Determination of the asymptotic behaviour of the heavy flavour coefficient functions in deep inelastic scattering

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Using renormalization group techniques we have derived analytic formulae for the next-to-leading order heavy-quark coefficient functions in deep inelastic lepton hadron scattering. These formulae are only valid in the kinematic regime $Q^2 \gg m^2$, where Q^2 and m^2 stand for the masses squared of the virtual photon and heavy quark respectively. Some of the applications of these asymptotic formulae will be discussed.

1. INTRODUCTION

Deep inelastic electroproduction of heavy flavours is given by

$$e^-(\ell_1)+P(p) \to e^-(\ell_2)+Q(p_1)(\,\overline{Q}(p_2)\,)+'X'\,.\quad(1)$$

When the virtuality $-q^2 = Q^2 > 0$ ($q = \ell_1 - \ell_2$) of the exchanged vector bosons is not too large ($Q^2 \ll M_Z^2$) the above reaction only gets a contribution from the virtual photon and we can neglect any weak effects caused by the exchange of the Z-boson. If the process is inclusive with respect to the hadronic state $'X'$ as well as the heavy flavours $Q(\bar{Q})$, the unpolarized cross section is given by

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{(Q^2)^2}S\Big[\{1+(1-y)^2\}F_2(x,Q^2,m^2) -y^2F_L(x,Q^2,m^2)\Big]\,, \qquad (2)$$

where S denotes the square of the c.m. energy of the electron proton system and the variables x and y are defined as

$$x=\frac{Q^2}{2p.q}\,(0<x\le 1)\,,\; y=\frac{p.q}{p.\ell_1}\quad(0<y<1)\,,\qquad(3)$$

with

$$-q^2 = Q^2 = xyS\;. \qquad (4)$$

In the QCD improved parton model the heavy flavour contribution to the hadronic structure functions, denoted by $F_i(x,Q^2,m^2)$ $(i=2,L)$, where m stands for the heavy quark mass, can be expressed as integrals over the partonic scaling variable. This yields the following results

$$F_i(x,Q^2,m^2) = x\int_x^{z_{max}}\frac{dz}{z}\Big[\frac{1}{n_f}\sum_{k=1}^{n_f}e_k^2 \times\Big\{\Sigma\Big(\frac{x}{z},\mu^2\Big)L_{i,q}^{\rm S}\Big(z,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\Big) +G\Big(\frac{x}{z},\mu^2\Big)L_{i,g}\Big(z,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\Big)\Big\} +\Delta\Big(\frac{x}{z},\mu^2\Big)L_{i,q}^{\rm NS}\Big(z,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\Big)\Big]$$

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$$+x\, e_H^2 \int_x^{z_{max}} \frac{dz}{z}$$
$$\times\Big\{\Sigma\Big(\frac{x}{z},\mu^2\Big) H_{i,q}\Big(z,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\Big)$$
$$+G\Big(\frac{x}{z},\mu^2\Big) H_{i,g}\Big(z,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\Big)\Big\}\,, \tag{5}$$

where $z = Q^2/(s+Q^2)$ and s is the square of the photon-parton centre-of-mass energy ($s \geq 4m^2$). Here the upper boundary of the integration is given by $z_{max} = Q^2/(4m^2+Q^2)$.

The function $G(z,\mu^2)$ stands for the gluon density whereas the flavour singlet and flavour non-singlet combinations of the quark densities are given by $\Sigma(z,\mu^2)$ and $\Delta(z,\mu^2)$ respectively. In the above expressions the charges of the light quark and the heavy quark are denoted by e_i and e_H respectively. Furthermore, n_f stands for the number of light quarks and μ denotes the mass factorization scale, which we choose to be equal to the renormalization scale. The latter shows up in the running coupling constant defined by $\alpha_s(\mu^2)$. The heavy quark coefficient functions are denoted by $L_{i,j}$ and $H_{i,j}$ ($i = 2, L$; $j = q, g$). The distinction between them can be traced back to the different photon-parton production processes from which they originate. The functions $L_{i,j}$ are attributed to the reactions where the virtual photon couples to the light quark, whereas the $H_{i,j}$ originate from the reactions where the virtual photon couples to the heavy quark. Hence $L_{i,j}$ and $H_{i,j}$ in (5) are multiplied by e_i^2 and e_Q^2 respectively. The superscripts NS and S on the heavy quark coefficient functions refer to flavour non-singlet and flavour singlet respectively. Furthermore the singlet quark coefficient functions $L_{i,q}^{\rm S}$ and $H_{i,q}^{\rm S}$ can be split into non-singlet and purely singlet (PS) parts,i.e.,

$$L_{i,q}^{\rm S} = L_{i,q}^{\rm NS} + L_{i,q}^{\rm PS}\,, \tag{6}$$

$$H_{i,q}^{\rm S} = H_{i,q}^{\rm NS} + H_{i,q}^{\rm PS}\,, \tag{7}$$

with $H_{i,q}^{\rm NS} = 0$ in all orders of perturbation theory.

