View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Lublin University of Technology Journals

DOI: 10.5604/01.3001.0011.6007

DEAD TIME MEASUREMENT BY TWO-SOURCE METHOD – OPTIMIZATION OF MEASUREMENT TIME DIVISION

Grzegorz Domański, Bogumił Konarzewski, Robert Kurjata, Krzysztof Zaremba, Janusz Marzec, Michał Dziewiecki, Marcin Ziembicki, Andrzej Rychter, Waldemar Smolik, Roman Szabatin, Piotr Brzeski
Warsaw University of Technology Institute of Radioelectronics and Multimedia Technology

Abstract. *The article presents the analysis of the dead time measurement using two sources for a non-paralyzable detector. It determined the optimum division of count rate measurement time between both source measurement and a single source one. Results of the work can be used to optimize dead time measurement for systems which count photons or particles.*

Keywords: dead time, count rate, two-source method

POMIAR CZASU MARTWEGO METODĄ DWÓCH ŹRÓDEŁ – OPTYMIZACJA PODZIAŁU CZASU POMIARU

Streszczenie*. W artykule zaprezentowano analizę pomiaru czasu martwego detektora nieparaliżowalnego metodą dwóch źródeł. Wyznaczono optymalny podział czasu pomiaru częstości zliczeń dla pomiaru jednym i dwoma źródłami. Wyniki pracy mogą być wykorzystane do optymalizacji systemów zliczających fotony lub cząstki.*

Słowa kluczowe: czas martwy, częstość zliczeń, metoda dwóch źródeł

Introduction

In many problems of nuclear techniques it is important to determine the detector dead time. In practice, there are two types of detectors: non-paralyzable and paralyzable (Chapter 4 in [1]). For the measurement of the dead time, there are two methods: two-source method [2, 3] and the method of short-lived single-source [1].

1. Theory

Two-source method of detector dead time measurement involves measuring the count rates for two radioactive sources separately, and then measuring the count rate from both radioactive sources together. For non-paralyzable detector it allows to easily determine the dead time.

Symbols:
m – count rate recorded by a detector of dead time τ,
n – count rate recorded by an ideal detector with zero dead time.

The following relationships occur:

$$m = \frac{n}{1 + n\tau}, \qquad n = \frac{m}{1 - m\tau} \tag{1}$$

The following indexes are added for the count rate symbols: 1 – measurement for the first source, 2 – measurement for the second source, 12 – measurement for both sources together. For the measurement procedure used to measure the dead time one can write equation:

$$n_1 + n_2 = n_{12} \tag{2}$$

The count rate n can be replaced by expressions dependent on the respective count rate m:

$$\frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau} = \frac{m_{12}}{1 - m_{12}\tau} \tag{3}$$

After simple transformations quadratic equation form can be obtained:

$$m_1 m_2 m_{12}\tau^2 - 2m_1 m_2\tau + (m_1 + m_2 - m_{12}) = 0 \tag{4}$$

The solution to this equation are the two roots:

$$\tau' = \frac{1 - \sqrt{\left(1 - \frac{m_{12}}{m_1}\right)\left(1 - \frac{m_{12}}{m_2}\right)}}{m_{12}}, \qquad \tau'' = \frac{1 + \sqrt{\left(1 - \frac{m_{12}}{m_1}\right)\left(1 - \frac{m_{12}}{m_2}\right)}}{m_{12}} \tag{5}$$

Due to the physical interpretation, only the first root is correct. It can be represented as

$$\tau = \frac{m_1 m_2 - \sqrt{m_1 m_2 (m_{12} - m_1)(m_{12} - m_2)}}{m_1 m_2 m_{12}} \tag{6}$$

Equation (6) allows to determine the dead time of the detector based on the count rate measurements m_1, m_2 and m_{12}.

To estimate the uncertainty of the dead time (the random error) the variance of the random variable τ need to be determined. However, use of a strict equation (6) leads to a very complex expression. Therefore, to evaluate the random error simplified formula is used:

$$\tau = \frac{m_1 + m_2 - m_{12}}{2m_1 m_2} = \frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{m_{12}}{m_1 m_2}\right) \tag{7}$$

obtained by assuming that in quadratic equation (4) the component τ_2 can be neglected.

