to the number of consonant cognitions and reduces the number of dissonant ones.

(2) *Dissonance resulting from effort* - When a voluntarily chosen experience turns out badly, the fact that we chose it motivates us to try to think that it actually turned out well. The greater the sacrifice or hardship associated with the choice, the greater the dissonance and, therefore, the greater the pressure towards attitude change (the suffering-leads-to-liking effect).

(3) *Engaging in counter-attitudinal behaviour* - This aspect of cognitive dissonance theory is of most relevance to the relationship between attitudes and behaviour. It suggests that a person can reduce or eliminate dissonance by changing the existing elements of knowledge to make the previous cognitive system and newly obtained knowledge consistent, with resultant change in attitudes and behaviour.

The point is that whichever method or way of reducing dissonance is applied, it is possible to say that the resulting attitudes lead to greater internal mental consistency.

Experiencing dissonance, feeling uncomfortable with the previously held attitude position and working towards restoring the condition of balance and stability, the person will readjust the system of cognitions and adopt the attitude which makes him feel comfortable, and which he will be able to defend (Skryabina, 2000). Dissonance seems to be a natural process of life. From time to time, a person will be exposed to new information or experiences which will disturb the consonant attitudes held. There will be a need to reduce the dissonance. Looked at logically, there a several ways to do this:

(a) Ignore the new dissonant input (information or experience).
(b) Emphasise as many consonant elements of information and experience so that the new information or experience is, in relative terms, small in impact.
(c) Keep the new dissonant information or experience separate from previous information or experience or attitudes (known as compartmentalisation).
(d) Allow attitudes to develop so that the new information or experience is now consonant.

A learner in the classroom can be put into a learning environment which can cause some dissonance in his system of cognitions (previously held beliefs or attitudes). For example, a learner was forced to learn a mathematical procedures which he did not like, that is, his attitude towards the procedures was negative. Later, a learner might find that the mathematical procedures were easy, interesting, and that the teacher was enthusiastic. The procedures and the real atmosphere of the lessons do not match the learner's beliefs about the learning, and this may cause dissonance.

The learner might, following the list above,

(a) Decide to ignore the new positive experience, perhaps arguing that this is not typical;
(b) Remind himself of the many bad experiences in mathematics classes;
(c) Keep the new dissonant information or experience separate from previous information or experience or attitudes simply by arguing that mathematics is good with this teacher but that is not the norm; and
(d) Develop more positive attitudes in relation to mathematics.

Persuasion is considered as a method related to the external way of attitude change. It is important to recognise that psychologists use the word '*persuasion*', without any manipulative or pejorative overtones. They talk of '*messages*' given to subjects which may or may not '*persuade*' them to change attitudes.

Laswell (1948) argued that, in order to understand and predict the effectiveness of one person's attempt to change the attitude of another, we need to know. "*Who says what in which channel to whom and with what effect*". Similarly, Hovland and Janis (1959) say that we need to study the:

- Source of the persuasive communication: the communicator
- The message itself
- Recipient of the message or the audience
- Situation or context

Gross (2005) outlines a basic paradigm in laboratory attitude-change research. This involves three stages:

(1) Measure people's attitude towards the attitude object (pre-test).
(2) Expose them to a *persuasive communication* (verbal or visual communication, experience).
(3) Measure their attitude again (post-test).

If there is a difference between pre-test and post-test measures, then the persuasive communication is judged to have 'worked'.

This approach began with Hovland *et al.,*'s (1953) proposal that the impact of persuasive messages can be understood in terms of a sequence of process:

Attention to message → Comprehension of message → Acceptance of conclusion

If any of these fails to occur, persuasion is unlikely to be achieved.

McGuire (1969) proposed a longer chain of processes. We should ask if the recipient:

(1) Attended to the message;
(2) Comprehended it;
(3) Yielded to it (accepted it);
(4) Retained it;
(5) Acted as a result.

Similarly, the failure in any one of these steps will cause the sequence to be broken. In looking at the five processes, it follows that attitude change is brought about by means of a new message and this incoming information should be comprehended by the recipient, then it is evaluated and integrated. Of course, related cognition and attitudes stored in the long-term memory are retrieved and influenced in this process. However, it has to be noted that the final stage is nothing to do with an attitude. This is the outcome arising from an attitude, perhaps providing evidence that the attitudes has changed.

The two main routes of multi-store information processing (central and peripheral routes) discussed in Chapter 2 are influenced in the process. Individuals who follow the central route work hard at evaluating the message logically. Attitude change through the central route is due to careful reasoning and argument quality. Conversely, the individual who follows the peripheral route depends on peripheral cues to evaluate the message. According to Jung (2005), this route does not involve any active thinking and attitude can be changed just under the influence of emotions or impressions such as source attractiveness and prestige.

Jung was primarily thinking of what might be called the social attitudes arising from various topics in the curriculum of biology, chemistry and physics. Today, these might include: the ethical implications of genetics research, the problems of production economics and the environment as well as nuclear energy. Of course, there are also attitudes in relation to specific disciplines and to learning in general.

In the context of mathematics, it is important that learners develop positive attitudes towards their study and most of the research has focussed on this. Mathematics does not generate the same kinds of social issues which are so apparent in the sciences.

Nonetheless, it is important that the learners see where and how mathematics fits into lifestyle and society, to appreciate that without mathematical insights, many of the great and important developments in society could not have taken place. This is an area which has not been studied much.

In context of learning science, and helping students to develop positive attitude, Jung (2005) suggested that the teacher should try to make their teaching materials fulfil the following necessary conditions:

"• *The message source must have high credibility.*
- *The message must be of high quality.*
- *The perceived relevance must be high to encourage good comprehension.*
- *Time and opportunity must be allowed for message-related thinking.*
- *Message related thinking to give attitude change only occurs when subject possesses sufficient motivation and ability to process message."*

These suggestions could also be practical to mathematics teachers in the development of positive attitudes. In particular, many researchers (e.g. Germann, 1988; Reid and Skryabina, 2002a; Jung, 2005) have investigated and found that teachers and their instructional materials play a prominent role in forming and developing students' attitudes towards school subjects. This is likely to be true also for mathematics, which indeed demonstrates the impart of teachers on attitudes.

4.6 Dimensions of Attitudes

Triandis (1971) summarised the dimensional characteristics of attitude: the *positive* versus *negative* affect, and *seeking* versus *avoiding* contact. They both describe the topology of behaviours on how the individual is going to respond to learning. Figure 4.4 shows this conceptualisation and includes some behaviours to illustrate how they would be positioned in this two-dimensional space. Any behaviour can be conceived as involving a certain amount of:

(a) Seeking or avoiding contact, and
(b) Positive or negative affect.

These two dimensions have a major impact in the learning process as learners would either develop positive/negative attitude (or interest or lack of it). Positive attitudes, beliefs, or actions associated with learning are more likely to help the learner to seek to learn more; however, negative affect would lead to avoidance of learning. Each of these two dimensions can also range from extremely positive to extremely negative (extremely seeking or extremely avoiding contact).

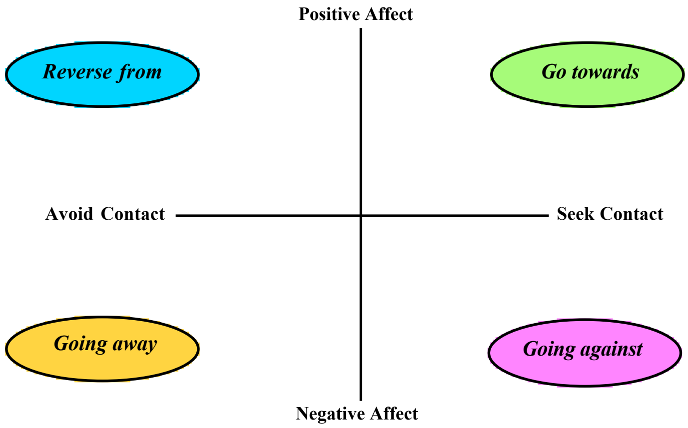


Figure 4.4 The Two Basic Dimensions of Behaviour Toward Attitude Objects (based on Triandis, 1971)

The two dimensions (positive-negative affect, and seeking-avoiding contact) are considered as independent. A person's attitude determines his behavioural intention, that is, what he would do toward an attitude object.

Overall, these dimensions might be able to describe individual learning characteristics. Any observant mathematics teacher will be able to see that different members of the class seem to work and behave in different kinds of ways. Some find mathematics more interesting and enjoyable, while others seek to avoid it because of their held attitudes.

4.7 Attitude Evaluation

In Section 4.4, attitude models were discussed. Attitudes seem to involve the cognitive, affective and behavioural responses, as Figure 4.3 shows.

> "*Attitudes is an evaluative state that intervenes between certain classes of stimuli and certain classes of response*" (Chaiken and Eagly, 1993).

People respond to stimuli that denote attitude objects with evaluation. Evaluative responses are those that express approval or disapproval, favour or disfavour, liking or disliking, approach or avoidance, attraction or aversion, or similar reactions. This categorisation is similar to the attitude dimensions discussed in the previous section. In the process of bringing evaluative response to attitude object, people develop their attitude and this process can be done on the cognitive, affective or behavioural basis or a combination of them.

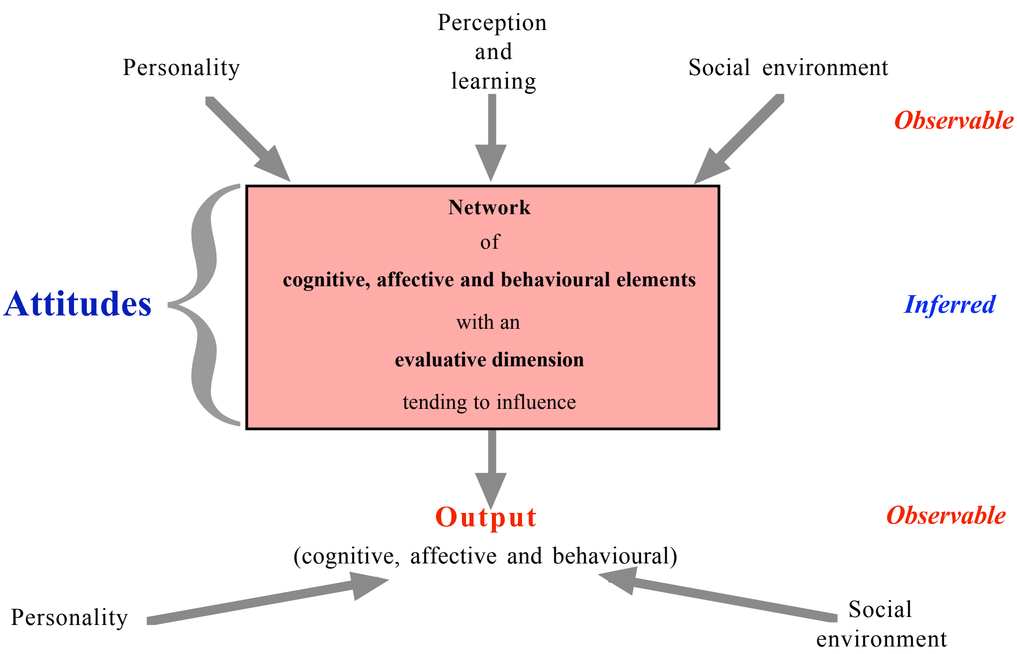


Figure 4.5 Attitudes, their Nature and their Measurement (derived from Reid, 1978)

The key thing is that, while attitudes may be influenced in many ways and may influence behaviour in quite complex ways as Figure 4.5 shows, they are constructs. Attitudes, therefore, cannot be measured directly. They can only be inferred by outcomes, to be seen in terms of behaviour. They cannot be measured directly since they are latent constructs. The only way to evaluate the attitudes of people is to observe their responses under certain stimuli regarding an attitude object.

The cognitive category contains thoughts, knowledge, or beliefs people have about the attitude object. Beliefs can be defined as "*associations or linkages that people establish between attitude objects and their various attributions*" (Fishbein and Ajzen, 1975). Learners can build cognitive attitudes directly or indirectly by the experiences they go through in their studies or normal life. Direct experience might be as a result of direct involvement with an attitude object. For example, if mathematics as a subject is considered as an attitude object, then a student learning about the subject believes about what he is doing and experiencing. His direct involvement might be associated with beliefs like: mathematics involves problem solving; relevant for modern society; applicable to real world; describes nature and its law; too abstract; too difficult. Indirect experience is the knowledge gained about an attitude object without engaging in direct relationship with the attitude object. An example of indirect experience is a student who obtains information about mathematics from friends, watching TV programmes or family without experiencing any classes in mathematics.

In general, a student who likes mathematics will be likely to say that it is very good to develop mathematics skills, learn how to apply mathematics, learn about nature and its laws. On the other hand, a student who dislike mathematics is more likely to associate it with negative attributes. Evaluation of these beliefs can be carried out on a scale from extremely positive to extremely negative.

The affective process is another type of response by which attitudes can be evaluated. The component consists of feelings or emotions that people have in relation to attitude object. Again feelings and emotions can range from very positive to extremely negative and therefore, have an evaluative meaning.

Chaiken and Eagly (1993) noted that people who "*evaluate an attitude object favourably are likely to experience positive affective reactions with it and unlikely to experience negative affective reactions, whereas people who evaluate an attitude object unfavourably are likely to experience negative affective reactions with it, but unlikely to experience positive reactions*". For example, students' responses about their mathematics lessons can be considered: '*I like mathematics lessons because: lessons are interesting, the lessons are easy, I like the teacher'. 'I dislike mathematics lessons because: lessons are boring and unenjoyable'.* All of these are affective ways of attitude manifestation.

The behavioural component emphasises person's actions in relation to attitude object, usually generated by past experience. Like cognitive and affective domains, because these responses range from extremely positive to extremely negative, they can be located on the evaluative dimension of meaning too. Behavioural intentions can also be considered as types of behavioural responses although they are not necessarily expressed in overt behaviour. As a typical example, by observing a student doing mathematics in a lesson, it may be possible to evaluate what kind of attitude towards the subject this student held. People who evaluate an attitude object favourably tend to engage in behaviour that support it, whereas people who evaluate an attitude object unfavourably tend to have the opposite tendency (Jung, 2005).

Not many teachers are enthused by behaviourists models, in which the focus of teaching is primarily observable behaviours rather than mental processing. The behavioural objectives movement has been particularly influential in special education (Ainscow and Tweddle, 1988), and in mainstream practice there has also been a trend towards setting and assessing precise learning goals and targets. The sterility and mechanistic nature of such approaches, however, has resulted in renewed interest in cognitive processes that appear to underpin learning (Elliot, 2000).

4.8 Attitudes and Achievement

Attitudes can be thought of as a blend or integration of beliefs and values. Beliefs represent the knowledge or information we have about the world (although these may be inaccurate or incomplete) and, in themselves, are *non-evaluative.* Research on attitude establishes the influence beliefs and values have on learning and students' achievement across academic discipline and at all levels of schooling. Many have found a relationship between attitude to learning and achievement (Eisenhardt,1977; Fraser, 1982; Schibeci and Riley, 1983).

In the work of Schibeci (1984), he argued that the student who achieves a good level in any subject because he/she has positive feelings means that a positive stimulus was held by the student to achieve that level. Such a positive stimulus may have arisen from positive attitudes based on previous learning. Future learning may well lead to further positive attitudes. Schibeci (1984) described this as a "*two-way relationship between attitude and achievement*". While it is very apparent that positive attitudes and success are related, however, it is more difficult to ascertain 'what is influencing what'. Indeed, they might simply influence each other or happen to be present together in many individuals.

Skryabina (2000) noted that both processes 'attitude influences achievement' and 'achievement influences attitude' could be explained using the theories from social psychology: achievement in the subject forms positive feelings about the subject and thus can be associated with positive stimulus. Positive stimulus associated with the attitude object (mathematics subject) will likely form positive attitudes towards the subject (operand conditioning), while positive attitudes retain attention, interest and motivation to study (mathematics), and this may lead to a good achievement. Thus, a good academic achievement might contribute to positive attitudes. Conversely, in the same way, as tall people tend to have big feet, then it may simply be that academic success and positive attitudes towards learning go together in general (Oraif, 2006).

Several other education researchers (e.g. Heider, 1946) have attempted to account for the correlation between attitudes and achievement. Very early, Heider argued that, if a person has a positive attitude towards learning, there will be a tendency for the person to achieve. On the other hand, a negative attitude will tend to yield negative results - known as law of cognitive balance or cognitive consistency.

Numerous studies followed where correlations between positive attitudes and higher achievement were often, but not invariably, found. However, the correlation values were not very high. Wilson (1980) conducted a meta-study in attitude-achievement interrelationships and found a mean correlation coefficient of 0.11 in 123 studies. In the light of this, Fraser (1982) concluded that, "*if teachers want to improve achievement, they would be well advised to concentrate on achievement 'per se' instead of trying to improve achievement scores by improving attitudes*." However, the low correlations may simply arise because attitude measurement, at an individual level, is a very imprecise science.

There is an alternative explanation: both attitudes and achievement are related to working memory capacity. Attitudes which are held in the long-term memory may influence the perception filter and can also affect what the learner allows to enter and process in their working memory because learners learn only what they want to perceive. Such attitudes held can influence future learning and achievement. Attitudes are not only aspects of learning, but also of thinking and problem-solving.

The whole issue of the achievement-attitude linkage is perhaps not as fundamental. Reid (2003) argues that students' attitudes are far more important than simply their possible impact on achievements. The argument is that, while much of what is learned is soon forgotten, the attitudes developed towards subjects, learning and themes in the curriculum may well remain throughout life. Thus, positive attitudes towards mathematics and its role in life and society may be far more important than encouraging positive attitudes simply to encourage good examination grades.

While there is a general picture indicating a positive association between attitude towards the subject and achievement in this subject and, specifically, students who are favourably inclined towards mathematics tend to be relatively serious-oriented, realistic and independent (Borich, 2004), the whole area of attitude development is much wider and of longer term significance. The role of the teacher is critical. Thus, Petty (2004) found that teachers and their instructional methods play a significant role in moulding students' attitudes towards their subject. This attitude may have a long term significance for career choice and attitudes to study in general.

In general, learning situations should not only help learners to plan their future on the basis of true comprehension about mathematical knowledge but also allow them to develop attitudes on a sound cognitive basis. The two models occur simultaneously in real educational situation and interact with each other continuously.

4.9 Attitudes to Mathematics

The understanding of the nature of mathematics learning in the affective as well as cognitive domains is now widely recognised (Leder, 1985). Large scale surveys of student performance in mathematics, such as the (American) National Assessment of Educational Progress (NAEP) and the Third International Mathematics Studies (TIMS) include items designed to measure students' attitudes to mathematics, although their validity is open to question.

The influential handbook of research on mathematics teaching and learning (Grous, 1992), devoted considerable space to the impact on mathematics learning of affective factors, for example, student and teacher beliefs and attitudes. Documents such as the national statement on mathematics for Australian schools (Australian Education Council, 1991) also recognise the importance of students' attitudes towards learning.

In these settings, attitudes are perceived as being closely linked to beliefs, emotions, and motivation to engage in the subject. Kilpartrick (1992) has argued that research in mathematics education has been greatly influenced by the disciplines of mathematics and psychology. Certainly, those concerned with the link between students' attitudes towards, and the learning of, mathematics have often relied heavily on the work done in the earlier field.

4.10 Chapter Summary

This chapter has explored a little about the nature of attitude: the meaning of attitude, attitude and achievement, attitude evaluation, attitude formation, and the cognitive nature of attitudes.

The three-component model of attitude structure sees attitudes as comprising affective, cognitive and behavioural components. Attitudes have much in common with beliefs, interests and values, but they need to be distinguished.

Attitudes are the core human individuality. A learner's attitudes guide his perceptions, feelings, and behaviour towards a subject, which of course influences his attention in the class, his motivation of learning, his use of categories for processing information, and his interpretation about recall and judgement.

Again, in the context of academic learning, attitudes influence the learning process continuously. Learners' attitudes may determine whether they display their ability completely or almost not at all in the learning process. Indeed, the way a learner sees himself or herself in relation to learning is a dimension which might have an important role on the learning itself. Favourable attitudes make learners attend to new information positively. Learners who have positive or favourable attitudes towards the subject being studied are willing to spend time in studying that subject and make every effort to understand the message.

Moreover, attitudes can be affected by learning experiences the learner has although attitudes are resistant to change. Therefore, it is the desire of all educators to make their students interested in mathematics by encouraging learning situations which allow learners to develop attitudes on a sound cognitive basis. The way to make the most of this interaction should be developed in order to achieve both affective and cognitive objectives in mathematics education.

There has been no discussion here about method of attitude measurement and this is the focus of the next chapter.

Chapter Five

Methods of Attitude Measurements

5.1 Introduction

Attitude measurement has generated a considerable controversy for many years for it was thought that attitudes could not be measured. Nonetheless, "*if attitudes lead to behaviour, then we aim to measure behaviour and then deduce what the attitude might be.*" (Reid, 2003). Indeed, all modern advertising seeks to change attitudes so that purchasing habits are changed in favour of the advertised product. The attitude may not be seen. Nonetheless, the change of attitude can lead to changes in behaviour.

Being described as a 'latent construct', it is obvious that any knowledge about attitude can only be measured by inference from behaviour responses. This measurement must be able to offer an accurate and valid picture of learners' attitudes to some specific aspect of learning mathematics. This is the focus of this chapter but, before this is discussed, there is need for some clarifications on the relationship between attitudes and behaviour.

5.2 Attitudes and Behaviour

Most modern theories agree that attitudes are represented in the memory, and that an attitude's accessibility can exert a strong influence on behaviour (Fazio, 1986). By definition, strong attitudes exert more influence over behaviour, because they can be *automatically activated.* According to the MODE model ('motivation and opportunity as determinants': Fazio, 1986, 1990), spontaneous/automatic attitude-behaviour links occur when people hold highly accessible attitudes towards certain targets. These spontaneously guide behaviour, partly because they influence people's selective attention and perceptions of a particular target or situation.

Triandis (1971) noted that, "*Attitudes are inferred from what a person says about an attitude object, from the way he feels about it, and from the way he says he will behave toward it.*" However, to what extent are what he says, how he feels, and how he intends to behave consistent with what his attitudes actually are? This is a problem that is loosely referred to by social psychologists as the problem of the relationship between attitudes and behaviour. The theoretical basis for expecting a close relationship between attitudes and behaviour comes first from the very definitions of attitude, and second from theories of consistency.

If attitudes are to be seen in terms of the cognitive, affective and behavioural, the attitudes can be inferred by considering the cognitive, affective and behavioural responses. However, these are all essentially behavioural in nature. Thus, for example, a student's attitude toward mathematics does require some knowledge of what mathematics actually involves, what feelings the student has towards the subject, and it may lead to a commitment to take the next course in mathematics.

It might be expected that there will be a certain degree of consistency between the evaluative nature of attitudes and behavioural responses. Consistency principles permeate a lot of social psychology thinking (Frey and Gaska, 1993; Heider, 1946). Perhaps one of the most influential models of attitude research which relates held attitudes and behaviour was the *Theory of Reasoned Action.* Its authors (Ajzen and Fishbein 1977) argued that much human behaviour can be predicted and explained almost exclusively in terms of individual beliefs and attitudes. The model considered behaviour over which people have control: the world of rational decision. In other words, an individual's overt behaviour depends on his behavioural intentions. The strength or weakness of the intention will affect the performance of the behaviour accordingly.

Further modification was made to this model in 1985 by Ajzen by addition of perceived behavioural control, which is "*a person's belief as to how easy or difficult performance of the behaviour is likely to be and represents the extent to which the individual believes that behavioural performance is complicated by internal (skills, ability, knowledge) and external (co-operation of others, lack of resources) factors."* (Skryabina, 2000).

Ajzen called his theory the *Theory of Planned Behaviour* (see Figure 5.1). It only applies to behaviour that is intended or volitional. He drew together considerable evidence to show that the three factors did, in fact, account for the '*Intention to Behave*' which, in turn, did relate strongly to the actual observed behaviour.

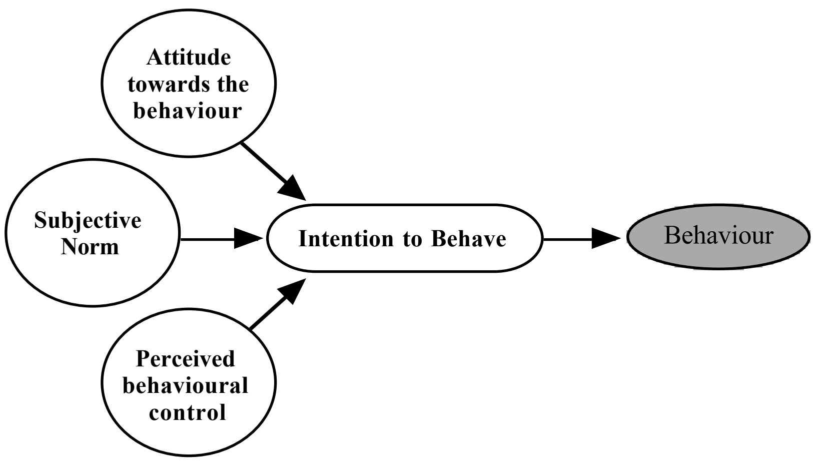


Figure 5.1 Theory of Planned Behaviour (after Eagly and Chaiken, 1993)

In essence, the behaviour of individuals is influenced by their attitude toward that particular behaviour, by what they think others (e.g. peers, family and mentors) think about the behaviour, and the individuals's control over the behaviour. Thus, in the context of choosing to study mathematics (behaviour), the individual's attitude to such study is very important along with what the individual considers others will think of undertaking the study (subjective norm) and whether the individual thinks the further study is possible (e.g. good enough marks in previous courses, time required, time tabling). These three factors were found by Ajzen to predict the person's intention to take some course of action, given an appropriate opportunity.

The model was employed in the prediction of secondary science students' intentions to enrol in physics in the USA. Information about the personal beliefs that determine the attitudes towards enrolling in physics, for example, "*can be utilised to develop systematically planned interventions for the initial secondary school science courses in order to improve students' attitudes towards physics enrolment and, at the same time, remove potential barriers in enrolment*" (Crawley and Black, 1992); and in studying the intentions of science teachers to use investigatory teaching methods in the USA (Crawley, 1990); and to predict college students' attendance at class lectures and achievement of grade 'A' in a course in the USA (Ajzen and Madden, 1985). The evidence obtained by Reid and Skryabina (2002a) in relation to attitudes towards a wide range of aspects of attitudes relating to physics also seemed to fit the model well.

Figure 5.1 also represents the network of causal relationships in which the attitude-behaviour relationship is embedded. Overall, attitudes can predict behaviour, provided that both are assessed at the same level of generality: there needs to be a high degree of compatibility (or correspondence) between them (Ajzen and Fishbein, 1977).

5.3 Approaches to Measurement

The importance of attitudes in the educational process at all levels emphasises the need for attitude measurement. This measurement must be able to offer an accurate and valid picture of the learners' attitudes to specific aspects of the learning (Reid, 2006).

In Chapter 4, it was mentioned that attitudes are latent constructs and cannot be measured directly or in any absolute way. In other words, all attitudes must be inferred from observed behaviour. Consequently, it is necessary to find adequate attitude indicators, and most methods of attitude measurement are based on assumption that they can be measured by people's beliefs or opinions about the attitude object.

Long ago, Cook and Selltiz (1964) categorised five techniques of attitude measurement and these techniques have stood the test of time:

- Physiological tests
- Surveys (questionnaires or interviews)
- Partially structured stimuli (projection tests)
- Observation of overt behaviour
- Performance tasks (congenial material learned rapidly).

These techniques can be considered under two broad types of categories:

(1) *Direct Approach*: Self report on one's attitude. The problem with this is that the participants may be reluctant to reveal their true feelings. This can produce the effects of social desirability in which participants give answers they think are expected or 'proper'.

(2) *Indirect Approach*: Report extracted from the set of indirect investigations (observations) when the subject is unaware that he is under investigation.

One major advantage of the indirect method of measuring attitudes is that the method minimises the subject's concern about 'appropriate' or 'desirable' responses.

In education, direct measurement of attitude is the most common approach used. The advantage of this method is that it allows the collection of data from a large number of people over a reasonably short period of time. On the other hand, an indirect approach has some disadvantages:

- Often involves considerable time for construction
- Final results are often open to misinterpretation

Both methods are capable of providing a broad spectrum of information, however, neither of them is perfect. It is therefore, dangerous to rely in research on only one of these approaches (Cook and Selltiz, 1964). Even within the limitations of pencil and paper tests, a wide variety of approach is available, and should be used whenever possible (Reid, 1978).

The most widely used techniques employed in educational research to investigate various aspects of learners' attitudes are *surveys* and *interviews*. When well-constructed, they are capable of providing insights into how learners think and the way they evaluate situations and experiences (Reid, 2003). Surveys can handle large numbers of respondents in schools and universities. There are two approaches: learners can either be asked to tell in writing or at interview what they think about their learning, or tests can be devised in which their responses can shed light on their attitude to learning. The former are open to error in that self-reporting may be skewed by such things as a wish to give 'desirable' answers, while the latter kind of questions are more difficult to devise Reid (2006).

These two widely used approaches in measuring attitudes will be discussed in the following section in that they are employed in the present study for data collection.

5.4 The Use of Surveys

A typical survey is a questionnaire, and questions used in a questionnaire can either be open or closed ones. Questionnaires which are open are easy to construct, and they give a freedom to the respondents to express their opinion in their own way or in their own words, although such questions may be difficult to answer, involving considerable time.

Closed questionnaires are more difficult to construct but easy and quicker to answer and analyse. The main drawback of this method is that the constructor needs to be extra careful in ensuring clarity and avoiding ambiguity so that responses are not restricted. Ali (2008) noted that a mixture of both types of questions in a questionnaire can often give a useful way forward to detect a specific attitude.

Thurstone (1929) was the first researcher to develop a systematic approach to attitude measurement using surveys when he developed equal appearing interval scales for constructing an attitude questionnaire. First, about 100 statements are collected about the attitude object. These statements range from extreme positive to extreme negative, and usually short and unambiguous. Next, about 100 'judges' (representative of the population for whom the scale is intended) evaluate the statements on an eleven-point scale, assuming an equal interval scale. Any statements (item) which produce substantial disagreements are discarded, until 22 remain (two for each of the eleven points on the scale: eleven favourable eleven unfavourable). The average numerical scale position of each statement is noted. Finally, the 22 statements are given, in random order, to participants who are asked to indicate every statement with which they agree. The final attitude score is the mean scale value for these statements chosen.

Though revolutionary in its time, the Thurstone scale is rarely used today, partly because it is time-consuming, and partly because of the assumption that it is an interval (as opposed to an ordinal) scale (Gross, 2005).

Following the breakthrough paper of Thurstone (1929), Likert (1932), developed what has become one of the dominant methods of attitude measurement. The Likert approach involves a number of statements, for each of which respondents indicate whether they strongly agree/ agree/ undecided/ disagree/ strongly disagree. If possible, statements are selected so that for half 'agree' represents a positive attitude and for the other half a negative attitude. This controls for any acquiescence response set (the tendency to agree or disagree with items consistently).

It is one of the most popular ways to measure attitudes, partly because it is claimed to be more statistically reliable than the Thurstone scale, and partly because it is easier to construct (Gross, 2005). The Likert approach makes no assumption about equal intervals but that is a source of a serious problem, to be discussed later. An example drawn as part of the study illustrates the format of the technique:

What are your feelings about mathematics tutorials?

	Strongly Agree	*Agree*	*Neutral*	*Disagree*	*Strongly Disagree*
I find the discussion boring	☐	☐	☐	☐	☐
I enjoy studying mathematics with members of my group	☐	☐	☐	☐	☐
Most of the ideas from other members of the group are not helpful	☐	☐	☐	☐	☐
Most of the ideas come from one person	☐	☐	☐	☐	☐
Studying as a group makes it easier for us to understand mathematics	☐	☐	☐	☐	☐
I do not respect ideas from other students since they are always wrong	☐	☐	☐	☐	☐

It is the way that questions are selected and the traditional scoring approaches which are open to considerable criticism. The usual way is to gather many items related to the attitude object (say, mathematics). These are then applied to a sample of the population and inter-item correlations computed, the items which give the lowest correlations being discarded. This is done by scoring a 'strongly agree' as 5, and 'agree' as 4, a 'neutral' as 3, a 'disagree' as 2 and a 'strongly disagree' as 1. A moment's thought shows the weakness of this approach. It implies that an 'agree' is worth twice as much as a 'disagree' and this is difficult to sustain. Scoring problems will be discussed later in the chapter.

There is also the Guttman Scalogram method. This is based on the assumption that a single, unidimensional trait can be measured by a set of statements that are ordered along a continuum of difficulty of acceptance. The statements range from those that are easy for most people to accept, to those that most people could not endorse. Such scale items are also cumulative, since accepting one item implies acceptance of all those 'below' it. It is constructed so that responses follow a step-like order (Hogg and Vaughan, 1995).

An example illustrates the approach. Respondents tick all the statements with which they agree. Their score for their attitude towards mixed-ethnic housing is then the **most** positive statement endorsed.

How acceptable	Statement
Least	Generally speaking, people should be able to live where they want.
	Real estate agencies should not discriminate against minority group.
	The local council should actively support the idea of open housing.
	There should be local review board that looks into housing discrimination
Most	There should be laws to enforce mixed-ethnic housing.

Figure 5.2 Illustrating a Guttman scale

Much later, Osgood *et al.,* (1957) were working on what they called *semantic* space. They were trying to explore the meaning of words and ideas and represent them in some hypothetical space. In a vast factor analysis of their data, they were surprised to find that semantic meaning seemed to be able to be reduced to three dimensional space, with the three dimensions loosely to be thought as:

good	....	bad	(the *evaluative* factor)
strong	....	weak	(the *potency* factor)
active	....	passive	(the *activity* factor)

In simple terms, meaning is to be thought of in terms of its location in three dimensional space where the three axes are defined as above.

Osgood *et al*., (1957) appreciated that the first dimension was strongly attitudinal in character in that evaluation was the key basis of attitude formation and development. They then developed a way of attitude measurement which was based on a scale set between adjectival word pairs. Five, six or seven point scales could be used and again, scoring and adding was often used for data analysis (Oraif, 2006), in a way similar to that employed with Likert scales.

An example (from Ali, 2008) illustrates the approach but adjectival pairs have been partly replaced by statements or adjectival phrases.

What is your opinion about the **subject mathematics?**
*Tick **one** box on each line*

I like mathematics	☐ ☐ ☐ ☐ ☐ ☐	I do not like mathematics
Useful in daily life	☐ ☐ ☐ ☐ ☐ ☐	Useless in daily life
Easy to understand	☐ ☐ ☐ ☐ ☐ ☐	Difficult to understand
Boring subject	☐ ☐ ☐ ☐ ☐ ☐	Interesting subject
I do not want to learn it but it is a compulsory subject	☐ ☐ ☐ ☐ ☐ ☐	I want to learn it because I enjoy it.

The Likert and the Osgood semantic differential methods of questionnaires were adopted in this study in that they have been widely used in educational research in recent years, but the scoring methods often used in the literature were rejected.

Reid (2003), summarised some few important points that should be taken into account while designing a questionnaire for attitude measurement.

"• *The attitude object must be specified, and the variety of stimulus which can help to elicit evaluation should be defined. For example, if consider mathematics as an attitude object, then the following stimulus might be considered: a teacher, classroom instructions, lesson activities, outdoor activities, mathematics TV programmes, etc.*
- *The appropriate techniques which can reflect the evaluative character of the attitude object should be used.*
- *Special attention should be placed on the validity and reliability of the methods used for attitude measurements.*
- *Time should be adequate, and pre-testing is helpful to check clarity and the format of the questionnaire."*

Cohen *et al.,* (2002) are very explicit in outlining a code of ethics (e.g. competence, voluntarism, full information and comprehension) required with regard to the participants of the questionnaires, and attempts were made to conform to this. According to them, *competence* implies that responsible, mature respondents will make correct decisions if they are given the relevant information. The questionnaires attempted to include valuable information to help with the completion. Although the completion of the questionnaires was done by all the students (simultaneously) chosen from particular year group(s) and supervised by their teachers, there was no pressure whatsoever placed on the students to complete the surveys, thereby addressing the requirement of *voluntarism*. The surveys were administered at an exploratory stage when it was not possible to ascertain exactly how the data would impact upon the next stage of the study. However, there was a clarification to students that data were being gathered with the intention of informing, shaping and improving teaching practice (see BERA, 2004: 11, Page 6).

Scoring is now discussed.

5.5 Scoring Problems

Although the Likert method (Likert, 1932) eliminates the role of judges used by Thurstone (1929) and allows the respondent to place himself on the evaluative scale according to the degree of his preference towards attitude object, the central assumption underlying the use of this technique is that the items in the scale must reflect a common construct. If this requirement is not met, the scoring procedure produces largely meaningless, uninterpretable data (Gardner, 1996). Thus, scoring can be considered as one of the major disadvantages of the Likert method (and the semantic differential method). In fact, Reid (2006) discussed his criticisms of this technique:

(1) The method of scoring assumes that every question reflects a common construct, and correlation is used to check this. However, correlations does not demonstrate that this is true.

(2) The ‘scores’ (5, 4, 3, 2 and 1) are ordinal numbers and cannot be added together.

(3) The final score is obtained by adding up scores from evaluation of different items, which may have different meanings. Gardner (1975) illustrates weakness by stating that, “*to add up the weight, the number of doors, the number of cylinders in a motor car to produce a single number would have a little meaning*”.

(4) It is assumed that the gaps between the ‘scores’ (5, 4, 3, 2, and 1) are equal and this cannot be checked.

(5) The method implies that, for example, an ‘agree’ (=4) is worth twice a ‘disagree’ (=2). This is meaningless.

(6) The responses to individual questions frequently give distributions very far from normal, making Pearson correlation inappropriate as this is based on the assumptions of approximate normality and the use of interval numbers.

(7) Responses to specific individual questions often offer valuable insights with scoring and adding obscure.

Similarly, the Osgood Semantic-Differential method can be criticised using the same arguments as above relative to the scoring in the Likert method. In addition, we cannot perform normal arithmetic on non-cardinal data. The only way forward, as suggested by Ali (2008), ‘‘*is to compare the response patterns, question by question, for two (or, perhaps more) groups (e.g. boys and girls) using the chi-square statistic as a contingency test*”. This statistic works well with frequency data and was employed in this study.

5.6 The Use of Interviews

Surveys in the form of questionnaires play a vital role in obtaining useful information. However, by their very nature, they tend to be somewhat prescriptive and do not allow the participants the opportunity to seek for clarification or to engage in a dialogue. This is one distinguishing feature of interviews. With interviews, some preliminary questions may be needed to enable the respondents to talk freely and openly. Surveys are drawn from a large number of respondents and the interview can be used to check outcomes from questionnaire data and to explore areas where the questionnaire happen to show interesting pattern of responses. “*The interviews were being used to amplify the questionnaire data while, of course, they can be used on their own ... only a small sample is needed for these purposes.*” (Reid, 2003).

In general, it is possible to learn a large amount by talking to students about their learning. In highly structured interviews, all the questions are decided beforehand and the interviewer controls the agenda. By contrast, in a totally open interview, the questions allow the interviewee to set the agenda and explore what she or he wants. However, there is the semi-structured interview in which some questions are decided beforehand, with opportunities to expand and develop to meet the way the interviewee responds. This is summarised in Figure 5.3.

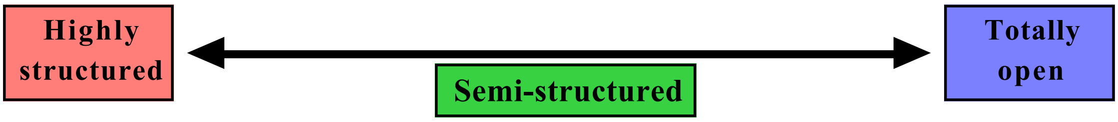


Figure 5.3 Spectrum of Interviews (source: Reid, 2003)

In this study, semi-structured interviews were employed where the researcher had planned questions but the students were given the opportunity to react to the questions in line with their own experiences and opinions, allowing issues important for the students to be explored.

The interview is a powerful research tool in gaining insights into learners attitudes. However, its major disadvantage is that it takes time both for the interviewer and the interviewee and "*it is difficult to translate evidence from interview into a neat summary.*" (Reid, 2006).

5.7 Reliability and Validity of Measurements

Both *reliability* and *validity* are very important issues in all measurement, including attitude measurement. Measurements should not only be able to reproduce results after a certain period of time (reliability) but also measure what is aimed to be measured (validity). Thus, if the aim is to measure an adult's height accurately, then an appropriate measuring tape might be used. The tape should be capable of measuring the height such that the same (or extremely similar) result is obtained on several consecutive days. The tape should not stretch to any significant extent, for example, with time. This means the measurement is reliable.

However, it is essential that the person stands totally upright, is not wearing shoes of different sizes and that the measuring tape is held exactly vertical. The aim is to measure the exact height and not an extended height by measuring at some angle to the vertical. This reflects the validity.

In education, when measuring performance in mathematics, a mathematics test may be used. If the teacher wants to measure the student ability to solve quadratic equations, then the test questions must involve the solving of quadratic equations. For example, being able to solve simultaneous equations may or may not reflect on the ability to solve quadratic equations. Validity is checked by seeking advice from colleagues as well as looking for any evidence to suggest that the students can get right answers without the skill of solving quadratic equations.

Validity is much more important than reliability but an unreliable test (in the sense that it gives different outcomes on different roughly equivalent occasions) will always be invalid. Germann (1988) notes that, "*Attitude research must clearly define the construct being investigated, describe the place of this construct within a large theoretical framework of relevant variables, and demonstrate the reliability and validity of instruments used to measure it*." These requirements are generally difficult in that there is no certainty that the measurement instruments developed are actually measuring the target object.

Nonetheless, some positive steps can be taken:

> "*The questionnaire items can be considered by 'experts' - those who have some knowledge of the field and who know the nature of the population to be tested. Adjustments can be made in the light of their comments. Secondly, the patterns of results from the questionnaire can be compared qualitatively to the outcomes from the interviews*".
>
> (Oraif, 2006).

Reliability in the sense of test: retest reliability is usually assured by using large samples and making the measurements under sensible conditions (adequate time, suitable environment, participants not feeling that they have to respond to 'impress' the investigator or their teacher). Reid (2003) notes that the, "*evidence suggests that reliability is high*" if these conditions are fulfilled and he refers to a number of examples (Reid, 2006).

However, it is important to note that, "*in the present state of knowledge, attitudes cannot be measured in any absolute sense, with any degree of certainty ... responses to attitude measures can be compared before and after some experience ...*" (Reid, 2006). Indeed, if a respondent ticks one box to the right or to the left of the 'correct' box in a typical item in a five point scale Likert format, the error will be ±20%. If the sample is large (>200), then the errors from individuals cancel out and the overall picture *for the population* will be remarkably stable (Reid, 2003). It is perhaps better to see questionnaire data as presenting a *picture* of the very complex perceptions of a population rather than trying to ascribe a 'score' to any individual.

In all research, it is an advantage to check one set of measurements with data from an independent source. Using both qualitative and quantitative data allows for some level of what is known as '*triangulation*': using different methods or approaches to study the key question. However, there is a controversy in the educational research literature involving the use of qualitative and quantitative methods. Some researchers (Muijs, 2004; Cohen and Morrison, 2007) support only one of these approaches, while others (Yin, 1994; Al-Enezi, 2008) advocate the usage of a combination of approaches which can offer different complementary strengths. Common sense argues for the latter as the combination of several approaches helps to overcome any weakness, bias and limitation in using just a single approach (Al-Enezi, 2006). Yin (1994, page 92) argued that "... *any finding or conclusion in a case study is likely to be much more convincing and accurate if it is based on several different sources of information* ...". Furthermore, the usage of a mixture of approaches helps in collecting more comprehensive and robust data, and helps to make the researcher to be more confident that his findings are valid (Cohen and Manion, 1994, pages 233-234).

The statistics techniques employed during the study is presented in Appendix H.

5.8 Attitude Research in Mathematics Education

In Chapter 4 of the present study, we noted that attitudes are often multidimensional and involve three components: cognitive (knowledge component); affective (feelings component); and behavioural (experiences, or tendency towards action component).

There is a considerable amount of attitude research in the field of mathematics education, much arising from concerns over declining numbers and apparently negative views of studies in mathematics in many countries. Many studies have focussed on specific variables. This section considers some of these variables and their effects on measurement of attitudes.

There are two categories of variables which can help teachers support their students in the acquisition of knowledge that is the province of formal education:

- Internal variables - this includes abilities, achievements, personality, gender, age.
- External or social variables - this includes the teacher characteristics, classroom and home environments, curriculum, instructional styles and culture.

Gardner (1975) argued that these two variables are by no means distinct and unrelated. According to him:

"(1) Some variables are difficult to classify unerringly (for example, achievement is a matter of school policies and practices, and not just a matter of learner's ability).

(2) Some internal variables (for example, personality) may themselves be the result of external variables (for example, socialisation practices).

(3) Some internal variables may interact with external variables in producing their effects (for example, certain type of teacher behaviour may exert varying effects upon learners of differing abilities and personalities.

(4) Internal perceptions of external variables may be more influential than the external variables per se (for example, a child's attitudes may be influenced by his beliefs about his parents' attitudes, and these beliefs may be unrelated to the attitudes which his parents actually hold."

5.8.1 Internal Variables on Attitude Formation in Mathematics Education

In looking at the internal or personal variables as an aspect in which attitude towards mathematics can be developed or formed, much research has shown that learners often perceived mathematics as a difficult subject (Cockburn and Haylock, 1997; Haylock, 2001; Brown *et al.,* 2008). Mathematics is often portrayed as being abstract and unrelated to life, and that it is not possible for the teacher to explain every concept of mathematics to the learners in terms of physical representation or to relate it to their daily life. The evidence from many countries is that many pupils do not enjoy school mathematics and seek to avoid it later (McLeod, 1994).

For many people, mathematics is a subject which generates a feeling of unease and insecurity. Those of us who sometimes have to admit that we are in some sense 'mathematicians' get used to responses like:

- You must be very clever if you study or teach maths!
- I never really understood maths at school. I just learnt to do the tricks
- Oh no, not maths! I'm hopeless at maths

Various countries have tried to find ways to improve learners' attitudes to mathematics but with limited success. The only way forward is to enable learners to be excited with it; to derive enjoyment, satisfaction, and fulfilment from studying mathematics. One key aim might be to find and develop ways to do this even more effectively and efficiently (Ali, 2008).