In [1] the heavy quark coefficient functions $L_{i,j}$ and $H_{i,j}$ have been exactly calculated up to α_s^2. Expanding them in a power series in $(\alpha_s/4\pi)^k$ they receive contributions from the following parton subprocesses

$$\gamma^*(q) + g(k_1) \to Q(p_1) + \overline{Q}(p_2)\,, \tag{8}$$

$$\gamma^*(q) + g(k_1) \to g(k_2) + Q(p_1) + \overline{Q}(p_2)\,, \tag{9}$$

and

$$\gamma^*(q) + q(\overline{q})(k_1) \to q(\overline{q})(k_2) + Q(p_1) + \overline{Q}(p_2)\,. \tag{10}$$

For reaction (9) one has to include the virtual gluon corrections to the Born process (8). The contributions from (8) and (9) to the heavy quark coefficient functions are denoted by $H_{i,g}^{(1)}$ and $H_{i,g}^{(2)}$ respectively. The parton subprocess (10) has two different production mechanisms. The first one is given by the Bethe-Heitler process (see figs. 5a,5b in [1]) leading to $H_{i,q}^{\rm PS,(2)}$ and the second one can be attributed to the Compton reaction (see figs. 5c,5d in [1]). Notice that $L_{i,q}^{\rm PS}$ and $L_{i,g}^{\rm S}$ are zero through order α_s^2. Then, from (6), one infers that $L_{i,q}^{\rm NS,(2)} = L_{i,q}^{\rm S,(2)}$. Finally we want to make the remark that there are no interference terms between the Bethe-Heitler and the Compton reactions in (10) if one integrates over all final state momenta.

The complexity of the second order heavy quark coefficient functions prohibits publishing them in an analytic form, except for $L_{i,q}^{\rm NS,(2)}$, which is given in Appendix A of [2], so that they are only available in large computer programs [1], involving two-dimensional integrations. To shorten the long running time needed for the computation of the structure functions in (5) we have tabulated the coefficient functions in the form of a two dimensional array in the variables η and ξ in a different computer program [3]. These variables are defined by

$$\eta = \frac{(1-z)}{4z}\xi - 1 \qquad , \qquad \xi = \frac{Q^2}{m^2}\,. \tag{11}$$

However when $\xi \gg 1$ ($Q^2 \gg m^2$) numerical instabilities appear so that it is desirable to have analytic expressions for the heavy quark coefficient functions in that region. Moreover it turns out that for $\xi > 10$ the asymptotic expressions for $H_{2,g}^{(2)}$ (9) and $H_{2,q}^{\rm PS,(2)}$ (10) approach the exact ones so that the former can be used for charm production at the HERA collidier provided $Q^2 > 22.5$

(GeV/c^2) ($m_c = 1.5$ (GeV/c). Furthermore one can use these asymptotic formulae in the context of the variable flavour number scheme as has been explained in [4].

2. METHOD

We will now explain how the asymptotic form ($Q^2 \gg m^2$) of the heavy quark coefficient functions $H_{i,j}$, $L_{i,j}$ (5) can be derived using the renormalization group and the operator product expansion (OPE) techniques. Using these techniques one can avoid the computation of the cumbersome Feynman integrals and phase space integrals which arise in the calculation of the processes in (8) - (10). In the limit $Q^2 \gg m^2$ the heavy quark coefficient functions behave logarithmically as

$$H_{i,j}^{(k)}\Big(z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\Big) = \sum_{l=0}^{k} a_{i,j}^{(k),(l)}\Big(z, \frac{m^2}{\mu^2}\Big) \times \ln^l \frac{Q^2}{m^2} , \qquad (12)$$

with a similar expression for $\mathrm{L}_{i,j}^{(k)}$. The above large logarithms originate from collinear divergences which arise when Q^2 is kept fixed and $m^2 \to 0$. As has been shown in [2] each fixed order term in expression (12) can be written as

