STATISTICAL PHYSICS OF RUPTURE IN HETEROGENEOUS MEDIA

The damage and fracture of materials are technologically of enormous interest due to their economic and human cost. They cover a wide range of phenomena like e.g. cracking of glass, aging of concrete, the failure of fiber networks in the formation of paper and the breaking of a metal bar subject to an external load. Failure of composite systems is of utmost importance in naval, aeronautics and space industry (Reichhardt, 1996). By the term composite, we refer to materials with heterogeneous microscopic structures and also to assemblages of macroscopic elements forming a super-structure. Chemical and nuclear plants suffer from cracking due to corrosion either of chemical or radioactive origin, aided by thermal and/or mechanical stress.

Despite the large amount of experimental data and the considerable effort that has been undertaken by material scientists (Liebowitz, 1984), many questions about fracture have not been answered yet. There is no comprehensive understanding of rupture phenomena but only a partial classification in restricted and relatively simple situations. This lack of fundamental understanding is indeed reflected in the absence of reliable prediction methods for rupture, based on a suitable monitoring of the stressed system. Not only is there a lack of non-empirical understanding of the reliability of a system, but also the empirical laws themselves have often limited value. The difficulties stem from the complex interplay between heterogeneities and modes of damage and the possible existence of a hierarchy of characteristic scales (static and dynamic).

Many material ruptures occur by a 'one crack' mechanism and a lot of effort is being devoted to the understanding, detection and prevention of the nucleation of the crack (Fineberg and Marder, 1999; Bouchaud, 2003). Exceptions to the 'one crack' rupture mechanism are heterogeneous materials such as fiber composites, rocks, concrete under compression, ice, tough ceramics and materials with large distributed residual stresses. The common property shared by these systems is the existence of large inhomogeneities, that often limit the use of homogeneization or effective medium theories for the elastic and more generally the mechanical properties. In these systems, failure may occur as the culmination of a progressive damage involving complex interactions between multiple defects and growing micro-cracks. In addition, other relaxation, creep, ductile, or plastic behaviors, possibly coupled with corrosion effects may come into play. Many important practical applications involve the coupling between mechanical and chemical effects with the competition between several characteristic time scales. Application of stress may act as a catalyst of chemical reactions (Gilman, 1996) or, reciprocally, chemical reactions may lead to bond weakening (Westwood, Ahearn and Mills, 1981) and thus promote failure. A dramatic example is the aging of present aircrafts due to repeating loading in a corrosive environment (National Research Council, 1997). The interaction between multiple defects and the existence of several characteristic scales present a considerable challenge to the modeling and prediction of rupture. Those are the systems and problems on which the interdisciplinary marriage with statistical physics has brought new fruitful ideas that we now briefly present.

Creep rupture

There are many different conditions under which a material can rupture: constant strain rate, or stress, or stress rate, or more complex strain/stress histories (involving also other control parameters such as temperature, water content, chemical activity, and so on). The situation in which a stress is imposed is very frequent in mechanics (constant weight) and leads to the phenomenon of creep (also known as 'static fatigue'). A stress step leads in general to a strain response and other observable changes such as acoustic emissions (see for a review (El Guerjouma et al., 2001)). Understanding damage and rupture of a material subjected to a constant stress is thus a good starting point. For industrial applications, creep experiments are not always practical because they require adjusting the stress to sub-critical levels such that one does not wait too long before interesting processes (including eventually rupture) are monitored. Accelerated tests, which yield information more quickly, include step-stress and ramp-stress loading (Nelson, 1990).

As we said, time-dependent deformation of a material subjected to a constant stress level is known as creep. In creep, the stress is below the mechanical strength of the material, so that the rupture does not occur upon application of the load. It is by waiting a sufficiently long time that the cumulative strain may finally end in catastrophic rupture. Creep is all the more important, the larger the applied stress and the higher the temperature. The time to creep rupture is found in a large variety of materials to be controlled by the stress sign and magnitude, temperature and microstructure.

Creep is often divided into three regimes: (i) the primary creep regime corresponds to a decay of the strain rate following the application of the constant stress, which can often be described by the so-called Andrade's law (a power law decay with time); (ii) the secondary regime describes an (often very long) cross over, characterized by an approximately constant strain rate, towards the (iii) tertiary creep regime in which the strain rate accelerates up to rupture. Andrade's law for the strain rate is similar to the power-law relaxation of the aftershock seismic activity triggered by the stress change induced by a previous earthquake, known as Omori's law (Omori, 1894). In creep experiments, Omori's law describes the decay of the rate of acoustic emissions in the primary regime. Creep experiments are thus interesting both because they constitute standard mechanical tests of long-time properties of structures and because of the power laws reminiscent of the critical behavior of complex self-organizing systems that have become popular paradigms, as discussed below.