Piburn and Baker (1993) attempted to find causes for the general tendency that interest in science decreases with age. Attitudes tend to become more negative as pupils move from primary to secondary schools. According to their results, the origins of the decline in attitude towards science are in the nature of "*learning styles and the relationships among* *people in classroom*". Attitudes to mathematics are no exception. For example, Duffin and Simpson (2000) found that attitudes tend to deteriorate with age simply because work becomes more demanding and also because, as the pupils get older, they may start to think that they will not need mathematics in future.

Although children begin school doing mathematics because it is a compulsory subject, no sooner than later, they become increasingly uncomfortable with 'open-ended' activities (Haylock, 1991). To be able to learn, children need instructions, assessment and feedback about their work. Moreover, as abstraction and complexity of mathematics grow with age, this generates a clear negative effect on attitudes toward mathematics. Overall, learners need pleasure and security in their learning. The learning atmosphere must be positively supported in order to relate their studies to their experience. Mathematics syllabuses have frequently not taken these factors into account.

Figure 5.4 summarises the processes of attitude development. The initial aspect is the input stage, where the characteristics of content are provided. Although what is presented in the content and how it is presented are important, however, no content can affect learners' attitude if it does not motivate them to be willing to receive.

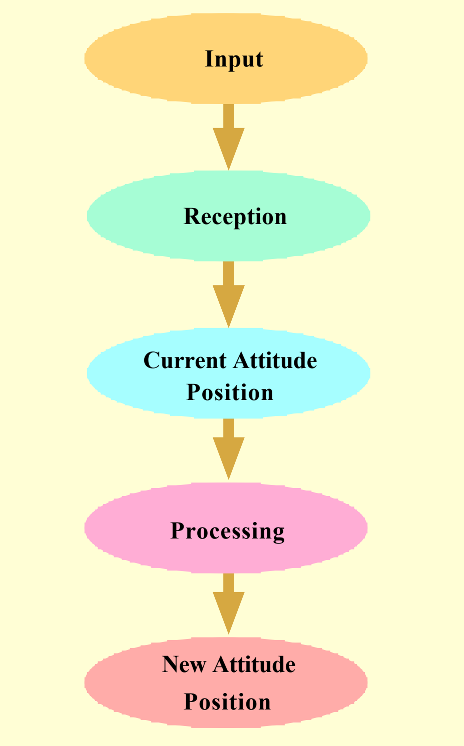


Figure 5.4 Attitude Development

The processing stage is where the new input is comprehended and assessed. Jung (2005), pointed out that "*attitude is not merely developed by acquisition of relevant information ... it needs to be internalised or made one's own and may have to be connected to other related attitudes to make a coherent whole*" in bringing about new attitude. Therefore, there is need to understand how learners perceived the relevance of the subject and how they process the content in the learning situation.

According to Oraif (2006), the starting point in building students' understanding of mathematics is to help them establish confidence in success, which is reflected in the speed of learning, understanding and examination success. This confidence leads to more positive attitudes in that, if the pupils see success as elusive, then it is unlikely that attitudes will remain positive (Ali, 2008). This has an enormous implications for mathematics teachers. If learners are asked to complete tasks which are beyond them, success will be rapidly eroded. This is where working memory limitations may be critical.

In general, it is difficult to summarise all findings in few words. Nevertheless, the general picture which emerges are that:

- Mathematics was not seen as an easy subject (Alhmali, 2008).
- Learners who are favourably inclined towards mathematics tend to be relatively serious and achievement-oriented, realistic and independent (Gardner, 1975).
- If the tasks learners have to do place too much strain on working memory, then understanding cannot take place fully (Johnstone and El-Banna (1989); Johnstone, 1993; Jung, 2005).
- While difficulty in learning does not neatly relate to positive or negative attitudes (Reid, 1978), excessive difficulty in learning could cause problems when a task is perceived as being so difficult that the effort is not justified by the rewards. Attitudes seem to deteriorate (Reid and Skryabina, 2000).
- Students who do not do well in a subject may develop a negative attitude towards that subject and blame their teacher, and male and female college students who dislike mathematics blame their school teachers (Aiken, 1961; Aiken and Dreger, 1961). However, Ali (2008) argued that there is always a tendency for students to blame their teachers if they do not like a subject; if a student is not doing well in mathematics or has a negative attitude towards it, then there are many other factors that may be involved and the teacher cannot be solely blamed for these.
- Those students who are more confident of their ability to learn mathematics are more likely to be successful and continue studying mathematics even when it becomes optional (Oraif, 2007).

5.8.2 External Variables on Attitude Formation in Mathematics Education

A number of factors such as the teacher characteristics, home and classroom environments, curriculum, culture and instructional styles, can indeed influence an individual's attitudes towards mathematics and achievement in mathematics. Many researchers (Johnstone and Reid, 1981; Germann, 1988; Ponte, 1991; Reid and Skryabina, 2002a; Ali, 2008) found that teachers and their instructional techniques play a vital role in shaping students' attitudes in their learning process. They also noted that a teacher's attitude may have the most enormous effect on a pupil, especially if the pupil's attitude towards the teacher is strong, either positively or negatively. In particular, Reid and Skryabina (2002a,b) found that the teacher's attitudes and effectiveness in a particular subject are important determinants of students attitudes and performance in the subject (especially with younger learners). It is impossible to separate the relationship between the views and attitudes of the teachers and those of the students (Ali, 2008).

There are numerous studies on external variables affecting attitudes development in mathematics education:

- General attitude of a class towards mathematics is related to the quality of teaching and the social psychological climate (the general way a class is run) of the class (Hannula, 2002);
- Classroom environment is important and, ideally, should be pleasant, encouraging and thought provoking (Orton and Wain, 1994);
- Mathematics curriculum should be related to life (as much as possible) and future needs (Alhmali, 2008);
- The relationship between parental education and mathematics attitude may be more culturally diverse than the relationship between parental education and mathematics achievement (Xin Ma and Kishore, 1997).

However, we have focused on the teacher characteristics in that the influence of the teacher is considered "*as ONE key factor in encouraging the development of positive attitudes*" (Ali, 2008). According to her, the key role of teachers can be seen as:

"• *To make mathematics accessible in terms of understanding, as well as 'getting it right'.*
- *To give the learners some sense of fun and challenge.*
- *To give the learners the feeling they are being supported and not condemned.*
- *To motivate every learner by giving a sense of achievement according to each learner's mathematics abilities."*

5.9 Interaction: Cognitive and Attitude Models in Mathematics Education

Recent studies have explored the difficulties of learning in the area of science education and made some practical suggestions (e.g. Johnstone, 1997; Skryabina, 2000; Haylock, 2001; Jung, 2005). In this section, an attempt is made to discuss them very briefly in that they have some parallels with mathematics education.

Gardner (1975) suggests that there are key questions which are important in order to obtain information on students' interest in science:

"To what extent are students interested in using science to meet personal needs?
To what extent do they want to learn about (scientific) issues which affect society at large?
How strongly are they motivated by the possibility of pursuing academic work in science?
How willing are they to consider the possibility of a scientific or technological career?"

In looking at the four key discussions, it is clear that cognitive content is an essential component to raise students' interest in science.

However, Jung (2005) questioned that, if students have no previous knowledge and skill about scientific aspects of their every day life, scientific issues of society, and industrial and economic application of scientific outcomes, how can their interest in science grow? This is the reason why many students state that they do not want to continue with science because it is perceived as too mathematical, too abstract and too difficult. These reasons are parallel with mathematics. Hence, the role of mathematics teachers is to educate in mathematics, its ways of thought and enquiry, its application in many areas of life, and its contributions to human understanding of the world.

It is important to know whether learners can comprehend well the nature of mathematics, and the role of mathematics in society or not. If learners know more about mathematics and how it is used, their attitude may improve although this cannot be guaranteed. Therefore, cognitive comprehension is an indispensable factor of attitude development. If this is neglected, the whole world may be closed off for students.

In general, unsuccessful learning experiences can cause students to lose learning intention. This may bring about negative influences on attitudes or loss of ability to study because there is little or no prior knowledge in the long-term memory. Such knowledge is important in making sense of new learning.

There are ten variables that may influence learners attitudes (Chapter 4). Among these, classroom climate and instructional strategy are within the teacher's control. But others are predetermined or are controlled by those well beyond the school. This means that students develop attitude towards mathematics by the way it is assessed (which may be out of the control of the teacher), on the way the curriculum is presented, and on the way the classroom climate is set up. The process of this development is summarised in Figure 5.4 based on discussions of Chapter 2 and is derived from Jung (2005, page 38).

The input stage characterises the content presented: what is to be presented, and how it is presented that allows it to be stored and remembered. No content can have an impact on learners' attitude unless it motivates them to be willing to receive it and to manipulate it.

In looking at the processing stage, the input has to be comprehended, assessed and retained. Attitude is not merely developed by acquisition of relevant information, it needs to be internalised or 'made one's own' (Jung, 2005). This is the crucial distinction between *availability* (whether or not the content has been stored) and *accessibility* (whether or not the content can be retrieved), which is especially relevant to theories of forgetting.

In the final part of the chain, the output, the attitude may have to be connected meaningfully to other related attitudes to make a coherent whole. For this reason, learners should participate actively in the learning process to achieve a meaningful outcome.

5.10 Age, Gender and Attitudes towards Mathematics

Research on gender differences in academic achievement offers school educators thought-provoking information on implications and guidance on specific directions to take. The accumulated literature on this topic covers students' confidence in learning mathematics, sex-typed expectations for performance in mathematics and science, age and self-estimations of ability to learn mathematics and science. Only a few studies were found which were devoted to the problem of the relationship between age, gender and attitudes towards learning. Attitudes tend to become more negative as pupils move from elementary (primary) to secondary schools (McLeod, 1994).

From the work done in the field of science, it appeared that the patterns of students' attitudes towards science with age are similar: as pupils grow up their attitudes towards science decline (Piburn and Baker, 1993; Ramsden, 1998). Simpson and Oliver (1985) reported that attitude towards science declines sharply from the beginning of the year to the middle of the year and more gradually from the middle to the end. In particular to this, attitudes decline steadily from grade six through grade ten. Attitudes towards mathematics are no exception. Attitudes tend to decline with age simply because as work becomes more demanding and difficult as learners get older, interest and motivation steadily decline.

There are three possible explanations for this decline as a result of age: the classroom instructions, the relationships among people in the classroom and poor grades. Children began school liking mathematics, and many of the mathematics activities they engaged in at this stage are mostly action-oriented and open-ended. Later on, in the junior and senior secondary schools, children became increasingly uncomfortable with complex and abstract mathematics lessons with resultant poor in grade. Piburn and Baker (1993) suggested that if the isolation of students as they move through the grades and the number of opportunities for student-student and student-teacher interactions, both academic and social decline, negative attitudes increase. This is likely to have a clear negative influence on attitude towards mathematics.

In looking at gender differences and attitudes towards study, Gardner (1975) observed that "*sex is probably the single most important variable related to learners attitudes to science*". This statement is not surprising as there is considerable evidence to support it. A report of science and mathematics in state schools in England and Wales (according to Ofsted, 1994) indicated that "*by 1993 only marginal improvement of girls could be seen taking-up physics courses. The proportion of A-level physics passes achieved by girls was still only 21 per cent*". Thus, Cheng *et al.,* (1995) concluded that "*sex-gap in take-up of physical sciences remain as wide as before*".

A report on the Future of Mathematics Education by National Research Council (1994, USA) found that as girls and boys progress through the mathematics curriculum, they show little difference in ability, effort or interest until the adolescent years. Then, as social pressure increase, girls tend to exert less effort in studying mathematics, which progressively limits their future education and, eventually, their career choices. The report also noted that gender differences in mathematics performance result from accumulated effects of sex-stereotyping perpetrated by families, schools and society.

In Scotland, there appears to be little gender difference. The entries and pass grades awards for the Higher Grade (university entry qualification) in mathematics for 2008 shows the following patterns (see Table 5.1).

2008	Entries	Awards			Total Awards
		A	B	C	
Numbers					
Girls	9496	2225	2339	2286	6850
Boys	10140	2514	2435	2376	7325
Percentages					
Girls	48	23	25	24	72
Boys	52	25	24	23	72

Table 5.1 Gender and Mathematics in Scotland (Source: SQA, undated)

The general conclusion from numerous research studies devoted to the gender issue and attitude towards study in the sciences can be formulated as follows: "*boys show more positive attitudes towards physical science than girls*" (Skryabina, 2000). Focussing on mathematics, the University of Minnessota Talented Youth Mathematics Program Research (1996, USA) found that males showed significantly higher levels of motivation, confidence and interest in mathematics than females.

It is difficult to pinpoint where the problem lies with girls. Early childhood experience, such as the environment at home or in the local community, and exposure to the media, play a vital role in shaping a child's interest and self-image (Murphy, 1990). This early socialisation may lead girls away from science and other related subjects. The nature of boys and girls also tend to differ, with boys relatively more interested in physical science and girls more interested in biological and social sciences topics (Clarke, 1972; McGuffin, 1973). A possible explanation is by taking into account the personality differences and social concepts of girls and boys: girls are more person-oriented, socially responsible, friendly and co-operative; while boys tend to be more independent, achievement-oriented and dominant (Smithers and Hill, 1987). However, none of this really applies to mathematics.

The Scottish experience suggests that there are no intrinsic reasons why girls should not be equally interested in and committed to mathematics.

5.11 Teachers' Impact of Attitudes on Learning

Academic learning is an expectation of the provision of education in schools. In reality, the results of education are more specifically achieved in classrooms within the schools, and the classroom is the nucleus where influences on students achievement and other results from their education are found. It is now well recognised that classroom influences consistently explain a large proportion of the variance in students outcome (Webster and Fisher, 2001). Classroom influences include: the students' opportunities to learn within the classroom; the instructional practices and beliefs of the teacher; the climate of the classroom; and teacher and peer relationships within the classroom.

Teachers and their instructional approaches are fundamental in building students' understanding. Ball *et al.,* (2001) argued that "*it is the teacher that makes the difference ... it is clear that the structural and social influences are minor, what the student brings in terms of achievement and disposition to learn are powerful, (and the) teaching process is paramount ... this must lead to the conclusion (that) teachers make the difference, but only teachers who teach in certain ways*".

Primary among their many duties and responsibilities, teachers structure and guide the pace of individual, small-group and whole-class work to present new material, engage students in learning tasks, and help deepen students' grasp of the content and concepts. There are many theories about instructional effectiveness and these differ in the amount of direction teachers should provide for learners.

During the last decade, a blending of approaches to instructional effectiveness has taken place. In looking at research on instructional practices and the influence of these factors on students achievement, quality of instruction is high if students learn the material as rapidly as their abilities and levels of prior knowledge allow. Webster and Fisher (2001) described teaching and learning in the classroom from three general perspectives: firstly, there is the perspective of teaching involving imparting information, knowledge and understanding, teacher-directed learning or transmission of knowledge (*instruction*); secondly, there is the perspective of teaching as meeting the needs of students, involving student participation (*participation*); thirdly, there is more scientific perspective in which the learning is seen as a process of investigation of practical work and open-ended inquiry learning (*investigation*). Indeed, the attitudes of the teachers, their experiences and the way they deliver the curriculum all contribute to the attitudes and the learning environment of students, which in turn will have an effect on students' mathematics outcome.

5.12 Parents' Influence on Attitude Development towards Mathematics

Parents have the wonderful opportunity and responsibility for nurturing children. This nurturing process takes place in several areas of development and experiences: physical, emotional, and intellectual. These experiences give rise to the findings from a study funded by the National Institutes of Child Health and Human Development (2006, USA) which reveals that a child's family life characteristics has more influence on a child's development and behaviour. No one would deny that parents might play an important role on a child's attitudes to mathematics. "*Parents' attitudes toward mathematics have an impart on children's attitudes. Children whose parents show an interest in and enthusiasm for mathematics around the home will be more likely to develop that enthusiasm themselves*" (USA Department of Education, 2006).

Poffenberger and Norton (1959) note that parents can affect the children's attitudes and achievement in various ways:

- By parental encouragement
- By parents' own attitudes
- By parental expectations of child's achievement

The study by Poffenberger and Norton was designed to probe students' own attitudes, and the attitude and expectations of their parents towards mathematics. The outcomes of their study were that the students' attitudes towards mathematics were positively related to how they rated their parents' attitudes towards mathematics, and their attitudes was also related to their reports of the level of achievement in mathematics their parents expected of them. Career aspiration, social and cultural trends may influence parents' expectations and it can indirectly affect their child's attitudes towards mathematics (Ali, 2008).

Besides the mathematics learning that takes place at the parents' initiative, there are many opportunities for parents and teachers to work co-operatively in enriching children's experience with mathematics. These situations are likely to be the most profitable for two reasons. First, young children generally want to please both their parents and their teachers. If they see that mathematics is important to both their parents and their teacher, they will consider it important for themselves too. Second, extending mathematical concepts from the classroom to home will establish the idea that mathematics is not just a school subject, but an everyday subject that makes life more interesting and understandable.

Parents who want to become more involved in their child's mathematical education, but who are hesitant to take the initiative on their own, may want to look to the teacher for guidance.

Teachers can provide assistance in:

- Setting up a system of own study.
- Helping parents understand the sequencing of mathematical skill development.
- Suggesting material and activities that are entertaining and suitable for the child's cognitive level which can be done in a reasonable amount of time.
- Providing clear guidance on how to use materials.
- Giving feedback on the successes and failures of home activities and knowing when to stop working to avoid overloading the child's limited working memory capacity.

In general, parental influence can play a prominent role in developing their child's attitudes towards mathematics. Several studies (Matthews and Pepper, 2005; Kyriacou and Goulding, 2006) suggest the same, and children often look to their parents when they form their habits. If any message about difficulty in relation to future struggle comes from parents then it may lead the child to develop negative attitude towards mathematics.

5.13 Chapter Summary

Up to the early part of the 20th century, measurement of attitudes was not considered possible until Thurstone demonstrated the possibility of achieving it successfully following his breakthrough paper published in 1929 entitled '*Attitudes can be measured*'. His technique is rarely used today because it is very cumbersome. However, his work paved way for a new area and he was quickly followed by Likert (1932) and later by Osgood *et al.,* (1957).

The model for the nature of attitudes given in Section 4.7 provides a useful basis for discussing attitudes measurement. Most researchers have made attempts to measure attitudes - usually by questionnaires - and have inferred the attitudes from behaviour, on the assumption that personality and social environment effects are irrelevant. This latter assumption may have some basis if the samples being employed are large, and are random samples of the population. However, the implicit assumption that a questionnaire is a valid measure of a particular attitude must also be exposed to question. Even if a person replies honestly, both consciously and subconsciously, there is no guarantee that his reply will be valid when he is placed in any particular situation relating to his expressed attitudes (Reid, 1978).

The key approach to attitude measurement within education is either to ask questions verbally or in writing. The survey and the interview are the most widely used approaches although direct observations of behaviour can sometimes be very useful. In every case, the approach relies on being able to infer an attitude from some kind of behaviour. Indeed, attitudes can predict behaviour, provided there is a close correspondence between the way the two variables are defined and measured.

The theory of planned behaviour has shown that an attitude is one factor influencing the intention to behave (see Figure 5.1). If we assume that a questionnaire or interview gives insights into intentions towards various aspects of behaviour then we can deduce the attitude which underlies the behaviour provided that the subjective norm (what the person thinks others think of the behaviour) and the perceived behavioural control (any hindrances to the behaviour) are kept to the minimum. By ensuring that respondents see no hidden agenda and that, therefore, what others think is unimportant as well as avoiding any restrictions of the behaviour, then this perhaps can be achieved.

Much work has analysed a questionnaire by adding up scores together (which are ordinal numbers). This has been rejected and the responses to each question will be examined separately in this study.

The next stage is to look at ways in which learners differ from each other to see if that throws any light on the way learners undertake their studies in mathematics.

Chapter Six

Cognitive Learning Styles

6.1 Introduction

Two of the most important functions of the memory system is to receive information and store it for recall. We store information which is potentially important, or interesting, or useful. We ignore information which is more trivial or unimportant. This is a personal process and for that purpose memory uses a variety of functions such as: pattern recognition, rehearsal, elaborating and organisation. "*We seek for patterns as we try to connect the new information with existing information in order to 'make sense'. We discard the new information when it does not 'make sense' to us*" (Johnstone, 1997).

As we noted in Chapter two, learning and memory represent two sides of the same coin: learning depends on memory for its *permanence*, and memory would have no *content* without learning. Hence, we could define memory as the retention of learning and experience. As Blakemore (1988) says:

> *"In the broadest sense, learning is the acquisition of knowledge and memory is the storage of an internal representation of that knowledge..."*

Blakemore expresses the fundamental importance of memory like this:

> *"... without the capacity to remember and learn, it is difficult to imagine what life would be like, whether it could be called living at all. Without memory, we would be servants of the moment, with nothing but our innate reflexes to help us deal with the world. There could be no language, no art, no science, no culture. Civilisation itself is the distillation of human memory..."*

Several researchers (Pask, 1976; Perkins, 1985; Al-Naeme, 1991; Riding and Rayner, 1999; Oraif, 2006) have used the word *style* in describing differences between people in the way they think, learn, process, store, and recall information. The idea of style is used also in a variety of contexts: in high street fashion, the sports arena, the arts, the media, and in many academic disciplines including psychology. It has a wide appeal which reflects an enduring versatility, but this same appeal can lead to overuse which unsurprisingly creates a difficulty for definition and understanding. Yet, the notion of style remains an important and popular expressions of individuality. It is used time and again, to describe a set of individual qualities, activities or behaviour sustained over a period of time.

The significance of an awareness of style is its potential for enhancing and improving human performance in a variety of contexts. For example, in psychology it has been developed in a number of different areas such as: personality, cognition, communication, motivation, perception, learning and behaviour. Unfortunately, the widespread use of the term 'style' has led to workers in the field often adopting different definitions and terminology. Consequently, Riding and Rayner (1999) noted that those workers interested in reaching agreement, in terms of definition or use of an accepted nomenclature for a theory of style, have faced considerable difficulty.

Style in educational psychology has nevertheless been recognised as a key 'construct' (a psychological idea or notion) in the area of individual differences in the learning context. Riding (1997), for example, suggested that cognitive style reflected the fundamental make-up of a person. He argued that style has a physical basis and can and does control the way in which individuals respond to the events and ideas they experience. Importantly, he identified the 'temporal stability' of style, suggesting that it is a constant aspect of a person's psychology which does not appear to change. It is impossible, according to Riding, for a person to 'switch off' their style. However, this stability is open to question. It is perfectly possible for a learner to adopt a 'style' which suits a particular task or a particular occasion.

Cognitive styles may prove important if they affect, directly or indirectly, aspects of the learning process: an individual's learning performance, subject attainment and social behaviour. The challenge then becomes one of identifying the dimensions of variation of explaining individual differences. It is argued that cognitive style is a distinct construct (Riding and Rayner, 1999) but it might be better to think in terms of cognitive styles. Hindal and Reid (2009) have avoided the phrase 'cognitive styles' altogether and talk in terms of learner characteristics.

Cognitive style as a subject includes several aspects of 'differential psychology' associated with individual differences in the learner and the learning environment (Jonassen and Grabowski, 1993). In this, it is suggested that every learner has a preferred style of collecting, organising, processing and storing information into beneficial knowledge. For example, there are learners who feel more comfortable with manipulation of abstract materials, while others prefer concrete ones. The same is true with mathematics, some students have preference for geometry because they find it easier to visualise, and they solve the problems by drawing shapes, in contrast to algebra which is more symbolic.

This approach poses problems when set alongside the assertion by Riding (1997) that there is a 'stability' to style. Stability suggests something which is strongly 'genetic' in origin while the word 'preferred' suggests a strong element of choice on the part of the individual. There is a third aspect to cognitive learning styles: is it possible that these styles are largely learned? The three aspects, of course, are not mutually exclusive. A person may have a genetically inherited approach to learning. Experiences in life, including formal learning, may confirm and enhance such styles or they may undermine them. Equally, the person may choose a specific approach in a specific situation. For example, it has been argued (Sternberg and Zhang, 2001) that many children have a strong genetic bias towards learning visually. However, the education system in schools often tends to emphasise the use of symbolics (language and number). The person is hindered but may well adapt their learning style to cope.

The fundamental question arises as to whether style is inbuilt (nature), or develops in response to experiences (nurture). This is a question which is difficult to answer, particularly since the assessment of style in infants and young children presents difficulties. Casual observation of young children of one year upwards suggests that they show consistent behaviour from an early age which does not change; a quiet child stays quiet and a talkative child continue to be verbally fluent (Rayner and Riding, 1999). No longitudinal studies of the effect of age by assessing subjects at different ages have been undertaken. However, where samples were from a wide age range, no significant relationship between age and style was observed.

Overall, cognitive learning styles may be influenced by any or all of:

Genetic factors Learning and experience Personal preference or by choice

The issue is important in that learning and experience can be adjusted to encourage the development of learning styles which are advantageous in gaining success while encouragement can be given to choose a style which is of greater benefit.

Another issue exists. Willing (1988) notes that, before the 1970s, individual differences tended to be synonymous with differences in ability. Later, a clear separation developed. Sternberg and Grigorenko (1997) considers this issue in detail, observing that two words are very critical: *level* and *manner*. Thus, ability focuses on *level* of performance while the *manner* of performance is the focus on style. This is a very useful distinction. Very often, styles are seen as bipolar (e.g. field dependent-field independent) while abilities are unipolar. However, this distinction is not as clear as might be supposed in that the opposite of ability is simply lack of ability in the same way as lack of field independence is field dependence.

Usually style is seen as value-free while ability has some kind of value attached to it. However, this is not always true. It has been found that those who are field-independent and those who are divergent always seem to have some advantage in performance seen in terms of typical academic tests and examinations (e.g. Bahar, 1999; Danili and Reid, 2005). The key distinction of seeing ability as focussing on *level* of performance while the *manner* of performance is focussing on style is the most useful way forward. This study utilised cognitive learning styles as a research tool, which defined cognitive processing characteristics based on task-relevent measures.

6.2 Meaning of Cognitive Learning Styles

It is not surprising, then, to discover that researchers in the learning-centred tradition often use the term *learning style* but mean something entirely different from one another. The learning-centred tradition use of the term *style* is in a strict sense different to the definition adopted by workers in the cognitive-centred approach (Riding and Cheema, 1991; Kirton, 1994; Riding and Rayner, 1997). Support for a distinction between cognitive style and learning style is found in work which attempted to integrate previous development of cognitive style and information processing into a theory of learning and instruction (Entwistle, 1981; Ramsden, 1988). Indeed, Schmeck (1988) quite carefully argued this distinction, while apologetically retaining the use of the term, 'learning style', to describe particular aspects of individual difference in learner's approach to learning.

Following this distinction, many definitions appear in the literature in the use of the term *cognitive style (or learning style)*. Numerous authors use the term interchangeably. (Riding and Rayner, 1997) note that learning style has been used as a description for the individual cognitive process of thinking, perceiving, and remembering information, or his preferred approach to using information to solve problems.

McFadden (1986) states that most definitions of learning style as well as cognitive style, illustrate variations in individual information processing and that no single definition for learning style or cognitive style has been identified. He goes on to note various descriptions of cognitive styles:

> *"the fundamental make-up of a person."* (Riding, 1997)
>
> *"a consistent pattern of behaviour within a range of individual variability."* (Brumby, 1982)
>
> *"a student's consistent way of responding to and using stimuli in a learning environment."* (Claxton and Ralston, 1978)
>
> *"how individual process information and prefer to learn."* (Riding and Rayner, 1997)

"the way individual organise information and experiences."
(Jonassen and Grabowski, 1993)

"a person's characteristic style of acquiring and using information."
(Sternberg and Zhang, 2001)

"an expression of psychological differentiation within characteristic modes of information processing." (Witkin and Goodenough, 1981)

"*the cognitive, affective, and psychological traits that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment."* (Zarghani, 1988)

These descriptions capture, at least implicitly, the problems of knowing whether cognitive styles are genetic, learned or matter of choice. There are also elements of *affect* or feeling, *behaviour* or doing and *cognition* or knowing. These primary elements in an individual personal psychology are structured and organised by an individual's cognitive style. This psychological process, in turn, is reflected in the way that a person builds a generalised approach to learning, still recognising that the person may choose to move from this generalised way in specific circumstances.

It is this dynamic approach which involves the individual in a lifelong process - the building up of a repertoire of learning strategies which combine with cognitive style - to contribute to an individual's *personal learning style.* Apart from this process, these primary elements of personal psychology interact with cognitive style to influence the formation of attitudes, skills, understanding, and a general level of competence realised in the learning process.

Saracho (1991) brings some of this together when he states that, "*cognitive styles identify the ways individual react to different situations and they include stable attitudes, preferences, or habitual strategies that distinguish the individual styles of perceiving, remembering, thinking and problem solving*".

Letteri (1980) described learning as an exercise in information processing involving the storage and retrieval of information. The process of learning was categorised into six stages ranging from initial perception to long-term memory. A failure to process information in any one of these stages was interpreted as a deficit in cognitive skill acquisition. The teaching of cognitive skills or 'augmentation' as Reinert (1976) described the process of cognitive skills training, formed the basis for assessing and developing learning style and intellectual development.

Letteri's style construct is important for its presumption that assessment and style awareness should be used to change a student's cognitive profile and learning style. The question of whether a development of style can be 'forced' is relevant to our own notion of strategy formation and a strategy approach to the learning process. The idea has also played a continuing part in discussions about operationalising learning style in the learning context.

In general, cognitive learning styles are the information processing habits of an individual. Unlike individual differences in abilities, cognition describes a person's typical mode of thinking, perceiving, remembering, or problem solving. It is usually described as a personality dimension which influences attitudes, values, and social interaction. For example, ask yourself how you process experiences and knowledge and how you organise and retain information. Do you need to visualise the task before starting? Do you approach learning and teaching sequentially or randomly? Do you work quickly or deliberately? These are cognitive learning style characteristics.

The whole question of the extent of the genetic nature and the extent of the learned or chosen nature of cognitive styles is addressed by some in using the term learning strategies.

6.3 Cognitive Style and Learning Strategy

It is useful to distinguish between *style* and *strategy.* Style probably may have a physiological basis and is fairly fixed for the individual. By contrast, strategies are ways that may be learned and developed to cope with situations and tasks, and particularly methods of utilising styles to make the best of situations for which the styles are not ideally suited. Within the literature, the term learning style is sometimes used to refer to what here is considered to be learning strategies. Overall, there is considerable confusion.

Personal style describes the way in which a person habitually approaches or responds to the learning task. Riding and Rayner (1999) address the problem by suggesting that personal style comprises two fundamental aspects: first, the *cognitive style,* which reflects the way in which the individual thinks; second, *learning strategy*, which reflects those processes which are used by the learner to respond to the demands of a learning activity.

Riding and Rayner consider a person's cognitive style as probably an inbuilt and automatic way of responding to information and situations that may be present at birth or at any rate fixed early on in life. It is thought to be deeply pervasive, affecting a wide range of individual functioning. A person's cognitive style is a relatively fixed aspect of learning performance and influences a person's general attainment or achievement in learning situations. This again suggest a strong genetic basis.

The problem is that any attempt to measure cognitive style under this kind of description will tend to measure cognitive style *plus* any experience and learning preferences. To deal with this problem, Hindal and Reid (2009) used the phrase '*learner characteristics*'. This allows for the genetic, the learned, and the element of choice. In any measurement, it is more or less impossible to separate these.

The implications of cognitive style for the educator and trainer are far-reaching, but to date conspicuously underdeveloped in working practice. Hamblin (1981) commented that constructive teaching of 'study skills', with the aim of raising the level of achievement, should not be regarded as a search for single correct 'way to do it'. Nor should 'study skills' or 'learning to learn' be left to random chance, individual adaptiveness, or a haphazard management of pedagogy. Hamblin advised that teachers' work is about

> *"Encouraging pupils to engage in a long-term process of building a style of learning which is meaningful and productive. Pastoral care embodies the ethic of a found respect for individuality. To try to impose a learning style is the pedagogic equivalent of imposing a false self upon someone - an act which is inevitably as destructive in the long run."* (Hamblin, 1981: 21)

While the evidence from information processing shows that all learn in essentially the same way: information is selected, processed in the working memory, and stored in the long term memory, however, there are differences in *how* information is selected, *how* it is processed and *how* it is stored. These differences constituted what might be known as learner characteristics (Hindal and Reid, 2009). These differences are extremely important. For optimal learning, the process must be as consistent as possible with the individual learner's characteristics. Given a class of students, this places an impossible burden on any teacher.

There is another issue. Is it possible to reduce the myriad of learner characteristics into a small number of dimensions. Here, controversy exists whether cognitive style has a single or multiple dimension of human personality. This will be discussed later.

6.4 The Development of Theory of Cognitive Style

Since the mid-1940s, there have been extensive list of style labels research. It is helpful to trace the development of a style construct from its various beginnings. An early interest in cognitive style as a construct is associated with the work of several areas of psychology with some writers approaching style from an organising perspective of *differential psychology* (Jonassen and Grabowski, 1993; Messick, 1996), while others have been cognitive psychologists interested in the process and abilities in cognition (e.g. Merriam and Caffarella, 1991; Swanson, 1995; Sternberg and Grigorenko, 1995; Riding, 1997).

According to Vernon (1973), the primary antecedents of style can be traced back to description of personality in classical Greek literature. Messick (1996) also suggested this same origin for style and argued that the idea that different individuals have contrasting personalities that differentially influence their modes of cognition and behavioural expressions could be traced back to ancient classifications of temperament and physique. The typology to which he referred was an early model of human personality created by Hippocrates. This typology consisted of four personality types: the melancholic, the sanguine, the phlegmatic and the choleric.

Over the last one hundred years, various traditions of psychology have contributed to the emerging field of cognitive style. Allport (1961), in his work which developed the idea of *life-styles,* was probably the first researcher to deliberately use the 'style' construct in association with cognition.

There have been several streams of work contributing to the development of cognitive style. A contemporary theory of style appears to flow from four areas of psychology:

- Perception
- Cognitive controls and cognitive processes
- Mental imagery
- Personality constructs

Each of these areas will be briefly discussed here.

Perception

The first influence in an emerging theory of cognitive style was in the psychology of perception, exemplified by the work of Witkin and co-workers, which began in the 1940s. Experimental work, reflecting an emphasis on the regularities of information processing-derived from the gestalt school of perceptual psychology, led to an early development of the concept of field-dependence and field-independence (Witkin *et al.,* 1962; Witkin, 1964; Witkin and Goodenough, 1981). In essence those who are more field-independent are able to separate out what is important from the surrounding field of perception. This construct will be discussed more fully later.

Cognitive controls and cognitive process

The second was the study of cognitive processes related to individual adaptation to the environment exemplified by the work of Gardner (1953). This work was influenced, originally, by studies focussed on variables in ego adaptation to the environment. This led to the identification of several cognitive processes including perceptual attitudes, cognitive attitudes, and cognitive controls. Further work related to this area led to several stylistic labels and models and supported the general notion of a cognitive style (Messick, 1996).

Mental Imagery

A third area involved work looking at mental representation. Early in the scientific study of psychology, attention was given to the notion that some people have a predominantly verbal way of representing information in thought, while others are more visual or imaginal (Galton, 1883; James, 1890). Paivio (1971) further developed this idea with a dual coding approach to the measurement of mental imagery. Riding and Taylor (1976) identified, as fundamental to the construct of cognitive style, the verbal-imagery dimension of cognitive style.

Personality Constructs

A fourth area of work involved researchers utilising personality-based constructs to develop a model of learning style (Myers, 1978). Much of this approach is attributed to a psycho dynamic perspective on the question of individuality. The simple most significant contemporary example of this kind of construct is the assessment model presented by Myers-Briggs (Myers, 1978).

While we will deliberately not give attention to this stream of development, it is a style model which has been adopted by several researchers in the field.

6.5 The Cognitive Style Dimensions

There have been many attempts to reduce the complexity of what were called learning styles to a smaller number of dimensions. Between the early 1940s and the 1980s, various investigators observed what they felt represented style dimensions. Generally, the researchers worked in their own contexts, in isolation from one another, developed their own instruments for assessment and gave their own labels to the styles they were studying with little reference to the work of others. Not surprisingly, this led to the development of a large and confusing variety of style labels. A number of workers have suggested that many of these are simply different conceptions of the same dimensions (Coan, 1974; Fowler, 1980; Brumby, 1982; Miller, 1987; Riding and Buckle, 1990).

Riding and Cheema (1991) found over 30 labels and, after reviewing the descriptions, correlations between them, methods of assessment, and effect on behaviour, concluded that they could be grouped into two principal cognitive style dimensions: the holistic-analytic and the verbal-imagery style dimensions.

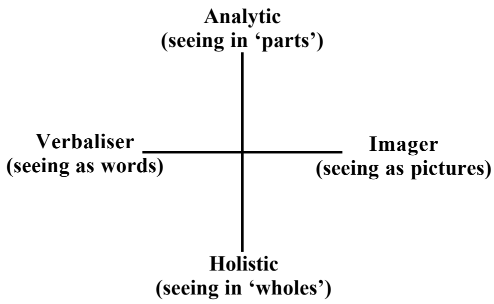


Figure 6.1: The Cognitive Style Dimensions (after Riding and Rayner, 1999)

Riding and Rayner (1999) found further evidence to support this conclusion. Their analysis can be summarised:

- The holistic-analytic style dimension of whether an individual tends to *organise* information into wholes or parts.
- The verbal-imagery style dimension of whether an individual is inclined to *represent* information during thinking verbally or in mental pictures.

However, there are major problems with this kind of analysis. It suggests that there are two dimensions. A person's learning style lies somewhere along each axis, giving each person a point in the two dimensions which best represents their learning style. However, is it possible that a person is strong at *both* ends of one dimension? Thus, an individual can see in parts well and can also see in wholes well. Another is strong in *both* verbal and imagery styles. A person who is average in each lies at the midpoint but where does the person strong in each find their representation? This model is based on an assumption of the linearity of each axis. This may or may not be true.

The categorisation of the models described in Figure 6.1 has been made on the basis of an identification of two fundamental dimensions of cognitive style originally identified by Riding and Cheema (1991). This categorisation of models of cognitive style will assist in the integration of various constructs or labels of cognitive style. In particular, it is helpful to attempt this kind of synthesis on the basis of identifying fundamental dimensions of cognitive style. The research and development associated with this categorisation can be further organised into three groups of models or labels which:

- Relate principally to cognitive organisation - the holistic-analytic style dimension.
- Relate principally to mental representation - the verbal-imagery style dimension.
- Reflect a deliberate attempt to integrate both the holist-analytic and verbal-imagery dimensions of cognitive style.

Thus, many authors have used the model shown in Figure 6.1 to encompass a wide diversity of learning styles (Table 6.1).

The holistic-analytic dimension		
Field dependency-independency	Dependency on a perceptual field when analysing a structure or form which is part of the field.	Witkin and Asch, (1948); Witkin et al (1971, 1977)
Levelling-sharpening	A tendency to assimilate detail rapidly and lose detail or emphasise detail and changes in new information.	Klein (1954); Gardner et., al (1959)
Impulsive-reflectiveness	Tendency for quick as against a deliberate response.	Kagan et., al (1964); Kagan (1966)
Converging-diverging thinking	Narrow, focused, logical, deductive thinking rather than broad, open-ended, associational thinking to solve problems.	Guilford (1967); Hudson (1966, 1968)
Holist-serialist thinking	The tendency to work through learning tasks or problem solving incrementally or globally and assimilate detail.	Pask and Scott (1972) Pask (1976)
Concrete sequential/ concrete random/ abstract abstract random	The learner learns through experience concrete and abstraction either randomly or sequentially.	Gregorc (1982)
Assimilator-explorer	Preferences for seeking familiarity or novelty in the process of problem solving creativity.	Kaufmann (1989)
Adaptors-innovators	Adaptors prefer conventional, established procedures and innovators restructuring or new perspective in problem solving.	Kirton (1976, 1982)
Reasoning-intuitive-contemplative	Preference for developing understanding through reasoning and or by spontaneity or insight and learning activity which allows active participation reflection	Allinson and Hayes (1996)
The verbal-imagery dimension		
Abstract versus concrete thinker	referred level and capacity of abstraction	Harvey et., al (1961)
Verbaliser-Visualiser	The extent to which verbal or visual strategies are used to represent knowledge and thinking.	Paivio (1971); Riding and Taylor (1976); Richardson (1977) Riding and Calvey (1981)
An integration of the	***holistic-analytic and verbal-imagery dimensions***	
Holistic-analytic	Tendency for the individual to process information in parts or as a whole and think in words or pictures.	Riding (1991, 1994) Riding and Cheema (1991)

Table 6.1 Dimensions of Cognitive Style ***(from Rayner and Riding, 1999)***

The models are grouped according to the dimension of style to which they relate, which in turn reflects the integration of style family described by Riding and Cheema (1991). With so many dimensions of cognitive style available, Lewis (1976) remarked that the diversity of style theory was unhelpful and misleading if the theory of style was ever to prove useful in practice. He stated that:

> *"In my opinion, the right thing to do is to focus ... on the search for individual differences which are basic, in the sense that they underlie (and to that extent, explain), a whole range of more readily observable differences."*

Indeed, the sheer complexity of the analysis make the idea of reducing learning styles to two dimensions unhelpful.

In this study, only three aspects of learner characteristics will be explored in relation to the learning of mathematics: the field-dependent/field-independent characteristic, the divergent/convergent characteristic, and creativity. Each is now discussed briefly.

6.6 Field Dependency Characteristics

Field dependency is most popular among the numerous studies in cognitive learning style from researchers (Witkin and Asch, 1948; Witkin and Goodenough, 1981; Al-Naeme, 1991; Riding and Rayner, 1997; Oraif, 2006) in that it has the broadest applications in the problem solving. Witkin and Asch, (1948) found that some individuals show remarkable consistency to different types of cues. Jonassen and Grabowski (1993) noted that field dependency describes the extent to which:

"• *The surrounding framework dominates the perception of items within it.*
- *The surrounding organised field influences a person's perception of items within it.*
- *A person perceives part of the field as a discrete form.*
- *The organisation of the prevailing field determines the perception of its components, or*
- *A person perceives analytically."*

Jonassen and Grabowski, (1993, Page 87)

The learning styles of individual characteristic and of group can be located on a continuum between a '*global style*' and an '*articulated style*'. People who use a global style tend to view the world holistically; they see first a bundle of relationships and only later the bits and pieces that are related. They are said to be field dependent. By contrast, people who use an articulated style tend to break up the world into smaller and smaller pieces, which can then be organised into larger chunks. They also tend to see a sharp boundary between their own bodies and the outside world. People using an articulated style are able to consider whatever they happen to be paying attention to apart from its context and so are said to be field-independent

Originally, most people in the Western societies were thought to be field-independent, whereas most people in most non-Western cultures were thought to be field-dependent. However, more detailed research by Cole *et al.,* (1971) shows that these generalisations are misleading. For instance, the preferred cognitive style of an individual often varies from task to task and from context to context. People who use articulated styles for some tasks also use global styles for other tasks. In fact, they may bring a range of different styles to bear on a single task. This again challenges the notion that cognitive styles are fairly fixed.

Witkin and Goodenough (1981) described a field-dependent individual as someone who has difficulty in separating an item from its context, whereas a field-independent individual is someone who can easily break up an organised field and separate relevant material from its context, that is, an individual who can distinguish between the *signal* and *noise.* Subjects with middle performance are called field-intermediate.

Research led by Witkin and Asch (1948) focused initially on perception, as they identified differences in individuals who were deciding whether an object was upright in space. Research into field dependency (this term will be used from now on) led to an awareness that competence at disembedding shapes and objects was strongly associated with competence at disembedding in other non-perceptual, problem solving tasks. This resulted in the construct being broadened to encompass both perceptual and intellectual activities and was referred to as the *global-articulated* dimension. Later, with additional evidence on self-consistency, extending to the areas of body concept, sense of self, and controls and defences, the construct became even more comprehensive and was labelled as 'psychological differentiation' (Witkin *et al.,* 1962; Witkin, 1964; Witkin and Goodenough, 1981).

In general, Al-Naeme (1991) notes that,

> *"field-independent people have the ability to overcome embedding contexts in perceptual functioning. This ability may give them a sense of separate identity, with internalised values and standards that allow them to operate with a degree of independence of the social field. In contrast, field-dependent people do not have the ability to overcome embedding contexts in perceptual functioning."*

The theoretical background to this construct involved an interest in individual differences in learning behaviour which is linked to personality, and it was deemed to be useful as a means of accessing and understanding the individual differences in students' learning behaviour in the classroom. These differences are summarised in Table 6.2.

Independent	***Dependent***
Works on his own	Relies upon the teacher for guidance
Complete tasks	Requires support and extrinsic motivation
Responsive	Non-responsive and little curiosity
Free-think	Does the minimum and follows the lead

Table 6.2 Field Dependency Characteristics

Weinstein and Van Materstone (1996) provided a useful description of the field-independent learner. They argued that field-independent learners can be identified by the way in which they use and apply the knowledge:

- About themselves as learners
- About different types of academic tasks
- About strategies and tactics for acquiring, integrating, and applying new learning
- Of both present and future contexts in which their knowledge could be useful

All of these are cognitive behaviours and make huge demands on working memory.

Several studies on field dependency characteristics (e.g. Ghani, 2004; Chu, 2007) noted that there are five factors that influence the extent or degree to which a learner is either field-dependent or field-independent.

(i) *Age*: Children are generally field-dependent, but their field-independence increases as they grow older to become adults. For example, adult learners are more field-independent (Gurley, 1984). After that time, the field-independence gradually decreases throughout the remainder of life, with the older tending to be field-dependent than their younger cohorts (Witkins *et al*., 1971).

(ii) *Gender*: Studies found that males perform slightly better in the hidden figure tests (tests of field-dependent/field-independent) but the effect of sex on the field-dependent/field-independent is so small that this factor is practically insignificant (Musser, 1998).

(iii) *Hemispheric Laterisation*: Pizzamiglio (1974) and Silverman *et al*., (1966) found that left-handed individuals are more field-dependent while the right-handed individuals are field-independent.

(iv) *Socio-Economic Status*: Students from higher socio-economic background are found to be more field-independent than those from lower socio-economic background (Forns-Santacana, *et al.,* 1993).

(v) *Childhood Upbringing*: Children from families where there is encouragement for them to develop separate, autonomous functions are relatively field-independent, while others who showed emphasis to parental authority and guidance are likely to become relatively field-dependent (Korchin, 1986).

This list of factors is very revealing. It suggests that extent of field dependency is open to development by means of experiences and, perhaps, formal learning. This will be discussed further in chapters 8 and 9.

The implication of field dependency, memory, and learning, along with assessment of field dependency which will be used as a basis for field experiment will be discussed in this chapter. However, in the following section, some research findings on cognitive field dependency, academic achievement, and gender are discussed and synthesised.

