STRUCTURAL MODELS FOR THE PRICING OF CORPORATE SECURITIES AND FINANCIAL SYNERGIES

Applications with Stochastic Processes Including Arithmetic Brownian Motion

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Abstract

Mergers are the combining of two or more firms to create synergies. These synergies may come from various sources such as operational synergies come from economies of scale or financial synergies come from increased value of securities of the firm. There are vast amount of studies analysing operational synergies of mergers. This study analyses the financial ones. This way the dynamics of purely financial synergies can be revealed. Purely financial synergies can be transformed into financial instruments such as securitization.

While analysing financial synergies the puzzle of distribution of financial synergies between claimholders is investigated. Previous literature on mergers showed that bondholders may gain more than existing shareholders of the merging firms. This may become rather controversial. A merger may be synergistic but it does not necessarily mean that shareholders' wealth will increase. Managers and/or shareholders are the parties making the merger decision. If managers are acting to the best interest of shareholders then they would try to increase shareholders' wealth. To solve this problem first the dynamics of mergers were analysed and then new strategies developed and demonstrated to transfer the financial synergies to the shareholders.

Declaration of Authorship

I, Ali Ferda ARIKAN, declare that this thesis titled, 'Structural Models for the Pricing of Corporate Securities and Financial Synergies' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: ________________________________

Date: ________________________________

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Dedicated to my family...

Chapter 1

Introduction

1.1 Introduction

This study analyses the distribution of merger gains between bondholders and shareholders of merging firms. Current merger literature focuses either on bondholder gains or shareholder gains (Bernile, Lyandres, and Zhdanov, 2007; Billett, King, and Mauer, 2004; Hackbarth and Morellec, 2008; Morellec and Zhdanov, 2005). The channels and mechanisms of merger gains are examined in an integrated model of bondholders and shareholders. An integrated approach may enable to identify and analyse the transfer mechanisms between bondholders and shareholders. Including shareholders and bondholders in the same framework will reveal potential gains that can be transferred from one party to other.

Leland (2007) develops a unified framework which can be used to analyse the distribution of merger gains between shareholders and bondholders. In a single period merger model of financial synergies, the Leland (2007) model produces an interesting result such that bondholders not only fully extract the financial synergy gains but also enjoy a transfer of wealth from shareholders. This raises the question of why shareholders, the owners of the company, should decide in favour of mergers where the wealth may be transferred to bondholders. In an ideal world shareholders would not approve a merger when their gains are negative. Modelling merger gains of shareholders and bondholders in a dynamic multi-period setting will reveal the sources and identifiers of merger synergies.

Taking this as a starting point and building on Leland (2007)'s model, all security holders' gains are modelled in a dynamic framework where the model developed by Ammann and Genser (2004) for corporate securities is applied to the case of mergers. The claims of shareholders and bondholders on companies are dynamic. Thus, financial instruments representing these claims may also have a dynamic nature. Shares of a company indeed represents a claim on the future cash flows of the firm. A dynamic framework is required for two reasons. A dynamic setting will not only capture the dynamic nature of claims over the company but also the dynamic structure of the financial instruments in financial markets. Bonds with different maturities can be a good example to verify the need of a dynamic setting to analyse mergers. As shareholders can use short term bonds to increase their merger gains, at least two periods are needed to model short term and long term bonds in an integrated framework.

Shareholders have the means to extract some of the gains from bondholders. As pointed out in Leland (2007), these means are short term bonds and call provisions on bonds. Call provisions enable shareholders to call the bonds back if the bond price exceeds a pre-specified value. Short term bonds can also be used in a similar way as long as merger timing and short term bonds' maturity match to extract all merger gains. However, these means are neither modelled nor demonstrated explicitly in the Leland (2007) framework. The one period framework of the Leland (2007) model prevents incorporating call provisions and short term bonds. That is why a dynamic multi-period framework is required to capture the extensions proposed by Leland (2007).

This chapter first provides a brief review of previous studies into mergers and identifies the gaps in the literature. It then reveals the preliminary analysis carried out to understand the nature of the problem. Finally the research design and the analytical framework used in this study are presented.

1.2 Previous Studies and Research Motivation

There appears to be an ongoing interest to mergers and acquisitions (M&A) deals around the world. U.S. firms announced 10,574 and European firms 13,842 M&A deals in 2007[1]. Despite the possible different motives, mergers are generally observed through waves. These waves are driven by economic, regulatory or technological shocks (Harford, 2005). Although mergers are generally observed when the economy is growing, they are not forgotten during recession times. Among various motives[2] behind mergers, bankruptcy avoidance is also an important motive as shown by Shrieves and Stevens (1979). Even the most trusted Wall Street or City banks were forced to merge because of the economic crisis observed following the credit crunch. Reasons like bankruptcy avoidance will help mergers keep their popularity even when the economy is under pressure.

Although M&A's are popular devices for firms at all times regardless of the economic situation, there are still some critical questions to be answered in the corporate and academic world. Who benefits from mergers? How can merger gains be increased(maximised/optimised)? Which one is better, to merge or to spin-off? Which company to merge with, the one with a higher credit rating or the one with a lower credit rating? All these questions require modelling and pricing of merger processes.

Finance literature has vastly investigated mergers and acquisitions analysing different aspects in different settings. The financial synergies and their valuation are the main interests of this study. More specifically merger gains of (existing) security holders are analysed and methods to maximise shareholder gains are investigated through financial synergies. Maximising shareholder gains implies that all of the

[1]Merger data compiled from www.mergerstat.com

[2]Undervaluation, diversification, synergy, poorly managed firms and managerial self interest are some of those. For a detailed list see Damodaran (2002).

existing securities of the merging firms need to be valued (priced). To our knowledge there are no previous studies on this specific problem [3]. A model which can provide pre- and post-merger firm values while pricing all existing securities of the merging firms, may be able to answer all the questions addressed above.

Mergers affect *capital structure* due to tax benefits of debt and also change the default risk of the merging firms. Modigliani and Miller (1958) argue that capital structure changes will not affect firm value. However, in the presence of bankruptcy costs and taxes this argument does not hold. Through the coinsurance effect and tax benefits of debt, firm value changes with a change in capital structure. If merging firms have optimal capital structures, the change in capital structure after a merger will be equal to financial synergies.

Stiglitz (1972) shows that the possibility of bankruptcy has strong implications for firm behaviour which leads to the *possibility of optimal debt-equity ratio*. Probability of default limits the debt capacity. However Lewellen (1971) argues that combining companies with lower correlations of cash flows will increase the debt capacity by decreasing the relative variability of cash flows compared to individual firms. The decrease in the default risk increases debt limits of the company which also increases the potential tax savings. Similarly Flannery, Houston, and Venkataraman (1993) argue that combining activities with not highly *correlated cash flows* creates value. By combining diverse activities, firms also reduce default risk on debt and improve capital structure in favour of their debt with coinsurance effect. Lewellen (1971) builds his arguments on coinsurance effect which will let lenders establish a new aggregate limit on lending, for the merged company. This new aggregate limit may exceed the sum of individual limits. Companies can also benefit from higher amounts of tax savings obtained by higher borrowing limits. Higher amounts of borrowing will lead to a higher optimal leverage level.

[3] There are a number of authors who studied bondholder and stockholder gains, however they do not have an explicit model and they do not try to increase stockholder gains (Higgins and Schall, 1975; Scott, 1977; Shastri, 1990; Stapleton, 1982) .

Lewellen (1971) results are in line with the recent studies except for the possibility of negative future cash flows. A company's future cash flows can well be negative. With the *limited liability* of owners, negative future cash flows provide an option to walk away (Sarig, 1985). This option defines the value of limited liability. According to Sarig (1985), if limited liability has a value, then having two options is more valuable than having one. This defines the loss of value after merger which is called *limited liability effect*.

Sarig (1985) defines a corporation as an option with an exercise price of zero where owners have the option to walk away in case of negative future returns. In case of a merger, owners give up two separate options for a less valuable single option. This reduces the total value of outstanding securities of the merging firms. Having a single option, instead of two, decreases the value of outstanding shares. On the contrary, as Sarig (1985) concludes, *"divesting economically unrelated lines of business should increase the aggregate value of corporate securities"(pp.388)*.

Shareholders would ideally not let a merger happen if it is going to decrease their share prices. On the other hand, they can force the management to decrease the bondholders' gains in favour of shareholders. This can be done using short term debt or by issuing bonds with call options (Leland, 2007). Short term debt can be used by issuing debt maturing before the merger and issuing new debt after the merger so that the increase in value of outstanding debt can be kept to the benefit of shareholders. Issuing bonds with call options can be used similarly to short term debt. In this case outstanding debt can be called back by the company after merger with a previously specified price to distribute the increase in the outstanding debt value to shareholders.

Hackbarth and Miao (2008) use a real options approach like Morellec and Zhdanov (2005) and build an industry equilibrium model that analyses how product market competition affects the gains from mergers. They specify a tangible asset that helps increase output for a given average cost instead, assuming presence of exogenous operating synergies. They focus on shareholder value but not debt value after merger. Their findings focus on the relationship between product market competition, industry concentration and takeover returns.

Lambrecht (2004) analyses timing and terms of mergers incorporating economies of scale with a one factor model which mostly applies to horizontal mergers. The paper uses the real options framework which relies on a strong form of market efficiency. The model, where timing is endogenous, assumes merger decisions are made with the objective of maximising shareholder value. Share prices values move smoothly to their pre-merger values. This is slightly different from the Leland (2007) framework where firms are valued as the sum of share and debt value and optimal capital structure is calculated by maximising this total.

Fluck and Lynch (1999) argue that firms are motivated to merge due to the fact that some marginally profitable projects may not be financed by themselves alone. Their theory may however have some limitations. For instance in their setup one of the firms must be financially distressed. Also, their theory only applies when there are agency problems between the managers and the shareholders of the target firm. They see divestitures as good news showing the merger-financed projects ability to survive as a single firm.

Morellec and Zhdanov (2005) develop a *dynamic model* of takeovers incorporating imperfect information, learning and competition and show how all three elements interact to determine abnormal returns to stockholders around takeover announcements. Timing of mergers is endogenous in this model. They make some predictions on shareholder returns of bidding and target companies. Their model does not make predictions about bond prices of the merging firms. However, the total merger gains may not be determined only by analysing the share returns. The company value may not be directly related to the market value of its shares. It can be argued that market value of shares affects the total value but in this case there is not an explicit solution for the debt value.

One of the results of Morellec and Zhdanov (2005) is the increase in stock returns with the lower correlation between the returns of merging firms. Lower correlation between firms' returns or cash flows will increase the coinsurance effect which will increase the merged firm value. However the valuation of the company still has some missing parts. The effects of coinsurance are not studied on outstanding debt values in their model. Bernile, Lyandres, and Zhdanov (2007) analysed how the industry demand affects firms' decisions on mergers using a real options framework. They also extend their model to investigate how operating synergies, merger costs and variations in the industry structure affect mergers. In their model, merger decision is endogenous in a dynamic framework and the effects of demand shocks to merger decisions are demonstrated. Furthermore Billett, King, and Mauer (2004) analyse bondholder wealth effects in mergers. They focus on bond values of merging firms and find strong evidence of a coinsurance effect for target bonds.

As mentioned above, existing literature does not provide valuation models for existing securities of merging firms. The Leland (2007) model which shows higher merger returns for the bondholders can be stated as the only model that makes such an attempt. Although there are various motives behind mergers, firms do not really merge to increase bondholders' value at the expense of shareholders. This thesis also try to find a solution to this problem. How can the shareholders' merger gains can be increased?

Leland (2007) successfully quantifies and demonstrates merger dynamics. However, the framework covers only one period and is difficult to apply to real world problems. For instance, the model does not assume operational synergies between the merged firms. Yet, the absence of operational synergies helps the model easily capture the securitization[4] case. Finance literature has vast amounts of studies on mergers but securitizations have not been studied in great detail. Modelling financial synergies through mergers will also lead to modelling of securitization in the absence of operational synergies.

This study develops a generalised merger and securitization valuation model which can be used both by practitioners and academics. Current literature does not provide a merger valuation model which prices all existing securities of merging firms. Theoretical (model) prices of merging firms' securities can be used by firms or investors, to identify the arbitrage opportunities. The results may have implications on merger decisions and also shed light to further enquiry of mergers and structured finance instruments.

[4]Securitization is simply transferring high quality assets of a firm to a 'special purpose vehicle', which has a separate body from the originating firm, and issuing bonds backed by these high quality assets (Gorton and Souleles, 2005). Asset backed securities and mortgage backed securities are subclasses of securitization.

1.3 Preliminary Analyses

Preliminary work is carried out to investigate and understand the dynamics of the problem and then the study is built on accordingly. The work consists of three parts. First, the relationships of financial synergies and ratings are explored. Second, the work tests the model against Credit Default Swap (CDS) value . Finally, the third part looks into the effects of callable bonds on mergers.

1.3.1 Ratings, Operating Synergies and Mergers

This part analyses the effects of different credit ratings in mergers on merger gains. The motivation is to reveal the effects of credit risk in mergers. Hackbarth and Morellec (2008) analyse stock returns in mergers and acquisitions. They focus on asset pricing implications of mergers and demonstrate that depending on the relative risks of the merging firms the beta of target firm might increase or decrease. The original Leland (2007) framework uses two identical firms and analyses their security values. However in practice this is a rare event; merging companies may have similar ratings but it is almost impossible to find two identical companies to merge. The original model uses two identical BBB rated companies. The question is what happens if an AAA rated firm and BBB rated firm merge. A merger matrix is constructed as a sort of sensitivity analysis to include companies with different credit ratings (Aaa, Aa, A, Baa, Ba, and B).

Credit ratings can be defined by using model parameters. These parameters can be set to match credit ratings by using leverage ratios and default probabilities as proxies defined by Huang and Huang (2003) for certain rating groups. Keeping tax rate, debt maturity, risk free rate the same for all companies, α, default cost, μ, expected future operational cash flows and σ, cash flow volatility are used to calculate target leverage ratios and default probabilities. After calibrating all parameters for Aaa, Aa, A, Baa, Ba, B rating groups, a merger matrix can be built.

Preliminary results obtained in this work indicate that merging companies with lower ratings than Baa does not create positive financial synergies. In the case of a merger, post-merger effects are mostly negative, favouring spin-offs.

1.3.2 Testing Against Credit Default Swaps

This part tests the model behaviour and its dynamics using a widely employed credit risk transfer instrument called Credit Default Swap (CDS). CDS is a tradable contract which transfers default risk of a bond issuer to the issuer of CDS. It is an insurance against default risk of a reference company. CDS contracts are used to analyse financial synergies while a short position in CDS can be used to increase shareholders benefits from merger compared to bondholders' gains. Using Hull and White (2000) model, payoff of a CDS is

$$P - RP[1 + A(t)] = P[1 - R - A],$$

where P is the notional principal, R is recovery rate and A is accrued interest which is equal to zero because of zero coupon bonds. The bondholders will claim principal amount of zero coupon bond in case of a default. Payoff of the CDS will then become

$$P[1 - R].$$

Expected value of default is equal to, default probability times loss in default

$$ED = DP * \text{Loss in Default.}$$

Loss in default is $(1 - R) * P$ so the default probability can be found using

$$DP = ED/\text{Loss in Default.}$$

Finally payoff of a CDS contract is $L(1 - R)$ in the case of a zero coupon bond. The expected value of CDS is given by $E(CDS) = Payoff * DP$.

In a merger, default probability of merging companies decreases compared to the sum of two firms. The merged company has a lower CDS value than the sum of the two companies due to a decrease in default probability. Value of CDS is equal to the present value of the expected payoff minus the payments made by the buyer

$$\frac{(1 - R)\int_{-\infty}^{X^d} dF(X) - sP\int_{X^d}^{\infty} dF(X)}{(1 + r_T)},$$

where P is principal and s is the CDS premium.

As a result it is found that shareholders of the merging companies can take a short position on a CDS contract to increase their benefits from a merger. A short position in a CDS contract is equivalent to a long position on company bonds. Instead of a cash position CDS contracts can be used while a short position will not require any cash payments and can be leveraged by shareholders to the benefit of the firm.

1.3.3 Shareholder Wealth Effect of Callable Bonds

The preliminary findings show that shareholders' gains can be increased by embedding call options to bonds of merging firms. Call options also enhance firm value and decrease the cost of debt by decreasing the default risk of outstanding bonds. When two companies merge, their debt becomes less risky than two separate firms' debts which increases debt capacity of the merged firm by decreasing default probability of the debt. This is the coinsurance effect. When two companies merge, through coinsurance effect, the value of outstanding debt increases at the expense of shareholders. To overcome this problem call options are embedded to bonds and the new debt is defined by

$$V = D + E - \text{Call Option}, \tag{1.1}$$

where V is total firm value, D is market value of debt and E is market value of equity. This equation represents company value with call option on bonds. The sign of call option is negative because callable bonds are represented in two different parts; bond and call option. The company sells bonds and buys call options on bonds. Total value is combination of a positive cash flow for bond sales and a negative cash flow for option purchase.

1.4 Research Design

Dynamics of mergers and acquisitions are demonstrated by using an analytical framework which is in the form of a theoretical model. An example to the framework can be the value of a firm given by

$$V(\tau, \mu, \sigma, X, \rho, ...), \tag{1.2}$$

where V is firm value, τ is marginal tax rate, μ is cash flows of merging firms, σ is volatility of cash flows, X is unlevered firm value (or expected value of cash flows), ρ is correlation of cash flows. Two firms $V_1(.)$ and $V_2(.)$ need to be valued including their securities before after the merger. Here a merger operator is also required as

$$V_{merged}(.) = V_1(.) \otimes V_2(.), \tag{1.3}$$

where $\otimes$ defines the merger operator which will map and identify merger synergies.

The task is to construct an applied model considering the trade-off between abstraction and realistic modelling. An applied model is an explicit, simplified representation of more general theories which are designed to apply to specific real-world problems or situations (Boland, 1989). The constructed model will be the research tool. The dynamics of the constructed model will measured and analysed to explain the real world phenomena.

The model must be testable for verification purposes. Control variables may be the output of existing literature, or observed data or benchmark models. To test the model's efficiency and external validity, model parameters/variables will be calibrated to replicate observed values of the variables. The proposed model must be able to value all outstanding securities of merging firms before and after the merger. The company valuation model employed in this study is contingent claim valuation as applied in Brennan (1979) or in Merton (1973)[5]. External validity of the model has a key importance. A non-generalizable model cannot be used as an applied model. Internal validity will be provided by following empirical and theoretical literature.

1.4.1 Analytical Framework

The type of modelling needs to be defined before constructing the analytical framework. Valuation of the existing securities of the merging firms involves the valuation of contingent claims. These claims can have a positive or negative future value. Thus, contingency introduces default risk to the model. Black and Scholes (1973) model corporate liabilities as combinations of options. They demonstrate valuation of common stocks, warrants and bonds. The framework used in this study is similar to theirs in terms of modelling corporate liabilities as options.

Credit risk models are also employed which handle contingency. There are two classes of credit risk models; reduced form models and structural models. Reduced form models do not use company specific data and try to model a group of firms' behaviour. Reduced form models can also be used in merger modelling. In this class of models defaults are caused by exogenous shocks and modelled using hazard functions. For example, merger waves can be analysed under an external economic shock. However structural models use company specific data and try to make company specific valuations for existing securities of firms. The latter of the two modelling frameworks best suits to this study so that the structural class of models is used in this thesis. This will enable to use firms' default risk, cash flows, volatility of cash flows and other critical parameters for merger modelling.

[5]Contingent claim is an asset whose payoff depends upon another "underlying" asset (Brennan, 1979).

This framework needs some unobservable data like volatility of cash flows, thus estimation methods will be employed.

The Leland (2007)'s framework is first used and some of the extensions on the model are demonstrated. Consequently, having used the Ammann and Genser (2004) model, a dynamic merger model is constructed. After demonstrating the single period model the next step is to extend the model to run in a multi-period setting. This is not for the sake of a comparison of these two frameworks, rather it is complementary. These models are chosen because of their ability to explain values of existing securities pre- and post-merger with their simple and intuitive designs.

Calibration method will be employed in the modelling phase. Models will be calibrated to produce observed real world parameters. For example, to model a BBB rated firm in the model, observed values of cashflows, default cost and default risk for a BBB rated firm will be used. This way, models will be available for empirical testing. Model outputs can easily be compared with observed values to test the model. If the model does not fail these tests, then it can be used to make contributions to existing theory by generalisations.

1.5 Contributions

This thesis is the first study to analyse mergers in such detail and also is the first study to model wealth transfer strategies for shareholders. For example, previous studies (Higgins and Schall, 1975; Leland, 2007) pointed out that calling existing bonds may increase shareholders gains in case of a merger however it has never been modeled explicitly.

To our knowledge this thesis is the first study which uses an Arithmetic Brownian Motion (ABM) process for earnings before interest and tax (EBIT) of the firm. This has two advantages. First, ABM process can produce negative realisations for the EBIT. Second, combination of two normal distributions will give a normal distribution therefore creates a mathematical convenience.

Using structural credit risk models, all existing security prices and term structures are modeled with closed form solutions. After using a one period model this thesis also employs a multi period model to analyse interactions of bonds with different maturities in case of a merger.

The main contribution of this thesis is it shows that shareholders can increase their merger returns using various strategies. These strategies include using callable bonds instead of plain bonds, issuing bonds with short maturities or timing of bond reissues. Each method listed here enhances financial synergy gains of shareholders. Increasing shareholder gains is important as it will prevent a conflict between managers and shareholders.

Call provisions on debt increases shareholder gains and firm value after in case of a merger. Even though a risk premium is offered to bond holders firm and shareholders from a synergistic merger.

Debt maturity also affects the gains of shareholders. As debt maturity gets shorter gains of shareholders increase. One extreme case is a merger when merging firms have perpetual bonds outstanding. In this case most of the merger gains are realised by perpetual bond holders.

Firm may also change the merger timing so that it merges before a debt issue and preferably after the maturity of short term debt. This can be used if the firm cannot call the bonds back without any cost. The goal of the firm must be to merge when the outstanding bond portfolio is small with a short maturity.

1.6 Conclusion

Some preliminary models were constructed to investigate the problem further. A merger matrix was used to understand the relation of credit ratings and financial synergies. A CDS contract is modelled to investigate the relations between credit risk and mergers. Callable bonds were used to change the distribution of financial synergies. These showed that there may be a conflict between shareholders and managers as in some cases mergers may decrease the wealth of shareholders. To prevent a wealth decrease shareholders may use callable bonds or CDS contracts. This thesis investigates the distribution of financial synergies between bondholders and shareholders in a structural credit risk model. While investigating this, strategies and instruments were developed to increase the wealth of shareholders.

Rest of this thesis is as follows. Chapter 2 looks at the existing literature and shows the gap in existing literature. Chapter 3 introduces the research methodology and briefly demonstrates some of the methods used in this thesis. Chapter 4 analyses financial synergies of mergers using a one period model. Chapter 5 introduces a new multi-period model and analyses its dynamics without a merger. Chapter 6 introduces mergers to the multi period model introduced in Chapter 5 and analyses the financial synergies of mergers in a multi-period setting with a complex tax and capital structure. Chapter 7 concludes.

Chapter 2

Literature Review

2.1 Introduction

Economic and financial aspects of mergers have been widely analysed in many theoretical and empirical studies. Undervaluation, diversification, synergy, poorly managed firms and managerial self interest are some motives behind mergers. The focus of this study will be on the financial effects of mergers which is a sub-category of synergy motive. Synergy is the sum of gains generated by merging firms, which cannot be created by individual firms. Synergy is the key motive and can be divided into two main parts: operating synergies and financial synergies. Operating synergies have been analysed in great detail but there are few papers on the analysis of financial synergies. Recently the interest has shifted to financial synergies in parallel with the emergence of the structured finance products. Structured finance products can be modelled as spin-offs which are the reverse of mergers. This reflects the fact that reversing the analysis of financial synergies is nothing but the analysis of structured finance instruments such as securitization, and project finance. Modelling structured finance and mergers as the reverse of each other gives the flexibility to analyse both activities using the same framework. Risky asset payoffs cannot be known with certainty thus claims on these assets are contingent. Analysis of financial synergies requires valuation of risky assets and/or companies. Contingent claims valuation methods satisfy these requirements.

This chapter first gives a brief literature review of the main capital structure theories. It then provides a detailed coverage of previous studies into mergers and financial synergies. It finally concludes.

2.2 Capital Structure

As capital structure literature is extensive, only the most relevant or influential pieces of literature are included here. The review starts with the famous Modigliani Miller theorem and then extends to others.

2.2.1 Modigliani-Miller (MM) Theory

The theory of optimal capital structure starts with Modigliani and Miller (1958)'s paper. In an equilibrium setting they showed that the market value of any firm is independent of its capital structure. Their firm valuation depends on the left side of the balance sheet, namely assets in place and growth opportunities (Constantinides, Harris, and Stulz, 2003, p. 219). In a series of papers they looked at the relationships between debt equity ratio-market value, leverage-weighted average cost of capital, market value-dividend policy and equity holder-firms' financial policy and propose an absence of relationship between them.

In a perfect capital market, value irrelevance may be true, however with the existence of costs and taxes, results will be different. Later Modigliani and Miller (1963) accepted that the amount of tax savings from debt will be higher than they suggested. They note that the difference between their valuations and traditional view is narrowed.

Financial innovations and new instruments can work against MM propositions. A simple example may be given by asset securitization. Asset securitization is like slicing a pie. Although overall value will not change just by slicing the pie, it is obvious that some slices will have higher values than the others while the sum of slices may still be different than the whole pie. Some slices may have fruits on them, some may not. If this is the case then these slices with higher values can be used to finance other slices as their average prices (cost of capitals) will be higher (lower).

Another example against MM theory is the existence of taxes. Firms can benefit from tax savings by issuing debt. The increase in tax savings will increase the overall firm value. The irrelevance argument of MM theory was under perfect capital markets. If imperfections exist then the reason for relevance must be these imperfections. Trade-off theory, agency theories and The Pecking-Order theory relax the assumptions of MM and try to explain the capital structure choices.

2.2.1.1 MM Propositions

MM Proposition I (Modigliani and Miller, 1958) states that *the market value of any firm is independent of its capital structure and is given by capitalising its expected return at the rate $\rho_k$appropriate to its class* . According to Proposition I in equilibrium $V_j = S_j + D_j = \overline{X_j}/\rho_k$where ρ_k is the expected rate of return, $\overline{X_j}$ is expected profit before interest, D_j is market value of debt, S_j is market value of equity and finally V_j is the market value of the firm. Any discrepancies will be corrected by investors through arbitrage. The process is also called **homemade leverage**, where investors can borrow at the same rate with firms. If levered firms are priced too highly then an investor can borrow money and buy shares of a fully equity financed firm to duplicate the effects of corporate leverage.

MM Proposition II states that *the expected yield of a share of stock is equal to the appropriate capitalization rate ρ_k for a pure equity stream in the class, plus a premium related to financial risk.* $i_j = \rho_k + (\rho_k - r)D_j/S_j$, where i is yield on the stock of any company j belonging to the kth class. In other words this proposition says that the required return on equity increases with debt ratio.

2.2.2 The Trade-off Theory

This theory is contrary to MM Proposition I. Firm value changes as capital structure changes. The source of value change comes from benefits of leverage. A firm's capital structure is determined by finding its optimal debt ratio. Increasing the amount of debt is optimal when the marginal benefits from borrowing is higher than the marginal costs. The costs stems from the bankruptcy costs of increased debt. The difference between an unlevered firm value and leveraged firm value is the cost undertaken to leverage the firm and the tax savings from interest payments.

$$V_L = D + E = V_U + TS - DC$$

where V is firm value and subscripts L and U denote levered and unlevered firm values. TS is tax savings and DC is default cost. Managers maximise firm value by choosing the optimal amount of debt. This mechanism will only work while the tax benefits of debt is higher than the financial distress it causes. If tax shields are not sufficiently high, because of the corporate tax system, then firms will not have this opportunity.

Graham (2000) analysed the tax advantages of debt and found that tax savings are about \$0.20 per dollar of pretax income. Graham (2000) concludes that growth firms that produce unique products and large, profitable, liquid firms use debt conservatively.

2.2.3 The Pecking-Order Theory

This theory is developed by Myers and Majluf (1984). They assume perfect financial markets except for the information asymmetry. Financing decision is an outcome of the timing of debt or equity issuance. Managers will have more information about the firm than investors. An equity issuance will be optimal when managers think the market price is right. The right price for the equity issuance is the price at which managers think the stock is overvalued. And investors will respond by not investing to the new stock issue by thinking the firm will not issue stock if it is not overvalued. Therefore investors will avoid buying the stocks causing a decrease in stock price.

According to this theory firms will first prefer to use internal financing then they will issue debt and finally they will use equity issuance.

There is no target amount of leverage as in trade-off theory. In trade-off theory firms issue debt as long as marginal benefit of debt is positive. However, in pecking order theory there is no limit for leverage. In the pure form firms may be 100 percent debt financed.

If a firm is profitable then it will use less debt according to pecking order theory. However, profitable firms may like to increase their profits by increasing leverage. This is just the opposite of trade-off theory where higher profits increase debt capacity and increased debt amount will increase the amount of tax savings.

2.2.4 Agency Theories

These types of theories say that managers (agents) and principals (shareholders) may have different interests and therefore there may be a conflict of interests. Managers may start to act on their own interests rather than acting on the best interests of shareholders or the firm. Capital structure is not optimised or aligned according to a target. Managers may want to invest in negative net present value (NPV) projects for their own interests. Jensen and Meckling (1976) define agency costs as the sum of monitoring expenditures, bonding costs and residual loss. These costs are created by the agency problems.

2.3 Contingent Claim Valuation

Black and Scholes (1973) and Merton (1974) used option valuation techniques to value contingent claims on firms' assets. They treated firm securities as options, for example, debt is a risk free bond and a put option written on firms' assets and equity is a call option. This study is similar to this line of research which uses options framework to value firm securities. Contingent claim valuation methods were applied to different lines of research. Brennan (1979) analysed contingent claim valuation models in discrete time, Ingersoll (1977) of convertible securities, and Heath, Jarrow, and Morton (1992) to bond pricing.

2.4 Merger and Financial Synergies

Merton (1974) developed a method for pricing corporate liabilities using an options pricing framework. Merton (1974)'s model used provisions of the indenture and limited liability of claims as the source of option. If a company cannot pay its debt, shareholders have the right to default the company to debt holders. Thus shareholders' liabilities will be limited.

Modigliani and Miller (1958) showed that capital structure does not affect firms' total value. Without taxes and default costs their results are correct, however, it is not the case in the presence of taxes and default costs. If there is a risk of default, bonds of the firm will become risky assets and yield spread of bonds will be related to the firms' debt ratio (Stiglitz, 1972). In this case managers can maximise firm value by choosing optimal capital structure. This argument also defines the yield spread difference between issuers with different credit ratings. A government bond, which is generally assumed risk-free, will have lower yield than a corporate bond [1]. This feature of debt is also one of the sources of financial synergies.

Higgins and Schall (1975) studied gains of bondholders and stockholders. They focus on two issues. First, they investigate if a merger benefits stockholders by reducing bankruptcy probability. Second, they want to analyse effects of merger on firm value when bankruptcy costs are positive. They use an equilibrium type of model to analyse mergers and shareholder and bondholder gains. Their results state that while total firm value is not affected from merger equity value may decline if existing debt is not recalled. This result is also confirmed in this thesis and call provisions were employed to increase the gains of shareholders.

Scott (1977) use a state preference model to analyse mergers. Their results are inline with the results of this thesis. They state that the tax structure encourages mergers for firms with outstanding bonds and equities. They also comment on bankruptcy costs saying they can work in either direction in a merger. The models presented in this thesis confirm these results. Tax savings increase firm value due to an increase in amount of outstanding debt while bankruptcy costs decrease firm value due to newly taken debt.

Using option pricing theory Stapleton (1982) shows that debt capacity may also be affected when merging firms are perfectly correlated. The author also states that debt capacity effects of merger can be underestimated using simple risk-neutral models. Therefore in this thesis a one period model is first used to analyse the dynamics of mergers and then a multi period model is introduced which can handle complex capital structures and multiple tax rates.

Shastri (1990) analyses financial effects of an exchange offer merger of firms with different risks and capital structures. Shastri (1990) uses the techniques for valuing securities as simple and compound options. The author classifies possible wealth

[1] It is important to note that some companies have higher credit ratings than some countries.

transfer effects by the security type and maturity (i.e., stockholders to long-term debt holders). Each of these cases are considered and analysed in Chapter 6 of this thesis.

There are different models analysing mergers with different focuses. Lambrecht (2004) analysed timing and terms of mergers with a one factor model motivated by economies of scale which mostly applies to horizontal mergers. The author has used the continuous-time real options framework which relies on a strong form of market efficiency. Strong form of market efficiency is not a realistic assumption, generally markets are accepted to be semi-strong-form efficient. Timing of mergers is endogenous in the model thus the model tries to identify when to merge. The author considers merger transaction costs like legal fees, fees to investment banks and other merger promotors, and costs of restructuring and integrating two companies defining each firm's pay-off as an option. Besides synergies Lambrecht (2004) sees creation of market power as a reason to merge. The merger resembles a call option for each merging firm and the author analyses division of merger surplus. Merger decision is taken in two steps by the managers of the merging firms. In the first step they make decisions on timing of merger and net present value of total merger surplus and in the second step they decide on how to share the surplus.

A dynamic real options model, used by Morellec and Zhdanov (2005), analyses abnormal returns to shareholders with endogenous timing. Although their model makes some predictions on shareholder returns, it does not make predictions about bond values of the merging firms. Their model is complementary to the model which Lambrecht (2004) used. Morellec and Zhdanov (2005) use a reduced form model for takeovers with a risk neutrality assumption.

Another real options approach is used by Hackbarth and Miao (2008), similar to Morellec and Zhdanov (2005). They build an industry equilibrium model using a real options framework in continuous-time to analyse how product market competition affects the gains from mergers. Unlike Lambrecht (2004), they assume a production function with constant returns to scale where Lambrecht (2004) assumed economies of scale. Merger costs are considered in the model. They specify a tangible asset that helps increase output for a given average cost instead, assuming a presence of exogenous operating synergies. They focus on shareholder value but on debt value after merger. Their main focus is on the relationship between product market competition, industry concentration and takeover returns. Increased product market competition among heterogeneous firms does not speed up the acquisition process (Hackbarth and Miao, 2008). They also relate merger activities and magnitude of merger returns to industry concentration. They model demand shocks as an identifier of mergers. However Leland (2007) analysed purely financial synergies in the absence of operational ones, using a structural model.

Fluck and Lynch (1999) analysed financial synergies, motives behind mergers and divestitures. However, their model has some limitations. First, one of the firms must be financially distressed in their model. Another limitation is that it only applies when there are agency problems between the managers and the shareholders of the target firm. They see mergers as a source of financing for projects where firms cannot finance themselves. Financial or economic synergies are the main advantages of mergers. They also focus on stock price response to a divestiture decision, however, it is the stock market response. Their theory is consistent with the argument that mergers increase the combined value of acquirer and target firms.

2.4.1 Sources of Gains

After a merger the combined firm becomes financially stronger than before the merger. Thus it is expected that merged firms' debt will be less risky than separate firms'. Firms can provide a mutual guarantee after merger; if one company's debt defaults it can be guaranteed by the other firm's cash flows. This argument brings attention to the merging firms' correlation of cash flows. Lower correlations between cash flows of two merging companies will provide a higher level of insurance. Merging two perfectly correlated firms will be less synergistic in terms of this insurance. This is called as coinsurance effect. Coinsurance effect will let lenders establish a new aggregate limit on lending for the merged company, which can exceed the sum of individual limits of separate firms. Thus, in a world where capital structure decisions affect firm value, managers will have the choice to change the capital structure. If changing leverage ratio increases firm value they will prefer to change it. Lewellen (1971) builds his model on coinsurance effect. Companies also can benefit from higher amounts of tax savings obtained by higher borrowing limits. Higher amounts of borrowing will lead to a higher optimal leverage level. Lewellen (1971) argues that combining companies with lower correlations will increase debt capacity by decreasing volatility of cash flows compared to individual firms. The decrease in the default risk increases debt limits of the company which also increases the potential tax savings. These findings are in line with recent studies. Limited liability has no value for Lewellen (1971) in the absence of "negative future cash flows". In an open economy all companies may face bankruptcy; unless it is a default-proof government organisation.