Studies of the creep rupture phenomena have been performed through direct experiments (Agbossou, Cohen and Muller, 1995; Liu and Ross, 1996; Guarino et al., 2002; Lockner, 1998) as well as through different models (Miguel et al., 2002; Ciliberto, Guarino and Scorretti,

2001; Kun et al., 2003; Hidalgo, Kun and Herrmann, 2002; Main, 2000; Politi, Ciliberto and Scorretti, 2002; Pradhan and Chakrabarti, 2003; Turcotte, Newman and Shcherbakov, 2003; Shcherbakov and Turcotte, 2003; Saichev and Sornette, 2005). If a lot of works were devoted to homogeneous materials like metals and ceramics, many recent studies are concerned with heterogeneous materials like composites and rocks (Agbossou, Cohen and Muller, 1995; Liu and Ross, 1996; Guarino et al., 2002; Lockner, 1998). The knowledge of the failure properties of composite materials are of great importance because of the increasing number of applications for composites in engineering structures. The long-term behavior of these materials, especially polymer matrix composites is a critical issue for many modern engineering applications such as aerospace, biomedical and civil engineering infrastructure. The primary concerns in long-term performance of composite materials are in obtaining critical engineering properties that extend over the projected lifetime of the structure. Viscoelastic creep and creep-rupture behaviors are among the critical properties needed to assess longterm performance of polymer-based composite materials. The knowledge of these critical properties is also required to design material microstructures which can be used to construct highly reliable components. For heterogeneous materials, the underlying microscopic failure mechanism of creep rupture is very complex depending on several characteristics of the specific types of materials. Beyond the development of analytical and numerical models, which predict the damage history in terms of the specific parameters of the constituents, another approach is to study the similarity of creep rupture with phase transitions phenomena as summarized here. This approach tackles the large range of scales involved in the damage evolution by using coarse-grained models describing the mechanism of creep, damage and precursory rupture by averaging over the microscopic degrees of freedom to retain only a few major ingredients that are thought to be the most relevant. By comparing the predictions of a hierarchy of models, from simple to elaborate, it is then possible to assess what are the relevant ingredients.

A recent experimental work on heterogenenous structural materials, conducted in GEMPPM at INSA Lyon, illustrates this approach (Nechad et al., 2005). Figure 1 shows a rapid and continuous decrease of the strain rate de/dt in the primary creep regime, which can be described by Andrade's law (Omori's law for the acoustic emissions)

$$\frac{de}{dt} \sim \frac{1}{t^p} , \qquad (1)$$

with an exponent p smaller than or equal to one. A quasi-constant strain rate (steady-state or secondary creep) is observed over an important part of the total creep time and then followed by an increasing creep rate (tertiary creep regime) culminating in fracture. Creep strains at fracture are large with values from a few percent up to 40% for such composite samples. The acceleration of the strain rate before failure is well fitted by a power-law singularity

$$\frac{de}{dt} \sim \frac{1}{(t_c - t)^{p'}} , \qquad (2)$$

with an exponent p' smaller than or equal to one. The critical time t_c determined from the fit of the data with expression (2) is generally close to the observed failure time (within a few seconds). The same temporal evolution is generally obtained for the acoustic emission activity as for the strain rate. The same patterns are obtained when plotting the acoustic emission event rate or the rate of acoustic emission energy. There are much larger fluctuations for the energy rate than for the event rate, due to the large distribution of acoustic emission energies, but the crossover time between primary creep and tertiary creep, and the values of p and p' are similar for the acoustic emission event rate and for the acoustic emission energy rate. This suggests that the amplitude distribution does not depend on time, a conclusion which is verified experimentally. How can one rationalize all these observations?

The role of heterogeneities and disorder

First, we need to define more precisely what is meant by 'heterogeneity' or 'disorder'. Disorder may describe the existence of a distribution (say Weibull-like) of material strength, and/or of their elastic properties, as well as the presence of internal surfaces such as fibermatrix interfaces, voids and microcracks (or internal microdefects). In this sense, a kevlarmatrix or carbon-matrix composite would behave more like a heterogeneous system than a homogeneous matrix. There is not a unique way of defining the amplitude of disorder, since the classification depends on how the mechanics and physics respond to the heterogeneity. It can in fact be shown from a theorem of Von Neumann and Morgenstern (1947) that the existence of possible correlations in the disorder prevents the existence of a unique absolute measure of disorder amplitude. In other words, the measure of disorder is relative to the problem. In practice, it can usually be quantified by some measure of the contrast between material and strength properties of components of the systems, weighted by their relative concentrations and their scales. When disorder is uncorrelated in space, a reasonable measure of its amplitude is the width or standard deviation (when it exists) of its distribution. The correlation length of the disorder and the characteristic sizes and their distribution are also important variables as they control the length scales that are relevant for the stress heterogeneity. A consequence is the size/volume effect, which is a very important practical subject.