6.7 Field Dependency and Academic Achievement

Many studies (e.g. Pascual-Leone, 1970; El-Banna, 1987; Riding and Rayner, 1999) have tried to relate field dependency to other cognitive factors such as learning and memory. Their findings showed that in problem-solving activities, when the solution depends on using an object in an unfamiliar way, field-independent learners are more likely to achieve better than field-dependant learners.

Field dependency has been reported to be one of the significant factors that may impact students' achievement on various school subjects (see, Witkin *et al.,* 1977; Al-Naeme, 1991; Uz-Zaman, 1996; Danili, 2001; Oraif, 2006). In a study, Dwyer and Moore (1995) investigated the effect of field-dependency on achievement with 179 students who enrolled in an introductory education course at two universities in the United States. They found that the field-independent learners tend to be superior to the field-dependent learners on tests measuring different educational objectives. The researchers concluded that field dependency had a significant relationship with students' academic achievement.

Tinajero and Paramo (1998) investigated the relationship between field dependency and students' achievement in several subject domains (English, mathematics, Spanish, natural science, social science, and Galician). With the sample of 408 middle school students, the researchers found that field dependency was a significant source of variation in overall performance of students. That is, field-independent students outperformed their field-dependent counterparts.

In another study, Meece (1981) sought to determine the relationship between academic achievement and field dependency of 63 undergraduate Canadian students in information management programme. They found that the field-independent students performed better than the field-dependent students only on one of the technical courses. For the other three courses, the two groups performed similarly.

To conclude, the construct of field dependency has been treated as a promising variable which may explain differences observed among students' academic achievement on various subjects and provide educators a better understanding of students' achievement by investigating the interaction and causal effects of affective variables. The current findings would help instructional designers and practitioners develop better quality instructional delivery methods from the standpoint of field dependency and attitudes towards mathematics.

6.8 Gender Differences and Information Processing in Field Dependency

The specific interest on the variance of gender differences in mathematics teaching, learning and achievement is explained on the basis of gender differences on cognition and brain lateralisation (Fennema and Leder, 1990; Mondoh, 2001). These differences have implication on instructional procedures to be adopted for purposes of setting up appropriate teaching and learning environment for mathematics instruction that is suitable for both genders.

Mondoh argues that people differ in learning according to how they perceive and process reality. The key argument describes a combination of perceiving and processing techniques that result in the formation of three unique learning styles. This system is largely associated with gender, and produces three types of learners. Each of these types of learner is characterised by certain attributes that are either compatible or non-compatible with the requisite expectations for learning and understanding mathematics.

Type one learners perceive information concretely and process it reflectively. They learn best by personal involvement, listening and sharing ideas. Their favourite question is 'why'. Teachers, therefore, need to provide learners with reasons for learning a particular concept. It is recommended by Mondoh (2001) that this type of learner be taught using group work approach. Girls are more likely to be in this category of learners (Barmao, 2006). The implication of this is that in a situation where mathematics lessons are teacher-dominated and individualised, boys are likely to perform better than girls.

Type two learners are observers and thinkers, who are best taught using experimental methods, which are practical and require the use of mathematics laboratories and instructional aids (FAWE, 1997). Mondo (2001) claimed that majority of girls are inclined to learning through this approach, so that "*in cases where such instructional* *environment is unavailable, they are likely to be disadvantaged*" (Nwosu and Omeje, 2008).

Type three learners perceive information abstractly and process it actively. They like trying things out for themselves. Mondoh (2001) notes that their favourite question is 'how' and concludes that this category of learner can apply concepts to new situations and cope with lots of homework individually. He concluded that boys are recognised to have these attributes, which are also favourable for learning mathematics.

In addition, peoples' ways of thinking and learning have been identified in ten categories (Riding and Rayner, 1997). These are reflective versus impulsive; serial versus holistic; field-independent versus field-dependent; convergent versus divergent; and confidence versus caution cognitive styles. Generally, these cognitive styles affect different learners differently. Hence, compatibility or incompatibility between between boys and girls in their preferred thinking style is likely to affect understanding and achievement in mathematics. For example, according to Costello (1991), boys are impulsive, holistic, field-independent, have convergent attributes and are confident, while girls are reflective, serialist, field-dependent, divergent in thinking and cautious in the process of dealing with matters. These different cognitive attributes affects boys and girls differently, especially with regard to confidence levels, attitudes, ability to take risks, interaction and intellectual dexterity. Some of these attributes favour boys more, while others may tend to favour girls more in the areas of learning and understanding.

Brain lateralisation has also been used to explain the cognitive differences that lead to differences which are in favour of boys' higher achievement in mathematics (Bryden, 1979). The explanation given has been that the right hemisphere, which controls spatial related activities, develops earlier in boys compared to girls. Spatial or visualisation is the ability to visualise movement of geometric figures in one's mind. Hence, a person with greater competence in spatial related activities is likely to perform well in science and mathematics. This explains why, given a similar age cohort of students, boys are more likely to be good in science and mathematics compared to girls (Bosire *et al.,* 2008).

In general, research findings on gender differences and cognitive styles are inconsistent and inconclusive. Some studies (Mallam, 1993; Colley *et al.,* 1994) have found gaps favouring girls as being more field-independent than boys in single-sex schools, but once these findings were adjusted, the differences diminished.

6.9 Assessment of Field Dependency

The early work of measurement of field dependency was done with the use of the first Body Adjustment Test with an attempt to replicate those conditions experienced by pilots in fighter aircraft flying through low cloud formation. The early version of the test involved the person being seated on a tilted chair, in a tilted room, and being asked to adjust the body to the upright. A further version of the test, called the Rod and Frame Test, involved the individual being seated in a completely darkened room. The person was asked to view a tilted luminous rod, within a tilted luminous frame, however, the individual was then asked to disregard the frame, and adjust the rod until it was in a totally upright position. Interest was focused on the relationship between a person's visual and kinaesthetic abilities, and the levels of dependence on the visual context displayed.

As the implications of the concept of field dependency became more apparent, a paper and pencil assessment, was developed reflecting earlier work on the discrimination of shape from its surrounding field carried out by Thurstone (1944). This was later developed into the Group Embedded Figure Test (GEFT); used for a group of adults in which the format is very similar to the Embedded Figure Test (Witkin *et al.,* 1971, 1977). When many shapes are identified correctly, the person is described as field-independent; when few shapes are identified correctly, the person is described as field-dependent. This test was used in the present study in order to obtain data from a group of students about how they learn, process and retrieve information.

6.10 Field Dependency and Memory Processes

In the 1970s and 1980s, many researchers (e.g. Pascual-Leone, 1970; Case and Globerson, 1974; Witkin, *et al.,* 1974, 1977; Witkin and Goodenough, 1981) attempted to study the relationship between extent of field dependency and memory processes in relation to other cognitive factors such as intelligence, learning and memory. They noted that differences exists in the way field-dependent and field-independent people use their working memory. The results of these studies support the hypotheses that some intellectual and perceptual abilities have influence on embedding contexts, which in turn shows that the larger the working memory capacity of the learner, the more likely for the learner to be field-independent.

The investigation carried out by El-Banna (1987) on the relationship between performance in chemistry examinations of low, medium, and high memory capacity students and field-dependent shows that among students with the same working memory capacity, the performance declines when the student is more field-dependent. A possible explanation of these results according to Johnstone and Al-Naeme (1991) could be the fact that "*students with low working memory capacity are not in position to devote any working space to the irrelevant information, and consequently field-independent low working memory capacity students would possibly perform better than the field-dependent low working memory capacity students*".

In a study carried out by Christou (2001), he found little difference in performance in a chemistry examination between low working memory capacity field-independent students and high working memory capacity field-dependent students. His results are shown in Table 6.3.

	Group	Field Dependent	Field Intermediate	Field Independent
Working	Capacity = 5	5.0	6.1	7.8
Memory	Capacity = 6	5.9	7.3	8.3
Capacity	Capacity = 7	7.3	7.3	8.4

Table 6.3 Mean Mathematics Performance related to Working Memory Capacity and Extent of Field Dependency (from Christou, 2001)

A possible explanation of these results can be obtained using suggestions made by Johnstone *et al.,* (1993). According to them, students with a high working memory space capacity and who are field-dependent are occupied with 'noise' as well as 'signal' because of the field dependent characteristic. Conversely, low capacity and field-independent students will receive only the 'signal', tending to ignore the 'noise', and they can use all their limited low working memory space for useful processing. Hence, high working memory capacity field-dependent students cannot benefit from their larger working memory because the working memory capacity is effectively reduced by the presence of 'useless' information.

As far as the relationship between field-dependence and academic performance is concerned, the investigations by Goodenough, 1976; Witkin *et al.,* (1977); Frank and Davies, (1982); El-Banna, (1987); Johnstone and Al-Naeme, (1991); Uz-Zaman, (1996) and Danili, (2001) have suggested that:

- Field-dependent and field-independent individuals differ in the cognitive processes that they employ as well as in the effectiveness of their performance.
- Field-independents score significantly higher than field-dependents in almost every field of science and mathematics.

- Field-independent people tend to be more 'self-sufficient' than field-dependent people who tend to depend more on the external environment.
- Those who are more field-independent in ability tend to show a higher performance in tests measuring working memory capacity.
- Field-dependent individuals encounter difficulties in recalling encoded information unless retrieval cues are directly relevant to the way in which the information was coded. The relevant cues could be considered as 'bridge' to gain access to the stored information.
- Field-dependent individuals exhibit less efficient memory strategies than field-independent individuals when they encounter a problem. The explanation of the poor memory of field-dependent individuals is that they process information in a rigid way which may be the result of an inefficient response to cues which would facilitate their recollection of the past information.
- Field-independent individuals are more capable of demonstrating cognitive structuring skills than field-dependent individuals. The procedure of cognitive restructuring involves the ability to:

  (a) Break up a task into its basic elements;
  (b) Manufacture a structure from an ambiguous stimulus which will be the outcome of such procedures; and
  (c) Make a different organisation of the task than its initial structure in the complex stimulus.

So far it has been emphasised in many parts of the present chapter that field-dependent learners have a less efficient memory process than field-independent learners, and a such, field-dependent learners may display a low performance in many tasks because of their inability to break up tasks and restructure them into basic elements.

6.11 Convergency and Divergency

This dimension was proposed by Guilford (1967). The dimension reflects a type of thinking and associated strategies for problem-solving. The learner will typically attack a problem or task by *thinking* in a way which is either open-ended and exploratory, or close-ended and highly focussed.

The theory was further developed by Hudson (1966, 1968) and its implications for the process of teaching and learning were more fully explored. Hudson reported that learners who were *convergers* preferred formal problems and structured tasks demanding logical method. On the other hand, learners who were *divergers* preferred more open-ended tasks which require creativity. The divergent thinker was far more likely to react negatively to routine, or to the task involving the familiar or expected and requiring a correct answer. It was thought that convergers are likely to be analytic and divergers holist in style.

Hudson (1966) developed tests where a high score indicated a divergent thinker and a low score was thought to indicate a convergent thinker. The items of the test invite the students to generate ideas based on some criteria. The more ideas generated in a fixed time, the more divergent the person was. Hudson did not view the characteristic as fixed, arguing (1968, Page 91) that,

> "*No one was, or was ever expected to be consistently convergent or consistently divergent. I have never seen why someone should not drift slowly over a period of years from divergence to convergence, or vice versa. Nor why someone should not be divergent in some moods and convergent in others. Nor why someone might not be convergent (or divergent).*"

Hindal and Reid (2009) took this further, arguing that it was possible to be divergent, convergent, both or neither: indeed, to be convergent and divergent in any combination of proportions. They went on to develop a test for convergency, designed to follow a similar structure to the established test for divergency. Attempts were made to validate the test by interviewing school pupils immediately after they had completed it.

To her surprise, Hindal (2007) found that school performance in six school subjects correlated strongly with *both* being convergent and being divergent. Indeed, extent of convergency correlated positively with extent of divergency. These outcomes were later confirmed by Badgaish (2008) in the specific area of mathematics. It is possible to suggest that the new convergency test was not valid but the interviews carried out strongly supported the validity of the test.

Haddon and Lytton (1968) studied the effects of differing primary school teaching strategies on divergent thinking abilities. Their results showed that pupils from the 'informal' schools were significantly better in divergent thinking, compared with 'formal' schools. Further support for the view that certain approaches foster convergent thinking and others divergent thinking comes from studies such as Crutchfield (1965), Barker-Lunn (1970), Covington *et al.,* (1974). This suggests that convergent and divergent thinking can be enhanced by means of teaching approaches, undermining the idea that it was an example of a fairly fixed learner characteristic.

Bransford *et al.,* (1977) reported a correlation of 0.44 with Sternberg's (1999) Remote Associates Test, a measure of creativity (creativity is the ability to combine ideas, things, techniques, or approaches in a new way). Contrary to expectation, field-independent students have sometimes been found to be more creative than the field-dependent students. Studies indicate that although in all subjects those who score highly on field-independence test are not necessarily creative; those scoring high on divergent thinking tend to score higher on field-independence test (Blooberg, 1971; Hindal, 2007).

Furthermore, it has been argued that learners studying supposedly more creative disciplines such as philosophy and theology have been found to be more field-independent than learners studying less creative subjects such as business. For example, Bergum (1977), and Moris and Bergum (1978) found that students of architecture regarded themselves as more creative than business students and were also more field-independent than business students.

In general, Hudson has found that convergers prefer formal problems and tasks that are better structured and demand greater logical ability than the more open-ended problems favoured by divergers. According to Austin (1971), convergent learners apparently are more emotionally inhibited than divergent learners, and appear to keep the different aspects of their lives 'compartmentalised'. One explanation of this is that convergers prefer to structure their experience at all levels more than divergers do and are more capable of utilising any structure present.

Overall, evidence seems to suggest that while divergent thinking is an invaluable cognitive quality, socially it is considered as irritating, disruptive and even threatening by teachers. Indeed, Getzels and Jackson (1962) found that teachers preferred learners who were low in divergent thinking (that is, conformist and orderly) to those higher in divergent thinking, even though all the learners were of similar intelligence, and even though the divergent thinkers produced more imaginative and original responses. Where many schools are inherently rule-bound and conservative, this inevitably means that much of the divergent thinking (creativity) is likely to contrast or even conflict with what is routine, familiar, expected and 'correct'.

A further concern was raised by Al-Naeme (1991) concerning the effectiveness of teaching convergent students by convergent teachers, and the teaching of divergent students by divergent teachers. It was found that students who had low convergent scores or high divergent scores tended to do poorly in examinations, if they were taught by convergent teachers. These students were found to perform significantly better in some examinations than others if they were taught by divergent teachers. The implication of this is that it seems beyond prediction what effect a teacher's thinking style will have on his students' academic achievement.

The review of Danili and Reid (2006) shows that all the evidence supports the idea that being divergent is an advantage in normal school and university tests and examinations. However, it has been suggested that those who were more convergent tended to choose the science subject (Hudson, 1966) although those who are divergent tended to perform significantly better. There is some uncertainty about this as Bahar (1999) found. In her study on problem-solving in biology at university level, Al-Qasmi (2006) found strong evidence to show that those who are highly divergent did best at open-ended problems. She also found evidence that being able to think laterally and, thus, be creative was an advantage. She interpreted this in terms of the way ideas were linked in the long-term memory: the more accessible links present, the greater the success in problem-solving.

6.12 Creativity in Problem Solving

We engage in problem solving when we need to overcome obstacles in order to answer a question or to achieve a goal. If we can quickly retrieve an answer from memory, we do not have a problem. If we cannot retrieve an immediate answer, then we have a problem to be solved. Real-life problems tend to be ill-defined. At school and university, most problems tend to be well-defined. They tend not to be open-ended and they focus on one right answer. Hayes and Allinson, (1996) defined a problem as what exists, "*whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap*".

Many have tried to describe the set of procedures which a problem solver should use. Thus, Bransford *et al.,* (1993) described the steps of *problem-solving cycle,* which include problem identification, problem definition, strategy formulation, organisation of information, allocation of resources, monitoring and evaluation.

Figure 6.2 shows what Sternberg (1999) considered as the steps in a problem solving cycle.

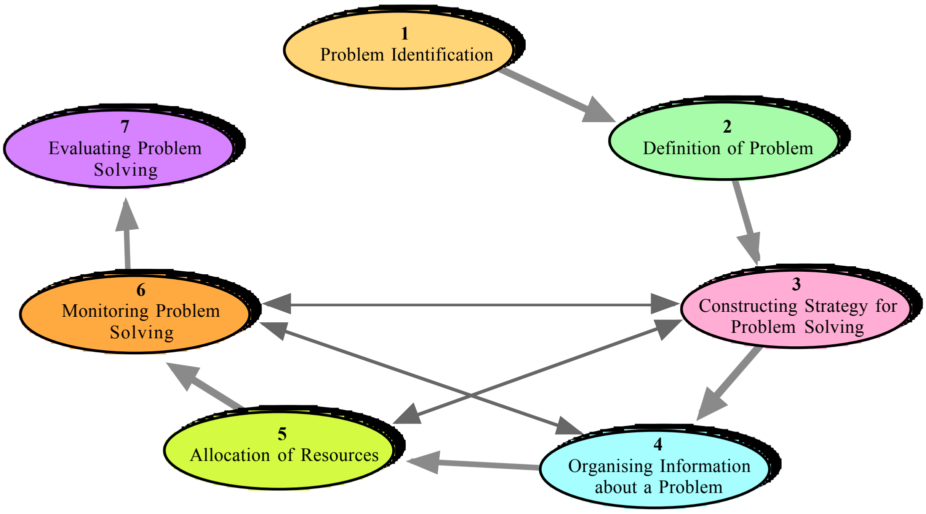


Figure 6.2 Problem-solving cycle (after Sternberg, 1999)

A problem cycle may offer some advantages in close problems, such as mathematical exercises where a taught procedures is applied to some new data to obtain a requested answer. However, successful problem solving may involve occasionally tolerating some ambiguity regarding how best to proceed. Rarely can we solve problems by following any one optimal sequence of problem-solving steps. Moreover, we may go back and forth through the steps, change their order as need be, or even skip or add steps as it seems appropriate.

The study of Reid and Yang (2002b) showed very clearly that, at school level, students did not and would not follow a plan (they would not even attempt thinking of a plan) for solving open-ended problems in chemistry. Bodner (1991) confirmed this very clearly in his many studies on open-ended problem solving at university level.

Mathematical exercises tend to follow set procedures, and teaching a set plan may be advantageous here. However, the study by Reid and Yang (2002b) concluded that there was considerable doubt if such planning and structure brought any advantage when moving into more open-ended problems. Indeed, there is considerable emphasis in instructional theories indicating that learners must find their own ways of problem solving, formulating hypotheses, and generalisations which would allow them to achieve their goal with a genuine feeling of having created and discovered ideas for their own benefit. However, there is the question: is there a place for creativity in mathematics?

Creativity is the process of producing something that is both original and worthwhile. The *something* could be a theory, a dance, a chemical, a process or procedure, a story, or anything else. Sternberg (1999) defined creativity as the ability to combine ideas, things, techniques, or approaches in a new way. This offers a good description.

There was a move towards enhancing creative thinking in problem-solving situations in the 1960s. The change of methods in mathematics teaching moved towards some emphases on creativity, during which learners would be involved in a problem-solving situations. It was believed that a problem-solving situation could provide the best opportunity for learners to develop their creative thinking.

To enhance creative thinking in a mathematics lesson, a problem would be given to learners for which they are not yet learned a method of reaching a solution, or the problem itself should not have yet been identified by the learners. It was believed that problem situation like this would encourage the development of skill which are considered to be original and worthwhile or useful. Encouragement for this kind of thinking activity may be found in several research studies (de Bono, 1976; Perkins *et al.,* 1985; Al-Naeme, 1991; Donovan and Bransford, 2005; Moseley *et al.,* 2005).

Of these, de Bono, whose articulation of a set thinking strategies, such as those set out in his Cognitive Research Trust (CoRT) programme, has been widely applied in both educational and vocational contexts in encouraging individuals to become thoughtful and reflective. His tools are designed to broaden the natural flow of thinking and learning and, redirect it away from well-worn and predictable channels. The programme provides a framework which can be used deliberately in everyday life and in the classroom to enable innovative thinking and cross-situational problem-solving.

Winocur (1985) developed what he called 'The Universe of Creative Thinking Skills,' circulated by the California-based Project Impact. This aimed to represent the basis for its new creative thinking program. Creative Thinking was conceptualised as involving logical reasoning and evaluation, involving the processes of analysing, inferring and questioning. Many such skills and abilities have been suggested, specific, broad, or general in nature. For example, Moseley *et al.,* (2005) conclude that creative thinking skills programmes typically involve five related types of thinking: metacognition; critical thinking; cognitive processes such as problem solving and decision making; core thinking skills such as representation and summarising; and understanding the role of content knowledge. However, there is no evidence that school students follow this kind of approach when showing creative behaviour.

In another analysis of creativity, Garrett (1989) combined utility (usefulness) and originality as major components and drawing a diagram between them as shown on Figure 6.3. A positive correlation between the axes (originality versus utility) indicates the existence of creativity in a task. The degree of creativity would be obtainable from the degree of positive correlation between originality and utility.

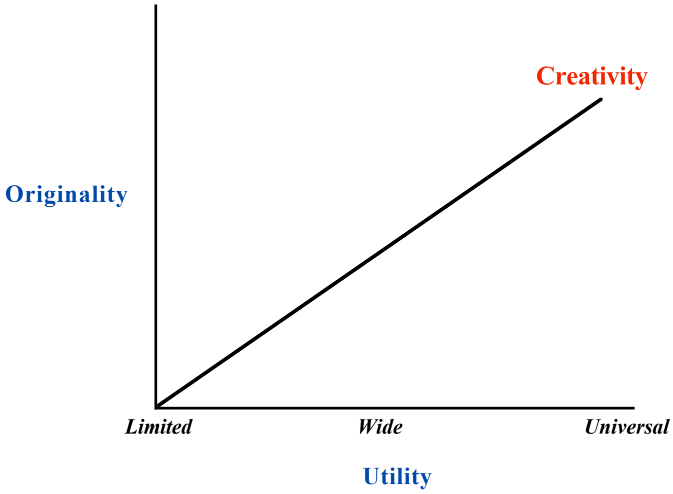


Figure 6.3 Garrett Relationship between Originality and Creativity

Factors that characterise creative individuals are:

- Extreme high motivation to be creative in a particular field of endeavour (e.g. for the sheer enjoyment of the creative process);
- Both nonconformity in violating conventions that might inhibit the creative work and dedication in maintaining standards of excellence and self-discipline related to creative work;
- Deep belief in the value of the creative work, as well as willingness to criticise and improve the work;
- Careful choice of the problem or subjects on which to focus creative attention;
- Thought processes characterised by both insight and divergent thinking;
- Risk taking;
- Extensive knowledge of the relevant domain; and
- Profound commitment to the creative endeavour.

Also involved in the development of '*thinking*' approaches to teaching and learning across curriculum has been a strong orientation to the teaching of strategies for learning in an explicit fashion. Research studies have highlighted the gains that can be achieved when specific cognitive and metacognitive strategies are embedded in the teaching of academic subjects such as reading, science and mathematics (e.g. de Corte *et al.,* 2001; Fuchs *et al.,* 2003). Much early work in this area was undertaken in the fields of memory (Cohen and Nealon, 1979) and reading comprehension (Palincsar and Brown, 1984; Meyer *et al.,* 1989). Such *learning to learn* initiatives were greatly strengthened by increasing teacher familiarity with the constructs of *metacognition* (specifying learning goals, monitoring execution of knowledge, clarity and accuracy) and *self-regulation* (examining self-efficacy, ability to learn emotion and motivation). As a result, the importance for learners of considering about how best to approach tasks involving such cognitive processes as memorising, problem-solving, and applying existing knowledge and skills to new areas (transfer), has become become widely recognised by teachers and other educators (Moseley *et al.,* 2005).

6.13 Chapter Summary

The information processing model discussed in Chapter 2 and the cognitive learning styles examined in the present chapter together offer a comprehensive model to describe what are the essential features of all learning at all levels. The information processing model has been found to be highly predictive and much recent work has shown that simply reorganising the learning situation in line with the insights offered by them will generate much improve understanding (Danili and Reid, 2004; Chu, 2008; Hussein and Reid, 2009) although only one of these studies relates specifically to mathematics.

Although all learners learn in essentially the same way, individual have different ways of collecting, organising and processing information depending upon their cognitive structure and what they already know. These differences which exist in cognitive structure and in psychological functioning enable individuals to have different learning styles.

It is interesting to use the information processing model of Johnstone (1993, 1997) and consider how this offers a possible interpretation for some of the learner characteristics described here. Hindal (2007) attempted this. In her study, working with very large samples aged about 13, she found that those with higher school examination performance tended to be those who were:

(a) Of high working memory capacity;
(b) Field independent;
(c) More visually-spatially skilled;
(d) Divergent;
(e) Convergent.

She argued that (page 211),

> "*It is well established that working memory can often be a rate-controlling feature in the way information is processed, understood and accessed. The student with a high working memory capacity will always have an advantage when faced with situations when understanding, thinking, and searching long-term memory are involved. Field dependency has been related to the way working memory is used. The field independent person can select more efficiently and working memory overload is much less likely. The inter-correlations between field dependency and working memory capacity measures would seem to confirm this (see Danili and Reid, 2004). The field independent person is using the perception filter more efficiently and effectively. This*

> *filter is controlled by what is already known in long-term memory. Clearly, the person who knows more may be able to select better and this may offer an explanation of why the field dependency relates to examination performance in a recall situation.*"

She saw the visual-spatially skilled as tending to see things as pictures or diagrams which can be seen as one but may hold much information and the information may be linked together in a meaningful way if the picture has meaning. Thus, the strong relationship between the extent of divergency and recall skills probably arises because the "*student who can use links between ideas has a considerable advantage in being able to find answers in a recall situation*" (Hindal, 2007).

Hindal went on the explore divergency and suggested that the divergent thinker was able, in some way, to search relentlessly for 'right' answers and this had obvious advantages in examination success. This could well reflect the way the working memory managed the search of the long-term memory.

Although much of this is speculative, it does make some kind of intuitive sense. The learner characteristics are related to variations in the ways information is processed, stored and recalled.

The way the study was conducted and the outcomes obtained are now discussed.

Chapter Seven

Exploring Attitudes and Learning Difficulties

7.1 Introduction

In many countries, mathematics is facing problems at school level. Nigeria is no exception to this and there are recent reports of school students finding mathematics difficult and turning away from further study (Haylock, 2006). We find in the schools now that students are ready to announce proudly that they cannot do better in mathematics while they will be ashamed to admit the same of other subjects such as business studies, history, geography, citizen education, modern language, etc.

It is true that every field of knowledge (including mathematics) has its own distinctive ways of learning and reasoning. The central problem of mathematical teaching then becomes one of relating the logic sequence being taught to the psychological or intellectual structures necessary to understand it. If we are to understand mathematics we need to have a good grasp of not just the terms, concepts and principles that are used in the subject but also the distinctive ways in which mathematics makes and justifies its assertions. In terms of school or university mathematics learning, some key aspects of mathematical reasoning are identified: the process of mathematical modelling; making connections; comparison and ordering; equivalence and transformation; classification; making and testing generalisations; explaining, convincing and proving; application and problem-solving; and thinking creatively.

Many of these aspects have reoccurred as themes throughout this study. For example, the significance of making connections was described in Figure 3.1 of Chapter 3 and then used as a framework for other chapters to reflect a kind of aspects of mathematical thinking and learning. The National Curriculum for mathematics in England has an attainment target called ‘using and applying mathematics’. This recognises that the skills, concepts and principles of mathematics that learners master should be used and applied to solve problems. It is the nature of the subject that applying what we learn in solving problems must always be central component of mathematical skills.

A problem, as opposed to something that is merely an exercise for practising a mathematical skill, is a situation in which we have some givens and we have a goal, but the route from the givens to the goal is not immediately apparent. This means, of course, that what is a problem for one person may not be a problem for another. For a task to be a problem, there must be for the person concerned a gap between the givens and the goals, without an immediately obvious way for the gap to be bridged.

Trying to understand how people think and learn in a problem-solving situation is in some ways an impossible challenge, since we can only try to understand these things by using the very processes that we do not fully understand. In such circumstances, choices are available. We can choose to focus on measurable aspects of human behaviour rather than on lived experience; or we can resort to metaphors which have personal or group appeal; or we can do what scientists have often done when entering a new and complex field - look for patterns and regularities between situations. All the three approaches are evident in the taxonomic approaches to thinking and learning that are described in Chapter one and they all involve classification. Moreover, they all result in simplified accounts, since the human memory can only operate consciously with limited amounts of information.

The first step of the study was to identify the problem areas and explore the nature of difficulty in mathematics. It was also important to consider how the students saw things themselves during the process of their learning. This chapter outlines how this initial survey was conducted. Different age groups were considered: approximately ages 12, 14, 17 and 19. The first three age groups were investigated together (see Section 7.4), and the last group was discussed sparately as further exploratory study in Section 7.10. A range of measurements was made:

(a) Collection of examination data;
(b) Measurement of working memory capacity;
(c) Survey of areas of difficulty; and
(d) Survey of student perceptions.

The aim in all was to gain an overview of the problems. Working memory capacity is known to be an important factor in much learning (Reid, 2009a,b). It was possible to explore this in relation to examination performance in mathematics. Most of the experiments in this chapter were drawn from Nigeria, however, few were from England. The study also aimed to focus on learners perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties.

Overall, this chapter attempts to explore four questions:

(1) What are students' attitudes towards mathematics?
(2) What cognitive demand does learning mathematics place on the learners?
(3) What are the areas of of mathematics difficulty?
(4) Are there any relationship between students' working memory capacity and their achievement in mathematics?

7.2 Measurement of Working Memory Capacity

To explore how working memory capacity is related to students' performance in mathematics, the Digit Span Backward Test was used. This test measures the working memory capacity of students and relates the result to their ability to hold and process information. The test consists of a set of digits numbers. These are read out to the respondents who were asked to recall and write down the numbers in reverse order. For example, the number '6 9 7 2' would be written as '2 7 9 6'. Two chances were given for each level of testing and the number of digits was increased by one, until 8 digits.

Every digit was read to the students at the rate of one digit per second, and the same time was given to recall after the reading of the whole set of numbers. The students were not permitted merely to write backwards. The precise details of how the test was conducted, along with the sets of random numbers used, are given in the Appendix A.

For this study, working memory space was chosen in that numerous studies have suggested that this was one of the most important factors affecting school achievement. Reid (2009b) has listed a number of studies in various subject areas and the table from that paper is given here along with data from some studies in mathematics:

Age	*Country*	*Sample*	*Subject*	*Test Used*	*Pearson Correlation*	*Probability*	*Source*
13-15	India	454	Science	DSBT	0.34	$p < 0.001$	Pidikiti, 2005
13	Kuwait	641	Science	FIT	0.23	$p < 0.001$	Hindal, 2007
15	Greece	105	Chemistry	FIT	0.34	$p < 0.001$	Danili, 2004
13	Taiwan	151	Physics	FIT	0.30	$p < 0.001$	Chen, 2005
13	Taiwan	141	Biology	FIT	0.25	$p < 0.001$	Chu, 2008
13	Taiwan	141	Genetics	FIT	0.62	$p < 0.001$	Chu, 2008
16-17	The Emirates	809	Physics	DSBT	0.11	$p < 0.01$	Al-Ahmadi, 2008
16-17	The Emirates	349	Physics	DSBT	0.32	$p < 0.001$	Al-Ahmadi, 2008
16	Greece	90	Mathematics	DSBT	0.40	$p < 0.001$	Christou, 2000
14-15	Kuwait	874	Mathematics	DSBT	0.52	$p < 0.001$	Al-Enezi, 2004
14-15	Kuwait	472	Mathematics	DSBT	0.24	$p < 0.001$	Al-Enezi, 2008
14-15	Kuwait	874	Mathematics	DSBT	0.36	$p < 0.001$	Al-Enezi, 2008
10	Pakistan (Urdu)	150	Mathematics	FIT	0.69	$p < 0.001$	Ali. 2008
10	Pakistan (English)	150	Mathematics	FIT	0.43	$p < 0.001$	Ali. 2008

Table 7.1 Summary of Some Data
(Source: Reid, 2009b and other papers)

The size of the working memory capacity was taken as the highest number of digits that a student was able to recall correctly. When the students failed to recall both items at one level with the same number of digits, they were given the score of the previous level. In practice, while most were straightforward to mark, there were a few cases where allocating the mark was not easy.

The results of the working memory capacity test were correlated with the mock examination scores in mathematics for the senior secondary students. Correlation only shows if two variables are associated. It does *not* indicate causality. However, the key experiments of Johnstone and El-Banna (1987) show that the relationship is, indeed, causality. Previous work had, therefore, indicated that limited working memory capacity was one factor which influenced success in mathematics examinations. This study aimed to explore the extent of importance of this factor in a Nigerian context.

The Digit Span Backwards Test is known to offer reliable and valid outcomes (El-Banna, 1987, page 62). El-Banna found that the Digit Span Backwards Test gave identical results for 92% of his sample when compared to the measurements made by the Figural Intersection Test, the two tests using very different approaches in measuring working memory capacity. In this study, the absolute values are not particularly important as correlation is based on the order of the results. Nonetheless, the procedures used here followed established procedures very closely and there is confidence that the working memory capacities measured are probably close to absolute. However, it has to be noted that the digit span backwards test tends to give outcomes one less than the actual capacity in that a 'space' is used for reversing the numbers.

7.3 Survey Data

The first three of the four age levels chosen were drawn from primary, junior secondary and senior secondary stages in the Nigerian education system. The senior students were offered a list of topics drawn from their current mathematics curriculum. For each topic, they were asked to tick one box:

☐	Easy	I understood this first time
☐	Moderate	I found it difficult but I understand it now
☐	Difficult	I still do not understand it
☐	Not studied	I have never studied this topic

In order to explore the perceptions of the students in relation to their studies in mathematics, students were invited to respond to a survey. This had the following features.

Secondary (senior and junior secondary):

(a) Analyses of students' attitudes towards various aspects of their secondary mathematics course were performed:

- Attitudes towards some mathematics topics
- Attitudes towards their general aspect of learning mathematics

- Self-evaluation of the personal progress and growth
- Perception about mathematics
- Career aspirations
- Choice of school subject

(b) Analyses of students' areas of mathematics difficulties to identify some mathematical topics which are causing most difficulty with the senior secondary students.

Primary

Pupils' attitudes towards various aspects of their mathematics lessons were analysed:

- Attitudes towards mathematics learning
- Self-evaluation of the personal progress and growth
- Perception about mathematics
- Future career opportunities
- Reason for studying mathematics

Apart from the questions aimed at gathering information of an evaluative character, questionnaires (for both groups) contained closed and open-ended questions which aimed to find out:

- Students' interest in mathematics
- Perceptions of being a mathematician
- Opinion about general aspects of learning mathematics

Questions used were:

(1) Multiple tick questions, where students could choose as many options as they desire.
(2) Yes or No questions.
(3) Preference ranking questions, where students could choose from a list the things they feel most appropriate.

The Semantic Differential method (Osgood *et al.*, 1957) was employed to construct the questions which were aimed at obtaining information of an evaluative character. This method is very often used in attitude surveys by education researchers. To find out students' views about studies in mathematics, a six point Semantic Differential scale was used. It was designed with several set of bipolar word-pairs placed at the opposite ends of the scale for students to rank their evaluation. It has the advantage of enabling the respondents to express their evaluative opinion without having to write their views in words. Another main advantage of this method is that respondents can finish it in high speed, and even children can do it (Reid, 1978). El-Sawaf (2007) has a full discussion on the use of the semantic differential method and outlines evidence for its usefulness and reliability.

Example of survey forms related to attitudes to study used with primary school students is shown on pages 145 and 146. However, questions 1 and 2 which contain information about respondents' gender and school are omitted. This survey is identical for junior and senior students with the exception of question 3 which show a different list of topics, reflecting the curriculum at these stages. All the three surveys are shown in full in the Appendix B. However, the survey form for difficult mathematics topics for *senior* school students is shown in Appendix C.

For the last age group (approximately 19) chosen, the survey data were both obtained in England and Nigeria from A level and first year university students respectively. The major differences between this survey and that employed in the previous age levels are that it employed method developed by Likert (Chapter 5) for which respondents indicate whether they 'strongly agree', 'agree', 'undecided', 'disagree', and 'strongly disagree' to statements about their studies in mathematics, and the survey does not show a list of mathematics topics at this stage. Again, the full survey is shown in the last part of Appendix B.

Mathematics
Where are the Difficulties?

(3) Here are some topics you may have studied in mathematics.
Which of them interest you?
*Tick **as many** as you like.*

- ☐ Prime numbers
- ☐ Factors
- ☐ Decimal
- ☐ Ratio
- ☐ Writing numbers in words
- ☐ Finding missing numbers
- ☐ Multiplication division of numbers
- ☐ Multiples
- ☐ Fractions
- ☐ Percentages
- ☐ Powers and roots
- ☐ Writing numbers in figures
- ☐ Addition and subtraction of numbers
- ☐ Area and perimeter

(4) What are your opinions about **mathematics lessons**?
Tick one box on each line

I like mathematics lessons	☐	☐	☐	☐	☐	☐	I hate mathematics lessons
Boring lessons	☐	☐	☐	☐	☐	☐	Interesting lessons
I enjoy the lessons	☐	☐	☐	☐	☐	☐	I do not enjoy the lessons
Easy lessons	☐	☐	☐	☐	☐	☐	Complicated lessons
Not essential for life	☐	☐	☐	☐	☐	☐	Essential for life
Best learned from a textbook	☐	☐	☐	☐	☐	☐	Best learned from a teacher
Relates to the events of daily life	☐	☐	☐	☐	☐	☐	Does not relate to the events of daily life
Very important for gaining employment	☐	☐	☐	☐	☐	☐	Not very important for gaining employment

(5) How do you feel about your **mathematics course** at school?
*Tick **one box** on each line*

I feel I am coping	☐	☐	☐	☐	☐	☐	I feel I am NOT coping well
I learn a lot new things	☐	☐	☐	☐	☐	☐	I learn nothing new in mathematics lessons
I am NOT obtaining new skills	☐	☐	☐	☐	☐	☐	I am obtaining a lot of new skills
I like the teacher	☐	☐	☐	☐	☐	☐	I dislike the teacher
Mathematics is important	☐	☐	☐	☐	☐	☐	Mathematics is unimportant subject
I hate doing home work	☐	☐	☐	☐	☐	☐	I enjoy doing homework

(6) Would you like doing more mathematics in secondary school?

Yes, because ………………………………………………………………………………………
No, because ………………………………………………………………………………………

(7) We should like to know what you think about **people who work using mathematics**
In your opinion, do you think the following statements are true or false?
Tick on box on each line.

True	***False***	
☐	☐	All mathematicians are very intelligent people
☐	☐	Being a mathematician is very interesting
☐	☐	Mathematicians don't dress well
☐	☐	Being a mathematician is hard
☐	☐	Mathematicians usually are rich people
☐	☐	Girls don't like being mathematicians
☐	☐	Mathematicians work to make discoveries

(8) Which of these do you think is going to be **most interesting** to do in secondary school?
Tick as many as you wish.

- ☐ Playing in the school sports team
- ☐ Cooking or metalwork
- ☐ Playing musical instruments
- ☐ Learning mathematics
- ☐ Solving different kinds of problems
- ☐ Painting pictures
- ☐ Learning foreign language
- ☐ Doing science experiments
- ☐ Learning commerce
- ☐ Learning foreign languages

(9) What would you like most like to **do when you leave secondary school**?
*Tick **TWO** boxes to show your top two choices*

- ☐ TV news reader
- ☐ Airline stewardess
- ☐ Car mechanic
- ☐ Doctor
- ☐ Lawyer
- ☐ Scientist
- ☐ Airline pilot
- ☐ Hairdresser
- ☐ Businessman or businesswoman
- ☐ Professional sportsman or sportswoman
- ☐ Engineer
- ☐ Teacher
- ☐ Bricklayer
- ☐ Making clothes

(10) Which two school subjects are the best for helping you get a job when you leave school?
*Tick **TWO** boxes*

- ☐ English
- ☐ Literature in English
- ☐ Geography
- ☐ Science
- ☐ Music
- ☐ History
- ☐ Mathematics
- ☐ Craft, design
- ☐ Technology
- ☐ Home economics

(11) I became interested in mathematics thanks to:
Tick as many as you like.

- ☐ Mathematics TV programs
- ☐ Mathematics lessons
- ☐ My teacher
- ☐ Things I have read
- ☐ My parents
- ☐ Exhibitions, demonstrations
- ☐ My friends
- ☐ Other: (please indicate)
  ……………..................…………

(12) What do you most look forward to learning in your mathematics lessons?

……
……

7.4 Samples Involved in the School Study

The total population of the school students involved in the research consisted of :

- Senior secondary (Grade 3) students (age 16/17)
- Junior secondary(Grade 3) students (age 13/14)
- Primary school pupils from upper primary (age 11/12)

Table 7.2 shows the sample of students participating in the research according to their year group, age and sex. A total of 360 responses were obtained with 189 boys and 171 girls. The aim was to obtain a sample of 450 students (150 at each level) but it was not possible to reach all the students because some had completed their mid-term examination, and decided not to come to school.

Year Group	Age	Boys	Girls	Total
Senior Secondary	16	70	50	**120**
Junior Secondary	13	58	62	**120**
Primary	11	61	59	**120**
Total		**189**	**171**	**360**

Table 7.2 Samples Sizes

In particular, it was necessary to select the senior secondary students for the survey on difficulties and achievements in mathematics because at this level, they should have chosen their subjects for their final examinations, and also have already formed their opinion about mathematics. Thus, the sample of those participating in the difficulty survey constitutes only those of this group. The difficulty survey which covered 31 topics in the current mathematics syllabus was aimed at identifying topics in mathematics perceived to be difficult from the students' perspective. Students were asked to tick in the appropriate columns containing, Easy, Moderate or Difficult for each topic, and also provide written comments why they find it difficult to understand. The results are analysed by means of tables and simple statistics. Responses from the students about the topics that they have studied provides information that is needed for further studies.

It was also decided to work with these groups for the following reasons:

(a) Students at these stages have completed much mathematics;
(b) Students at these stages are about to participate in national mathematics examination but are not yet under intense examination pressures.

The students were drawn from typical Nigerian schools. It was also the intention to carry out this study in the university. This was not possible. Despite numerous attempts to gain access, the complex bureaucracy proved insurmountable. Again the attitudes and difficulty surveys are shown in full in Appendices B and C respectively.

The aim of this section of the study is to:

(a) Relate examination data in mathematics to measured working memory capacity;
(b) Identify the main areas where students say they have the greatest difficulties in understanding mathematics
(c) Explore students perceptions of the studies in mathematics at various ages.

7.5 Analysis of Data

For each student involved in the study, the following data were obtained: working memory capacity; examination marks; perceived areas of difficulty, responses about their attitudes and perceptions relating to their studies in mathematics, although some of the data being gathered only from senior secondary students.

The data from the difficulty survey was summarised to give totals and percentages. All the remaining data were entered into a spreadsheet. Overall frequencies for each year group and for gender were obtained. For the comparison of attitudes towards mathematics between the primary, junior secondary and senior secondary students, the distributions of frequencies of responses were examined for each particular statement in a question. All statistical analyses were carried out using the raw data but the response patterns are summarised as percentages for clarity.

By using a cross-age analysis (measurement at only one time with students of different age groups) comparison is made between the groups to see how their attitudes change. The Chi-square (χ^2) test was used to judge the statistical significant differences in responses of two groups in turn (primary/junior, junior/senior). There are two different applications of the chi-square test: in the '*goodness-of-fit test*', a frequency distribution is compared to a distribution from a control group. In the '*contingency test',* two or more independent samples are compared, for example, year groups or gender. In this study, the latter was used (see Appendix H).

One main aim of this study was to explore the relationship between academic performance and students' working memory space capacity. In doing this, students' mock examination scores from the senior secondary school were correlated with their scores obtained from the digit span backwards test.

Finally, correlation coefficients were calculated to determine if attitudes to learning is related to students' working memory capacity. In calculating a correlation coefficient, there are three ways depending on the nature of the measurement data:

- For data with integer values (for example, examination marks), Pearson correlation is used. This assumes an approximately normal distribution.
- With ordered data (for example, examination grades), Spearman correlation is employed, which does not assume normal distribution.
- With ordered data where there are only a small number of categories, Kendall's Tau-b correlation is used, which does not assume a normal distribution.

Kendall's tau-b method of correlation was employed in part of this study in that the data from the attitudes surveys are ordinal and are often far from normal distributions. Kendall's tau-b also handles 'ties' more appropriately. There is a summary of the use of this method in the Appendix H. However, relationships between marks and working memory capacity were explored using Pearson correlation as these variables are integers with an approximately normal distribution.

The aim was to see how attitudes developed with age and related to each other among the three age groups (primary, junior and senior secondary), and to explore any relationships with working memory capacity in order to gain insights as to whether attitude is associated with learning.

Each question from the attitude survey is now discussed in turn.

Note that in this chapter and in the remaining chapters:

- 'S' denotes senior secondary school
- 'J' means junior secondary school
- 'P' is primary school
- 'df' is "degree of freedom"
- 'ns' stands for "not significant"

7.6 Mathematics Performance and Working Memory

In order to investigate whether working memory space influences performance, students examination marks were correlated with their scores from the digit span backwards test. Table 7.3 below shows the correlation of students' mathematics examination marks with working memory capacity for senior secondary students only.

	Sample	Exam Data		Digit-Span Backwards		Pearson Correlation	
		Mean	Standard Deviation	Mean	Standard Deviation	r	p
Senior Secondary	N = 120	61.5	13.4	5.7	1.6	0.55	< 0.001

Table 7.3 Working Memory Correlation

The mean outcome from the digit span backwards test is approximately what might be expected for students aged about 16 when the average working memory should be about 7. The backwards test gives outcomes approximately one less than the actual working memory capacity. It can seen that there is a positive relationship between students' working memory capacity and mathematics achievement. Indeed, a value of 0.55 is quite high, indicating that over 30% [0.55^2, as %] of the variance of the mathematics performance was being caused by the working memory capacity. The work of Johnstone and El-Banna (1987) shows that the correlation is a cause-and-effect relationship.

The sample of 120 was divided into three groups. The 'average' group are those whose working memory falls within one half of a standard deviation from the mean, with the other two groups lying outside that range. This method gives three very approximately equal groups (this approach is discussed on page 174). The effect on examination performance is large and is illustrated in Table 7.4.