Sarig (1985) assumes presence of negative future cash flows in his model. With negative future cash flows, limited-liability provides an option to walk away for owners. This option defines the value of limited liability. According to Sarig (1985), if limited liability has a value, then having two options is more valuable than having one. Loss of value after merger through this mechanism is called limited liability effect. The explained decrease in value is generally compensated by other effects and mergers become profitable. Sarig (1985) only focuses on limited liability effect and argues that mergers reduce the aggregate value of merging firms' securities. Without tax benefits and coinsurance effect, this argument is true but after a merger all three effects react and generally create a positive value for the merged firm. Recent studies also showed that sum of these three effects are positive. Flannery, Houston, and Venkataraman (1993) similar to others argue that combining activities with not highly correlated cash flows creates value. Their model depends on operational synergies. Leland (2007) disregarded operational synergies to analyse purely financial synergies.

Analysing only the returns to shares will not reveal the total return of mergers. To analyse total financial effects, debt values must be included in the model. One of the results of Morellec and Zhdanov (2005) is that stock returns increase as there is a lower correlation between the stock returns of merging firms. This is inline with the literature discussed here however the valuation of the company still has some missing parts. The effects of merger was not analysed on outstanding debt instruments.

2.4.2 Distribution of Gains

Shareholders, as an authority for the merger decision, will not allow a merger which will decrease share prices or increase debt holders' gains. On the other hand they can force the management to decrease the bondholders' gains in favour of shareholders, in other words, to redistribute the merger gains. This can be done by using short term debt or by issuing bonds with call options (Leland, 2007). Short term debt can be used by issuing debt instruments which mature before a merger. After the merger new debt instruments can be issued. The company can prevent bondholders from over-benefiting from the merger. Issuing bonds with call options can be used similarly as in short term debt where outstanding debt can be called by the company after the merger with a previously specified price to distribute the increase in outstanding debt value to shareholders. Instead of redistributing merger gains, Lambrecht (2004) takes maximising shareholder value as an objective function. This is slightly different to (Leland, 2007)'s model where firms are valued as a total of share value and debt value, while optimal capital structure is calculated by maximising this total.

Leland (2007), with his one period model, analysed both debt and share values of a theoretical firm before and after merger. First he starts with a simple capital structure model which optimises bond and share values to maximise total company value. Leland (2007) assumes that operational cash flows of combined activities are non-synergistic. Previous studies, analysing distribution of merger gains between extant bondholders and stockholders, do not have an explicit model of optimal capital structure. The author shows that most of the financial synergies are gained by extant bondholders. The model analyses mergers in a single period timeframe, hence, it is not convenient to make extensions. At least there must be two time periods for firms to use short term debt or bonds with call options.

Divestitures are also analysed in previous studies. Fluck and Lynch (1999) see divestitures as good news showing the merger-financed projects' ability to survive as a standalone firm. Divestitures are very similar to a structured finance tool known as securitization. Some quality assets of a company -here merger financed projects- are pooled. Then this pool is converted to a standalone company. This standalone company is named as special purpose vehicle (SPV) (Gorton and Souleles, 2005). By definition SPV's are default-proof and they have a separate body than the originating firm. This is similar to divesting the profitable project from the merged company. Merging imperfectly correlated cash flows creates synergy by decreasing the default risk and increasing the potential tax savings. To apply this rule to a single firm, a company's cash flows can be grouped into different risk groups. Then a separate firm can be created with some high quality assets of the main company. The new company with high quality assets will have a higher credit rating than the main company. This process is just the reverse of a merger and is a tool of structured finance called securitization, similar to 'divestiture' in Fluck and Lynch (1999). Securitization is conducted by some characteristic stages. These can be defined as; pooling and transferring. Pooling means to create a set of high quality assets (cash flows) of the main company. Transferring means to transfer the higher quality assets to a SPV. Leland (2007) also analyses separation with his closed form model. In his model, securitization (a form of separation) is defined as the reverse of a merger. Other aspects of securitization such as agency costs are not included in this analysis but the model still correctly shows the dynamics of securitization. Gorton and Souleles (2005) focus on special purpose vehicles and securitization and finally build a theoretical model of SPV's.

2.5 Conclusion

Literature discussed here shows strengths and weaknesses of existing studies on mergers. To summarise, modelling purely financial synergies requires exogenous timing of mergers, absence of operational synergies and extensions to cover the re-distribution of merger gains. Different aspects of mergers, including returns to stockholders have been widely analysed in the existing literature. However, there is a gap in the analysis of shareholders' and bondholders' gains together. Even some authors (Higgins and Schall, 1975; Scott, 1977; Shastri, 1990; Stapleton, 1982) studied security holder returns after merger however, they either do not explicitly model the problem or do not develop strategies to redistribute the merger gains. Market value of shares will show the gains through mergers while in real life it is not always possible to filter external price effects from share prices. So market prices should not be used in stock valuation. This valuation must also cover the value of existing or newly issued debt instruments to reveal gains of debt holders and shareholders of merging companies.

Chapter 3

Methodology

3.1 Analysis of Financial Synergies

Mergers and structured finance instruments are directly linked together as they are the reverse of each other. In a broader sense they are both elements of company restructuring. Quantitative analysis of both activities require valuation of firms and their securities. Pre- and post-restructuring event firm valuation along with all existing securities will reveal the financial synergies.

Traditional valuation methods state that the price of any asset is the present value of its cash flows. However it is not possible to know a cash flow from the future. Cash flows are not definite, thus, the firm's survival cannot be definite. This creates a randomness for the claimholders of a firm, thus, they hold contingent claims on the company. A contingent claim is an asset which has a pay-off created by another underlying asset. The underlying asset here is the firm which depends on the cash flows. With a probability of, p, claim holders will receive their claims in full and with probability of, $1-p$, claimholders will lose some or all of their claims.

For example, shareholders' claims on a company can be modelled as an option written on assets of the company. Here the problem of firm valuation becomes an option pricing problem. Availability of the option is provided by limited liability of claims. Under limited liability, shareholders have the option to default the company to bondholders. Option valuation methods can be employed for valuation of these contingencies. Moreover, option pricing models provide a valuation for flexibility, however, conventional company valuation methods generally do not provide valuation of flexibility (and randomness) explicitly. A well known example is Merton (1974). Using a diffusion type stochastic process [1] Merton (1974) defined the dynamics of a company. The value of the company was defined as the sum of its debt

[1] $dV = (\alpha V - C)dt + \sigma V dz$, α is the instantaneous expected rate of return on the firm per unit time, C is the total dollar payouts by the firm per unit time to either its shareholders or bondholders, σ^2 is the instantaneous variance of the return on the firm per unit time, dz is a standard Gauss-Wiener process.

and equity. [2] Merton (1974) proposed a structural credit risk model by defining equity as a European call option on its assets. Model requires the current value of the assets, the volatility of the assets, the outstanding debt, and the debt maturity as inputs.

The rest of this chapter discusses the available methods and the choices of methods for the proposed research. Firm and securities valuation, option pricing, numerical methods, firm restructuring are the methods that are presented in this section.

3.2 Company Valuation Methods

Discounted cash flow (DCF) valuation depends on the assumption that the value of any asset is the present value of expected cash flows generated by the asset. The DCF method is not capable of handling uncertainty. Traditional DCF assumes discount rates and cash flows are known with certainty.

$$Firm\ Value\ = \sum_{t=1}^{t=n} \frac{E[CF_t]}{E[r]} \tag{3.1}$$

Where n is life of asset, CF_t is the cash flow in period t and r is the discount rate. Using an options framework rather than the traditional DCF methods considers all future investment opportunities and provides flexibility to management (Miller and Park, 2002). Company valuation methods other than real options framework do not handle flexibility or contingency explicitly. Real options approach (ROA) is an adaptation of financial option valuation models to investment decisions. These models can capture the flexibility of managers which DCF do not.

ROA is similar to financial options, the only difference being the underlying asset. Financial options use stocks or bonds, real options use real assets. The underlying

[2] $V \equiv F(V,\tau) + f(V,\tau)$, τ is time, F is bond value, and f is equity value

asset of the option can be seen as the project and its outcome as well as the liabilities of the firm. DCF method makes all the decisions at the beginning of the analysis without any flexibility thus DCF's accuracy highly depends on these initial decisions. Real options methods allow flexibility and consider different decision sets. Options pricing methods used by ROA will be covered in section 3.3.

3.3 Option Pricing

Options are tradable financial contracts that give the holder the right to buy or sell a specific security at a specific price. Option pricing methods introduced randomness in to the existing pricing models. Therefore they can be extended to areas where randomness is present.

Option pricing models generally assume a stochastic process to model the underlying assets price movements. The process is defined as; $dX = \mu X dt + \sigma X dW$. W is a Brownian motion stochastic process, μ is the expected return on asset in the time interval dt and σ is standard deviation of expected returns, which also states that the asset returns also follow a Brownian motion.

Using Itô's lemma, no arbitrage condition and with some algebra random component can be eliminated to get;

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V}{\partial X^2} + rX\frac{\partial V}{\partial X} - rV = 0. \tag{3.2}$$

This is the fundamental pricing equation. This equation is also known as Black and Scholes (1973)' equation. The solution for this partial differential equation will give an expression for the option price in terms of its underlying asset. Under certain assumptions Black and Scholes (1973) found a closed form solution for this equation which is a special case.

A good review of option pricing methods is provided by Broadie and Detemple (2004). Options can be grouped into two categories by their exercise terms. Options which only allow an exercise at maturity date are called European options and options which allow an early exercise are called American options. American and European are two extreme cases in terms of exercise time; there are other types of options which fall between them.

As mentioned in the previous section Merton (1974) uses a European option to model equity which only allows default at maturity date of the bonds. Firms are assumed to have infinite life span and may default at any time. An option with European properties will not serve well to firm valuation needs as they will not allow early exercise.

3.3.1 Barrier Options

Barrier options' pay-off, depends on whether underlying asset price hits a barrier or not. Down-and-out barrier option ceases to exist when underlying assets price reaches the barrier and down-and-in option comes into existence when the underlying asset price reaches the barrier (Hull, 2003). "Out" options are also called "knock-out" options and "in" options are also called as "knock-in" options.

The payoff of a down-and-out call is $max(S(T) - K, 0)$ if $z > L$ and 0 otherwise at maturity T where S is underlying assets price, K is strike price, L is lower barrier and z is the minimum price of underlying asset (Back, 2005). The value of a continuously-sampled down-and-out call option with barrier L is given by (Back, 2005)

$$\begin{aligned} &\mathrm{e}^{-qT} S(0)[N(d_1) - (\tfrac{L}{S(0)})^{2(r-q+\frac{1}{2}\sigma^2)/\sigma^2} N(d_1')] \\ &-\mathrm{e}^{-rT} K[N(d_2) - (\tfrac{L}{S(0)})^{2(r-q-\frac{1}{2}\sigma^2)/\sigma^2} N(d_2')] \end{aligned} \tag{3.3}$$

$$\begin{aligned} &for K > L \\ &d_1 = \frac{\log(\frac{S(0)}{K}) + (r-q+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\ &d_2 = d_1 - \sigma\sqrt{T} \\ &d_1' = \frac{\log(\frac{L^2}{KS(0)}) + (r-q+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\ &d_2' = d_1' - \sigma\sqrt{T} \end{aligned} \tag{3.4}$$

$$
\begin{aligned}
&\text{for} K \leq L \\
&d_1 = \frac{\log(\frac{S(0)}{L}) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
&d_2 = d_1 - \sigma\sqrt{T} \\
&d_1' = \frac{\log(\frac{L^2}{S(0)}) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
&d_2' = d_1' - \sigma\sqrt{T}
\end{aligned}
\tag{3.5}
$$

Haug (2001) shows pricing of an American barrier option using a plain vanilla American option formula utilising the reflection principle which enables fast and accurate pricing.

Examples to applications of barrier options to firm valuation are Ericsson and Reneby (1998) and Ericsson and Reneby (2003). They see corporate securities as a portfolio of three basic claims: a down-and-out call option, a down-and out binary option and a unit down-and-in claim. Using this definition they construct a framework for valuing corporate securities.

3.4 Numerical Methods for Option Pricing

Derivative values are often expressed with differential equations in terms of an underlying asset or assets. Analytical solutions of these equations are not always possible. Numerical methods use simulations or approximations to solve differential equations of derivatives. Lattice methods, Monte Carlo simulations and finite difference methods are the most commonly used numerical methods.

3.4.1 Lattice Methods

The idea behind lattice methods comes from generating the price distribution of asset price in a discrete timeframe. Lattice trees are used to generate the prices. Each step on the tree corresponds to time steps and as the number of steps increases the model converges to its continuous time equivalent. These models assume risk neutrality and use discrete timeframe with hedging availability and no arbitrage condition. Although there are various types of lattice methods they all use the following work flow; First, price trees for assets are generated to simulate stock price behaviour. Then using these underlying asset prices derivative prices are calculated backwards by starting at the end of the tree. This requires availability of pay-off function of the derivative being priced thus the price of derivative at the maturity is equal to its pay-off.

3.4.1.1 Binomial Methods

An options underlying asset price is modelled with two outcomes on each time step of its life. The asset price S may go up to Su or may go down to Sd on each time step N. The original model proposed by Cox, Ross, and Rubinstein (1979) defines u and d as follows

$$u = e^{\sigma\sqrt{\Delta t}},$$

$$d = e^{-\sigma\sqrt{\Delta t}}.$$

CRR model recovers the volatility only for infinite N and fits the physical process to the binomial model. There are many variations of binomial models, for example, Jarrow and Rudd define u and d as;

$$u = e^{\mu\Delta t + \sigma\sqrt{\Delta t}}$$

$$d = e^{\mu\Delta t - \sigma\sqrt{\Delta t}}$$

where $\mu = r - \frac{1}{2}\sigma^2$.

CRR tree is symmetric while $ud = 1$ but the up and down probabilities are not equal. However in the Jarrow-Rudd (JR) (Jarrow and Rudd, 1983) model the probabilities are equal.

American options (early exercise availability) can be priced using binomial trees. To improve the accuracy of the pricing of an American option, control variate technique can be used (Hull and White, 1988). This method simply calculates the value of the American option and European option using the same tree and compares the European option price with the corresponding Black-Scholes option price. The error given by tree for the European option is assumed the same for the American option.

There are different types of lattice trees which use more than two states on each node. They will be referred to as multinomial trees. Multinomial trees can produce identical values to binomial tree results with a faster convergence, however multinomial lattices are more complicated than binomials and require higher computational time. Binomial trees cannot handle volatility changes and more than one asset or mean reverting process, thus, multinomial trees are required. An example of a lattice model with two underlying assets is Boyle (1988).

3.4.1.2 Multinomial

The binomial method is now extended to the multinomial case as in Kamrad and Ritchken (1991). Assume that the underlying asset follows a Geometric Wiener Process with a drift of $\mu = r - \sigma^2/2$ where r is the risk free rate and σ is the instantaneous volatility.

The distribution $\xi(t)$ is approximated with $\xi^a(t)$ over the period $[t, t + \Delta t]$. Discrete random variable $\xi^a(t)$ takes following values

$$\xi^a(t) \begin{cases} v, & p_1 \\ 0, & p_2 \\ -v, & p_3 \end{cases}, \tag{3.6}$$

where $p_{1,2,3}$ represent probabilities and $v = \lambda\sigma\sqrt{\delta t}$ and $\lambda \geq 1$.

Choosing the same mean and variance with $\xi(t)$ for approximating distribution $\xi^a(t)$ gives

$$E\{\xi^a(t)\} = v(p_1 - p_3) = \mu\Delta t,$$

$$Var\{\xi^a(t)\} = v^2(p_1 + p_3) = \sigma^2\Delta t + O(\Delta t).$$

Using $v = \lambda\sigma\sqrt{\Delta t}$ and $p_1 + p_2 + p_3 = 1$ probabilities can be found as

$$p_1 = \frac{1}{2\lambda^2} + \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma}, \tag{3.7}$$

$$p_2 = 1 - \frac{1}{\lambda^2} = 1 - p_1 - p_3, \tag{3.8}$$

$$p_3 = \frac{1}{2\lambda^2} - \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma}. \tag{3.9}$$

The probabilities calculated here are the probability of different outcomes at each node. At any node probability of moving up, down and staying the same is p_1, p_2 and p_3 respectively. This model can be extended to include more probable states for each time step as shown by Kamrad and Ritchken (1991).

3.4.2 Monte Carlo Simulation

The Monte Carlo method simulates underlying assets' price behaviour. The price of an option is defined as the present value of the expected pay-off at expiry date with a risk neutral random walk for the underlying. Simulating risk-neutral random walk will give the price path for the underlying asset. If the price process of underlying asset is a stochastic process with σ volatility and r expected return under risk neutrality;

$$dS = rSdt + \sigma SdX \tag{3.10}$$

Option value is defined as

$$Option\ Value = e^{-r(T-t)}E[Payoff(S)]. \tag{3.11}$$

Random numbers generated from a standardized normal distribution can be used to update asset price S using $\delta S = rS\delta t + \sigma S\sqrt{\delta t}\phi$, where ϕ is drawn from a standardized normal distribution. This discreet simulation of time series is called the Euler method.

However, for lognormal random walk an exact solution can be found for S. Equation 3.10 can be written as

$$d(logS) = (r - \frac{1}{2}\sigma^2)dt + \sigma dX, \tag{3.12}$$

which at the end gives the exact solution for S

$$S(t + \delta t) = S(t)exp((r - \frac{1}{2}\sigma^2)\delta t + \sigma\sqrt{\delta t}\phi) \tag{3.13}$$

thus it is possible to calculate $S(T)$ from $S(0)$. The Monte Carlo simulation allows the calculation of the whole path of an asset price. In some cases pay-off of the derivative may depend on the price path not only the price at expiry. Pay-offs may occur not only at the end of the term but also at several different times. They may also depend on several different market variables leading to a more realistic modelling of a company valuation. As the number of stochastic processes increases Monte Carlo simulations become more efficient. For instance, the required time to calculate Monte Carlo simulations increases linearly unlike other methods where required time increases exponentially. The precision of Monte Carlo simulations is enhanced as the number of sample paths is first increased and then their average pay-offs for the option price is employed. One of the disadvantages of MC simulations is that it can be computationally time consuming especially when the number of samples is greater. MC simulations cannot easily handle early exercise opportunities.

3.4.2.1 Variance Reduction

Increasing the number of samples increases accuracy, however it comes with the cost of increased computation time. Variance reduction procedures aim to increase accuracy for lower number of samples. The Antithetic variable technique calculates two values for a derivative. The first is calculated the usual way f_1 and the other is calculated by changing signs of all random samples f_2. The value of derivative is the average of f_1 and f_2. Control variate technique assumes the same error for a similar derivative which is calculated analytically and numerically. Importance sampling tries to use samples which will not lead to zero pay-off, thus it saves computation time. Stratified sampling and quasi-random sampling methods use controlled sampling techniques instead of random sampling.

3.4.3 Finite Difference Methods (FD)

Different to the MC simulations and lattice methods, finite difference (FD) methods try to solve the differential equation of the derivative. The differential equation is converted into difference equations and solved iteratively. Fundamental pricing equation (eq. 3.2) is an example equation for this. FD methods resemble lattice methods, however, this time stock prices are not generated by a process. The stock prices are obtained by dividing maximum stock price into equal intervals for every equally divided time step through options life. This maximum stock price is typically three or four times the value of the asset at which there is some important behaviour. Using equal price intervals and time intervals FD grid can be constructed. For asset values $S = i\delta S$ and for times $t = T - k\delta t$. The terms in equation 3.2 can be approximated using this grid. Moving up or down on the grid will correspond to different option prices corresponding to different stock prices and moving horizontally on the grid will give option price change due to time change.

An option price on the grid can be defined as V_i^k where i is for asset price interval index and k is time interval index. As time step k increases, time decreases. The terms of the fundamental pricing equation can be approximated using a grid. For example, option theta is given by

$$\Theta = \frac{\partial V}{\partial t} = \frac{V_i^k - V_i^{k+1}}{\delta t}. \tag{3.14}$$

Using pay-off function, the option price at maturity date can be calculated and the type of the option will specify the option prices at the boundaries of the grid, for example, a call option value when $S = 0$ gives $V = 0$. For all points on grid option value can be calculated using approximations however iterative calculations must start from a point where option prices are known. Using three points at time step k and calculating option values at time step $k + 1$ as a function of time step k is known as explicit finite difference method. Explicit finite difference method can also be interpreted as a trinomial tree. Implicit finite difference method calculates option prices at $k+1$ using one option price at time step k. Implicit finite difference can be best described as a reverse trinomial tree where, this time, instead of calculating connection point of branches using three values, three branches are calculated using one value.

One of the advantages of the explicit method is that it is easier to implement, easy to track instability, it copes well with coefficients that are asset and/or time-dependent, and it is easy to incorporate accurate one-sided differences. The disadvantage of the explicit method is the restrictions on the time step so the method can be slower than other methods.

It is worth mentioning the Crank-Nicolson (CN) method. It is simply the average of implicit and explicit FD methods. CN method becomes more complicated compared to other schemes, however, it has a better stability and accuracy.

3.5 Credit Risk

Giesecke (2004) defines credit as the distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement. A clearer definition is provided by Ammann (2001). Credit risk is the possibility of a loss caused by a contractual counterparty not meeting its obligations (Ammann, 2001). Bond valuation is one of the areas in which credit risk plays an important role. Higher credit risk means higher possibility of loss, therefore investors will require higher returns or a lower price for the security.

Credit spread is defined as the difference between yield of a security issued by a defaultable issuer and the yield of security issued by a benchmark, generally default free, issuer. Credit spreads of corporate securities are measured by taking the yield difference between the government bonds with same terms. While credit risk has an important role on corporate securities valuation, through financial innovation new instruments were introduced to financial markets. Credit default swap is one of these instruments to hedge the default risk of an issuer. In the next section these instruments and their pricing are presented.

3.5.1 Credit Default Swaps

Credit Default Swaps (CDS) are financial instruments used to transfer default risk of a bond of particular issuer. The buyer pays the seller periodic premiums and in exchange the buyer is protected against the default risk of reference entity. The life of CDS is bound by the credit event or the life of underlying security. Credit events are defined by the International Swaps and Derivatives Association (ISDA). Credit event has a broader definition than just the default. It also includes firm restructuring.

Pricing credit default swaps are covered in detail by Hull and White (2000) and Hull and White (2001).

The risk neutral probability, $1-\pi$, that a credit event will occur by maturity, T, is given by

$$\pi = 1 - \int_0^T q(t)dt,$$

where $q(t)$ is risk neutral probability density at time t.

Payments are received by the seller until the termination of the contract. This may be due to a credit event or expiration of the contract. The present value of payments at time t in case of a default is $w[u(t)+e(t)]$. If there is no default the present value of payments is $wu(T)$. Taking the expected value of both mutually exclusive states will give the expected value of payments as

$$w\int_0^T q(t)[u(t)+e(t)]dt + w\pi u(T),$$

where $u(t)$ is present value of payments at the rate of \$1 per year on payment dates between time zero and time t, $e(t)$ is present value of an accrual payment at time t equal to $t - t*$ where $t*$ is the payment date immediately preceding time t and w is total payments per year made by credit default swap buyer.

The pay-off of CDS contract in default is face value L minus the market value of the bond after default. The market value of the bond after default can be found by adding the recovery value and the accrued interest on bonds, given by

$$1 - [1 + A(t)]\hat{R} = 1 - \hat{R} - A(t)\hat{R}.$$

The present value of the expected payoff is given by

$$\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt.$$

The value of CDS to the buyer is given by present value of expected pay-off minus the present value of payments made by the user.

$$\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt$$
$$-w \int_0^T q(t)[u(t) + e(t)]dt - w\pi u(T)$$

The CDS spread is defined by the total payments $w*$ made by the buyer which is

$$w* = \frac{\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt}{\int_0^T q(t)[u(t) + e(t)]dt + \pi u(T)} \tag{3.15}$$

equal to zero (Hull and White, 2001).

3.6 Proposed Research Design

The main focus of this study will be financial synergies of mergers which also leads to analysis of structured finance, securitization, and firm valuation. After a merger, generally, resulting company has a higher value than the sum of separate companies which is caused by synergies created through merger. These synergies may be operational or financial. Operational synergies are generally created by economies of scale or other mechanisms which are out of scope of this study. Financial synergies are created by financial aspects of the firm and reflected through changes in values of securities of the merging firms. The securities are bonds and equities which, in sum, constitutes firm value, thus measuring financial synergies problem becomes a valuation problem. Recently Leland (2007) with a simple one period model with two dates, correctly showed the dynamics of mergers. The model involves valuation of assets of the firms which reveals the gains of bondholders and shareholders after merger. The problem arises at this point as bondholders' merger gains are higher than those of equity holders'. Modelling financial synergies will also require modelling of merger gains of claim holders. Shareholders will not be in favour of a merger which will increase debt holders' value more than their gains. Literature discussed in Chapter 2 does not address how to change distribution of financial synergies of mergers. There are some recommendations but it is not explicitly demonstrated. This study aims to investigate how to distribute financial synergies of mergers between bondholders and equity holders.

3.6.1 Model of Financial Synergies

Following Merton (1974) and using the framework in Leland (2007) X is defined as random future operational cash flow of a company. Using risk neutrality X_0 is its discounted expected value.

$$X_0 = \frac{1}{(1 + r_T)} \int_{-\infty}^{\infty} X dF(X) \tag{3.16}$$

where r is risk free rate and $F(X)$ is cumulative probability distribution. Then pre-tax value of limited liability is

$$H_0 = \frac{1}{(1 + r_T)} \int_{0}^{\infty} X dF(X), \tag{3.17}$$

which creates a value for limited liability $L_0 = H_0 - X_0$. After tax payment, τ, the value of firm with limited liability is $(1 - \tau)H_0$. Firms can issue zero coupon bonds and interest is tax deductable thus zero-tax level, X^Z is;

$$X^Z = P - D_0(P)$$

where P is principal value and $D_0(P)$ is market value of debt at $t = 0$. Another important threshold is default triggering cash flow level X^d which is defined as;

$$X^d = P + \frac{\tau}{(1 - \tau)} D_0$$

After preliminary definitions market value of debt and equity can be modelled as;

Market value of debt, $D_0(P)$ is given by

$$D_0(P) = \frac{P}{1 + r_T} \int_{X^d}^{\infty} dF(X) + (1 - \alpha) \int_{0}^{X^d} X dF(X) - \tau \int_{X^Z}^{X^d} (X - X^Z) dF(X).$$

Value of equity, $E_0(P)$ is given by

$$E_0(P) = \frac{1}{1+r_T}(\int_{X^d}^{\infty}(X-P)dF(X) - \tau\int_{X^d}^{\infty}(X-X^Z)dF(X)).$$

Firm value is defined as sum of debt and equity values; $v_0(P) = D_0(P) + E_0(P)$ where P gives the optimal capital structure where $v_0(P)$ is at maximum. Optimal capital structure is solved iteratively while D_0 and P must be solved simultaneously.

Closed form solutions for the integrations are given as

$$G(x,y) \equiv \int_x^y zpr(z)dz = \mu(N(d(y)) - N(d(x))) - \sigma(n(d(y)) - n(d(x))), \quad (3.18)$$

where $d(y) = (y-\mu)/\sigma$, $N(\bullet)$ is the standard normal cumulative distribution function, and $n(\bullet)$ is the standard normal density function.

Two optimally leveraged companies can be merged to analyse financial synergies of mergers. Presence of operational synergies will increase the synergy amount observed thus ignored here. Sources of financial synergies are default costs, DC and tax savings, TS and total change can be observed by using;

$$\Delta = v_m - v_1 - v_2$$

which is the difference between optimally leveraged merged firm and sum of optimally leveraged separate firms. Same method can be used to identify merger effects on existing securities of the companies. It is possible to calculate all required values to analyse financial synergies like yield spread, recovery rates, default probability, default costs, company value, equity value and debt values.

The results of this model show that after merger, shareholders' gains are lower than bondholders'. Theoretically managers try to increase the overall company value which is in line with the results of the model, however, in practice shareholders will not be in favour of a merger in this case. In practice, managers try to increase shareholder value. Managers will not decide on a merger unless it increases shareholder value.

3.6.2 Distribution of Synergy

The model mentioned in section 3.6.1 is capable of measuring financial synergies and is able to demonstrate dynamics of a merger or a securitization. However it still needs some extensions to increase the shareholders gains after a merger. Leland (2007) has identified two methods for this without explicitly demonstrating them. The first method argues that using short term bonds will increase the shareholders' gains. After a merger, companies can issue new bonds with lower yield spreads and preserve the merger gains for shareholders. The addition of bonds with different terms to the model requires at least two periods. It is not possible to incorporate short term bonds to the current framework which is a one period model with two time points. The second method requires call provisions on debt, then the firm can call its bonds to prevent the gain flow to bondholders. This method is applicable within the current framework. The adaptation of a callable bond to the model can be done by modifying the bond equation to incorporate call provision.

The model will also be extended to incorporate credit derivatives like credit default swaps[3] (CDS). CDS contracts can be used to test the model in case of a credit event or to hedge the risks of bondholders.

[3]A protection against default of a company Hull and White (2000).

3.7 Weaknesses of Proposed Research Design

Presence of taxes and default costs and probabilities were introduced to the model and parameter values were chosen to represent observed values. Although a realistic modelling approach was used, applicability of the model to real life may not always be practical. Volatility of cash flows of the company is one of the key parameters of the model. Volatility of an asset may be relatively easier but identifying volatility of cash flows of a company may not always be possible. Especially identifying correlation of cash flows of two companies may be harder than identifying their volatilities which requires all the cash flow information of the involved companies.

Operational synergies are generally the main motives of mergers, however, financial synergies are the main motive for structured finance and similar company restructuring tools, thus, applying the proposed research design to valuation of merger activities requires presence of operational synergies. Although this may seem a weakness of the model, however, it is a strength when applying the model to structured finance.

Transaction costs may not be a big proportion of operating synergies, however, they may eliminate the financial synergies. Incorporating transaction costs requires inclusion of operational synergies unless it is not a structured finance model in which case no operational synergies are expected.

Although the proposed model is a more flexible and realistic model, it is not possible to build a fully flexible one. The proposed model assumes constant values for some of the parameters like volatility of cash flows or the risk free interest rate. This modelling approach is used by many authors (Ammann and Genser, 2004; Black and Scholes, 1973; Leland, 2007) previously which does not affect the validity of this kind of research. For the sake of simplicity and tractability the models in this thesis does employ variable interest rates or variable volatility.

3.8 Conclusion

Structured finance is a sub branch of risk transfer instruments which are specifically filed under credit risk. Credit risk models are grouped into two main parts; structural models and reduced form models. Structural models (Leland, 1994, e.g.) deal with a particular firm and its credit risk whereas reduced form models (Jarrow and Turnbull, 1995, e.g.) do not model the company and its assets. Inevitably the model type will be structural. Investigating structured finance requires modelling of companies and their securities. When the total value of a company is defined as the sum of its debt and equity, modelling the securities will also give the total firm value. The framework proposed in section 3.6.1, models the firm and its securities as options. Besides handling randomness and its ability to price flexibility, options models provide more realistic modelling. As the model becomes realistic empirical testing will be easier and the model will be practically possible to implement. The proposed model also uses calibration method to produce the observed values for parameters. The model provides results for mergers and structured finance without modification while still being simple enough to have a closed form solution and complicated enough to give results for all parameters required for credit risk modelling.

Chapter 4

One Period Model

4.1 Introduction

This section introduces a one period model to analyse mergers. A one period model is used due to its simple design. Despite its simple setup, the model can successfully demonstrate the dynamics of mergers. In particular, having closed form solutions for all equations increases the strength of the model.

The model is a structural credit risk model which uses firm specific parameters as inputs. By using different parameter combinations model separate firms and their merger can be modelled. This task would not be possible in a reduced form model.

The assumption is that firms operate at an optimal capital structure. The optimal capital structure is calculated by finding the optimal debt ratio which maximises the firm value. Pre and post merger optimal capital structure levels are required to find the gains of the claimholders of the firm.

The following section introduces the Leland (2007) model and then the extensions of this study are introduced. The extensions are: (1) The use of merger matrix; (2) Adding call provisions to debt; (3) Financial synergies and CDS value; and (4) Arithmetic Brownian Motion solution.

4.2 One Period Model of Financial Synergies

A recent study by Leland (2007) analysed financial synergies in a one period framework. The model can both capture mergers and spin-offs. Spin-offs are modelled in form of structured finance instrument; securitisation. The beauty of the model comes from the ability to price all existing securities of the firm and determine the capital structure without losing the simplicity. This section introduces the Leland

(2007) model and then extensions to the base model are demonstrated in the following sections.

4.2.1 Assumptions

The focus of this thesis is on financial synergies, therefore operational ones are ignored. Operational cash flows of the combined activities are non-synergistic. This rules out the existence of operational synergies, keeping the focus on financial synergies. Financial synergies may disappear if operational ones are included in the model.

A risk neutral environment is assumed therefore risk free rate can be used for discounting.

Only interest expenses are tax deductible. Financial losses are not taxed therefore there is no need to account for this. As operational decisions are ignored, operational tax deductible costs are not included.

Bondholders have priority in a default case. This is the actual case in most countries. Bond holders have priority over shareholders. The shareholders generally get the residual value, or nothing, in the case of a default.

There is a possibility of negative future operational cash flows. This is an important assumption. Without this limited liability will be a less valuable option. Besides the probability of default will be biased.

4.2.2 One Period Model of Financial Synergies

An activity generates a random future operational cash flow X at time $t = T$. At a risk neutral environment the value X_0 of the operational cash flow at $t = 0$ is its discounted expected value;

$$X_0 = \frac{1}{(1+r_T)} \int_{-\infty}^{\infty} X dF(X),$$

where $F(X)$ is the cumulative probability distribution of X at $t = T$. With limited liability owners can "walk away" from negative cash flows through the bankruptcy process.

The pre-tax **value of the activity** with limited liability is

$$H_0 = \frac{1}{(1+r_T)} \int_{0}^{\infty} X dF(X),$$

and the pre-tax **value of limited liability** is

$$L_0 = H_0 - X_0.$$

The after tax value of the unlevered firm is

$$V_0 = \frac{1}{(1+r_T)} \int_{0}^{\infty} (1-\tau) X dF(X)$$

Zero tax level, X^Z, is

$$X^Z = I = P - D_0,$$

where D_0 is the market value of debt at t=0, P is principal, I is interest rate.

The firm faces bankruptcy if the form value hits a certain threshold. Default triggering level of cash flow is

$$X^d = P + \tau Max[X - X^Z, 0].$$

Default cost can be calculated by taking a fraction, α, of the firm value when default occurs. α is the fraction of firm value which will be paid to lawyers and other costs in case of a default. Present value of default cost is

$$DC(P) = \frac{\alpha}{1 + r_T}(\int_0^{X^d} X dF(X)).$$

Tax savings are calculated by taking the difference between tax payments by levered and unlevered firm values. τH_0 gives the amount of tax payments by unlevered firm. The second term in the equation below gives the present value of taxes to be paid by the levered firm. Therefore tax savings are given by

$$TS_0(P) = \tau H_0 - \frac{\tau}{1 + r_T} \int_{X^Z}^{\infty} (X - X^Z) dF(X).$$

The equations for equity and bond values of the firm shown here, however for a complete definition of the model see Leland (2007). Following Merton (1974) levered firm value is defined as the sum of outstanding debt and equity values.

$$V(P) = D(P) + E(P), \tag{4.1}$$

where P is the debt principal which maximises total firm value $V(P)$. $D(P)$ is the market value of debt and $E(P)$ is the market value of equity.

Debt value has three components. The first one is the discounted expected value of principal, P. The second one is the recovery value in case of a default. The third one is the tax payments in case of a default if there is any taxable income. Here, the government has priority over debt payments in case of a default. The value of debt is given by

$$D_0(P) = \frac{P\int_{X^d}^{\infty} dF(X) + (1-\alpha)\int_0^{X^d} X dF(X) - \tau\int_{X^Z}^{X^d}(X - X^Z)dF(X)}{1 + r_T}. \quad (4.2)$$

The equity value is calculated as a residual value. The firm first pays the principal, P and the taxes. Anything left belongs to shareholders. The value of equity is given by

$$E_0(P) = \frac{\int_{X^d}^{\infty}(X - P)dF(X) - \tau\int_{X^d}^{\infty}(X - X^Z)dF(X)}{1 + r_T}, \quad (4.3)$$

where X is operational cash flows of the company, X^d is the default threshold, X^z is the zero tax level, α is the default cost, τ is the tax rate and $F(X)$ is the cumulative probability distribution of X at $t = T$.

