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ISOTROPY ANALYSIS OF METAMATERIALS

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Abstract. *Metamaterials – artificial periodic structures with subwavelength feature sizes – can be characterized with macroscopic electromagnetic properties, like refractive index and wave impedance, just like homogeneous materials. However, in contrast to homogeneous materials, their properties depend strongly on the angle of incidence. 3D metamaterial structures can exhibit homogeneous behaviour, but their fabrication is challenging compared to layered (2D) metamaterials. Here we demonstrate with an example, that layered (2D) metamaterials can also exhibit homogeneous behaviour despite of their anisotropic geometry.*

Keywords: homogenization, metamaterials, electromagnetic wave propagation

ANALIZA IZOTROPII METAMATERIAŁÓW

Streszczenie. *Metamateriały – sztuczne struktury periodyczne o wymiarze charakterystycznym porównywalnym z długością fali – można scharakteryzować za pomocą tych samych właściwości jak dla materiałów homogenicznych np. współczynnikiem załamania czy impedancją falową. Jednakże, w przeciwieństwie do materiałów homogenicznych, ic parametry silnie zależą od kąta padania fali. Trójwymiarowe struktury metamateriałów mogą wykazywać jednorodne zachowanie, ale ich wykonanie jest trudne w porównaniu do metamateriałów warstwowych (2D). W artykule pokazujemy na przykładzie, że metamateriały warstwowe (2D) mogą również wykazywać jednorodne zachowanie pomimo ich anizotropowej geometrii.*

Słowa kluczowe: homogenizacja, metamateriały, propagacja fali elektromagnetycznej

Introduction

Metamaterials [2] are artificial, mostly periodic structures with subwavelength feature sizes, which are applied to control the propagation of electromagnetic waves from radio frequencies to visible light. Due to their deep subwavelength feature sizes, their fine structure cannot be resolved by the electromagnetic wave propagating through them and they behave as homogeneous materials in the frequency range where the wavelength is much larger than the characteristic sizes of the structure. Homogeneous materials, like most natural materials, can be described with macroscopic (bulk) material parameters, like electric permittivity and magnetic permeability or refractive index and wave impedance, hence metamaterials can be characterized with the same quantities. However, in contrast to natural materials, metamaterials can be designed to have resonances at arbitrary frequencies and thus their electromagnetic properties can be altered to our will by changing the geometry of the structure. Metamaterials are also able to exhibit extraordinary electromagnetic properties like near-zero or negative refractive index [4, 7], which cannot be found in nature. This opens up new opportunities in optics and radio technology.

Unfortunately, these extraordinary properties are limited to a narrow frequency range and/or to a narrow incident angle range. The former is due to the nature of resonances, the latter is caused by the structure of metamaterials. Metamaterials are mostly layered structures, i.e. 2D structures are fabricated and stacked up, as shown in Fig. 1a. This anisotropic structure leads to a strong angle dependence of macroscopic parameters, which is often an undesired effect. Isotropic behaviour can be achieved by fabricating 3D structures [5] (see Fig. 1b) which is however rather challenging. Hence it is worth examining if there are layered structures that show isotropic behaviour despite of their anisotropic structure.

In this paper we study the so called Fishnet structure and show that despite being a 2D structure, it can represent a quasi-isotropic behaviour in a given frequency range. Additionally, the examined structure has a near-zero refractive index in the same frequency range which can be exploited in different applications.

1. Retrieving effective parameters of metamaterials

Because of the subwavelength feature sizes, the farfield of a metamaterial slab will be the same as that of a homogeneous slab. The metamaterial can be, hence, substituted with an ideal homogeneous slab. The material parameters of this slab is called the effective parameters of the metamaterial.