As already mentioned, the key parameter controlling the nature of damage and rupture is the degree and nature of disorder. This was considered early by Mogi (1969), who showed experimentally on a variety of materials that, the larger the disorder, the stronger and more useful are the precursors to rupture. For a long time, the Japanese research effort for earthquake prediction and risk assessment was based on this very idea (Mogi, 1995). A quantification of this idea of the role of heterogeneities on the nature of rupture has been obtained with a two-dimensional spring-block model with stress transfer over a limited range and with the democratic fiber bundle model (Andersen, Sornette and Leung, 1997). These models do not claim realism but attempt rather to capture the interplay of heterogeneity and of the stress transfer mechanism. It was found that heterogeneity plays the role of a relevant field (in the language of the statistical physics of critical phase transitions): systems with limited stress amplification exhibit a tri-critical transition (Aharony, 1983), from a Griffithtype abrupt rupture (first-order) regime to a progressive damage (critical) regime as the disorder increases. In the two-dimensional spring-block model of surface fracture (Andersen, Sornette and Leung, 1997), the stress can be released by spring breaks and block slips. This spring-block model may represent schematically the experimental situation where a balloon covered with paint or dry resin is progressively inflated. An industrial application may be for instance a metallic tank with carbon or kevlar fibers impregnated in a resin matrix wrapped up around it which is slowly pressurized (Anifrani et al., 1995). As a consequence, it elastically deforms, transferring tensile stress to the overlayer. Slipping (called fiber-matrix delamination) and cracking can thus occur in the overlayer. In (Andersen, Sornette and Leung, 1997), this process is modeled by an array of blocks which represents the overlayer on a coarse grained scale in contact with a surface with solid friction contact. The solid friction will limit stress amplification. The fact that the disorder is so relevant as to create the analog of a tri-critical behavior can be traced back to the existence of solid friction on the blocks which ensures that the elastic forces in the springs are carried over a bounded distance (equal to the size of a slipping 'avalanche') during the stress transfer induced by block motions. There are similarities between this model and models of quasi-periodic matrix cracking in fibrous composites and of fragmentation of fibers in the so-called 'single-filamentcomposite' test. This last model has been extensively developed and extended in a global and local load-sharing framework (Curtin, 1991; 1998; Ibnabdeljalil and Curtin, 1998).

In the presence of long-range elasticity, disorder is found to be always relevant leading to a critical rupture. However, the disorder controls the width of the critical region (Sornette and Andersen, 1998). The smaller it is, the smaller will be the critical region, which may become too small to play any role in practice. This has been confirmed by simulations of the 'thermal fuse model' described below (Sornette and Vanneste, 1992): the damage rate on the approach to failure for different disorder can be rescaled onto a universal master curve.

Qualitative physical scenario: from diffuse damage to global failure

The following qualitative physical picture for the progressive damage of an heterogeneous system leading to global failure has emerged from a large variety of theoretical, numerical and experimental works (see for instance (Lei et al., 1999; 2000; Tang et al., 2000)). First, single isolated defects and microcracks nucleate which then, with the increase of load or time of loading, both grow and multiply leading to an increase of the density of defects per

unit volume. As a consequence, defects begin to merge until a 'critical density' is reached. Uncorrelated percolation (Stauffer and Aharony, 1992) provides a starting modeling point valid in the limit of very large disorder (Roux et al., 1988). For realistic systems, long-range correlations transported by the stress field around defects and cracks make the problem much more subtle. Time dependence is expected to be a crucial aspect in the process of correlation building in these processes. As the damage increases, a new 'phase' appears, where microcracks begin to merge leading to screening and other cooperative effects. Finally, the main fracture is formed causing global failure. The nature of this global failure may be abrupt ('first-order') or 'critical' depending of the strength of heterogeneity as well as load transfer and stress relaxation mechanisms. In the 'critical' case, the failure of composite systems may often be viewed, in simple intuitive terms, as the result of a 'correlated percolation processes.' However, the challenge is to describe the transition from damage and corrosion processes at a microscopic level to macroscopic failure.