Working Memory Capacity	Number of Students	Average Examination Mark (%)
Above average	42	67.2
Average	45	64.3
Below average	33	50.3

Table 7.4 Marks and Working Memory Capacity

The difference in performance between the average in the lower working memory capacity group and the upper working memory capacity group is nearly 17%. This difference might be caused during the learning process or might simply reflect the types of questions asked in this particular examination, or both.

Of course, Pearson Correlation is only appropriate if the data are interval in nature, with an approximation to a normal distribution. Figure 7.1 shows that this is approximately so.

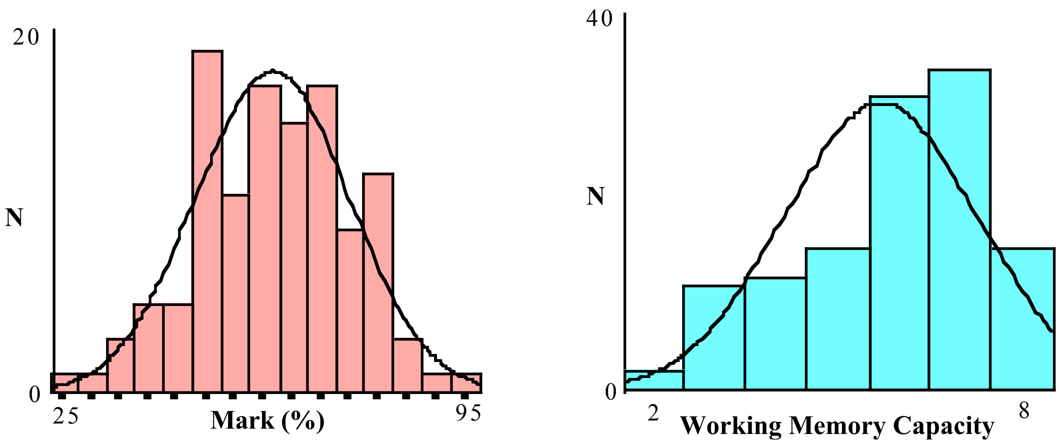


Figure 7.1 Data Distribution

7.7 Students' Perceptions of Mathematics

In this section and in all further analyses, data are presented percentages for clarity but all statistical analyses are carried out on actual data. On occasions, totals do not add up to 100% because of rounding errors.

Question 3: Students' Choice of Mathematics Topics

(a) **Primary**

(3) Which topics interest you?	
50	Prime numbers
55	Factors
51	Decimal
48	Ratio
59	Writing numbers in words
52	Finding missing numbers
53	Multiplication division of numbers
65	Multiples
44	Fractions
48	Percentages
29	Powers and roots
45	Writing numbers in figures
60	Addition and subtraction of numbers
48	Area and perimeter

Table 7.5 Data for Question 3 (Primary)

Table 7.5 shows that, overall, all topics seem to interest them with the exception of topics related to powers and roots. Powers and roots are very abstract notions, especially at primary stages. Therefore, they may have little meaning for many of the students. In addition, to make any sense of these ideas may put demands on working memory which cannot be sustained. Many symbolisms are used in addition to the demands made in understanding.

At age 11, the average working memory capacity of the students is likely to be about 4 or 5. The learner may have to cope with grasping the meaning of the symbol, understanding what is required and remembering the procedure to be adopted - all at the same time. Working memory will almost certainly overload.

(b) **Junior secondary**

(3) Which topics interest you?	
67	Indices
38	Sequences
30	Transformations
78	Fractions
78	Equations
54	Standard Form
44	Plane geometry
52	Directed numbers
48	Inequalities
65	Ratio
44	Construction
36	Probability
38	Measures of central tendency
44	Pythagoras' Theorem

Table 7.6 Data for Question 3 (Junior Secondary)

Overall, in Table 7.6, topics such as: fractions, equations and ratio tend to dominate their interest. This is because they are already familiar with these topics in the primary school. Thus, building new concepts based on previous knowledge is important in the teaching and learning of mathematics.

(c) **Senior secondary**

(3) Which topics interest you?	
86	Indices
69	Sequences
37	Transformations
52	Bearings
82	Equations
68	Standard Form
65	Plane geometry
69	Mensurations
67	Inequalities
58	Logic and set theory
55	Loci and construction
79	Probability
65	Measures of central tendency
63	Circle geometry

Table 7.7 Data for Question 3 (Senior Secondary)

It is obvious from Table 7.7 that students are more interested in equations, indices and probability. They have less interest in transformations probably because it is not an examinable area in the current mathematics school syllabus. However, students do not find bearings attractive because it requires high mental and problem solving ability.

Question 4 Students' Opinion About their Mathematics Lessons

A six-point scale semantic differential method was used to judge the student's attitude in their study of mathematics. Table 7.8 shows the distribution of P, J and S students' responses to different aspects of studies in mathematics, and the chi-square values to compare the difference of attitude between the groups (P and J; J and S).

For clarity, the data are shown as percentages, but rounded to the nearest whole number but statistical calculations are carried out using the frequency data. Thus, adding up all the figures in some items does not always come exactly to a hundred because of rounding errors and because a few students failed to answer some questions.

(4) What are your opinions about mathematics lessons?									
I like mathematics lessons	***P***	22	20	33	11	8	7		*I hate mathematics lessons*
	J	59	16	14	9	2	2	**P/J**	35.5 (df3), p < 0.001
	S	50	20	10	6	4	9	**J/S**	4.1 (df3), ns.
Boring lessons	***P***	17	7	21	21	16	18		*Interesting lessons*
	J	9	9	26	23	10	25	**P/J**	1.8 (df3), ns.
	S	19	15	8	9	37	13	**J/S**	24.1 (df2), p < 0.001
I enjoy the lessons	***P***	23	22	29	13	9	3		*I do not enjoy the lessons*
	J	31	21	24	13	8	3	**P/J**	1.6 (df4), ns.
	S	23	33	13	7	13	11	**J/S**	8.3 (df3), 0.05
Easy lessons	***P***	16	16	32	21	8	7		*Complicated lessons*
	J	20	14	32	22	9	3	**P/J**	0.6 (df4), ns.
	S	21	23	11	12	22	10	**J/S**	23.5 (df4), p < 0.001
Not essential for life	***J***	3	8	18	10	16	44		*Essential for life*
	S	13	11	10	12	38	17	**J/S**	27.9 (df4), p < 0.001
Best learned from a textbook	***J***	9	2	21	27	20	21		*Best learned from a teacher*
	S	14	10	10	8	27	31	**J/S**	21.4 (df2), p < 0.001
Relates to the events of daily life	***J***	36	24	16	13	6	6		*Does not relate to .... daily life*
	S	15	16	11	8	27	24	**J/S**	35.3 (df3), p < 0.001
Very important ... employment	***J***	41	24	14	12	3	7		*Not very important ... employment*
	S	39	21	14	5	9	13	**J/S**	5.9 (df2), ns.

Table 7.8 Data for Question 4

In general, views from Table 7.8 tend to be positive. However, in several questions, the opinions of those in the junior schools are more positive than those in the senior schools. It is clear that lessons become considerably more attractive when moving from primary to junior secondary. This may be because the teachers in the junior secondary schools are mathematicians and are, therefore, much more committed to mathematics as a subject. The junior secondary pupils think that lessons are more interesting, more enjoyable, and more related and essential to life when compared to those in the senior secondary schools. This may simply reflect the greater demands of mathematics at older ages and pupils are thinking of choosing other subjects for more long term study

In looking at whether lessons are easy or complicated as well as whether lessons are best centred on a textbook or a teacher, the views of the senior secondary pupils are much more polarised than junior secondary pupils. This pattern of polarisation with age is quite common and was seen in a recent study by Alhmali (2008).

Question 5 Students' Views on their feelings of Mathematics Studies

Table 7.9 below shows the distribution of P, J and S students' responses to question about their views of mathematics courses. This question explores the students' feeling about their school mathematics courses.

(5) How do you feel about your mathematics course at school?									
I feel I am coping	***P***	23	27	35	7	5	4		*I feel I am not coping*
	J	28	23	22	15	5	6	**P/J**	0.8 (df2), ns.
	S	43	21	9	6	9	12	**J/S**	14.2 (df2), $p < 0.001$
I learn a lot new things	***P***	21	23	33	11	8	3		*I learn nothing new*
	J	24	33	24	11	5	3	**P/J**	3.9 (df3), ns.
	S	29	33	13	10	5	12	**J/S**	5.0 (df3), ns.
I am not obtaining new skills	***P***	9	6	20	27	15	21		*I am obtaining a lot new skills*
	J	7	16	13	23	23	19	**P/J**	5.7 (df4), ns.
	S	13	20	11	6	38	13	**J/S**	9.5 (df2), $p < 0.05$
I like the teacher	***P***	45	18	21	7	4	5		*I dislike the teacher*
	J	40	19	17	14	3	7	**P/J**	2.5 (df3), ns.
	S	25	31	16	5	12	12	**J/S**	6.1 (df3), ns.
Mathematics is important	***P***	30	27	28	10	2	4		Mathematics is unimportant subject
	J	72	13	6	2	3	5	**P/J**	42.6 (df3), $p < 0.001$
	S	37	26	13	5	9	10	**J/S**	27.4 (df2), $p < 0.001$
I hate doing homework	***P***	37	18	9	15	6	16		I enjoy doing homework
	J	26	17	20	2	12	12	**P/J**	8.6 (df4) ns.
	S	20	13	6	11	25	25	**J/S**	0.2 (df2), ns.
Do not inspire me to think	***J***	5	21	18	21	15	20		*Inspire me to think*
	S	18	12	12	13	26	18	**J/S**	6.2 (df4), ns.
Easy to apply to real life	***J***	26	21	32	14	6	1		*Difficult to apply to real life*
	S	24	15	10	14	14	22	**J/S**	25.1 (df3), $p < 0.001$

Table 7.9 Data for Question 5

Clearly, students have positive opinions about their mathematics course at school, with the exception that they do not like mathematics homework. Their views that mathematics inspires them to think are fairly well spread. In looking at how they cope, this develops with age but there is increased polarisation of view with the seniors. Similarly, for obtaining new skills their views are becoming increasingly polarised with age.

They all feel they are learning new things but a comparison between the primary and seniors shows that this is growing slightly with age ($\chi^2 = 14.4$, df5, $p < 0.05$). Their teachers are generally liked although this, understandably, declines with age slightly (from primary to senior: $\chi^2 = 18.8$, df5, $p < 0.01$). Mathematics is important for all age groups but the pattern for the junior students is very different from the other groups. There is no obvious explanation for this but, perhaps, the junior groups were studying some topic at the time which they saw as particularly important.

In looking at whether mathematics is easy to apply to real life, the senior secondary students are more of the view that mathematics is difficult to apply to real life than those of the junior group. Perhaps, this reflects increased realism with age.

Question 6 Students' interest in continuing a course in mathematics in future

Table 7.10 shows the percentage of students at the three stages who would like to continue a course in mathematics in future. Views about going on with mathematics are shown in Table 7.10

(6) Would you like doing more mathematics?		
	Yes	No
Primary	93	7
Junior Secondary	57	43
Senior Secondary	38	62

Table 7.10 Data for Question 6

Sadly, this table shows clearly that, as students grow older, their idea and interest in mathematics moves towards the negative direction, making them less interested in further studies. Perhaps, this is inevitable as the older students see other career possibilities. Nonetheless, the very strong interest at primary stages is steadily being lost and this is to be regretted.

Question 7 Students' view about Mathematicians

Table 7.11 shows students' responses to 'true and false' questions asking their views about mathematicians.

(7) Do you think the following statements are true or false?	% Stating this is True			
	Primary N = 120	Junior N = 120	Senior N = 120	Average N = 360
All mathematicians are very intelligent people	48	70	53	57
Being a mathematician is very interesting	41	42	53	45
Mathematicians don't dress well	37	29	26	31
Being a mathematician is hard	49	46	52	49
Mathematicians usually are rich people	29	19	47	32
Girls don't like being mathematicians	25	27	27	26
Mathematicians work to make discoveries	45	47	44	45

Table 7.11 Data for Question 7

Clearly, students have the opinion that mathematics is meant for the intelligent people and they recognise that the subject is hard. Perhaps mathematics is interesting as it can be used as a basis for discovery. Students tend to feel that mathematicians are not usually rich people, perhaps in comparison to doctors, pharmacists and lawyers. The idea is that it is believed in Nigeria that mathematicians work only in the classroom, and are not very well paid. While about one quarter observed that female students do not like to become mathematicians, it is encouraging that the majority did not tick this box.

It has to be recognised that the students have met few of many mathematicians apart from their school teachers. Therefore, the responses largely reveal how they see the image of their mathematics teachers.

Question 8 Students' General Interest about various Aspects of learning

Tables 7.12 and 7.13 show the general tendency of interests obtained separately for primary, junior and senior secondary students. There were two versions of the question, one for primary pupils and the other for the remainder.

Question 8 (Primary Schools) Which is most interesting to do in the secondary school?	
%	
67	Playing in the school sports team
32	Cooking or metalwork
32	Playing musical instruments
44	Learning mathematics
43	Solving different kinds of problems
44	Painting pictures
50	Learning foreign language
40	Doing science experiments
27	Learning commerce
43	Using a computer

Table 7.12 Data for Question 8 (Primary)

The strongest preferences are for sports and learning foreign language. Practical and business skills are least attractive. Perhaps, they are not fully aware of what might be involved in commerce. Encouragingly, doing mathematics comes quite high.

Question 8	**(%)**	Which of the following aspects would you like to devote more attention and time in your mathematics course?
Junior	**Senior**	
40	45	Studying the theory deeper
62	54	Practising mathematics
18	49	Preparing for a career
44	42	Studying about mathematics application in social life
23	38	Learning about modern perspectives of mathematics for development
40	53	Studying more about practical application
46	45	Learning about modern discoveries in mathematics
63	56	Studying more mathematics for sciences
25	29	Studying environmental problems and ways of solving them

Table 7.13 Data for Question 8 (Junior and Senior Secondary)

The options chosen most are shown in purple. The interesting in practising mathematic is somewhat surprising while the place of mathematics in the sciences is clearly very important. Perhaps, they appreciate that practising their skills in mathematics exercises tends to lead to examination success. The next four choices are shown in yellow and demonstrate quite a commitment to mathematics.

Question 9 Students' desire about Future Career

Students' responses to the question , "*What would you like most to do when you leave school?*" are enumerated in the Table 7.14.

(9) What would you like most like to do when you leave school?				
% Primary	% Junior	% Senior	% Average	
9	11	19	13	TV news reader
12	4	13	10	Airline
2	1	9	4	Car mechanic
25	32	20	26	Doctor
17	23	23	21	Lawyer
14	24	9	16	Scientist
12	6	12	10	Airline
11	5	10	9	Hairdresser
20	16	19	18	Businessman or businesswoman
15	7	13	12	Professional sportsman or sportswoman
16	40	15	24	Engineer
18	11	20	16	Teacher
2	1	3	2	Bricklayer

Table 7.14 Data for Question 9

The top three choices (overall) are shown in darker red and are all high status professional careers. This shows that Nigerians have the same kind of prestige ladder of jobs or careers as those found in other nations of the world (see Jung, 2005, page 53). The next three choices (overall) are shown in pale pink: businessman or woman, scientist, teacher, again all tending to have high status, especially in a third world context.

Question 10 Students' Choice of Subject for future career Opportunity

The table below shows choice of subjects at all levels.

(10) Which two school subjects are the best for helping you get a job when you leave school?				
% Primary	% Junior	% Senior	% Average	
53	6	49	36	English
4	10	24	13	Literature in English
10	9	15	11	Geography
31	75	14	40	Science
4	3	15	7	Music
5	2	10	6	History
51	55	26	44	Mathematics
11	4	10	8	Craft, design
12	7	9	9	Technology

Table 7.15 Data for Question 10

It is interesting that three subject areas stand out so strongly. As in many countries, the mother tongue, mathematics and the sciences are key to many job opportunities. It is obvious that students have an overwhelming preference for mathematics and this illustrates the power of mathematics to open career opportunities. This is because to study medicine, engineering, pharmacy, accounting, law agriculture, etc., it is compulsory to have an acceptable grade pass in mathematics. Job opportunity also linked with success in mathematics. As Al-Enezi (2006) puts it, "*mathematics is needed to keep the world running and maintain satisfactory standard of living*".

Question 11 Students' favourite method of Instruction

Finally, Table 7.16 presents students' choice of instruction.

(11) I became interested in mathematics thanks to:				
% Primary	% Junior	% Senior	% Average	
16	35	48	33	Mathematics TV programs
46	46	57	51	Mathematics lessons
62	74	61	66	My teacher
28	29	30	29	Things I have read
37	62	46	48	My parents
10	9	19	13	Exhibitions, demonstration
27	27	49	34	My friends
13	12	13	13	Other:

Table 7.16 Data for Question 11

In general, looking at all the three groups (colour-coded) together, the key influence is the teacher. This is followed by mathematics lessons and parents. In her detailed study of what attracts students into physics, Skryabina (2000) found that teachers and the quality of the learning experience were two of the strongest factors. The influence of parents was small but her study was conducted in Scotland. Her findings were consistent with those of Hadden and Johnstone (1982) in relation to the sciences.

There is a key message. To attract students towards mathematics requires a quality curriculum experience and this depends critically on the teacher quality, at least in part. However, teachers rarely influence the actual curriculum to be taught.

Question 12 does not reveal anything as most of the students did not respond to it.

7.8 Attitudes, Working Memory and Performance

The senior group offered data for working memory capacity, examination performance and responses to the survey. It is possible to relate their responses to the survey to the working memory capacity and to the examination performance. This could be carried out using Kendall's Tau-b correlation but this only works for questions 4 and 5. Almost no significant correlations were found.

The only exceptions with examination performance were that those who like mathematics ($r = 0.15$, $p < 0.05$) and those felt they were coping ($r = 0.16$, $p < 0.05$) tended to be those who obtained better marks. The only exception with working memory capacity was that those who saw mathematics as essential for life tended to have higher working memory capacities ($r = 0.16$, $p < 0.05$). The last outcome is difficult to interpret.

7.9 Difficulty Survey

In this survey, the senior secondary students were invited to tick a column if they had found that a particular mathematics topic was ‘easy’, ‘moderate’ or ‘difficult to understand’. Table 7.17 gives the results of this survey. The difficult column is considered mainly. Raw data is converted to percentage for clarity. The total percentage on each topic does not add up to 100 because some students did not tick all the subject areas.

N = 120	Percentage		
Topic	**Easy**	**Moderate**	**Difficult**
Indices	41	40	17
Directed Numbers	26	57	12
Logarithms	32	42	22
Approximations	29	53	14
Ratio	41	52	8
Fractions	45	46	8
Percentages	38	48	10
Sequences	38	52	9
Series	23	63	12
Algebraic Equations	29	54	17
Algorithms	16	58	22
Inequalities	25	57	16
Transformations	26	54	19
Pythagoras’Theorem	24	52	18
Algebraic Graphs	26	53	17
Loci and Construction	22	51	24
Bearings	28	52	24
Standard Form	38	45	16
Logic and Set Theory	21	57	19
Circle Geometry	22	51	20
Polynomials	21	52	20
Mensurations	30	47	16
Measures of Central Tendency	21	55	14
Measures of Dispersions	22	53	18
Graphical Representation of Data	23	56	15
Plane Geometry	22	57	14
Surds	22	54	20
Probability Theory	22	56	16
Longitude and Latitude	18	50	27
Elevation and Depression	20	51	25
Trigonometry	12	48	37

Table 7.17 Difficult Mathematics Topics

The areas perceived to be more difficult are highlighted in red while those which are more easy in green. This is arbitrary (difficulty ≥ 20%; easy ≥ 35%) and not based on any statistical analysis.

The more ‘*easy*’ areas include: ratio, fractions, percentages, sequences, indices and standard form. Many of these were met first at primary stages and are now well established. Others, like indices and standard form can be handled by following a simple set of procedures.

The more '*difficult*' areas include: trigonometry, circle geometry, longitude and latitude, bearings, elevation and depression, loci and construction, algorithms, logarithms, and surds and polynomials.

There may be several reasons why these topics are considered difficult. Firstly, perhaps they have been covered more recently and the ideas are not yet established clearly. Secondly, these areas are perhaps taught too early when learners lack sufficient experience of life and, perhaps, underlying mathematical ideas. Thirdly, the curriculum may require a too demanding approach. However, many of these topics are likely to place considerable demand on limited working memory capacity resources.

7.10 Further Explorations

Having gained an overview of the situation in Nigeria at three stages (ages 11-12, 13-14, 16-17), the aim was to conduct a more detailed survey looking at students in the oldest age group and students taking mathematics at university to see to what extent those committed to mathematics at university in Nigeria differed from those at school who had elected to continue with mathematics to age 17-19. The university data were obtained but they would not permit any examination data to be given and were unwilling to allow any working memory capacity measurements to be made. To make matters worse, further access to schools proved impossible. However, it did prove possible to gain data from the survey with students aged 17-18 (first year A Level) in England. However, comparisons are meaningless.

The data are now shown with the discussion of each question shown under the data for each question. However, the discussion is brief in that the original purpose of the study was not possible.

7.10.1 University Data

The data were gathered from 110 first year university mathematics students in Nigeria. Again data are shown as percentages for clarity. The full questionnaire is shown in Appendix B.

(1) Think about ***mathematics***
*Tick **one box** on each line*

Exciting	46	23	16	7	1	8	Boring
Not essential for living	5	4	15	9	20	48	Essential for living
Best learned from a textbook	4	4	14	14	21	43	Best learned from a teacher
Relates to events of daily life	46	18	16	10	4	6	Does not relate to events of daily life
Important for the future of a nation	63	14	12	5	4	2	Not important for the future of a nation
Not important for my personal development	6	6	10	13	13	52	Important for my personal development
Very necessary for gaining employment	50	16	13	8	7	8	Not important for gaining employment

Their view of mathematics tend to be very positive. As they have chosen to take this course, this is to be expected.

(2) Think about your **lectures in mathematics**
*Tick **one box** on each line*

Boring	15	6	11	17	25	27	Interesting
Help me to work out solutions to problems	31	29	24	7	5	6	Do not help me to work out solutions to problems
Relate mathematics to daily life events	38	19	23	7	6	8	Not relating mathematics to daily life events
Make me like mathematics even more	35	21	28	3	5	9	Make me dislike mathematics even more
Do not inspire me to think	6	5	6	15	17	52	Inspire me to think
Does not show me clearly what to study	9	6	10	19	26	31	Shows me clearly what to study
Easy to apply to real life	22	12	21	16	10	19	Difficult to apply to real life
Complicated to follow	16	11	17	23	21	12	Easy to follow
Very important for me	43	25	17	7	5	4	Not very important for me

They are also positive about their mathematics lectures at university. They recognise that the course is difficult and that application of ideas is not easy.

(3) Think about **yourself and mathematics**
*Tick **one box** on each line*

I find the course very easy	9	32	22	19	7	11	I find the course very hard.
I am growing intellectually	32	35	21	6	2	4	I am not growing intellectually
I am obtaining a lot of new skills	39	26	20	7	4	5	I am not obtaining a lot of new skills
I am getting worse at the subject	4	8	9	18	35	27	I am getting better at the subject
It is definitely "my subject"	30	29	18	8	4	11	It is definitely not "my subject"
Memorising is the key to success	11	8	4	9	10	60	Understanding is the key to success
I aim to memorise mathematical procedures	5	10	9	4	18	55	I try to understand how to do things
I think in terms of pictures, diagrams, graphs	19	14	12	10	18	27	I think in terms of written ideas
I spend much time revising just before exams	50	22	7	7	8	8	I do not spend much time revising just before exams

It is interesting to note that they are aware that mathematics is not always easy but they very strongly want to understand what they are being taught, seeing understanding as the key to success. Their views about the value of the more visual compared to written ideas are very polarised.

(4) Here are some statements about **the way I like to work**.
*Tick **one box** on each line to show how you like to work.*

I plan what I am going to do	65	23	5	4	1	2	I do not plan what I am going to do
I care a lot about what others think of me	33	14	22	8	11	13	I do not care a lot about what others think of me
I learn more through listening	37	18	9	8	5	24	I learn more through reading
I think in terms of pictures, diagrams, graphs	23	13	12	10	16	25	I think in terms of written ideas
I understand something better after I try it out.	56	20	6	6	2	11	I understand something better after I think it through
I gain much information from diagrams and graphs	38	23	18	9	5	8	I gain little information from diagrams and graphs

It is clear that many students much favour listening while others much favour reading. The power of diagrams and graphs to offer information is apparent. Of greatest importance is their observation that they want to try things out before attempting to understand. This reveals the importance of practice to master procedural skills before seeking to understand the meaning of what they have done.

(5) Think about **examinations and tests in mathematics.**
*Tick **one box** on each line to show your opinion*
.

I do not like short answer questions, as they do not give me the chance to explain what I know and understand	15	10	14	9	16	37	I prefer to learn the facts and then be tested on them in short answer questions.
In exams, I like questions that give me the scope to go beyond what is covered and shows my ability to think.	9	13	4	7	13	52	In exams I prefer questions that are based on what the lecturer covered
I believed that what should matter in exams is the quality of my answers, not on how	57	10	5	3	4	20	In exams I expect to be rewarded for giving as much information as possible.
My main task in an examination is to write down all I have been taught	11	1	3	7	7	71	My main task in an examination is to show that I understand what I have been taught
Examinations in mathematics should test my ability to work things out for myself.	38	18	13	2	10	20	Examinations in mathematics should test my ability to remember mathematics facts

In some questions their views are highly polarised. However, they tend to want to work within the security of the taught material and a strong view in favour of understanding as the key outcome to be tested.

(6) What are your feelings about ***tutorials?***
*Tick **one box** on each line to show your view.*

	Strongly agree	*agree*	*neutral*	*disagree*	*Strongly disagree*
I find the discussions boring	16	10	22	23	30
I enjoy studying mathematics with members of my group	41	38	13	4	4
Most of the ideas from other members of the group are not helpful	3	15	15	32	35
Most of the ideas come from one person	9	17	24	34	16
Studying as a group makes it easier for us to understand mathematics	51	37	10	1	2
I do not respect ideas from others students since they are always wrong	6	7	16	33	38

Views tend to be positive and they are strongly in favour of group activities.

(7) What do you **enjoy most in learning mathematics**?
*Tick **all** the reasons that apply.*

30 Studying the theory
32 Studying how mathematics can make our lives better
60 Studying about the natural phenomenon
66 Finding relevance of mathematics knowledge to our daily life
64 Doing practical work
71 Studying making equipment
19 Solving everyday problems

They clearly want mathematics to be practical and related to life.

(8) Think about ***yourself.***
*Tick **FIVE boxes** which **MOST apply to yourself***

33	I prefer studying mathematics alone	53	I enjoy chatting with my classmates
32	I can concentrate on my study for a long time.	49	I am influenced easily by other people's opinion
16	I can easily influence other people's emotions	53	My emotions are easily influenced by others
42	I have high expectation on myself	48	If I work hard, I will be successful.
20	I cannot be happy unless everyone likes me.	25	I am fond of solving problems with new methods.
75	I am always eager to try new ideas.	36	I can accept new concept quickly
31	I can organise my time well.	42	I am confident I find a solution when I encounter problems

This question does not reveal anything too unexpected. However, it does reveal the power of practice to master new ideas, concepts and procedural skills.

(9) Imagine you are faced with a new and demanding type of problem in your studies.
What is your likely reaction?
*Tick **as many** as you wish*

38	Worry about passing the examinations	10	See it as a challenge
49	Start to panic	19	Seek help from books
75	I have coped in the past - I'll manage now	73	Think of changing my course
10	Enjoy it because it is new	71	Seek help from others

Their reactions are much as expected.

(10) Here are some descriptions of the way students approach mathematics.
Tick as many as are true for you.

34	It is all my fault if I cannot study mathematics well	42	I think mathematics helps me understand the world better.
60	I study mathematics for interest	47	I study mathematics for credits.
45	If I study mathematics well I will have a bright future	66	I can grasp the main points in mathematics easily
51	I enjoy studying in mathematics classes	29	Diagrams help me understand mathematics better
28	I have many friends sharing the same interests in mathematics	42	Mathematics are meant for people with exceptional ability
18	Assignments help me understand mathematics more	43	Examination scores in mathematics reflect my understanding of mathematics well

Interest, enjoyment and success are the key factors which characterise the approach of these students.

7.10.2 School Data

The data were gathered from 165 (89 boys, 76 girls) first year A Level mathematics students in England. The response patterns for boys and girls were compared using chi-square as a contingency test. Very few significant differences were found and these are discussed as they arise.

(1) Think about ***mathematics***
*Tick **one box** on each line*

Exciting	8	21	25	23	13	10	Boring
Not essential for living	2	2	8	18	26	44	Essential for living
Best learned from a textbook	2	2	9	28	22	37	Best learned from a teacher
Relates to events of daily life	37	23	20	12	6	3	Does not relate to events of daily life
Important for the future of a nation	24	21	26	16	9	4	Not important for the future of a nation
Not important for my personal development	3	7	7	18	24	42	Important for my personal development
Very necessary for gaining employment	54	27	9	3	4	2	Not important for gaining employment

Their view of mathematics tend to be very positive. As they have chosen to take this course, this is to be expected. They are very perceptive is seeing the importance of mathematics for future employment.

(2) Think about your **lessons in mathematics**
*Tick **one box** on each line*

Boring	15	13	17	24	21	10	Interesting
Help me to work out solutions to problems	29	48	14	6	3	1	Do not help me to work out solutions to problems
Relate mathematics to daily life events	29	39	24	9	3	3	Not relating mathematics to daily life events
Make me like mathematics even more	12	17	27	27	9	8	Make me dislike mathematics even more
Do not inspire me to think	3	8	12	25	29	23	Inspire me to think
Does not show me clearly what to study	6	3	19	30	28	14	Shows me clearly what to study
Easy to apply to real life	29	30	20	10	6	4	Difficult to apply to real life
Complicated to follow	11	9	18	23	23	16	Easy to follow
Very important for me	38	21	21	13	4	2	Not very important for me

They are also positive about their mathematics lessons at school although their views on boring-interesting are well spread.

(3) Think about **yourself and mathematics**
*Tick **one box** on each line*

I find the course very easy	12	23	39	15	9	3	I find the course very hard.
I am growing intellectually	27	37	28	5	3	1	I am not growing intellectually
I am obtaining a lot of new skills	33	35	22	7	3	1	I am not obtaining a lot of new skills
I am getting worse at the subject	3	4	9	17	30	38	I am getting better at the subject
It is definitely "my subject"	14	13	29	13	12	20	It is definitely not "my subject"
Memorising is the key to success	11	17	14	14	20	24	Understanding is the key to success
I aim to memorise mathematical procedures	9	14	19	17	15	27	I try to understand how to do things
I think in terms of pictures, diagrams, graphs	19	18	27	24	5	8	I think in terms of written ideas
I spend much time revising just before exams	10	19	20	19	19	13	I do not spend much time revising just before exams

It is interesting to note that they tend to want to understand what they are being taught, seeing understanding as the key to success. Their views about the value of the more visual are positive. In thinking about memorising mathematical procedures or trying to understand how to do things, the girls tend to try more to understand ($\chi^2 = 14.6$ (4), $p < 0.01$).

(4) Here are some statements about **the way I like to work**.
*Tick **one box** on each line to show how you like to work.*

I plan what I am going to do	18	21	22	21	8	9	I do not plan what I am going to do
I care a lot about what others think of me	29	24	15	15	8	10	I do not care a lot about what others think of me
I learn more through listening	26	20	23	11	11	9	I learn more through reading
I think in terms of pictures, diagrams, graphs	22	18	26	17	8	10	I think in terms of written ideas
I understand something better after I try it out.	31	31	15	9	5	9	I understand something better after I think it through
I gain much information from diagrams and graphs	21	32	24	15	3	5	I gain little information from diagrams and graphs

It is clear that many students much favour listening. The power of diagrams and graphs to offer information is apparent. Of greatest importance is their observation that they want to try things out before attempting to understand. This reveals the importance of practice to master procedural skills before seeking to understand the meaning of what they have done.

(5) Think about **examinations and tests in mathematics.**
*Tick **one box** on each line to show your opinion.*

I do not like short answer questions, as they do not give me the chance to explain what I	5	8	14	19	20	34	I prefer to learn the facts and then be tested on them in short answer questions.
In exams, I like questions that give me the scope to go beyond what is covered and	8	11	16	18	17	30	In exams I prefer questions that are based on what the teacher covered
I believed that what should matter in exams is the quality of my answers, not on how	39	23	14	12	5	7	In exams I expect to be rewarded for giving as much information as possible.
My main task in an examination is to write down all I have been taught	12	13	5	11	26	33	My main task in an examination is to show that I understand what I have been taught
Examinations in mathematics should test my ability to work things out for myself.	26	27	25	11	5	6	Examinations in mathematics should test my ability to remember mathematics facts

They tend to want to work within the security of the taught material and be tested using short answer questions. The girls are less sure of the use of short answers (χ^2 = 12.7 (4), $p < 0.05$). Boys and girls hold a strong view in favour of understanding as the key outcome to be tested. They want to be able to work things out but this probably means getting mathematics problems right. However, the boys are happier to be tested on what they have memorised (χ^2 = 14.1 (3), $p < 0.001$).

(6) What do you **enjoy most in learning mathematics**?
*Tick **all** the reasons that apply.*

23 Studying the theory
18 Studying how mathematics can make our lives better
68 Studying about the natural phenomenon
67 Finding relevance of mathematics knowledge to our daily life
43 Doing practical work
48 Studying making equipment
36 Solving everyday problems

It is clear that seeing mathematics related to life around and natural phenomena are very important features. The importance of natural phenomena is even more marked for the boys (χ^2 = 10.7 (1), $p < 0.001$). Boys were also very slightly more interested in studying how mathematics can make our lives better (χ^2 = 4.3 (1), $p < 0.05$).

(7) Think about ***yourself.***
*Tick **FIVE boxes** which **MOST apply to yourself***

30	I prefer studying mathematics alone	17	I enjoy chatting with my classmates
25	I can concentrate on my study for a long time.	47	I am influenced easily by other people's opinion
15	I can easily influence other people's emotions	47	My emotions are easily influenced by others
41	I have high expectation on myself	68	If I work hard, I will be successful.
21	I cannot be happy unless everyone likes me.	22	I am fond of solving problems with new methods.
69	I am always eager to try new ideas.	29	I can accept new concept quickly
29	I can organise my time well.	34	I am confident I find a solution when I encounter problems

Working hard for success and willingness to try new ideas rank very highly. As might be expected boys have very slightly higher expectations of themselves (χ^2 = 6.1 (1), p < 0.05) but girls see themselves as better time organisers (χ^2 = 6.8 (1), p < 0.01).

(8) Imagine you are faced with a new and demanding type of problem in your studies.
What is your likely reaction?
*Tick **as many** as you wish*

38	Worry about passing the examinations	28	See it as a challenge
36	Start to panic	30	Seek help from books
64	I have coped in the past - I'll manage now	36	Think of changing my course
11	Enjoy it because it is new	47	Seek help from others

Faced with problems, the highest chosen reaction relate to past experiences of coping as well as willingness to seek help from others. Boys tend to be more confident because they have coped in the past (χ^2 = 7.3 (1), p < 0.01).

(9) Here are some descriptions of the way students approach mathematics.
Tick as many as are true for you.

22	It is all my fault if I cannot study mathematics well	24	I think mathematics helps me understand the world better.
59	I study mathematics for interest	35	I study mathematics for credits.
25	If I study mathematics well I will have a bright future	35	I can grasp the main points in mathematics easily
36	I enjoy studying in mathematics classes	21	Diagrams help me understand mathematics better
46	I have many friends sharing the same interests in mathematics	46	Mathematics are meant for people with exceptional ability
18	Assignments help me understand mathematics more	62	Examination scores in mathematics reflect my understanding of mathematics well

Interest and past examination scores are the strongest factors here. This is the one question where several gender differences were found. In every case, boys chose the option very slightly more:

If I study mathematics well I will have a bright future	χ^2 = 4.0 (1), p < 0.05
Assignments help me understand mathematics more	χ^2 = 7.5 (1), p < 0.01
I think mathematics helps me understand the world better	χ^2 = 5.0 (1), p < 0.05
I can grasp the main points in mathematics easily	χ^2 = 4.7 (1), p < 0.05
Mathematics are meant for people with exceptional ability	χ^2 = 5.5 (1), p < 0.05

7.11 Chapter Summary

(1) The majority of primary, junior secondary and senior secondary Nigerian students thought:

- Mathematics is useful and important
- Mathematics topics are relevant to them
- Mathematics is difficult and is meant for intelligent, clever people
- They learn a lot from practising mathematics

(2) A lot of students thought:

- They enjoy their mathematics lessons
- They are coping with their mathematics study
- Homework is not appealing
- Teachers play a prominent role in the education of the learner
- Mathematics, science and English are important for personal development and for future career opportunities

(3) Attitudes towards the learning of mathematics change with age. As students grow older, the belief that mathematics is interesting and relevant to them is weakened, although many still think positively about the importance of mathematics.

(4) Students prefer prestigious jobs like doctors, engineers, teachers and lawyers, perhaps reflecting the same kind of prestige ladder jobs or careers people prefer in other nations of the world .

(5) Students prefer topics relating to application of mathematics knowledge and skill.

(6) In looking at areas of greatest perceived difficulty in mathematics, the following conclusions can be made: at senior secondary level, the topics regarded as 'difficult' tend to be those which require some kind of conceptual understanding. Topics like trigonometry, algorithm and angles of elevation and depression are part of the main problems. In these topics, many ideas and concepts need to be connected from previously learned experience and have to be handled cognitively at the same time, and working memory capacity cannot cope easily. Thus, topics of greatest difficulty seem to be those where the demand on limited working memory is greatest.

(7) There is a positive relationship between students' working memory capacity and mathematics achievement. One explanation of this result is that students who have high working memory capacity tend to try to *understand* mathematics knowledge (rules, concepts and theory) as much as they can, while students who have a lower memory capacity tend to try to memorise mathematics knowledge (see Jung and Reid, 2009).

(8) The outcomes from the longer surveys (Section 7.10) used with Nigerian university mathematics students and English school students starting A Level cannot really be compared in that the two groups are so different in nature, culture and context. However, some observations seem to be very similar for both groups:

- With both groups, the importance of applications and seeing mathematics in a life context are important.
- It is clear that both groups see understanding following the actual ‘doing’ of the mathematics [both groups see this strongly although the student group are slightly more marked in their view [$\chi^2 = 7.1$ (2), $p < 0.05$]
- The importance of the visual is emphasised by both groups. Perhaps, being self-selected, graphs and diagrams are seen as very helpful.
- There is clearly a greater place for group work in studying mathematics at the undergraduate level.

The second stage of the research was carried out totally in England to explore ways by which working memory can be used more efficiently, specifically, in the area of field dependency. Initially, all experimental data was planned to be obtained in Nigeria. However, this was not possible because access to schools proved too difficult. Thus, in the following two chapters, experiments were conducted in England where the investigator was teaching.

Chapter Eight

Measurement of Field Dependency

8.1 Introduction

In Chapter 6 of the present study, field dependency was discussed. It was noted that individuals have different ways of collecting, organising and processing information depending upon their cognitive structure and what they already know. The differences which exist in cognitive structure and in psychological functioning enable individuals to have different cognitive styles. Field dependency is regarded as one of many learner characteristics and the field-independent learner is more capable of restructuring a field by breaking it up into separate items to make a number of changes to the field, and selecting what is important for the task in hand from the context of surrounding information. A field-dependent learner has the difficulty in separating an item from its context and is inclined to respond to the dominant properties of a field presented to him.

One of the results from the previous experiment revealed that students' working memory capacity correlates with their academic performance in mathematics. This finding suggests that working memory capacity is a rate-controlling factor in learning and assessment (see, for example, Johnstone, 1997).

Working memory capacity is fixed genetically (Miller, 1956). However, the capacity a learner possesses may be used more efficiently. One of the main ways by which limited capacity is used more efficiently is by means of what Miller called 'chunking'. Ideas or units of information can be grouped together in that they are seen as one 'chunk'. Less space in the working memory, therefore, is used, leaving more space for processing or handling other ideas. Another way by which the limited capacity of working memory can be used more efficiently is by avoiding information that is not essential for the task in hand. The concept of field dependency helps here.

The field-independent learner is capable of selecting in only that which is essential for the task in hand. This means that the working memory is less likely to overload. Field dependency is a learner characteristic and there is no clear evidence of how the extent of field dependency in a learner arises: it could be genetic in origin, a learned characteristic or a characteristic adopted by some element of choice (see Figure 8.1 overleaf). Of course, the development of this characteristic may arise from any combination of these three.

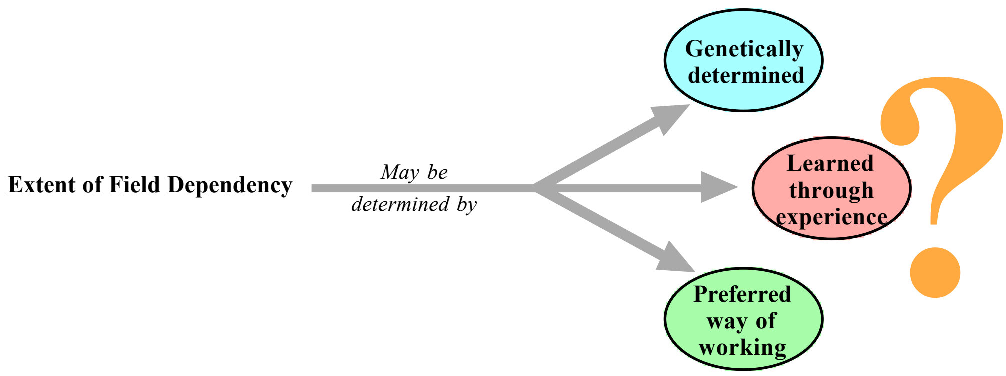


Figure 8.1 Nature of Field Dependency

Looking back to page 124 where some of the factors which had been found to influence extent of field dependency were discussed, Figure 8.1 can be interpreted to some extent. Korchin (1986) had found that children from families where there is encouragement for them to develop separate, autonomous functions are relatively field-independent, while others who showed emphasis to parental authority and guidance are likely to become relatively field-dependent. This does suggest that extent of field dependency can be influenced by life experiences as does the finding of Forns-Santacana, *et al.,* (1993) relating to socio-economic status. The age factor Gurley (1984) can be interpreted in terms of cognitive development or experiences in life while the gender factor was reported as very small (Musser, 1998).

It is interesting to note that every study of the relationship of the extent of field dependency with academic performance shows that being field-dependent is never an advantage (see Danili and Reid, 2006). However, this may merely reflect the types of assessment questions used in conventional examinations although the generalisation applies to all subjects, not just the sciences and the mathematical areas of the curriculum (Hindal, 2007).

The key issue is this. The limitations of working memory capacity constitute one key factor in success in mathematics assessments. However, working memory capacity is fixed for an individual. To improve performance, the assessments may need to be changed so that those with higher working memory capacities do not have the advantage and Reid (2002) has shown that this is possible. Alternatively, steps need to be taken to develop ways by which the limited working memory capacity can be used more efficiently. Field dependency may offer assistance here, but only if this characteristic can be developed in some practical and acceptable way. Thus, if the extent of field dependency is open to development by means of experiences in the learning situation, then it becomes an interestingly possible that such experiences might be integrated into normal school programmes, thus raising the prospect of improved examination performance.

8.2 Student Sample Involved in the Research

Originally, the entire study was planned to focus on students in Nigeria. However, this was not possible because of difficulties of getting all of students to follow and complete every single procedure applied in the research. In 2007, a decision was made to select a sample of students from English secondary schools. The schools which were chosen were from the North East of England where the investigator happened to be teaching. Thus, this allowed careful monitoring of everything that was going on as the data were gathered.

Table 8.1 shows the sample of 547 students participating in the research according to their year group, age and sex. All the samples were obtained from three state-maintained schools. Since the research has been carried out in England, the nature and types of school were discussed in Chapter 1 of the present study. The schools are drawn from a diversity of areas and social background. Consequently, the sample again contains all kinds of students, from those who have independent thinking ability to those who need support. Analysis of students' sample was performed using the responses from the hidden figure test and mathematics examination/test scores.

Year Group	Approximate Age	Boys	Girls	Total
Year 7	13	43	49	92
Year 8	14	85	83	168
Year 9	15	52	50	102
Year 10	16	79	63	142
Year 11	17	20	23	43
Totals		**279**	**268**	**547**

Table 8.1 Students' Sample Sizes

8.3 Measurement of Extent of Field Dependency

To measure the students' degree of field-dependence or field-independence, a modified version of the Witkin *et al.,* (1977) Group Embedded Figure Test, called the Hidden Figure Test was used to measure the extent of field dependency of all 547 students. In order to classify the students of the sample into learning style categories (field-dependent, field-independent, field-intermediate), the same method used by a number of researchers (Al-Naeme, 1991; Danili, 2001) was employed. The test was not used as an absolute measure of field dependency. The basic purpose of this test is to measure the students' degree of field-dependence/field-independence by placing students along a continuum ranging from very field-dependent to very field-independent. It is possible to analyse any relationships between extent of field dependency and performance in mathematics using correlation. It is also possible to illustrate any relationship by dividing the sample into three groups:

(a) *Field-dependent* students are those who are unable to break up an organised perceptual field and separate readily an item from its content or incapable of restructuring the field as required by the task so that they tend to accept the organisation of the field as given.

(b) *Field-independent* students are those capable of restructuring the field as is required and have relatively analytical cognitive style which allow their experiences to be analysed and developed.

(c) *Field-intermediate* students are those who may be located in between the above two categories along the continuum ranging.

The division into these groups seeks to generate groups of approximately equal size. This is done by considering the normal distribution curve in Figure 8.2.