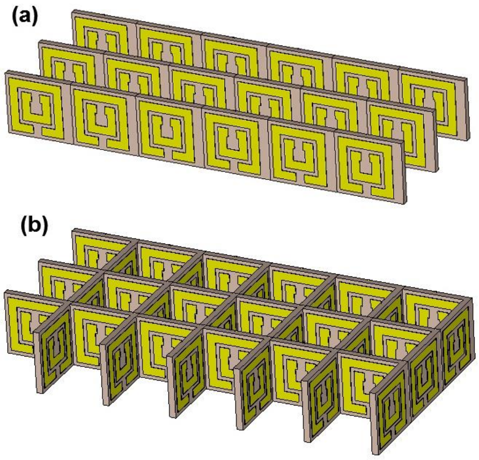

Fig. 1. (a) Anisotropic metamaterial consisting of 2D layers. (b) isotropic 3D metamaterial structure

The effective parameters of a metamaterial slab can be determined from its response to a plane wave excitation. If a metamaterial slab is excited with a plane wave, then the phase and intensity of the transmitted/reflected wave will give the S-parameters of the slab. Also, the S-parameters of an ideal homogeneous slab can be calculated, according to [7], as:

$$S_{11}=\frac{(1-\zeta^2)e^{jk_zd}-(1-\zeta^2)e^{-jk_zd}}{(1+\zeta)^2e^{jk_zd}-(1-\zeta)^2e^{-jk_zd}}, \quad (1)$$

$$S_{21}=\frac{4\zeta}{(1+\zeta)^2e^{jk_zd}-(1-\zeta)^2e^{-jk_zd}}, \quad (2)$$

where k_z is the normal wavenumber within the slab, ζ is the generalized wave impedance of the slab and d is the thickness of the slab. For normal incidence:

$$k_z = nk_0, \quad (3)$$

$$\zeta = Z, \quad (4)$$

where n is the refractive index of the slab, k_0 is the wavenumber in free space and Z is the wave impedance of the slab. The effective parameters of the metamaterial slab, i.e. the material parameters of the ideal homogeneous slab with which the metamaterial can be substituted, can be retrieved from the S-parameters by inverting these expressions [1]. For normal incidence

$$Z=\pm\sqrt{\frac{(1+S_{11})^2-S_{21}{}^2}{(1-S_{11})^2-S_{21}{}^2}}, \quad (5)$$

$$n = \frac{\mathrm{Im}\{\ln(\frac{S_{21}}{1-S_{11}\Gamma})\} + 2m\pi}{k_0 d} - i\frac{\mathrm{Re}\left\{\ln(\frac{S_{21}}{1-S_{11}\Gamma})\right\}}{k_0 d}, m \in \mathrm{Z}, \quad (6)$$

where Γ is the reflection coefficient computed from Z and the free space wave impedance Z_0

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}. \quad (7)$$

For oblique incidence (3) and (4) do not hold and only k_z and ζ can be retrieved [3]:

$$\zeta = \pm\sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \quad (8)$$

$$k_z = \frac{\mathrm{Im}\{\ln(\frac{S_{21}}{1-S_{11}\Gamma})\} + 2m\pi}{d} - i\frac{\mathrm{Re}\left\{\ln(\frac{S_{21}}{1-S_{11}\Gamma})\right\}}{d}, m \in \mathrm{Z}. \quad (9)$$

As it can be seen, these expressions contain ambiguities. The sign of ζ is not fixed and k_z has multiple values. The first problem is solved by assuming that the metamaterial is passive, i.e. $\mathrm{Re}\{Z\} > 0$. The second problem is resolved by utilising the Kramers-Krönig equation as described in [6].

2. Dispersion relation of metamaterials

The dispersion relation of a material gives the relationship between ω and $\boldsymbol{k}$, where ω is the angular frequency and $\boldsymbol{k} = (k_x, k_y, k_z)$ is the wave vector of the wave propagating in the medium. If the frequency is fixed, the dispersion relation gives the relationship between the spatial components of $\boldsymbol{k}$. In case of isotropic materials

$$k_x^2 + k_y^2 + k_z^2 = n^2 k_0^2, \quad (10)$$

which is the equation of a sphere.

