Resonance model for πN scattering and η-meson production in the S_{11} channel

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Abstract

A model for πN scattering and η-meson production in the S_{11} channel is presented. The model includes πN-scattering Born terms as well as the N^* resonances $S_{11}(1535)$ and $S_{11}(1610)$. The T-matrix is computed in the K-matrix approximation. The parameters of the model are determined by fitting the elastic πN-scattering T-matrix to empirical data. We find an excellent fit for all energies up to $\sqrt{s} = 1.75$ GeV. Furthermore, a good description of the cross section for $\pi^- + p \rightarrow \eta + n$ is obtained without further adjustment of parameters.

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1 Introduction

During the last few years there has been renewed interest in the physics of the η-meson. New data on η-production in different reactions are becoming available. For example, the photoproduction of η-mesons is being studied at MAMI [1] and ELSA [2], while η-production in hadronic and heavy-ion collisions is under investigation at SATURNE [3] and GSI [4], respectively. Calculations of these processes have been performed by several groups (see e.g. refs. [5-9]).

The ultimate aim of our investigation is a better understanding of the η-properties in an hadronic environment and the possible consequences for η-production in heavy-ion collisions. However, before attacking the relatively complex problem of medium effects, we need a better understanding of more elementary processes, like e.g. η-production in hadronic collisions and in photo-induced reactions. New accurate data on these processes will in the near future provide very useful constraints on models for the elementary η-meson–hadron interactions and the properties of the η-meson at normal nuclear matter density and below. The investigation of possible η-meson bound states in nuclei could also yield information on η-meson–nucleon interactions at low densities [10]. Since the η-meson couples strongly to the $\mathrm{S}_{11}(1535)$ resonance, the in-medium properties of the η-meson and this resonance are strongly interdependent. Thus, η-production on nuclei also probes the in-medium properties of the $\mathrm{S}_{11}(1535)$ resonance.

A useful framework for the study of hadrons in nuclear matter is an effective field theory with hadronic degrees of freedom. In such theories one needs as input the hadronic coupling constants. It is the aim of this letter to construct a model for pion-induced η-production on the nucleon. The parameters, in particular the $\eta\mathrm{NN}^*$ coupling constant, are determined by fitting data on elastic $\pi\mathrm{N}$ scattering. The consistency of our model is then checked by comparing the cross section for the inelastic channel $\pi\mathrm{N}\rightarrow\eta\mathrm{N}$ with experiment. The present calculation is a first step in a systematic investigation of η-interactions with hadrons and hadronic matter, as outlined above.

2 Model lagrangian

Pion-nucleon scattering in the S_{11} channel at center-of-mass (c.m.) energies below $\sqrt{s}$ =1.8 GeV involves two N^* resonances [11, 12]. The lower one, the $\mathrm{S}_{11}(1535)$ resonance, decays into $\pi\mathrm{N}$, $\eta\mathrm{N}$ and $\pi\pi\mathrm{N}$ with the branching ratios 35–55 %, 30–50 % and 5–20 %, respectively [13]. The upper one, i.e. the $\mathrm{S}_{11}(1650)$, couples strongly to the $\pi\mathrm{N}$ and $\pi\pi\mathrm{N}$ channels with branching ratios of 60–80 % and 5–20 %, respectively, but only weakly to the $\eta\mathrm{N}$ channel with a probability of approximately 1 %.

In order to describe these experimental facts we include the following interaction terms in the lagrangian:

$$
\begin{aligned}
\mathcal{L}_I = & -ig_{\pi \mathrm{NN}}\overline{\Psi}_{\mathrm{N}}\gamma_5\vec{\tau}\Psi_{\mathrm{N}}\vec{\pi} - g_{\sigma\mathrm{NN}}\overline{\Psi}_{\mathrm{N}}\Psi_{\mathrm{N}}\sigma - g_{\sigma\pi\pi}\vec{\pi}^2\sigma \\
& -ig_{\pi\mathrm{NN}_1^*}\overline{\Psi}_{\mathrm{N}_1^*}\vec{\tau}\Psi_{\mathrm{N}}\vec{\pi} + \mathrm{h.c.} - ig_{\pi\mathrm{NN}_2^*}\overline{\Psi}_{\mathrm{N}_2^*}\vec{\tau}\Psi_{\mathrm{N}}\vec{\pi} + \mathrm{h.c.} \\
& -ig_{\eta\mathrm{NN}_1^*}\overline{\Psi}_{\mathrm{N}_1^*}\Psi_{\mathrm{N}}\eta + \mathrm{h.c.} \\
& -ig_{\zeta\mathrm{NN}_1^*}\overline{\Psi}_{\mathrm{N}_1^*}\gamma_5\Psi_{\mathrm{N}}\zeta + \mathrm{h.c.} - ig_{\zeta\mathrm{NN}_2^*}\overline{\Psi}_{\mathrm{N}_2^*}\gamma_5\Psi_{\mathrm{N}}\zeta + \mathrm{h.c.}.
\end{aligned} \tag{1}
$$

The first line contains the interaction terms of the linear sigma-model [14, 15] which is chirally invariant and describes low-energy πN scattering reasonably well. In this model the following relations between the coupling constants hold: $g_{\sigma\mathrm{NN}} = g_{\pi\mathrm{NN}}$ and $g_{\sigma\pi\pi} = g_{\pi\mathrm{NN}}(m_\sigma^2 - m_\pi^2)/2m_{\mathrm{N}}$, where m_σ, m_π and m_{N} denote the masses of σ, π and nucleon, respectively. The interaction terms in the second line describe the πNN* interaction for the two N* resonances (N$_1^*$ $\equiv$S$_{11}$(1535), N$_2^*$ $\equiv$S$_{11}$(1650)) whereas those in the third line account for the ηNN$_1^*$ coupling. The weak ηNN$_2^*$ and ηNN couplings are neglected [16].

In principle, one could attempt to describe the $\pi\pi$N decay of the S$_{11}$ resonances by the two-step process N$^* \to \sigma$N $\to \pi\pi$N. This would require only two additional coupling terms of the resonances to the σN channel. The subsequent decay of the σ into two pions would be described by the third term in (1). However, there are other intermediate states, like ρN and $\pi\Delta$, which also contribute to the $\pi\pi$N decay channel and should be handled on the same footing as the σN process. Thus, a proper treatment of this decay channel, which plays a somewhat peripheral role in our investigation, would be unduly complex. Consequently, we choose a phenomenological approach, where the physical two-pion continuum is parametrized by means of an effective scalar field ζ of positive parity, mass m$_\zeta$=400 MeV and zero width. The field ζ interacts only through the ζNN* couplings, which are given in the last line of (1).

3 T-matrix

The T-matrix is related to the K-matrix by the integral equation

$$
T = K - i\pi K\delta(E - H_0)T, \tag{2}
$$

where H_0 describes the free motion of the two particles in one of the three coupled channels πN, ηN and ζN. The K-matrix can be derived from the lagrangian by solving another, more complicated, integral equation. However, since any approximation, which leaves the K-matrix hermitian, yields a unitary S-matrix [17], we circumvent this step by identifying the K-matrix elements with the diagrams shown in fig. 1 (c.f. ref. [18]). In the tree approximation to the linear σ-model the direct and crossed nucleon pole terms and the σ-exchange term are included [14, 15]. These terms, the so called Born terms, correspond to the first three diagrams of $K_{\pi\pi}$ in fig. 1. We also include the direct contributions from the resonances. The corresponding crossed diagrams are suppressed due to the large masses in intermediate states.

In the diagrams for the K-matrix the intermediate state particles propagate with their physical masses but with zero widths; e.g., for the $\mathrm{S}_{11}(1535)$ resonance in the intermediate state the corresponding inverse propagator is given by $S(p)^{-1} = \not{p} - m_1$, where m_1 is real. The summation of the processes implied by eq. (2) generates the imaginary parts needed to satisfy unitarity, and consequently also the correct resonance widths. In this approximation the K-matrix, defined by the diagrams in fig. 1, is obviously free of divergences. Since the intermediate meson-nucleon states in eq. (2) are restricted to be on-shell, this also applies to the T-matrix. Consequently, there is no need to renormalize e.g. the coupling constants. Furthermore, since the integral equation (2) reduces to an algebraic one after projection on partial waves, one can, given a suitable approximation to the K-matrix, write down the T-matrix analytically.