Scaling and critical point

Motivated by the multi-scale nature of ruptures in heterogeneous systems and by analogies with the percolation model (Stauffer and Aharony, 1992), statistical physicists suggested in the mid-1980s that rupture of sufficiently heterogeneous media would exhibit some universal properties, in a way maybe similar to critical phase transitions (de Arcangelis, Redner and Herrmann, 1985; Duxbury, Beale and Leath, 1986; Gilabert et al., 1987). The idea was to build on the knowledge accumulated in statistical physics on the so-called N—body problem and cooperative effects in order to describe multiple interactions between defects. However, most of the models were drastically simplified and essentially all of them quasi-static with rather unrealistic loading rules (Herrmann and Roux, 1990; Meakin, 1991). Suggestive scaling laws, including multifractality, were found to describe size effects and damage properties (Herrmann and Roux, 1990; Hansen, Hinrichsen and Roux, 1991), but the relevance to real materials was not convincingly demonstrated with a few exceptions (e.g., percolation theory to explain the experimentally based Coffin-Manson law of low cycle fatigue (Bréchet, Magnin and Sornette, 1992) or the Portevin-Le Chatelier effect in diluted alloys (Bharathi and Ananthakrishna, 2002)).

With numerical simulations and perturbation expansions, Hansen, Hinrichsen and Roux (1991) (see also Herrmann and Roux, 1990) have used this class of quasi-static rupture models (with short-range as well as long-range interactions) to classify three possible rupture regimes, as a function of the concentrations of weak versus strong elements in the system. Specifically, the distribution p(x) of rupture thresholds x of elements of the discretized systems was parameterized as follows: $p(x) \sim x^{\phi_0-1}$ for $x \to 0$ and $p(x) \sim x^{-(1+\phi_\infty)}$ for $x \to +\infty$. Then, the three regimes depend on the relative value of the exponents ϕ_0 and ϕ_∞ compared with two critical values ϕ_0^c and ϕ_∞^c . The 'weak disorder' regime occurs for $\phi_0 > \phi_0^c$ (few weak elements)

and $\phi_{\infty} > \phi_{\infty}^{c}$ (few strong elements) and boils down essentially to the nucleation of a 'onecrack' run-away. For $\phi_{0} \leq \phi_{0}^{c}$ (many weak elements) and $\phi_{\infty} > \phi_{\infty}^{c}$ (few strong elements), the rupture is controlled by the weak elements, with important size effects. The damage is diffuse but presents a structuration at large scales. For $\phi_{0} > \phi_{0}^{c}$ (few weak elements) and $\phi_{\infty} \leq \phi_{\infty}^{c}$ (many strong elements), the rupture is controlled by the strong elements: the final damage is diffuse and the density of broken elements goes to a non-vanishing constant. This third case is very similar to the percolation models of rupture: Roux et al. (1988) have indeed shown that percolation is retrieved in the limit of very large disorder.

Beyond quasi-static models, the 'thermal fuse model' of Sornette and Vanneste (1992) was the first one with a realistic dynamical evolution law for the damage field. It was initially formulated in the framework of electric breakdown: when subjected to a given current, all fuses in a network heat up due to a generalized Joule effect (with exponent b); in the presence of heterogeneity in the conductances of the fuses, one of them will eventually breaks down first when its temperature reaches the melting threshold. Its current is then immediately distributed in the remaining fuses according to Kirchoff law. The model was later reformulated by showing that it is exactly equivalent to a (scalar) antiplane mechanical model of rupture with elastic interaction in which the temperature becomes a local damage variable (Sornette and Vanneste, 1994). This model accounts for space-dependent elastic and rupture properties, has a realistic loading (constant stress applied at the beginning of the simulation, for instance) and produces growing interacting micro-cracks with an organization which is a function of the damage-stress law. This model is thus a statistical generalization with quenched disorder of homogeneization theories of damage (Chaboche, 1995; Maire and Chaboche, 1997). In a creep experiment (constant applied stress), the total rate of damage in the late stage of evolution, as measured for instance by the elastic energy released per unit time dE/dt, is found on average to increase as a power law similar to expression (2),

$$dE/dt \sim 1/(t_c - t)^{\alpha} , \qquad (3)$$

of the time-to-failure $t_c - t$ in the later stage. This behavior reproduces the tertiary creep regime culminating in the global rupture at t_c . In this model, rupture is found to occur as the culmination of the progressive nucleation, growth and fusion between microcracks, leading to a fractal network. Interestingly, the critical exponents (such as $\alpha > 0$) are non-universal and vary as a function of the damage law (exponent b). This model has since then been found to describe correctly the experiments on the electric breakdown of insulator-conducting composites (Lamaignère, Carmona and Sornette, 1996). Another application of the thermal fuse model is the damage by electromigration of polycristalline metal films (Bradley and Wu, 1994). See also (Sornette and Vanneste, 1994) for relations with dendrites and fronts propagation.