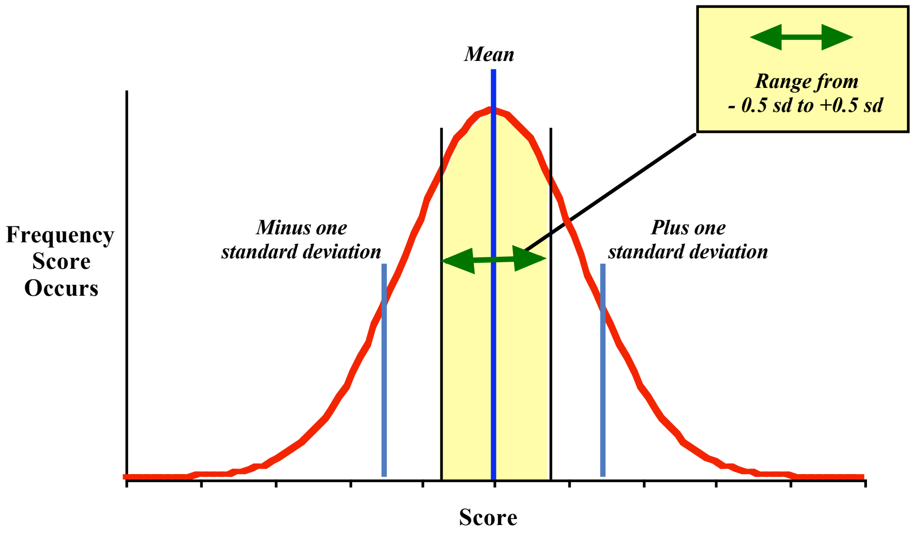


Figure 8.2 Normal Distribution

Approximately, one third of the population will lie *below* one half of one standard deviation below the mean, while approximately, one third of the population will lie *above* one half of one standard deviation below the mean. By using division points at half of one standard deviation above and below, three approximately equal groups will be obtained. This approach has bean widely used, the first use being by Case (1974)**.**

The conditions for carrying out the test were as follows:

1. A total time of 20 minutes were allocated to complete the test.
2. Tasks should be addressed in the order in which they appeared in the test booklet.
3. Students can refer to the page of simple shapes as often as necessary.
4. The target shapes must be traced in the same size, same proportions, and faces in the same direction as they appeared alone in the page of the booklet.

5. Only the required target shape should be traced, ignoring any of the other shapes in each complex figure.
6. Students are not allowed to use a ruler or any other means to measure the size of the simple shape in the complex figures.

In every task, the students were asked to recognise and identify a simple geometric shape in a complex figure, by tracing its outline with a pen or pencil against the lines of the complex complex figure. The whole test consists of 20 tasks. Thus, the possible maximum score that can be obtained is 20, although there are two additional introductory figures that were used as examples. The complete test is shown in Appendix D, and the answers to the tasks are shown in Appendix E.

The main scoring scheme for the tests is to give one point for a correct simple shape embedded in a complex figure. Students who attempted the hidden figure test were divided into three categories according to their scores: field-dependent, field-intermediate and field-independent. Such a classification of students is shown in Table 8.3 (page 178). The criterion used for such division is based upon the method which was employed by Case (1974) and Scardamalia (1977).

8.4 Research Questions

Four research questions were surveyed in this chapter:

(1) Are there significant relationships between field-dependence and mathematics performance?
(2) Does field dependency grow with age?
(3) Are there any gender differences related to field dependency?

8.5 Analysis of Data

Examination marks for each year group were obtained. The marks reflected the work of the previous year covering the same curriculum in all three schools for each year group. Marks were standardised for each year group (mean 50, standard deviation 12). The extent of field dependency was measured for each of the 547 students.

The data gave approximately normal distributions (Figures 8.3 and 8.4).

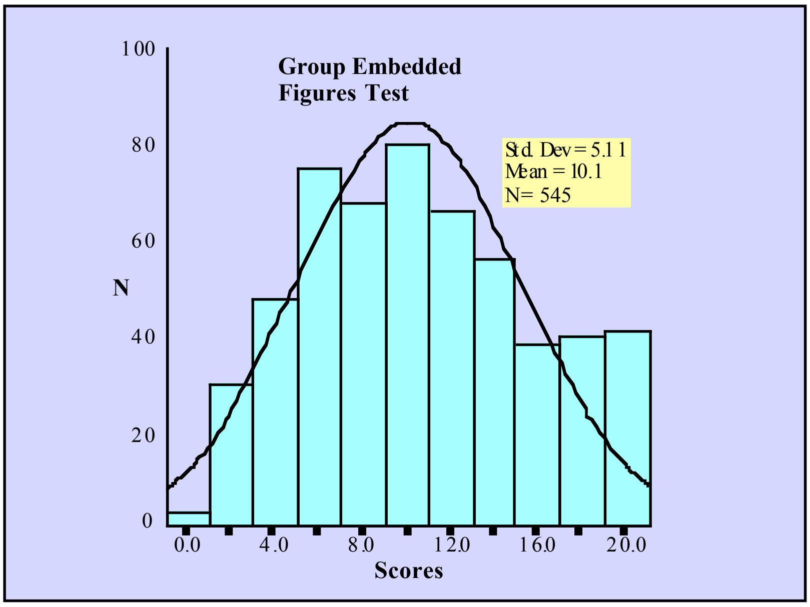


Figure 8.3 Field Dependency Data (total sample)

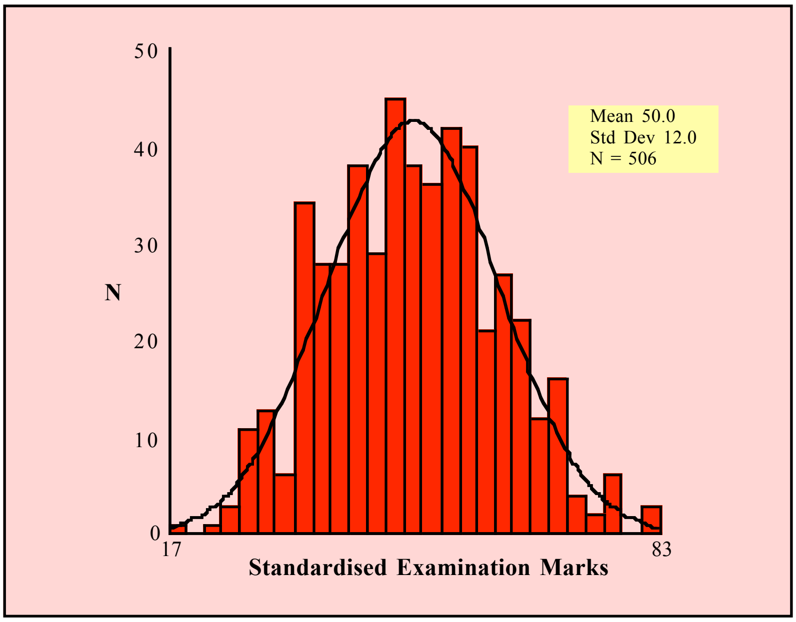


Figure 8.4 Mathematics Examination Marks (total sample)

8.5.1 Performance in Mathematics Test Scores and Hidden Figure Scores

The extent of field dependency was correlated with the mathematics performance for each year group. The mathematics test data were obtained for 506 of the 547 students. Because the data are approximately normally distributed, Pearson correlation was used.

Firstly, each year group is considered in turn (Table 8.2).

Pearson Correlation: Field Dependency and Mathematics Scores			
Age	***Sample Size***	***r***	***p***
13	92	0.54	< 0.001
14	168	0.22	< 0.01
15	102	0.52	< 0.001
16	142	0.25	< 0.01
17	43	0.44	< 0.01
Total	**547**	**0.32**	**< 0.001**

Table 8.2 Field Dependency and Performance in Mathematics

The variations in correlation will reflect the actual examination papers set, the way the questions were asked and the topics being tested. Thus, it is possible to set questions where the student has no problem in seeing what is important or what has to be done first. Equally, it is possible to set questions where the skill of being able to see what is important or what has to be done first are important. Nonetheless, the extent of field dependency seems to relate highly with mathematics performance.

It is possible to look at the entire sample. The overall correlation of mathematics marks (standardised) and extent of field dependency is 0.32 ($p < 0.001$), in line with the findings of Al-Enezi (2006). Thus, if the standardised mathematics marks reflect some kind of general ability in mathematics, then this ability correlates with the measured extent of field dependency (as shown in Figure 8.5). Of course, correlation does not necessarily imply cause and effect.

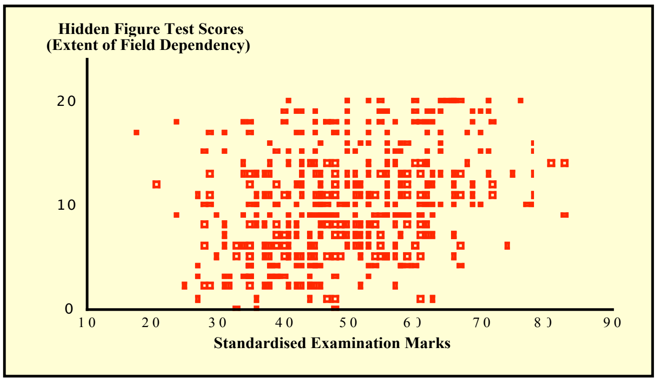


Figure 8.5 Scatter gram

The power of the relationship can be illustrated by dividing the entire sample into three groups as described in section 8.3 before (but shown in Table 8.3)

Group	Number of students	Field Dependency Score Range	Mean Mark in Mathematics
Field Dependent	191	0-7	45.6
Field Intermediate	177	8-12	50.9
Field Independent	179	13-20	53.5
Total	**547**		

Table 8.3 Classification of sample

This table reveals that the average performance in mathematics for the more field independent group is 8% higher than the average performance of the more field dependent group.

8.5.2 Field Dependency and Age

The outcomes for the whole sample can also be analysed using ANOVA (Table 8.4). The aim here is to see if the measured extent of field dependency changes with age. It was observed that, when the hidden figure test was applied on students sample of ages between thirteen and fourteen, they had difficulty in recognising the simple shape from the complex patterns, and more than 25% of them required further examples to start the test. The findings show that there is a significant growth of field dependency with age (Table 8.4).

Age	*Sample Size*	*Mean FD Score*	*Standard Deviation*	*Analysis of Variance*	
				F	*p*
13	92	6.9	3.4	**17.6**	**< 0.001**
14	168	9.6	4.7		
15	102	10.6	5.2		
16	142	11.8	5.3		
17	43	12.3	5.2		
TOTAL	**547**	**10.1**	**5.1**		

Table 8.4 Field Dependency and Age

It is difficult to interpret Table 8.4. Is it possible that the extent of field dependency changes with age, simply on the basis of cognitive development, in parallel with the growth in working memory capacity with age (one unit for every two years, approximately, to about age 16). Alternatively, is the measured growth with age a reflection of learning or more general life experiences? If this is so, then is it possible (and easy) to encourage the development of this learner characteristic by specific teaching strategies?

Thus, Ausubel (1968) found that what a learner already knew was absolutely critical in influencing success at the next stage of learning. This finding is reflected in the Johnstone Information Processing Model (Johnstone, 1997) in what he called a 'feedback loop'. Knowledge held in long-term memory was influencing the perception filter. It is possible that this is a key feature of what is meant by field dependency. The long-term memory is enabling the filter to work more efficiently in the more field-independent person: previous knowledge and experience allows a more efficient selection, thus reducing the load on working memory.

This idea was followed up by Sirhan and Reid (2001) in their development of what became known as 'chemorganisers'. These were designed to bring previous knowledge to the forefront so that this information could assist the perception filter in making more efficient selection on facing new knowledge. They saw this in terms of working memory efficiency but another way to interpret their findings is to see them in terms of field dependency.

It is possible to show the growth in field independency with age as a graph (Figure 8.6)

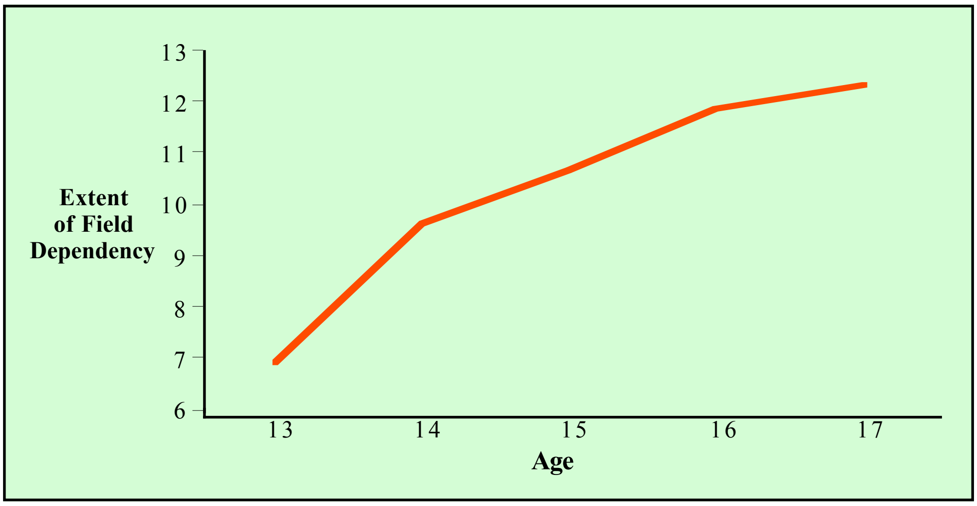


Figure 8.6 Extent of Field Dependency and Age

From the Figure 8.6, it is interesting to see that extent of field dependency increases with age. However, the graph suggests that the rate of growth declines with age. Of course, this may simply reflect cognitive development, essentially genetic in nature. It seems intrinsically more likely that the growth is brought about by learning and experience. If this is so, then the development of the skill should take place with younger age groups.

8.5.3 Gender Issues and Field Dependency

It has been observed by Ali (2008) that there are many gender differences in terms of students' overall cognitive structure, perception and understanding mathematics. On one hand, girls seem to dominate in understanding and general commitment, and on the other hand, boys tend to dominate in terms of perception and showing strong relationships. The gender differences of correlations between items also show many gender differences. In the present study, the overall field dependency measurements were divided by gender and the results are shown in Table 8.5.

Descriptive Statistics for Field Dependency Measurements							
	Sample	**Maximum**	**Minimum**	**Mean**	**Standard Deviation**	**t**	**p**
Boys	279	20	0	9.6	4.9	**2.5**	**< 0.01**
Girls	268	20	0	10.7	5.2		
Total	**547**	**20**	**0**	**11.2**	**5.1**		

Table 8.5 Gender and Field Dependency

It is surprising and very interesting to note that the girls tend to be more field independent than the boys. While there are differences in field dependency for girls as compared with boys, that does not mean that all girls learn essentially one way and all boys learn another way.

Sex differences in field dependency may derive from basic physiological differences, such as in the ability to hear, perceive and process information, and also from differences in innate potential. This occurrence of gender interactions needs more careful study since these suggest a possible fundamental difference in information processing between males and females and if these were better understood then both sexes might be helped to learn more effectively. However, the explanation might simply lie in cognitive development where girls at these ages are often markedly more mature than boys. This may lead to a greater attention to detail, to study and greater commitment to look for what is right.

8.6 Chapter Summary

There are various cognitive styles available in the literature, among which are field-dependence and field-independence. Although various forms of cognitive styles have been introduced and different instruments have been developed to assess them, Witkin *et al.*,'s (1971) Group Embedded Figure Test (GEFT) has been applied most commonly. There are two reasons for choosing GEFT in this study. First, the instrument is a non-verbal test and requires only a minimum level of language skill for performing the tasks (Cakan, 2003). Another reason is that psychometrical properties of the instrument have been investigated in cross-cultural settings and accepted as quite reasonable.

According to Witkin and Goodenough (1981), people are considered field-independent if they are able to abstract an element from its context, or background field. In that case, they tend to be more analytic and approach problems in a more analytical way. Field-dependent people, on the other hand, are more likely to be better at recalling social information such as conversation and relationships. They approach problems in a more global way by perceiving the total picture in a given context.

Field dependency may be determined by an individual's genetic origin, a learned characteristic as a result of experience, or as a characteristic adopted by some element of choice. It may involve all three. The fact that extent of field dependency grows with age does not necessarily show that it is experience related. Working memory is known to be genetic in nature and it grows with age. However, it does seem likely that experience is a factor. Teachers do encourage their students, especially in subjects like mathematics, to focus in on the key information. This suggests that their experience shows that this tactic brings benefits to the student, thus implying some kind of learning of field dependency.

Overall, the following results were obtained as a result of data analysis:

(1) There is a significant relationship between field dependency, students' perception about mathematics and achievement. Field-independent students had better performance than field-dependent students for all five age groups.

(2) Field dependency increases with age, and the rate of growth declines with age reflecting cognitive develoment as essentially genetic in nature. There are several possible reasons for this although it is highly likely that formal learning and life experiences may enhance the field dependency skill.

(3) Girls tend to be more field-independent than the boys but it is difficult to explain this. However, this does not necessarily mean that girls learn essentially the same way and boys in another way. There is need for this to be explored.

Chapter Nine

Further Exploration on Field Dependency and Working Memory

9.1 Introduction

Many studies have explored the difficulties in the sciences and mathematics, and made many practical suggestions (e.g. Schminke *et al.,* 1978; Johnstone and Wham, 1982; Copeland, 1984; Liebeck, 1990; Johnstone, 1997; Haylock, 2006). The main conclusion gained from these studies is that many students have difficulties during the learning process, and a significant number have specific difficulties. Such difficulties appear to be equally common in boys and girls. This study has considered students' attitudinal problems in relation to mathematics, the role of working memory and the extent of field dependency, set in a context of the way they process information. In this chapter, an attempt is made to bring together the latter two areas in an effort to explore the exact impact of field dependency by considering specific mathematics problems.

Working memory and field dependency influence the learning process continuously in that they are linked with the individual's ways of perceiving, selecting, processing and retrieving information. They may well control and determine whether students display their ability completely or almost not at all in learning activities. Therefore, the way to make the most of this interaction should be developed in order to achieve both cognitive and affective objectives in mathematics education.

Chapters 7 and 8 of the present study explored the relationship between attitudes towards learning, working memory capacity, and field dependency characteristics (known to be major factors in performance). The findings suggest that attitudes towards learning relate to mathematics achievement, and achievement relates to the amount of information students are able to process in relation to the extent of field dependency. The results also revealed that mathematics is perceived less easy as students grow older, with likely reduction in understanding. In particular, the overload of students' working memory space is considered as the main factor causing learning difficulty and, in consequence, learning failure.

In this chapter, the aim is to gain further evidence that the field dependency characteristic is actually causing the differences in mathematics understanding and achievement. The focus is on school mathematics in English schools. The experimental structure along with students' sample involved in the research is now described.

9.2 Experimental Structure

The third and final experiment took place at the start of the third year of the research. The total sample of students involved was 117 from four year 10, GCSE classes of varying abilities. This sample can be regarded as typical of the year group and, therefore, provided results that offered potential generalisability for the year group as a whole. However, in these classes, the gender balance was not exact. Thus, the average number of students per class is 29 with more boys than girls (boys = 71, girls = 46).

The aim was to check if the questionnaire experiments (in Chapters 7 and 8) were valid, and also to obtain extra insights into the precise nature of the field dependency and working memory problems in mathematics.

The following experimental procedures were employed:

(1) Measurement of extent of field dependency of the whole sample of 117 students of four year 10 classes (aged 15 to 16). The method and procedure of this measurement are explained in Chapter 8. Their hidden figure test was marked by the investigator and scores were correlated with their mathematics examination/test scores, duly standardised.

(2) Six students were selected from the sample. The aim was to select 3 who were *highest* and 3 who were *lowest* on the field dependency test. An individual interview of 20 minutes was undertaken with these students. The interview was fairly well structured and was conducted in a quiet informal relaxed atmosphere with five basic questions being explored:

    Do the field independent group,-

    (a) Focus more quickly on the key task?
    (b) Spot the key information more quickly?
    (c) Show greater ability to pull together key information, ignoring what is less relevant?
    (d) Focus on the first step, then the second, and so on, more clearly?
    (e) Tend to ignore the peripheral more easily?

    Before the interview, each student completed a short mathematics test and they were invited to discuss their problems in relation to this test. Their verbal responses to the questions were related to the way they tackled these mathematics problems. There was opportunity for further more open-ended discussion when appropriate. The mathematics test employed is shown in Appendix F.

(3) In order to explore whether it was the extent of field dependency and/or working memory capacity was directly relating to difficulties, a short survey was employed. This survey was administered to 117 participants with the aim of considering the characteristics of students in the aspects of their understanding and perception of mathematics learning difficulties.

The questionnaire contained three questions. The first two questions had four-point difficulty scales (always a problem, frequently a problem, sometimes a problem, never a problem). This method was favoured for the reasons that attract other researchers: it offered participants a flexible response to ease differentiation, it offered the investigator a ready means by which to generate quantitative data from a participating group. This is not to claim, however, that in employing this method, the boxes that were ticked were completely accurate interpretation of their views.

The last question was open-ended to allow students to express their views and make comments about a solution to a specific mathematics problem. The time allowed for the questionnaire to be answered was 10 minutes. 12 students submitted their responses but did not give their names or identity number, making it impossible to include them in the experiment. The survey was considered critically by a number of experienced mathematics teachers and minor adjustments were made. It is shown in full in Appendix G.

This survey was designed to be friendly and unthreatening. It was hoped that the students would respond openly and freely and that they offer insights into the precise nature of the source of the difficulties.

It was possible to gain some kind of overall impression of the general approach of each interviewee. Notes were taken throughout and these were studied and compared to their responses to the interview mathematics questions to seek to classify each student in relation to different field-dependency characteristics and working memory capacity.

The overall aim in this part of the study was to explore in more detail how those who were very highly field-independent attempted some problems in mathematics. Is there clear evidence that the field dependency characteristic is specifically related to the way students solve mathematics problems?

9.3 Analysis of Data

The response pattern from the hidden figure test was entered into a spreadsheet before analysis, and correlated (using Pearson correlation) with their mathematics examination scores. The results show that there is a significant relationship between field dependency and mathematics performance with a correlation of 0.56 ($p < 0.001$), confirming the previous results in chapter 8 where the field-independent students had a better performance than field-dependent students. This corresponds to earlier work by Vaidya and Chansky (1980) who found that "*field-independent students correlated with higher mathematics achievement, especially for concepts and application learning*".

As before, data are shown as rounded percentages for clarity but any statistical analyses used actual frequencies. The responses to the survey shown on page 260 (in the Appendix) are now discussed in some detail.

(1) Think of your ***class work and homework*** in mathematics
*Tick **one box** on each line*

Age ~ 15-16, N = 117, all data as %	*Always a problem*	*Frequently a problem*	*Sometimes a problem*	*Never a problem*	*Always + Frequently*	*Always + frequently + sometimes*
(a) Grasping what is required	7	12	77	5	19	96
(b) Understanding the symbols used (like x and y)	8	11	55	21	19	74
(c) The most difficult thing is knowing where to start	12	15	39	34	27	66
(d) Trying to take in all the information	15	33	44	8	48	92
(e) Seeing what is important	8	21	57	13	29	86
(f) Not understanding the instructions	10	15	54	21	25	79
(g) Not seeing why an answer is needed anyway	10	16	36	38	26	62
(h) Selecting what is most important	3	22	58	17	25	83
(i) Coping with all the information in the question	10	16	48	26	26	74
(j) Remembering what to do	12	18	46	25	30	76
(k) Trying to interpret the English to know what to do.	8	16	51	25	24	75

Table 9.1 Question 1

The options which are chosen most are highlighted (red for the highest and orange for the next group). This is a very revealing question although there is some degree of polarisation of views. Table 9.1 shows that much of the difficulties in learning mathematics in the classroom and practising it at home by a way of assignment are linked with failures in grasping what is required, difficulties in taking in too many information, perhaps at the same time, and not being able to select what is important from the question (as can be seen from the items colour-coded). These are all working memory and field dependency characteristic problems. When this occurs, as might be expected, students are more likely to become overwhelmed, frustrated with the whole instruction process, or confused by something that might be quite basic to handle. Moreover, lack of confidence may lead to learning failures.

Indeed, the highest percentage choosing the first two boxes (*always a problem* and *frequently a problem*) is for the statement: '*Trying to take in all the information'*. This seems to be directly a working memory or field dependency problem, perhaps both. This offers some clear evidence to support the idea that the correlation between extent of field dependency and performance is cause and effect.

(2) Think of ***tests and examinations*** in mathematics.
*Tick **one box** on each line*

Age ~ 15-16, N = 117, all data as %	*Always a problem*	*Frequently a problem*	*Sometimes a problem*	*Never a problem*	*Always + Frequently*	*Always + frequently + sometimes*
(a) Grasping what is required	0	12	68	12	12	80
(b) Understanding the symbols used (like x and y)	9	17	59	16	26	85
(c) The most difficult thing is knowing where to start	11	12	42	35	23	65
(d) Trying to take in all the information	12	23	49	16	35	84
(e) Seeing what is important	11	16	54	19	27	81
(f) Not understanding the instructions	9	18	53	21	27	80
(g) Not seeing why an answer is needed anyway	15	13	46	27	28	74
(h) Selecting what is most important	9	16	53	23	25	78
(i) Coping with all the information in the question	11	14	50	25	25	75
(j) Remembering what to do	13	9	50	29	22	72
(k) Trying to interpret the English to know what to do.	9	25	43	23	34	77

Table 9.2 Question 2

Again, colour shading picks out the highest reasons. The findings from Table 9.2 show clearly that the problem areas in tests and examinations are related to student inability to grasp what is required, lack of understanding of mathematics symbols, struggling to take in too many information, or trying to interpret the English to know what to do, and not able to see what is important from the item of the question. Again, the difficulties are linked with working memory and learning style characteristics. The very contextual nature of mathematics (language and symbols characteristics) also place so much demand on working memory. When this occurs, understanding becomes almost impossible.

9.4 Interviews

The interview was well structured, allowing opportunities for follow up questions and general discussions. The structured part of the interview was based on four questions which were related to each mathematics problem in the test:

(a) What did you do first?
(b) What was the key task?
(c) What was important?
(d) What was irrelevant at the start?

A sample of six students (3 who are *very* field-independent and 3 who are *very* field-dependent on the field dependency test) were interviewed, discussing how they individually tackle some mathematics problems. Notes were taken throughout and these were studied to seek to classify each interviewee in relation to extent of field dependency. The key goal was to get evidence that the field dependency is making a key difference to the way students understand and approach some questions in mathematics. In simple terms, do the highly field independent students approach the questions in different ways when compared to the highly field dependent students which then suggest that it is the specific skill of field dependency which influences mathematics achievement directly.

The interviews were tailored to obtain responses to the five basic questions.

These questions were designed to look for:

(a) Do the field-independent group focus more easily on the key task?
(b) Do the field-independent group spot the key information more easily?
(c) Do the field-independent group show greater ability to pull together key tasks, ignoring what is less important?
(d) Do the field-independent group focus on first step, then second step and so on more clearly?
(e) Do the field-independent group tend to ignore the peripheral more easily?

The test questions and comments are now discussed. The interviews followed the test completion immediately and the discussion was built around the actual questions and how the students saw the ways they had tried to gain answers to the mathematics questions.

Key Results

(1) Solve the following equation for x.

$$\frac{2x}{3} + 5 = 2$$

The three field-independent students were all able to start this problem easily. They all had the capacity to focus in on the numbers (5 and 2) and combine them to a value of -3 on the right hand side. Two of them were able to multiply both sides of the equation by 3 to clear the fraction and then complete the problem. None of the three field-dependent students was able to start at all. They needed a little opportunity for some items of the questions to be explained to them before they could make an attempt. It seems that the skill of being able to see the first step involves the combination of the two numbers is critical. Thus, it is likely that the ability to see the first step and focus in on it is critical for success.

(2) Solve the following equations for x and y.

$$3x + 2y = 5$$
$$2x + 3y = 0$$

The three field-independent students all were able to start this problem easily. They all managed to multiply the equations correctly, this being the first critical step. They all then could see that subtraction was the next stage. Nonetheless, mathematical errors crept in. Two of the field-independent students made slips in the final stages, with only one gaining a fully correct answer. Of the three field-dependent students, only one started the task and there was clear difficulty. The idea of multiplying coefficients was tried but executed incorrectly.

Discussions during the interview revealed that the field-independent students knew exactly what was expected of them and could focus easily on the tasks in the right order. Their responses reflected what they had actually achieved. In contrast, the field-dependent students seemed to have little idea of what was required, or where they should start. They expressed a sense of 'lostness', with more or less no idea of where they were going or what was important.

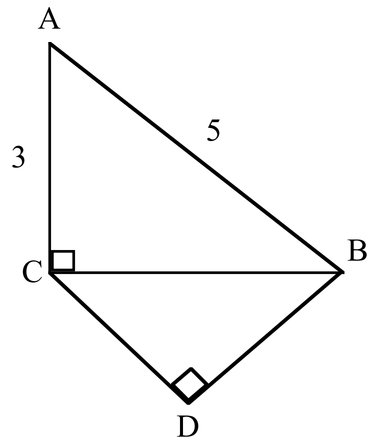
(3) Solve the following equation for x.

$$2x^2 - x - 3 = 0$$

Surprisingly, no one could attempt this question at all. All the students admitted freely that they simply had no idea of where to start or what to do.

(4) Look at the diagram:

If CD = BD, find the length of CD.



This question revealed little in that two from each group failed to make a start at all. These students revealed that, with all the information in front of them they were unsure of what to do first.

Comments

With a small sample, it is difficult to draw clear conclusions. However, the test and the discussions suggest that academic performance is related to field dependency in a cause and effect sense. Thus, those who are more field-independent tend to perform better than others because they seemed to know what was required of them and they were much more able to focus exactly on the series of steps required to gain correct answers.

In addition, the general impression left with the interviewer was that those who were highly field-dependent had little idea of what the question was really asking. They seemed to have difficulty in interpreting this into an action. They did not know what to focus in on in the questions. All of this suggests that field dependency is a key characteristic which enables success to take place.

However, not too much can be drawn from the interviews although they gave a generally consistent pattern when compared to much of the questionnaire data. It is perhaps possible that the questionnaire offers a more accurate picture in that, in the interviews, individuals could be identified.

9.5 Chapter Summary

This experiment was designed to offer extra insights of some kind of overview of field dependency and working memory characteristics in relation to mathematics understanding and achievement. The key observations are that learners have difficulties in:

(1) Grasping what is required (in mathematics tasks).
(2) Understanding mathematics symbols (such as x and y).
(3) Trying to take in all or too many information.
(4) Trying to interpret the English to know what to do.
(5) Remembering what to do.
(6) Seeing what is important, selecting what is important and ignoring what is irrelevant.

Clearly, in looking at the above list, items 1 to 5 are all working memory problems, while item 6 is a field dependency characteristic. Thus, learning and understanding tend to relate with cognitive behaviours: what learners are able to perceive, process, select and recall. It was also observed that mathematics performance significantly correlates with field dependency. The more field-independent a learner is, the better he or she is able to achieve.

Again, it is possible that those who are field-independent are able to use their limited working memory capacity more efficiently and this means that they are more successful in terms of understanding. Such understanding generates greater overall satisfaction, confidence and a sense of achievement. Looking at the two effects together (working memory capacity and extent of field dependency), they are clearly powerful correlates of success. Interviews tended to confirm the outcomes from the surveys. Again, mathematics achievement is related to working memory and field dependency. High working memory and more field-independent students show greater ability to pull together key information more quickly, ignoring what is irrelevant.

Chapter Ten

Summary and Conclusions

10.1 Review of the Study

Learning with understanding is more desirable than learning by memorisation. Information that is simply memorised tends to be less accessible later while understanding potentially enables the learner to apply usefully what is learned. Indeed, a possible description of understanding is that genuine understanding is demonstrated when the person can apply what is learned in a novel situation, with some prospect of success.

Of course, learning with understanding involves actual knowledge and the development of the rich links within what is known in order to make a coherent whole. Competent performance is built on neither factual nor conceptual understanding alone; the concepts take on meaning in the knowledge-rich contexts in which they are applied.

In looking at learning with understanding, one controversy centres on the question of how much and what kind of guidance should be provided to learners. Those favouring learning by discovery or invention (e.g. Piaget, 1971; Bruner, 1966) advocate maximum opportunity for physical exploration by the learner. Those preferring guided learning (e.g. Gagné, 1965) emphasise the importance of carefully sequenced instructional experiences (information processing) through maximum guidance by teacher or instructional materials.

The procedure for Gagné is to begin with a task analysis - what do you want the learner to be able to do? The capability must be stated '*specifically*' and '*behaviourally*'. It can be conceived as a terminal behaviour and placed at the top of what will become a pyramid-like network of cognitive skills. Hence, a pyramid or hierarchy is built. If the capability with which we begin is problem-solving, the learner must first know '*principles*'. But to understand certain principles, he must know specific concepts, and prerequisite for the '*concepts*' are particular associations, connections and facts. Logical processes such as mathematics must be based on the psychological structures available to the learner. These structures change as the learner matures physiologically and as the learner has necessary experiences in the physical world (Copeland, 1984).

It has to be recognised that simply exposing learners to the world of mathematics and allowing them to explore and discover is not realistic. However, following the logic advocated by Gagné (1985) may not provide the answer either. While the concepts of mathematics do build on each other with an elegant logic, there is no assurance that the learner works in such a logical way. The logic may be apparent only with hindsight. It is at this point that information processing offers some insights.

One of the key outcomes from information processing is that the capacity of working memory is a rate-controlling step in all understanding. Thus, the working memory capacity limitation offers a clear rationale in interpreting and explaining the nature of many difficulties in learning. Learning in mathematics involves the mastery of procedures as well as an understanding of what is being done. This places enormous pressure on working memory and the two aspects almost certainly must be taught sequentially to avoid excessive overload.

In one of the most famous early studies in comparing the effect of 'learning a procedure' with 'learning with understanding,' two groups of children practised throwing darts at a target under water (Judd, 1908). One group received an explanation of refraction of light, which causes the apparent location of the target to be deceptive. The other group only practised dart throwing, without the explanation. Both groups did equally well on the practice task, which involved a target 12 inches under water. But the group that had been instructed about the abstract principle did much better when they had to transfer to a situation in which the target was under only 4 inches of water. Because they understood what they were doing, the group that had received instruction about the refraction of light could adjust their behaviour to the new task. The picture that is painted here is that learners need to be guided to be able to understand, to make connections from previous knowledge and to manipulate new situations.

Learning mathematics with understanding is a 'sense-making' activity. Some researchers (e.g. Perkins, 1985) view understanding more like an ability and less like a mental model or mental representation of knowledge. In this view, understanding is the ability to think and act flexibly with what one knows. Understanding develops when a person uses what he or she knows to construct meaning out of new situation. It involves making connections among ideas and procedures; these connections are considered to facilitate the transfer of prior knowledge to novel situations. As a person renders new information sensibly, his or her knowledge about the topic not only increases quantitatively, but also changes qualitatively by becoming more differentiated and elaborated. The result is a representation or mental model that structures the conceptual knowledge.

When mathematical ideas are thoroughly understood, they are set within a matrix of other concepts. The ideas can then be applied. This leads to the whole area of transferability. The evidence suggests that ideas tend to be very much set in the contexts within which they have been acquired.

Thus, the ability of transfer (the ability to use what have been learned in new and unfamiliar problems) is a very difficult skill to achieve in that so much is context-dependent. This led Reid and Yang (2002b), in their study of solving open-ended problems (and thus unfamiliar in nature), to conclude that problem-solving was not a generic skill. It was learned in one context and then tended to be applied in a similar context. They suggested that this was because it was the accessibility of the links in long-term memory between the ideas *in that specific context* which was critical. This was later supported strongly by further studies (Al-Qasmi, 2006). Transfer of learning is clearly a most desirable outcome. Achieving it may prove to be much more elusive than is often suggested.

In contrast, rote learning or learning by memorisation is a process in which the learner tries to copy new information into memory. Although the individual may be able to replicate the material, he or she does not necessarily grasp the relationships among the ideas and facts. The chance of being able to use that knowledge in novel situations is highly unlikely.

Anxiety about mathematics and feelings of inadequacy in this subject seem to be widespread amongst students in many countries. Although attitudinal outcomes of education have been underdeveloped, there are signs today that there is an increasing concern with such outcomes. However, there is an element of suspicion among teachers when attitudinal topics are discussed (Reid, 1978). It has been suggested (Section 4.2) that part of the problem of mathematics underachievement may be due to the lack of useful teaching approaches for developing attitude growth among learners as well as insecurity in the measurement of such attitudes. On the basis of social psychology usage, attitudes have been demonstrated to be made up of cognitive, affective and behavioural elements and, in the context of mathematics education, the concept of '*cognitive attitude'* has been suggested (Section 4.7). These are attitudes of interest, awareness and appreciation, which have occurred as a result of cognitive growth. They stand in contrast to attitudes which have been developed as a result of feeling and emotions, based on partial or inadequate cognition.

Learners develop attitudes by means of input, reception and processing, as with all learning. The key task is to identify the fundamental features of learning experiences which tend to lead to the development of positive attitudes. The work of Reid and Skryabina (2002a) has shown that, at least in relation to physics, positive attitudes depend on the nature of the curriculum. Specifically, if what is taught is perceived by the learners to be related to them in the context of their lifestyle and environment, then attitudes tend to be very positive. Later work by Jung and Reid (2009) suggested that being able to understand was also critical. Much work has shown the importance of the teacher (see Reid and Skryabina, 2002a). Perhaps, the key rests in the ability of the teacher to present material in ways to enable learners to see how their learning relates to themselves as well as enabling understanding to occur.

Much of the research about learning and understanding has been related to sciences (chemistry, biology, physics). If it is also true for mathematics that major problems arise during the learning process, then it is of much benefit to teach mathematics so that it is perceived by learners to be related to them in the context of their lifestyle and environment. The limitations of working memory may make this very difficult because of the procedural nature of mathematics, a point stressed by Al-Enezi (2006). However, taking the limitations of working memory capacity into account is important in enabling greater understanding.

Memory is often thought of as a storing of information, but this oversimplifies its true meaning. Once we admit new information through the sensory memory, it is transferred to the working memory where the manipulative processes (interpreting, rearranging, comparing, holding and storing) take place in order to make sense. The important feature to recognise is that working memory is where the person thinks and solve problems, it has limited capacity and is, therefore, a major limiting factor in learning with understanding.

Success in developing mathematics education, therefore, depends on taking into account the cognitive aspects of learning (and, specifically, the key role of working memory) as well as the attitudinal aspects of learning (specifically, the need for learners to perceive the significance of what they are learning in the context of their lifestyle and environment).

All understanding takes place in the same way: information is selected into the working memory, processed and then linked to previously learned material to make a richer and more coherent whole. However, learners differ in many respects. We are probably more aware of physical differences in appearance than we are of different styles of thinking, understanding and representing information. This is where field dependency comes in. In a problem solving situation, the learner may encounter a problem which is not familiar to him, especially if the problem is an open-ended one. Learners could face such a situation in a mathematical classroom, in a school laboratory, or even in the outside world manifesting itself as an everyday problem. The extent of their field dependency may well influence success in the problem greatly, as this study has shown. Understanding something of these individual differences is important for teachers as they seek to enable their students to understand themselves better and develop their full potential and be guided into more appropriate learning outcomes.

The aim of the study has been to explore aspects of the mathematics learning experiences in the primary and secondary schools of ages approximately 11-17. The study began with the intention of looking at cognitive and attitudinal aspects of learning mathematics. This has involved looking at students' perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties. In order to explore these areas, it was decided to investigate the relationship between attitudes, working memory capacity, and extent of field dependency.

Despite many checks on validity and reliability of the questionnaire and interview experiments, the results must be interpreted with caution, as there is no certainty that they can be generalised. However, all the evidence obtained does give rise to cautious optimism. Mathematical performance has relied mainly on school tests and examinations, recognising that these are imperfect measuring tools.

In this chapter, the results of this study are summarised. This is followed by comments on the strengths and weaknesses of the entire work. Conclusions are associated with reflection and suggestions for further research.

10.2 Main Findings

As a result of questionnaire analysis, working memory and hidden figure tests, and examination performance data, the following outcomes are obtained from the three experiments conducted; the generalisations may only apply in these countries studied:

(1) Students vary considerably in their reaction to different topics in mathematics and can hold widely varying attitudes related to their experiences in learning mathematics depending on their cognitive structure.

(2) Students are positive towards the more cognitive elements of attitude to mathematics (e.g. "mathematics is important"; "mathematics lessons are essential to me"). However, they are more negative towards affective elements (for examples, "I enjoy my mathematics lessons"; "mathematics lessons are interesting"). Thus, they are very realistic about the value of mathematics but find their experiences of learning it much more daunting. Thus, the majority of students thought mathematics is useful, important and relevant to them.

(3) Those who like mathematics and those who felt they were coping tended to be those who obtained better marks.

(4) Attitudes towards the learning of mathematics change with time or age. As students grow older, the belief that mathematics is interesting and relevant to them is weakened, although many still think positively about the importance of mathematics.

(5) Loss of interest in mathematics is perhaps due to difficulties in learning mathematics: 'difficult content', 'method of instruction', 'inability to grasp what is required', 'trying to take in too many information', 'inability to select what is important', and the 'complexity nature of mathematics in relation to abstract language and symbols'. All of these features relate to working memory overload, with the skills in the field dependency area being important.

(6) There is a positive relationship between working memory capacity and mathematics achievement. Perhaps, students who have high working memory capacity tend to try to understand mathematics knowledge as much as they can, while students who have low memory capacity tend to have difficulty in understanding mathematical knowledge and thus resort to memorisation of procedures. Recent work (Jung and Reid, 2009) has found this to be true in sciences where either or both teaching and assessment make more demand on the working memory.

(7) Some topics are regarded as 'difficult'. Major areas relate to trigonometry, longitude and latitude, elevation and depression, loci and construction, algorithms, and surds. Topics of greatest difficulty seem to be those where the demand on limited working memory resources is greatest. These topics, require ideas and concepts which have to be handled cognitively at the same time, and working memory capacity cannot cope easily in this case.

(8) There is a significant positive correlation between field dependency and mathematics examination scores. Performance in the mathematics test is best for those who are more field-independent. Overall, Field dependency has a cause and effect on mathematics achievement and success.

(9) Field dependency grows with age. Thus, as students grow older (at least between 12 and about 17), they tend to become more field-independent-suggesting that field dependency may be genetic, learned or an element of choice.

(10) Girls tend to be more field-independent than boys, perhaps reflecting maturity or their greater commitment during the years of adolescence to undertake their work with care, although boys may still catch up later.

10.3 Strengths and Weaknesses of the Study

It was interesting to note that the students were enthusiastic to complete the surveys, perhaps appreciating the opportunity to express their views in relation to aspects of learning and success. Overall, data from large samples were obtained, making it likely that there is highly reliability in the measurements made. This gives confidence that the outcomes are generalisable. Several experiments were employed to give extra insights that the surveys employed were valid.

The validity of measurements is always open to question. Encouragingly, the tests used to measure working memory capacity and the field dependency are well established and there is great confidence that they do, in fact, measure these characteristics correctly. It was not possible to have enough access to the students to test their mathematical skills with tests devised for the purpose. Instead, there was reliance on marks from school examinations. It would have been helpful to gain more information about working memory capacity in the schools where the second and third experiments were carried out but it was difficult due to access to the students. Nonetheless, the analyses from the data on both working memory capacity and field dependency were consistent with the patterns gained from other research studies.

Surveys of attitudes are always open to criticism. In looking at the responses from the attitude survey, there is no certainty that students responded to reflect the reality of their views but their responses may have indicated their aspirations. However, there was a considerable consistency of views related to their attitudes and mathematics achievement and the general patterns of responses are consistent with the researcher's experience as a teacher.

The interview experiment was disappointing. Time for access to students prevented increasing the numbers interviewed and they were hesitant, or perhaps incapable, of revealing more about their difficulties. Nonetheless, the observations made were entirely consistent with the view that the ability to focus in on the 'message' and ignore the 'noise' (field dependency) was one key factor in success in much mathematics.

10.4 Suggestions for Improving Teaching and Learning

There are some useful key points from this study if teachers and other educators want to motivate and develop students' positive attitude to mathematics. The following suggestions should be applied:

- There is a limitation to student working memory space and, whenever an overload occurs, student performance declines markedly. Thus, there is a limit to what learners can handle at the same time. In the teaching and testing situations, teachers should be careful with the way they give instructions, the language used, and the amount of information they pass on to students. The skill of controlling the amount of information is not an easy one. The teacher needs to be able to see it from the learner's perspective and this is never easy.

- The limitations of working memory capacity apply even more markedly in formal examination settings. It is quite possible for national examinations to test the capacity of working memory and extent of field dependency as much as testing the knowledge and skills in mathematics.

- When ideas and procedures are strongly linked in meaningful ways in the long-term memory, they tend to be remembered easily understood and enough to be used in applications. One of the key features of mathematics is the way exercises are used to enable learners to be able to carry out procedures in a fairly automated way. Once this automation is achieved, it is important to stress the way such procedures relate to each other, what they mean and how they can be applied.

- Effective learning comes to mind when new information is linked correctly to information that is already stored. This requires the teacher to know fairly precisely what has been taught previously, the problems learners are likely to have had, and the way it was taught. The links must be overt and clearly spelled out.

- When it comes to the development of positive attitudes, two factors seem to be important: seeing how the mathematics relate to real-life and being able to understand what is being done. Aiming for understanding must always be the goal but making the application of mathematics real is not easy for there is great potential for working memory overload. The key must lie in establishing procedures first and then, when these are secure, giving the students insights into understanding. When that is established, perhaps applications can be then be added. However, this is the opposite order to that found in making the sciences attractive where the work starts with applications and then reveals the science understandings which make sense of what is happening. This needs much more exploration in mathematics.

Many of the outcomes from the information processing model were later modified in a set of ten principles as a way of improving learning by Johnstone (1997). These were later called the 'Ten Commandments for Learning' by Gray (1997):

"(1) What you learn is controlled by what you already know and understand.
(2) How you learn is controlled by how you have successfully learned in the past.
(3) If learning is to be meaningful it has to link on to existing knowledge and skills enriching and extending both.
(4) The amount of material to be processed in unit of time is limited.
(5) Feedback and reassurance are necessary for comfortable learning, and assessment should be humane.
(6) Cognisance should be taken of learning styles and motivation.
(7) Students should consolidate their learning by asking themselves about what is going on in their heads.
(8) There should be room for problem solving in its fullest sense to exercise and strengthen linkages.
(9) There should be room to create, defend, try out, and hypothesise.
(10) There should be opportunity given to teach (you don't really learn till you teach)."

This set of principles captures the findings of vast amounts of research evidence. It addresses the working memory problem (item (4)) and emphases the importance of attitudes (5). The way understanding actually occurs in the long term memory is included (3 and 8). The field dependency issue is included within (6).

10.5 Recommendation for Further Study

This study has shown that both working memory capacity and extent of field dependency correlate highly with performance in examinations in mathematics. Previous studies have shown that the working memory capacity effect is, in fact, cause and effect: the size of the learner's working memory strongly influences success in examinations in mathematics. This study has indicated that the same may well be true for extent of field dependency: the extent of field dependency strongly influences success in examinations in mathematics.

The next stage is to explore how mathematics teaching and assessment can be carried out so that these factors do not have such a strong influence on mathematics performance. This may involve changes in the way the curriculum is ordered. It may involve changes in the way the curriculum is taught. Almost certainly, it will involve changes in the way the curriculum is assessed. The last has been studied but only with younger children (Reid, 2002) and found to be possible to achieve.