4.3 Extension 1: Analysis of Financial Synergies Using Merger Matrix

The previous section analysed financial synergies of mergers when two merging firms are identical. Although the analysis is valid, in terms of theory merger of identical firms is a rare event in the real world. Therefore, in this section merging firms are not necessarily identical.

The focus is on asset quality of the firms which is identified by the volatility of cash flows and therefore default probabilities. Credit rating is a good proxy for asset quality and default risk of the firm as volatility of firms' cash flows has more effect on asset quality than size.

Instead of using arbitrary firms with different credit ratings are generated.

It is assumed that all of the firms are operating in the same country and they are all subject to the same tax rate and other legal regulations.

This analysis helps to understand financial dynamics of merging companies with different credit ratings and also gives insight into applications of the model to real world problems.

4.3.1 Calibration of Model Parameters

The model presented in Section 4.2 is calibrated to match different credit ratings. Average leverage ratios and default probabilities compiled in Huang and Huang (2003, p.46) are used as a proxy to credit ratings (Table 4.1). For each credit rating level one firm is calibrated.

Calibration is carried out by changing default cost and volatility of cash flows to meet the target parameters which are default probability and leverage ratio of the firm.

Credit rating	Leverage ratio (%)	Cumulative Default Prob (%) 4 years
Aaa	13.08	0.04
Aa	21.18	0.23
A	31.98	0.35
Baa	43.28	1.24
Ba	53.53	8.51
B	65.70	23.32

Table 4.1: Target Rating Parameters as compiled by Huang and Huang (2003).

In table 4.1 default probability of firms increases as their credit rating decreases which is consistent with the current model and is as expected. However, leverage ratio decreases as credit rating increases opposite to the model discussed here. This may be due to the effect of debt ratio on credit ratings as an increase in amount of debt has a negative effect on credit ratings. If firms are operating with optimal capital structures an increase on debt ratio is expected as credit rating increases. The model here uses optimal capital structure, therefore it is not possible to match the leverage ratios in table 4.1 exactly. The target capital structures are not optimal. One example to this may be a firm with a high credit rating does not want to benefit from increased leverage while risking its credit quality. The firms may have enough cash and therefore they may choose to operate with a suboptimal capital structure. In this model calibrated firms operate at an optimal capital structure therefore purely financial synergies of mergers can be identified. Otherwise some of the gains or losses may be caused by arbitrary capital structure decisions. Calibration results are presented in Table 4.2. Default probabilities are almost perfectly matched to target parameters.

Credit rating	Leverage ratio (%)	Cumulative Default Prob (%) 4 years
Aaa	94.60	0.04
Aa	77.54	0.23
A	78.59	0.34
Baa	47.12	1.24
Ba	47.23	8.44
B	63.93	23.45

Table 4.2: Model results after calibration

For demonstration purposes, the tax rate is $\tau = 20\%$, correlation between cash flows is $\rho = 20\%$ and risk free rate is $r = 5\%$. However these parameters can also be freely chosen.

ΔV_0		σ	1.00%	5.00%	5.00%	17.00%	51.00%	69.00%
		α	49%	49%	34%	37%	28%	22%
σ	α		**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**
1.00%	49%	**Aaa**	**0.04%**	**0.04%**	**0.03%**	-0.22%	-4.02%	-7.54%
5.00%	49%	**Aa**	**0.04%**	**0.18%**	**0.17%**	**0.04%**	-3.65%	-7.11%
5.00%	34%	**A**	**0.04%**	**0.19%**	**0.17%**	**0.09%**	-3.46%	-6.82%
17.00%	37%	**Baa**	-0.17%	**0.09%**	**0.07%**	**0.33%**	-2.71%	-5.84%
51.00%	28%	**Ba**	-3.67%	-3.30%	-3.32%	-2.50%	-3.36%	-5.47%
69.00%	22%	**B**	-6.69%	-6.25%	-6.26%	-5.16%	-4.94%	-6.55%

Table 4.3: Δ Total Value. σ is volatility of cash flows and α is the default costs as a fraction of firm value.

Table 4.3 shows the post merger firm value changes. The merger of firms are optimal when mergers create positive synergies. These combination of firms are marked with the bold font in the table. In other areas the firm value decreases after merger therefore merging is not optimal.

ΔD		σ	1.00%	5.00%	5.00%	17.00%	51.00%	69.00%
		α	49%	49%	34%	37%	28%	22%
σ	α		**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**
1.00%	49%	**Aaa**	**1.22%**	**1.15%**	**0.54%**	-9.74%	-122.60%	-189.03%
5.00%	49%	**Aa**	**1.15%**	**5.56%**	**4.90%**	**0.47%**	-97.48%	-158.90%
5.00%	34%	**A**	**1.13%**	**5.83%**	**5.18%**	**3.02%**	-64.73%	-100.79%
17.00%	37%	**Baa**	-7.22%	**3.07%**	**2.23%**	**13.90%**	-32.32%	-67.22%
51.00%	28%	**Ba**	-67.48%	-47.42%	-48.67%	-12.33%	-10.27%	-27.72%
69.00%	22%	**B**	-61.55%	-43.70%	-44.73%	-11.34%	-2.10%	-13.49%

Table 4.4: Δ Debt. σ is volatility of cash flows and α is the default costs as a fraction of firm value.

Table 4.4 shows critical parameters like cash flow volatility and default cost of firms and their corresponding credit ratings and debt value change after a merger. In certain areas debt holders are always gaining, where in other areas, they are losing money after a merger. If merging firms have higher than Baa rating and if merging firms have closer ratings to each other then the merger creates synergies for existing bondholders (Table 4.4).

$\Delta Spr.$		σ	1.00%	5.00%	5.00%	17.00%	51.00%	69.00%
		α	49%	49%	34%	37%	28%	22%
σ	α		**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**
1.00%	49%	**Aaa**	**-0.001**	**-0.006**	**-0.007**	-0.121	-2.572	-4.094
5.00%	49%	**Aa**	**-0.006**	**-0.012**	**-0.013**	**-0.124**	-2.522	-4.021
5.00%	34%	**A**	**-0.006**	**-0.012**	**-0.013**	**-0.116**	-2.308	-3.562
17.00%	37%	**Baa**	-0.116	**-0.118**	**-0.119**	**-0.195**	-2.182	-3.446
51.00%	28%	**Ba**	-2.211	-2.139	-2.140	-1.890	-2.850	-3.988
69.00%	22%	**B**	-2.611	-2.488	-2.489	-2.066	-2.659	-3.734

Table 4.5: Δ Debt Spread. σ is volatility of cash flows and α is the default costs as a fraction of firm value.

Table 4.5 shows the change in debt spread. Spread is calculated by taking the difference between debt yield and risk free rate in percentages. Debt spread follows the same pattern as the debt value. As debt value increases debt spread decreases as expected.

The area where debt value is increasing is marked with bold font for tractability. In the next table debt spread can be observed after each merger which has the same bold area, and debt spread change is consistent with the debt value change area.

Post merger equity value change is shown in Table 4.6. In the bold area of the table equity holders are negatively affected after the merger. Shareholders wealth is only increasing outside of the bold area. However Table 4.3 shows that when shareholders are making money from a merger the total effect is negative for the firm. Shareholders will benefit from a non-synergistic merger however this will create a similar problem to asset substitution.

ΔE		σ	1.00%	5.00%	5.00%	17.00%	51.00%	69.00%
		α	49%	49%	34%	37%	28%	22%
σ	α		**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**
1.00%	49%	**Aaa**	**-26.18%**	**-7.42%**	**-3.41%**	17.41%	54.31%	67.25%
5.00%	49%	**Aa**	**-7.42%**	**-24.21%**	**-21.34%**	**-0.70%**	41.72%	55.19%
5.00%	34%	**A**	**-7.63%**	**-26.81%**	**-23.80%**	**-5.32%**	36.26%	50.01%
17.00%	37%	**Baa**	13.78%	**-5.29%**	**-3.82%**	**-15.97%**	14.39%	27.92%
51.00%	28%	**Ba**	45.81%	30.69%	31.63%	4.93%	2.13%	13.60%
69.00%	22%	**B**	52.27%	34.34%	35.42%	1.80%	-8.77%	3.87%

Table 4.6: Δ Equity. σ is volatility of cash flows and α is the default costs as a fraction of firm value.

Interestingly, the change in total firm value in Table 4.3 only becomes positive in the bold area. The bold area represents where the bond value increases and the opposite happens for the equity value. This demonstrates that if firms do not use wealth transfer strategies bondholders always gain more than shareholders after a merger.

4.3.2 Conclusion

Merger matrix can be seen as a strategic map of mergers. The map both shows when the mergers are synergistic and when the claim holders' wealth increases. This map can be used as a strategy guide. For example, if a firm has a credit rating below a certain level than it will not be financially optimal to merge. Using the firm specific data firms can identify if a merger will be synergistic or not. With current parameters set, the lowest rating level where the mergers are synergistic is Baa. This level is identified for current parameter combination which may represent a country or an industry.

Besides the usefulness of the merger matrix, the interesting point is that the distribution of financial synergies of mergers is in favour of bondholders in all cases. This observation is possible when no wealth transfer strategies are involved. The wealth transfer strategies can be defined as any financial strategy trying to transfer wealth from one party to another. These parties are the claimholders of the asset in question. Some of the strategies are using call provisions on bonds or using short term bonds. The next section will look at call provisions and its effects on merger gains.

4.4 Extension 2: Adding Call Provisions to Debt

The previous section introduced the basic model which demonstrates financial synergies using plain bonds. This section extends the model to include call provisions on debt and repeats the analysis. The goal is to analyse the effects of call provisions on financial synergies and credit risk.

The previous section also revealed a potential problem which managers may face. If managers are acting in the best interests of shareholders then they should, at least, preserve the value of equity after a merger. However, when the total gains from a merger are positive, bondholders gain more than shareholders and in some cases shareholders lose money while bondholders are benefiting from a merger.

Two strategies may be used to deal with this problem. The first one is the use of short term bonds to prevent a wealth transfer to bondholders. The second one is to use call provisions on debt which practically lets the firm retire the debt before maturity. This can be seen as bonds with a flexible maturity date. Bonds can be called just before the merger so the firm has the flexibility to decide when the merger happens. In the first strategy, the firm must wait for the short term bonds to mature to keep all the merger gains for the shareholders. The first strategy will be demonstrated in the multi period section of this thesis as the current model does not allow bond reissues and complex capital structures. The second strategy will be implemented into the current model.

This strategy works if merging firms have existing callable bonds before the merger. Calling the bonds before a merger will prevent a wealth transfer to bondholders, and shareholders can benefit from all merger gains. After the merger, firms can issue new bonds with a new credit spread. If the merger is a successful one then the new credit spread will be lower than the pre-merger credit spread.

4.4.1 Callable Bond Pricing

A call provision gives the issuer the right to retire the debt before its maturity. This brings a protection for the issuer, however, this can be seen as an increased risk for the buyer as their probability of getting the full interest at maturity decreases. Because of this feature, callable bonds must sell with a premium compared to plain bonds.

A callable bond is a combination of two assets. The plain bond and a call option. The call options' writer is the bondholder in this case, and the firm is the buyer of the option, therefore the bondholder must receive the option premium. When a callable bond is issued the buyer buys the bond and sells the call option on the bonds to the issuer. The bondholder must pay

$$\text{Callable Bond} = \text{Plain Bond} - \text{Call Option},$$

to the issuer. Receiving the call premium is an incentive for the bondholder to buy the callable bond instead of plain bonds. However, on the other hand, bondholders will not buy the callable bond if they do not have a return which will cover the risks of buying the callable bond. Otherwise investors will choose to invest in plain bonds or the risk free asset instead of callable bonds as their returns will be uncertain in the case of callable bonds. The risk premium rp is added to bond yield to ensure that, the investors will get a return over the risk-free rate. Setting a fixed call price may increase the risk premium of the bonds as it increases the uncertainty of the bondholders' returns from the investment.

4.4.2 Adding Callable Bonds to Base Model

Adding call provisions to bonds will enable a wealth transfer from bondholders to shareholders. A new debt value D^c is proposed for the company to increase shareholders' wealth after a merger. By issuing bonds, the firm takes a short position in bonds and a long position in call options written on bonds. The value of debt D^c with call option C is given by

$$D^c = D_0(P) - C. \tag{4.4}$$

The firm is the buyer of the option and bondholders are the writer of the option. From this assumption, subtracting call price C from plain debt value $D_0(P)$, as it is short position for debt holders, gives the price of a callable bond. Shareholders are in a long position in this option, thus, C should have positive effect on equity value. In terms of cash flows, selling the bond is a positive/negative cash flow for the firm/buyer and buying/selling the call option is a negative/positive cash flow for the firm/buyer.

To find the value of D^c, call provision on the bond must be priced. Using the following payoff function of option and its probability, the price of the callable bond will be calculated. A payoff function for this option is defined as

$$\max[P - D_0(1 + r + rp), 0], \tag{4.5}$$

where P is principal value of the bond, D is market price of the bond, r is risk-free rate and rp is risk premium for investors. By introducing rp, a higher return than the risk-free rate to investors is guaranteed. If $rp = 0$ then investors will decide to invest in risk-free assets.

Pay-off function in eq.4.5 states that investors will earn a risk-free rate plus risk premium if the bond is called. Since the bond has no coupons $D(1 + r + rp) \leq P$. If the bond is called, investors will get

$$D(1 + r + rp). \tag{4.6}$$

This way the firm will have an option to keep the excess gain of bondholders in the company. Risk premium for investors rp can be empirically identified by calculating yield spreads between callable bonds and plain bonds of equally rated companies. For the calculations here it is assumed that $rp = 1\%$.

The firm will use its operational cash flows to pay the call price. It is assumed that the company is aware of an upcoming merger and wants to call the bonds whenever the firm can afford to call the bonds back. Therefore, call threshold is defined as a function of operational cash flows. The company calls the bond when operational cash flows are higher than call price. Whenever bond price is higher than the call price and the firm has enough cash to pay the call price, they choose to call the bonds. Thus, call triggering cash flow level X^c is defined as

$$X^c = D(1 + r + rp). \tag{4.7}$$

Normally companies prefer not to use all their operational cash flows to call their bonds, therefore another level of call threshold can be identified. However, here managers are aware of an upcoming merger and are willing to call their bonds before the merger to transfer the wealth from bondholders to equity holders when they have enough cash. This way the company will keep the merger gains for the shareholders and will be able to issue new bonds after the merger with a new credit spread.

The value of callable bond D^c is defined as,

$$D^c(P) = D_0(P) - \frac{[P - D_0(P)(1 + r + rp)]}{1 + r_T} \int_{X^c}^{\infty} dF(X), \tag{4.8}$$

where principal P is fixed. Through the life of the bond, as D_0 decreases, call option becomes more valuable. All bonds of the company have equal credit risks so investors will prefer bonds with higher yields thus options on these bonds will be more valuable. As D_0 increases, the value of the option decreases and r and rp change the option price similarly. As call threshold X^c increases, the probability of option exercise decreases.

4.4.3 Numerical Results

Base case scenario parameters, given in Table 4.7, represent a BBB rated firm[1]. Here two different merger scenarios are considered. Scenario 1 demonstrates a merger with plain vanilla bonds and Scenario 2 demonstrates a merger with callable bonds. Comparison of the two scenarios will demonstrate the effects of call provisions on bonds in the case of a merger.

4.4.4 Scenario 1: Merger with Plain Bonds

This scenario is the exact replication of the original model. Using equations (4.1), (4.2) and (4.3) before and after merger values of two identical firms with optimal capital structures are obtained.

Scenario 1 results are displayed in Table 4.8. Although the firm value is increasing after merger, shareholder value is decreasing. This may cause a manager/shareholder conflict over the merger decision. Shareholders will not want a merger which

[1] Using the same inputs used by Leland (2007).

Variable	Values
Annual risk-free rate (r)	5%
Time Period (T)	5
Expected operational cash flow at T (μ)	127.63
Cash flow volatility as T (σ)	49.19
Tax rate (τ)	20%
Value of unlevered firm w/limited liability (V_0)	80.05
Value of limited liability after tax ($(1-\tau)L_0$)	0.05

Table 4.7: Base Case Parameters as defined in Leland (2007)

will decrease their value. To overcome this problem Scenario 2 can be used, which will both increase firm value and shareholders' value at the expense of bondholders.

Variable	Symbols	Sum of Firms	Merged Firm	Δ
Value of debt	D	84.46	89.40	4.94
Value of equity	E	78.47	73.74	-4.73
Total Value	$V(P)$	162.94	163.15	0.21

Table 4.8: Scenario 1: Financial Synergies when bonds are not callable

4.4.5 Scenario 2: Merger with Callable Bonds

In this scenario plain bonds are replaced by bonds with call provisions. Instead of using Eq. (4.2) for debt value now Eq. (4.8) is used to calculate callable debt value. Firm value $V(P)$ becomes,

$$V(P) = D^c(P) + E(P) \tag{4.9}$$

or

$$V(P) = D(P) + E(P) - \text{Call Option.} \tag{4.10}$$

Variable	Sum of Firms	Merged Firm	Δ
Value of debt D^c	24.12	20.27	-3.86
Value of equity E	144.79	152.87	8.08
Total Value $V(P)$	168.91	173.14	4.22

Table 4.9: Scenario 2: Financial Synergies with Call provision on bonds

Table 4.9 shows the security values of Scenario 2. The loss in equity value is recovered and also equity increased is almost double the total firm value increase. Value of plain bonds is the difference between the debt value and the value of call provision. Therefore option adjusted yields/prices must be calculated to find the value of plain bonds.

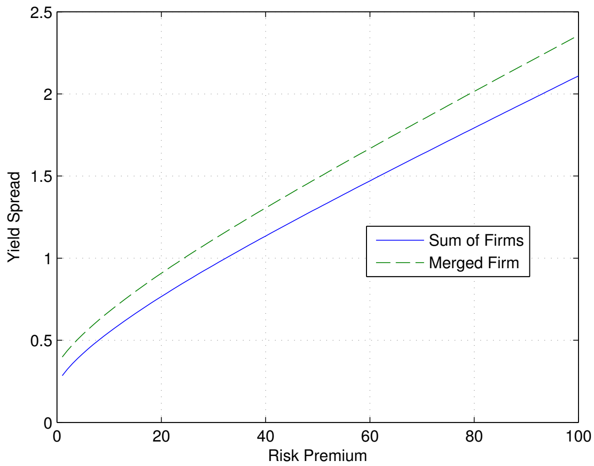


Figure 4.1: Scenario 2: Yield Spread

Yield spread of the bonds increases after merger for all risk premium levels. Figure 4.1 shows that the change in yield spread is higher for higher risk premium levels.

Figure 4.2 shows the leverage ratio and risk premium relation. Leverage decreases as risk premium increases. This is mainly due to the embedded option in bonds. As mentioned earlier, option adjusted leverage will be higher than the one shown here.

Figure 4.3 shows merger gains of security holders of the firm. Merger gains of shareholders are positive for all risk premium levels. Call provisions transfer wealth

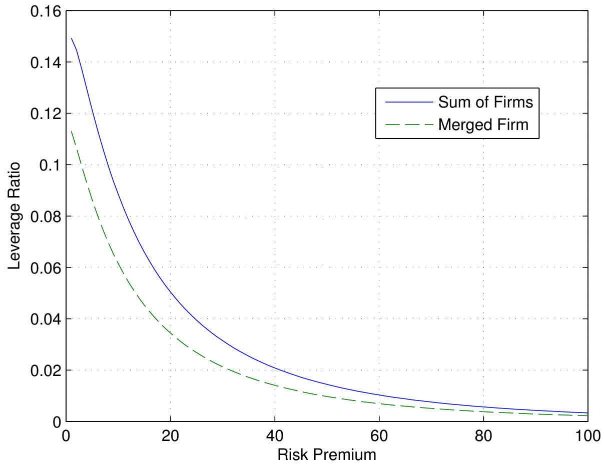


Figure 4.2: Scenario 2: Leverage Ratio

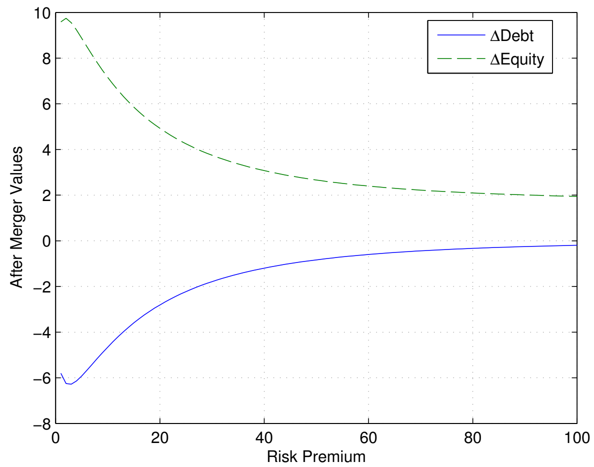


Figure 4.3: Scenario 2: Debt and Equity Values

from bondholders to shareholders. Shareholder gains decrease as risk premium increases and bondholder gains increase as risk premium increases but never becomes positive.

Risk premium and total firm value are shown in Figure 4.4. Firm value increases as risk premium increases. The increase is caused by the increased call option value.

4.4.6 Option Adjusting

Callable bond purchase D^c can be separated into two transactions. The firm sells bonds and buys call options on bonds. This explains the very low value of callable

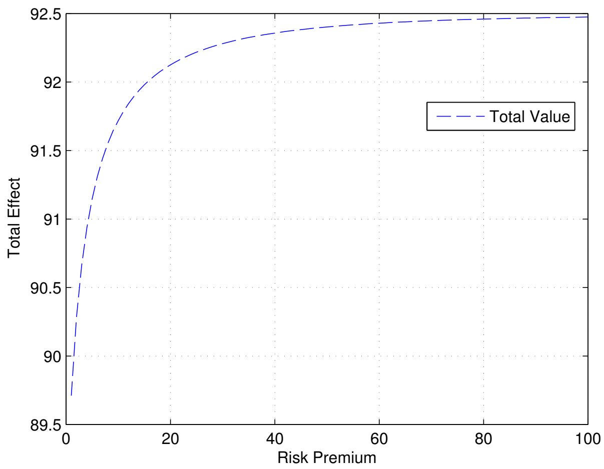


Figure 4.4: Scenario 2: Firm Values

bond D^c thus very high bond yield. To calculate the real yield of bonds, option adjusting is required. Option adjusted bond price can be calculated using equation 4.11

$$D_0(P) = D^c - (\text{Call Price}). \tag{4.11}$$

The price of callable bonds will be lower than the price of plain bonds because of the cash flow created by the call premium which is received by the investors. This will result in a higher bond yield compared to plain bonds. However, it is expected that the callable bonds credit spread will be lower than plain bonds after adjusting the prices for the call option. Firstly, this effect can be explained by the existence of call option and gives a company flexibility in meeting its liabilities. This makes the firm less risky as it creates an extra cash flow for the firm to meet its liabilities. The extra cash flows come from the difference between call price and the face value of the bonds $P - D(1 + r + rp)$. Secondly, the firm has a valuable option to pay $min[P, D(1 + r + rp)]$ which increases the credit quality of the firm.

Option adjusted bond values are displayed in Table 4.10. Option adjusted debt value and yield gives the price of the plain bonds and correct leverage ratio.

Variable	Unadjusted	Adjusted
Debt value	12.06	32.07
Debt yield	28.33%	5.53%
Leverage ratio	14.28%	30.70%

Table 4.10: Option Adjusted Values of debt value, yield spread and leverage ratio

Variable	Single Firm	Merged Firm
Scenario 1	6.23%	5.60%
Scenario 2	5.53%	5.27%

Table 4.11: Bond Yields: Call provisions decrease yield spread of single firms and merged firm. Without a call provision bond holders gains are higher after a merger.

4.4.7 Conclusion

This section shows that using callable bonds increases both firm value and share values after merger. Timing of the merger is exogenous to the model, thus, firms can merge at any given time. Another effect of callable bonds is a decrease in cost of debt. Call option on bonds provides extra cash flows to the firm. Much of the risk premium provided to the investors goes to the shareholders through call options causing a lower bond yield compared with provided risk premium. An increase in the risk premium also increases firm value.

The results obtained here concludes that shareholders can benefit from a merger if outstanding bonds are callable. Without a call option on bonds shareholders wealth is decreasing in case of a synergistic merger as shown in Table 4.6 and 4.3. Call provisions on bonds not only increase the wealth of existing shareholders it also enhances the firm value after the merger.

A synergistic merger may increase the merged firm value however it is not always beneficiary for the shareholders. The contribution of this section is that it shows that shareholders can benefit from a merger if existing bonds are callable.

4.5 Extension 3: Testing Financial Synergies Against Credit Default Swap Value

This section aims to investigate the distribution of purely financial synergies of mergers using the framework developed by Leland (2007) with the addition of credit derivatives to the original model. Through **coinsurance** effect merging firms create financial synergy. Leland (2007) shows that bondholders' financial gains are higher than those of shareholders' after a merger. This study will try to demonstrate an alternative way to distribute merger gains using credit derivatives.

4.5.1 Introduction

There are many studies analysing operational synergies of mergers. Recently Leland (2007), in a simple two-period model, has analyzed the effects of purely financial synergies on firm value. Leland (2007), considers activities with non-synergistic operational cash flows and examines the purely financial benefits of separation versus merger. Firm value, is defined as the sum of debt and equity values, maximised by calculating optimal capital structure. He breaks down the financial effects into two parts; default costs (ΔDC) and tax savings (ΔTS). One of the benefits of his analysis is that it captures the structural finance cases. For example, securitization is applied by transferring some part of the higher quality assets of the firm to a special purpose entity (SPE) which is modelled as the reverse of the merger event.

4.5.2 The Model

4.5.2.1 A Quantitative Analysis of Financial Synergies

An activity generates a random future operational cash flow X at time $t = T$. In a risk neutral environment the value X_0 of the operational cash flow at $t = 0$ is its discounted expected value

$$X_0 = \frac{1}{(1 + r_T)} \int_{-\infty}^{\infty} X dF(X),$$

where $F(X)$ is the cumulative probability distribution of X at $t = T$.

With limited liability owners can "walk away" from negative cash flows through the bankruptcy process.

The pre-tax **value of the activity** with limited liability is

$$H_0 = \frac{1}{(1 + r_T)} \int_{0}^{\infty} X dF(X)$$

and the pre-tax **value of limited liability** is

$$L_0 = H_0 - X_0.$$

The after tax value of the unlevered firm is

$$V_0 = \frac{1}{(1 + r_T)} \int_{0}^{\infty} (1 - \tau) X dF(X).$$

Zero tax level, X^Z, is defined as

$$X^Z = I = P - D_0,$$

where $D_0 =$Market value of debt at t=0, P=Principal, I=Interest

Default triggering level of cash flow, X^d, is,

$$X^d = P + \tau Max[X - X^Z, 0].$$

Debt, $D_0(P)$, and equity, $E_0(P)$, values of the firm are defined as

$$D_0(P) = \frac{P \int_{X^d}^{\infty} dF(X) + (1-\alpha) \int_0^{X^d} X dF(X) - \tau \int_{X^Z}^{X^d} (X - X^Z) dF(X)}{1 + r_T},$$

and

$$E_0(P) = \frac{1}{1 + r_T} \left(\int_{X^d}^{\infty} (X - P) dF(X) - \tau \int_{X^d}^{\infty} (X - X^Z) dF(X) \right).$$

The optimal capital structure is the debt P that maximises total firm value $v_0(P)$.

$$v_0(P) = D_0(P) + E_0(P),$$

After finding optimal capital structure with normally distributed cash flows, two identical companies are merged with a correlation between the activities' cash flows.

The merged firm is compared with sum of two firms to observe financial synergies after the merger.

4.5.3 Credit Default Swaps

Credit default swap (CDS) is an insurance against default risk of a reference company. CDS contracts will be used here to analyse financial synergies, while a short position in CDS can be used to increase shareholders' benefits from a merger compared to bondholders' gains. Following Hull and White (2000), pay-off of a CDS is

$$P - RP[1 + A(t)] = P[1 - R - A],$$

where P is the notional principal, R is recovery rate and A is accrued interest which will be ignored here. Bondholders will claim principal amount of zero coupon bond in the case of a default. Payoff of the CDS will then become

$$P[1 - R]$$

Expected value of default is

$$ED = (\int_{-\infty}^{X^d} XdF(X)),$$

which is equal to, default probability times loss in default

$$ED = p_D(1 - R)P.$$

Loss in default (for bondholder) is $P(1 - R)$ so default probability is

$$p_D = \frac{ED}{(1 - R)P}.$$

Finally pay-off of a CDS contract is $P(1-R)$ in the case of a zero coupon bond. The expected value of CDS is given by $E(CDS) = P(1-R) * p_D$.

In a merger, default probability of merging companies decreases compared to the sum of two firms. The merged company has a lower CDS value than the sum of two companies due to a decrease in default probability.

The value of CDS is equal to the present value of expected pay-off minus the payments made by the buyer

$$\frac{(1-R)\int_{-\infty}^{X^d} dF(X) - sP\int_{X^d}^{\infty} dF(X)}{(1+r_T)},$$

P is principal and s is the CDS premium,

$$s^* = \frac{(1-R)\int_{-\infty}^{X^d} dF(X)}{\int_{X^d}^{\infty} dF(X)}$$

4.5.4 Model Application

The same parameters used in previous section are used for the calculations. The analysis is conducted through sensitivity analysis. The parameters used for the analysis are volatility and interest rate.

4.5.4.1 Effects of Volatility

Claim holders gains vary by the volatility of cash flows. Figure 4.5 shows $\Delta Debt$ and $\Delta Equity$ for volatility levels between 0-100. After 40% volatility both claimholders' gains are increasing with a higher gain for bondholders. For volatilities lower than 40% ΔD decreases and ΔE increases. For volatilities lower than 35% ΔD is negative and ΔE is positive. For volatilities higher than 35% ΔD is always positive and ΔE is negative up to 75% and positive for higher volatilities.

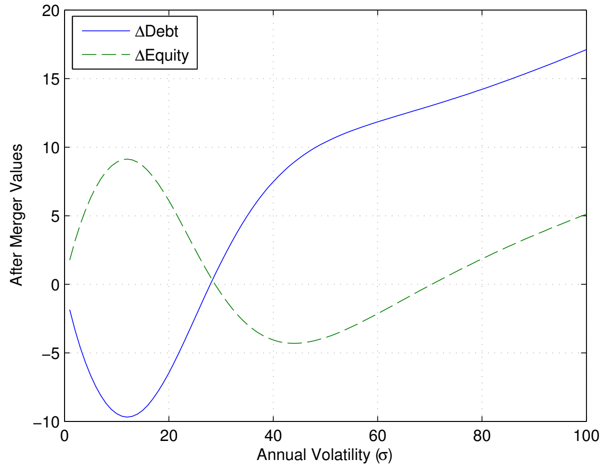


Figure 4.5: After merger values

As expected, CDS spread increases with volatility. However, for very high volatility levels CDS spread decreases after merger. Figure 4.6 shows CDS spread change for merged and single firms for different volatility levels. For lower volatility firms, a merger decreases CDS spread and for the rest merger increases the CDS spread.

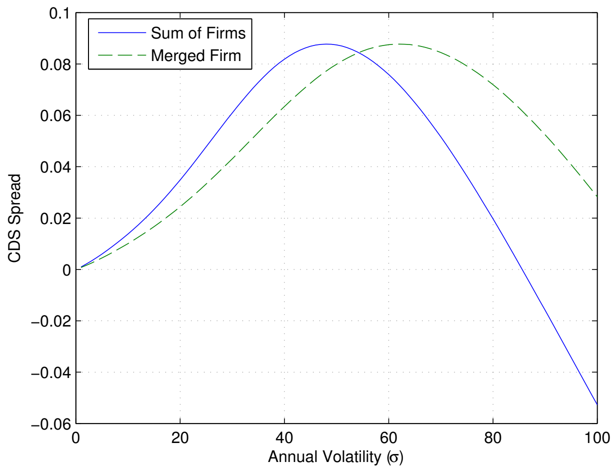


Figure 4.6: CDS Spread

Recovery rates decrease as volatility increases. Recovery rate is important as it identifies the amount of money which will be paid by the CDS seller in case of a default. The reversing of CDS spread after a certain volatility level is not related to recovery rate as a similar pattern is not observed here.

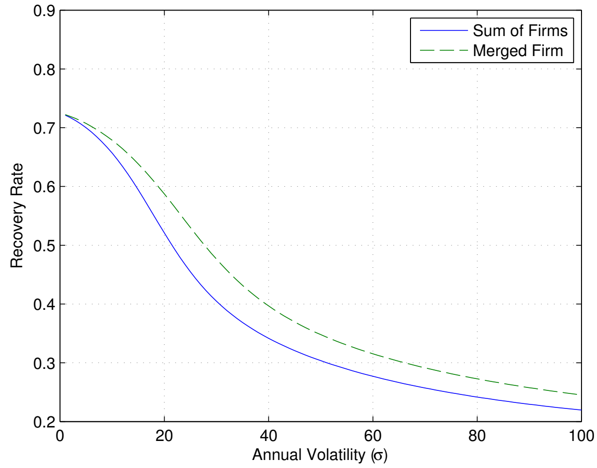


Figure 4.7: Recovery Rate

Figure 4.8 depicts a similar pattern with CDS spread change which shows the difference between tax savings and default costs, $TS - DC$. This difference gives the net gains from the merger where TS is expected to be positive, creating an incentive for merger, and DC has a negative effect on net gains as it is the increased default costs by increased debt amount. For volatility levels higher than 30%, net gains increase and for lower volatilities net gains decrease.

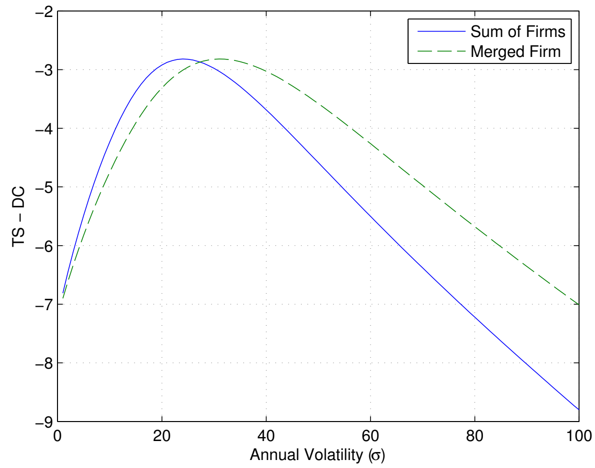


Figure 4.8: TS-DC

4.5.4.2 Effects of Interest Rate

CDS spread decreases as the risk-free rate increases, approaching zero as the risk free rate, r, approaches 100. Here, CDS value change is presented when risk free interest rate changes.

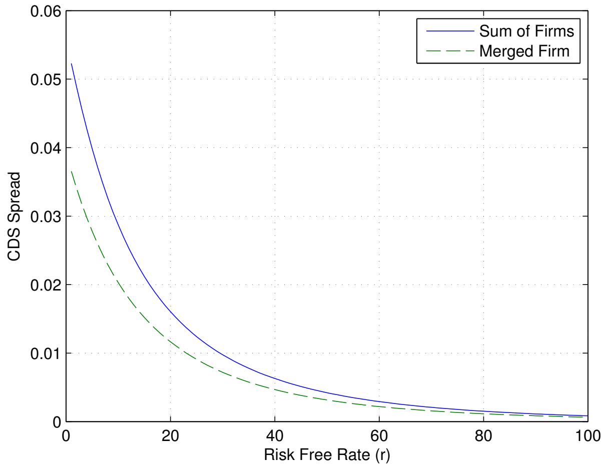


Figure 4.9: CDS Spread

4.5.5 Conclusion

Shareholders of the merging companies can take a short position on a CDS contract to increase their benefits from a merger. A short position in a CDS contract is equivalent to a long position on company bonds (Backshall, 2004). A short position in CDS of companies or a long position in bonds is in line with the results of Leland (2007) where bondholders gain more than shareholders after a merger. Instead of a cash position CDS contracts can be used while a short position will not require an cash payments and can be leveraged by shareholders to the benefit of the company.

4.6 Extension 4: ABM Solution for Leland Model

This section develops an Arithmetic Brownian Motion solution for the Leland (2007) model. Geometric Brownian Motion can also be used, however, it will not allow modelling of mergers because of lognormality. Whereas ABM will allow the modelling of mergers with its normal distribution property. Levered and unlevered firm values are calculated separately to keep the original model setup.