The concept that rupture in heterogenous materials is a genuine critical point, in the sense of

phase transitions in statistical physics, was first articulated by Anifrani et al. (1995), based on experiments on industrial composite structures. In this framework, the power law (3) is interpreted as analogous to a diverging susceptibility in critical phase transitions. It was found that the critical behavior may correspond to an acceleration of the rate of energy release or to a deceleration, depending on the nature and range of the stress transfer mechanism and on the loading procedure. Symmetry arguments as well as the concept of a hierarchical cascade of damage events led in addition to suggest theoretically and verify experimentally that the power law behavior (3) of the time-to-failure analysis should be corrected for the presence of log-periodic modulations (Anifrani et al., 1995). This 'log-periodicity' can be shown to be the signature of a hierarchy of characteristic scales in the rupture process. This hierarchy can be generated dynamically by a cascade of sub-harmonic bifurcations (Huang et al., 1997). These log-periodic corrections to scaling amount mathematically to taking the critical exponent $\alpha = \alpha' + i\alpha''$ complex, where $i^2 = -1$ (Sornette, 1998). This has led to the development of a powerful predictive scheme ((Le Floc'h and Sornette, 2003) and see below). The critical rupture concept can be seen as a non-trivial generalization of the dimension analysis based on Buckingham theorem and the asymptotic matching method proposed by Bazant (1997) to model size effect in complex materials, in the same way that Barenblatt (1987)'s second-order similitude generalizes the naive similitude of first-order (or simple analytical behavior) of standard dimensional analysis, or in the same way the non-analytical behavior characterizing critical phase transitions generalizes the mean-field behavior of Landau-Ginzburg theory. Acharyya and Chakrabarti (1996) have shown how to define a "breakdown susceptibility" during the progressive damage of model systems when subjected to local short-duration impulses and how the breakdown point can then be located in advance by extrapolating this breakdown susceptibility.

Numerical simulations on two-dimensional heterogeneous systems of elastic-brittle elements have confirmed that, near the global failure point, the cumulative elastic energy released during fracturing of heterogeneous solids with long-range elastic interactions follows a power law with log-periodic corrections to the leading term (Sahimi and Arbati, 1996). The presence of log-periodic correction to scaling in the elastic energy released has also been demonstrated numerically for the thermal fuse model (Johansen and Sornette, 1998) using a novel averaging procedure, called the 'canonical ensemble averaging.' This averaging technique accounts for the dependence of the critical rupture time t_c on the specific disorder realization of each sample. A recent experimental study of rupture of fiber-glass composites has also confirmed the critical scenario (Garcimartin et al., 1997). A systematic analysis of industrial pressure tanks brought to rupture has also confirmed the critical rupture concept and the presence of significant log-periodic structures, that are useful for prediction (Johansen and Sornette, 2000). Through a series of computer and laboratory simulations and table-top experiments, Chakrabarti and Benguigui (1997) have presented a useful synthesis of basic modeling principles borrowing from statistical physics putting in perspective three case studies: electrical failures like fuse and dielectric breakdown, mechanical fractures, and earthquakes. Their work also emphasizes the critical rupture concept (Acharyya and Chakrabarti, 1996; Banerjee and Chakrabarti, 2001; Pradhan and Chakrabarti, 2002).

Let us also mention the work of Ramanathan and Fisher (1998): using analytical calculations and by numerical simulations, they compare the nature of the onset of a single crack motion in an heterogeneous material when neglecting or taking into account the dynamical wave stress transfer mechanism. In the quasistatic limit with instantaneous stress transfer, the crack front is found to undergo a dynamic critical phenomenon, with a second-order-like transition from a pinned to a moving phase as the applied load is increased through a critical value. Real elastic waves lead to overshoots in the stresses above their eventual static value when one part of the crack front moves forward. Simplified models of these stress overshoots showed an apparent jump in the velocity of the crack front directly to a nonzero value. In finite systems, the velocity also shows hysteretic behavior as a function of the loading. These results suggest a first-order-like transition (Ramanathan and Fisher, 1998).