One of the major issues focusses on the mathematics tetrahedron (Figure 3.1, page 53). In mathematics, there are four aspects to the learning process: mastering the procedures, being familiar with representations, understanding the concepts and being able to apply the understandings. The limitations of working memory make it more or less impossible to achieve in all four areas. The major research question is how to achieve in all four areas: is this to be done by some sequential presentation? If so, what sequence, at what rate or time gap, and how can it be presented?

The whole area of attitudes towards mathematics needs explored further. The fact that attitudes and performance correlate in some way does not necessarily imply cause and effect. The work of Reid and Jung (2009) in the sciences suggests that working memory limitations underpin the whole area. Is it possible that those with lower than average working memories find difficulty in understanding, resort to memorisation of procedures and then lose interest in mathematics as a result? This needs researched in relation to mathematics.

A final area of research relates to extent of field dependency. Is there any direct way by which teachers of mathematics can enhance this skill with learners and thus make mathematics more accessible to them?

10.6 Reflection on the Study

Mathematics is one of the most fundamental subjects in the curriculum at school level. However, it is important to understand the goals for learning mathematics and to develop specific learning objectives and precise assessment items to meet these goals. In this, it is essential to see mathematics globally. It is also important to understand the nature of the learners in terms of the ways by which they can understand mathematics.

The goal of meaningful learning is to help students to understand and not to memorise if positive attitudes are to be retained and applied. On the other hand, if the working memory is overloaded too often, the students may well turn to memorisation of procedures simply to pass examination. Again, the way a learner sees himself or herself in the learning process is a dimension which might have an important role in relation to confidence and understanding. Accordingly, offering learners positive experiences is a fundamental issue that teachers and other educators have to bear in mind during the learning process.

Like so many pieces of research work, this study, while answering its main questions, has raised numerous other questions. If it has identified specific understanding difficulties, provided a means for more effective mathematics education in the near future, as well as indicating relevant mathematical problems to be faced, then it has served its intended objective.

References

Aiken, (1961) Attitudes towards Mathematics: The Effect of Attitudes on Performance in Mathematics, *Journal of Research in Mathematics Education, **40**(4):551.*

Aiken, and Dreger (1961) Updates on Attitudes and other Affective Variable in Learning: Two Scale of Attitudes toward Mathematics, *Journal of Research in Mathematics Education, 5, 67-71.*

Ainscow, W. and Tweddle, D. (1988) *Encouraging Success*, London: David Fulton.

Ajzen, I. (1985) *From Intention to Action: A Theory of Planned Behaviour Action Control: From Cognition to Behaviour*, New York: Springer-Verlag.

Ajzen, I. and Fishbein, M. (1975) *Beliefs, Attitude, Intention and Behaviour: An Introduction to Theory and Research*, Reading, MA: Addison - Wesley.

Ajzen, I. and Fishbein, M. (1977) Attitude-Behaviour Relations; A Theoretical Analysis and Review of Empirical Research, *Psychological Bulletin*, 84, 888-918.

Ajzen, I. and Madden, T. J. (1985). Prediction of Goal Directed Behaviour: Attitudes, Intentions and Perceived Behavioural Control, *Journal of Experimental Social Psychology*, 22: 453-74.

Akhtar, R. (2008) *Effects of Language on Working Memory in Relation to Students' Mathematics Achievement*, MEd Thesis, University of Glasgow, Glasgow.

Al-Enezi, D. (2006) *Difficulties Associated with Teaching and Learning Mathematics: A Study of Psychological Factors Affecting Pupils' Performance*, PhD thesis, Glasgow, University of Glasgow.

Al-Naeme, F. F. (1988) *A Study of Some Psychological Factors Affecting Performance in Chemistry at Secondary and Tertiary Levels*, MSc thesis, Glasgow: Glasgow University.

Al-Naeme, F. F. (1991) *The Influence of Various Leaning Style on Practical Problem-Solving in Chemistry in Scottish Secondary Schools,* PhD thesis, Glasgow: University of Glasgow.

Al-Qasmi, S. A. (2006) *Problem Solving in Biology at University Level*, PhD thesis, Glasgow: University of Glasgow.

Alhmali, R. A. (2008) *Student Attitudes in the Context of the Curriculum in Libyan Education in Middle and High Schools,* PhD thesis, Glasgow: University of Glasgow.

Ali, A. A. (2008) *Perceptions, Difficulties and Working Memory Capacity Related to Mathematics Performance,* MSc thesis, Glasgow: University of Glasgow.

Allinson, J. and Hayes, C. (1996) The Cognitive Style Index: A Measure of Intuition-Analysis for Organisational Research, *Journal of Management Studies,* 33, 119-135.

Allport, G. W. (1935) *Attitudes*, In C.M. Murchison (Ed.) Handbook of Social Psychology Worchester, MA: Clark University Press.

Allport, G. W. (1961) *Pattern and Growth in Personality*, New York: Holt Rinehart Winston.

Anderson, J. R. (1995) *Learning and Memory: An Intelligent Approach*, New York: Wiley.

Ashcraft, H. M. (1994) *Human Memory and Cognition,* New York, Harper Collins College Publishers.

Atkinson, R. C. and Shiffrin, R. M. (1968) *Human Memory: A proposed system and its Control Processes*, In K. W. Spence and J. T. Spence (Eds.), The Psychology of Learning and Motivation, Volume 2, London: Academic Press.

Atkinson, R. C. and Shiffrin, R. M. (1971) The Control of Short-term Memory (1971), *Scientific American*, 224, 82-90.

Austin, M. (1971) Dream Recall and the Bias of Intellectual Ability, *Nature*, 231, 59.

Australian Education Council (1991) *A National Statement on Mathematics for Australian Schools*, Carlton, Victoria: Curriculum Corporation.

Ausubel, D. P. (1968) *Educational Psychology a Cognitive View*, New York, Holt, Rinehart and Winston.

Ausubel, D. P. (1978) *Educational Psychology,* New York: Holt, Rinehart and Winston.

Ausubel, D. P. and Robinson, F. G. (1969) *School Learning: An Introduction to Educational Psychology*, New York: Holt, Rinehart and Winston.

Ausubel, D. P., Novak., J. D. and Hanesian, H. (1978) *Educational Psychology: A Cognitive View*, (2nd Ed.), New York: Holt, Rinehart and Winston.

Baddeley, A. D and Hitch, G. (1977) *Recency Re-Examined,* in: Dornic, S. (Ed.) Attention and Performance, Volume 6, Hillsdale, NJ: Lawrence Erlbaum.

Baddeley, A. D. (2000) The Phonological Loop and the Irrelevant Speech Effect: Some Comments on Neath. Psychonomic Bulletin and Review, 7, 544-549.

Baddeley, A. D. and Hitch, G. (1974) *Working Memory*, in G. H. Bower (ed.), Recent Advances in Learning and Motivation, Volume 8, New York: Academic Press.

Baddeley, A. D., Thomson, N. and Buchanan, M. (1975) Word Length and the Structure of Short Term Memory, *Journal of Verbal Learning and Verbal Behaviour*, 14, 575-589.

Badgaish, M. (2008) *Attitudes and Extent of Divergency and Convergency related to Performance of Saudi Pupils in Mathematics,* MSc Thesis, University of Glasgow, Glasgow.

Bagozzi, R. P. and Burnkrant, R. E. (1979) Attitude Organisation and the Attitude-Behaviour Relationship, *Journal of Personality and Social Psychology*, 37, 913-929.

Bahar, M. (1999) *Investigations of Biology Students' Cognitive Structure Through Word Association Tests, Mind Maps and Structural Communication Grids*, PhD Thesis, University of Glasgow, Glasgow.

Ball, D. L. and Bass, H. (2003) *Making Mathematics Reasonable in Schools,* in: Kilpatrick, J., Martin, W.G. and Shifter, D. (Eds.), A Research Companion for Principles and Standards for School Mathematics (pp 27-44). Reston, VA: National Council of Teachers.

Ball, D. L., Lubienski, S. and Mewborn, D. (2001) *Research on Teaching Mathematics: The Unsolved Problem of Teachers' Mathematical Knowledge*, in: Richardson, V. (Ed.) Handbook of Research on Teaching. 4th Ed. (pp. 433-456). New York: McMillan.

Barber, P. (1988) *Applied Cognitive Psychology,* London, Methuen and Co. Ltd.

Barker-Lunn, J. C. (1970) *Streaming in the Primary School,* Slough: National Foundation for Educational Research.

Barmao, A. C. (2006) *The Effect of Streaming on Students' Performance in Mathematics in Secondary School Mathematics: A Case of Nakuru District, Kenya,* MEd Dissertation. Njoro: Egerton University.

Barwell, R. (2002) *Understanding EAL Issues in Mathematics*, in: Leung, C. (Ed.) Language and Additional/Second Language Issues for School Education (pp 69-80). Watford: NALDIC.

Bergum, B. O. (1977) Undergraduate Self-perceptions of Creativity and Independence. *Perceptual and Motor Skills*, 44, 187-190.

Blakemore, C. (1988) *The Mind Machine,* London: BBC Publication.

Bloom, B. S. (Ed.), Englehart, M. D., Frust, E. J., Hill, W. H and Krathwohl, D. R (1956) *Taxonomy of Educational Objectives: The Classification of Educational Goal,* Handbook 1: Cognitive Domain. New York: Longmans, Green and Co.

Bloomberg, M. (1971) Creativity as Related to Field-independence and Mobility, *Journal of Genetic Psychology*, 118, 3-12.

Bodner, G. M. (1991) I Have Found Your Argument: The Conceptual Knowledge of Beginning Chemistry Graduate Students, *Journal of Chemical Education*, 86(5), 384-388

Borasi, R. (1990) *The Invisible Hand Operating Mathematics Instruction: Students' Conception and Expectations,* in: Cooney, T. J. (Ed.) Teaching and Learning Mathematics in the 1990s (NCTM Yearbook) (pp 174-182). Reston: NCTM.

Borich, G. D (2004) *Effective Teaching Methods,* Pearson Education Inc.

Bosire, J., Mondoh, H. and Barmao (2008) Effect of Streaming by Gender on Students Achivement in Mathematics in Secondary Schools in Kenya. South African Journal of Education, 28, 595-607.

Bower, G. H. (1975) *Cognitive Psychology: An Introduction,* in: W. Estes (Ed.), *Handbook of Learning and Cognitive Processes*, Volume1, Hillsdale, Jew Jersey: Lawrence Erlbaum Associates Inc.

Bowler, P. J. (1992) *The Environmental Sciences,* London: Fontana.

Brainerd, C. J. and Reyna, V. F. (1988) Generic Resources, Reconstructive Processing and Children's Mental Arithmetic, *Developmental Psychology*, 24, 324-334.

Bransford, J. D., Stein, B. S., Arbitman-Smith, R. and Vye, N. J. (1985) *Three Approaches to Improving Thinking and Learning Skills*, in: R. Segal, S. Chipman and R. Glaser (Eds.), Thinking and Learning Skills: Relating Instruction to Basic Research (Volume1).

British Educational Research Association (2004) *Revised Ethical Guidelines for Educational Research*, Available at: http://www.bera.ac.uk/blog/category/publications/guidelines/

Broadbent, D. E. (1958) *Perception and Communication*, Oxford: Pergamon.

Brown, M., Brown, P. and Bibby, T. (2008) I would Rather Die - Reasons given by 16-Year-Olds for not Continuing their Study of Mathematics, *Research in Mathematics Education*, 10(1), 3-18.

Brown, R. and McNeill, D. (1966) The 'Tip of the Tongue' Phenomenon. Journal of Verbal Learning and Verbal Behaviour, 5, 325-337.

Brownell, W. A. (1945) When is Arithmetic Meaningful? J*ournal of Education Research*, 38, 481-498.

Brownell, W. A. (1947) The Place of Meaning in the Teaching of Arithmetic, *Elementary School Journal*, 47, 256-265.

Brumby, M. N. (1982) Consistent Differences in Cognitive Styles shown for Qualitative Biological Problem Solving, *British Journal of Educational Psychology,* 52, 244-257.

Bruner, J. S. (1966) *Toward the Theory of Instruction*, Cambridge, MA, Harvard University Press.

Bryden, M. P. (1979) *Evidence of Sex-related Differences in Cerebral Organisation*, in: Wilting, M. A. and Patteron, A. C. (Eds.) Sex Related Differences in Cognitive Functioning, New York: Academic Press.

Cakan, M. (2003) Psychometric Data on the Group Embedded Figure Tests for Turkish Undergraduate Students, *Perceptual and Motor Skills*, 96, 993-1004.

Carpenter, T. P., Fennema, E. Fuson, K., Hiebert, J, Human, P, Murray, H., Oliver, A. and Wearne, D. (1994) *Teaching Mathematics for Learning with Understanding in the Primary Grades*, in: Brunning, R., Schraw, G. and Ronning, R. (1995) Cognitive Psychology.

Case, R. (1974) Structures and Structures, Some Functional Limitations on the Course of Cognitive Growth, *Cognitive Psychology*, 6, 544-574.

Case, R. and Globerson, T. (1974) Field Independence and Control Computing Space, *Child Development* , 45, 772-778.

Cassells, J. R. and Johnstone, A. H. (1982) The Effect of Language on Students' Performance on Multiple Choice Tests in Chemistry, *Journal of Chemical Education*, 6 (7), 613-615.

Chaiken, S. H. and Eagly, A. (1993) *The Psychology of Attitudes*, Harcourt Brace College Publishers, New York.

Chaiken, S. and Stangor, C. (1987). Attitudes and Attitude Change, Annual Review of Psychology, 38: 575-630.

Chen, W. C. (2004) *An Analysis of Pupil Difficulties in Physics in Relation to Working Memory Space*, Glasgow: University of Glasgow.

Cheng, Y. Payne, J. and Witherspoon, S. (1995) *England and Wales Youth Cohort Study*, London: Department for Education and Employment.

Child Development, Quarterly Report (U.S. Department of Education, 2006).

Child, D. (1993) *Psychology and the Teacher*, Fifth Edition, London, Cassell.

Chin, S. J. and Ashcroft, J. R. (1998) *Mathematics for Dyslexics: A Teaching Handbook*, London: Whurr Publishing.

Choppin, B. and Frankel, R. (1976) The three Most Interesting things in Educational Evaluation, *Journal of Education*, 2, 57-61.

Christina, T., Lacey, P. Porter, J. and Christopher, R. (2000) *Pupils with LearningDifficulties in Mainstream Schools*, David Fulton Publishers Ltd.

Christou, K. (2001) *Difficulties in Solving Algebra Story Problems with Secondary Pupils,* Glasgow: Centre for Science Education, University of Glasgow.

Chu, Y. C. (2007) *Learning difficulties in Genetics and the Development of Related Attitudes in Taiwanese Junior high*, PhD Thesis, University of Glasgow, Glasgow.

Clarke, C. O. (1972) A Determination of Commonalities of Science Interests held by Intermediate Grade Children in Inner-city Suburban and Rural Schools, *Science Education*, 56, 125-136.

Claxton, C. S. and Ralston, Y. (1978) *Learning Styles: Their Impact on Teaching and Administration* (Higher Education Report), Washington D C: American Association for Higher Education.

Clements, D. H. and Battista, M. T. (1992b) *Geometry and Spatial Reasoning*, in: Grouws, D. A. (Ed.) Handbook of Research on Mathematics Teaching and Learning (pp 420-464), New York: MacMillan.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z. and Sarama, J. (1999) Young Children's Concept of Shape, *Journal for Research in Mathematics Education*, 30(2), 192-212.

Coan, R. (1974) *The Optimal Personality: An Empirical and Theoretical Analysis*, New York: Columbia University Press.

Cockburn, A. and Haylock, D. (1997) *Understanding of Mathematics in the Lower Primary Years*, London: Paul Chapman Publishing Ltd.

Cockcroft Report (1981) *Mathematics Counts*, HMSO.

Cohen, L. and Manion, L. (1994) *Research Methods in Education*, London, Routledge.

Cohen, L. M. and Morrison, K. (2007) *Research Methods in Education*, London and New York, Routledge: Taylor and Francis Group.

Cohen, L., Manion, L. and Morrison, K. (2002, 5th Edition) Research Methods in Education, London, Routledge Falmer.

Cole, M., Gray, J., Glick, J and Sharp, D. (1971) *The Cultural Context of Learning and Thinking*, New York: Basic Books.

Colley, A. Comber, C. and Hargreaves, D. J. (1994) School Subject Preferences of Pupils in Single-sex and Co-educational Secondary Schools, *Educational Studies,* 20, 379-385.

Confrey J. (1994) A Theory of Intellectual Development for the Learning of Mathematics (Part 1), *Journal of Mathematics Education* 14(3), 2-8.

Cook, S. W. and Selltiz, C. (1964) *Physiological Bulletin*, 62, 36-55.

Copeland, R. W. (1984) *How Children Learn Mathematics: Teaching Implication of Piaget's Research*, Collier Macmillan Canada, Inc.

Costello, J. (1991) *Teaching and Learning Mathematics*, (11-16 years), New York: Routledge.

Covington, M., Crutchfield, R, Davis, L. and Otton, R. (1974) *The Productive Thinking Programme: A Course in Learning to Think*. Columbus: Merril.

Craik, F. I. and Lockhart, R. (1972) Levels of Processing, *Journal of Verbal Learning and Verbal Behaviour*, 11, 671-684.

Craik, F.I.M. and Watkins, M. J. (1973) The Role of Rehearsal in Short-Term Memory, *Journal of Verbal Learning and Verbal Behaviour*, 12, 599-607.

Crawley, F. (1990) Intentions of Science Teachers to use Investigative Teaching Methods: A Test of the Theory of Planned Behaviour, *Journal of Research in Science Teaching*, Volume 27, Number 7, Pages 685-697.

Crawley, F. and Black, C. (1992) Causal Modelling of Secondary Science Students' Intentions to Enrol in Physics, *Journal of Research in Science Teaching*, 29(6), 585-599.

Cretchley, P., Fuller, A. and McDonald, C. (2000) *Influences on the Choice to Study Mathematics at University*, Proceedings of MERGA 23, Freemantle: MERGA Inc.

Crutchfield, R. S. (1965) *Creativity Thinking in Children: Its Teaching and Testing*, in: Brun, O. J., Crutchfield, R. S., Holzman, W. H. (Eds.) Intelligence Perspectives 1965: The Terman-Otis Memorial Lectures, New York: Harcourt, Brace and World.

Danili, E. (2001) *New Teaching Materials for Secondary School Chemistry: A Study of Psychological factors Affecting Pupil Performance,* MSc Thesis, Glasgow, University of Glasgow.

Danili, E. and Reid, N. (2004) Some Strategies to Improve Performance in School Chemistry Based on two Cognitive Factors, *Research in Science and Technological Education,* **22**(2), 203-226.

Danili, E. and Reid, N. (2005) Assessment Formats: Do they make a Difference? *Chemistry Education Research and Practice,* **6**(4), 198-206.

Darwin, C. J., Turvey, M. T. and Crowder, R. G. (1972) An Auditory Analogue of the Sperling Partial Report Procedure: Evidence for Brief Auditory Storage, *Cognitive Psychology*, **3**, 255-267.

De Bono, E. (1976) *Teaching Thinking*, London: Temple Smith.

De Corte, E. Verschaffel, L. and van de Ven, A. (2001) Improving Text Comprehension Strategies in Upper Primary School Children: A Design Experiment, *British Journal of Educational Psychology*, **71**, 531-559.

Donaldson, M. C. (1987) *Children's Minds*, London: Fontana Press.

Donovan, M. S. and Bransford, J. D. (2005) *How Students Learn Mathematics in the Classroom*, Washington, D C: The National Academic Press.

Driver, R. Guesne, E. and Tiberghien, A. (1985) *Children's Ideas in Science*, Milton Keynes: Open University Press.

Duffin, J. and Simpson, T. (2000) Understanding their Thinking: The Tension Between the Cognitive and the Affective, in: Diana Coben (2000), *Perspectives on Adult Learning Mathematics*, 83-99.

Durkin, K. (1991) *Language in Mathematics Education, an Introduction*, in: Durkin, K. and Shire, B. (1992) Language in Mathematical Education, Buckingham: Open University Press.

Durrani, M. (1998) Standards in Mathematics and Chemistry Exams in England and Wales, *Journal of Education Psychology*, **64**, 419-428.

Dwyer and Moore (1995) *Effect of field-dependent/independent on Achievement*, New York.

Ebbinghaus, H. (1885) *On Memory*, Leipzig: Duncken.

Eisenhardt. W. B. (1977) A Search for Predominant Casual Sequence in the Interrelationship of Interest in Academic Subjects and Academic Achievement, A Cross-lagged Panel Correlation Study, *Dissertation Abstracts International*, **37,** 4225A.

El-Banna, H. (1987) *The Development of a Predictive Theory of Science Education Based upon Information Processing Theory*, PhD Thesis, Glasgow: University of Glasgow.

El-Sawaf, M. M. F. (2007) *Education beliefs Development with Pre- and In-Service Teachers, using Perry's Model*, PhD Thesis, University of Glasgow, Glasgow.

Elliot, J. G. (2000) The Psychological Assessment of Children with Learning Difficulties, *British Journal of Special Education*, **27**(2), 59-66.

Entwistle, N. J. (1981) *Styles of Teaching and Learning: An Integrated Outline of Educational Psychology for Students, Teachers and Lecturers*, Chichester: Wiley.

Eysenck, M. (1998) *Psychology an Integrated Approach*, New York: Addison Wesley Longman Inc., New York.

Fazio, R. H. (1986) *How do Attitudes Guide Behaviour?* in: R. M. Sorrentino and E. T. Higgins (Eds.) Handbook of Motivation and Cognition: Foundations of Social Behaviour, New York: Guilford.

Fazio, R. H. (1990) *Multiple Processes by Which Attitudes Guide Behaviour: The MODE model as an Integrative Framework*, in: M. P. Zanna (Ed.) Advances in Experimental Social Psychology (Volume 23), San Diego, CA: Academic Press.

Fennema, E. and Leder, G. (1990) *Mathematics and Gender, Influence on Teachers and Students*, New York: Teachers College Press.

Festinger, L. (1957) *Theory of Cognitive Dissonance*, Stanford, CA: Stanford University Press.

Fishbein, M. and Ajzen, I. (1975) *Beliefs, Attitude, Intention and Behaviour: An Introduction to theory and Research*, Reading, MA: Addison-Wesley.

Forns-Santacana, M., Amador-Campos, J. A. and Roig-Lopez, F. (1993) Differences in Field dependence/independence Cognitive Styles as a Function of Socio-Economic Status, Sex and Cognitive Competence, *Psychology in the Schools*, **30**, 176-186.

Forum for African Women Educationalists (FAWE) (1997) *Girls' Education: An Agenda for Change*, Nairobi.

Fowler, W. (1980) Cognitive Differentiation and Developmental Learning, in: Rees, H. and Lipsitt, L. (Eds.) *Advances in Child Development and Behaviour*, **15**, 163-206. New York: Academic Press.

Fraser, B. (1982), How Strongly are Attitude and Achievement Related? *School Science Review*, **63**, 557-559.

Frederickson, N. and Cline, T. (2009) *Special Educational Needs, Inclusion and Diversity*: A Textbook (2nd Ed.) Buckingham: Open University Press.

Frey, D. and Gaska, A. (1993) The Theory of Cognitive Dissonance, (2), 275-326.

Fuchs, L.S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R. and Shroeter, K. (2003), Enhancing Third-Grade Students Mathematical Problem Solving with Self-Regulated Learning Strategies, *Journal of Educational Psychology*, **95**, 306-315.

Gagné, R. M. (1965) *Conditions of Learning*, First Edition, New York: Holt, Rinehart and Winston.

Gagné, R. M. (1974) *Principles of Instructional Design*, USA: Holt, Rinehart and Winston.

Gagné, R. M. (1985) Conditions of Learning, Second Edition, New York: Holt, Rinehart and Winston.

Gagnon, G. and Colay, M. (2001) *Designing for Learning: Six Elements in Constructivist Classrooms*, Thousand Oaks, CA: Corwin Press.

Galton, F. (1883) *Inquiries into Human Faculty and its Development*, London: MacMillan.

Gardner, H. (1983) *Frames of Mind: The Theory of Multiple Inteligences*, New York: Basic Books.

Gardner, H. (1998) A Multiplicity of Intelligences, Scientific American Presents: Exploring Intelligence, **9**(4), 18-23.

Gardner, P. L. (1975) Attitude to Science: A Review Studies in Science Education, **2**, 1-41.

Gardner, P. L. (1996) The Dimensionality of Attitude Scales: A Widely Misunderstood Idea, *International Journal of Science Education*, **18**,(8), 913-919.

Gardner, R. (1953) Cognitive Style in Categorising Behaviour, *Journal of Perceptual and Motor Skills*, **22**, 214-233.

Gardner, R. W., Holzman, P., Klein, G., Linton, H. and Spence, D. (1959) *Cognitive Control: A Study of Individual Differences in Cognitive Behaviour*, Psychological Issue 1, Monograph 4.

Garrett, R. M. (1989) Promoting Creativity Through a Problem-solving Science Curriculum, *School Science Review*, **70**(252), 127.

Germann, P. J. (1988) Development of the Attitude towards Science in School Assessment and its Use to Investigate the Relationship Between Science Achievement and Attitude toward Science in School, *Journal of Research in Science Teaching*, **25**(8), 689-703.

Getzels, J. W. and Jackson, P. W. (1962) *Creativity and Intelligence*, New York: Wiley.

Ghani, S. A. (2004) *Attitudes, Performance, and Psychological Characteristics of Student Teachers Learning Introductory Statistics*, PhD Thesis, Glasgow, University of Glasgow.

Goos, M., Brown, R., and Makar, K. (2008) *Advancing Research into Affective Factors in Mathematics Learning: Clarifying Key Factors, Terminology and Measurement*, Proceeding of the Mathematics Education Research Group of Australia: MERGA (Inc.).

Gray, C. (1997) *A Study of Factors Affecting Curriculum Innovation in University*, PhD Thesis, Glasgow, University of Glasgow.

Gregorc, A. R. (1982) *Style Delineator*, Maynard, MA: Gabriel Systems.

Sternberg, R. J. and Grigorenko, E. L. (1995) *Thinking Styles*, in: Saklofske, D. H. and Zeidner, M. (Eds.) Internation Handbook of Personality of Intelligence, 205-230. New York: Plenum Press.

Gross, R. (2005) *Psychology The Science of Mind and Behaviour* (Fifth Edition), London: British Library Cataloguing in Publication Data.

Grous, D. A. (Ed.) (1992) *Handbook of Research on Mathematics Teaching and Learning*, New York: MacMillan.

Guilford, J. P. (1967) *The Nature of Human Intelligence*, New York: McGraw-Hill.

Gurley, M. P. (1984) *Characteristics of Motivation, Field-independence, Personality Type, Learning Style and Teaching Preference of the Adult Learner as Compared with Traditional Age College Students*. (Unpublished Doctoral Dissertation, Catholic University of America), *Dissertation Abstracts International*, 145, 1268.

Hadden, R. A and Jonhstone, A. H. (1982) Primary School Pupils' Attitude to Science: The Year of Formation, *European Journal of Science Education*, **4**(4), 397-407.

Haddon, F. A and Lytton, H. (1968) Teaching Approach in the Development of Divergent Thinking Abilities in Primary School. British Journal of Educational Psychology, 38, 171-180.

Hamachek, D. (1995) *Psychology in Teaching, learning and Growth*, New York, USA, Allyn and Bacon.

Hamblin, D. H. (1981) *Teaching Study Skills*, Oxford: Basil Balckwell.

Hannula, M. (2002) Attitudes Towards Mathematics: Emotions, Expectations and Values, *Educational Studies in Mathematics*, 49, 25-46. Netherlands: Kluwer Academic Publishers.

Harvey, O. J., Hunt, D. E. and Schroder, H. M. (1961) *Conceptual Systems and Personality Organisation*, New York: Wiley.

Hayes, J. and Allinson, C. W. (1996) The Implications of Learning Styles for Training and Development: A Discussion of the Matching Hypothesis, *British Journal of Management*, **5**, 53-71.

Haylock, D. (1991) *Teaching Mathematics to Low Attainers*, (8-12), London: Paul Chapman Publishing Ltd.

Haylock, D. (2001) *Mathematics Explained for Primary Teachers* (2nd Ed.), London: Paul Chapman Publishing.

Haylock, D. (2006) *Mathematics Explained for Primary Teachers*, (Third Edition), SAGE Publishing Ltd.

Hebb, D. O. (1949) *The Organisation of Behaviour*, New York: Wiley.

Heider, F. (1946) Attitudes and Cognitive Organisation, *Journal of Psychology*, 21: 107-12.

Heider, F. (1958) *The Psychology of Interpersonal Relations*, New York: Wiley.

Herron, J. D. (1975) Piaget for Chemists, Explaining What Good Students Cannot Understand, *Journal of Chemical Education*, **52**(3), 146-150.

Hiebert, J. and Carpenter, T. P. (1992) *Learning and Teaching with Understanding*, in: Grous, D. A. (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp 65-97), New York: McMillan.

Hindal, H. (2007) *Cognitive Characteristics of Students in Middle Schools in State of Kuwait, with Emphasis on High Achievement,* PhD Thesis, Glasgow, University of Glasgow.

Hindal, H. and Reid, M. (2009) Working Memory, Performance and Learning Characteristics, *Research in Science and Technology Education*, **27**(2), 187-204.

Hitch, G. J. (1978) The Role of Short-term Working Memory in Mental Arithmetic. *Cognitive Psychology*,10, 302-323.

Hogg, M. A. and Vaughan, G. M. (1995) *Social Psychology: An Introduction*, Hemel Hempsteed: Prentice Hall/Harvester Wheatsheaf.

Hovland, C. I. and Janis, I. L. (1959) *Personality and Persuasibility*, New Haven, CT: Yale University Press.

Hovland, C. I., Janis, I. L. and Kelly, J. J. (1953) *Communication and Persuasion*, New Haven: Yale University Press.

Hudson, L. (1966) *Contrary Imaginations,* London: Methuen.

Hudson, L. (1968) *Frames of Mind*, Harmondsworth: Penguin Books.

Humans Sciences Research Council (2008), Business Day, 14 July 2008 (Linda Chisholm and Martin Carnoy), DST/SAMF/SAMS Workshop 31 July 2008.

Hussein, F. and Reid, N. (2009) Working Memory and Difficulties in School Chemistry, *Research in Science and Technological Education,* **27**(2), 161-186.

James, W. (1890), *The Principles of Psychology*, New York: Henry Holt and Company.

Jenkins, J. J. (1974) Remember That Old Theory of Memory? Well, Forget It! *American Psychologist,* 29, 785-795.

Johnstone, A. H. (1984) New Stars for the Teacher to Steer by? *Journal of Chemical Education*, **61**(10), 847-9.

Johnstone, A. H. (1991) Why is Science Difficult to Learn? Things are Seldom What they seem, *Journal of Computer Assisted Learning,* 7, 75-83.

Johnstone, A. H. (1993) The Development of Chemistry Teaching - A Changing Response to Changing Demand, *Journal of Chemical Education*, **70**(9), 701-705.

Johnstone, A. H. (1997) Chemistry Teaching, Science or Alchemy? *Journal of Chemistry Education*, **74**(3), 262-268.

Johnstone, A. H. (2000) Teaching Chemistry-Logical or Psychological? *Chemistry Educational Research and Practice in Europe*, **1**(1), 9-15.

Johnstone, A. H. and El-Banna, H. (1986) Capacities, Demands and Processes: a Predictive Model for Science Education, *Education in Chemistry*, **23**(3), 1986, 80-84.

Johnstone, A. H. and El-Banna, H. (1989) Understanding Learning Difficulties - A Predictive Research Model, *Studies in Higher Education*, **14**(2), 159-68.

Johnstone, A. H. and Kellett, N. C. (1980) Leaning Difficulties in School Science - Towards a Working Hypothesis, *European Journal of Science Education*, **2**(2), 175-181.

Johnstone, A. H. and Reid, N. (1981) Toward a Model of Attitude Change, *European Journal of Science Education*, **3**(2), 205-212.

Johnstone, A. H. and Selepeng, D. (2001), A Language Problem Revisited, *Chemistry Education Research and Practice in Europe*, **2**(1), 19-29.

Johnstone, A. H., and Wham, A. J. B. (1982) Demands of Practical Work, *Education in Chemistry* **19** (3), 71-73.

Johnstone, A. H., Sleet, R. T. and Vianna, J. F. (1994) An Information Processing Model of Learning: Its Application to an Undergraduate Laboratory Course in Chemistry *Studies in Higher Education*, 99, 77-87.

Johnstone, A. H., Watt, A. and Zaman, T. U. (1998) The Students' Attitude and Cognition Change to a Physics Laboratory. *Physics Education*, **1**(1), 9-15.

Jonassen, D. H. and Grabowski, B. L. (1993) *Handbook of Individual Differences, Learning and Instruction*, Hillsdale, NJ: Lawrence Erlbaum.

Judd, C. H. (1908) The Relation of Special Training to General Intelligence, *Educational Reviews*: 36, 28-42.

Jung, E. S. (2005) *Attitudes and Learning Difficulties in Middle School Science in South Korea*, MSc Thesis, Glasgow, University of Glasgow.

Jung, E. S. and Reid, N. (2009), Working Memory and Attitudes, *Research in Science and Technological Education*, **27**(2), 205-224.

Kagan, J. (1966) *Developmental Studies in Reflection and Analysis*, in: Kidd, A. H. and Rivoire, J. L. (Eds.) Perceptual Development in Children. New York: International University Press.

Kagan, J., Rosman, B., Day, D., Albert, J. and Philips, W. (1964) Information Processing and the Child: Significance of Analytic and Reflective Attitudes, *Psychological Monographs*, 78, 578.

Kato, Y., Kamii, C., Ozaki, K. and Nagahiro, M. (2002) Young Children's Representations of Groups of Objects: The Relationship Between Abstraction and Representation, *Journal of Research in Maths Education*, **33**(1), 30-45.

Katz, D. and Sarnoff, I. (1954) The Motivational Basis of Attitude Change, Journal of Abnormal and Social Psychology, 49, 115-124.

Katz, D. and Stotland, E. (1959) in: Koch, S. (Ed.) Psychology: A Study of a Science, 3, 423-475.

Kaufmann, G. (1989) *The Assimilator-Explorer Inventory*, in: Martinsen, O. Cognitive Style and Insight, PhD Thesis, Faculty of Psychology, University of Bergen, Norway.

Khan, S. B. and Weiss, J. (1973) (Second Edition) *Handbook of Research in Teaching*, (Ed.), Traver, Chicago, Rand McNally.

Kilpatirck, J. (1992) *A History of Research in Mathematics Education*, in: Grouws, D. A. (Ed.) Handbook of Research on Mathematics Teaching and Learning (pp 3-38), New York McMillan.

Kintsch, W. and Greeno. J. G. (1985) Understanding and Solving Arithmetic Word Problems. *Psychological Review*, 92, 109-129.

Kirschner, P. A., Sweller, J. and Clark, R. E. (2006) Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-based, Experiential, and Inquiry-based Teaching, *Educationalist Psychologist*, **42**(2), 115 – 212.

Kirton, J. W. (Ed.) (1994) Adaptors and Innovators: A Description and Measure, *Journal of Applied Psychology*, 61, 622-9.

Kirton, M. J. (1976) *Kirton Adaption-Innovation Inventory (KAI) Manual*, (2nd Ed.) Hatfield, UK: Occupational Research Centre.

Klein, G. A. (1954) *Need and Regulation*, in: Jone, M.P. (Ed.) Nebraska Symposium on Motivation, Lincoln, NB: University of Nebraska Press.

Korchin, S. J. (1986) *Field-Dependence, Personality Theory and Clinical Research*, in: Bertini, M., Pizzamiglio, L. and Wapner, S. (Ed.), Field-dependence in Psychological Theory, Research, and Application, New York: Cambridge University Press.

Kyriacou, C. and Goulding, M. (2006) *A Systematic review of Strategies to raise Pupils' Motivation Effort in Key Stage 4 Mathematics*, EPPI Centre, Institute of Education, London.

Lampert, M. (1990) When the Problem is Not the Question and the Solution is Not the Answer: Mathematical Knowing and Teaching, *American Educational Research Journal*, **27**(1), 29-63.

Langford, T. (1989) *The Process of Learning*. In: Langfor, P., Educational Psychology: An Australian Perspective (pp 43-64), Melbourne: Longman Cheshire.

Laswell, H. D. (1948) *The Structures and Function of Communication in Society*, in: L. Bryson (Ed.), Communication of Ideas, New York : Harper.

Leder, G. and Grootenboer, P. (2005) Affect and Mathematics Education. *Mathematics Education Research Journal*, **17**(2), 1-8.

Leder, G. C. (1985) Measurement of Attitudes for the Learning of Mathematics, **5**, 18-21, 34.

Letteri, C. A. (1980) Cognitive Profile: Basic Determinant of Academic Achievement, *Journal of Educational Research*, 73, 195-9.

Lewis, B. N. (1976) *Avoidance of Aptitude-treatment Trivialities*, in: Messick, S. (Ed.) Individuality in Learning, 301-308. San Francisco, CA: Jossey-Bass.

Liebeck, P. (1990) *How Children Learn Mathematics - A Guide for Parents and Teachers*, USA: Pelican Books.

Likert, R. (1932) A Technique for the Measurement of Attitudes, *Archives of Psychology*, **22**, 140: 5-53.

Lockhart, R. S (1989) *Consciousness and the Function of Remembered Episodes*, in: H. L. Roediger and F. I. M Craik (Eds.), Varieties of Memory and Consciousness (pages 423-430) Hillsdale, NJ: Lawrence Erlbaum Associates.

Lumb, D. and Papendick, A. (1978) *Mathematics: A Source book of Guidance, Information and Ideas for Teachers,* London: John Murray Publishers Ltd.

Xin M. and Kishore, N. (1997) Assessing the Relationship between attitude towards Mathematics and Achievement in Mathematics: A Meta Analysis, *Journal for Research in Mathematics Education,* **28**(1), 26-47.

Mager, R. F. (1962) *Preparing Instructional Objectives,* Belmont, CA: Fearon.

Mahdi, M. A. (1995) Primary School Maths and Logical Inferences. Glasgow: Centre for Science Education, University of Glasgow.

Malacinski, G. (1994) Impediments to Developing Collaborative Learning Strategies; The Input vs Output Conflict, *Journal of Science Education and Technology*, **3**(2), 107-114.

Mallam, W. A. (1993) Impact on School-type and Sex of the Teacher on Female Students' Attitudes Towards Mathematics in Nigerian Secondary Schools. *Educational Studies in Mathematics*, 24, 223-229.

Matthews, A. and Pepper, D. (2005) *Evaluation of Participation in A level Mathematics*: Interim Report Qualifications and Curriculum Agency, London.

Mc Leod, D. B. (1994) Research on Affect and Mathematics Learning in the JRME: 1970 to the present, *Journal for Research in Mathematics Education*, 24, 637-647.

McFadden, E. A. (1986) *Clinical Decision Making and its Relationship to Learning Style and Personality Type*, (Unpublished Doctoral Dissertation) University of Maryland.

McGuffin, S. J. (1973) What Northern Ireland Pupils Think of SCISP, *Education in Science,* November, 24-27.

McGuire, W. J. (1969) *The Nature of Attitudes and Attitude Change*, in: Lindzey and E. Aronson (Eds.), Handbook of Social Psychology (2nd ed., Vol 3, pp 136-314), Reading, MA: Addison - Wesley.

McKnight, I. V. S. (1987) *The Underachieving Curriculum,* Champaign, The Champaign: Stipes.

Meece, J. L. (1981) *Individual Differences in the Cognitive Reactions of Students to Mathematics*, Doctoral Dissertation, University of Michigan.

Merriam, S.B. and Caffarella, R. S. 1991 *Learning in Adulthood*, San Francisco, CA: Jossey-Bass.

Messick, S. (1996) *Cognitive Styles and Learning*, in: De Corte, E. and Weinert, F. E. (Eds.) International Encyclopaedia of Developmental Psychology (pp 638-641). London: Pergamon.

Meyer, B. J. F., Young, C. J. and Bartlett, B.J. (1989) *Memory Improved: Reading and Memory Enhancement Across the Life Span through Strategic Text Structures*, Hillsdale, NJ: Erlbaum.

Miller, A. (1987) Personality Types, Learning Styles and Educational Goals. Educational Psychology, 11, 217-238.

Miller, G. D. (1956) The Magical Number Seven Plus or Minus Two: Some Limits on Our Capacity for Processing Information, *Psychological Reviews*, 63, 81-97.

Miller, P. H. (1993) *Theories of Developmental Psychology*, New York, W.H. Freeman and Company.

Mondoh, H. O. (2001) *Cognitive Differences in a Mathematics Class*. Paper presented during a workshop on in-servicing science and mathematics teachers of Nakuru District, April 9-12, Nakuru High School, Kenya.

Morris, T. L. and Bergum, B. O. (1978) A note on the Relationship Between Field-independence and Creativity, *Perceptual and Motor Skills*, **46**(1), 114.

Moseley, D., Baumfield, V., Elliot, J., Gregson, M., Higgins, S., Miller, J., and Newton, D. P. (2005) *Framework for Thinking: A Handbook for Teaching and Learning*, Cambridge University Press.

Muijs, D. (2004) *Doing Quantitative Research in Education with SPSS,* London, Sage.

Murphy, P. (1990) *Gender Gap in the National Curriculum*, Physics World, 3, 1, 11.

Musser, T. (1998) *Individual Differences, How Field-Dependence-Independence Affect Learners* (From htt: //www.personal.psu.edu/staff/t/x/txm4/paper1.htm1)

Myer, I. (1978) *Myers-Briggs Type Indicator*, Palo Alto, CA: Consulting Psychologists Press.

National Centre for Research in Mathematics Sciences Education (1994) Statistics and School Mathematics. NCRMSE Research Review, ***3***(2), 1-5.

National Institutes of Child Health and Human Development (2006, USA), *Journal of Applied Developmental Science, Child Development*, Quarterly Research on Early Childhood.

National Policy on Education (1978-79:21), Nigeria, in Uche, U.W. (1986), *Sociology of Nigeria Education for Universities and Colleges,* New Africa Publishing Co. Ltd.

National Training Laboratories (2005) Bethel, Maine, USA, in National Research Council of the National Academies (www.nap.edu). National Academy of Sciences.

Novak, D. J. (1978) An Alternative to Piagetian Psychology for Science and Mathematics Education, *Studies in Science Education*, 5, 1-30.

Novak, J. (1998) *Learning, Creating and Using Knowledge - Concept Map as Facilitative Tools in Schools and in Corporations*, London: Lawrence Erlbaum.

Novak, J. (2002) Meaningful Learning: The Essential Factor for Conceptual Change in Limited or Inappropriate Prepositional Hierarchies Leading to Empowerment of Learners, *Science Education*, **86**(4), 548-571.

Nwosu, E. N. and Omeje, J. C. (2008) Gender Differences in Communication Patterns of Females in Single Sex and Mixed Sex Schools in Nnewi Education Zone, *African Research Review*, 2, 103-115.

Ofsted (Office for Standards in Education) (1994) *Curriculum Organisation and Classroom Practice in Schools*: A Follow-up Report, London: HMSO.

Oppenheim, A. N. (1992) Questionnaire Design, Interviewing and Attitude Measurement, Pinter Publishers London and New York.

Oraif, F. A. (2006) *An Exploration of Confidence Related to Formal Learning in Saudi Arabia*, PhD Thesis, Glasgow, University of Glasgow.

Orton, A. and Wain, G. (1994) *Issues in Teaching Mathematics*, London: Cassell.

Osgood, C. E., Suci, P. H. and Percy, H. T. (1957) *The Measurement of Meaning*, Urbana, USA: University of Illinois Press.

Oskamps, S. (1991) *Attitudes and Opinions* (Ed.), Prentice-Hall, Inc.

Paivio, A. (1971) Style and Strategies of Learning, *British Journal of Educational Psychology*, 46, 128-148.

Palincsar, A. S and Brown, A. L (1984) Reciprocal Teaching of Comprehension-Fostering and Comprehension-Monitoring Activities, *Cognition and Instruction*, 1, 117-175.

Pascual-Leone, J. (1970) A Mathematical Model for the Transition Rule in Piaget's Developmental Stages, *Acta Psychologica*, 32: 301-345.

Pask, G. (1976) Styles and Strategies of Learning, *British Journal of Educational Psychology*. **46**(2), 128-148.

Pask, G. and Scott, B. C. E. (1972) Learning Strategies and Individual Competence, *International Journal of Man-Machine Studies*, 4, 217-253.

Perkins, D. N. (1985) *General Cognitive Skills: What Not?* in: Chipman, S.F., Segal, J.W., Glaser, R. (Eds.) Thinking and Learning Skills (pp. 339-364), Hillsdale, NJ: Lawrence Erlbaum.

Perry, W. G. (1999) *Forms of Ethical and Intellectual Development in the College Years*: *A Scheme,* San Francisco, CA: Jossey-Bass Publishers.

Petty, G. (2004) *A Practical Guide: Teaching Today (3rd Edition)*, Cheltenham: Nelson Thoms Ltd.

Petty, R. E. and Cacioppo, J. T. (1981) *Attitudes and Persuasion; Classic and Contemporary Approaches,* Dubuque, Iowa: Brown.

Piaget, J. (1963) *The Child's Conception of the World*, in: Peterson, N. J.: Little Field, Adams.

Piaget, J. (1971) *Science of Education and the Psychology of the Child*, New York: The Viking Press, Inc.

Piaget, J. (1973) *To Understand is to Invent*, New York: Grossman.

Piburn, M. and Baker, D. (1993). If I were the Teacher ... Qualitative Study of Attitude Toward Science, *Science Education*, **77** (4), 393-406.

Pizzamiglio, L. (1974) Handedness, Preferences and Field-Dependence, *Perceptual and Motor Skills*, 38, 700-702

Poffenburger, T. and Norton, D. A. (1959) Factors in the Formation of Attitudes Towards Mathematics, *Journal of Education Research*, 52, 171-176.

Ponte, J. P., Matos, J. F. Guimaraes, H. M., Leal, L. C, and Canavarro, A. P. (1991). *Students' Views and Attitudes Towards Mathematics Teaching and Learning: A Case of a Curriculum Experience*.

Prosser, P. and Trigwell, K. (1998) *Improving Students' Learning Through a Focus on the Teaching Context*, in: Gibbs, G. (Ed.) Improving Student Learning, Oxford: Oxford Centre for Staff Development.

Ramsden, J. M. (1998) Mission Impossible?: Can Anything Be Done About Attitudes to Science? *International Journal of Science Education*, **20**(2) 125-137.

Ramsden, P. (1988) *Context and Strategy: Situational Influences on Learning*, in: Schmeck, R. R (Ed.) Learning Strategies and Learning Styles, 159-84, New York: Plenum Press.