We introduce form factors for the off-shell nucleon at the πNN vertex [19]

$$F = \frac{\Lambda^4}{\Lambda^4 + (p_{\mathrm{N}}^2 - m_{\mathrm{N}}^2)^2} \tag{3}$$

and for the off-shell σ-meson at the $\sigma\pi\pi$ and σNN vertices

$$G_{1,2} = \frac{\lambda_{1,2}^2}{\lambda_{1,2}^2 - p_\sigma^2}. \tag{4}$$

Here p_{N} and p_σ are the 4-momenta of the off-shell nucleon and σ-meson, respectively.

The T-matrix in the S_{11} channel is obtained by projecting eq. (2) onto total angular momentum $j = 1/2$, isospin $t = 1/2$ and negative parity. Since the π- and η-mesons have negative parity, only $l = 0$ for the relative angular momentum is allowed in the πN and ηN intermediate states. On the other hand, the ζN intermediate state must be p-wave, since the ζ-meson has positive parity. With our choice of form factors the projection can be done analytically.

The resulting T-matrix can be given in closed form. However, since the full expression, keeping all three channels is quite lengthy, we do not present it here. Instead, we show the solution for the simpler case, where the $\pi\pi$N channel is neglected. For elastic πN scattering in the S_{11} channel we then find

$$T_{\pi\pi}^0 = \frac{-im_{\mathrm{N}}q}{(4\pi)^2\sqrt{s}} \left[\frac{-\left[F(k)\gamma - v\right]\left[K^B w + \beta\right] + \alpha w}{\left[F(k)\gamma - v)\right]\left[F(q)(K^B w + \beta) - w\right] - \alpha w F(q)} \right], \tag{5}$$

where $\sqrt{s}$ is the invariant mass in the πN channel. The absolute values of the relative three-momenta in the c.m.-system of the πN and ηN system are denoted by q and k, respectively. Furthermore, $v = \sqrt{s} - m_1$, $w = \sqrt{s} - m_2$, where m_1 and m_2 denote the masses of the two resonances and $F(k) = m_{\rm N} k / \left(4(2\pi^2)\sqrt{s}\right)$. The other quantities are given by

$$\begin{aligned} \alpha &= \frac{-6i\pi g^2_{\pi {\rm NN}^*_1}}{m_{\rm N}}(E_q + m_{\rm N}),\ \beta = \frac{-6i\pi g^2_{\pi {\rm NN}^*_2}}{m_{\rm N}}(E_q + m_{\rm N}), \\ \gamma &= \frac{-2i\pi g^2_{\eta {\rm NN}^*_1}}{m_{\rm N}}(E_k + m_{\rm N}) \end{aligned} \tag{6}$$

where $E_q = \sqrt{q^2 + m^2_{\rm N}}$ and $E_k = \sqrt{k^2 + m^2_{\rm N}}$. The projection of the non-resonant πN Born terms onto the S_{11} channel is denoted by K^B. In order to obtain the empirical scattering lengths in the tree approximation to the linear σ-model, the πNN coupling constant must be renormalized [15]. This renormalization, which effectively accounts for loop corrections, is usually done by fixing the axial coupling constant to its empirical value $g_A \approx 1.3$. We choose a pragmatic approach and adjust the effective πNN coupling constant to the s-wave πN scattering length a_1 for isospin=1 in the t-channel, where the σ-exchange term does not contribute. The mass of the σ-meson is then fixed by the isospin=0 scattering length a_0. The remaining five coupling constants as well as the masses of the two resonances are then determined by fitting the elastic πN-scattering data up to $\sqrt{s} = 1.84$ GeV.