$$H_{i,j}\Big(\frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\Big) = A_{kj}\Big(\frac{m^2}{\mu^2}\Big) \otimes C_{i,k}\Big(\frac{Q^2}{\mu^2}\Big), \qquad (13)$$

where the power of α_s has to match on the left and right hand sides. There is a similar expression for $L_{i,j}$ ($i = 2, L$; $k, j = q, g$). Notice that we have suppressed the dependence on the scaling variable z in (12) and the convolution symbol $\otimes$ is defined by

$$\Big(f \otimes g\Big)(z) = \int_0^1 dz_1 \int_0^1 dz_2 \; \delta(z - z_1 z_2) \times f(z_1) g(z_2) \; . \qquad (14)$$

The light quark and gluon coefficient functions $C_{i,k}$ have been calculated up to order α_s^2 in [5]. The operator matrix elements (OME's) A_{kj} are now also known up to the same order in perturbation theory (see [2]). They are defined by

$$A_{kj}\Big(\frac{m^2}{\mu^2}\Big) = < j|O_k|j > , \qquad (15)$$

where O_k represent the local operators which show up in the operator product expansion of the two electromagnetic currents which appear in the calculation of the process (1). Notice that the OME's in (15) are finite which means that all renormalizations and mass factorizations have already been carried out. The last operation is needed because of the collinear divergences which appear in the OME's when the external on-mass-shell massless quarks and gluons are coupled to internal massless quanta. The ultraviolet and collinear divergences are regulated by using the method of n-dimensional regularization. The removal of these divergences has been done in the $\overline{\mathrm{MS}}$-scheme. For the computation of the heavy quark coefficient functions $H_{i,g}^{(1)}$ (8) $H_{i,g}^{(2)}$ (9) we need the OME's $A_{Qg}^{(1)}$ and $A_{Qg}^{(2)}$ respectively, which are given by the Feynman graphs in figs.1,2 of [2]. The Bethe-Heitler coefficient functions $H_{i,q}^{\mathrm{PS},(2)}$ (10) requires the calculation of $A_{Qq}^{\mathrm{PS},(2)}$ whereas for the Compton coefficient function $L_{Qq}^{\mathrm{NS},(2)}$ (10) one has to compute $A_{qq}^{\mathrm{NS},(2)}$ The results for these OME's can be found in appendix C of [2]. Substitution of A_{kj} and $C_{i,k}$ in (13) leads to the asymptotic heavy quark coefficient functions which are presented in Appendix D of [2].

3. RESULTS

We are now interested to find out at which values of ξ (11) or Q^2 the asymptotic coefficient functions approach the exact ones computed in [1] and [3]. For that purpose we define the ratio $R_{i,j}^{(\ell)}$ which is given by

$$R_{i,j}^{(\ell)}\Big(z, \xi, \frac{m^2}{\mu^2}\Big) = \frac{H_{i,j}^{\mathrm{exact},(\ell)}(z, \xi, m^2/\mu^2)}{H_{i,j}^{\mathrm{asymp},(\ell)}(z, \xi, m^2/\mu^2)} , \qquad (16)$$

where $H_{i,j}^{\mathrm{exact}}$ and $H_{i,j}^{\mathrm{asymp}}$ stand for the exact [1], [3] and asymptotic [2] heavy quark coefficient functions respectively. Choosing $\mu^2 = m^2$ and the range $5 < \xi < 10^5$, we have plotted as an example $R_{L,g}^{(2)}$ in fig.1 and $R_{2,g}^{(2)}$ in fig.2 for $z = 10^{-2}$ and $z = 10^{-4}$. The reason that we have chosen these two ratios is that the coefficient functions $H_{L,g}^{(2)}$ and $H_{2,g}^{(2)}$ (9) constitute the