Firm value will depend on the cash flows which follow an ABM process given by $d\eta = \mu dt + \sigma_\eta dz^Q$. In Ammann and Genser (2004)'s model firm value is calculated and then this value is distributed among claimholders. Here firm value is found by adding the value of debt and equity. Firm value given by taking the discounted expected value of cash flows can be used as unlevered firm value as it does not include any information on leverage of the firm. All the solutions and definitions used here are explained in Chapter 5 of this thesis, therefore they are not repeated here.

4.6.1 Unlevered Firm Value

Firm value is the discounted expected value of operational cash flows, η, and is given by

$$V = E^Q_{t_0} \int_{t_0}^{\infty} \eta_s e^{-r(s-t_0)} ds,$$

where η is future random operational cash flows between time t_0 and infinity. The firm value, V, represents the value of all equity financed firms as it does not know anything about the firm's capital structure. This equation also implies that firms with the same operational cash flows will have the same unlevered firm values. However, when capital structure comes into effect, values of firms will differ according to their default costs which is identified by the amount of outstanding debt.

4.6.2 Levered Firm Value

The value of leverage is defined by the (unlevered) firm value minus the cost to lever it up, plus tax savings. Here it is assumed that there are no transaction costs. The only cost to levering the firm is the increase in the default cost of the firm. V_L, levered firm value is given by

$$V_L = V_U - DC + TS,$$

where DC is default cost and TS is tax savings. As default cost increase debt issue will be more expensive for the firm as the credit spreads will increase. And also an increased default cost will offset the gains from tax savings making net marginal gains from leveraging zero.

Default cost is defined by,

$$DC = VB\alpha pB(),$$

where α is the ratio of firm value which will be lost when the firm goes bankrupt. VB is the default threshold and $pB()$ is the price of Arrow-Debreu security which pays 1 unit of currency when the firm defaults. Default threshold, VB is assumed to be equal to the amount of debt principal as, if the firm cannot pay its debt principal, then firm will be forced into bankruptcy by its bondholders.

4.6.3 Claims on Levered Firm Value

Levered firm value is separated between three claim holders. These are bond holders, equity holders and government. Assume that the firm only has infinite maturity bonds. The present value of perpetual cash flows is given by

$$V_C^+ = \frac{C}{r}.$$

Interest payments are not taxed and they are tax deductible by the firm. The receivers of coupon payments, investors, are taxed by τ^d. The after tax value of the debt is given by

$$D^+ = (1 - \tau^d)V_C^+,$$

shareholders receive the residual claim after the interest and tax payment. The value of equity is given by

$$E^+ = (1 - \tau^e)(1 - \tau^c)(V_L^+ - D^+),$$

$$= (1 - \tau^{eff})(V_L^+ - V_C^+),$$

where τ^e is the tax rate for dividend payments and τ^c is the corporate tax rate.

Government has a claim to the firm's assets. The value of this claim is identified by the total taxes paid by investors and the firm. The below equation shows that the firm first pays the coupon payments and the rest is taxed by τ^{eff} and the taxes paid by bond investors are also added to this value.

$$G^+ = \tau^{eff}(V_L^+ - V_C^+) + \tau^d V_C^+$$

Solvent firm value can be easily obtained after defining the three claims on the firm

$$V_L^+ = D^+ + G^+ + E^+.$$

4.6.4 Default Claims

The firm declares bankruptcy when the value of assets fall below a certain level, V_B. Insolvent firm value is given by

$$V^- = V_B p_B,$$

where p_B is the value of Arrow-Debreu security paying 1 unit of currency in the case of default. Adding solvent and insolvent firm value gives total firm value, V_L,

$$V_L = V_L^+ + V_L^-.$$

In case of default bondholders have priority over shareholders. Bond holders receive the amount after bankruptcy costs and taxes. This value is also the recovery value of debt,

$$D^- = (1-\alpha)(1-\tau^{eff})V_L^-.$$

The government receives taxes after bankruptcy costs, αV_L^-

$$G^- = (1-\alpha)\tau^{eff} V_L^-.$$

Insolvent firm value is given as the sum of three insolvent claims,

$$V_L^- = D^- + G^- + BC.$$

4.6.5 Conclusion

Unlevered firm value is given by the discounted value of EBIT flows. Adding tax advantage of debt and subtracting the bankruptcy costs will give the value of levered firm value. Here equations are not solved explicitly and these are left to the multi period part of this thesis. Instead of assuming presence of levered and unlevered firm values at the same time, the Ammann and Genser (2004) method is followed and unlevered firm value is ignored.

Chapter 5

Multi Period Model

5.1 Introduction

In the previous section a one period model is used to analyse financial synergies. The model used in the previous section has some limitations such as that it cannot handle interactions of bonds with different maturities or complex capital structure choices. A dynamic model is needed which can price both equity and debt of the merging firms. The pricing model should be able to price both solvent and insolvent values of these securities, therefore the source of gains can be analysed. Operational decisions are taken as given and not required in the current setting.

In the next section, two recent EBIT based models will be demonstrated and for the rest of this thesis Ammann and Genser (2004)'s model, which is one of the two, will be used with some extensions.

5.2 EBIT Based Models

EBIT based modelling has some advantages to modelling equity or $equity + debt$ as the underlying process. In the case of EBIT based modelling the EBIT flows runs independently of how it is distributed among its claim holders (Goldstein, Ju, and Leland, 2001). In EBIT based models, debt, equity and taxes are modelled as separate claims to EBIT flows. The sum of these three claims will give the total firm value or total claims. Due to this separation all claims are taxed with different tax rates which represents most of the tax systems better than models with a single tax rate.

The following two sections will demonstrate two EBIT based models. The first model is the Goldstein, Ju, and Leland (2001) model and the second one is Ammann and Genser (2004)'s model. The former is an introductory model to EBIT Based modelling and the latter is used for the rest of this thesis.

5.2.1 Goldstein(2001) Model

This is a dynamic capital structure model with only upward debt adjustments. The authors note that the model can be easily modified to include downward debt alignments. They start with payout flows of a single firm which follows a Geometric Brownian Motion process, given by

$$\frac{d\delta}{\delta} = \mu_P dt + \sigma dz,$$

where μ_P and σ are constants.

The value of total claim can be found by taking the discounted expected value of the cash flows under risk-neutral measure. Therefore the value of the claim to the entire payout flow is

$$V(t) = E_t^Q(\int_t^\infty \delta_s dse^{-rs})$$
$$= \tfrac{\delta_t}{r-\mu}$$

$$\mu = (\mu_p - \theta\sigma)$$

where θ is risk premium and r is risk-free rate.

$$\frac{d\delta}{\delta} = \mu dt + \sigma dz^Q.$$

r and μ are constants. This implies that both V and δ share the same dynamics

$$\frac{dV}{V} = \mu dt + \sigma dz^Q$$

$$\frac{dV + \delta dt}{V} = rdt + \sigma dz^Q$$

This implies that the expected return on the claim is risk-free rate under risk neutrality.

The authors assume a simple tax structure with personal and corporate taxes. Interest payments to investors are taxed at a personal rate τ_i, effective dividends are taxed at τ_d, and corporate profits are taxed at τ_c with full loss offset provisions.

Claimants of a debtless firm with value V_0 are equity and government. Assuming that the current management refuses to take on any debt and that no takeover is likely, then, the firm value is divided between equity and government as

$$E = (1 - \tau_{eff})V_0$$

$$G = \tau_{eff} V_0$$

where effective tax rate is $(1 - \tau_{eff}) = (1 - \tau_c)(1 - \tau_d)$

The authors assume a firm with a static debt level that will maximise the wealth of current equity holders. The firm will issue a perpetual bond with constant coupon, C, and will pay these coupons as long as the firm is solvent.

Due to issuance of perpetuity, the threshold at which the firm chooses to default is time independent (Goldstein, Ju, and Leland, 2001). This threshold is defined as V_B. When firm value reaches V_B then an amount αV_B will be lost to bankruptcy costs.

In general any claim must satisfy;

$$\mu V F_v + \frac{\sigma^2}{2} V^2 F_{vv} + F_t + P = rF \tag{5.1}$$

P is payout flow,

Due to issuance of perpetual debt, all claims will be time-independent. Thus PDE reduces to an ODE;

$$0 = \mu V F_v + \frac{\sigma^2}{2} V^2 F_{vv} + P - rF \tag{5.2}$$

The general solution to;

$$0 = \mu V F_v + \frac{\sigma^2}{2} V^2 F_{vv} - rF \tag{5.3}$$

is given by;

$$F_{GS} = A_1 V^{-y} + A_2 V^{-x} \tag{5.4}$$

First and second order derivatives are;

$$\begin{aligned} F_v &= -yA_1 V^{(-y-1)} - xA_2 V^{(-x-1)} \\ F_{vv} &= y(y+1)A_1 V^{(-y-2)} + x(x+1)A_2 V^{(-x-2)} \end{aligned}$$

Substituting derivatives and F into eq.5.3 gives

$$\begin{aligned} 0 = \quad & \mu V[-yA_1 V^{(-y-1)} - xA_2 V^{(-x-1)}] \\ + \quad & \tfrac{\sigma^2}{2} V^2 [y(y+1)A_1 V^{(-y-2)} + x(x+1)A_2 V^{(-x-2)}] \; . \\ - \quad & r[A_1 V^{-y} + A_2 V^{-x}] \end{aligned}$$

Multiplying with the coefficients gives

$$\begin{aligned} & -\mu V y A_1 V^{(-y-1)} - \mu V x A_2 V^{(-x-1)}] \\ & + \tfrac{\sigma^2}{2} V^2 (y^2 + y) A_1 V^{(-y-2)} + \tfrac{\sigma^2}{2} V^2 (x^2 + x) A_2 V^{(-x-2)} \; . \\ & -rA_1 V^{-y} - rA_2 V^{-x} = 0 \end{aligned}$$

Rearranging and grouping A_1 and A_2 terms together gives

$$
\begin{aligned}
0 = \quad & -\mu y A_1 V^{-y} + \tfrac{\sigma^2}{2}(y^2 + y) A_1 V^{-y} - r A_1 V^{-y} \\
& -\mu x A_2 V^{-x} + \tfrac{\sigma^2}{2}(x^2 + x) A_2 V^{-x} - r A_2 V^{-x}
\end{aligned},
$$

$$
\begin{aligned}
0 = \quad & A_1 V^{-y}(-\mu y + \tfrac{\sigma^2}{2}(y^2 + y) - r) \\
+ \quad & A_2 V^{-x}(-\mu x + \tfrac{\sigma^2}{2}(x^2 + x) - r)
\end{aligned},
$$

where x is positive while y is negative. A_1 equals 0 for all claims of interest.

Thus x and y can be solved from;

$$
\begin{aligned}
0 = \quad & (-\mu y + \tfrac{\sigma^2}{2}(y^2 + y) - r) \\
0 = \quad & (-\mu x + \tfrac{\sigma^2}{2}(x^2 + x) - r)
\end{aligned}.
$$

Using following quadratic formula the roots of x and y are

$$
\begin{aligned}
& Quadratic \\
& ax^2 + bx + c = 0 \\
& \tfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}.
$$

Rearranging

$$
\frac{\sigma^2}{2}x^2 + (\frac{\sigma^2}{2} - \mu)x - r = 0,
$$

$$
\frac{\sigma^2}{2}y^2 + (\frac{\sigma^2}{2} - \mu)y - r = 0,
$$

$$
x = \frac{(\mu - \frac{\sigma^2}{2}) + \sqrt{(\frac{\sigma^2}{2} - \mu)^2 - 2r\sigma^2}}{\sigma^2},
$$

$$
y = \frac{(\mu - \frac{\sigma^2}{2}) - \sqrt{(\frac{\sigma^2}{2} - \mu)^2 - 2r\sigma^2}}{\sigma^2}.
$$

The value of x is always positive (see Appendix C).

Particular solutions to F; If the relevant cash flow is the entire payout, $P = \delta = V(r-\mu)$ then;

From

$$V(t) = E_t^Q(\int_t^\infty \delta_s dse^{-rs}) = \frac{\delta_t}{r-\mu},$$

$$F = \frac{\delta}{r-\mu}, \quad \frac{V(r-\mu)}{r-\mu} = V, \quad F_{PS}^\delta = V \ .$$

If the relevant payout is the coupon payment $P = C$ then the particular solution to eq.5.2 is;

Using the relation;

$$V(t) = E_t^Q(\int_t^\infty \delta_s dse^{-rs}) = \frac{\delta_t}{r-\mu}$$

Coupon payments C have $\mu = 0$ drift, hence the particular solution for coupon payments is

$$F_{PS}^C = \frac{C}{r},$$

where $p_B(V)$ is defined as the present value of a claim that pays \$1 contingent on firm value reaching V_B. From eq.5.4, $p_B(V)$ will be of the form

$$p_B(V) = A_1 V^{-y} + A_2 V^{-x}.$$

Boundary conditions are

$$\lim_{V\to\infty} p_B(V) = 0, \lim_{V\to V_B} p_B(V) = 1, \tag{5.5}$$

Using $A_1 V_B^{-y} + A_2 V_B^{-x} = 1$, $A_2 = V_B^x$ is obtained and using this in $p_B(V)$ gives

$$p_B(V) = (\frac{V}{V_B})^{-x}.$$

While the firm is solvent equity, government and debt share the payoutδ through dividends, taxes and coupon payments. When the firm is solvent the value of entire payout claims are, V_{solv};

$$V_{solv} = V + A_1 V^{-y} + A_2 V^{-x}$$

For $V >> V_B$ this claim must approach total firm value V. This implies $A_1 = 0$. For $V = V_B$ the value of this claim vanishes, this constraint determines A_2, giving;

$V_{solv} = V_B + A_2 V_B^{-x} = 0$ thus $A_2 = -V_B / V_B^{-x}$

substituting A_2 into $V_{solv} = V - \frac{V_B}{V_B^{-x}} V^{-x}$

$$V_{solv} = V - V_B p_B(V) \tag{5.6}$$

Claim to interest payments V_{int} is of the form

$$V_{int} = \frac{C}{r} + A_1 V^{-y} + A_2 V_B^{-x}.$$

For $V >> V_B$, $D_c \rightarrow C/r$ implying $A_1 = 0$. This claim vanishes at $V = V_B$,

$$V_{int} = \frac{C}{r} + A_2 V_B^{-x}$$

$A_2 = -\frac{C/r}{V_B^{-x}}$ substituting into $V_{int} = \frac{C}{r} - \frac{C/r}{V_B^{-x}} =$ gives,

$$V_{int} = \frac{C}{r}[1 - p_B(V)]. \tag{5.7}$$

Separating the value of continuing operations between E,D and G gives;

$$E_{solv}(V) = (1 - \tau_{eff})(V_{solv} - V_{int})$$

$$G_{solv}(V) = \tau_{eff}(V_{solv} - V_{int}) + \tau_i V_{int}$$

$$D_{solv}(V) = (1 - \tau_i)V_{int}$$

The sum of these claims gives V_{solv}

From the Feynman-Kac theorem value of V_{solv} in eq.5.6 can be obtained from the risk neutral expectation

$$V_{solv}(V_0) = E_0^Q(\int_0^{T^\sim} dse^{-rs}\delta_s),$$

where $T^\sim$ is the (random) bankruptcy time. Similarly V_{int} in eq.5.7 can be obtained from

$$V_{int}(V_0) = E_0^Q(\int_0^{T^\sim} dse^{-rs}C).$$

Using this E_{solv} is written as

$$E_{solv}(V_0) = (1 - \tau_{eff})[E_0^Q(\int_0^{T^\sim} dse^{-rs}(\delta_s - C))].$$

This equation implies that, at each instant, s, equity has a claim to $(1-\tau_{eff})(\delta_s - C)1_{T^{\sim}>s}$. That is after the coupon payment is made, what remains is divided between equity and government according to the tax code. (Similar interpretations hold for other claims)

If payout level falls below the promised coupon payments the firm may prefer to sell some assets. However, bonds have protective covenants, therefore firm cannot sell no part of itself. Shareholders have right to infuse cash to the firm to avoid bankruptcy. This has a limit however, therefore shareholders must believe that they will get their money back if they invest more. If the firm is in a very poor state then the shareholders will choose to default the firm. This means using their option to walk away. If this happens the shareholders' claim is equal to zero and the firm is divided between debt, government and bankruptcy costs.

PV of default claim, $V_{def}(V)$ can be written as

$$V_{def}(V) = A_1 V^{-y} + A_2 V^{-x}.$$

For $V >> V_B$, the value of this claim must vanish, so again $A_1 = 0$. Boundary condition $V_{def}(V = V_B) = V_B$ implies

$$V_{def}(V) = V_B p_B(V).$$

Using $A_1 = 0$ and $A_2 = V_B / V_B^{-x}$

$$\begin{aligned} D_{def}(V) &= (1-\alpha)(1-\tau_{eff})V_{def}(V) \\ G_{def}(V) &= (1-\alpha)\tau_{eff}V_{def}(V) \\ BC_{def}(V) &= \alpha V_{def}(V) \end{aligned} \quad .$$

Restructuring costs, which are deducted from the proceeds of the debt issuance before distribution to equity occurs can be found by

$$RC(V_0) = q[D_{solv}(V_0) + D_{def}(V_0)].$$

5.2.1.1 Optimal Default Level

Management acts in the best interest of shareholders. Managers can choose optimal coupon level, C, and the bankruptcy level, V_B, to maximise their wealth. Optimal bankruptcy level, V_B, can be found by using smooth pasting condition

$$\frac{\partial E}{\partial V} \mid_{V=V_B} = 0, \tag{5.8}$$

$$E_{solv}(V) = (1 - \tau_{eff})(V_{solv} - V_{int}),$$

$$E_{solv}(V) = (1 - \tau_{eff})(V - V_B p_B(V) - \frac{C}{r} + \frac{C}{r} p_B(V)),$$

$$E_{solv}(V) = (1 - \tau_{eff})[V - p_B(V)(V_B - \frac{C}{r}) - \frac{C}{r}],$$

$$E_{solv}(V) = (1 - \tau_{eff})[V - \frac{V^{-x}}{V_B^{-x}}(V_B - \frac{C}{r}) - \frac{C}{r}].$$

Differentiating gives

$$0 = (1 - \tau_{eff})[1 + x\frac{V^{-x-1}}{V_B^{-x}}(V_B - \frac{C}{r})].$$

When $V = V_B$

$$(1-\tau_{eff})[1 + x\frac{1}{V_B}(V_B - \frac{C}{r})] = 0,$$

$$[1 + x\frac{1}{V_B}(V_B - \frac{C}{r})] = 0,$$

$$[1 + x(1 - \frac{C}{r}\frac{1}{V_B})] = 0,$$

$$[1 + x - x\frac{C}{r}\frac{1}{V_B}] = 0,$$

$$V_B = \frac{x}{x+1}\frac{C}{r}. \tag{5.9}$$

5.2.1.2 Optimal Coupon

The objective of management is to maximise shareholder wealth. Optimal coupon is the coupon rate on bonds which maximises the shareholder wealth. The objective function is given by

$$\max_C \{(1-q)D[V_0, C, V_B(C)] + E[V_0, C, V_B(C)]\}. \tag{5.10}$$

Differentiating Eq.5.10 with respect to C and setting the equation equal to zero gives the optimal coupon level where q is restructuring costs.

$$(1-q)(D_{solv} + D_{def}) + (1-\tau_{eff})(V_{solv} - V_{int}),$$

$$(1-q)[(1-\tau_i)V_{int}+(1-\alpha)(1-\tau_{eff})V_{def}]+(1-\tau_{eff})(V-V_Bp_B(V)-\frac{C}{r}(1-p_B(V)),$$

$$\begin{array}{l}(1-q)[(1-\tau_i)(\frac{C}{r}(1-p_B(V)))+(1-\alpha)(1-\tau_{eff})V_Bp_B(V)] \\ +(1-\tau_{eff})(V-V_Bp_B(V)-\frac{C}{r}+\frac{C}{r}(\frac{V}{V_B})^{-x})\end{array},$$

$$\begin{array}{l}a=(1-q)(1-\tau_i)\\ b=(1-q)(1-\alpha)(1-\tau_{eff})\\ d=(1-\tau_{eff})\end{array},$$

$$a(\frac{C}{r}-\frac{C}{r}p_B(V))+bV_Bp_B(V)+dV-dV_Bp_B(V)-d\frac{C}{r}+d\frac{C}{r}p_B(V),$$

$$a\frac{C}{r}-a\frac{C}{r}p_B(V)+bV_Bp_B(V)+dV-dV_Bp_B(V)-d\frac{C}{r}+d\frac{C}{r}p_B(V),$$

$$a\frac{C}{r}-a\frac{C}{r}(\frac{V}{V_B})^{-x}+bV_B(\frac{V}{V_B})^{-x}+dV-dV_B(\frac{V}{V_B})^{-x}-d\frac{C}{r}+d\frac{C}{r}(\frac{V}{V_B})^{-x},$$

$$V_B=\lambda\frac{C}{r},$$

$$a\frac{C}{r}-a\frac{C}{r}(\frac{\lambda C}{Vr})^x+bV_B(\frac{\lambda C}{Vr})^x+dV-dV_B(\frac{\lambda C}{Vr})^x-d\frac{C}{r}+d\frac{C}{r}(\frac{\lambda C}{Vr})^x,$$

$$a\frac{C}{r}-a\frac{C}{r}(\frac{\lambda C}{Vr})^x+b\frac{\lambda C}{r}(\frac{\lambda C}{Vr})^x+dV-d\frac{\lambda C}{r}(\frac{\lambda C}{Vr})^x-d\frac{C}{r}+d\frac{C}{r}(\frac{\lambda C}{Vr})^x,$$

$$a\frac{C}{r} + dV - d\frac{C}{r} + (\frac{\lambda C}{Vr})^x[d\frac{C}{r} - a\frac{C}{r} + b\frac{\lambda C}{r} - d\frac{\lambda C}{r}],$$

$$a\frac{C}{r} + dV - d\frac{C}{r} + (\frac{\lambda}{Vr})^x C^{x+1}[\frac{d - a + \lambda b - \lambda d}{r}].$$

Differentiating gives

$$\frac{a-d}{r} + (x+1)(\frac{\lambda}{Vr})^x C^x[\frac{d - a + \lambda b - \lambda d}{r}] = 0,$$

$$C^x = (\frac{d-a}{r})\frac{1}{(x+1)}(\frac{Vr}{\lambda})^x[\frac{r}{d - a + \lambda b - \lambda d}],$$

$$C = (\frac{Vr}{\lambda})(\frac{1}{(x+1)}\frac{(d-a)}{(d - a + \lambda b - \lambda d)})^{1/x},$$

$$C^* = V_0(\frac{r}{\lambda})[(\frac{1}{1+x})(\frac{A}{A+B})]^{(1/x)}$$

$$A = \lambda(1-\tau_{eff})[1-(1-q)(1-\alpha)]$$

$$B = (1-q)(1-\tau_i) - (1-\tau_{eff}).$$

For there to be tax advantage to debt, A must be positive.

5.2.1.3 Tax Advantage to Debt

Plugging optimal default and coupon level to objective function gives the value of the equity claim just before the debt issuance

$$E(V_{0-}) = \{(1-q)D[V_0, C^*, V_B(C^*)] + E[V_0, C^*, V_B(C^*)]\}, \tag{5.11}$$

$$= V_0[(1-\tau_{eff}) + AQ],$$

$$Q = [(\frac{A}{A+B})(\frac{x}{1+x})]^{\frac{1}{2}}.$$

Eq.5.11 can be rewritten as

$$E(V_{0-}) + RC = \{D[V_0, C^*, V_B(C^*)] + E[V_0, C^*, V_B(C^*)]\}, \tag{5.12}$$

where $RC = qD[V_0, C^*, V_B(C^*)]$.

5.2.1.4 Conclusion

This model shows how to obtain closed form solutions for a dynamic EBIT based model. This model has some limitations. Some of them are that: this model does not handle complex capital structures; the dynamic capital structure only supports upward alignments; and the process followed by EBIT is Geometric Brownian motion. In the next section a new model is presented which overcomes many of the problems presented here.

5.2.2 Ammann (2004) Model

Adding on to the previous model, this model can handle complex capital structures, both finite and infinite maturity bonds and more importantly underlying process is an Arithmetic Brownian Motion (ABM). Having ABM as an underlying process has several benefits compared to Geometric Brownian Motion. Firstly, ABM allows negative outcomes for the underlying process, therefore, it is more realistic than GBM in case of EBIT modelling. Secondly, working with normal distribution rather than lognormal distribution is more convenient in certain cases i.e. mergers.

Firms have infinite life with stochastic EBIT (η) flows. The underlying process is defined by

$$d\eta = \mu(\eta, t)dt + \sigma_{\eta}(\eta, t)dz^{Q},$$

where μ is the instantaneous drift, σ is the volatility of the process and z^Q is a Brownian motion under the risk-neutral martingale measure Q. It is assumed that all stochastic integrals exist and are well adapted to probability space $(\Omega, Q, F_t(\eta_t))$.

Discounted expected value of these flows gives total firm value or the sum of claims on the firm value

$$V = E_{t_0}^{Q} \int_{t_0}^{\infty} \eta_s e^{-r(s-t_0)} ds,$$

where r is risk-free interest rate.

5.3 Process Selection

Selection of the underlying process has critical importance on the success of the proposed model. Two processes, one more popular than the other can be named as Geometric Brownian Motion (GBM) and Arithmetic Brownian Motion (ABM). Although the latter process is more suitable for this kind of research, most of the time the former is used due to its convenience. The major flow of the GBM process comes into effect in cases of negativity. GBM process does not allow negative values for the underlying asset. This is convenient in pricing securities as negative security prices are not observed and expected. However in this case, the underlying asset is the cash flow of the firm and it can be positive or negative. Both ABM and GBM solutions for the proposed model demonstrated in following sections and rest of this study uses the ABM process.

GBM solution for the firm value is given by

$$\overline{V}_t = \frac{\overline{\eta}_t}{r - \overline{\mu}},$$

and the ABM solution for the firm value is given by

$$V = \frac{\mu}{r^2} + \frac{\eta_{t_0}}{r}.$$

Both solutions are included in Appendix D of this thesis. The solution for the ABM process is presented here. The underlying process is defined by

$$d\eta = \mu dt + \sigma dz.$$

A claim V that receives $\eta dt, t > t_0$ forever must have a law of motion by Ito's lemma

$$dV = \frac{\partial V}{\partial \eta} d\eta + \frac{1}{2} \frac{\partial^2 V}{\partial \eta^2} d\eta^2.$$

The expected capital gain on the claim V must equal a risk-free return so that

$$rVdt = dV + \eta dt.$$

A solution to the stochastic partial differential equation is guessed as

$$V = A + B\eta. \quad (5.13)$$

Solution 1:

The present value of infinite flows η is given by $\frac{\eta}{r}$, therefore $B = \frac{1}{r}$ if the guessed equation holds.

$\frac{\partial V}{\partial \eta} = B = \frac{1}{r}$ and $\frac{\partial^2 V}{\partial \eta^2} = 0$

Inserting the derivatives into value dynamics equation gives

$$dV = B\mu dt + B\sigma dz.$$

Substituting B,

$$dV = \frac{1}{r}\mu dt + \frac{1}{r}\sigma dz,$$

$$rVdt - \eta dt = \frac{1}{r}\mu dt + \frac{1}{r}\sigma dz,$$

$$rVdt = \frac{1}{r}\mu dt + \frac{1}{r}\sigma dz + \eta dt.$$

Setting $dz = 0$ gives

$$rVdt = \frac{1}{r}\mu dt + \eta dt.$$

Dividing both sides with r

$$Vdt = \frac{1}{r^2}\mu dt + \frac{\eta}{r}dt,$$

gives $A = \frac{\mu}{r^2}$ therefore

$$V = \frac{\mu}{r^2} + \frac{\eta_{t_0}}{r}.$$

Solution 2: Inserting derivatives and dV into $dV = \frac{\partial V}{\partial \eta}d\eta + \frac{1}{2}\frac{\partial^2 V}{\partial \eta^2}d\eta^2$ gives $rVdt - \eta dt = Bd\eta$.

Substituting $d\eta$ yields $rVdt - \eta dt = B\mu dt + B\sigma dz$.

Rearranging gives

$rVdt - \eta dt = B\mu dt,$

$V = \frac{B\mu}{r} + \frac{\eta}{r}.$

From guessed equation

$V = A + B\eta,$

B is $\frac{1}{r}$and $A$is $\frac{\mu}{r^2}$.

5.4 Valuation of Firm and its Securities

Firm value is defined as discounted value of future EBIT flows. EBIT values can be negative as well as positive. For sufficiently low values of EBIT the firm will declare bankruptcy at a random time τ. At $t = \tau$, equity owners receive the residual value after all bankruptcy claims and costs are paid. The density of η_T that the firm survives until time T is $\phi(\cdot)$. The probability of the firm going bankrupt before T is given by $\Phi(\cdot)$. The price of Arrow-Debreu security paying 1 unit of currency at bankruptcy is given by $p_B(\cdot)$. Prices of particular claims are given in the following sections.

5.4.1 Bankruptcy

An Arrow-Debreu security is defined with price $pB(V_i)$. This security pays one unit of currency when a boundary V_B^i is reached for the first time. V_B^i can be named as the default threshold or default boundary. When firm value hits this threshold the firm will declare bankruptcy or reorganise its debt (Ammann and Genser, 2004). The security $pB(V_i)$ has no intermediate cash flows and maturity date. The only cash flow occurs when V_B^i is hit.

If firm value increases the value of this security will converge to 0 as the firm moves further away from being bankrupt. As firm value goes to V_B^i, $pB(V_i)$ must go to 1. Solving the general solution for these conditions $A_1 = 0$, $A_2 = exp(k_2 V_B^i)$ and

$$p_B(V_i) = e^{-k_2(V_i - V_i^B)}.$$

5.4.2 Firm Value

Firm value V_i is split into two parts. One is solvent part V_+^i and the other is insolvent part V_-^i (see Appendix D).

$$V_i = V_+^i + V_-^i.$$

This is done to ease the calculations as they are mutually exclusive events the linear combination of the two will give the total firm value.

In this model the firm has three claimants; bondholders D, shareholders E, and government G. The sum of these three claims gives the total firm value. Each claim has its solvent and insolvent parts as in definition of firm value.

5.4.3 Solvent Firm Value

There are two boundary conditions for the solvent firm value. For sufficiently high firm value V^- vanishes and for bankruptcy V_i^+ becomes zero. Therefore boundary conditions for V_+^i are

$$\lim_{V_i \to \infty} V_+^i = V_i,$$

and

$$\lim_{V_i \to V_B^i} V_+^i = 0.$$

These boundary conditions imply that

$$V_+^i = V_i - V_B^i pB(V_i).$$

5.4.4 Taxes

In this section taxes are introduced to the model. The firm has three claimants with different tax rates; equity investor with tax rate τ^e, debt investor with tax rate τ^d and the government with tax rate τ^c corporate tax rate. The after tax value of perpetual debt for the investors is given by

$$D_{C,\infty}^{i+} = (1-\tau^d)V_{C,\infty}^{i+}.$$

Firms' equity is the residual of firms solvent value after debt payment $V_E^{i+} = V_+^i - V_{C,\infty}^{i+}$. The after tax value of the equity is given by

$$(1-\tau^e)(1-\tau^c)(V_+^i - V_{C,\infty}^{i+})$$
$$= (1-\tau^{eff})(V_+^i - V_{C,\infty}^{i+}),$$

where τ^{eff} is the effective tax rate paid by an equity investor, which includes the corporate tax and the tax on dividend payments.

The government's claim from solvent firm value is

$$G_{i+} = \tau^{eff}(V_+^i - V_{C,\infty}^{i+}) + \tau^d V_{C,\infty}^{i+}.$$

The sum of these three claims is equal to solvent value of the firm $V_+^i = D_{C,\infty}^{i+} + G_{i+} + E_i^+$.

5.4.5 Perpetual Bonds

Although firms do not usually use perpetual debt to finance their operations, it is widely used in capital structure related literature ie. (Goldstein, Ju, and Leland, 2001; Leland, 1994). Using perpetual debt is convenient in terms of modelling as it makes the partial differential equation (PDE) an ordinary differential equation (ODE) by making it independent of time (Goldstein, Ju, and Leland, 2001).

Any claim F with a regular payment flow $f(\eta_t)$ to investors depending on EBIT η must satisfy the PDE

$$\mu F_\eta + \frac{(\sigma_\eta)^2}{2} F_{\eta\eta} + F_t + f(\eta_t) = rF. \tag{5.14}$$

Existence of perpetual debt makes all claims time-independent therefore PDE reduces to following ODE

$$\mu F_\eta + \frac{(\sigma_\eta)^2}{2} F_{\eta\eta} + f(\eta_t) = rF. \tag{5.15}$$

Without the cash flows to investors $f(\eta_t)$

$$\mu F_\eta + \frac{(\sigma_\eta)^2}{2} F_{\eta\eta} - rF = 0, \tag{5.16}$$

the general solution is given by

$$F = A_1 e^{-k_1 \eta_t} + A_2 e^{-k_2 \eta_t}, \tag{5.17}$$

where

$$k_{1/2} = \frac{\mu \mp \sqrt{\mu^2 + 2r(\sigma_\eta)^2}}{(\sigma_\eta)^2}. \tag{5.18}$$

k_1 is negative, k_2 is positive. From equation 5.17 as EBIT becomes larger, the first term goes to infinity and the second term converges to zero.

Cash flows to investors are not accounted for in the general solution. To find the price of a particular security, the present value of the payments to investors must be accounted for. A_1 and A_2 are constants determined by boundary conditions of the particular security being priced.

The value of perpetual coupon payments to investors is given by particular solution $F^*_{C,\infty} = C/r$. Firm value with perpetual debt issue is given by

$$V^{i+}_{C,\infty} = \frac{C}{r} + A_1 e^{-k_1 V_i} + A_2 e^{-k_2 V_i}. \tag{5.19}$$

Boundary conditions

$$\lim_{V_i \to \infty} V^{i+}_{C,\infty} = C/r,$$

and

$$\lim_{V_i \to V^i_B} V^{i+}_{C,\infty} = 0$$

gives

$$V^{i+}_{C,\infty} = \frac{C}{r}[1 - pB(V_i)]. \tag{5.20}$$

5.4.6 Finite Maturity Bonds

The price of a finite maturity bond is derived using the price of a perpetual bond. Simply the price of a finite bond is given by taking the difference between a perpetual issued today and a perpetual issued at the maturity of the finite bond. If there were no default risk involved this would be the price of a finite maturity bond. However, firms may go bankrupt at any time at or before maturity, therefore default terms and probabilities are required.

Bond value is a combination of a series of cash flows. These cash flows are coupon payments, principal payments and payments which only occur in case of a bankruptcy. These cash flows are as follows

$$\begin{array}{ll} -(1-\tau^d)\frac{C}{r} & \text{Perpetual risk-free coupon bond} \\ & \text{starting at } t \\ -(1-\tau^d)\frac{C}{r}e^{-r(T-t)} & \text{Perpetual risk-free bond} \\ & \text{starting at } T \\ e^{-r(T-t)}P & \text{Principal repayment at } T \end{array} \tag{5.21}$$

$$\begin{array}{ll} -(1-\tau^d)\frac{C}{r} & \text{Perpetual risk-free coupon bond} \\ & \text{starting at } \tau \\ (1-\tau^{eff})w(T) & \\ \min[(1-\alpha)V_B(T);\sum_{j=1}^{J}P_j] & \text{Recovery value at } \tau \end{array} \tag{5.22}$$

where t is today, T is maturity date, τ is the default time and $w(T)$ is the weight of the bond issue. If there is only one class of bonds outstanding then $w(T) = 1$, because all of the recovery value will belong to that bond.