4 Elastic πN-scattering data

There are several partial-wave analyses of the πN-scattering data available. We employ the solutions obtained by Höhler and the Karlsruhe-Helsinki group [11] and by Cutkosky and the CMU-LBL group [12].

In our least-square fit we find a fairly shallow, but nevertheless clear minimum. The resulting parameter sets are given in table 1 in rows 1 and 2.

Table 1. Parameter values obtained by fitting the Karlsruhe-Helsinki (KH) and CMU-LBL solutions.

	m_1 [GeV]	m_2 [GeV]	$g_{\pi {\rm NN}^*_1}$	$g_{\pi {\rm NN}^*_2}$	$g_{\eta {\rm NN}^*_1}$	$g_{\zeta {\rm NN}^*_1}$	$g_{\zeta {\rm NN}^*_2}$
KH	1.55	1.70	0.67	1.17	2.10	2.36	4.85
CMU-LBL	1.53	1.70	0.64	1.26	2.13	1.89	4.37
KH2	1.55	1.69	0.70	1.20	2.16	3.08	5.00

The πNN coupling constant $g_{\pi{\rm NN}}$=8.99 which corresponds to $g_A = 1.48$, and the σ-meson mass m_σ=640 MeV are fixed by fitting the scattering lengths, as described above. The cutoff masses $\Lambda = 3.1$ GeV, $\lambda_1 = 2.0$ GeV and $\lambda_2 = 2.1$ GeV are determined by fitting the data below the first resonance.

We note that the two fits are of similar quality and yield very similar parameter sets. In figs. 2 and 3 we show the fit to the Karlsruhe-Helsinki solution which is well reproduced up to and including the energy range of the second resonance. The discrepancy at energies beyond $\sqrt{s}$= 1.75 GeV is most likely due to other resonances, not included in our model.

Since the presence of form factors complicates the question of gauge invariance e.g. in the process $\gamma + p \to \eta + p$, we have also made a fit to the Karlsruhe-Helsinki solution without a form factor at the πNN vertex. This fit (c.f. figs. 2 and 3) deviates appreciably from the other ones only at high energies ($\sqrt{s} > 1.75$ GeV). The corresponding parameters are given in the third row of table 1.

As a further test of the model we have also calculated the cross section for the process $\pi^- + p \to \eta + n$

$$\sigma_{\pi\eta} = \frac{4\pi}{q^2}|T_{\pi\eta}|^2. \tag{7}$$

The T-matrix element $T_{\pi\eta}$ is approximated by its l=0 component obtained in our model. In fig. 4 the resulting cross section is compared with the experimental data compiled in ref. [20]. We find good agreement in the energy range of the $S_{11}(1535)$ resonance. At higher energies contributions of higher lying resonances become important (see below).

The $S_{11}(1650)$ resonance, which was not included in previous models [21, 5], plays an important role both in elastic πN scattering and η-production. The coupling strengths of one resonance are affected by the presence of the other one, since the two resonances overlap due to their relatively large widths. At an isolated resonance the real part generally goes through zero near the resonance energy. The behaviour of the real part of the T-matrix in the S_{11} channel can only be described by taking two overlapping resonances into account. Furthermore, the upper resonance, although it couples only weakly to the η-meson, plays a crucial role in the determination of $g_{\eta \mathrm{NN}_1^*}$. In order to demonstrate this, we have repeated the fit in a reduced model, where the upper resonance is neglected. A reasonable description of the data can be obtained for energies below $\sqrt{s} = 1.55$ GeV, where one would expect only a small contribution from the $S_{11}(1650)$. However, compared to the complete model we find a strong increase of both coupling constants ($g_{\pi \mathrm{NN}_1^*} = 0.80$ and $g_{\eta \mathrm{NN}_1^*} = 2.38$). Furthermore, the η-production data cannot be described within the reduced model; at its maximum the cross section is about 40 % too large.