Creep rupture: models

Let us come back to the experiments shown in Figure 1. There are many models, at the interface between standard mechanical approaches and statistical physics, which attempt to capture these observations. Vujosevic and Krajcinovic (1997), Turcotte, Newman and Shcherbakov (2003), Shcherbakov and Turcotte (2003) and Pradhan and Chakrabarti (2004) used systems of elements or fibers within a probabilistic framework (corresponding to socalled annealed or thermal disorder) with a hazard rate function controlling the probability of rupture for a given fiber as a function of the stress applied to that fiber. Turcotte, Newman and Shcherbakov (2003) obtained a finite-time singularity of the strain rate before failure in fiber bundle models by postulating a power law dependence of the hazard rate controlling the probability of rupture for a given fiber as a function of the stress applied to that fiber. Shcherbakov and Turcotte (2003) studied the same model and recovered a power-law singularity of the strain rate for systems subjected to constant or increasing stresses with an exponent p' = 4/3 larger than the experimental results. Using energy conservation and the requirement of non-negative entropy change, Lyakhovsky, Ben-zion and Agnon (1997) derived an evolution equation for the density of microcracks similar to that of Turcotte, Newman and Shcherbakov (2003) for a fiber bundle model. Ben-Zion and Lyakhovsky (2002) derived analytically the existence of power laws describing the timedependent increase of the singular strain and the accelerated energy release in the tertiary regime using the continuum-based damage approach of Lyakhovsky, Ben-zion and Agnon (1997). Sammis and Sornette (2002) give an exhaustive review of the mechanisms giving rise to the power law tertiary regime, with application to earthquakes. Vujosevic and Krajcinovic (1997) also found a power-law acceleration in two-dimensional simulations of elements and in a mean-field democratic load sharing model, using a stochastic hazard rate, but they do not obtain Andrade's law in the primary creep regime. Shcherbakov and Turcotte (2003) were able to obtain Andrade law only in the situation of a system subjected to a constant applied strain (stable regime). But then, they did not have a global rupture and they did not obtain the critical power law preceding rupture. Thus, the models described above do not reproduce at the same time Andrade's law for the primary regime and a power-law singularity before failure. Miguel et al. (2002) reproduced Andrade's law with $p \approx 2/3$ in a numerical model of interacting dislocations, but their model does not reproduce the tertiary creep regime (no global failure).

Several creep models consider the democratic fiber bundle model (DFBM) with thermally activated failures of fibers. Pradhan and Chakrabarti (2004) considered the DFBM and added a probability of failure per unit time for each fiber which depends on the amplitude of a thermal noise and on the applied stress. They computed the failure time as a function of the applied stress and noise level but they did not discuss the temporal evolution of the strain rate. Ciliberto, Guarino and Scorretti (2001) and Politi, Ciliberto and Scorretti (2002) considered the DFBM in which a random fluctuating force is added on each fiber to mimic the effect of thermal fluctuations. Ciliberto, Guarino and Scorretti (2001) showed that this simple model predicts a characteristic rupture time given by an Arrhenius law with an effective temperature renormalized (amplified) by the quenched disorder in the distribution of rupture thresholds. Saichev and Sornette (2005) showed that this model predicts Andrade's law as well as a power law time-to-failure for the rate of fiber rupture with p = p' = 1, with logarithm corrections (which may give apparent exponents p and p' smaller than 1).

A few other models reproduce both a power-law relaxation in the primary creep and a finite time singularity in the tertiary regime. Main (2000) reproduced a power-law relaxation (Andrade's law) followed by a power-law singularity of the strain rate before failure by superposing two processes of subcritical crack growth, with different parameters. A first mechanism with negative feedback dominates in the primary creep and the other mechanism with positive feedback gives the power-law singularity close to failure. Lockner (1998) gave an empirical expression for the strain rate as a function of the applied stress in rocks, which reproduces, among other properties, Andrade's law with p = 1 in the primary regime and a finite-time singularity leading to rupture.

Kun et al. (2003) and Hidalgo, Kun and Herrmann (2002) studied numerically and analytically a model of visco-elastic fibers, with deterministic dynamics and quenched disorder. They considered different ranges of interaction between fibers (local or democratic load sharing). Kun et al. (2003) derived the condition for global failure in the system and the evolution of the failure time as a function of the applied stress in the unstable regime, and analysed the statistics of inter-event times in numerical simulations of the model. Hidalgo, Kun and Herrmann (2002) derived analytically the expression for the strain rate as a function of time. This model reproduces a power-law singularity of the strain rate before failure with p' = 1/2 in the case of a uniform distribution of strengths, but is not able to explain Andrade's law for the primary creep. This model gives a power-law decay of the strain rate in the primary