The expected values were taken using their respective probabilities. The components in Eq. 5.21 will be weighted with the probability that the default occurs after T

$$1 - \phi(t_0, T_1, \eta_{t_0}, \eta(T_1)).$$

Eq. 5.22 will be weighted using Arrow-Debreu default prices

$$p_B(t_0, T_1, \eta_{t_0}, \eta_B(T_1)).$$

Putting everything together will yield the bond valuation formula for the model. The debt value of solvent firm is defined by

$$D^+ = (1-\tau^d)\frac{C}{r} + Pe^{-r(T-t)}(1-\Phi) - e^{-r(T-t)}(1-\tau^d)\frac{C}{r}(1-\Phi) - (1-\tau^d)\frac{C}{r}pB.$$

Total debt value with recovery value

$$D^i_{C_k,T_k} = e^{-r(T-t)}[P_k - (1-\tau^d)\frac{C_k}{r}][1-\Phi_k] + (1-\tau^d)\frac{C_k}{r}[1-pB_k] + D^{i-}_{C_k,T_k}.$$

Separate tax terms

$$D^i_{C_k,T_k} = P_k e^{-r(T-t)}(1-\Phi_k) - (1-\tau^d)\frac{C_k}{r}e^{-r(T-t)}(1-\Phi_k) + (1-\tau^d)\frac{C_k}{r}(1-pB_k) + D^{i-}_{C_k,T_k}.$$

The principal is not taxed and ignoring recovery value gives the tax amount on each bond

$$D^i_{C_k,T_k}(Tax) = (\tau^d)\frac{C_k}{r}(1-pB_k) - (\tau^d)\frac{C_k}{r}e^{-r(T-t)}(1-\Phi_k)$$

5.4.7 Insolvent Firm Value

A firm will declare bankruptcy when the bankruptcy barrier V_B^i is hit, therefore the value of insolvent firm is given by

$$V_-^i = V_B^i pB(V_i).$$

Insolvent values of the three claims are

$$D_{C,\infty}^{i-} = (1-\alpha)(1-\tau^{eff})V_-^i$$

$$G_{i-} = (1-\alpha)\tau^{eff}V_-^i$$

$$BC_i = \alpha V_-^i$$

and three claims add up to the insolvent value of the firm.

$$V_-^i = D_{C,\infty}^{i-} + G_{i-} + BC_i.$$

5.4.8 Equity

Shareholders get the residual value after debt and tax payments. Debt is taxed with investors' personal tax rate, τ^d. However, to find the equity value, debt must be taxed at corporate tax rate, τ^c. Interest payments are not taxed. This reduces the tax burden on a firm which would normally be paid at rate τ^c. Ignoring the default value a new debt value is obtained which is given by

$$\begin{aligned}D^{i,E+}_{C_j,T_j} &= e^{-r(T-t)}[P_k - (1-\tau^c)\tfrac{C_k}{r}][1-\Phi(T_k,-t;V^i,V^i_B)]\\ &+(1-\tau^c)\tfrac{C_k}{r}[1-pB(T_k,-t;V^i,V^i_B)].\end{aligned} \tag{5.23}$$

The value of equity is given by

$$\begin{aligned}E_i &= (1-\tau^e)[(1-\tau^c)V^i_+ - \sum_j D^{i,E+}_{C_j,T_j}] + E^{i-}\\ &-\tau^e\sum_j P_j e^{-r(T-t)}[1-\Phi(T_k,-t;V^i,V^i_B)],\end{aligned} \tag{5.24}$$

where the last line prevents shareholders to receive a tax subsidy on bond redemptions.

The tax on equity can be found by expanding the equation 5.24 as

$$\begin{aligned}E_i &= (1-\tau^e)[(1-\tau^c)V^i_+\\ &-\sum e^{-r(T-t)}[[1-\Phi(T_k,-t;V^i,V^i_B)]P_k - [1-\Phi(T_k,-t;V^i,V^i_B)](1-\tau^c)\tfrac{C_k}{r}]\\ &-\sum(1-\tau^c)\tfrac{C_k}{r}[1-pB(T_k,-t;V^i,V^i_B)]] - \tau^e\sum_j P_j e^{-r(T-t)}[1-\Phi(T_k,-t;V^i,V^i_B)],\end{aligned}$$

$$\begin{aligned}E_i &= (1-\tau^e)\{(1-\tau^c)V^i_+ - \sum e^{-r(T-t)}[(1-\Phi)P_k - (1-\Phi)(1-\tau^c)\tfrac{C_k}{r}]\\ &-\sum(1-\tau^c)\tfrac{C_k}{r}(1-pB)\} - \tau^e\sum_j P_j e^{-r(T-t)}(1-\Phi),\end{aligned}$$

$$\begin{aligned}E_i &= (1-\tau^e)\{(1-\tau^c)V^i_+ - \sum P_k e^{-r(T-t)}[(1-\Phi)] + (1-\tau^c)\sum e^{-r(T-t)}\tfrac{C_k}{r}(1-\Phi)\\ &-(1-\tau^c)\sum\tfrac{C_k}{r}(1-pB)\} - \tau^e\sum_j P_j e^{-r(T-t)}(1-\Phi).\end{aligned}$$

Taking all the tax terms together will give the tax on equity,

$$E_i(Tax) = (\tau^{eff})V_+^i - (\tau^e)\sum P_k e^{-r(T-t)}(1-\Phi_k) + (\tau^{eff})\sum e^{-r(T-t)}\tfrac{C_k}{r}(1-\Phi_k)$$
$$-(\tau^{eff})\sum \tfrac{C_k}{r}(1-pB_k) + \tau^e \sum_j P_j e^{-r(T-t)}(1-\Phi_k).$$

5.4.9 Government Claim

The government's claim on the firm is formed by total taxes. Adding the tax on equity and the debt will give the government's claim on the firm as

$$G^i = \sum \tau^d \tfrac{C_k}{r} - \sum e^{-r(T-t)}\tau^d \tfrac{C_k}{r}(1-\Phi_k) - \sum \tau^d \tfrac{C_k}{r} pB_k +$$
$$(\tau^{eff})V_+^i - (\tau^e)\sum P_k e^{-r(T-t)}(1-\Phi_k) + (\tau^{eff})\sum e^{-r(T-t)}\tfrac{C_k}{r}(1-\Phi_k)$$
$$-(\tau^{eff})\sum \tfrac{C_k}{r}(1-pB_k) + \tau^e \sum_j P_j e^{-r(T-t)}(1-\Phi_k).$$

5.4.10 Default Claim

The density of V hitting the barrier V_B^i before a time T is defined by $\phi(T-t; V^i, V_B^i)$ and its cumulative distribution function is defined by $\Phi(T-t; V^i, V_B^i) = \int_{V_B^i}^{\infty} \phi(T-t; V^i, V_B^i)dz$.

The solution for Φ can be found in Genser (2005, p.p. 47) which is given by

$$\Phi(T-t; V^i, V_B^i) = \int_{V_B^i}^{\infty} \phi(T-t; V^i, V_B^i)dz = N(h_1) + e^{-\frac{2(\mu_\eta^i - \theta_\eta^i \sigma_\eta^i)}{}r(V^i - V_B^i)} N(h_2),$$

with

$$h_{1/2} = \frac{-r(V^i - V_B^i) \mp (\mu_\eta^i - \theta_\eta^i \sigma_\eta^i)(T-t)}{\sigma_\eta^i \sqrt{T-t}},$$

where $N(.)$ denotes the cumulative standard normal distribution.

$\Phi(T-t; V^i, V_B^i)$ gives the probability of hitting the barrier V_B^i before or at maturity T. This can be interpreted by using barrier option terminology. A down-and-out barrier option which pays one unit of currency at maturity T can be priced with $e^{-r(T-t)}(1 - \Phi(T-t; V^i, V_B^i))$.

The value of a finite maturity claim which pays 1 unit of currency if the firm goes bankrupt until a specified time T and zero otherwise can be written as (Rubinstein and Reiner, 1991)

$$p_B(T-t; V^i, V_B^i) = \int_0^{T-t} e^{-rs}\phi(s; V^i, V_B^i)ds$$

$$= e^{-k_1(V_i - V_B^i)}N(q_1) + e^{-k_2(V^i - V_B^i)}N(q_2)$$

$$q_{1/2} = \frac{-r(V^i - V_B^i) \mp \sqrt{(\mu_\eta^i - \theta_\eta^i \sigma_\eta^i)^2 + 2r(\sigma_\eta^i)^2}(T-t)}{\sigma_\eta^i \sqrt{T-t}}$$

As $T \to \infty$ $q_1 \to -\infty$ and $q_2 \to +\infty$ making $N(q_1) \to 0$ and $N(q_2) \to 1$ which is a generalisation of infinite case.

The next consecutive claim between T and T' with $T < T'$ can be found by

$$p_B(T' - T; V^i - V_B^i | \mathcal{F}_t) = p_B(T' - t; V^i - V_B^i) - p_B(T - t; V^i - V_B^i)$$

P_B can be interpreted as the price of a finite maturity down-and-in barrier option which pays a currency unit if the barrier V_B^i is passed before or at time T.

5.4.10.1 Default Claim of Debt

In case of a default, bondholders receive a proportional amount of the insolvent firm value. Each bond issue receives an amount weighted by its principal amount, $\frac{P}{\Sigma P}$. Bondholders get the remaining firm value if it is less than total principal amount of outstanding bonds given by

$$V^{i-}_{C_k,T_1} = \min[(1-\alpha)V^i_B; \sum_{j=2}^{J} P_j] \frac{P_k}{\sum\limits_{j=2}^{J} P_j} pB(T_2, T_1; V^i, V^i_B|\mathcal{F}_t). \tag{5.25}$$

After tax value of default claim of debt is given by,

$$D^{i-}_{C_k,T_1} = (1-\tau^{eff})(V^{i-}_{C_k,T_1}). \tag{5.26}$$

5.4.10.2 Default Claim of Equity

Equity owners get the residual value after bankruptcy costs and debt repayments. The insolvent value of equity is given by

$$[V^-_{E,j=1} = \max[(1-\alpha)V_B(T_1) - \sum_{j=1}^{J} P_j; 0]p_B(t_0, T_1, \eta_{t_0}, \eta_B(T_1)). \tag{5.27}$$

The after tax value of insolvent equity value is

$$E^-_{j=1} = (1-\tau^{eff})V^-_{E,j=1}. \tag{5.28}$$

5.4.10.3 Default Claim of Government

The value of taxes paid to government in case of default is given by

$$G^{-}_{j=1} = \tau^{eff}(V^{-}_{E,j=1} + \sum_{j=1}^{J} V^{-}_{C_j,T_1}) = \tau^{eff}(1-\alpha)V_B(T_1)p_B(t_0, T_1, \eta_{t_0}, \eta_B(T_1)). \tag{5.29}$$

5.5 Simulation of the Firm

A recombining trinomial tree is used to simulate the model. A trinomial is used for its faster convergency. Having three possibilities at each time step is also more realistic than using a binomial tree.

5.5.1 Model Calibration

Although the static model has closed form solutions, multi-period version is solved numerically using a trinomial tree. To control and calibrate the numerical results a benchmark model is required. Leland (2007)'s model is a good candidate for this purpose as it has closed form solutions for the entire model and uses similar parameters with the multi-period model. The firm value provided by the trinomial tree has the growth rate as the firm in Leland (2007)'s model. Therefore both models can produce the same firm values for identical firms. The calibration is done by setting the correct tree parameters to produce the required outputs.

5.5.2 Node Probabilities

Kamrad and Ritchken (1991)'s multinomial method is applied to find the node probabilities. It is assumed that EBIT process follows Arithmetic Brownian Motion. Simulation of this process is done by $S(t + \Delta t) = S(t) + \xi(t)$ where ξ is a normal random variable with mean $\mu\Delta t$ and variance $\sigma^2\Delta t$.

The distribution $\xi(t)$ is approximated with $\xi^a(t)$ over the period $[t, t + \Delta t]$. Discrete random variable $\xi^a(t)$ takes the following values

$$\xi^a(t)\begin{cases} v, & p_1 \\ 0, & p_2 \\ -v, & p_3 \end{cases} \tag{5.30}$$

where $p_{1,2,3}$ represent probabilities and $v = \lambda\sigma\sqrt{\delta t}$ and $\lambda \geq 1$.

Choosing the same mean and variance with $\xi(t)$ for approximating distribution $\xi^a(t)$ gives

$$E\{\xi^a(t)\} = v(p_1 - p_3) = \mu\Delta t,$$

$$Var\{\xi^a(t)\} = v^2(p_1 + p_3) = \sigma^2\Delta t + O(\Delta t).$$

Using $v = \lambda\sigma\sqrt{\Delta t}$ and $p_1 + p_2 + p_3 = 1$ probabilities can be found as

$$p_1 = \frac{1}{2\lambda^2} + \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma}, \tag{5.31}$$

$$p_2 = 1 - \frac{1}{\lambda^2} = 1 - p_1 - p_3, \tag{5.32}$$

$$p_3 = \frac{1}{2\lambda^2} - \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma}. \tag{5.33}$$

The probabilities calculated here are the probabilities of different outcomes at each node. At any node, the probability of moving up, down and staying the same is p_1, p_2 and p_3 respectively.

5.5.3 Further Node Probabilities

The probabilities calculated in previous section are the transition probabilities. To calculate the probabilities of a specific node of trinomial tree, trinomial distribution mass function can be used. The following equation gives the probability mass function of a multinomial distribution

$$f(x_1,\ldots,x_k;n,p_1,\ldots,p_k) = \Pr(X_1 = x_1 \text{and} \ldots \text{and} X_k = x_k)$$

$$= \begin{cases} \frac{n!}{x_1!\cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}, & \text{when} \sum\limits_{i=1}^{k} x_i = n \\ \\ 0 & \text{otherwise,} \end{cases}$$

For the trinomial distribution the formula will be

$$f(a,b,c;n,p_a,p_b,p_c) = \Pr(A = a \text{and} B = b \text{and} C = c)$$

$$= \begin{cases} \frac{n!}{a!b!c!} p_a^a p_b^b p_c^c, & \text{when} a+b+c=n \\ \\ 0 & \text{otherwise,} \end{cases} \tag{5.34}$$

where a is up movement, b is middle, and c is down movement on the tree. The probability given by this formula is the probability of following the given path. It is not the probability of a specific node as there may be alternative routes reaching to a specific node on tree. The following movements will all reach the same node on the tree:

1 up, 1 middle, and 1 down; 3 middle.

Assuming all the probabilities are same $1/3$, the probabilities of the movements given can be found as, 0.2222, and 0.037. Sum of these two probabilities gives the probability of the node. As the trinomial tree is a recombining tree, there may be more than one path reaching to a particular node. The coefficients of trinomial expansion will help identify the probabilities correctly. However, the formula

$$\frac{n!}{x_1! \cdots x_k!},$$

gives the coefficients for a specific movement and requires calculation of each different path. For the example given above, the coefficient is 7 which is the sum of both paths' coefficients. If there are more paths reaching to same node then these should be included in the calculation. A new method is proposed here, to correctly identify the coefficients and to calculate the probabilities of each node. The probabilities can also be calculated by recursive methods starting from the first node of the tree while p_1, p_2 and p_3 are known. However, this method can be time consuming if one does not need to know the coefficients of all nodes. For example, one may need to know the probabilities of a node at a future time on tree.

Let n be the number of columns or time-steps of the tree. Having constant and equal step sizes on the tree ensures that only paths with an equal number of movements can reach to same column on tree. Therefore

$$a + b + c = n, \tag{5.35}$$

must be satisfied where a is up movement, b is middle node which states there is not any change in the state variable and c is down movement on the tree. $a, b, c \in \mathbb{Z}$

Using eq.5.35,

$$b = n - c - a \tag{5.36}$$

Each path satisfying $c - a = x$ condition leads to the same node on the tree which is x nodes above the central node. The smallest number that c can take is a.

Substituting c gives

$$b = n - 2a - x, \tag{5.37}$$

or

$$a = (n - x - b)/2.$$

The maximum value that a can take is given by $b = 0$, $x = 0$. For x to be minimum, $c = a$ is required. Solving Eq.5.37 for $b = 0$ yields

$$max(a) = (n - x)/2, \tag{5.38}$$

Therefore the values that a, b, and c can take are given by

c=x+a

b= N-c-a

where

N=0,1,2,...,T

x=0,1,2,...,N

a=0,1,2,...,(N-x)/2. (5.39)

The method proposed here allows the calculation of coefficients of any tree node without calculating the coefficients of previous nodes. This method also simplifies calculation of probabilities by calculating them using subsets of coefficients instead of using one big coefficient for each node.

A small example will calculate the probability of a node at time step $N = 3$ and distance to central node $X = 0$.

Using the relations

$$a = 0, 1,$$

$$c = X + a,$$

$$b = N - c - a,$$

the following table can be calculated

a	**b**	**c**	**N**	**X**	**P**
0	3	0	**3**	0	3.704E-02
1	1	1	**3**	0	2.222E-01
0	2	1	**3**	1	1.111E-01
1	0	2	**3**	1	1.111E-01
0	1	2	**3**	2	1.111E-01
0	0	3	**3**	3	3.704E-02

Table 5.1: Probability calculation for a tree node

applying the formula

$$\frac{n!}{a!b!c!} p_a^a p_b^b p_c^c, \quad ,$$

for each a, b, and c value combination and adding the results gives 0.2593. If the probabilities of each node at a specific time is required then the same formula should be used for every possible X as that time step.

A sample code can be found in Appendix C.

5.5.4 Simulation of the Firm with Finite Maturity Bonds

In this section a hypothetical firm is created with one class of finite maturity bond outstanding. The bond parameters are given in Table 5.3. The coupon rate is set higher than the risk-free rate. Parameters used the calculations are given in Table 5.2. Unless otherwise stated all parameters are constant in time. Later this assumption is relaxed.

A trinomial tree is used to simulate the firm in time. Security values and all other parameter values are obtained by taking the expected value of all nodes with each time step in the tree. To speed up the process a sampling scheme is employed which limits the expected value calculation to specified dates. The sampling interval is annual and the model only calculates the values at the end of each year. This speeds up the computation process without loss of any information.

Parameter	Symbol	Value
Alpha	α	0.8
Default Barrier	V_B	1500
EBIT	η	102.5
Mu	μ	1.5
Risk Free Rate	r	0.05
Sigma	σ	25
Tax Corporate	τ^c	0.35
Tax Debt	τ^d	0.1
Tax Efficient	τ^{eff}	0.415
Tax Equity	τ^e	0.1
Theta	θ	0.01
Time	t	0

Table 5.2: Base case parameters

Principal	**Coupon**	**Maturity**
1,500.00	0.06	12

Table 5.3: Bond Portfolio with one finite maturity bond

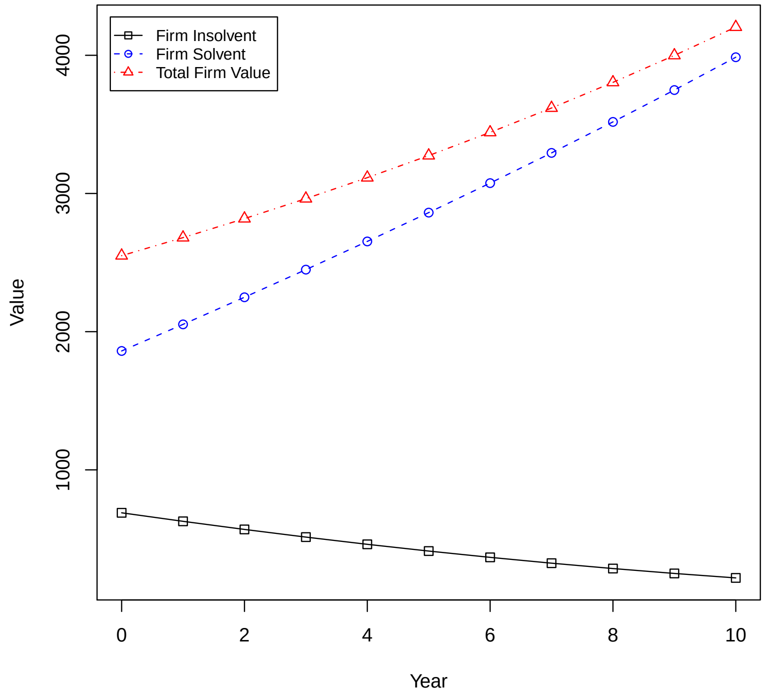


Figure 5.1: Firm Value: with finite maturity bond.

Total firm value is shown in Figure 5.1 with its solvent and insolvent parts together. Firm value grows linearly over time with an increase in the solvent part. As firm value increases, the firm becomes less exposed to default risk, therefore this decreases the insolvent firm value and increases solvent firm value. The probability of survival is increased. It is expected that equity value will increase with the increase in firm value. Figure 5.2, which shows debt and equity values, confirms this. Debt value is increasing until its maturity is reached due to the improved financial status of the firm. The firm has more cash to honour its liabilities.

Securities are modelled as the sum of different claims. This helps to analyse the source of value change in these securities. Figure 5.3 shows the components of equity value. The components that are analysed are 1) solvent value of equity 2) insolvent value of equity 3) the tax claim on equity. Total equity value is increasing, therefore tax claim on equity is increasing. The solvent part of equity is increasing,

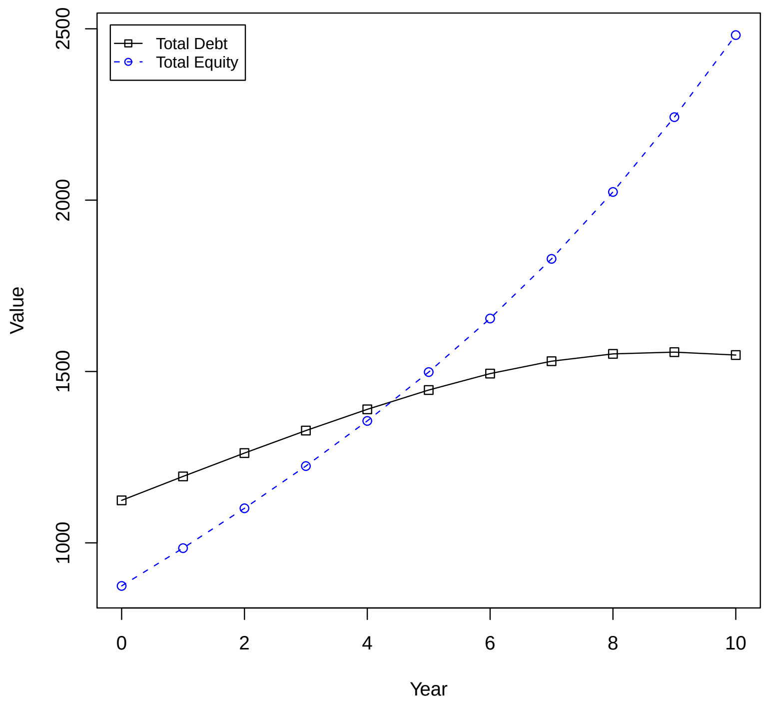


Figure 5.2: Debt and Equity Values with finite maturity bonds

however, the insolvent part of equity has a different pattern than the solvent part. Default claim increases until year 6 and starts decreasing thereafter. This change may be explained through the change in debt value. With a constant default barrier V_B^i the change in debt value will not change the credit quality of the firm.

As time passes debt value increases but its number of payable coupons decreases making it less risky for the firm. Secondly, as time gets closer to the maturity date of the bond it becomes easier to judge if the bond will be paid back or not. These two effects cause a decreasing default value for the debt. This default claim comes to a point that it negatively affects the default value of the equity. The change observed in default value of equity is not substantial with the current parameter combination, however, it is possible to observe a higher value for a different firm.

Equity owners will prefer to infuse cash into the firm if EBIT is negative when their marginal return from the investment is positive, and default will be optimal if their

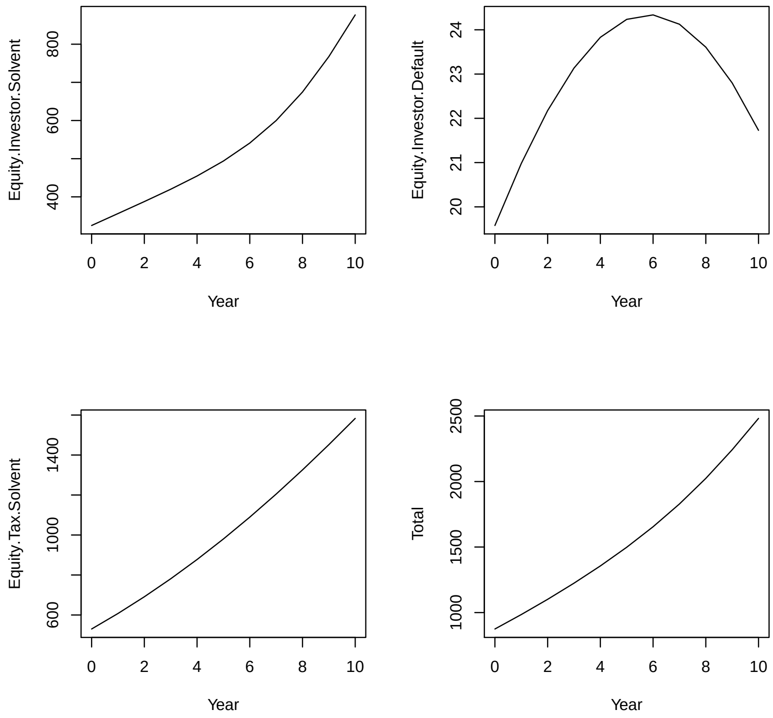


Figure 5.3: Components of Equity Value with finite maturity bonds

marginal return from the investment is equal to zero. Firgure 5.3 shows that the default claim on equity is still positive and the total equity value is also positive. For this firm default is not optimal.

The total bankruptcy cost shown in Figure 5.5 represents the total default cost for the firm. This is the sum of all bankruptcy claims on the firm, $V^- = D^- + E^- + G^-$. This shows the change in overall credit quality of the firm.

In this section how security values behave for a single firm with a constant default barrier is presented. A finite maturity bond is used to demonstrate the debt value changes. Although it is not common for firms to issue perpetual bonds, previous literature has used them extensively (Goldstein, Ju, and Leland, 2001; Leland, 1994). Instead of focusing on perpetual bonds, here it is preferred to focus on a more realistic capital structure with finite maturity bonds.

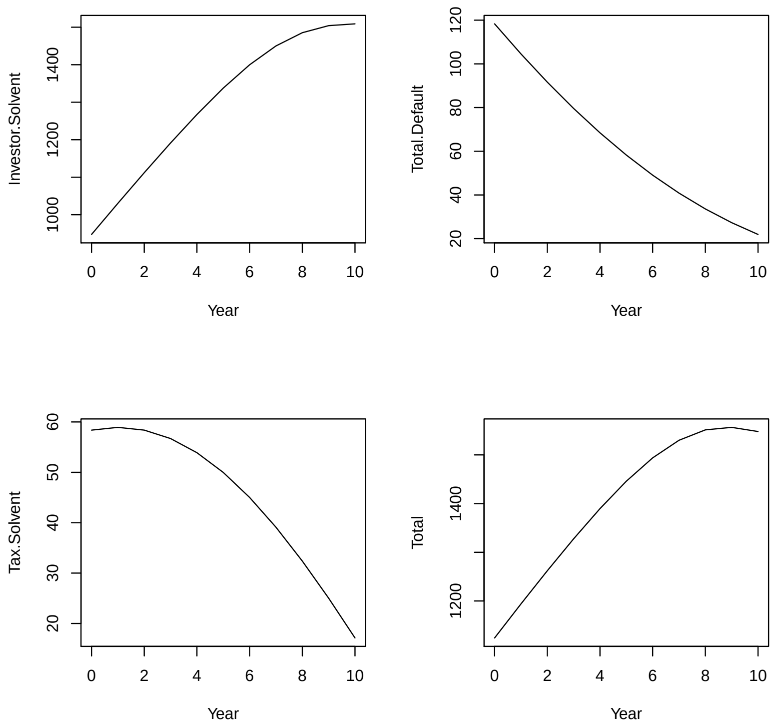


Figure 5.4: Components of Bond Value with finite maturity bonds

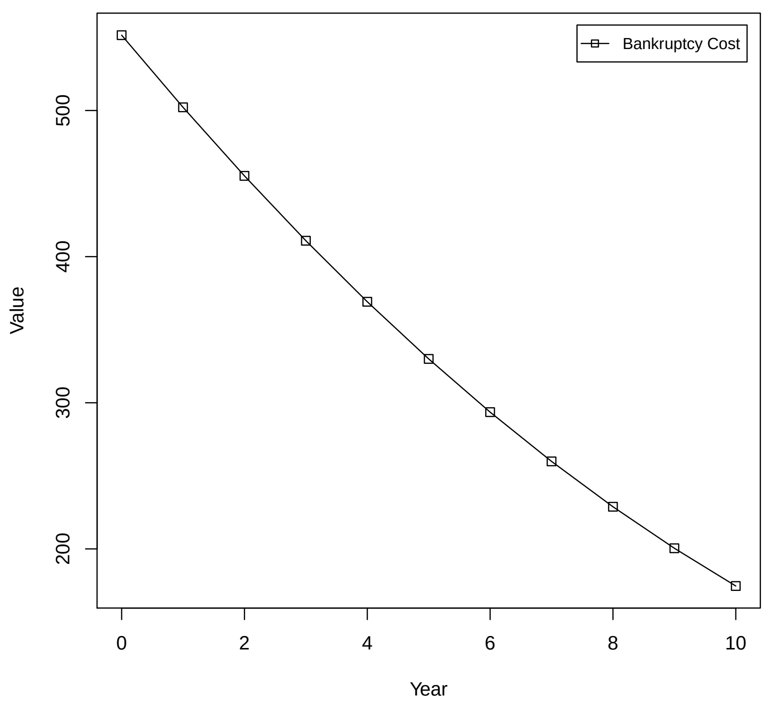


Figure 5.5: Bankruptcy cost with finite maturity bonds

In the next section the term structure of credit spreads is analysed where a complex capital structure is assumed.

5.6 Term Structure of Credit Spreads

This section focuses on bonds and their behaviour in a dynamic setting. A given capital structure with a given bond portfolio is assumed. Capital structure decision is exogenous to the model. However, an optimal capital structure can be found using the model by the classic trade-off theory to balance the tax savings with default costs incurred. The result from this calculation will not provide any information on the maturity structure of the firm. Rather than financial, it is a strategic decision. It requires some extra information such as maturity structure of operational cash flows or the time of a restructuring decision. Then, by looking at this information managers can decide on the maturity structure of the debt being issued. A classic example to this can be given by using the data in Table 5.4. The first column of the table shows the year and second shows the net cash flow at each year. If this information is known by the managers then they can balance their cash flows by buying or issuing bonds. For the sake of a short example it is assumed that firm does not have any other investment alternatives or does not want to make new investment with cash in hand and equity owners does not want to infuse cash to the firm. Therefore, the only external financing option is debt. At t_0 the firm buys a two-year bond for 150 and second year firm buys extra bonds. When the firm reaches year t_3 the matured bonds do not match the cash requirements and the firm decides to issue more debt. The maturity of debt issued at t_3 is a strategic decision. Managers can issue a short term debt or a long term debt. Both will meet the cash requirement of the firm at t_3, however only one of them will be a good fit for the future cash flows of the firm. Instead of focusing on how the maturity structure of the debt is formed, this decision is left to managers. The focus is on the spread changes on a given debt portfolio.

Year (t)	**Cash Flows**	**1Y Bond**	**2Y Bond**
0	+150.00		**-150.00**
1	+150.00	**-50.00**	**-100.00**
2	-200.00	+50.00	+150.00
3	-150.00	**+50.00**	+100.00

Table 5.4: Hedging Example with Bonds

Here Ammann and Genser (2004)'s model is modified and a new method of yield calculation is proposed. The reason for this is that firstly, their model shows yield calculation only for par bonds, and secondly, it does not produce correct yields for the bonds in the model.

Due to default risk inherent in corporate bonds investors will price riskless government bonds and risky corporate bonds differently. The difference between government debt price D^{Gov} and corporate debt D^{C} will be in favour of government bonds which is assumed to be riskless. A riskless government bond is priced using $D^{Gov}(C, P, r, T-t)$, where C is coupon, P is principal, r is risk free-rate and $T-t$ is time to maturity. r is used as yield as no risk is assumed for government bonds. If a formula for riskless government bonds can be developed then it can be used to find the yield of the risky debt for a given price. The yield from this formula will be the "default risk adjusted" yield for a corporate bond. All of the default risk will be reflected into the yield as the government bond formula does not have any "default risk" component. The yield will be the government bond equivalent yield for corporate bonds.

A government bond formula for the model will be derived here. Using the formula for corporate bond prices, the government bond pricing formula can be derived. The difference between the two is; default terms disappear in the government bond pricing formula. In the case of corporate debt the following price formula is used

$$D^{i}_{C_k,T_k} = P_k e^{-r(T-t)}(1-\Phi_k)-(1-\tau^d)\frac{C_k}{r}e^{-r(T-t)}(1-\Phi_k)+(1-\tau^d)\frac{C_k}{r}(1-pB_k)+D^{i-}_{C_k,T_k},$$

however in the case of the government debt, formula reduces to

$$D^{Gov,i}_{C_k,T_k} = P_k e^{-r(T-t)} - (1-\tau^d)\frac{C_k}{r^*}e^{-r(T-t)} + (1-\tau^d)\frac{C_k}{r^*},$$

by setting

$$\Phi_k = 0$$

$$pB_k = 0$$

$$D^{i-}_{C_k,T_k} = 0.$$

The insolvent part of the debt has no value in case of government bonds. Market convention for bonds is to calculate the yield before taxes, therefore, the tax rate is set to $\tau^d = 0$. Rearranging the formula gives the government bond price

$$D^{Gov,i}_{C_k,T_k} = P_k e^{-r(T-t)} - \frac{C_k}{r^*}e^{-r(T-t)} + \frac{C_k}{r^*},$$

which is assumed to be default free. Using this formula for corporate bonds by adjusting the r to be r^* will give the "default risk adjusted" yield of corporate bonds $D^C(C, P, r^*, T-t)$. Setting D and P to a defaultable corporate bonds price and principal and solving for r^* gives the required yield for a defaultable corporate bond. Without the default related terms, yield r^* will be the only yield to reflect the default-risk adjusted yield for the corporate bond. Yield spread can be calculated by using $s = r^* - r$.

5.6.1 Applications of the Model

In this section the firm is simulated with a given capital structure. The bond portfolio is arbitrarily chosen to be 5% coupon rate and 500 face value for all bonds with 2, 3, 5, 10, 20 and 30 year maturities. The same principal for all bonds have been chosen to see the effects of default risk on all bonds equally. In the case of a default each bonds recovery value is calculated as a proportion of its principal to the firms default value. This way, bonds with higher face values will have higher recovery values.

Maturity	**Yield**
2	0.023
3	0.026
5	0.0284
10	0.0282
20	0.027
30	0.0269

Table 5.5: $t = 0$ Coupon Rate is 5% and principal is 500 for all bonds

Table 5.5 shows the static $t = 0$ yields of the bond portfolio which is also shown in Figure 5.6. The yield curve is the yield curve for the firm and the dashed horizontal line shows the risk-free rate. Yield spreads can be observed from this chart but instead it is left to the next example and the focus here is on the shape of the yield curve. The yield is increasing until year 5 and bonds longer than that have a lower yield. This can be explained by the financial distress created by the short term debt. As the firm increases its short term borrowing it is increasing its default risk by pushing the yields of these bonds higher. By looking at this yield curve one can arrive at the conclusion that this firm must use long term debt rather than short term. Very short term debt has a lower yield, however, it has other risks involved like rollover risk or liquidity risk.

Now the example is extended to demonstrate yield spreads within a trinomial tree. The same trinomial tree is used to simulate the firm with a bond portfolio. The static results for the portfolio and its structure is given in Table 5.6. Static results exhibit a similar yield curve with the previous example. Notice that in this example, the shortest debt maturity is 5 years. Very short term debt will disappear in the tree quickly so bonds shorter than 5 years were not included.

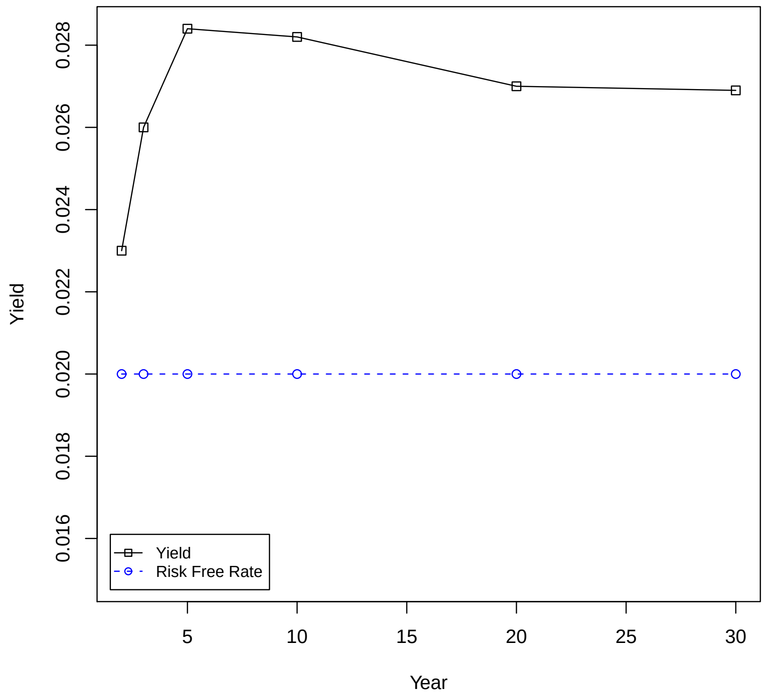


Figure 5.6: Term Structure: Yield curve at $t = 0$

Simulation results are shown in Figure 5.7. Each line represents the yield curve of the firm at a given year. It is assumed that the company does not issue the matured debt therefore the yield curve gets shorter as time passes. The yield curve shifts in Figure 5.7 can be explained by the increased credit quality of the firm. As firm value changes, the shape of yield curve is not affected but the position of the yield curve changes. An increase/decrease in the credit quality of the firm shifts the yield curve down/up.