5 Conclusions

We have constructed a model for πN scattering in the elastic as well as in the major inelastic S_{11} channels. The parameters of the model are determined by fitting only data for elastic scattering. Our best fit yields $g_{\eta \mathrm{NN}_1^*} = 2.10$, which differs considerably from the values obtained in most earlier calculations [5,10,21]. Vetter *et al.* [9] find a value close to ours by fitting the partial decay widths of the S_{11} resonance. However, we caution the reader that the coupling constants discussed here, in particular $g_{\eta \mathrm{NN}_1^*}$, may be model dependent (see e.g. the discussion at the end of section 4).

We find a total width of the $S_{11}(1535)$ of 162 MeV with branching ratios of 55 % to ηN, 41 % to πN and 4 % to $\pi\pi$N, respectively. The $S_{11}(1650)$ decays with 77 % to πN and with 23 % to $\pi\pi$N, with a total width of 293 MeV. The branching ratios of both resonances and the width of the $S_{11}(1535)$ are consistent with the values given by the particle data group [13], while the width of the $S_{11}(1650)$ is larger. We note, however, that the resonance parameters, in particular the width, strongly depend upon the treatment of the background. For example, in the reduced model, where the upper resonance is neglected, the total width of the $S_{11}(1535)$ is 204 MeV. The resonance parameters quoted in [13] are determined in a parametrization, which includes a phenomenological background. In our model, this background is, at least in part, included in the large width of the $S_{11}(1650)$.

In order to explore how reliable our determination of $g_{\eta \mathrm{NN}_1^*}$ is, we have performed constrained fits, where this coupling constant was fixed to the values 1.9 and 2.3. The quality of the fit to the elastic T-matrix is slightly worse than in the optimal case. In the figs. 2 and 3 these fits are almost indistinguishable from the original fit. However, the cross section for η-production depends strongly on the value of $g_{\eta \mathrm{NN}_1^*}$. This is illustrated by the band in fig. 4, where the upper edge corresponds to $g_{\eta \mathrm{NN}_1^*} = 2.3$ and the lower edge to $g_{\eta \mathrm{NN}_1^*} = 1.9$. Thus, the optimal fit to the elastic πN-scattering T-matrix yields a good description for the inelastic η-production channel *without any further adjustment of parameters.*

At energies beyond the $S_{11}(1535)$, the calculated η-production cross section is too low. The missing cross section is probably due to higher lying resonances, not included in our model. The coupling to the $P_{11}(1440)$ and $D_{13}(1520)$, which have been studied previously, seems to be rather weak [5]. This is consistent with our model, where there is little room for additional contributions at energies below 1.55 GeV. The next resonance which couples strongly to the η-meson [13] is the $P_{11}(1710)$. A rough estimate of its contribution is obtained by using [22]

$$\Delta\sigma_{\pi\eta} = \frac{4\pi}{q^2}\left(\frac{2}{3}\right)\frac{\frac{1}{4}\Gamma_{\pi \mathrm{N}}\Gamma_{\eta \mathrm{N}}}{(\sqrt{s}-m_R)^2+\frac{1}{4}(\Gamma_{tot})^2}, \tag{8}$$

where $m_R = 1.71$ GeV is the resonance energy and Γ_i and $\Gamma_{tot} = 50$-250 MeV are the partial and total widths of the $P_{11}(1710)$. We use $\Gamma_{tot} = 200$ MeV, while for the branching ratios we take the maximum values [13] $\Gamma_\pi/\Gamma_{tot} = 0.2$ and $\Gamma_\eta/\Gamma_{tot} = 0.4$. The energy dependence of the widths is accounted for by multiplying Γ_i and Γ_{tot} by the corresponding phase-space factors. Adding this contribution to the cross section yields the dash-dotted line in fig. 4.

In summary, we have performed a coupled-channel calculation of πN scattering in the S_{11} channel including Born diagrams and the two resonances $S_{11}(1535)$ and $S_{11}(1650)$. The parameters of our model were determined by fitting the T-matrix for elastic πN scattering to empirical data. An excellent fit is obtained for all energies up to $\sqrt{s} = 1.75$ GeV. Furthermore, with the same parameters the total cross section for the inelastic process $\pi^- + p \to \eta + n$ is also well reproduced in the region of the two resonances.