Ramsey, G. A. and Howe, R. W. (1969), *Science Teacher*, 36, 62-70.

Riding, R. J. and Rayner, S. (1997) Toward a Categorisation of Cognitive Styles and Learning Styles, *Educational Psychology*, 17, 5-28.

Reid, N. (1978) *Attitude Development Through a Science Curriculum*, PhD Thesis, Glasgow, University of Glasgow.

Reid, N. (1999) Towards an Application-led Curriculum, *Staff and Educational Development International,* **3**(1), 71-84.

Reid, N. (2000) The Presentation of Chemistry: Logically Driven or Application Led? *Chemistry Education Research and Practice in Europe*, **1**(3), 381-392.

Reid, N. (2003) *Getting Started in Pedagogical Research in the Physical Sciences*, LTSN Physical Sciences Practice Guide, Hull, LTSN.

Reid, N. (2006) Thoughts on Attitude Measurement, *Research in Science and Technological Education,* **24**(1) 3-27.

Reid, N. (2008) A Scientific Approach to the Teaching of Chemistry, (The Royal Society of Chemistry Nyholm Lecture, 2007-8), *Chemistry Education Research and Practice*, **9**(1), 51-19.

Reid, N. (2009a) The Concept of Working Memory, *Research in Science and Technological Education,* **27**(2), 131-138.

Reid, N. (2009b) Working Memory and Science Education, *Research in Science and Technological Education,* **27**(2), 245-250.

Reid, N. and Skryabina, E (2002a) Attitudes Towards Physics, *Research in Science and Technological Educatio*n, **20**(1), 67-81.

Reid, N. and Skryabina, E (2002b) Gender and Physics, *International Journal Science Education,* **25**(4), 509-536..

Reid, N. and Yang, M. J. (2002b) Problem Solving in Chemistry: The More Open Ended Problem, *Research in Science and Technological Education*, **20**(1), 83-98.

Reid, P. A. (2002) *Problem Solving by Primary Children with Particular Reference to Dyslexics,* MSc Thesis, Glasgow, University of Glasgow.

Reinert, H. (1976) One Picture is Worth a Thousand Words? Not Necessarily! The Modern Language Journal, 60, 160-8.

Reitman, J. S. (1974) Without Surreptitious Rehearsal, Information in Short-Term Memory Decays, *Journal of Verbal Learning and Verbal Behaviour*, 13: 365-77.

Rhine, R. J. (1958) A Concept Formation Approach to Attitude Acquisition, *Psychology Review*, 65, 362-370.

Richardson, A. (1977) Verbalizer-Visualiser: A Cognitive Dimension, J*ournal of Mental Imagery*, 1, 109-125.

Riding, R. and Rayner, S. (1999) *Cognitive Styles and Learning Strategies*, David Fulton Publishers Ltd.

Riding, R. J. (1991) *Cognitive Styles Analysis*, Birmingham: Learning and Training Technology.

Riding, R. J. (1994) *Personal Style Awareness and Personal Development*, Birmingham: Learning and Training Technology.

Riding, R. J. (1997) On the Nature of Cognitive Style, *Educational Psychology*, 17, 29-50.

Riding, R. J. and Buckle, C. F. (1990) *Learning Styles and Training Performance*, Sheffield: The Training Agency.

Riding, R. J. and Calvey, I. I. (1981) The Assessment of Verbal-Imagery Learning Styles and Their Effect on the Recall of Concrete and Abstract Prose Passages by Eleven Year Old Children, *British Journal of Psychology*, 72, 59-64.

Riding, R. J. and Cheema, I. (1991) Cognitive Styles: An Overview and Integration, *Educational Psychology*, 11, 193-215.

Riding, R. J. and Rayner, S. (1995) *Personal Style and Effective Teaching*, Birmingham: Learning and Training Technology.

Riding, R. J. and Taylor, E. M. (1976) Imagery Performance and Prose Comprehension in Seven Year old Children. *Educational Studies*, 2, 21-27.

Rokeach, M. (1968) *Beliefs, Attitudes and Values*, San Francisco: Jossey-Bass.

Romberg , T. A. (1992) *Problematic Features of the School Mathematics Curriculum*, in: Jackson, P. W. (Ed.), Handbook of Research on Curriculum, New York: MacMillan.

Rosenberg, M. J. and Hovland, C. I (1960) *Cognitive, Affective, and Behavioural Components of Attitudes*, in: M. J. Rosenberg, C. I. Hoveland, W. J. McGuire, R. P. Abelson and J. W. Brehm (Eds.), Attitude Organisation and Change: An Analysis of Consistency.

Russell, B. A. W. (1921) *Mysticism and Logic*: Allen and Unwin, in Orton, A., and Wain, G., (1994), Issues in Teaching Mathematics, London: Cassel.

Saracho, O. N. (1991) Students' Performance for Field-dependence-independence Teacher Characteristics, *Educational Psychology*, 11, 323-332.

Sawer, W. W. (1959) *Mathematician's Delight*, Harmondsworth: Penguin.

Scardamalia, M. (1977) *Information Processing Capacity and the Problem of Horizontal Decalage*: *A Demonstration using Combinatorial Reasoning Tasks, Child Development*.

Schibeci, R. A. (1984) Attitude to Science: An Update Studies in Science Education,11,26-9.

Schibeci, R. A. and Riley, J. P. (1983) *Influence of Student Background on Science Attitudes and Achievement*, Paper Presented to the Annual Meeting of the American Educational Research Association, Montreal Canada.

Schmeck, R. R. (1988) *An Introduction to Strategies and Styles of learning*, in: Schmeck, R. R. (Ed.) Learning Strategies and Learning Styles, 3-20, New York: Plenum Press.

Schmidt, W. H., McKnight, C. C. and Raizen, S. A. (1997) *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education*.

Schminke, C. W., Maertens, N. and Arnold, W. (1978) Teaching the Child Mathematics, 2nd Edition, New York: Holt, Rinehart and Winston.

Schminke, C. W., Norbert, M. and William, A. (1978) *Teaching the Child Mathematics,* Holt, Rinehart and Winston.

Schoenfeld, A. (1985) *Mathematical Problem Solving*, Orlando, FL: Academic Press.

Schoenfeld, A. (1994) *Mathematical Thinking and Problem Solving*. USA: LEA Press.

Scottish Qualifications Authority (http://www.sqa.org.uk/sqa/24759.html).

Skryabina, E. (2000) *Student Attitudes to Learning Physics at School and University Levels in Scotland,* PhD Thesis, University of Glasgow, Glasgow.

Simon, M. (1995) Reconstructing Mathematics Pedagogy from a Constructivist Perspective, *Journal of Research in Mathematics Education*, **26**(2), 114-145.

Simpson, B. J. (1966) The Classification of Educational Objection, Psychomotor Domain. *Illinois Teacher of Home Economics*, **10**(4), 110-144.

Sirhan, G and Reid, N. (2001) An Approach in Supporting University Chemistry Teaching, *Chemistry Education Research and Practice*, **3**, 65-75.

Slavin, R. E. (2000) *Educational Psychology* (6th Edition), Allyn and Bacon.

Smith, R. (2007) *Effective Teaching: A Practical Guide to Improving Your Teaching Skills*, Philip Allan Updates.

Smithers, A. and Hill, S. (1987) *Science Teaching, Education*, **170**, 375, 3-21

Solomon, J. (1994) The Rise and Fall of Constructivism, *Studies in Science Education*, 23, 1-19

Solso, R. L. (1995) *Cognitive Psychology* (4th Edition), Boston: Allyn and Bacon.

Spencer, H. (1862) *First Principles*, London, Williams and Norgate.

Sperling, G. (1960) The Information Available in Brief Visual Presentation, Psychological Monographs, 74 (page 498).

Stainton Rogers, R., Stenner, P., Gleeson, K. and Stainton Rogers, W. (1995) Social *Psychology: A Critical Agenda*, Cambridge: Polity Press.

Standing, L. (1973) Remembering Ten Thousand Pictures, Quarterly, *Journal of Experimental Psychology*, 25, 207-222.

Sternberg, R. J. and Grigorenko, E. L. (1997) Are Cognitive Style Still in Style? American Psychologist, 7(July),700-712.

Sternberg, R. J. and Zhang, L. F. (2001) *Perspective on Thinking , Learning, and Cognitive Styles* (Edited), NJ: Lawrence Erlbaum.

Swanson, L. J. (1995) *Learning Styles: Review of Literature*, Educational Research Information Centre.

Thurstone, L. L. (1929) Attitudes can be Measured, *American Journal of Sociology*, 33, 529-554.

Tinajero, C. and Paramo, M. (1998) Field dependence-independence, Cognitive Style and Academic Achievement: A Review of Research and Theory, *European Journal of Psychology of Education*, **13**(2), 227-251.

Treisman, A. M. (1964) Verbal Cues, Language and Meaning in Selective Attention, *American Journal of Psychology*, 77, 206-219.

Triandis, H. C. (1971) *Attitudes and Attitude Change*, New York; London: Wiley.

Tulving, E. (1972) *Episodic and Semantic Memory*, in: E. Tulving and W. Donaldson (Eds.), Organisation of Memory, London: Academic Press.

Tulving, E. and Thomson, D. M. (1973) Encoding Specificity and Retrieval Processes in Episodic Memory, *Psychological Review*, 80, 352-373.

University of Minnessota Talented Youth Mathematics Program Research (1996, USA)

Uz-Zaman, T. (1996) *The Use of an Information Processing Model to Design and Evaluate Physics Undergraduate Laboratory*, PhD Thesis, Glasgow: University of Glasgow.

Vaidya, S. and Chansky, N. (1980) Cognitive Development and Cognitive Style in Mathematics Achievement, *Journal of Educational Psychology*, 72, 326-30.

Vernon, P. E. (1973) *Multivariate Approaches to the Study of Cognitive Style*, in: Royce, J. R. (Ed.) Multivariate Analysis of Psychological Theory, 125-148, London: Academic Press.

Vygotsky, L. (1987) *Thinking and Speech*, in: Rieber, R. and Carlton, A. (Eds.), (N Minick, Trans.) Collected Works, 1, 39-285, New York: Plenum.

Waugh, N. C. and Norman, D. A. (1965) Primary Memory, *Psychological Review*, 72, 89-104.

Webster, B. and Fisher, D. L. (2001) *School-level Environment and Students' Outcomes in Mathematics Achievemen*t (Paper presented at the Australian Association for Research in Education) December 3-6, 2001, Freemantle, Perth.

Weinstein, F. E. and Van Materstone, G. (1996) *Learning Strategies and Learning to Learn*, in: De Corte, E. and Weinert, F. E. (Eds.) International Encyclopaedia of Developmental Psychology (pp 419-423), London: Pergamon.

White, R. (1998) *Learning Science*, Oxford: Basil Blackwell.

Wilson, V. L. (1980) *A Meta-analysis of the Relationship Between Students' Attitude and Achievement in Secondary School Science*: Paper Presented at the Annual Meeting of the National Association for Research in Science Teaching, Boston, April.

Willing, K. (1988) *Learning Styles in Adult Migrant Education*, Adelaide: NCRC Research.

Wingfield and Byrnes, D. L. (1981) *The Psychology of Human Learning*.

Winocur, S. L. (1985) *Project IMPACT*, in: A. L. Costa (Ed.)Developing Minds (pp210-211), Alexandria, VA: Association for Supervision and Curriculum Development.

Witkin , H. A. (1964) *Origins of Cognitive Style*, in: Sheerer, C. (Ed.) Cognition: Theory, Research and Promise, New York: Harper and Row.

Witkin, H. A. , Dyk, R. B., Faterson, H. F., Goodenough, D. R, Karp, S.A. (1962) *Psychological Differentiation*, New York: Wiley.

Witkin, H. A. and Asch, S. E. (1948) Studies in Space Orientation (III): Perception of the Upright in the Absence of Visual Field, *Journal of Experimental Psychology*, 38, 604-614.

Witkin, H. A. and Goodenough, D. R. (1981) *Cognitive Styles Essence and Origins: Field Dependence and Field Independence Psychological Issues,* New York: International University Press.

Witkin, H. A., Dyk, R. B. Faterson, H. F, Goodenough, D. R. and Karp, S. A. (1974) *Psychological Differentiation: Studies of Development,* New York: Lawrence Erlbaum Associates Inc.

Witkin, H. A., Moore, C., Goodenough, D. and Cox, P. (1977) Field-dependent and Field-independent Cognitive Styles and their Educational Implications, *Review of Educational Research*, 47, 1-64.

Witkin, H. A., Oltman, P, Raskin, E., Karp, S. (1971) *A Manual for Embedded Figure Test*, Palo Alto, CA: Consulting Psychologists Press.

Yackel, E. and Cobb, P. (1996) Sociomathematical Norms, Argumentation and Autonomy in Mathematics, *Journal of Research in Mathematics Education*, **27**(4), 458-477.

Yin, R. K. (1984) *Case Study Research: Design and Methods,* Beverly Hills, CA: Sage.

Yang, M. J. (2000) *Problem Solving in Chemistry at Secondary School*, Unpublished PhD Thesis, Glasgow: University of Glasgow.

Zanna, M. P. and Rempel, J. K. (1988) *Attitudes: A New Look at an Old Concept*, in: D. Bar-Tal and A. W. Kruglanski (Eds.), The Social Psychology of Knowledge, Cambridge, Cambridge University Press.

Zarghani, G. H. Z. (1988) *Identification of Learning Strategies Which enable College Students with Differing Personality Temperament to cope with Learning Blocks*. (Unpublished Doctoral Dissertation) University of Nebraska, Lincoln.

Zaslavsky, O. (2005) Seizing the Opportunity to create Uncertainty in Learning Mathematics, *Educational Studies in Mathematics*, **60**(3), 297-321.

List of Appendices

Appendix A

Digit Span Test
(Digit Span Backward Test)

Digit Span Test

This is carried out in the following way:

(1) Give each student a sheet with spaces for writing down answers
Instruct them to write their names, registration numbers or some other identifier.

(2) Read them the following instructions:

“This is an unusual test. It will not count for your marks or grades in any way. We are trying to find out more about the way you can study and this test will give us useful information. You will not be identified in any way from it.

I am going to say some numbers. You must not write as I speak. When I stop speaking, you will be asked to write the numbers down the boxes on your sheet. Are we ready? Let’s begin.

(3) You say the numbers exactly at a rate of one per second (use a stop watch or heart beat to keep your time right). You allow the same number of seconds for the students to write down the answers. Thus, if you gave the numbers: 5,3,8,6,2. You give them five seconds for writing them down. I follow the procedure:

“5,3,8,6,2 - ‘write’ - five seconds allowed for writing, then ‘next’”

(4) Here are the numbers used by El-Banna in his early work:

5 8 2
6 9 4

6 4 3 9
7 2 8 6

4 2 7 3 1
7 5 8 3 6

6 1 9 4 7 3
3 9 2 4 8 7

5 9 1 7 4 2 8
4 1 7 9 3 8 6

5 9 1 9 2 6 4 7
3 8 2 9 5 1 7 4

2 7 5 8 6 2 5 8 4
7 1 3 9 4 2 5 6 8

(5) You now give a second set of instructions.
“Now I am going to give you another set of numbers. However, there is an added complication!
When I have finished saying the numbers, I want you to write them down in reverse order.
For example, if I say “7,1,9”, you write it down as “9,1,7”.
Now, no cheating!! You must not write the numbers down backwards.
You listen carefully, turn the numbers round in your head and then write them down normally.
Have you got this? Let’s begin.”

(6) Here are the numbers:

2 4
5 8

6 2 9
4 1 5

3 2 7 9
4 9 6 8

1 5 2 8 6
6 1 8 4 3

5 3 9 4 1 8
7 2 4 8 5 6

8 1 2 9 3 6 5
4 7 3 9 1 2 8

9 4 3 7 6 2 5 8
7 2 8 1 9 6 5 3

(7) This is the version used for adults (those over 16). With younger children, it will need to be adjusted by removing the larger sets of numbers.

(8) Marking: the main thing is to be consistent. Ideally, if a person achieves at, say, 4,5,6 and 7 but fails at eight digits, then their working memory is 7. However, they can often fail an odd one (by simple slips) or succeed at one at, say, eight digits and fail at the other. Use the simple rule that, for a single failure followed by two correct answers, the failure is ignored. For those who fail at one and succeed at the other at one level, just be consistent: that level can be given.

Note also: check the number sequences above to see if any sequence of numbers has any pattern in your cultural setting (like a radio wavelength, a car registration code or whatever...)

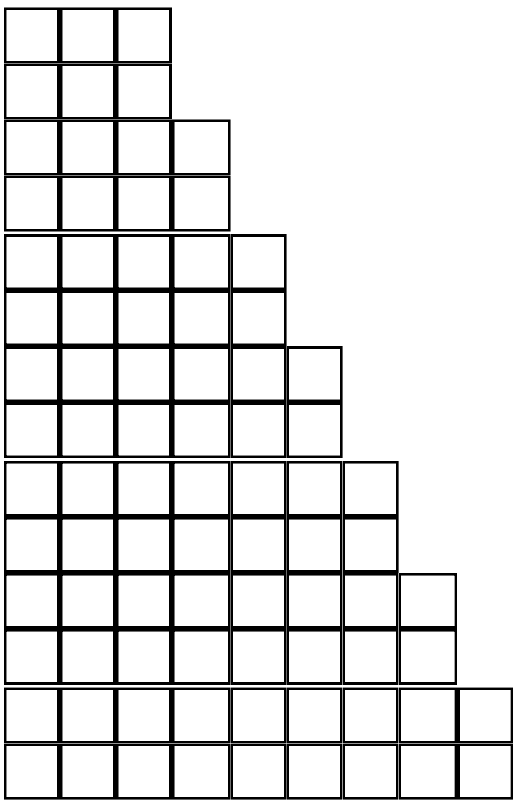
(9) The students' answer sheet is as shown below.

Student Answer Sheet

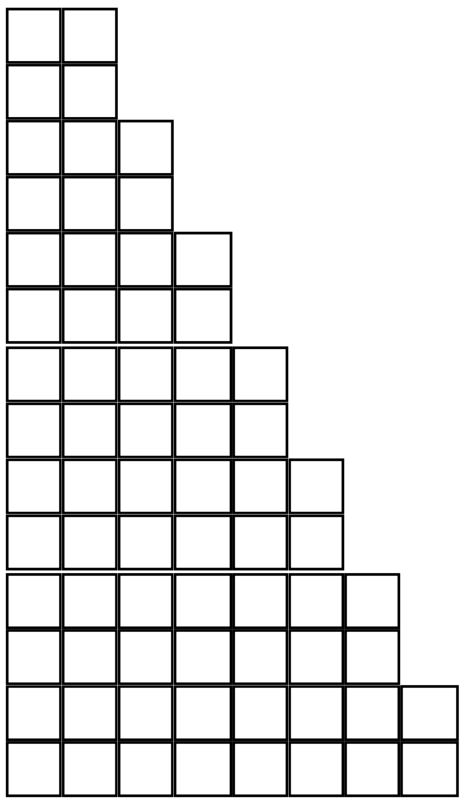
Your name: (or other identifier): ....

Write the numbers in the boxes below:

Digit Span Forwards



Digit Span Backwards



Appendix B

Attitudes Surveys

Mathematics
Where are the Difficulties?

> *This survey is designed to show your views about mathematics*
> *Please help us to improve learning.*
> *Your answers will NOT affect your mathematics marks in any way.*
> *Please be absolutely honest!!*

(1) Are you: ☐ boy ☐ girl

(2) Which primary school do you attend at moment? ………………………………………………

(3) Here are some topics you may have studied in mathematics.
Which of them interest you?
*Tick **as many** as you like.*

☐ Prime numbers	☐ Multiples
☐ Factors	☐ Fractions
☐ Decimal	☐ Percentages
☐ Ratio	☐ Powers and roots
☐ Writing numbers in words	☐ Writing numbers in figures
☐ Finding missing numbers	☐ Addition and subtraction of numbers
☐ Multiplication division of numbers	☐ Area and perimeter

Here is a way to describe a *sports car*.

quick	✓	☐	☐	☐	☐	☐	slow
important	☐	☐	✓	☐	☐	☐	unimportant
safe	☐	☐	☐	☐	✓	☐	dangerous

The positions of the ticks between the word pairs show that you consider it as very quick, slightly more important than unimportant and quite dangerous.

Use the same method to answer questions 4 and 5.

(4) What are your opinions about **mathematics lessons**?
Tick one box on each line

I like mathematics lessons	☐	☐	☐	☐	☐	☐	I hate mathematics lessons
Boring lessons	☐	☐	☐	☐	☐	☐	Interesting lessons
I enjoy the lessons	☐	☐	☐	☐	☐	☐	I do not enjoy the lessons
Easy lessons	☐	☐	☐	☐	☐	☐	Complicated lessons
Not essential for life	☐	☐	☐	☐	☐	☐	Essential for life
Best learned from a textbook	☐	☐	☐	☐	☐	☐	Best learned from a teacher
Relates to the events of daily life	☐	☐	☐	☐	☐	☐	Does not relate to the events of daily life
Very important for gaining employment	☐	☐	☐	☐	☐	☐	Not very important for gaining employment

(5) How do you feel about your **mathematics course** at school?
*Tick **one box** on each line*

I feel I am coping	☐	☐	☐	☐	☐	☐	I feel I am NOT coping well
I learn a lot new things	☐	☐	☐	☐	☐	☐	I learn nothing new in mathematics lessons
I am NOT obtaining new skills	☐	☐	☐	☐	☐	☐	I am obtaining a lot of new skills
I like the teacher	☐	☐	☐	☐	☐	☐	I dislike the teacher
Mathematics is important	☐	☐	☐	☐	☐	☐	Mathematics is unimportant subject
I hate doing home work	☐	☐	☐	☐	☐	☐	I enjoy doing homework

(6) Would you like doing more mathematics in secondary school?

Yes, because ………………………………………………………………………………

No, because ………………………………………………………………………………

(7) We should like to know what you think about **people who work using mathematics**
In your opinion, do you think the following statements are true or false?
Tick on box on each line.

True	***False***	
☐	☐	All mathematicians are very intelligent people
☐	☐	Being a mathematician is very interesting
☐	☐	Mathematicians don't dress well
☐	☐	Being a mathematician is hard
☐	☐	Mathematicians usually are rich people
☐	☐	Girls don't like being mathematicians
☐	☐	Mathematicians work to make discoveries

(8) Which of these do you think is going to be **most interesting** to do in secondary school?
Tick as many as you wish.

- ☐ Playing in the school sports team
- ☐ Cooking or metalwork
- ☐ Playing musical instruments
- ☐ Learning mathematics
- ☐ Solving different kinds of problems
- ☐ Painting pictures
- ☐ Learning foreign language
- ☐ Doing science experiments
- ☐ Learning commerce
- ☐ Learning foreign languages

(9) What would you like most like to **do when you leave secondary school**?
*Tick **TWO** boxes to show your top two choices*

- ☐ TV news reader
- ☐ Airline stewardess
- ☐ Car mechanic
- ☐ Doctor
- ☐ Lawyer
- ☐ Scientist
- ☐ Airline pilot
- ☐ Hairdresser
- ☐ Businessman or businesswoman
- ☐ Professional sportsman or sportswoman
- ☐ Engineer
- ☐ Teacher
- ☐ Bricklayer
- ☐ Making clothes

(10) Which two school subjects are the best for helping you get a job when you leave school?
*Tick **TWO** boxes*

- ☐ English
- ☐ Literature in English
- ☐ Geography
- ☐ Science
- ☐ Music
- ☐ History
- ☐ Mathematics
- ☐ Craft, design
- ☐ Technology
- ☐ Home economics

(11) I became interested in mathematics thanks to:
Tick as many as you like.

- ☐ Mathematics TV programs
- ☐ Mathematics lessons
- ☐ My teacher
- ☐ Things I have read
- ☐ My parents
- ☐ Exhibitions, demonstrations
- ☐ My friends
- ☐ Other: (please indicate) ……………………………………………

(12) What do you most look forward to learning in your mathematics lessons?

……………………………………………………………………………………………………

……………………………………………………………………………………………………

Mathematics
Where are the Difficulties?

This survey is designed to show your views about mathematics
Please help us to improve learning.
Your answers will NOT affect your mathematics marks in any way.
Please be absolutely honest!!

(1) Are you: ☐ boy ☐ girl

(2) Which junior secondary school do you attend at moment?
…………………………………………………

(3) Here are some topics you may have studied in mathematics.
Which of them interest you?
*Tick **as many** as you like.*

☐ Indices	☐ Directed numbers
☐ Sequences	☐ Inequalities
☐ Transformations	☐ Ratio
☐ Fractions	☐ Construction
☐ Equations	☐ Probability
☐ Standard form	☐ Measures of central tendency
☐ Plane geometry	☐ Pythagoras' Theorem

Here is a way to describe a *sports car*.

quick	✓	☐	☐	☐	☐	☐	slow
important	☐	☐	✓	☐	☐	☐	unimportant
safe	☐	☐	☐	☐	✓	☐	dangerous

The positions of the ticks between the word pairs show that you consider it as very quick, slightly more important than unimportant and quite dangerous.

Use the same method to answer questions 4 and 5.

(4) What are your opinions about **mathematics lessons**?
Tick one box on each line

I like mathematics lessons	☐	☐	☐	☐	☐	☐	I hate mathematics lessons
Boring lessons	☐	☐	☐	☐	☐	☐	Interesting lessons
I enjoy the lessons	☐	☐	☐	☐	☐	☐	I do not enjoy the lessons
Easy lessons	☐	☐	☐	☐	☐	☐	Complicated lessons
Not essential for life	☐	☐	☐	☐	☐	☐	Essential for life
Best learned from a textbook	☐	☐	☐	☐	☐	☐	Best learned from a teacher
Relates to the events of daily life	☐	☐	☐	☐	☐	☐	Does not relate to the events of daily life
Very important for gaining employment	☐	☐	☐	☐	☐	☐	Not very important for gaining employment

(5) How do you feel about your **mathematics course** at school?
*Tick **one box** on each line*

I feel I am coping	☐	☐	☐	☐	☐	☐	I feel I am NOT coping well
I learn a lot new things	☐	☐	☐	☐	☐	☐	I learn nothing new in mathematics lessons
I am NOT obtaining new skills	☐	☐	☐	☐	☐	☐	I am obtaining a lot of new skills
I like the teacher	☐	☐	☐	☐	☐	☐	I dislike the teacher
Mathematics is important	☐	☐	☐	☐	☐	☐	Mathematics is unimportant subject
I hate doing home work	☐	☐	☐	☐	☐	☐	I enjoy doing homework

(6) Would you like doing more mathematics in secondary school?

Yes, because ……………………………………………………………………………………
No, because ……………………………………………………………………………………

(7) We should like to know what you think about **people who work using mathematics**
In your opinion, do you think the following statements are true or false?
Tick on box on each line.

True	***False***	
☐	☐	All mathematicians are very intelligent people
☐	☐	Being a mathematician is very interesting
☐	☐	Mathematicians don't dress well
☐	☐	Being a mathematician is hard
☐	☐	Mathematicians usually are rich people
☐	☐	Girls don't like being mathematicians
☐	☐	Mathematicians work to make discoveries

(8) Which of these do you think is going to be **most interesting** to do in secondary school?
Tick as many as you wish.

- ☐ Playing in the school sports team
- ☐ Cooking or metalwork
- ☐ Playing musical instruments
- ☐ Learning mathematics
- ☐ Solving different kinds of problems
- ☐ Painting pictures
- ☐ Learning foreign language
- ☐ Doing science experiments
- ☐ Learning commerce
- ☐ Learning foreign languages

(9) What would you like most like to **do when you leave secondary school**?
*Tick **TWO** boxes to show your top two choices*

- ☐ TV news reader
- ☐ Airline stewardess
- ☐ Car mechanic
- ☐ Doctor
- ☐ Lawyer
- ☐ Scientist
- ☐ Airline pilot
- ☐ Hairdresser
- ☐ Businessman or businesswoman
- ☐ Professional sportsman or sportswoman
- ☐ Engineer
- ☐ Teacher
- ☐ Bricklayer
- ☐ Making clothes

(10) Which two school subjects are the best for helping you get a job when you leave school?
*Tick **TWO** boxes*

- ☐ English
- ☐ Literature in English
- ☐ Geography
- ☐ Science
- ☐ Music
- ☐ History
- ☐ Mathematics
- ☐ Craft, design
- ☐ Technology
- ☐ Home economics

(11) I became interested in mathematics thanks to:
Tick as many as you like.

- ☐ Mathematics TV programs
- ☐ Mathematics lessons
- ☐ My teacher
- ☐ Things I have read
- ☐ My parents
- ☐ Exhibitions, demonstrations
- ☐ My friends
- ☐ Other: (please indicate)

………………………………………………

(12) What do you most look forward to learning in your mathematics lessons?

………

Mathematics
Where are the Difficulties?

This survey is designed to show your views about mathematics
Please help us to improve learning.
Your answers will NOT affect your mathematics marks in any way.
Please be absolutely honest!!

(1) Are you: ☐ boy ☐ girl

(2) Which senior secondary school do you attend at moment?
…………………………………………………

(3) Here are some topics you may have studied in mathematics.
Which of them interest you?
*Tick **as many** as you like.*

☐ Indices	☐ Mensurations
☐ Sequences	☐ Inequalities
☐ Transformations	☐ Logic and set theory
☐ Bearings	☐ Loci and construction
☐ Equations	☐ Probability
☐ Standard Form	☐ Measures of central tendency
☐ Plane geometry	☐ Circle geometry

Here is a way to describe a *sports car*.

quick	✓	☐	☐	☐	☐	☐	slow
important	☐	☐	✓	☐	☐	☐	unimportant
safe	☐	☐	☐	☐	✓	☐	dangerous

The positions of the ticks between the word pairs show that you consider it as very quick, slightly more important than unimportant and quite dangerous.

Use the same method to answer questions 4 and 5.

(4) What are your opinions about **mathematics lessons**?
Tick one box on each line

I like mathematics lessons	☐	☐	☐	☐	☐	☐	I hate mathematics lessons
Boring lessons	☐	☐	☐	☐	☐	☐	Interesting lessons
I enjoy the lessons	☐	☐	☐	☐	☐	☐	I do not enjoy the lessons
Easy lessons	☐	☐	☐	☐	☐	☐	Complicated lessons
Not essential for life	☐	☐	☐	☐	☐	☐	Essential for life
Best learned from a textbook	☐	☐	☐	☐	☐	☐	Best learned from a teacher
Relates to the events of daily life	☐	☐	☐	☐	☐	☐	Does not relate to the events of daily life
Very important for gaining employment	☐	☐	☐	☐	☐	☐	Not very important for gaining employment

(5) How do you feel about your **mathematics course** at school?
*Tick **one box** on each line*

I feel I am coping	☐	☐	☐	☐	☐	☐	I feel I am NOT coping well
I learn a lot new things	☐	☐	☐	☐	☐	☐	I learn nothing new in mathematics lessons
I am NOT obtaining new skills	☐	☐	☐	☐	☐	☐	I am obtaining a lot of new skills
I like the teacher	☐	☐	☐	☐	☐	☐	I dislike the teacher
Mathematics is important	☐	☐	☐	☐	☐	☐	Mathematics is unimportant subject
I hate doing home work	☐	☐	☐	☐	☐	☐	I enjoy doing homework

(6) Would you like doing more mathematics in secondary school?

Yes, because ………………………………………………………………………………
No, because ………………………………………………………………………………

(7) We should like to know what you think about **people who work using mathematics**
In your opinion, do you think the following statements are true or false?
Tick on box on each line.

True	***False***	
☐	☐	All mathematicians are very intelligent people
☐	☐	Being a mathematician is very interesting
☐	☐	Mathematicians don't dress well
☐	☐	Being a mathematician is hard
☐	☐	Mathematicians usually are rich people
☐	☐	Girls don't like being mathematicians
☐	☐	Mathematicians work to make discoveries

(8) Which of these do you think is going to be **most interesting** to do in secondary school?
Tick as many as you wish.

☐ Playing in the school sports team	☐ Painting pictures
☐ Cooking or metalwork	☐ Learning foreign language
☐ Playing musical instruments	☐ Doing science experiments
☐ Learning mathematics	☐ Learning commerce
☐ Solving different kinds of problems	☐ Learning foreign languages

(9) What would you like most like to **do when you leave secondary school**?
*Tick **TWO** boxes to show your top two choices*

☐ TV news reader	☐ Hairdresser
☐ Airline stewardess	☐ Businessman or businesswoman
☐ Car mechanic	☐ Professional sportsman or sportswoman
☐ Doctor	☐ Engineer
☐ Lawyer	☐ Teacher
☐ Scientist	☐ Bricklayer
☐ Airline pilot	☐ Making clothes

(10) Which two school subjects are the best for helping you get a job when you leave school?
*Tick **TWO** boxes*

☐ English	☐ History
☐ Literature in English	☐ Mathematics
☐ Geography	☐ Craft, design
☐ Science	☐ Technology
☐ Music	☐ Home economics

(11) I became interested in mathematics thanks to:
Tick as many as you like.

☐ Mathematics TV programs	☐ My parents
☐ Mathematics lessons	☐ Exhibitions, demonstrations
☐ My teacher	☐ My friends
☐ Things I have read	☐ Other: (please indicate) ……………………………………

(12) What do you most look forward to learning in your mathematics lessons?

……………………………………………………………………………………………………
……………………………………………………………………………………………………

Your Studies in Mathematics
What Do You Think?

We should like to find out about your experiences when studying mathematics.
Please answer every question completely honestly!!
None of your answers will affect your school marks in any way.
We hope that the results will help us to see how learning mathematics can be made more interesting.

Here is a way to describe a *racing car*.

quick	☑	☐	☐	☐	☐	☐	slow
important	☐	☐	☑	☐	☐	☐	unimportant
safe	☐	☐	☐	☐	☑	☐	dangerous

The positions of the ticks between the word pairs show that you consider it as very quick, slightly more important than unimportant and quite dangerous.

Use the same method to answer questions 1 to 5.

(1) Think about ***mathematics***
*Tick **one box** on each line*

Exciting	☐	☐	☐	☐	☐	☐	Boring
Not essential for living	☐	☐	☐	☐	☐	☐	Essential for living
Best learned from a textbook	☐	☐	☐	☐	☐	☐	Best learned from a teacher
Relates to events of daily life	☐	☐	☐	☐	☐	☐	Does not relate to events of daily life
Important for the future of a nation	☐	☐	☐	☐	☐	☐	Not important for the future of a nation
Not important for my personal development	☐	☐	☐	☐	☐	☐	Important for my personal development
Very necessary for gaining employment	☐	☐	☐	☐	☐	☐	Not important for gaining employment

(2) Think about your **lessons in mathematics**
*Tick **one box** on each line*

Boring	☐	☐	☐	☐	☐	☐	Interesting
Help me to work out solutions to problems	☐	☐	☐	☐	☐	☐	Do not help me to work out solutions to problems
Relate mathematics to daily life events	☐	☐	☐	☐	☐	☐	Not relating mathematics to daily life events
Make me like mathematics even more	☐	☐	☐	☐	☐	☐	Make me dislike mathematics even more
Do not inspire me to think	☐	☐	☐	☐	☐	☐	Inspire me to think
Does not show me clearly what to study	☐	☐	☐	☐	☐	☐	Shows me clearly what to study
Easy to apply to real life	☐	☐	☐	☐	☐	☐	Difficult to apply to real life
Complicated to follow	☐	☐	☐	☐	☐	☐	Easy to follow
Very important for me	☐	☐	☐	☐	☐	☐	Not very important for me

(3) Think about **yourself and mathematics**
*Tick **one box** on each line*

I find the course very easy	☐	☐	☐	☐	☐	☐	I find the course very hard.
I am growing intellectually	☐	☐	☐	☐	☐	☐	I am not growing intellectually
I am obtaining a lot of new skills	☐	☐	☐	☐	☐	☐	I am not obtaining a lot of new skills
I am getting worse at the subject	☐	☐	☐	☐	☐	☐	I am getting better at the subject
It is definitely “my subject”	☐	☐	☐	☐	☐	☐	It is definitely not “my subject”
Memorising is the key to success	☐	☐	☐	☐	☐	☐	Understanding is the key to success
I aim to memorise mathematical procedures	☐	☐	☐	☐	☐	☐	I try to understand how to do things
I think in terms of pictures, diagrams, graphs	☐	☐	☐	☐	☐	☐	I think in terms of written ideas
I spend much time revising just before exams	☐	☐	☐	☐	☐	☐	I do not spend much time revising just before exams

(4) Here are some statements about **the way I like to work**.
*Tick **one box** on each line to show how you like to work.*

I plan what I am going to do	☐	☐	☐	☐	☐	☐	I do not plan what I am going to do
I care a lot about what others think of me	☐	☐	☐	☐	☐	☐	I do not care a lot about what others think of me
I learn more through listening	☐	☐	☐	☐	☐	☐	I learn more through reading
I think in terms of pictures, diagrams, graphs	☐	☐	☐	☐	☐	☐	I think in terms of written ideas
I understand something better after I try it out.	☐	☐	☐	☐	☐	☐	I understand something better after I think it through
I gain much information from diagrams & graphs	☐	☐	☐	☐	☐	☐	I gain little information from diagrams and graphs

(5) Think about **examinations and tests in mathematics.**
*Tick **one box** on each line to show your opinion.*

I do not like short answer questions, as they do not give me the chance to explain what I know and understand							I prefer to learn the facts and then be tested on them in short answer questions.
In exams, I like questions that give me the scope to go beyond what is covered and shows my ability to think.							In exams I prefer questions that are based on what the lecturer covered
I believed that what should matter in exams is the quality of my answers, not on how much I write							In exams I expect to be rewarded for giving as much information as possible.
My main task in an examination is to write down all I have been taught							My main task in an examination is to show that I understand what I have been taught
Examinations in mathematics should test my ability to work things out for myself.							Examinations in mathematics should test my ability to remember mathematics facts

(6) What do you **enjoy most in learning mathematics**?
*Tick **all** the reasons that apply.*

- ☐ Studying the theory
- ☐ Studying how mathematics can make our lives better
- ☐ Studying about the natural phenomenon
- ☐ Finding relevance of mathematics knowledge to our daily life
- ☐ Doing practical work
- ☐ Studying making equipment
- ☐ Solving everyday problems

(7) Think about ***yourself.***
*Tick **FIVE boxes** which **MOST apply to yourself***

- ☐ I prefer studying mathematics alone
- ☐ I can concentrate on my study for a long time
- ☐ I can easily influence other people's emotions
- ☐ I have high expectation on myself
- ☐ I cannot be happy unless everyone likes me
- ☐ I am always eager to try new ideas
- ☐ I can organise my time well
- ☐ I enjoy chatting with my classmates
- ☐ I am influenced easily by other people's opinion
- ☐ My emotions are easily influenced by others
- ☐ If I work hard, I will be successful
- ☐ I am fond of solving problems with new methods
- ☐ I can accept new concept quickly
- ☐ I am confident I find a solution when I encounter problems

(8) Imagine you are faced with a new and demanding type of problem in your studies.
What is your likely reaction?
*Tick **as many** as you wish*

- ☐ Worry about passing the examinations
- ☐ Start to panic
- ☐ I have coped in the past - I'll manage now
- ☐ Enjoy it because it is new
- ☐ See it as a challenge
- ☐ Seek help from books
- ☐ Think of changing my course
- ☐ Seek help from others

(9) Here are some descriptions of the way students approach mathematics.
Tick as many as are true for you.

- ☐ It is all my fault if I cannot study mathematics well
- ☐ I study mathematics for interest
- ☐ If I study mathematics well I will have a bright future
- ☐ I enjoy studying in mathematics classes
- ☐ I have many friends sharing the same interests in mathematics
- ☐ Assignments help me understand mathematics more
- ☐ I think mathematics helps me understand the world better
- ☐ I study mathematics for credits
- ☐ I can grasp the main points in mathematics easily
- ☐ Diagrams help me understand mathematics better
- ☐ Mathematics are meant for people with exceptional ability
- ☐ Examination scores in mathematics reflect my understanding of mathematics well

Appendix C

Difficulty Survey

Mathematics

Where are the Difficulties?

This survey is designed to show where you find difficulties in mathematics
Your participation may help us to improve learning.
Your answers will NOT affect your mathematics marks in any way.
Please be absolutely honest!!

☐	Easy	I understood this first time
☐	Moderate	I found it difficult but I understand it now
☐	Difficult	I still do not understand it

Please tick one box on each line to show how difficult you find each topic.

	Easy	***Moderate***	***Difficult***	*If difficult, please say why*
Indices	☐	☐	☐	....
Directed Numbers	☐	☐	☐	....
Logarithms	☐	☐	☐	....
Approximations	☐	☐	☐	....
Ratio	☐	☐	☐	....
Fractions	☐	☐	☐	....
Percentages	☐	☐	☐	....
Sequences	☐	☐	☐	....
Series	☐	☐	☐	....
Algebraic Equations	☐	☐	☐	....
Algorithms	☐	☐	☐	....
Inequalities	☐	☐	☐	....
Transformations	☐	☐	☐	....
Pythagoras Theorem	☐	☐	☐	....
Algebraic Graphs	☐	☐	☐	....
Loci and Construction	☐	☐	☐	....
Bearings	☐	☐	☐	....
Standard Form	☐	☐	☐	....
Logic and Set Theory	☐	☐	☐	....
Circle Geometry	☐	☐	☐	....
Polynomials	☐	☐	☐	....
Mensurations	☐	☐	☐	....
Measures of Central Tendency	☐	☐	☐	....
Measures of Dispersions	☐	☐	☐	....
Graphical Representation of Data	☐	☐	☐	....
Plane Geometry	☐	☐	☐	....
Surds	☐	☐	☐	....
Probability Theory	☐	☐	☐	....
Longitude and Latitude	☐	☐	☐	....
Elevation and Depression	☐	☐	☐	....
Trigonometry	☐	☐	☐	....

Any other comments about your course:

Appendix D

Hidden Figure Test
(Group Embedded Figure Test)

Shape recognition within complex patterns

This test seeks to find your ability in recognising shapes in complex patterns.
The results of the test will NOT affect your course assessment in any way.

Enjoy the challenge and do your very best.

The total time allowed is 20 minutes but you may not finish in this time.

About yourself

Your Name: ....

Your class: .... Boy ☐ Girl ☐

Instructions

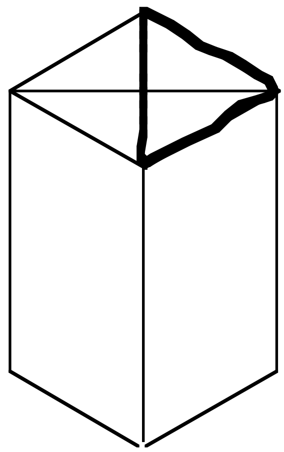
This is a test of your ability to find a simple form when it is hidden within a complex pattern.

Here is a simple example.

Can you find this triangle in the shape below?

Now Turn Over

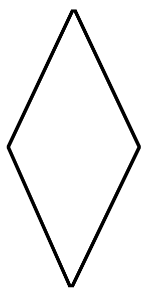
Did you spot the right answer ?



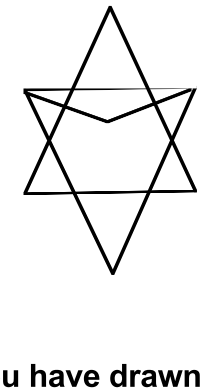
The triangle must be the same shape and facing the same way.

Here is another one.

Look for this diamond shape in the diagram below

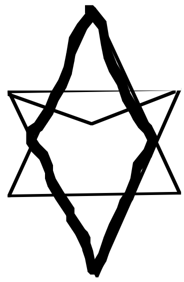


Find and trace the diamond shape in the diagram below using a pencil.



Once you have drawn in the diamond shape, turn over

Did you get it right ?



On the following pages, there are 20 puzzles like this. Each puzzle will have a letter underneath. You will be given a sheet with the shapes to find and these are labelled by letters.

(1) Do the puzzles in order. However, if you are completely stuck, move to the next puzzle.

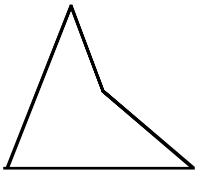
(2) Trace only ONE shape in each puzzle.

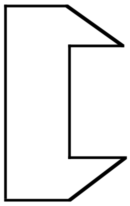
(3) The shapes you are to find in the puzzles are the SAME SIZE, the SAME PROPORTIONS, and FACING IN THE SAME DIRECTION as they appear on the sheet showing all the shapes.