The shape of the curve is related to the total amount of outstanding debt. As bonds mature short term bonds become less risky. This can also bee seen as realisation of the long term yield curve.

Maturity	**Yield**	**Spread**
5	0.1137	0.0637
6	0.11	0.06
7	0.1065	0.0565
8	0.1033	0.0533
9	0.1005	0.0505
10	0.0979	0.0479
15	0.0888	0.0388
20	0.0837	0.0337
25	0.0803	0.0303
30	0.0783	0.0283

Table 5.6: Coupon rate is 5% and principal is 200 for all bonds

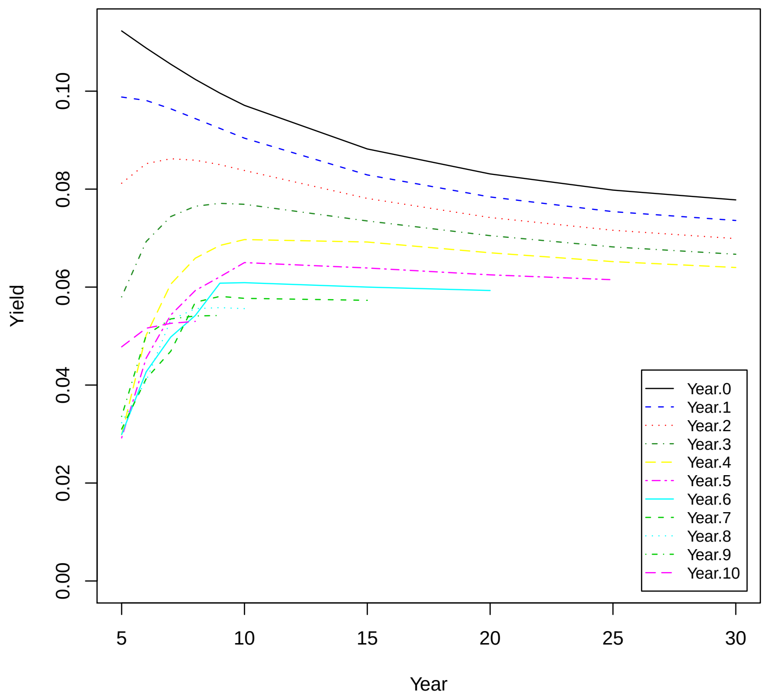


Figure 5.7: Term Structure: Evolution of yield curve within time

The shape of the yield curve is identified by the outstanding bond portfolio of the firm and its term structure. This can be observed from Figure 5.7. The firm has a declining yield curve at the first year $t = 0$, shorter maturity bonds have higher credit spread than the longer maturity bonds. As all the bonds have equal par values, long term debt puts less stress on the firm compared to short term debt via the time-value effect. Investors are more willing to lend to this firm over the longer term rather than short term, therefore they ask for a higher spread over the short term. At the third year, $t = 3$, the shortest term rates are lower than the long term rates. This is due to the decreased risk in the short term. Now investors are willing to lend money at the short term rather than long term. As the shortest maturity bonds mature, the firm has more space for borrowing at the shortest term.

Until now it is assumed that matured bonds are not replaced with new issues of debt. In the next section this assumption is relaxed and future bond issues were introduced to the model.

5.7 Reissue

In the current setting the firm does not have to reissue matured debt. Reissuance decision is exogenous to the model. Some capital structure models assume continuous reissuance (i.e. Leland and Toft, 1996). In this model it is not a necessity, rather it is a strategic or financial decision. Current firm value is not affected from future bond issues as current firm value is a function of EBIT, growth rate, volatility, risk premium and risk-free rate, $V(\eta, \mu, \sigma, \theta, r)$. However, security values are affected after the issuance occurs. The new issues changes the distribution of EBIT claims by changing the security values.

An indirect effect can be the capital structure changes due to future bond issues. Consider two identical firms with different strategies of bond issuance. One of them decides to issue one class of bonds for 12 years and does not want to issue debt again in near future. The second firm issues 6 year bonds and then plans to reissue the matured bonds after year 6. Security prices of these two firms will be different to each other. Although they are identical, their views on term structure changes the value of the securities. This is not directly caused by existence of future bond issues but it is a result of it.

Figure 5.8 shows the firm value change after bond issuance. At year 6 the firm reissues the matured debt. Insolvent firm value decreases until year 6 and then increases after year 6. Insolvent firm value is affected by the changes in default probability. The decrease in default probability can be explained by two factors. Firstly, as time passes a firm's liability on bonds decreases as the number of coupons decrease. Secondly, as time passes the growth of EBIT flows increases firm value, taking the firm farther away from being bankrupt. An increase in insolvent firm value does not necessarily mean default risk is increasing.

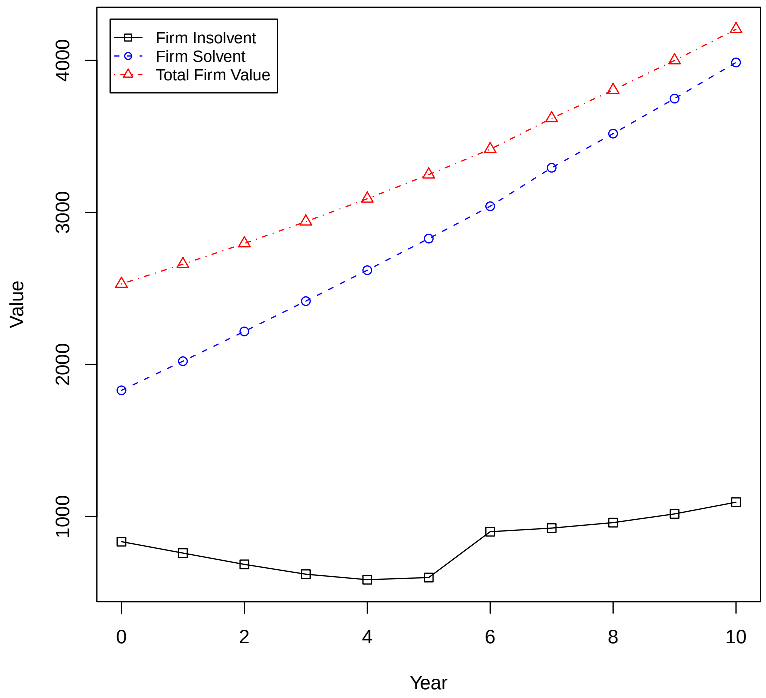


Figure 5.8: Firm Value with Future Bond Issue

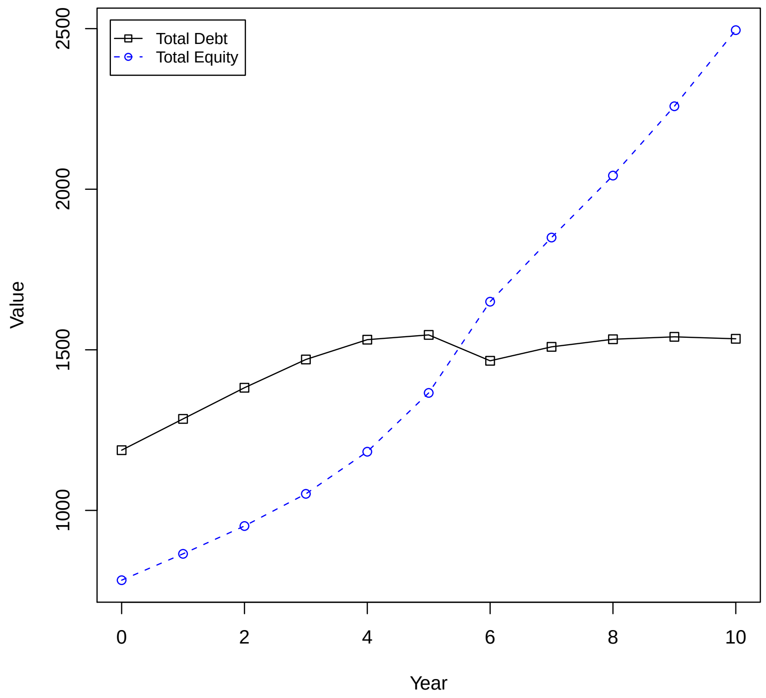


Figure 5.9: Debt and Equity with Future Bond Issue

Before and after reissuance, debt value follows its normal price path. It increases until maturity and starts with a new (lower) market price when new debt is issued. Equity value also follows its normal path until it reacts to bond reissue at year 6. The increase in equity value can be observed from Figure 5.9. The reason for the slight increase in equity can be the decreased value of debt. The value of the bonds before maturity date and newly issued bonds are compared. After reissue bondholders' claims from the EBIT flows are less than their pre-maturity claims.

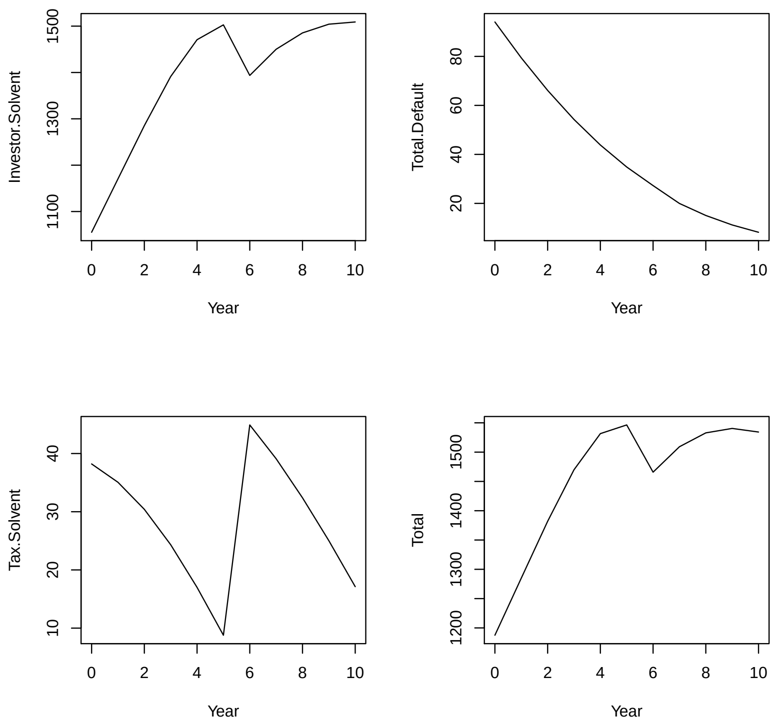


Figure 5.10: Components of Bond with Future Bond Issue

Components of debt are shown in Figure 5.10. Investor claim and Tax claim are moving as expected after the reissuance with a break at reissue date. The change in default claim is worth mentioning. Default claim is monotonically decreasing over time without being affected by the reissue or redemption of matured debt. Old debt and newly issued debt has the same principal. By definition default claim on debt (Equation 5.25) is a proportion of principal, P_j, of the bond therefore it does not affect the default claim (also, bankruptcy barrier V_B is constant).

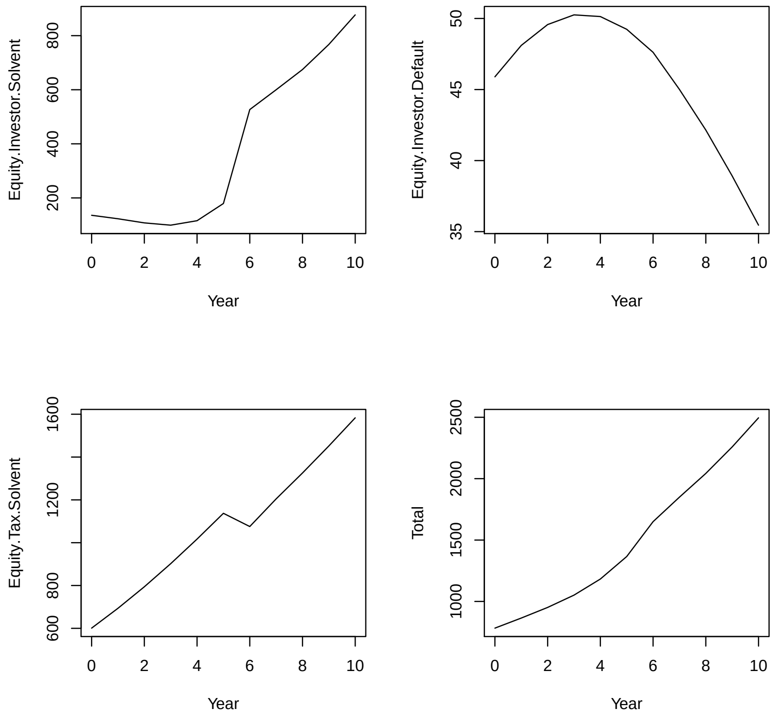


Figure 5.11: Components of Equity with Future Bond Issue

Figure 5.11 After the reissue, equity value slightly increases due to an increase in solvent value of equity. This increase is due to the decrease of debt value because of reissue. Debt claim is now replaced by equity. The default claim of equity is increasing until the maturity of first bond issue and starts decreasing after reissue.

Bankruptcy cost is decreasing without being affected by reissuance.

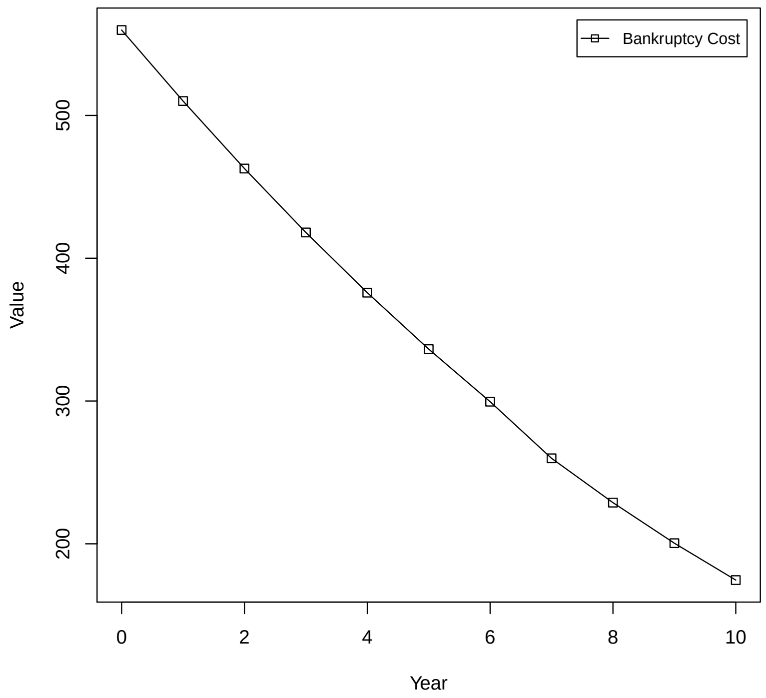


Figure 5.12: Bankruptcy Cost with Future Bond Issue

5.8 Dynamic Default Barrier

So far, the default barrier, V_B, is assumed to be constant through time. When default barrier V_B is equal to the total debt principal ΣP the default barrier changes as the outstanding debt principal changes over time. An increase in debt principal will increase the default risk as the firm's liabilities will be increased. With higher debt volume an increase in the default barrier will increase the probability of default therefore decrease the firm value. If matured debt is not replaced by new issues of debt then this will improve the credit quality of the firm and therefore increase security values. In such cases it will not be possible to distinguish the merger or reissue gains from gains caused by a change in default barrier. Therefore the analysis with dynamic default barrier is not included here. The results with dynamic barrier are displayed in Appendix B.

This chapter introduced the multi-period model and analysed its dynamics using a trinomial tree. Term structure of credit spreads were analysed using a complex capital structure. This showed how bond spreads evolve within time. The shape of yield curve is identified by the outstanding debt structure. Future bond issues increases equity value after reissue. The results obtained here are used in Chapter 6.

Chapter 6

Financial Synergies

6.1 Introduction

This section mainly focuses on financial synergies and its affects on firm value and its securities. The same trinomial tree in the previous chapter is employed to simulate the underlying process. The difference from the previous chapter is that the mergers are introduced to the model. So far all the required material has been introduced in previous chapters. Now, the mergers are introduced to the model and the analysis is concluded. Merger modelling follows the Leland (2007) model. The value of the merged company is the linear combination of two firms, given by $V_m = V_1 + V_2$ and synergies are observed when $V_m - V_1 - V_2 \neq 0$. Synergies can be positive or negative. The possibility of both cases are shown in Table 4.3. Here, the focus is on positive synergies which are generated when combining two non-perfectly correlated cash flows. Volatility of the combined cash flows is given by

$$\sigma_M(\rho) = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2\rho\sigma_1\sigma_2 w_1 w_2}, \tag{6.1}$$

where $\sigma_{1,2}$ shows the volatilities of merging firms, $w_{1,2}$ shows the weights of each firm and finally $\rho[-1, 1]$ is the correlation of the cash flows. When correlation of cash flows is equal to 1 merged volatility becomes the weighted average of separate volatilities.

Merger timing is exogenous and known by the managers, therefore they can set the maturity structure of debt accordingly.

6.1.1 Simulation of Merger

The trinomial method introduced in Chapter 5 is employed to simulate a single firm. The EBIT process is assumed to be an Arithmetic Brownian Motion defined by, $d\eta = \mu dt + \sigma dz$ where drift and volatility is constant. Merger is defined as the linear combination of single firms. Total EBIT of the merged firm is $\eta_m = \eta_1 + \eta_2$ and total firm value is $V_m = V_1 + V_2$ with the volatility given in Equation 6.1. Using a similar methodology with Leland and Skarabot (2003) the sum of firms simulated until merger. When merger occurs $V_1 + V_2$ becomes V_M and the simulation is continued with the merged firm. The drift is constant for the merged firm but the volatility changes by Equation 6.1. Therefore merger gains can be measured as in Leland (2007).

6.1.2 Base Case Scenario

For demonstration purposes the base case scenario shown in Table 6.1 is used. Unless otherwise stated the calculations in this chapter use the same parameters for the firms. For each identical firm effects of different capital structures on security values are analysed. These parameters are chosen by calibrating the dynamic model with the Leland (2007) model. A single tax rate for all claims is used, however, later this will be extended to incorporate different tax rates.

Variable	Symbol	Value
Risk-free rate	r	0.05
Corporate Tax	τ_c	0.2
Debt Investor Tax	τ_d	0.2
Equity Investor Tax	τ_e	0.2
EBIT	$EBIT$	100
Risk premium	θ	0.01
Volatility	σ	22
Loss in default	α	0.23
Growth rate	μ	1.5
Default Barrier	VB	1200

Table 6.1: Model parameters

6.2 Bond Maturity

The effects of bond maturity are analysed here. The analysis includes finite maturity bonds and perpetual bonds in the case of a merger and focuses on how they affect financial synergies.

6.2.1 Finite Maturity Bonds

Firm valuation models differ by the type of bonds they use. Some models such as Ammann and Genser (2004); Ericsson and Reneby (1998); Leland (2007) use finite maturity debt and some models like Goldstein, Ju, and Leland (2001); Merton (1974) use perpetual bonds. Use of perpetual bonds comes from mathematical convenience rather than an effort to make modeling more realistic. The model presented here supports finite and and infinite maturity bonds. This section analyses financial synergies in the presence of finite maturity bonds and the next section analyses perpetual bonds. The difference between the two scenarios will reveal the effects of bond maturity on merger gains.

Base case parameters displayed in Table 6.1 are used to calculate firm value. For demonstration purposes a simple capital structure with one class of outstanding finite maturity bond is assumed. The firm has one class of 12 years bonds given in Table 6.2 and merger will happen on the 6th year ($t = 5$). There are no bond reissues and bonds are plain bonds without any call provisions.

Issue Time	**Maturity**	**Coupon Rate**	**Principal**
0	12	0.045	1200

Table 6.2: Structure of Debt

The results of the 10 year simulation can be seen in Figure 6.1. The firm value is shown with its solvent part and insolvent part. It increases slightly after merger due to the decrease in total default risk. Merger creates a substantial decrease in insolvent value of the firm and this value decrease is offset by a slightly higher increase in solvent firm value, therefore, the total change is minimal. In this case, overall firm value does not change much, however, the merger changes the proportional value of claims.

A different approach may be to look at the solvent part of the firm alone as an insured claim on the total firm value. For example, for the equity owners this insurance is provided by the limited liability by creating an option to walk away in the case of bankruptcy. The value of this claim increases substantially after the merger.

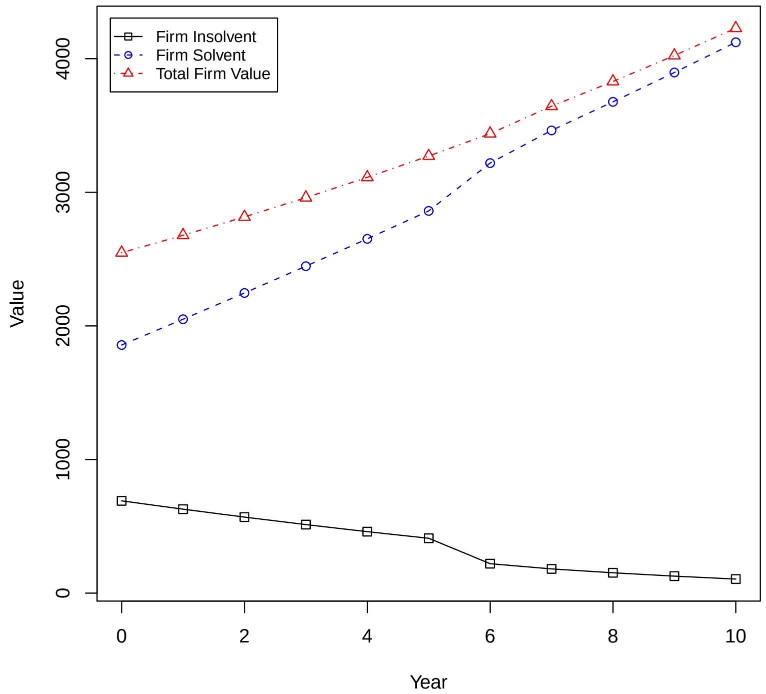


Figure 6.1: Firm Value change before and after merger with finite maturity bonds

Comparing debt and equity values will give further insight into value dynamics of claims on a firm. Figure 6.2 compares the value of equity and debt over time. Equity holders make a positive gain from this merger. The Equity value is shifted up after year 6 where the merger occurs. The value of bonds also increase after this merger reflecting gains of bondholders from the merger. In this case both claim holders made positive gains from the merger. This can be explained by two factors. Firstly, a lower leverage ratio helps equity owners gain more from mergers as their claim from total firm value will be higher. And secondly, in this case the firm does not re-structure its financing after the merger to benefit from the decrease in default cost. If the firm is allowed to restructure its debt then it will prefer to issue more debt to utilise the increased debt capacity. The new issue will have a lower credit spread than previously issued debt. The new debt will also shift down previous debts' credit spread as they will be both backed by same cash flows. If their covenants are the same then they will have the same credit quality. This way, existing bondholders will benefit from increased credit quality as their bonds will be more valuable now.

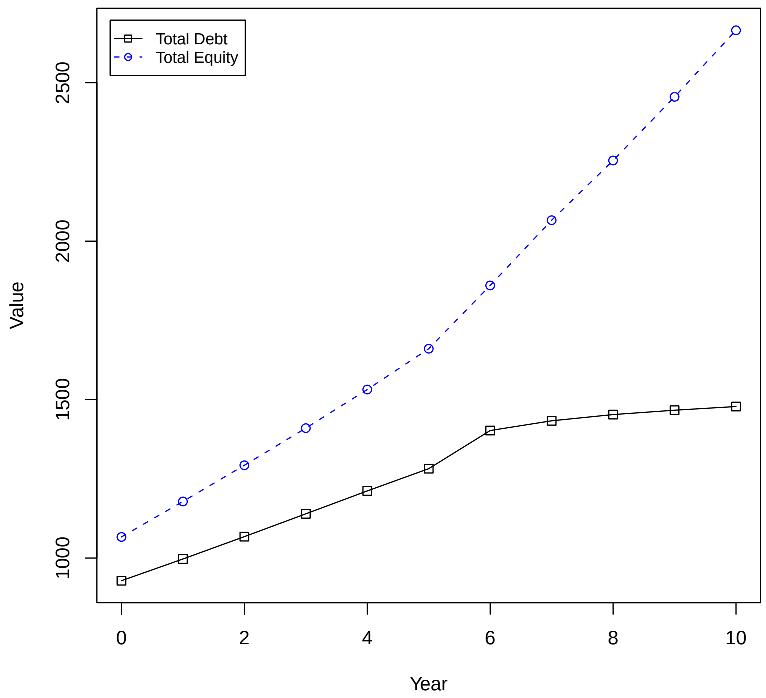


Figure 6.2: Debt and Equity value change before and after merger with finite maturity bonds

The main source of benefits from merger, for bondholders is the coinsurance effect. Merging two cash flows which are not perfectly correlated, decreases default risk of the bonds. These bonds now are backed by two different cash flows, therefore, they are now less exposed to default risk.

Looking at the components of each security value will provide information about the source of **synergies**. Bond and equity value is expressed in terms of different claims on them. Each security has solvent value, default value and tax value. The sum of these three claims gives the total value of each security. These components are shown for debt in Figure 6.4 and in Figure 6.3 for equity.

Equity value has the highest change in governments claim and secondly investors solvent claim. Increase in government claim means an increase in the amount of taxes paid to the government. This can be explained by the increased income of the equity owner.

As expected, default claim on equity decreases after the merger, with a decrease in firms default risk.

As seen in Figure 6.2 bondholders benefit from the merger and realise some of the financial synergies. Much of the increase in debt value is caused by the increase in the solvent part of the debt. Tax claim from debt decreases over time as the number of coupons decreases over time.

Figure 6.5 shows the change of bankruptcy cost over time. Bankruptcy cost decrease substantially after the merger as expected. Liabilities of the merged company are now backed by two different streams of cash flows after the merger.

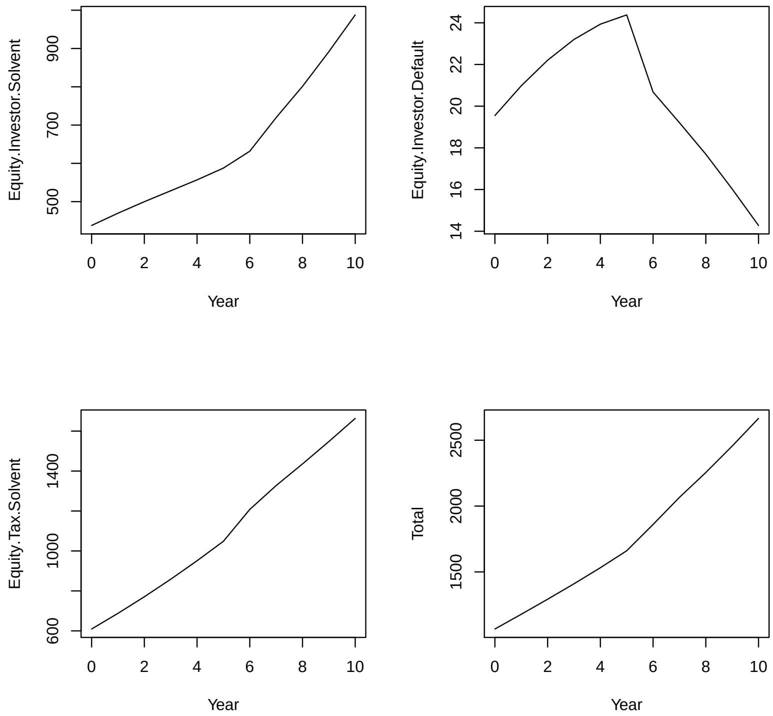


Figure 6.3: Components of Equity Value before and after merger with finite maturity bonds

6.2.2 Perpetual Bonds

The previous section analysed mergers with finite maturity bonds. This section analyses mergers of firms with outstanding perpetual bonds. A comparison of the two cases will be made at the end of this section.

Perpetual bonds are used more in theory than practice. It is not common for firms to issue perpetual bonds, although due to mathematical convenience they are often used in theoretical firm valuation models. When the bond is perpetual this means that the bond may be modelled without time dependence. The stochastic differential equation used to price the bond becomes an ordinary differential equation. Perpetual bonds are included in this analysis for two reasons. Firstly, to make the comparison of existing literature against this study model easier, and secondly,

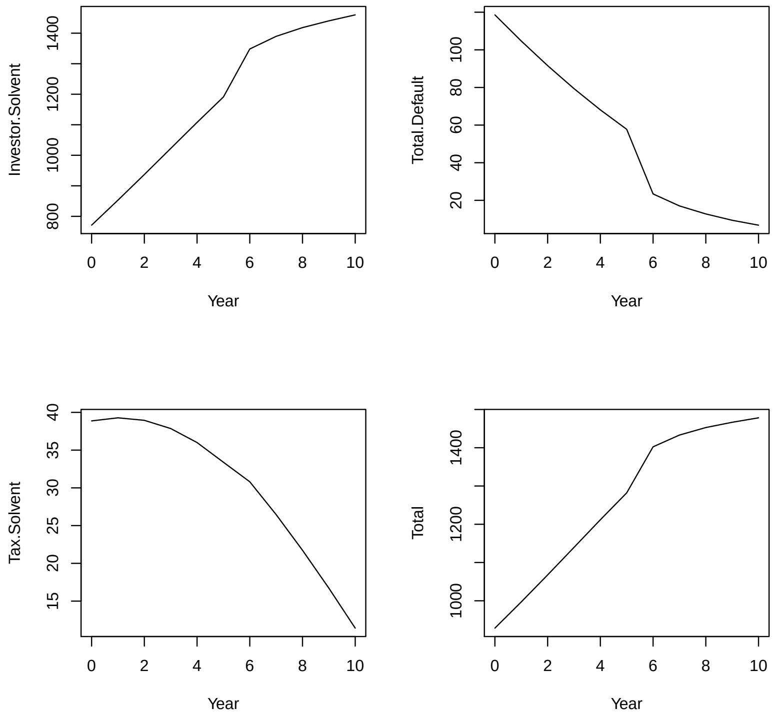


Figure 6.4: Components of Bond Value before and after merger with finite maturity bonds

although perpetual bonds are not widely used by firms, generally they are used by banks to strengthen their capital requirements [1].

A change in firm value is shown in Figure 6.6. As in the case of finite maturity, the increase in solvent firm value is offset by the decrease in insolvent value of the firm. The change in total firm value after merger is minimal, however, the proportion of claims are affected by the merger. Firm value increases with constant growth rate until the merger and the merger shifts the firm value upwards along its growth path as the risk of default is now lower.

A similar pattern may be observed by looking at debt and equity values in Figure 6.7. Debt and Equity values are shifted upwards from their normal growth paths.

[1] Perpetual debt is more like equity and because of this feature it is accepted as Tier 1 capital. Two recent large issues are: HSBC Holdings PLC issued \$3.4 billion of perpetual bonds with 8% coupon in June 2010. Credit Suisse Group AG issued \$3.5 billion of perpetual debt with 11% coupon in October 2008 (Bloomberg, 2010)

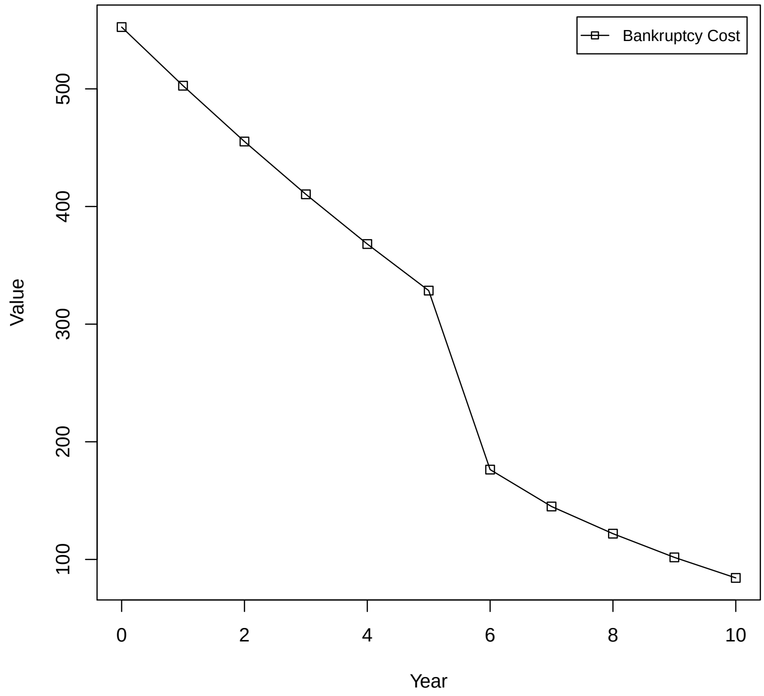


Figure 6.5: Bankruptcy Cost change before and after merger with finite maturity bonds

Both bondholders and shareholders benefit from this merger. The effects of a merger can be seen in Figures 6.9 and 6.8. Equity investors' default value is always zero with current parameter combination, which shows that all the remaining funds after default costs are received by bondholders, $\max[(1-\alpha)V_B - \Sigma P, 0]$.

Tax claims and investors' solvent claims are affected in the same way by the merger. They are both monotonically increasing with a small break at the time of merging.