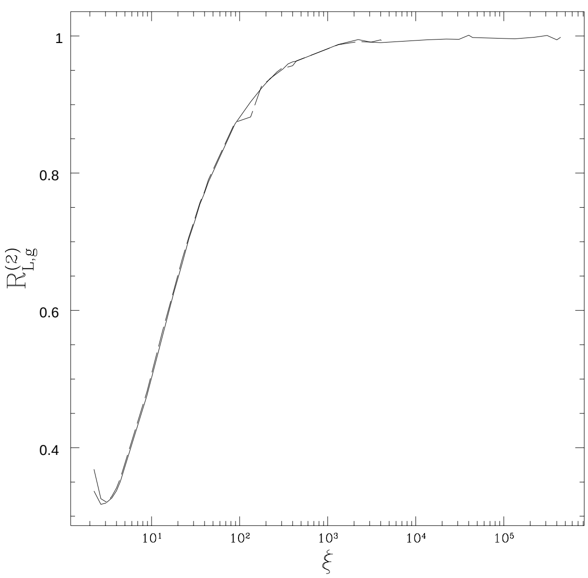

Figure 1. $R^{(2)}_{L,g}$ plotted as a function of ξ for fixed $z = 10^{-2}$ (solid line) and for $z = 10^{-4}$ (dashed line).

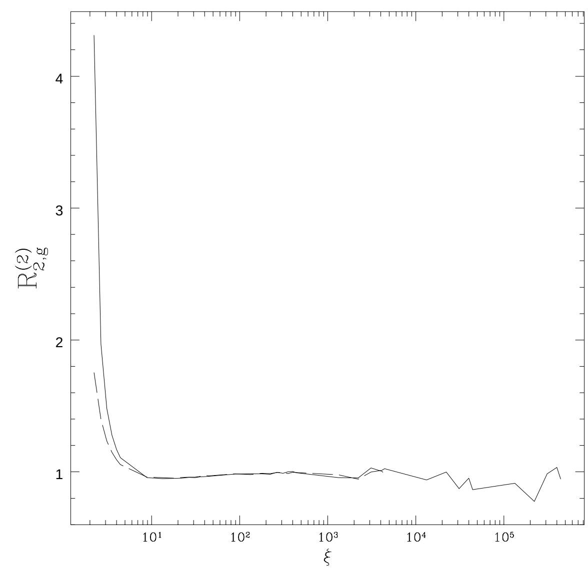

Figure 2. $R^{(2)}_{2,g}$ plotted as a function of ξ for fixed $z = 10^{-2}$ (solid line) and for $z = 10^{-4}$ (dashed line).

bulk of the radiative corrections to the Born reaction (8). From fig.1 we infer that $H^{\rm exact,(2)}_{L,g}$ and $H^{\rm asymp,(2)}_{L,g}$ coincide when $\xi \geq \xi_{\rm min} = 10^3$ and there is essentially no difference between the ratios for $z = 10^{-2}$ and $z = 10^{-4}$. In the case of $H^{\rm exact,(2)}_{2,g}$ and $H^{\rm asymp,(2)}_{2,g}$ (see fig.2) the above ξ-value is much smaller and both coefficient functions coincide when $\xi \geq \xi_{\rm min} = 10$, which is quite insensitive to the values chosen for z. The reason why the convergence of $R^{(2)}_{L,g}$ to one is so slow in comparison to $R^{(2)}_{2,g}$ is unclear. Apparently the logarithmic terms in $H^{\rm exact,(2)}_{L,g}$ start to dominate the coefficient functions at much larger values of ξ than is the case for $H^{\rm exact,(2)}_{2,g}$. A similar observation has been made for $H^{\rm PS,(2)}_{i,q}$. The small value found for $\xi_{\rm min}$ in the case of $H_{2,g}$ is very interesting for charm production where $F_2(x, Q^2, m_c^2)$ can be measured with much higher accuracy than $F_L(x, Q^2, m^2)$. Since $H^{(2)}_{2,g}$ dominates the radiative corrections to $F_2(x, Q^2, m_c^2)$ one can state that for $Q^2 > 22.5$ (GeV/c)^2 ($m_c = 1.5$ GeV/c) the exact coefficient functions can be replaced by their asymptotic ones. However before one can draw definite conclusions about the dominance of the terms $ln^l(Q^2/m^2)$ on the level of the structure functions ones first to convolute the heavy quark coefficient functions with the parton densities (see (5)). This will be done in future work. If it turns out that the above logarithms also dominate $F_k(x, Q^2, m^2)$, in particular for $k = 2$, then these terms have to be resummed using the renormalization group equations. This is done using the variable flavour number scheme approach [4]. One of the features of this method is that one has to define a charm density in the proton which is a convolution of the OME's $A_{k,j}$ (15) and the light parton densities Σ and G in (5). Hence for $Q^2 >> m_c^2$ the charm quark behaves like a light parton provided the large logarithmic terms dominate the proton structure functions in (5).

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