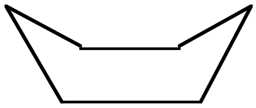
DO NOT TURN OVER

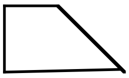
DO NOT START UNTIL YOU ARE TOLD TO DO SO

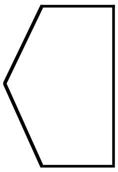
The shapes you have to find

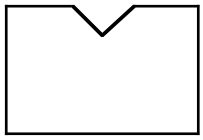

A

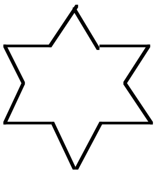

B

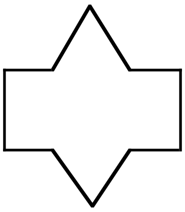

C

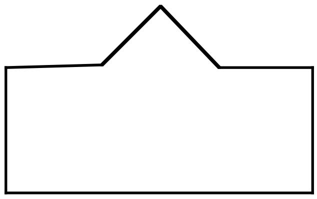

D

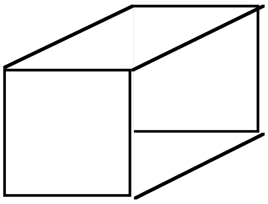

E

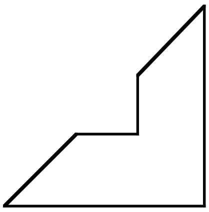

F

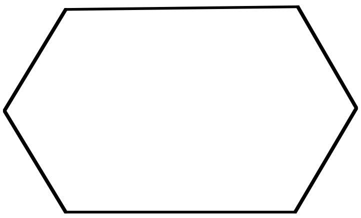

G

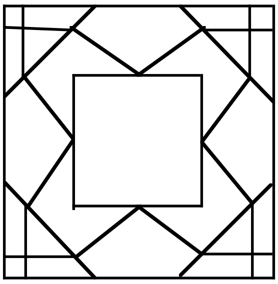

H


I

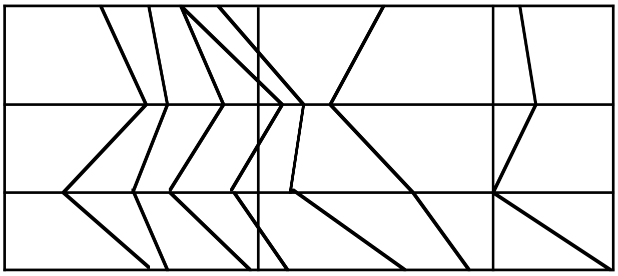

J


K

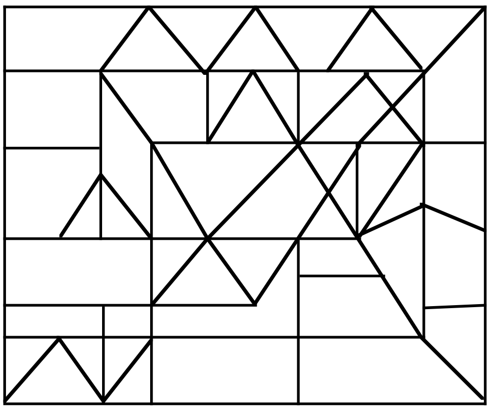

L



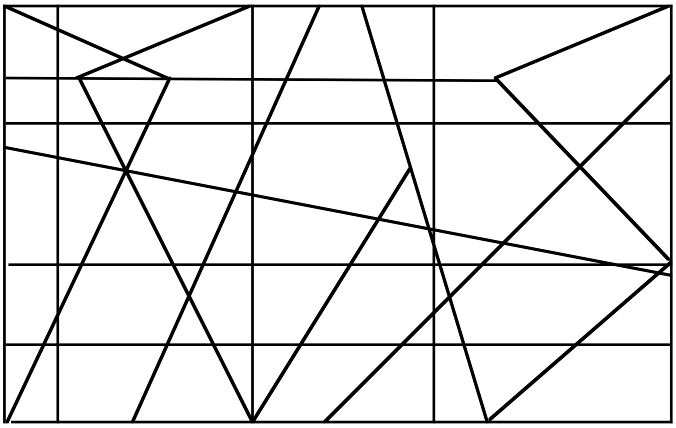
Find shape B



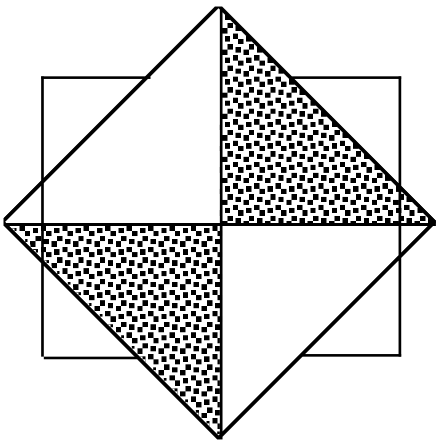
Find shape D



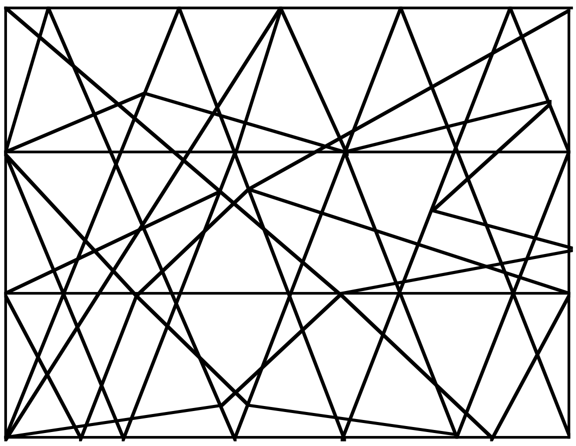
Find shape H



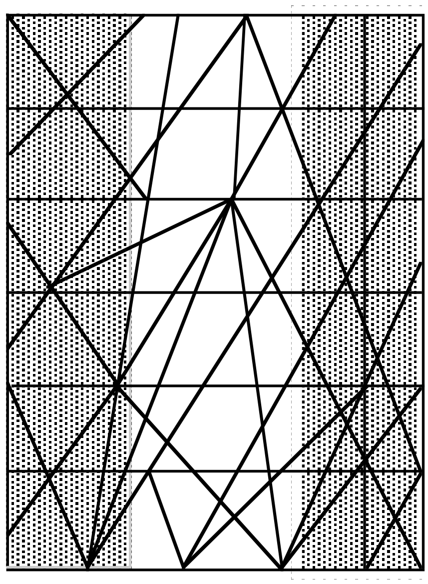
Find shape E



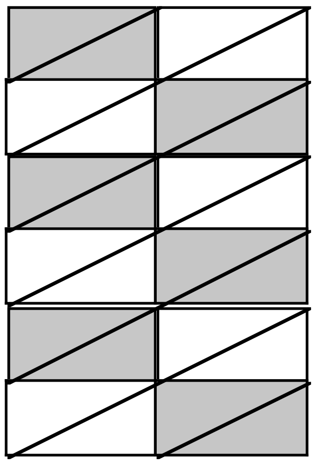
Find shape I



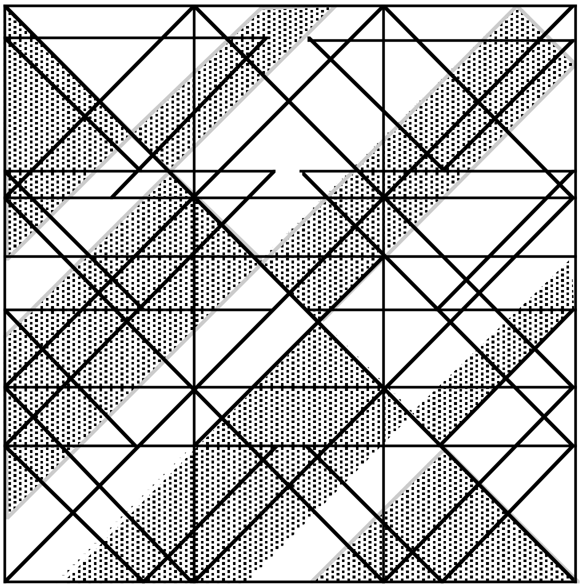
Find shape A



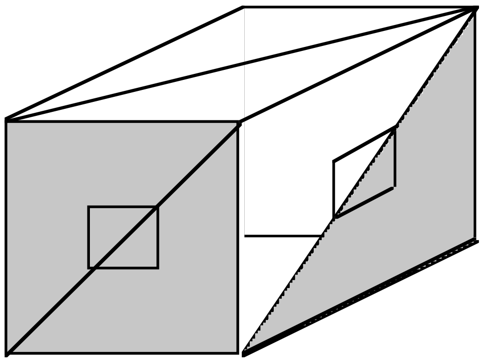
Find shape E



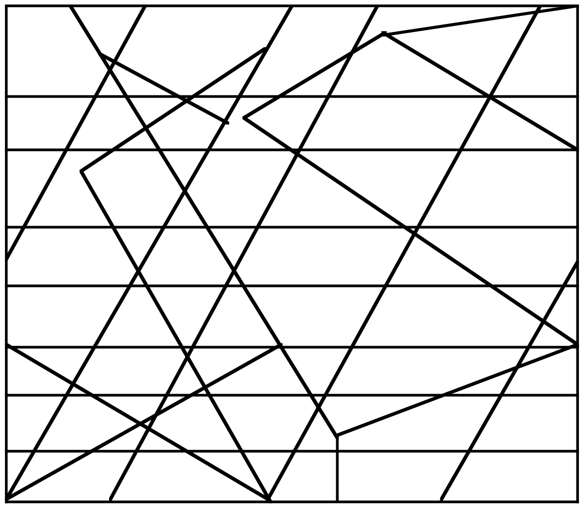
Find shape J



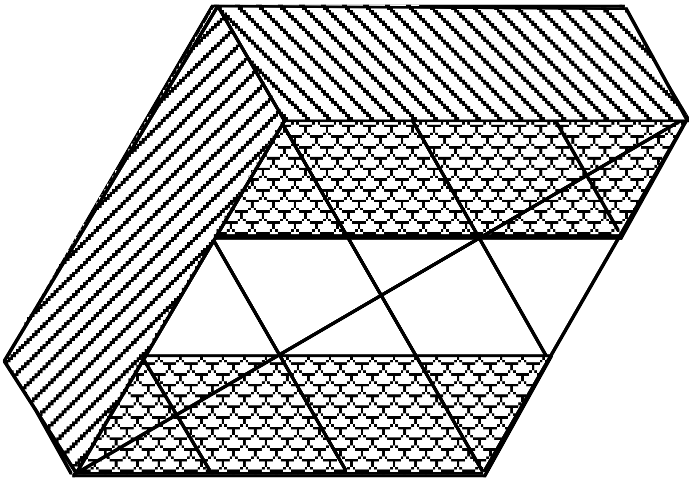
Find shape D



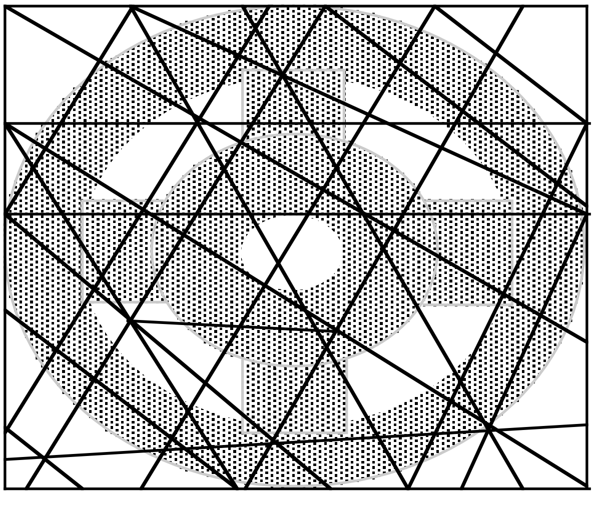
Find shape K



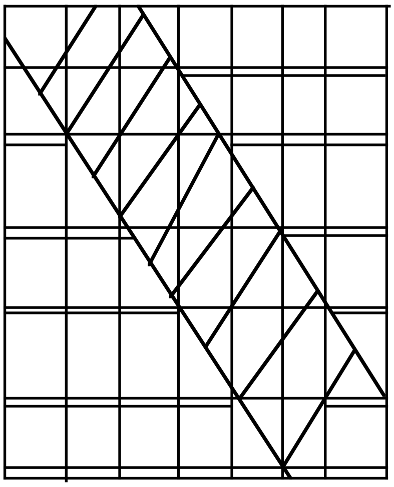
Find shape C



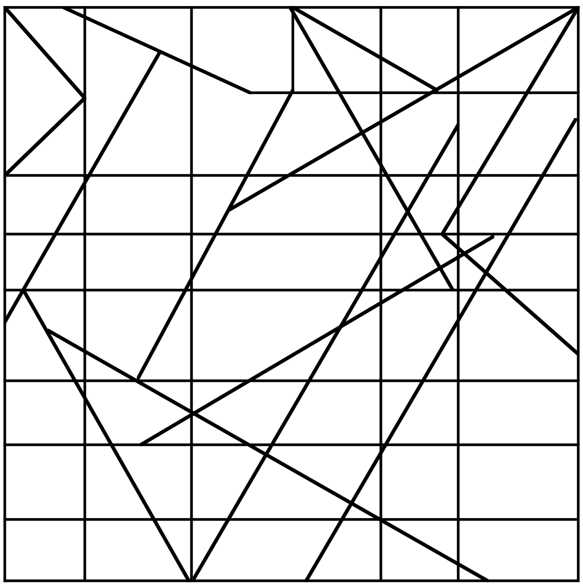
Find shape L



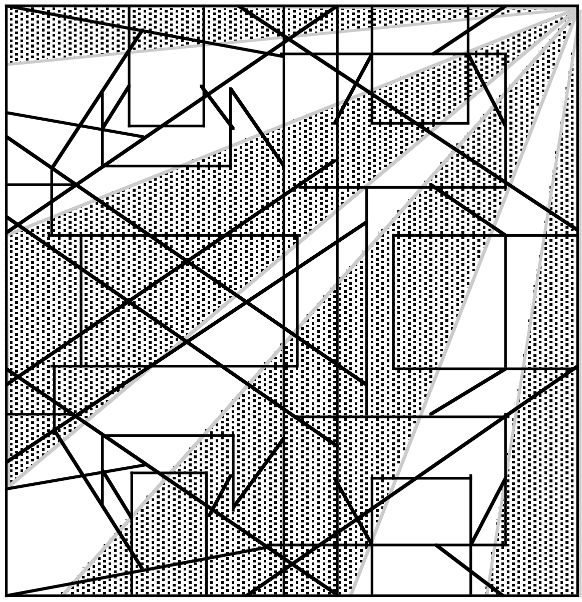
Find shape G



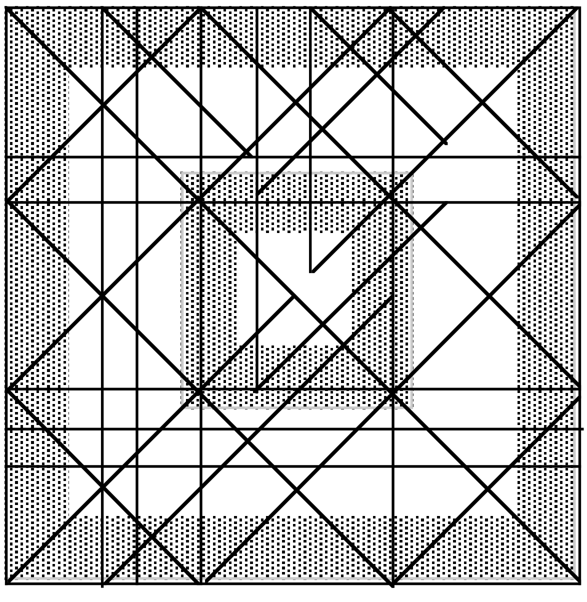
Find shape H



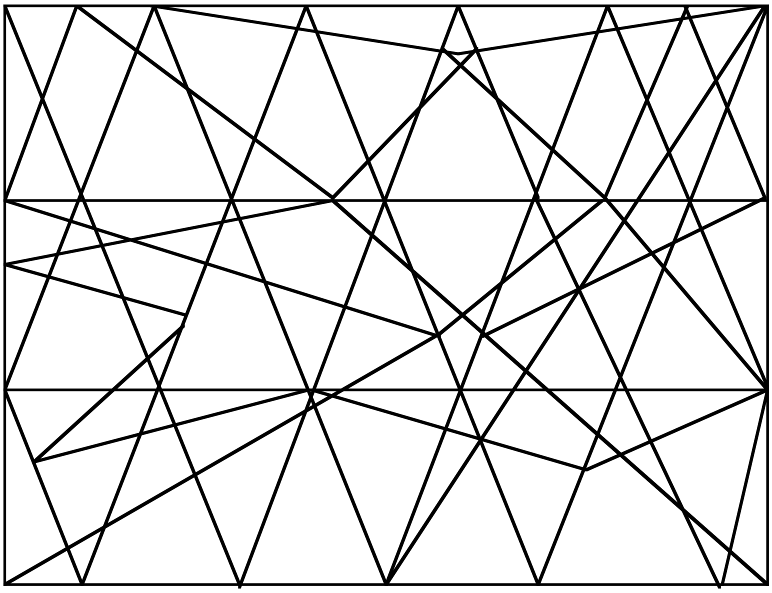
Find shape C



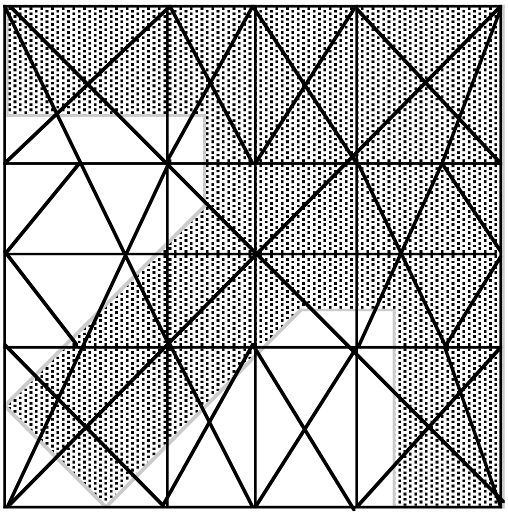
Find shape B



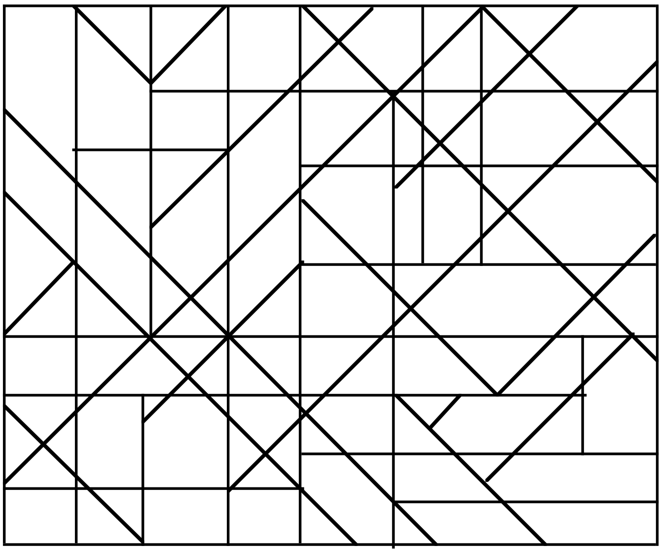
Find shape D



Find shape A



Find shape E



Find shape F

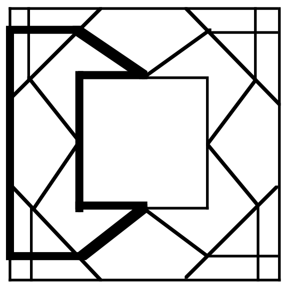
THIS IS THE LAST QUESTION

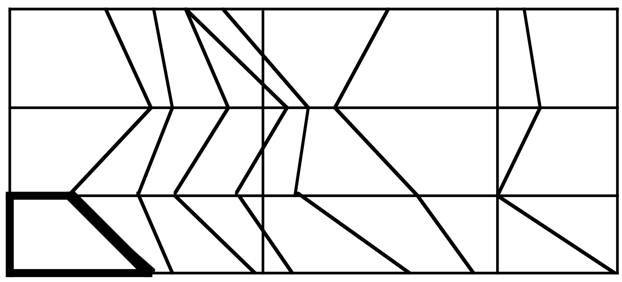
Appendix E

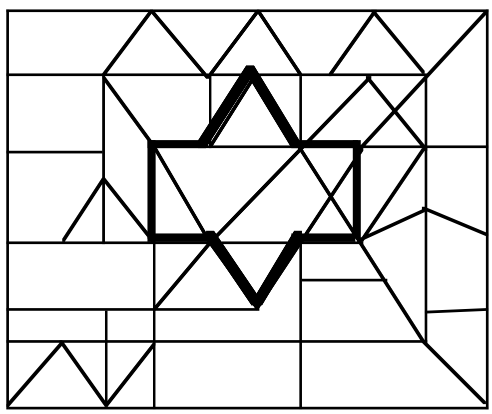
Answers to Hidden Figure Test

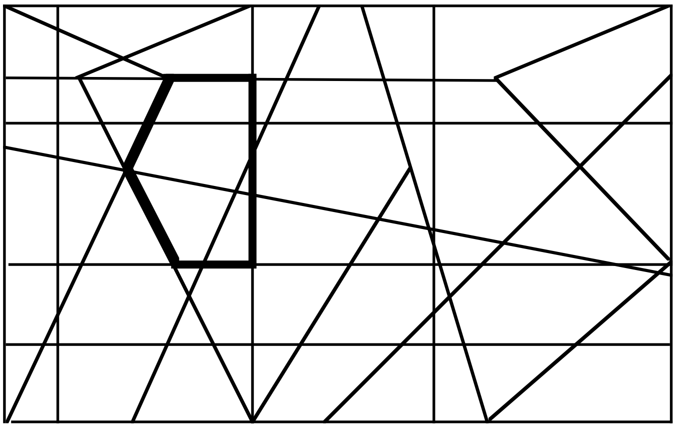
Shape recognition within complex patterns

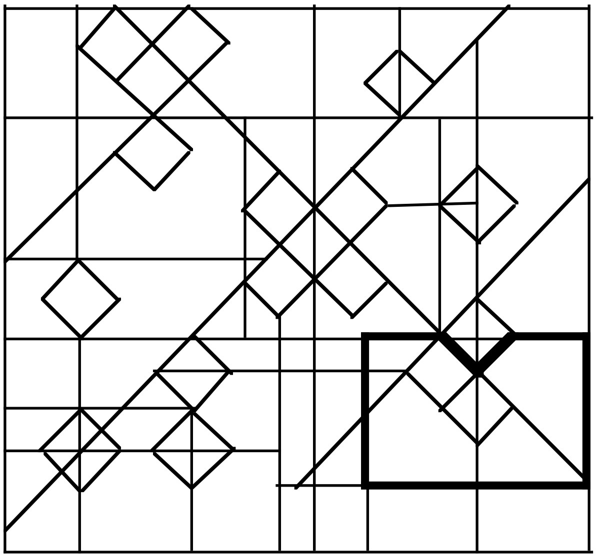
Answers

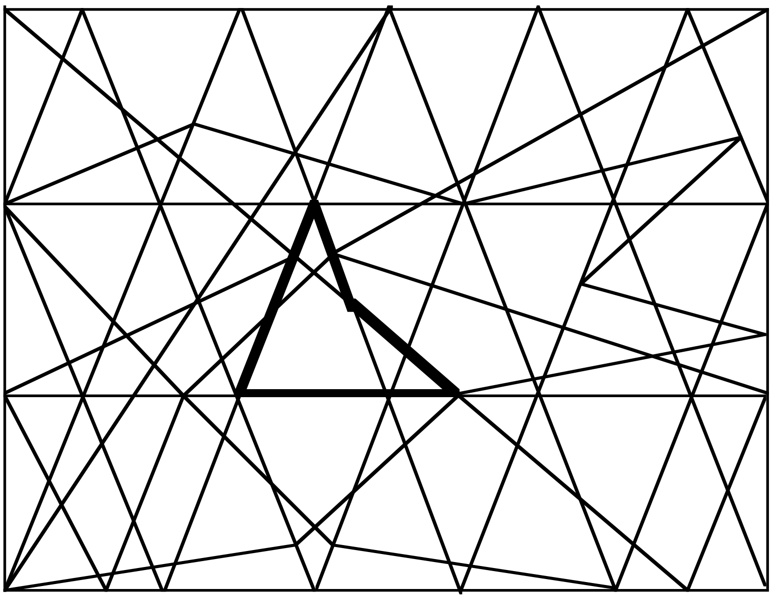


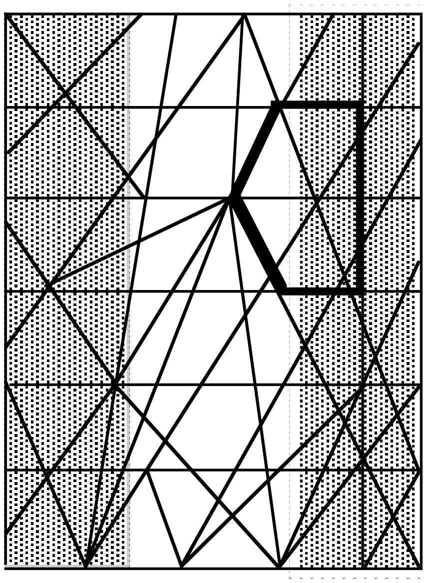


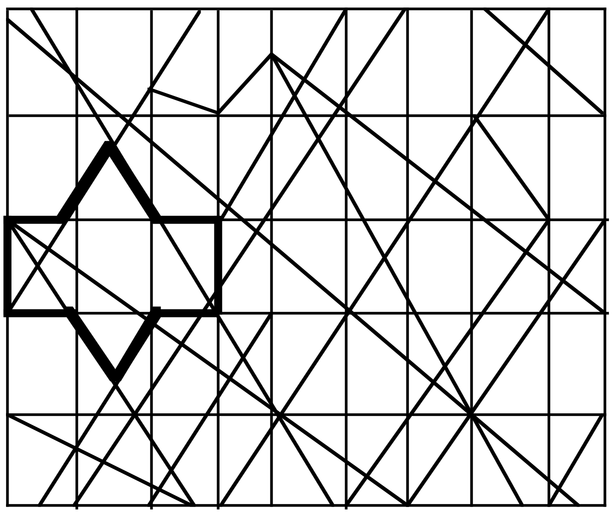


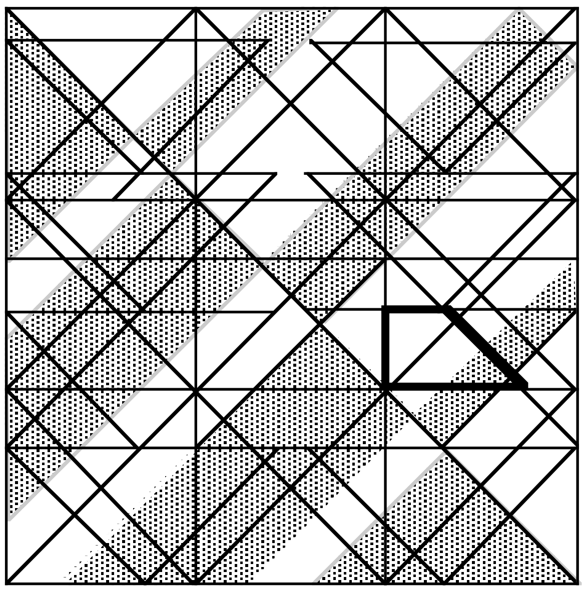


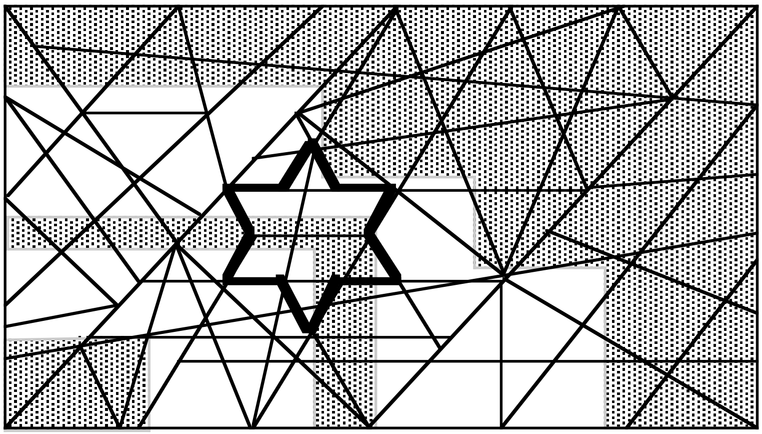


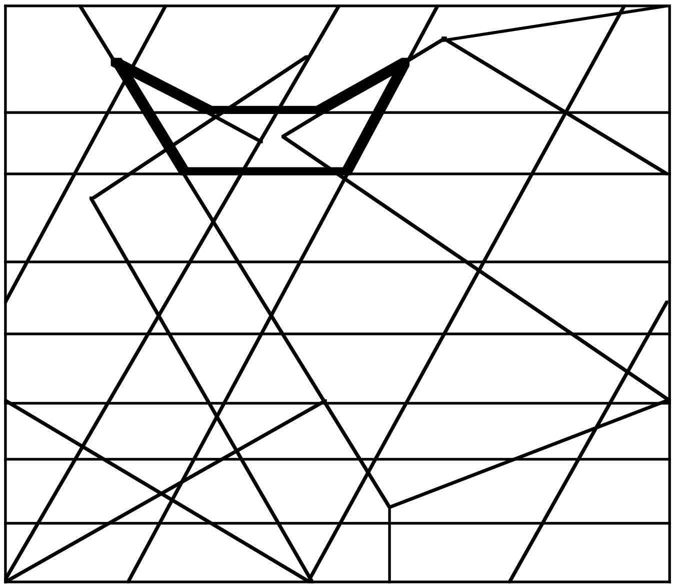


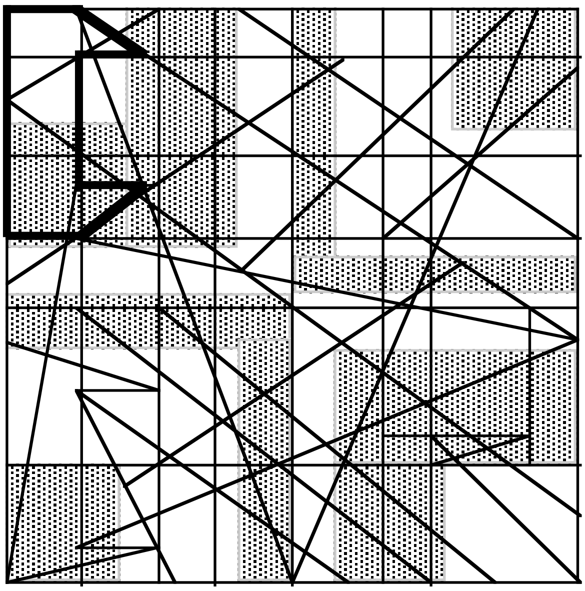


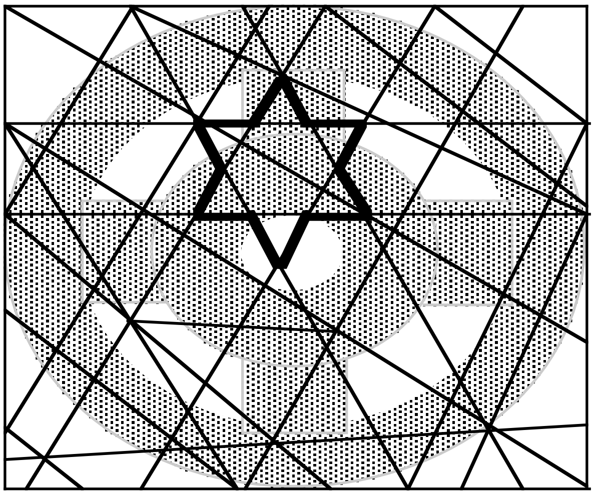


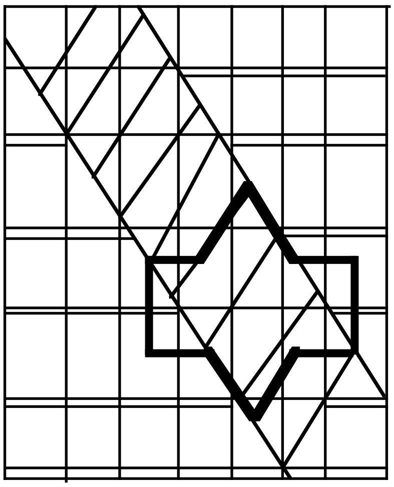


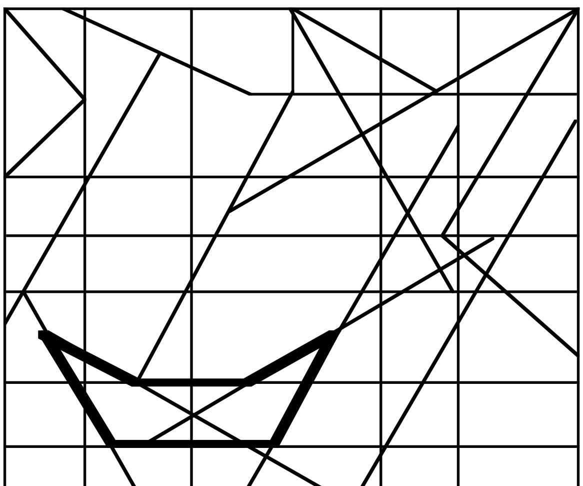


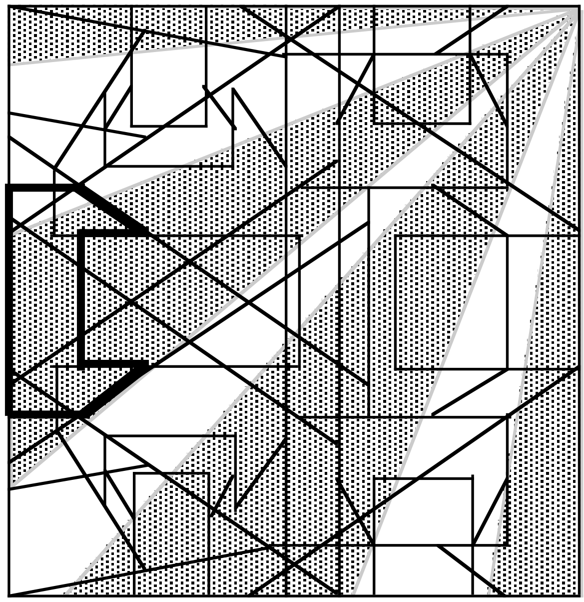


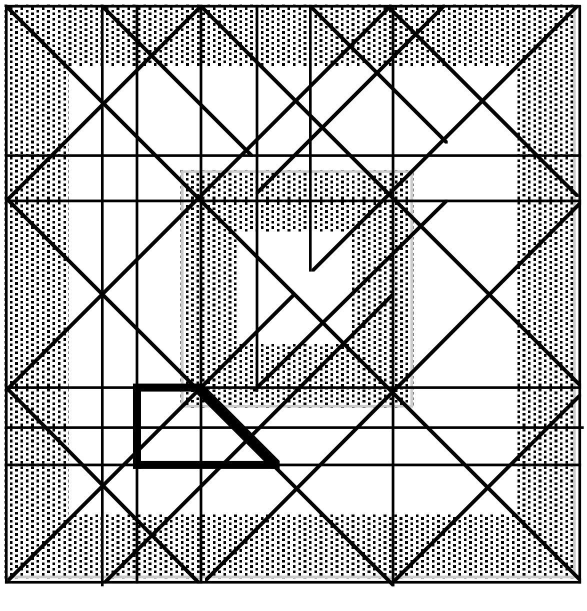


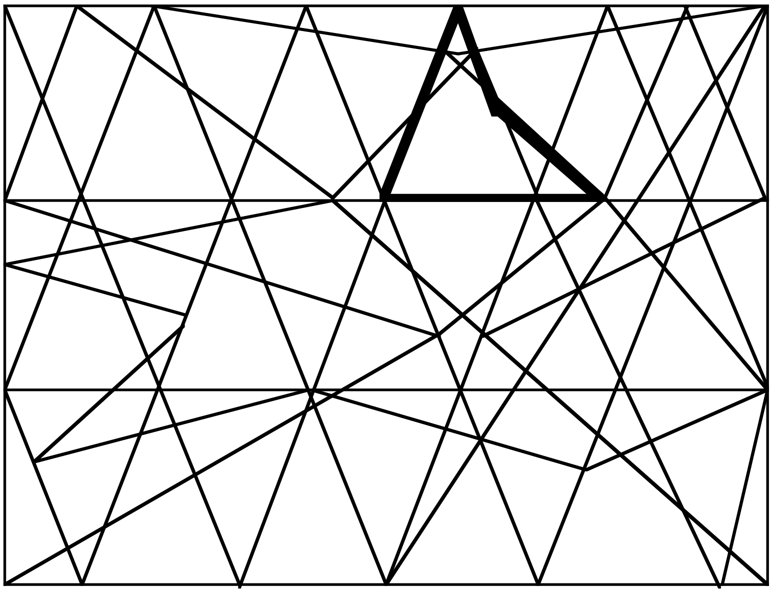


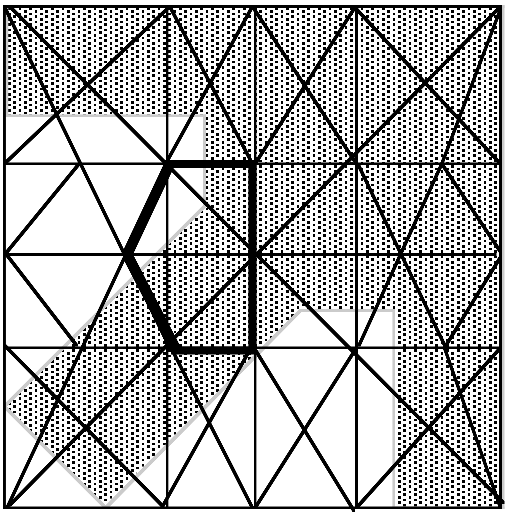


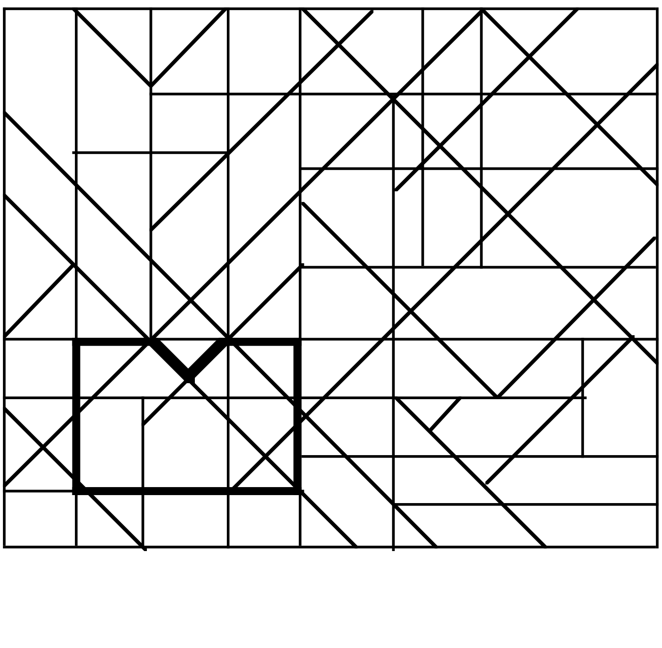












Appendix F

Mathematics Test Related to Learning Difficulties

Mathematics to Make you Think

This test does NOT count for any school marks.
The aim is to see where you find difficulties.

Name: .... Class: .... Sex: ....

(1) Solve the following equation for x.

$$\frac{2x}{3} + 5 = 2$$

(2) Solve the following equations for x and y.

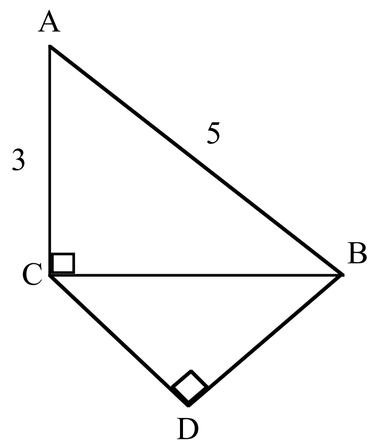
$$3x + 2y = 5$$
$$2x + 3y = 0$$

(3) Solve the following equation for x.

$$2x^2 - x - 3 = 0$$

(4) Look at the diagram:

If CD = BD, find the length of CD.



Appendix G

Questionnaire on Learning Difficulty
Related to Field Dependency
and
Working Memory Capacity

Why is Mathematics Often Difficult?

Name .... Class .... Boy/Girl

Please answer all the questions
Take care to tell us what you really think!
Your answers will not affect your school marks in any way

For the questions 1 and 2, please place ***one tick*** on each line to show what you think.

(1) Think of your ***class work and homework*** in mathematics
*Tick **one box** on each line*

		Always a problem	Frequently a problem	Sometimes a problem	Never a problem
(a)	Grasping what is required	☐	☐	☐	☐
(b)	Understanding the symbols used (like x and y)	☐	☐	☐	☐
(c)	The most difficult thing is knowing where to start	☐	☐	☐	☐
(d)	Trying to take in all the information	☐	☐	☐	☐
(e)	Seeing what is important	☐	☐	☐	☐
(f)	Not understanding the instructions	☐	☐	☐	☐
(g)	Not seeing why an answer is needed anyway	☐	☐	☐	☐
(h)	Selecting what is most important	☐	☐	☐	☐
(i)	Coping with all the information in the question	☐	☐	☐	☐
(j)	Remembering what to do	☐	☐	☐	☐
(k)	Trying to interpret the English to know what to do.	☐	☐	☐	☐

(2) Think of ***tests and examinations*** in mathematics.
*Tick **one box** on each line*

		Always a problem	Frequently a problem	Sometimes a problem	Never a problem
(a)	Grasping what is required	☐	☐	☐	☐
(b)	Understanding the symbols used (like x and y)	☐	☐	☐	☐
(c)	The most difficult thing is knowing where to start	☐	☐	☐	☐
(d)	Trying to take in all the information	☐	☐	☐	☐
(e)	Seeing what is important	☐	☐	☐	☐
(f)	Not understanding the instructions	☐	☐	☐	☐
(g)	Not seeing why an answer is needed anyway	☐	☐	☐	☐
(h)	Selecting what is most important	☐	☐	☐	☐
(i)	Coping with all the information in the question	☐	☐	☐	☐
(j)	Remembering what to do	☐	☐	☐	☐
(k)	Trying to interpret the English to know what to do.	☐	☐	☐	☐

(3) Look at the mathematics problem alongside:

$$3x + 2y = 5$$
$$2x + 3y = 0$$

Find the values for *x* and *y*.

Many students find this difficult.
You do **NOT** need to find an answer to the problem!
Just *think about **why*** it is so often difficult.

*In your own words, explain what you would do **first** to solve the problem*

*In your own words, explain **why** you find this kind of problem so difficult*

Summary of Data Obtained

Chapter	Group	Mathematics Marks	Working Memory Capacity	Attitudes	Difficulties	Field Dependency
7	Nigeria Primary	✔		✔		
7	Nigeria Junior Secondary	✔		✔		
7	Nigeria Senior Secondary	✔	✔	✔	Specific	
7	Nigeria University			✔		
7	England A level: Year 12	✔		✔		
8	England: Years 7 to 11	✔				✔
9	England: Year 10	✔		✔	General	✔
9	England small sample Year 10				Interview	

Appendix H

Statistics Techniques Employed

Correlation

Correlation is simply a measure of linear relationship between two scale of variables that relate to each other: a high value of the measurement in one is related with a high value in the other. Its focus is to establish if there is any association and whether that association is likely not have been brought about by chance. The degree to which any two relationships are related in this way is known by calculating the correlation coefficient denoted by (r). The important point is that correlation does not establish why the relationship exists nor does it imply cause and effect between the variables. For example, if the heights and weights of students in a class were measured, they would likely correlate highly, however, that does not mean that one causes the other.

There are three well known ways of calculating correlation coefficient:

(1) *Pearson correlation* - This is most popular and is used when the data come from measurements related to heights, weights or from examination marks. The Pearson correlation coefficient assumes an interval scale which is approximately normally distributed.

(2) *Spearman correlation* - This is used when one or both variables are not measured on an interval scale. It is commonly used when the data involved is in form of ranking (e.g. examination grades), and does not assume a normal distribution.

(3) *Kendall's Tau-b correlation* - This is used when there are a lot of tied values, or with ordered data where there are few points on each scale or small number of categories (e.g. five or six). This measurement does not assume a normal distribution.

For all the three measurements, the coefficient range between -1 and 1 and can be calculated using SPSS (a statistical software package). In this research, where survey data are employed, Kendall's Tau-b statistic was used.

As might be expected, when the calculation is done using SPSS, the probability is given out, which shows the possibility that correlation occurred simply as chance event. This probability is indicated as two-tailed or one-tailed. It is one-tailed when the direction of the relationship between the two variables is certain. This study employed two-tailed throughout.

Chi-Square

It is known to be one of the most widely non-parametric test used for statistical data to compare patterns of responses or frequencies, or to check if two response distributions differ from each other. For example, it can be employed to compare students' response to a survey item which was used before and after their learning experiences to see if their have changed. There are two main applications of chi-square: *Goodness of Fit* and *Contingency Tests*. The former is employed when it is appropriate to compare a pattern of responses to those of a control group. Here, the frequencies of responses are compared to some expected set of frequencies (e.g., like the sets of responses to Likert survey: from strongly agree to strongly disagree). The later is used to compare two patterns of responses when neither can be considered as a control group (e.g., comparing year groups or comparing boys and girls).

In this study, a contingency test was employed to:

- Compare year groups (primary, junior secondary and senior secondary.
- Compare gender (boys and girls)

(1) Goodness of Fit Test

This tests how well the experimental (sampling) distribution fits the control (hypothesised) distribution. An example of this could be a comparison between a group of experimentally observed responses to a group of control responses. For example,

	Positive	Neutral	Negative	
Experimental	55	95	23	N(experimental) = 173
Control	34	100	43	N(control) = 177
				(using raw numbers)

A calculation of observed and expected frequencies leads to:

	Positive	Neutral	Negative
fo = observed frequency	*55*	95	23
fe = expected frequency	33	97	42

Where *fe* = [N(experimental)/N(control)] X (control data) or (173/177) X (control data)

The degree of freedom (df) for this comparison is 2. This comparison is significant at two degrees of freedom at greater than 1%. (χ^2 critical at 1% level = 9.21)

(2) Contingency Test

This chi-square test is commonly used in analysing data where two groups or variables are compared. Each of the variable may have two or more categories which are independent from each other. The data for this comparison is generated from the frequencies in the categories. In this study, the chi-square as a contingency test was used, for example, to compare two or more independent samples like, year groups, gender, or ages. The data is generated from one population group. For example,

	Positive	Neutral	Negative
Male (experimental)	55	95	23
Female (experimental)	34	100	43

(Actual data above)

	Positive	Neutral	Negative	**N**
Male (experimental)	55 (44)	95 (96)	23 (33)	**173**
Female (experimental)	34 (45)	100 (97)	43 (33)	**177**
Totals	**89**	**195**	**66**	**350**

(Expected frequencies above in red)

The expected frequencies are shown in red in brackets ( ), and are calculated as follows:

e.g. 44 = (173/350) x 89

$$\chi^2 = 2.75 + 0.01 + 3.03 + 2.69 + 0.09 + 3.03 = \mathbf{11.60}$$

At two degrees of freedom, this is significant at 1%. (χ^2 critical at 1% level = 9.21)

The degree of freedom (df) must be stated for any calculated chi-square value. The value of the degree of freedom for any analysis is obtained from the following calculations:

df = (r-1) x (c-1)
*where **r** is the number of rows and **c** is the number of columns in the contingency table.*

Limitations on the Use of χ^2

It is known that when values within a category are small, there is a chance that the calculation of χ^2 may occasionally produce inflated results which may lead to wrong interpretations. It is safe to impose a 10 or 5% limit on all categories. When the category falls below either of these, then categories are grouped and the df falls accordingly.

t- Test

The t-test compares the means of two sets of measurements to see if they are significantly different. The test assumes data are interval and approximately normally distributed. There are various types of t-test available, for example, the independent-sample t-test and paired-sample t-test. If the comparison is between the mean scores of two different groups of people or category then the independent-sample t-test will be applied. However, if the comparison of mean scores of the same group of people or category on two different occasions, then the paired-sample t-test will be employed. The appropriate one used in this study is the independent-sample t-test in order to compare the results of various test questions presented in various formats.

Analysis of Variance (F-distribution)

This is called ANOVA. It compares the means of more than two samples to see if they are statistically different. It requires integer data, and approximately normally distributed.