creep regime only if the stress is at the critical point, but with an exponent p = 1/2 smaller than the experimental values. Nechad et al. (2005) developed a variant of this model in which a composite system is viewed as made of a large set of representative elements (RE), each representative element comprising many fibers with their interstitial matrix. Each RE is endowed with a visco-elasto-plastic rheology with parameters which may be different from one element to another. The parameters characterizing each RE are frozen and do not evolve with time (so-called quenched disorder). Specifically, each RE is modeled as an Eyring dashpot in parallel with a linear spring. The Eyring rheology is standard for fiber composites (Liu and Ross, 1996). It consists, at the microscopic level, in adapting to the matrix rheology the theory of reaction rates describing processes activated by crossing potential barriers. With these sole ingredients, the model recovers the three primary, secondary and tertiary regimes with exponents p = 1 (defined in expression (1)) and p' = 1 (defined in expression (2)). These solutions for the primary and tertiary regimes are basically of the same form with p = p' = 1 as the Langevin-type model solved by Saichev and Sornette (2005); this may not be surprising since the Evring rheology describes, at the microscopic level, processes activated by crossing potential barriers, which are explicitly accounted for in the thermal fluctuation force model (Saichev and Sornette, 2005). The key ingredients leading to these results are the broad (power law) distribution of rupture thresholds and the nonlinear Eyring rheology in a Kelvin element. Nechad et al.'s model is a macroscopic deterministic effective description of the experiments. In contrast, the modeling strategy of Ciliberto, Guarino and Scorretti (2001), of Politi, Ciliberto and Scorretti (2002) and of Saichev and Sornette (2005) emphasizes the interplay between microscopic thermal fluctuations and frozen heterogeneity. Qualitatively, Nechad et al.'s model is similar to a deterministic macroscopic Fokker-Planck description while the thermal models of Ciliberto, Guarino and Scorretti (2001), of Politi, Ciliberto and Scorretti (2002) and of Saichev and Sornette (2005) are reminiscent of stochastic Langevin models. It is well-known in statistical physics that Fokker-Planck equations and Langevin equations are exactly equivalent for systems at equilibrium and just constitute two different descriptions of the same processes, and their correspondence is associated with the general fluctuation-dissipation theorem. Similarly, the encompassing of both the Andrade relaxation law in the primary creep regime and of the time-to-failure power law singularity in the tertiary regime by Nechad et al.'s model and by the thermal model solved in (Saichev and Sornette, 2005) suggests a deep connection between these two levels of description for creep and damage processes.

Toward rupture prediction

There is a huge variability of the failure time from one sample to another one, for the same applied stress, as shown in Figure 2. This implies that one cannot predict the time to failure of a sample using an empirical relation between the applied stress and the time of failure. There is however another approach suggested by Figure 2 as proposed by Nechad et al. (2005). It shows the correlation between the transition time t_m (minimum of the strain rate) and the rupture time $t_c \approx t_{end}$ and shows that t_m is about 2/3 of the rupture time t_c . This suggests a way to predict the failure time from the observation of the strain rate during the primary and secondary creep regimes, before the acceleration of the damage during the tertiary creep regime leading to the rupture of the sample. As soon as a clear minimum is observed, the value of t_m can be measured and that of t_c deduced from the relationship shown in Figure 2. However, there are some cases where the minima is not well defined, for which the first (smoothed) minimum is followed by a second similar one. In this case, the application of the relationship shown in Figure 2 would lead to a pessimistic prediction for the lifetime of the composite.

The observation that the failure time is correlated with the *p*-value and the duration of the primary creep suggests that, either a single mechanism is responsible both for the decrease of the strain rate during primary creep and for the acceleration of the damage during the tertiary creep or, if the mechanisms are different nevertheless, the damage that occurs in the primary regime impacts on its subsequent evolution in the secondary and tertiary regime, and therefore on t_c . In contrast, using a fit of the acoustic emission activity by a power-law to estimate t_c according to formula (3) works only in the tertiary regime and thus does not exploit the information contained in the deformation and in the acoustic emissions of the primary and secondary regimes which cover 2/3 to 3/4 of the whole history. In practice, one needs at least one order of magnitude in the time $t_c - t$ to estimate accurately t_c and p', which means that, if the power-law acceleration regime starts immediately when the stress is applied (no primary creep), one cannot predict the rupture time using a fit of the damage rate by equation (3) before 90% of the failure time. If, as observed in the experiments of Nechad et al. (2005), the tertiary creep regime starts only at about 63% of t_c , then one cannot predict the rupture time using a fit of the damage rate before 96% of the failure time. This limitation was the motivations for the development of formulas that interpolate between the primary and tertiary regimes beyond the pure power law (3) using log-periodic corrections to scaling (Anifrani et al., 1995; Johansen and Sornette, 2000; Gluzman et al., 2001; Moura and Yukalov, 2002; Gauthier et al., 2002; Yukalov et al., 2004).