6.2.3 Perpetual vs Finite Maturity

In this section two merger cases is compared explicitly. Two different merger simulations are used for analysis. One of the mergers is done when merging firms

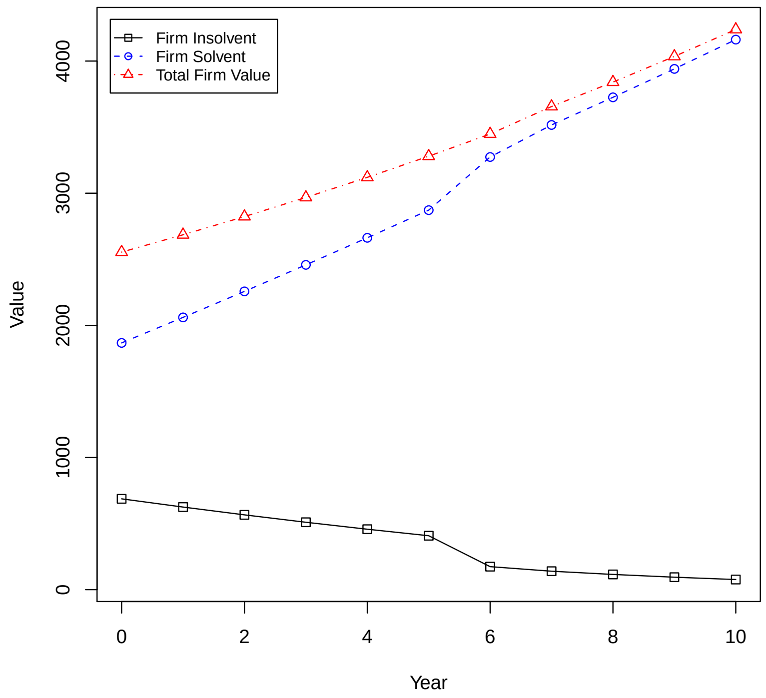


Figure 6.6: Firm Value with perpetual bonds

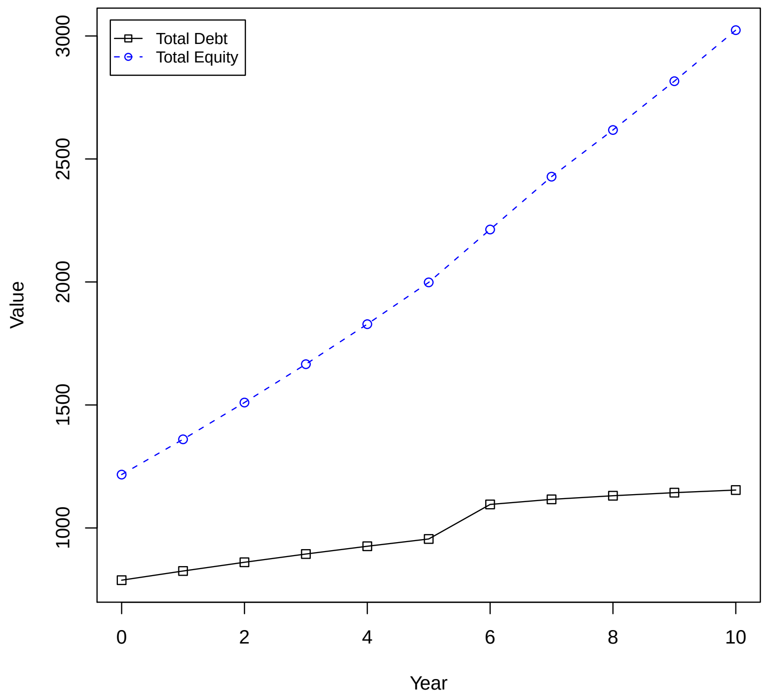


Figure 6.7: Debt and Equity Values with perpetual bonds

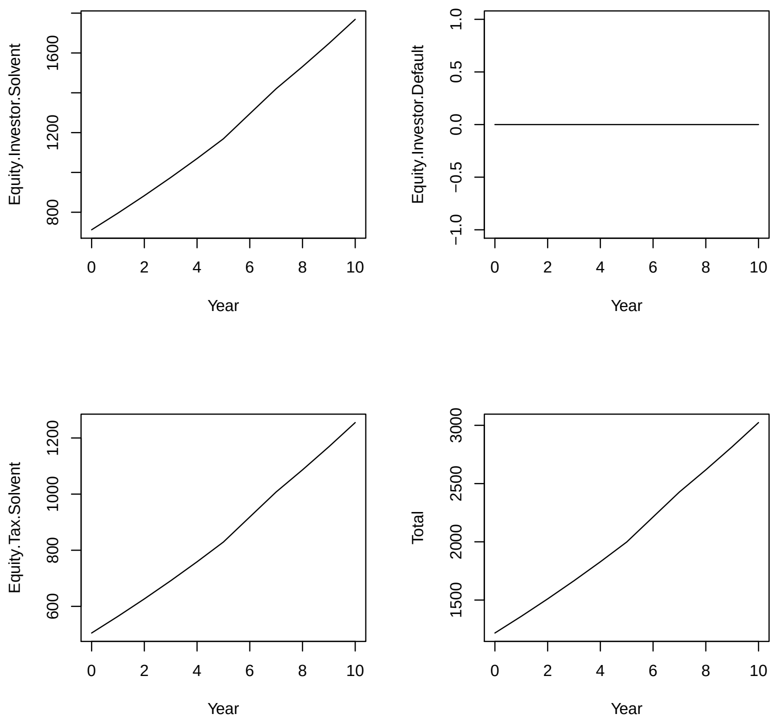


Figure 6.8: Components of Equity Value with perpetual bonds

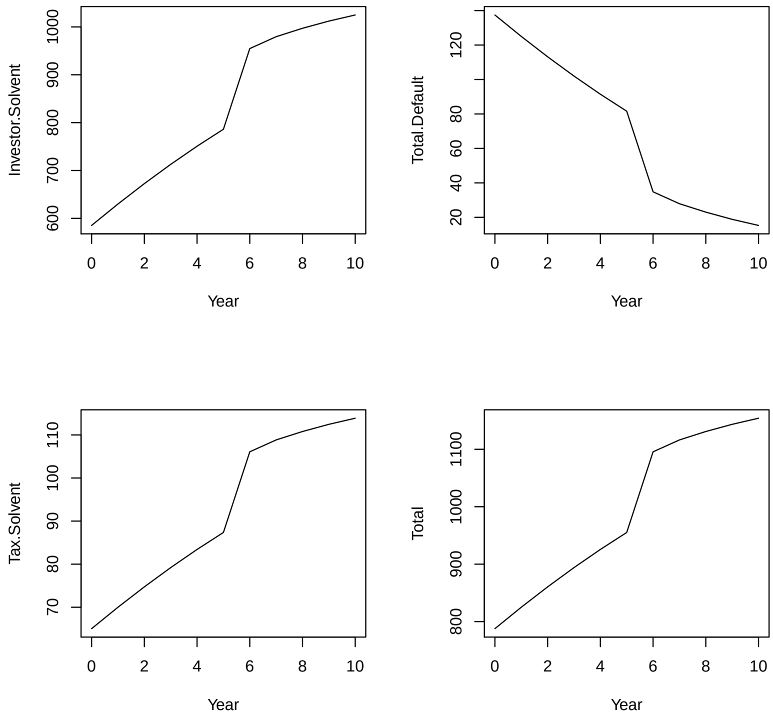


Figure 6.9: Components of Bond Value with perpetual bonds

have finite maturity bonds and the other is done when merging firms have perpetual bonds. As everything else is kept the same the difference between the results of the two mergers will reflect the effects of bond maturity. Here, the analysis covers results for one set of parameters but can easily be extended to work with any reasonable combination of parameters.

6.2.3.1 Debt Value

Merger gains are analysed by comparing the results from two simulations. Table 6.3 shows bond value changes in time when no merger takes place and Table 6.4 shows bond value changes when merger takes place at $t = 5$. When no merger takes place at time $t = 5$ yearly return is 2.87% for of perpetual bonds and 5.04% for finite maturity bonds (Table 6.3). When merger takes place (Table 6.4) perpetual bond price increases by 14.70% after merger and finite maturity bond price increases by 9.38%. In Table 6.4 from years 0 to 5 bond value increase varies between 3.2% and 4.74% due to firm value growth and the effect of time on bonds. Therefore, at least 10%-11% of the bond price increase at year 5 is due to the merger. These are the bondholders' gains from the merger.

t	Perpetual Debt	Finite Maturity Debt
0	4.71%	7.32%
1	4.27%	7.01%
2	3.88%	6.67%
3	3.51%	6.26%
4	3.18%	5.73%
5	2.87%	5.04%
6	2.58%	4.16%
7	2.32%	3.12%
8	2.07%	2.06%
9	1.85%	1.21%

Table 6.3: Perpetual and Finite Maturity Debt Value Change without merger

However after the merger with finite maturity debt the change in debt value is 9.38% and at least 3.5% of this is due to the merger. Perpetual bondholders gain more than finite maturity bondholders.

One interesting point is that after merger bond value increase slows down significantly in both cases. This can also be seen in Table 6.3 which shows bond price change over time when firms decide not to merge. The bond prices shown are the benchmark prices to identify the financial synergies. Depending on the bond maturity, debt and equity prices, both decrease. For debt, the change ranges from 4.71% to 1.85% with perpetual debt and for equity, the range is from 11.80% to 7.69%. Any shifts from these changes will be caused by mergers.

Tables 6.3 and 6.4 exhibit quite similar values up to the time of the merger. However, after the merger, debt price jumps by 14.7% for perpetual debt and 9.38% for finite maturity debt. Perpetual debt gains more than finite maturity debt. The longer term debt gains more than short term debt. Longer maturity has more coupons which are exposed to new credit spread of the firm, whereas short term debt has less exposure to new credit spread. From this result, the shorter the debt maturity the gains from the merger will be less for bondholders. Managers can balance the merger gains between claim holders by setting the debt maturity as required. If debt maturity is short enough to mature before the merger, then shareholders can benefit even more. This scenario will be analysed in next section under the subject of 'reissue'.

The slow down of debt value increase after merger comes from its design. Bond value reaches to a value that the value change caused by time becomes relatively very small.

t	Perpetual Debt	Finite Maturity Debt
0	4.74%	7.38%
1	4.30%	7.08%
2	3.90%	6.74%
3	3.53%	6.34%
4	3.20%	5.80%
5	14.70%	9.38%
6	1.89%	2.19%
7	1.31%	1.37%
8	1.11%	0.95%
9	0.93%	0.79%

Table 6.4: Bond value change before and after merger. Merger takes place at $t = 5$

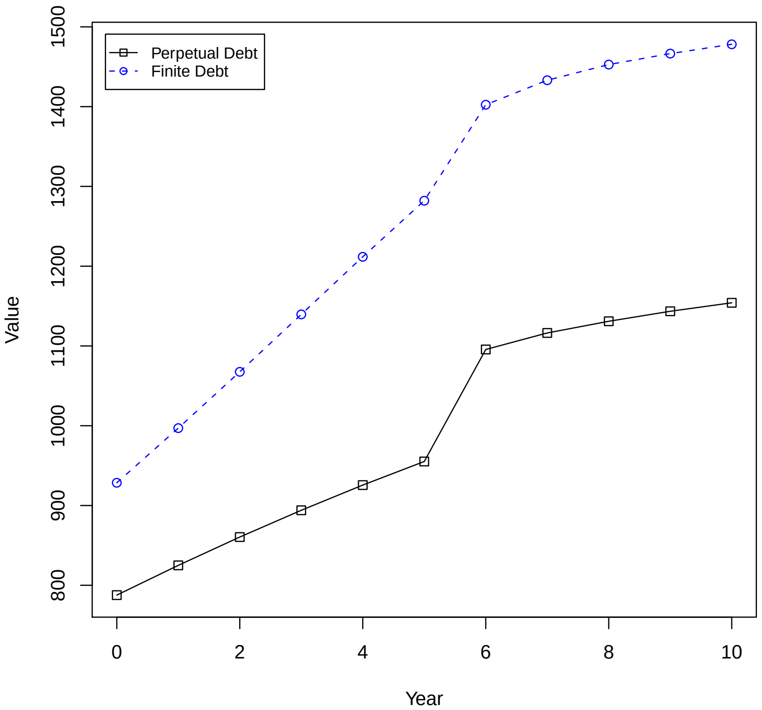


Figure 6.10: Debt Values

6.2.3.2 Equity Value

Equity value grows with the firm's growth. Table 6.5 shows equity value change with perpetual and finite maturity debt. For the perpetual case equity value change ranges from 11.8% to 7.7% and for the finite maturity case equity value change decreases from 10.5% to 9.2%. The growth rates shown here are linked with the firm's growth rate and the growth rate of liabilities. As debtholders have priority over equity therefore only the residual claim is received by shareholders. Merger case is displayed in Table 6.6. When firms merge the equity value increases by 10.76% which is 1.87% higher than the combination of separate firms. In this case the magnitude of increase is significantly lower than the increase in debt value. In this case most of the gains are realised by bondholders. With a different parameter combination (i.e. with a higher leverage ratio) shareholders gains may also be negative after the merger as shown in the merger matrix extension to one period model.

t	Equity with Perpetual Debt	Equity with Finite Maturity Debt
0	11.80%	10.49%
1	11.01%	9.69%
2	10.34%	9.08%
3	9.78%	8.66%
4	9.30%	8.43%
5	8.89%	8.40%
6	8.53%	8.55%
7	8.21%	8.82%
8	7.94%	9.09%
9	7.69%	9.16%

Table 6.5: Equity Value Change with Perpetual and Finite Maturity Debt (No Merger)

t	Equity with Perpetual Debt	Equity with Finite Maturity Debt
0	11.78%	10.50%
1	10.99%	9.69%
2	10.33%	9.07%
3	9.77%	8.64%
4	9.29%	8.41%
5	10.76%	12.02%
6	9.71%	11.07%
7	7.81%	9.12%
8	7.58%	8.91%
9	7.37%	8.56%

Table 6.6: Equity Change with merger. Merger takes place at $t = 5$

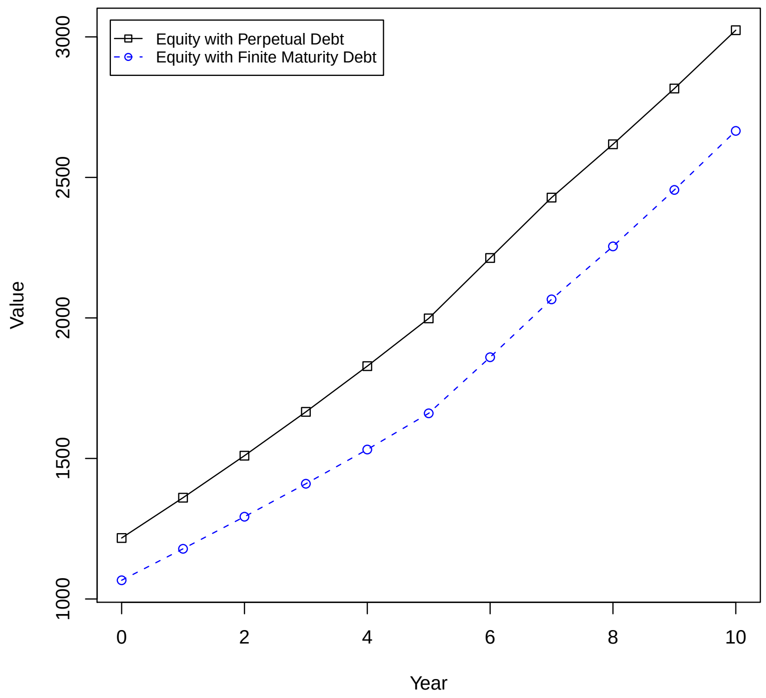


Figure 6.11: Equity Values

6.2.3.3 Sources of Merger Gains

The change in tax savings (ΔTS) and change in default cost (ΔDC) were identified as sources of merger gains. TS and DC are shown in Figure 6.12 for both cases. For the finite maturity case TS follows a decreasing trend and follows the same path, even after the merger without a change. Tax savings on a single bond is given by

$$TS = \tau^c \frac{C}{r}(1 - e^{-rT}(1 - \Phi) - p_B()). \tag{6.2}$$

This equation uses the same idea to price the finite maturity bonds and calculates finite maturity using perpetual cash flows. The principal is not tax exempt therefore it does not generate any tax savings, and hence it is not included in the equation. The number of coupons and the corporate tax rate, τ^c has the biggest impact on amount of tax savings. For finite maturity bonds, as time passes the number of coupons decreases and therefore tax savings follows a decreasing path. For perpetual bonds the term $e^{-rT}(1-\Phi)$ vanishes. As the price of Arrow-Debreu security $p_B()$ decreases tax savings increases when everything else remains constant. Tax savings from perpetual bonds change significantly after a merger, however, short term maturity bonds are not affected in the example presented. The sensitivity analysis shows that tax savings decrease by bond maturity as expected.

The credit quality of the firm improves for the current parameter set. This can also be the opposite with a different parameter combination but here it is assumed that managers act in the best interest of shareholders and are trying to increase share value. Therefore, they do not find it optimal to merge when gains are negative. Default cost decreases as credit quality improves and the opposite is true. Figure 6.12 shows default cost change for both perpetual and finite maturity cases. For both of these cases default cost decreases after merger with a slightly greater

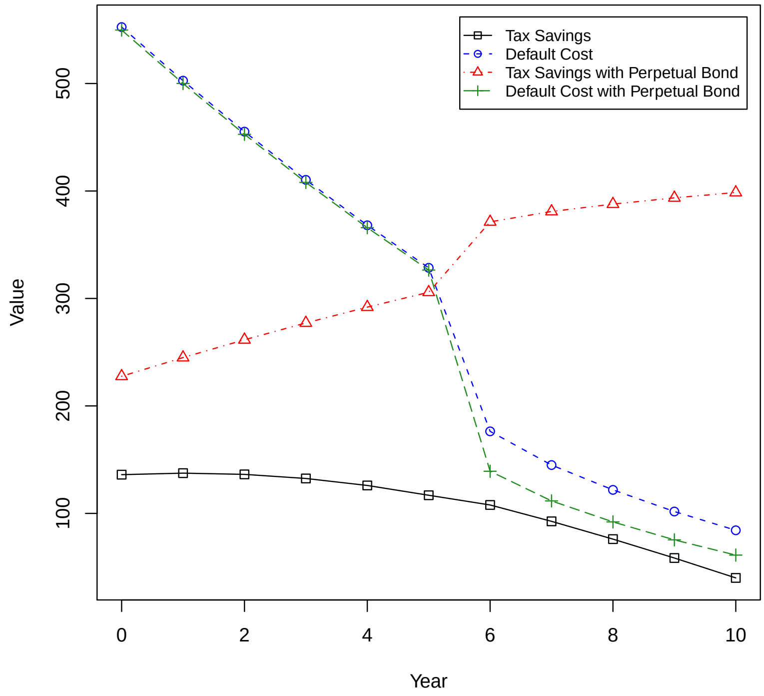


Figure 6.12: Tax Savings and Default Cost

decrease for the perpetual case. The reward for holding long maturity bonds is a greater decrease in default cost when credit quality improves, however, when the credit quality gets worse, then the increase in default cost will be higher.

6.3 Reissue

The previous section showed that it is optimal for the firm to hold shorter maturity debt rather than long maturity debt before a merger. Long maturity debt receives more of the financial synergies compared to shorter maturity debt. Here, a special case is analysed in which debt matures just before merger or the firm decides to merge just after its short term bond matures. This way the firm can keep all/most of the financial synergies as there will be no/less debt outstanding. After the merger, the firm can reissue the matured debt with new improved credit quality and can benefit from increased debt capacity.

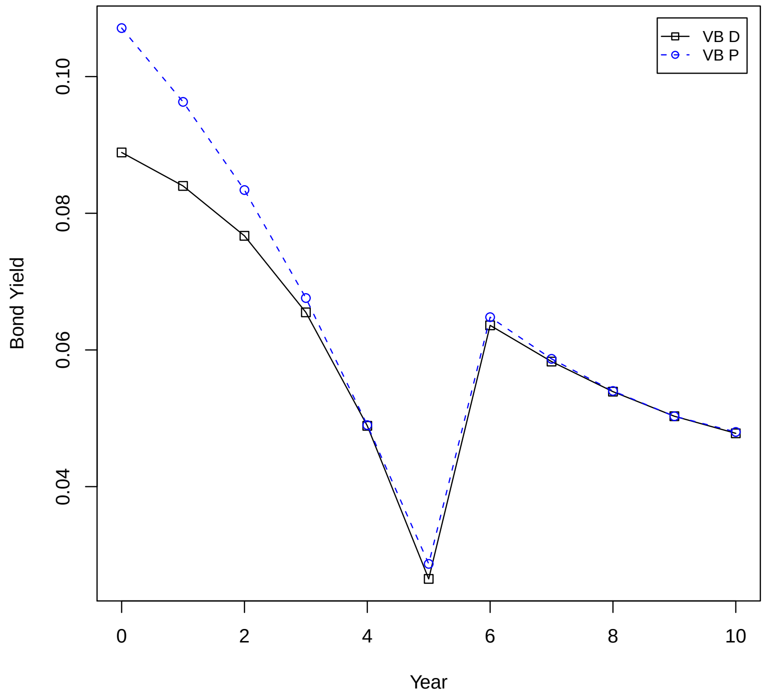


Figure 6.13: Bond Yield

Bond reissuance can be used as a mechanism to transfer financial synergies from bondholders to shareholders. This can be done by merging after the short term bonds' maturity or after calling the bonds back. After completing the merger shareholders will keep the financial synergy gains and then they can issue the same

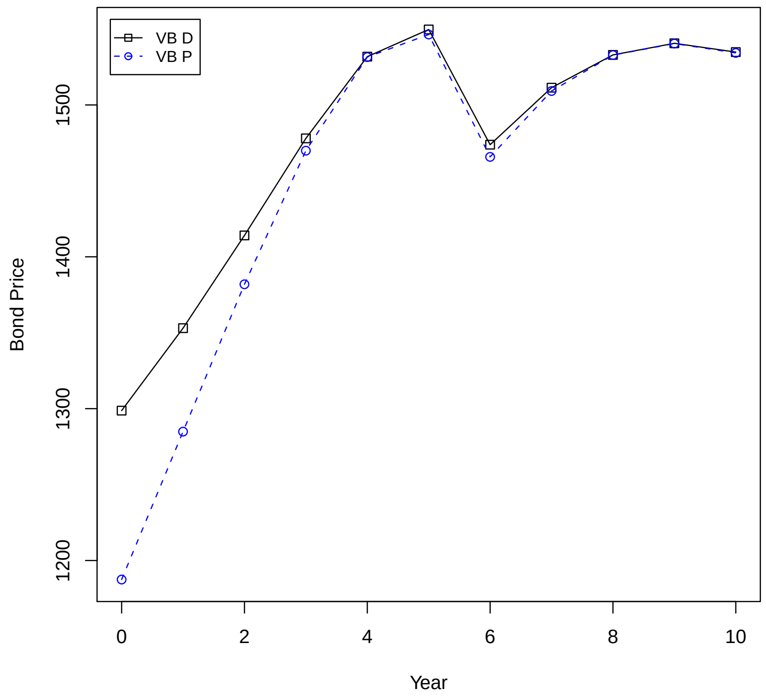


Figure 6.14: Bond Price

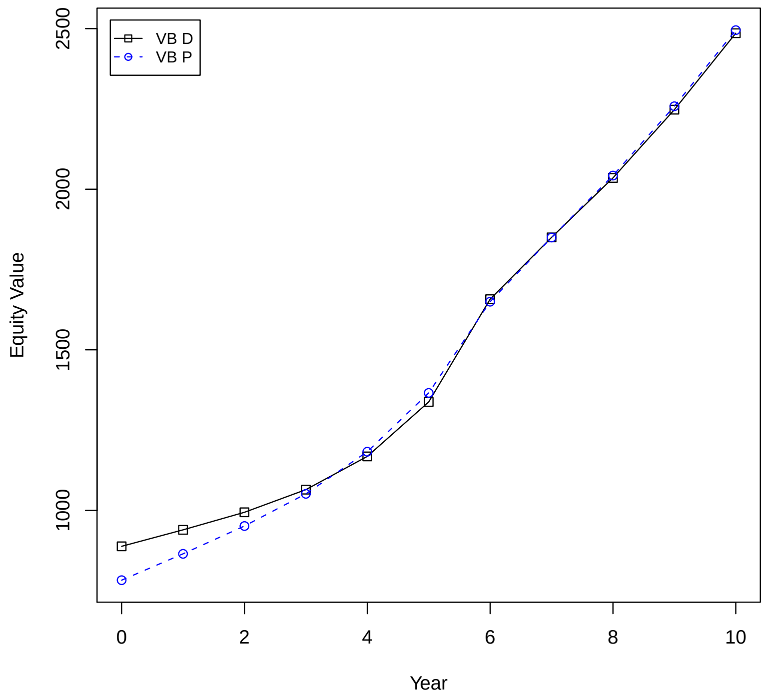


Figure 6.15: Equity Value

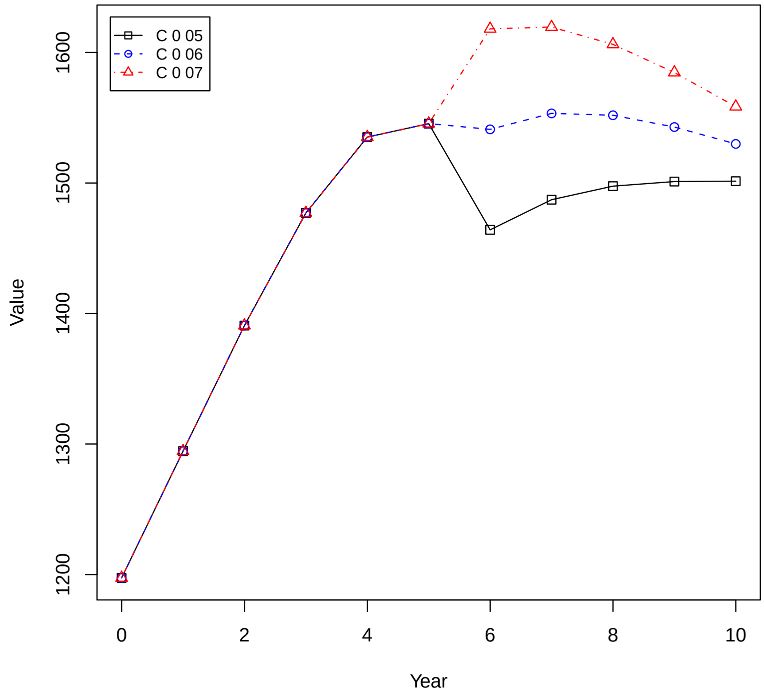


Figure 6.16: Debt Value

amount of bonds as before the merger. This mechanism is demonstrated in this section and results are analysed in a multi-period setting.

Instead of finding an optimal coupon for the debt issue three scenarios are used in which the merged firm finds it optimal to reissue the matured bond with higher, lower and the same coupon rates. Reissuing the bonds with the same coupon rate will prevent the new bondholders enjoy the new credit spread of the firm by keeping bond price at almost the same levels as with matured debt. A lower coupon rate will decrease and higher coupon rate will increase the bond value. Reissuing the bond with a higher coupon rate would not be optimal for the firm after a merger with positive financial synergies.

Figure 6.16 shows debt reissue with three different coupon rates. Short term debt has a 6% coupon rate for all three cases and after merger the bond is reissued with

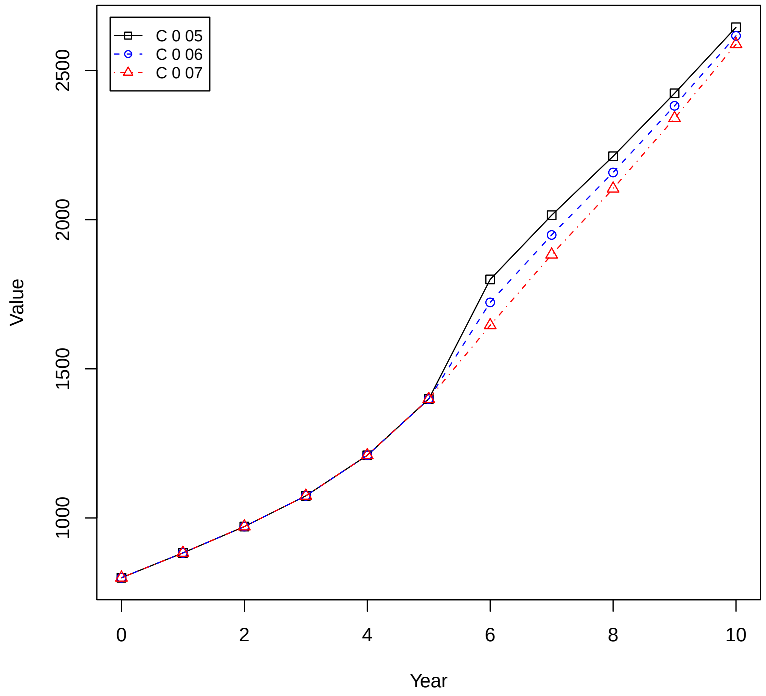


Figure 6.17: Equity Value

5%, 6% and 7% rates. The same or a lower coupon rate keeps or adds to merger gains where a higher coupon rate will transfer wealth.

Equity value increases as the coupon rate decreases. This is shown in Figure 6.17. Lower coupon payments leave more funds in the firm which may be claimed by shareholders.

Table 6.18 shows sum of debt and equity value for different coupon rates. Coupon rate doesn't affect the sum of debt and equity as the bonds with same covenants except their coupon rates will have the same credit spread therefore the three lines on Table 6.18 are identical. This can also be observed by looking at the yield spreads of the three scenarios. Table 6.19 shows the yield spreads of the three different bonds issued by the firm. Coupon rate does not change the yield spread of the bonds as they are backed by same cash flows and therefore their credit risk are identical.

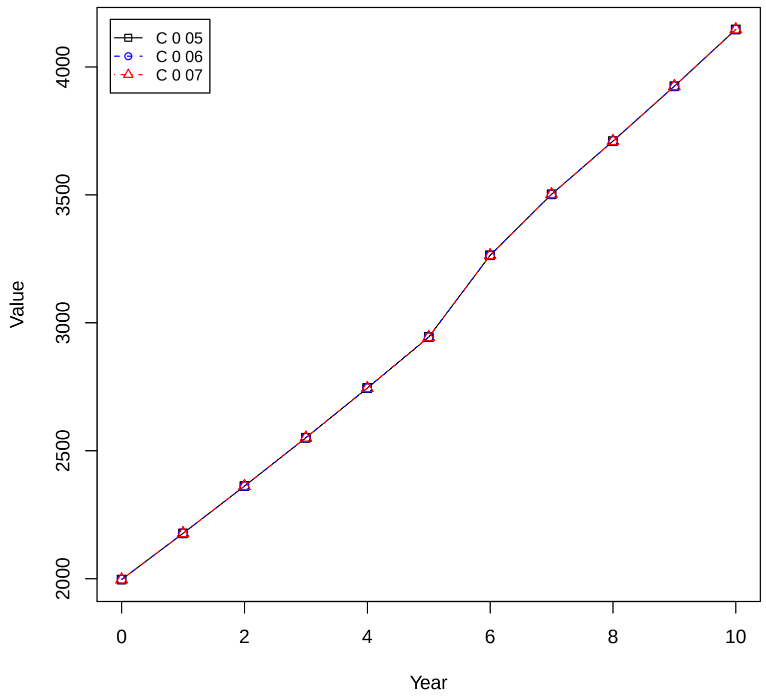


Figure 6.18: Debt + Equity Value

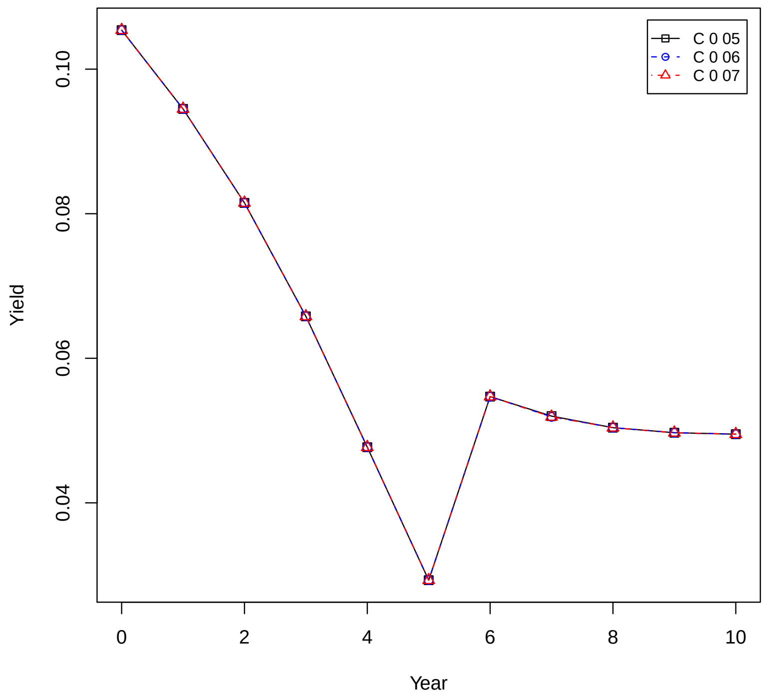


Figure 6.19: Yield Spread

When coupon rate changes it changes the bond price as it changes the present value of coupon payments. Each bond with similar covenants and maturity will have the same yield spread. The yield spread reflects the credit quality of the issuer and is not related to the coupon rate of the bond as all payments for a given period of time are backed by the same cash flows and exposed to the same default risks.

6.3.1 Timing

Managers can maximise firm value by setting the right time for the bond reissue. This can be achieved by either setting the reissue time or setting the merger timing. However, merger timing cannot be easily set by the managers as merging with another firm is a long and hard process compared to setting reissue timing.

Merger timing is exogenous to the model. Using an externally identified merger time the effects of merger timing and reissue timing to firms' securities are analysed. The timing of merger or reissuance is important in the sense that it affects the value of outstanding securities. Merger timing is kept the same and reissue timing is changed to create different scenarios. These scenarios are; reissue at merger time, reissue before merger and reissue after merger.

Figure 6.20 shows bond yields for different reissue scenarios. Bonds are matured and reissued at year 6 and merger takes place at years 4 to 8 for scenarios M4 to M8. MNO is the case where no merger takes place and is therefore the benchmark scenario. First case to consider is M8 at which merger takes place at year 8 and bonds are reissued at year 6. The bond yield is falling until maturity of first issue at year 6 reaching to 0 at maturity.

New bonds are issued at year 6 with a yield of 6.4% at year 6. After the merger at year 8, bond yield drops to 5% which means a price increase in bonds therefore

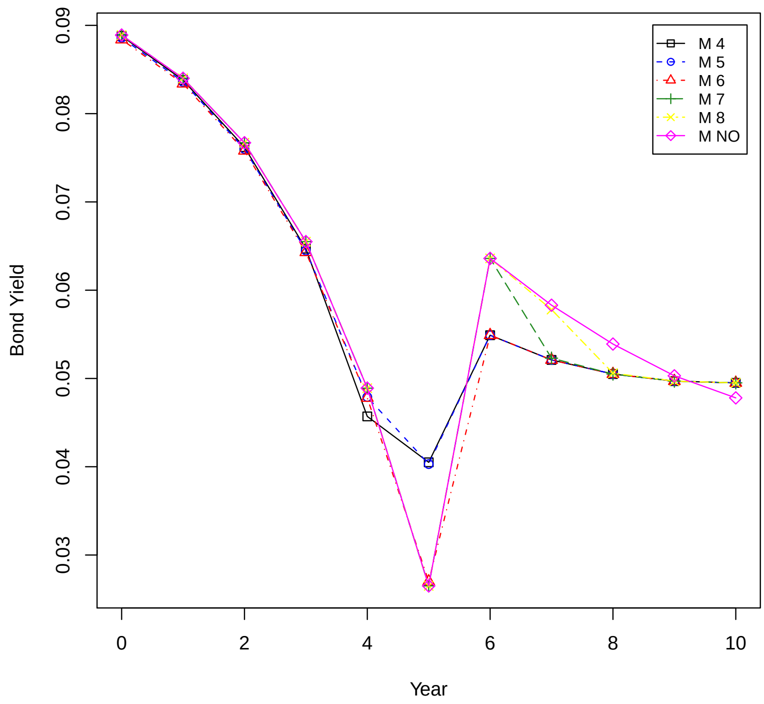


Figure 6.20: Bond Yield

bondholders are benefiting from the merger. A similar pattern is observed for the scenarios M6 and M7. For the scenarios M6, M7 and M8 the difference is the timing of yield decrease caused by timing of the merger.

M5 and M4 are different in the sense that bond yields are affected differently. In these two cases merger increases the bond yields. Actually, the yield is decreasing over time, however, it is higher than the benchmark case MNO. If the company has not merged at all bonds would have lower yields. As the credit quality improves through the merger, the company moves away from default implying a lower default probability for the firm. Lower default probability decreases the value of the default claim and increases the solvent claim on bonds. For scenarios M4 and M5 the net changes in solvent and insolvent claims are negative causing a value decrease. Solvent claim does not increase more than the value decrease in solvent claim. This can be explained by the number of coupons left to benefit from an improved credit quality.

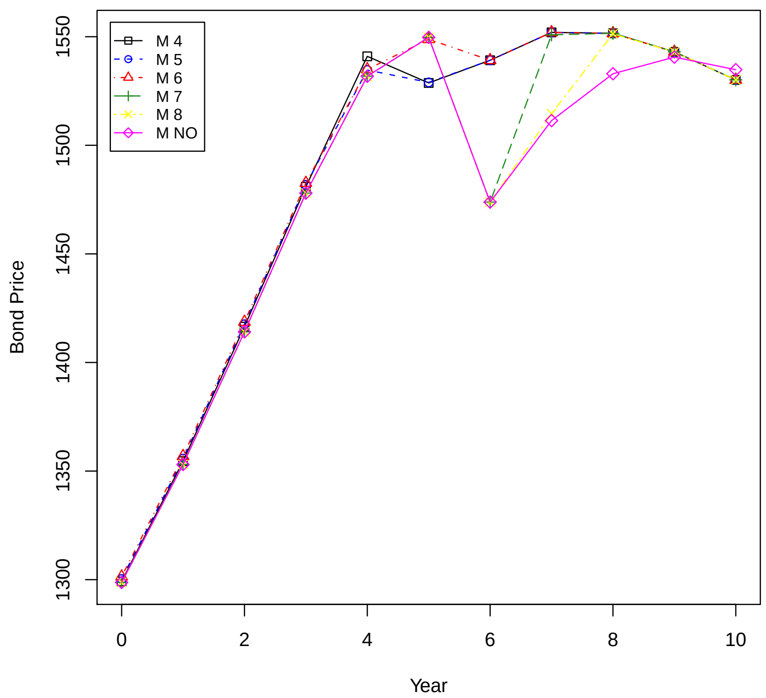


Figure 6.21: Bond Price

Managers may prevent a wealth transfer to bondholders by merging the firms at or just before maturity of bonds. Shorter term bonds benefit less from mergers, and in some cases short term bonds affected negatively by a synergistic merger. Bond prices for the scenarios are given in Figure 6.21. Bond prices increase after merger for scenarios M6, M7 and M8. For the M4 and M5 cases bond price decreases consistently with the yield increase.