Our model can be used in calculations of other processes, e.g. photoproduction of η-mesons or η-production in hadronic and heavy-ion collisions. Work on the photoproduction of η-mesons on protons is in progress. There the present model is used to describe the strong-interaction part of the T-matrix.

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Figures

Fig. 1: K-matrix elements for the three open channels, with $i = \eta, \zeta$.

Fig. 2: Imaginary part of the T-matrix amplitude for πN scattering in the elastic S_{11} channel as a function of the c.m. energy. The data are taken from the Karlsruhe-Helsinki analysis [11], the solid line is our best fit and the dash-dotted line shows our fit without a form factor at the πNN vertex.

Fig. 3: Same as fig. 2 for the real part of the T-matrix.

Fig. 4: Total cross section for the process $\pi^- + p \to \eta + n$ as a function of the laboratory pion momentum. The data are from ref. 20. The heavy solid line is computed with the parameters of our best fit to the elastic πN-scattering data. The boundaries of the shaded area correspond to $\pm 10\%$ deviations of $g_{\eta \mathrm{NN}_1^*}$ from its optimal value (for details see text). The dash-dotted line includes the contribution from the $P_{11}(1710)$ resonance. The arrows indicate the location of the $S_{11}(1535)$ and $P_{11}(1710)$ resonances.

References

[1] B. Krusche, preprint, to be published in *Proc. II TAPS Workshop*, Alicante, 1993

[2] B. Schoch, Acta Phys. Polonica **24** (1993) 1765

[3] E. Chiavassa *et al.*, preprint 1994

[4] F.-D. Berg *et al.* Z. Phys. **A340** (1991) 297; Phys. Rev. Lett. **72** (1994) 977

[5] C. Bennhold, H. Tanabe, Nucl. Phys. **A530** (1991) 625

[6] M. Benmerrouche and N. C. Mukhopadhyay, Phys. Rev. Lett. **67** (1991) 1070

[7] J. F. Germond and C. Wilkin, Nucl. Phys. **A518** (1990) 308

[8] J. M. Laget, F. Wellers, J. F. Lecolley, Phys. Lett. **B257** (1991) 254

[9] T. Vetter, A. Engel, T. Biro and U.Mosel, Phys. Lett. **B263** (1991) 153

[10] H. C. Chiang, E. Oset, L.C. Liu, Phys. Rev. **C44** (1991) 738

[11] G. Höhler, *Pion-Nucleon Scattering*, Landolt-Bornstein Vol.I/9b2, (Springer, Berlin, 1983) and private communication

[12] R. E. Cutkosky, Phys. Rev. **D20** (1979) 2804 and 2839

[13] Particle Data Group, *Review of Particle Properties*, Phys. Rev. **D45** (1992)

[14] B. Lee, *Chiral Dynamics*, (Gordon and Breach, New York, 1972)

[15] D. Campbell, Proceedings Les Houches 1977, *Nuclear Physics with heavy ions and mesons* (North-Holland, Amsterdam, 1978)

[16] W. Grein and P. Kroll, Nucl. Phys. **A338** (1980) 332

[17] C. J. Joachain, *Quantum Collision Theory*, (North-Holland, Amsterdam, 1975)

[18] G. E. Brown and W. Weise, Phys. Rep. **22** (1975) 279
E. Oset, H. Toki and W. Weise, Phys. Rep. **83** (1982) 282
T. Ericson and W. Weise, *Pions and Nuclei*, (Clarendon Press, Oxford, 1988)

[19] B. C. Pearce and B. K. Jennings, Nucl. Phys. **A528** (1991) 655

[20] M. Clajus, B. M. K. Nefkens in πN-Newsletter No.7, ed. R. E. Cutkosky, G. Höhler, W. Kluge, B. M. K. Nefkens, IEKP University of Karlsruhe and Department of Physics UCLA, Dec. 1992

[21] R. S. Bhalerao und L. C. Liu, Phys. Rev. Lett. **54** (1985) 865

[22] J.D. Jackson, Nuovo Cimento **34** (1964) 1645