In particular, Anifrani et al. (1995) have introduced a method based on log-periodic correction to the critical power law which has been used extensively by the European Aerospace company Aérospatiale (now EADS) on pressure tanks made of kevlar-matrix and carbonmatrix composites embarked on the European Ariane 4 and 5 rockets. In a nutshell, the method consists in this application in recording acoustic emissions under constant stress rate and the acoustic emission energy as a function of stress is fitted by the above log-periodic critical theory. One of the parameters is the time of failure and the fit thus provides a 'prediction' when the sample is not brought to failure in the first test (Gauthier et al., 2002). The results indicate that a precision of a few percent in the determination of the stress at rupture is typically obtained using acoustic emission recorded about 20 % below the stress at rupture. This has warranted the selection of this non-destructive evaluation technique as the routine qualifying procedure in the industrial fabrication process. This methodology and these experimental results have been guided by the theoretical research over the years using the critical rupture concept discussed above. In particular, there is now a better understanding of the conditions, the mathematical properties and physical mechanisms at the basis of log-periodic structures (Huang et al., 1997; Sornette, 1998; 2002; Ide and Sornette, 2002; Zhou and Sornette, 2002). Another noteworthy approach already mentioned above for the prediction of rupture, which is inspired by statistical physics, is the "breakdown susceptibility" introduced by Acharyya and Chakrabarti (1996). It requires monitoring the response of the system when subjected to local short-duration impulses whose nature depends upon the problem (stress, strain, temperature, electromagnetic and so on).

In summary, starting with the initial flurry of interest from the statistical physics community on problems of material rupture, a new awareness of the many-body nature of the rupture problem has blossomed. There is now a growing understanding in both communities of the need for an interdisciplinary approach, improving on the reductionist approach of both fields to tackle at the same time the difficult modelling of specific properties of the microscopic structures and their interactions leading to collective effects. Independently of the types of materials for given applications, this approach will be crucial in making progress on the optimisation of the lifetime of materials ("durability") and on the determination of the remaining life time of materials in use ("remaining potential").

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Didier Sornette

Institute of Geophysics and Planetary Physics and Department of Earth and Space Science University of California, Los Angeles, California, USA and CNRS and Université des Sciences Nice, France sornette@moho.ess.ucla.edu

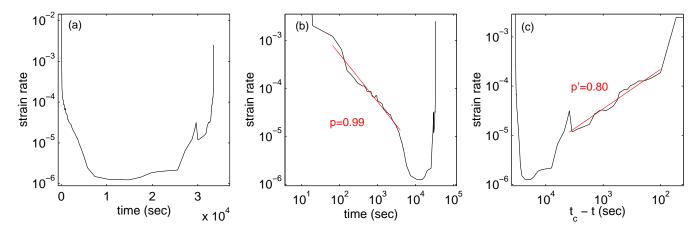


Figure 1: Creep strain rate for a Sheet Molding Compound (SMC) composite consisting in a combination of polyester resin, calcium carbonate filler, thermoplastic additive and random oriented short glass reinforced fibres. The creep experiment was performed at a stress of 48 MPa and a temperature $T = 100^{\circ}$ C, below the glass transition, at the GEMPPM, INSA LYON, Villeurbanne, France. The stress was increased progressively and reached a constant value after about 17 sec. Left panel: full history in linear time scale; middle panel: time is shown in logarithmic scale to test for the existence of a power law relaxation regime; right panel: time is shown in the logarithm of the time to rupture time t_c such that a time-to-failure power law (2) is qualified as a straight line. Reproduced from (Nechad et al., 2004).

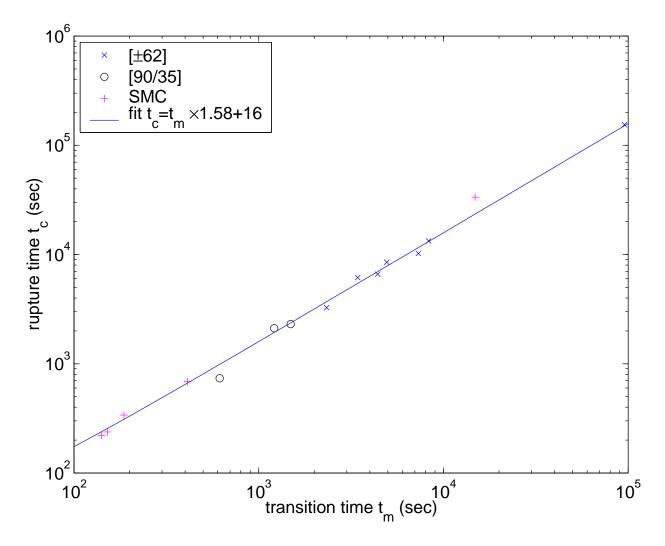


Figure 2: Relation between the time t_m of the minima of the strain rate and the rupture time t_c , for all samples investigated in (Nechad et al., 2004).