In case of metamaterials the relationship between k_x, k_y and k_z mostly cannot be described with an explicit function. Still, it can be depicted if k_z is retrieved for a number of incident angles. For each incident angle the normal wave number k_z is retrieved from the S-parameters, the lateral wave number k_x, which is preserved at the interface of the slab, is calculated as

$$k_x = k_0 \sin\alpha, \quad (11)$$

where α is the angle of incidence, while the lateral wave number k_y normal to the plane of incidence is 0. **Błąd! Nie można odnaleźć źródła odwołania.** shows the dispersion relation of a Fishnet type metamaterial compared to that of an ideal isotropic material with n = 0.3. The anisotropy can be well observed.

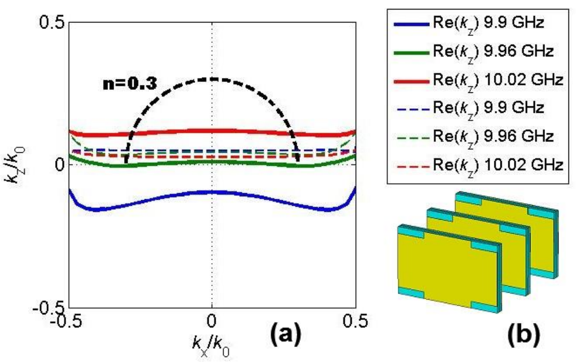

Fig 2. (a) Dispersion relation of the Fishnet type metamaterial compared to the dispersion relation of an isotropic material with n=0.3. The dispersion relation of the metamaterial is far from being circle-like. (b) The unit cell of the Fishnet structure

A number of papers in the literature retrieves the effective parameters only for normal incidence. This is, however, far not enough to characterize the metamaterial. If the metamaterial could be described with one parameter, it would assume, that this parameter would not change over incident angle. **Błąd! Nie można odnaleźć źródła odwołania.** shows, however, that neither n nor k_z is constant with respect to the incident angle.

It is also very important to note, that the response of metamaterials depends strongly on the polarization of the incident wave. Hence, to fully characterize a metamaterial it is necessary to retrieve the dispersion relation for both TE and TM mode.

3. Angle dependence of the refractive index

The refractive index n is defined as the ratio of $|\boldsymbol{k}|$ and k_0, i.e. the ratio of the wave numbers within the medium and within vacuum, hence the refractive index for a given angle of incidence can be calculated as

$$n(\alpha) = \frac{\sqrt{k_x^2 + k_y^2 + k_z^2}}{k_0} = \frac{\sqrt{k_0^2 \sin^2\alpha + k_z^2|_{k_x = k_0 \sin\alpha}}}{k_0}. \quad (12)$$

For isotropic materials this is a constant function, while for metamaterials this is an arbitrary function. Our goal is to find layered metamaterials that has a nearly constant $n(\alpha)$, i.e. a circle-like dispersion relation.

4. Anisotropic Fishnet structure with isotropic behaviour

A material has negative refractive index if its electric permittivity and magnetic permeability are negative simultaneously. While such materials do not exist in nature, metamaterials can be designed to satisfy this condition [[7]]. Similarly, a material can show zero refractive index with finite wave impedance if both the permittivity and permeability is zero. This condition can be satisfied with metamaterials, too [[4]].

Here we present a Fishnet structure that has both zero permittivity and zero permeability around 9.8GHz, as shown on **Błąd! Nie można odnaleźć źródła odwołania.**a. Zero permittivity near 9.8GHz is achieved by tuning the plasma frequency of the structure, while zero permeability is achieved by placing a strong magnetic resonance near 9.8 GHz. Both properties are determined by the geometry of the structure.

The Fishnet structure is a periodic metal-insulator-metal structure, the unit cell of which is shown on **Błąd! Nie można odnaleźć źródła odwołania.**b. The width of the unit cell is 20 mm, the height of the unit cell is 18 mm, the width of the vertical bars of the copper crosses are 9 mm, the height of the horizontal bars of the copper crosses are 11.5 mm, the thickness of the copper layers are 38 µm, the thickness of the ISOLA carrier layers are 760 µm and the thickness of the air gap between the ISOLA layers are 500 µm.