The all above count rate values are mean values. To estimate the dead time uncertainty (the random error) the variances of the count rates m_1, m_2 and m_{12} presented in formula (7), will be required. Count rate m can be expressed depending on the number of registered pulses M and the measurement time t:

$$m = \frac{M}{t} \tag{8}$$

Then the variance of the random variable m is:

$$\sigma^2 m = \frac{\sigma^2 M}{t^2} = \frac{\overline{M}}{t^2} \approx \frac{M}{t^2} = \frac{m}{t} \tag{9}$$

with the obvious assumption that the number of registered pulses M is subject to the Poisson distribution, and the best approximation of its average value (for single measurement) is measured value M. In fact, the variance of registered counts, with non-zero dead time is less than that resulting from the Poisson distribution, but we accept this approximation in view of the establishment of a short dead time. We assumed here that dead time is short compared to the average time between successive counts. In the case that the above assumption is not fulfilled the full formula for the variance of the variable m should be used:

$$\sigma^2 m = \frac{m}{t(1+n\tau)^2} \tag{10}$$

The variance of the random variable τ is

$$\sigma^2\tau = \left|\frac{\partial\tau}{\partial m_1}\right|^2 \sigma^2 m_1 + \left|\frac{\partial\tau}{\partial m_2}\right|^2 \sigma^2 m_2 + \left|\frac{\partial\tau}{\partial m_{12}}\right|^2 \sigma^2 m_{12} \tag{11}$$

where the partial derivatives are respectively:

$$\begin{aligned} \frac{\partial\tau}{\partial m_1} &= \frac{1}{2}\frac{m_{12}-m_2}{m_2 m_1^2} \\ \frac{\partial\tau}{\partial m_2} &= \frac{1}{2}\frac{m_{12}-m_1}{m_1 m_2^2} \\ \frac{\partial\tau}{\partial m_{12}} &= -\frac{1}{2m_1 m_2} \end{aligned} \tag{12}$$

The formula (11), after using the necessary numerical values of variables, estimates the variance of the random variable τ (that is, the random error of measurement).

However, in addition to the knowledge of τ dispersion, it is useful to be able to select optimal partition of count rate measurement time. If the measurement times t_1 and t_2 will be equal it raises the question of what the relationship should be between these times and t_{12} measuring time to get the minimum random error of measurement dead time for the total measurement time equal to t = const.

Symbols:

$$\begin{aligned} & t_1 = t_2 = t_0 \\ & t_1 + t_2 + t_{12} = t = const \\ & 2t_0 + t_{12} = t = const \\ & t_0 = kt \\ & t_{12} = (1-2k)t \end{aligned} \tag{13}$$

where k is a constant with a value in the range of (0,0.5).

Using the above designations variance of the random variable τ can be written as:

$$\sigma^2 m = \frac{m}{t(1+n\tau)^2} \tag{14}$$

Differentiating expression for the variance of the random variable τ with respect to k one can determine the division of the measurement time t, which will provide the minimum value of the measured random error of dead time τ. To further transformations there will be adopted the following substitutions that can simplify equations:

$$A = \frac{(m_{12}-m_2)^2}{4m_2^2 m_1^3}, B = \frac{(m_{12}-m_1)^2}{4m_1^2 m_2^3}, C = \frac{m_{12}}{m_1^2 m_2^2} \tag{15}$$

After using these substitutions the optimum allocation of given measurement time t can be determined by solving the equation of the form:

$$\frac{\partial\sigma^2\tau}{\partial k} = -\frac{A}{k^2 t} - \frac{B}{k^2 t} + \frac{2C}{(1-2k)^2 t} = 0 \tag{16}$$

After simple transformations one can obtain the quadratic equation form:

$$[2C - 4(A+B)]k^2 + 4(A+B)k - (A+B) = 0 \tag{17}$$

The solution to this equation are the two roots:

$$\begin{aligned} k' &= \frac{-1 - \frac{1}{2}\sqrt{\frac{2C}{A+B}}}{\frac{C}{A+B} - 2} \\ k'' &= \frac{-1 + \frac{1}{2}\sqrt{\frac{2C}{A+B}}}{\frac{C}{A+B} - 2} \end{aligned} \tag{18}$$

Due to the physical interpretation, only the second root is valid – it can be represented as:

$$\begin{aligned} k &= \frac{-1 + \frac{1}{2}\sqrt{2x}}{x-2}, \\ x &= \frac{4m_1 m_2 m_{12}}{(m_{12}-m_2)^2 m_2 + (m_{12}-m_1)^2 m_1} \end{aligned} \tag{19}$$

2. Results of measurements

Measurements of dead time in the spectrometric system that uses radiation detector type Ge(Li). A block diagram of the measurement system is shown in Figure 1.