We extracted the S-parameters of a 5-layer structure, where the layers were placed 10 mm apart. The S-parameters were extracted by full wave simulation of the unit cell. Since oblique incidence introduces a phase difference between the edges of the unit cell, Bloch's boundary condition was applied instead of periodic boundary condition on the edges of the unit cell to simulate the periodic structure.

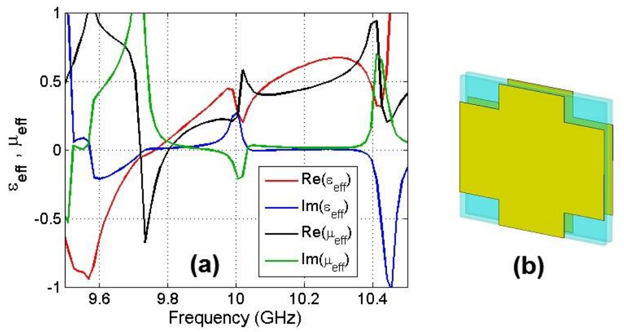

Fig. 3. (a) Effective relative permittivity and relative permeability of the fishnet structure. (b) Unit cell of the fishnet structure

Effective parameters and the dispersion relation were retrieved by the methods described above. Fig. 4 shows the dispersion relation in the negative refractive index region, while Fig. 5 shows the dispersion relation in the positive, near-zero refractive index region. The dispersion relation of an ideal isotropic material with n = -0.3 and n = 0.3 is also shown as a comparison. It can be well seen, that although in the negative refractive index region the metamaterial has deviations from the isotropic case, in the positive near-zero refractive index region the metamaterial can be well approximated with an isotropic material in spite of its anisotropic structure.

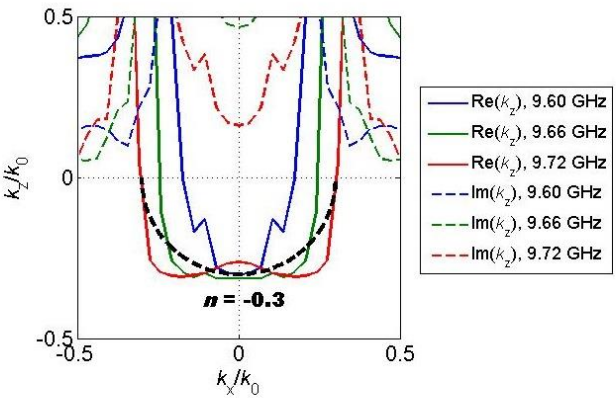

Fig. 4. Dispersion relation of the presented Fishnet structure in the negative refractive index regime. Deviations from isotropic case can be observed

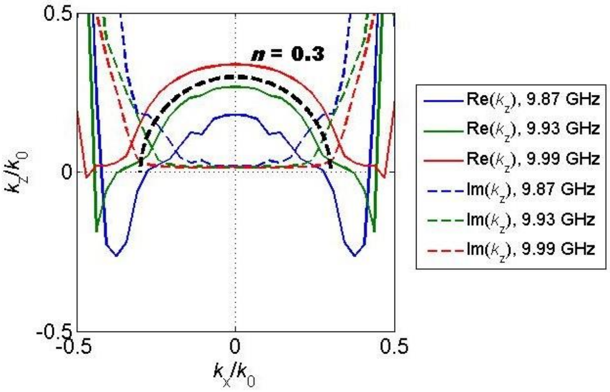

Fig. 5. Dispersion relation of the presented Fishnet structure in the positive near-zero refractive index regime. The dispersion relation shows a quasi-isotropic behaviour

5. Conclusion

We demonstrated with an example, that although layered (2D) metamaterials are mostly anisotropic, there exist layered metamaterial structures that show isotropic behaviour. Isotropy can be proved, however, only if the whole dispersion relation is retrieved, which means that the normal wave number has to be retrieved for a number of incident angles. After depicting the dispersion relation, isotropy can be easily determined.

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