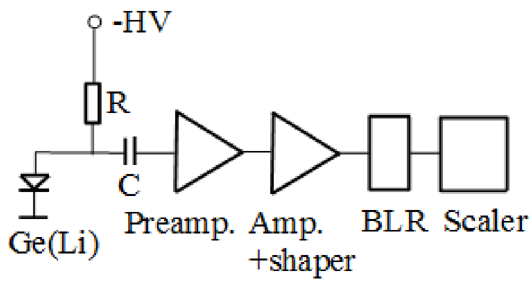

Fig. 1. The block schema of measurement system

It consists of a detector powered by the high voltage -1000V, preamplifier, shaping amplifier, base line restorer and scaler type PT-72. Three series of measurements using two Am-241 radioactive sources were made. The first series consisted of 10 measurements using the first source during 10 s, 10 measurements using the second one during 10 s and 10 measurements using both sources during 20 s. The second series consisted of 10 measurements using the first source during 1 s, 10 measurements using the second one during 1 s and 10 measurements using both sources during 40 s. The third series consisted of 10 measurements using the first source during 20 s, 10 measurements using the second one during 20 s and 10 measurements using both sources during 1 s. Mean count rates are presented in the Table 1.

The Table 2 shows the determined values of dead time and relative standard deviations for the three measurement series.

Table 1. The results of measurements of the count rates for the three measurement series

Serie	Source	Time [s]	Mean count rate [cps]
1	No. 1	10	12881
1	No. 2	10	6930
1	No. 1 + No. 2	20	15004
2	No. 1	1	12859
2	No. 2	1	6864.8
2	No. 1 + No. 2	40	14988
3	No. 1	20	12868
3	No. 2	20	6918.7
3	No. 1 + No. 2	1	14959

Table 2. The results of the determined values of dead time and relative standard deviation for the three measurement series

Serie	Total time [s]	Ratio k	Mean dead time τ [μs]	Relative standard deviation δ(τ)/τ
1	40	0.2500	37.437	0.0036
2	42	0.0238	37.185	0.0095
3	41	0.4878	37.792	0.0075

For the measured count rates and Poissonian approximation the optimum split of the time is determined by the ratio $k = 0.15$. For non-Poisson approximation the optimal value of k is about 0.28. The dependence of the relative standard deviation of the random variable τ with respect to the ratio k is shown in Figure 2 (assuming total measurement time of 40 s). Two curves – the first (solid line) calculated according to the formula (9) and the second (dashed line) calculated according to the formula (10) are shown. Location of the minimum on the first curve corresponds to the value calculated from the formula (19). The second curve was calculated using numerical approximation of derivatives for full accuracy formula (6). The relative standard deviation tends to infinity for two extreme cases, the parameter k close to zero or value of 0.5. In the first case, the measurement time for a single source tends to zero, and the relative error of count rate for a single source tends to infinity. In the second case, the value of the parameter k tends to 0.5 and the time of the measurement by both sources tends to zero, so the relative error of count rate for both sources also tends to infinity. The measured relative standard deviation values of the dead time are generally higher than predicted by the formula (11). This may be caused by instability of the measuring equipment.

3. Conclusions and discussion

The analysis of the dead time measurement uncertainty of non-paralyzable detector indicates the existence of an optimal allocation of measurement time between both sources. The formula for the optimal allocation of measurement time was derived assuming that the dead time is short compared to the average time between successive counts. Experimentally confirmed the existence of the optimal allocation of time. Results of the work can be used to optimize dead time measurement for systems which count photons or particles.

Bibliography

[1] Knoll G.F.: Radiation Detection and Measurement (4th ed.). Willey, 2010.
[2] Meeks C., Siegel P.B.: Dead time correction via the time series. Am. J. Phys. 76 (6), 2008, 589–590 [DOI: 10.1119/1.2870432].
[3] Myers R.T.: Dead Time of a Geiger-Mueller Tube by the Double-Source Method. Journal of Chemical Education 33(8), 1956, 395–396 [DOI: 10.1021/ed033p395].

Ph.D. Grzegorz Domański
e-mail: g.domanski@ire.pw.edu.pl

M.Sc. (1994), Ph.D. (2001); nuclear and medical electronics; Assistant Professor, Nuclear and Medical Electronics Division, Institute of Radioelectronics and Multimedia Technology, Faculty of Electronics and Information Technology, Warsaw University of Technology. Faculty Coordinator of Radiological Protection (2002-); Tutorial assistance of Biomedical and Nuclear Engineering Students Scientific Group (2013-).

Ph.D. Eng. Bogumił Konarzewski
Ph.D. Eng. Robert Kurjata
Prof. Krzysztof Zaremba
Prof. Janusz Marzec
Ph.D. Eng. Michał Dziewiecki
M.Sc. Eng. Marcin Ziembicki
Ph.D. Eng. Andrzej Rychter
Prof. Waldemar Smolik
Ph.D. Eng. Roman Szabatin
Ph.D. Eng. Piotr Brzeski

otrzymano/received: 15.06.2015 *przyjęto do druku/accepted: 15.02.2018*