Equity value is shown for each scenario in Figure 6.22. The MNO case is again the benchmark case as no merger takes place. The break between the 5th and 6th years is caused by the bond reissue taking place at year 6. Reissuing bonds positively affects the equity price. There are two reasons for this. First, the firm at year 6 is in a better condition compared to firm at year 0. At year 6 firm value is

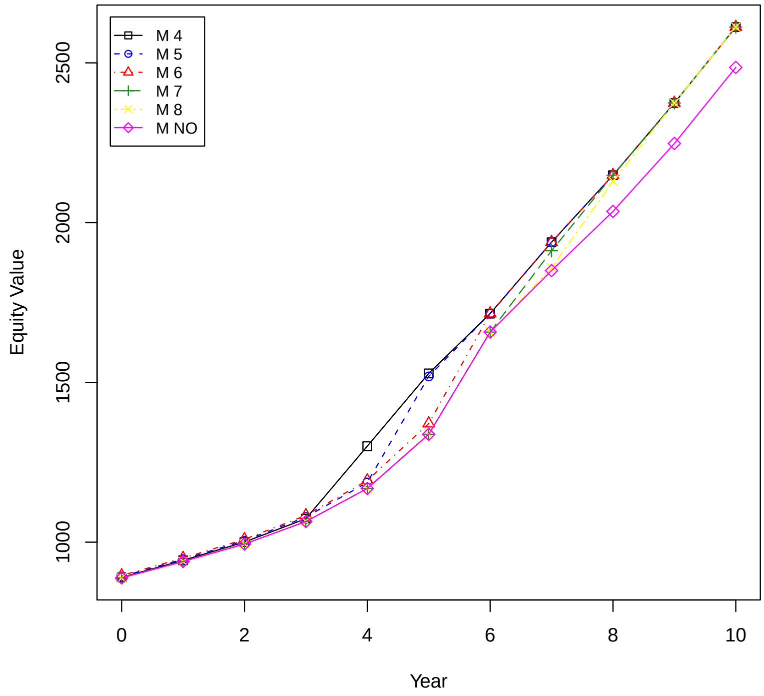


Figure 6.22: Equity Value

higher due to growth rate of the cash flows, therefore, has a lower default probability. Second, reissued bonds have lower value compared to matured bonds, leaving more residual claim for the equity.

Scenarios M4 to M8 give a higher equity value for years 4 to 10. The highest equity value is achieved by M4 and the lowest value is achieved by MNO scenarios. The difference between scenarios M4 to M8 comes from the timing of change in equity value. Other than the merger time, all scenarios except MNO reach to the same equity price. Two price paths can be defined for the equity price. One path is the lower path (runway) where no merger takes place which is demonstrated by MNO and the other is the higher path (flight route[2]) which is demonstrated by M4 case. Taking off from the runway is identified by the time of merger as it causes a jump from runway to flight route. M4 reaches to the flight route earlier than other

[2]This does not mean that the flight route is a better or desired condition for the firm or claim holders. This analogy is only used to define two different paths that equity follows.

scenarios and delaying the merger only delays the timing of a jump to the flight route.

This chapter looked at the financial synergies and distribution of them in case of a merger. The results show that shareholders of firms with short maturity bonds will make gains from mergers. As the bond maturity goes shorter the gains of shareholders increase. In case of the perpetual debt most of the gains are realised by bondholders of the firm.

Chapter 7

Conclusion

7.1 Introduction

This is the first study to analyse the financial synergies of mergers in such detail. Ignoring operational synergies reveals the purely financial motives of mergers. This does not mean that operational synergies are not important. Most of the time they are the main motive for mergers. However, studying purely financial synergies will help financial engineers/managers to design new financial products/strategies in the future. A good example to this is securitization which is a purely financial transaction. The motive behind securitization is to create financial synergies through spin-offs which are exactly reverse of a merger transaction.

7.2 Summary of Findings

A merger matrix, showing the merger of firms with different credit ratings is constructed. Credit ratings are chosen because it is a standardized measure of credit quality. If credit ratings were not used, then someone wanting to use the merger matrix should first set the parameters for the firms they are interested in and then make their calculations for their parameter combinations. It is not an easy task to estimate the volatility of cash flows of a firm. However, when using the credit ratings, for rated companies, one should only need to learn the credit rating of the firms.

Using the merger matrix the financial gains for claim holders and the firm are revealed. The results are in line with the results of Higgins and Schall (1975); Leland (2007). Findings show that bondholders gain more than shareholders of the firm. This may cause an agency conflict between managers and owners (shareholders). To increase the gains of shareholders an extension is proposed. Plain bonds can be replaced by callable bonds, therefore the firm may call the bonds back before the merger to prevent bondholders' wealth increasing.

Replacing plain bonds with bonds that have call provisions improves the gains of shareholders. The firm value also increases more after merger when callable bonds are used. The bonds are not explicitly called but the call provision embedded in the bonds affects the price of bonds such that the shareholders' gains increase.

Option adjusting is introduced to find the yield of the bonds. These yields are lower than the plain bond yields. This is due to the fact that call provision creates additional cash flows for the firm and therefore bonds become less risky. The firm pays the option premium upfront while issuing the bonds. Therefore it does not create an extra risk hence, having the option to call the bonds provides potential profits for the firm.

Credit default swaps are used to check the consistency of the model and to see how a credit derivative behaves in a merger. This analysis also helps to create some strategies to increase shareholder gains from mergers. CDS contracts can be used by the investors to exploit the merger event. The mergers will be reflected on the CDS spreads as the firms' credit quality shifts from one state to another.

Without explicitly solving a new solution for the one period model was proposed. The current one period model can be modified to use an Arithmetic Brownian Motion process. The main aim of this extension is to demonstrate the differences between the multi period model and the one period model used in this thesis.

After analysing the mergers using a one period model with closed form solutions a multi-period framework is modelled. This new model can handle complex capital structures and multiple tax rates. The multi-period model uses a more realistic process for the EBIT flows. EBIT flows follow an Arithmetic Brownian Motion process, which allows negative future EBIT values. Using ABM, all of the equations for the static form of the multi-period model has closed form solutions. However, numerical methods were employed to simulate the firm. Simulation allows addition of a dynamic default barrier and bond reissues.

Short term bonds can be used to increase shareholder gains from mergers. Merger is optimal for shareholders either after the short term bonds mature or when there is a very short time until maturity. In other cases most of the gains from mergers are received by bondholders. The leverage ratio is important as well. When the leverage ratio is low in most cases most of the gains goes to the shareholders because the distress created by debt is relatively minimal and this does not change the credit risk of the existing bonds sufficiently enough.

Merger timing is also analysed. But instead of trying to find an optimal time for the merger, the focus was on finding an optimal time to reissue debt. It is easier for firms to set the reissue time rather than setting the merger time. Merger timing is identified after long negotiations between two firms and is less flexible than setting bond reissue timing.

7.3 Weakness of the Research

Transaction costs were not included in the model. These costs will only decrease the financial synergies realised by the claim holders. The model can be extended to incorporate the transaction costs. The presence of transaction costs can be modelled as an increase in tax rates for every claim holder. As long as each claim holder is affected by the same amount, the analysis remains same. However, there will be multiple costs to include into the model. Transaction costs of mergers, bankruptcy, bond issue and calling the bonds. These costs will all be paid from earnings of the firm before paying the interest and taxes. Therefore the presence of these costs will only decrease the total amount of gains but will leave the distribution of the gains the same.

7.4 Future Work

This study can be extended to include transaction costs. The presence of transaction costs will make some of the mergers suboptimal. This way the model may become more realistic.

The model can be extended to use empirical data. Solving the model using observed debt and equity values of a firm will give an estimation of volatility or other parameters. These estimated parameters can be used to model the mergers.

Appendix A

Sensitivity Analysis for One Period Model

A.1 Sensitivity Analysis for Extension 3

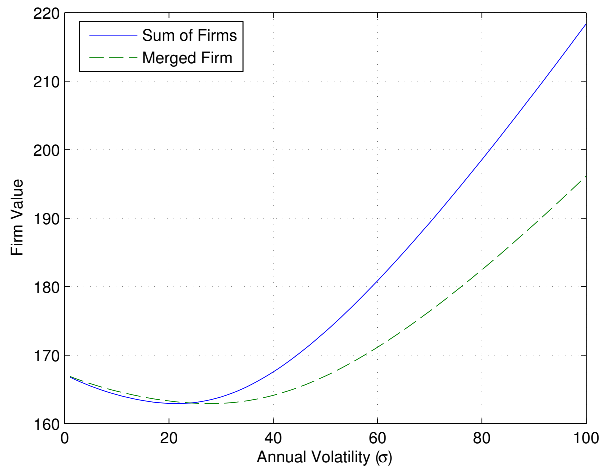


Figure A.1: Firm Value

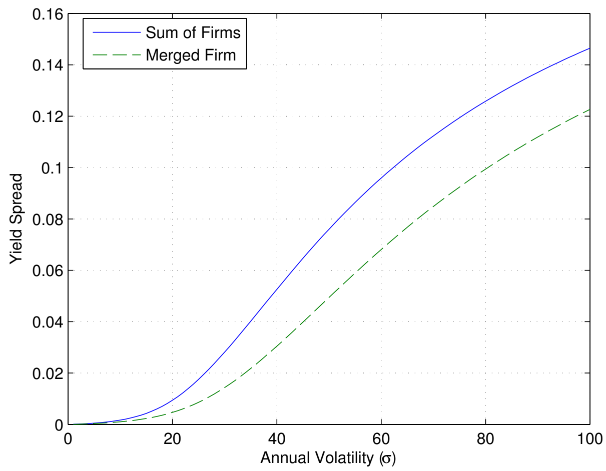


Figure A.2: Yield Spread

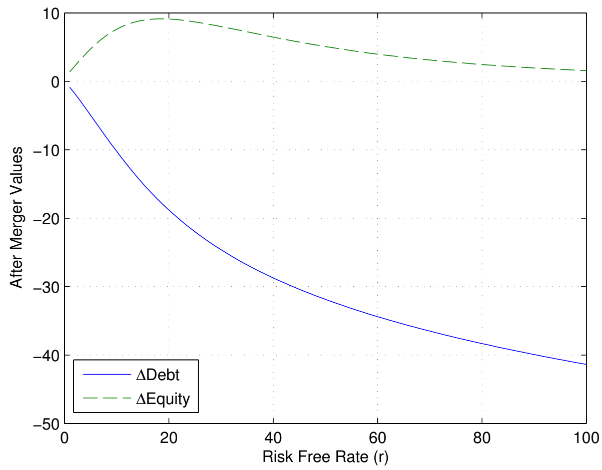


Figure A.3: After merger values

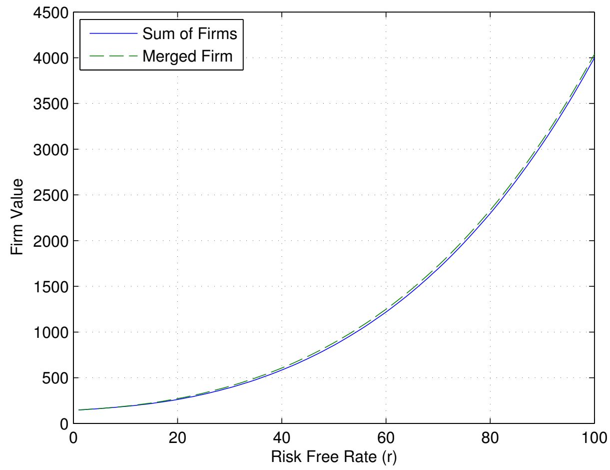


Figure A.4: Firm Value

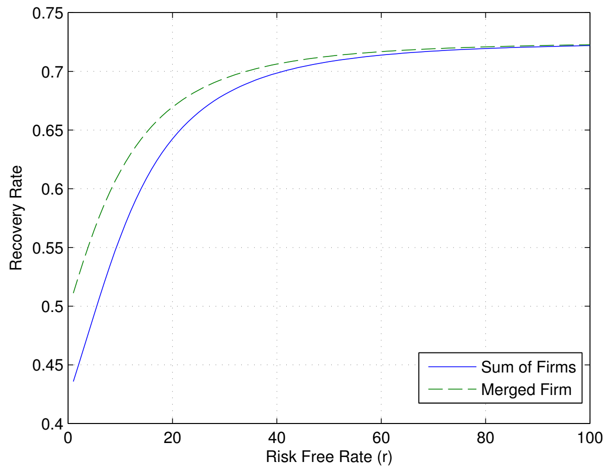


Figure A.5: Recovery Rate

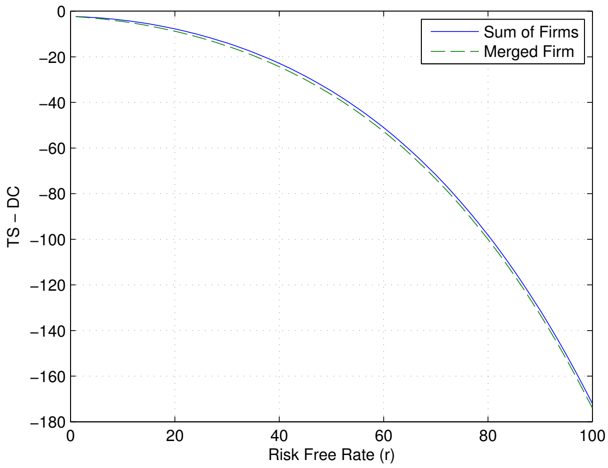


Figure A.6: TS-DC

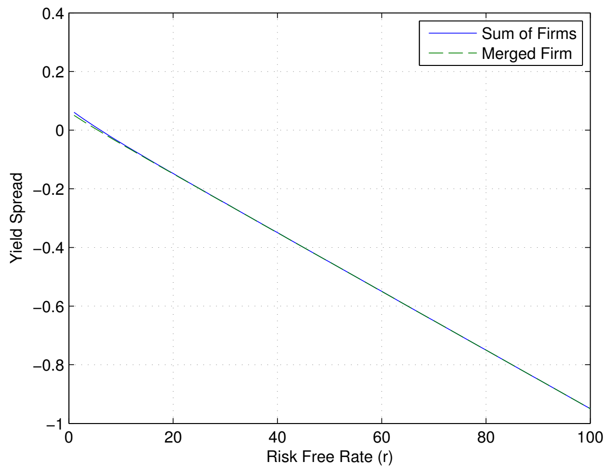


Figure A.7: Yield Spread

Appendix B

Sensitivity Analysis for Multi-Period Model

B.1 Correlation

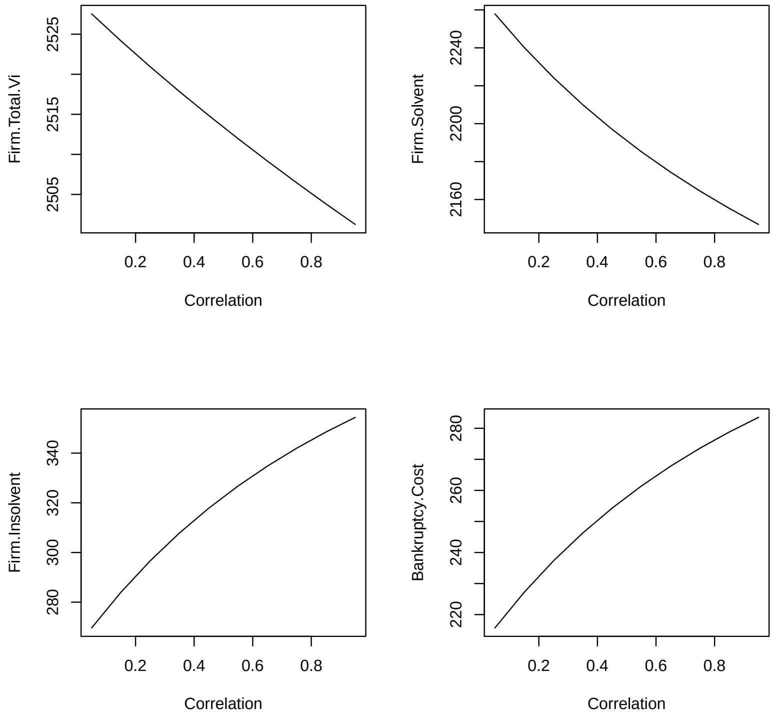


Figure B.1: Firm Value as a Function of Correlation

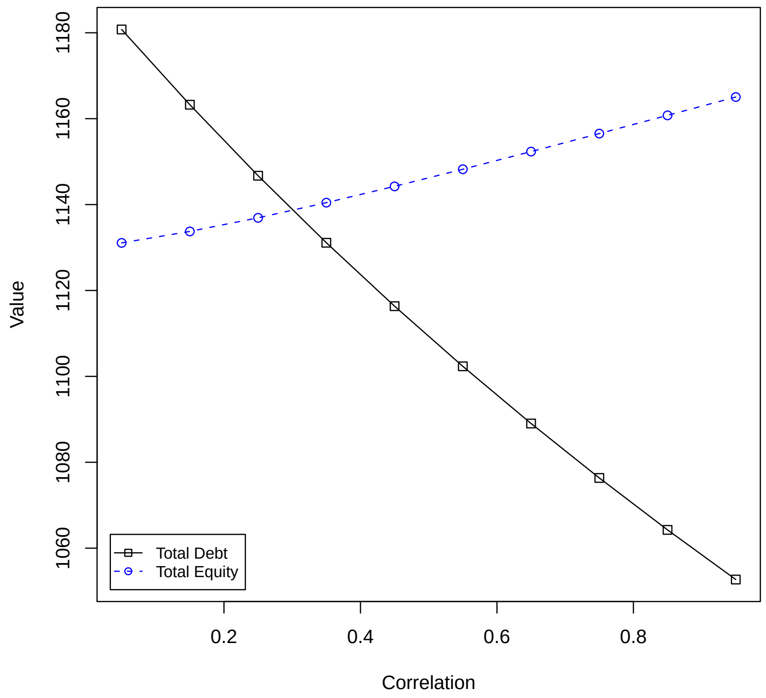


Figure B.2: Debt and Equity Values as a Function of Correlation

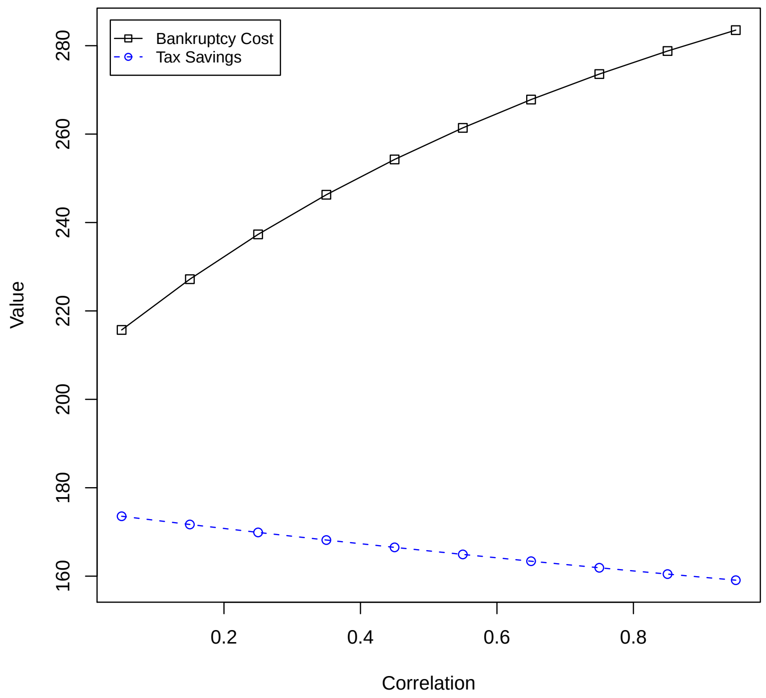


Figure B.3: Benefits of Merger (TS,BC) as a Function of Correlation

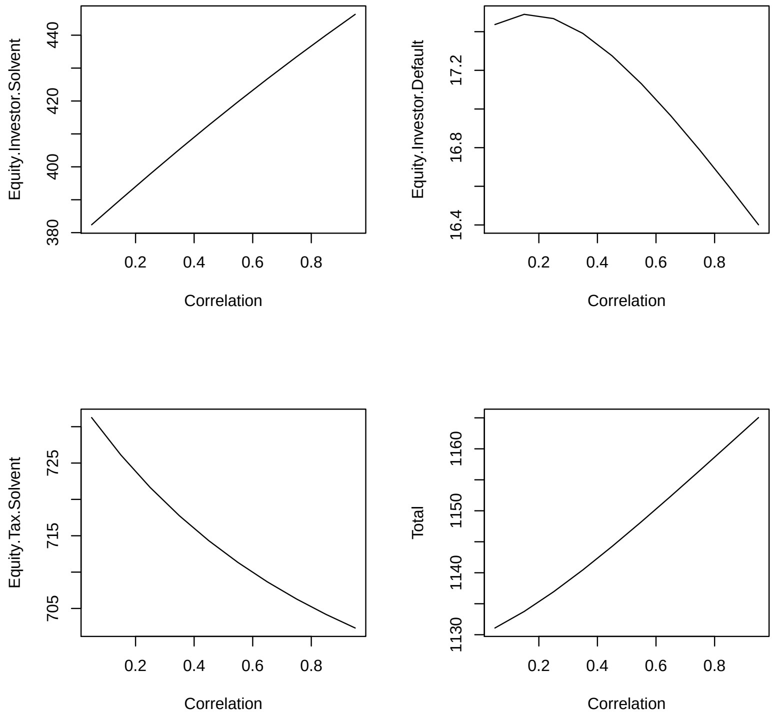


Figure B.4: Components of Equity as a Function of Correlation

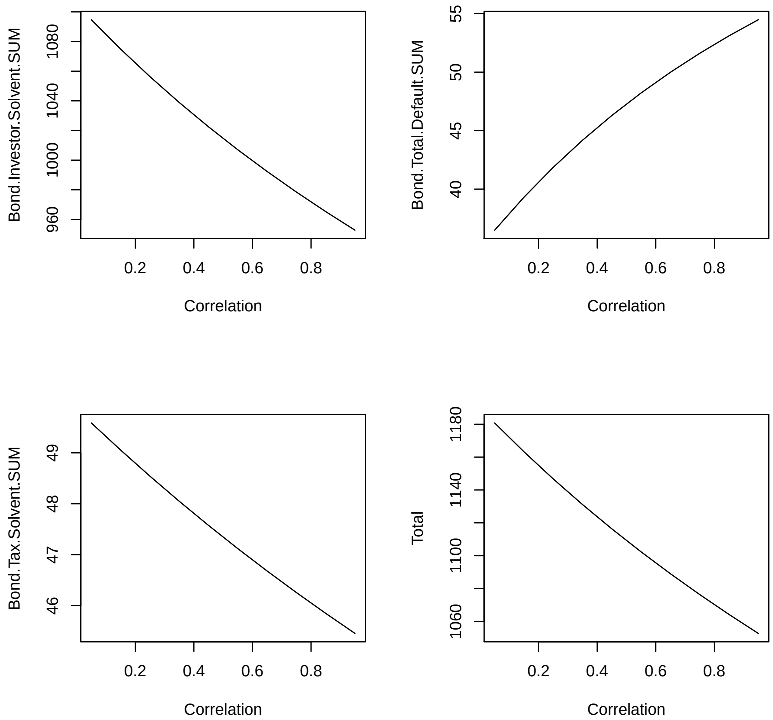


Figure B.5: Components of Debt as a Function of Correlation

B.2 Interest Rate

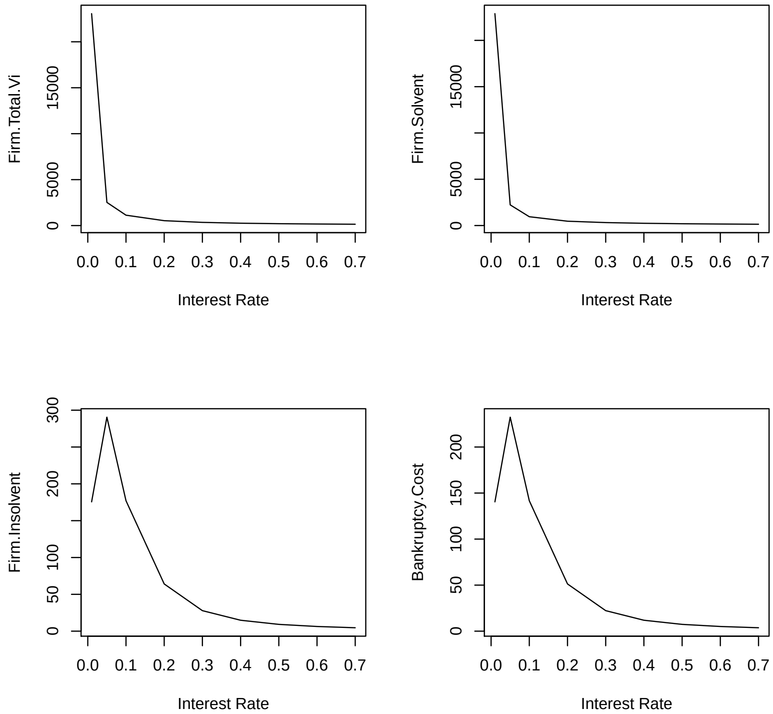


Figure B.6: Firm Value as a Function of Interest Rate

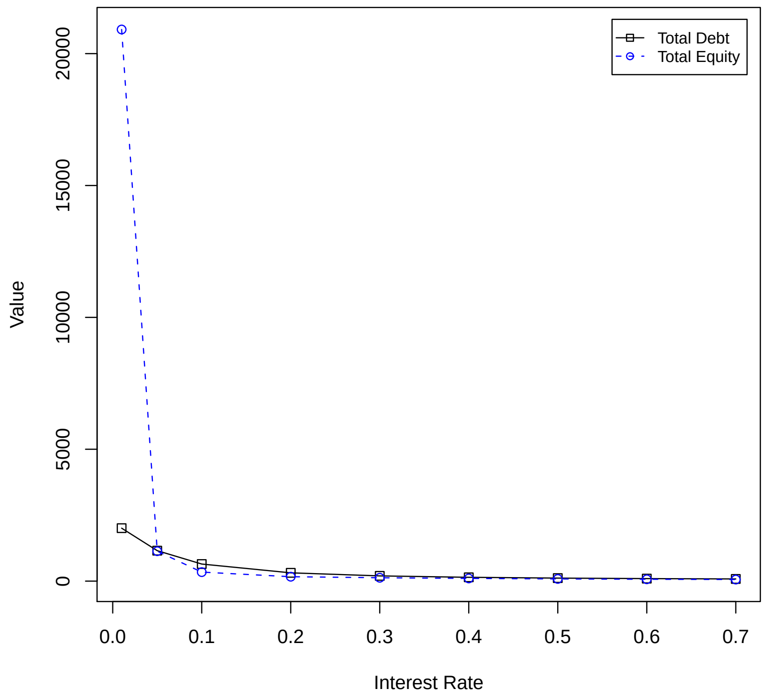


Figure B.7: Debt and Equity Values as a Function of Interest Rate

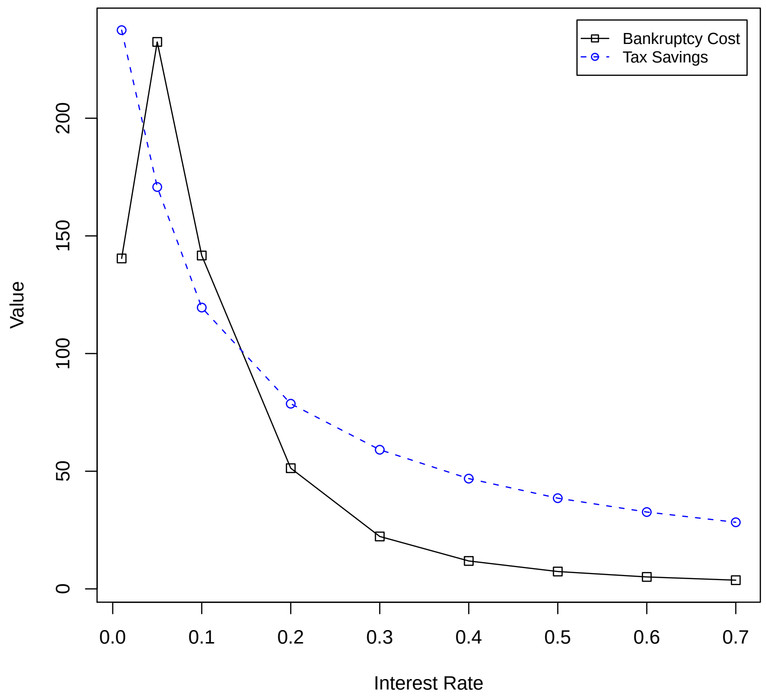


Figure B.8: Benefits of Merger (TS,BC) as a Function of Interest Rate

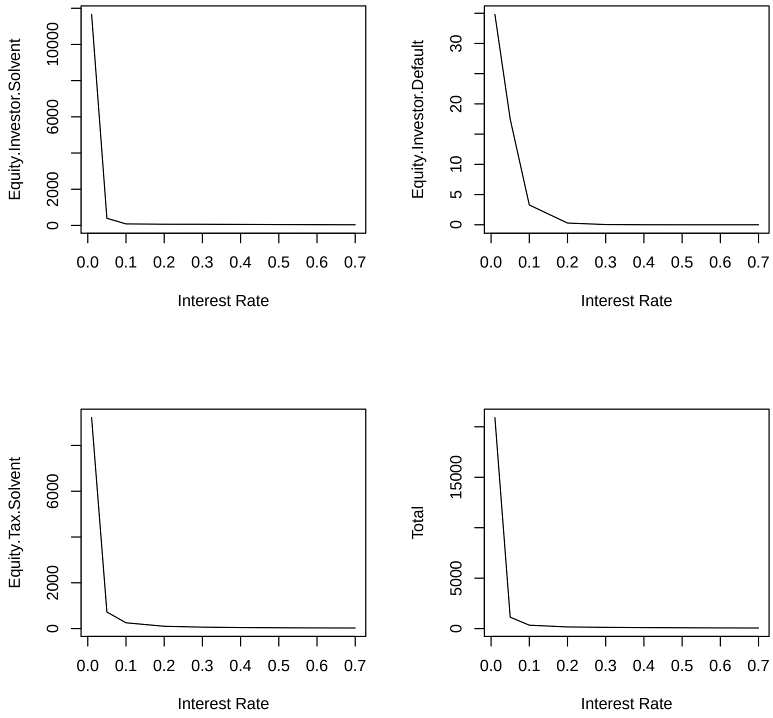


Figure B.9: Components of Equity as a Function of Interest Rate

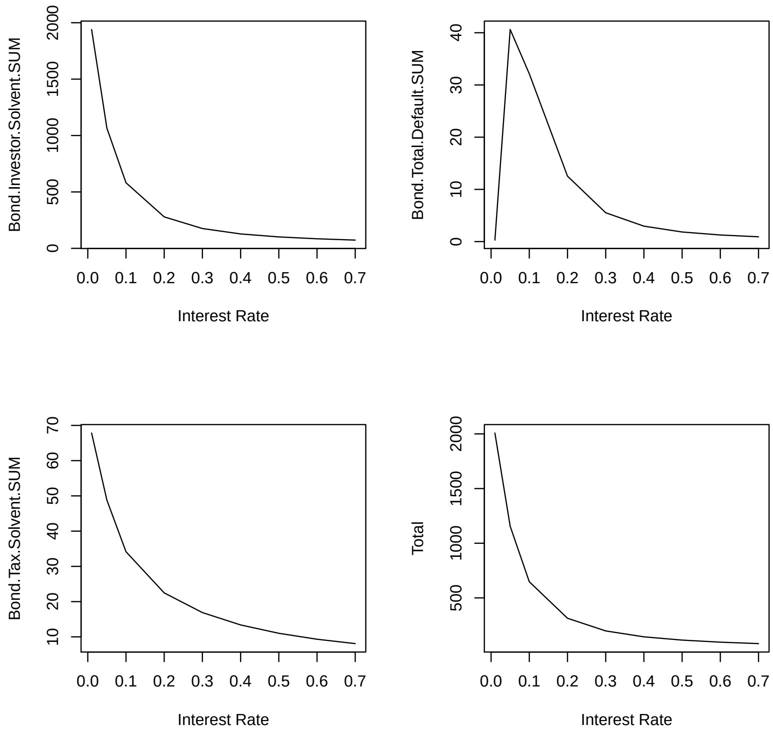


Figure B.10: Components of Debt as a Function of Interest Rate

B.3 Alpha

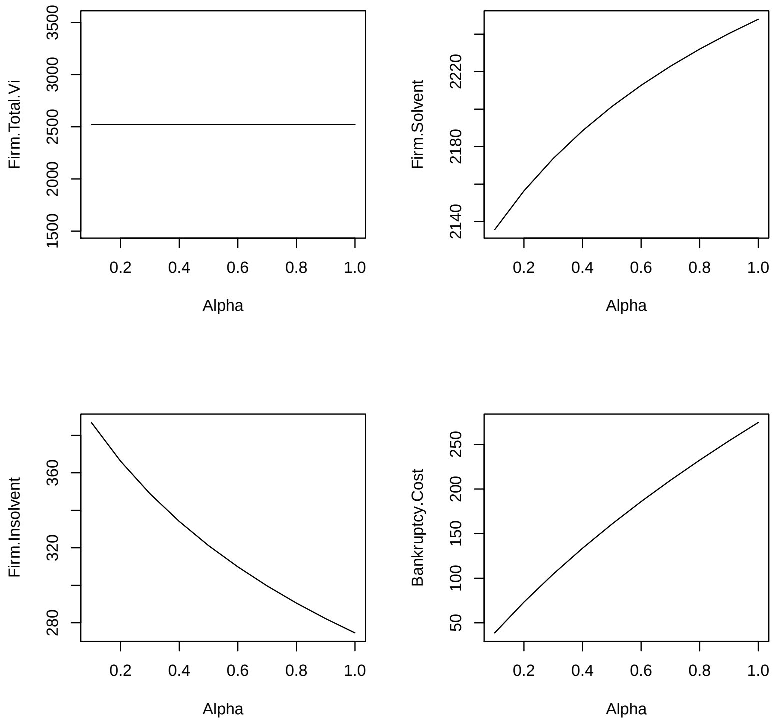


Figure B.11: Firm Value as a Function of Alpha

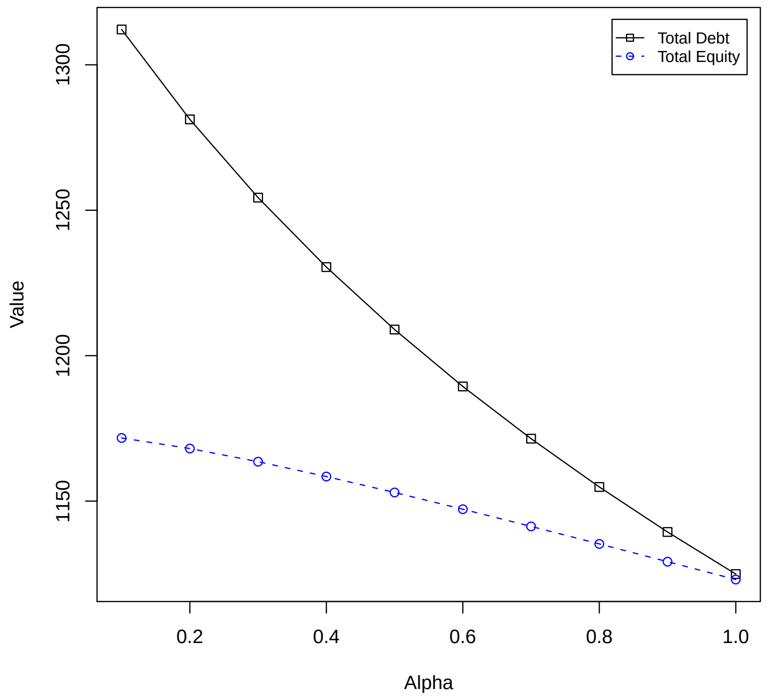


Figure B.12: Debt and Equity Values as a Function of Alpha

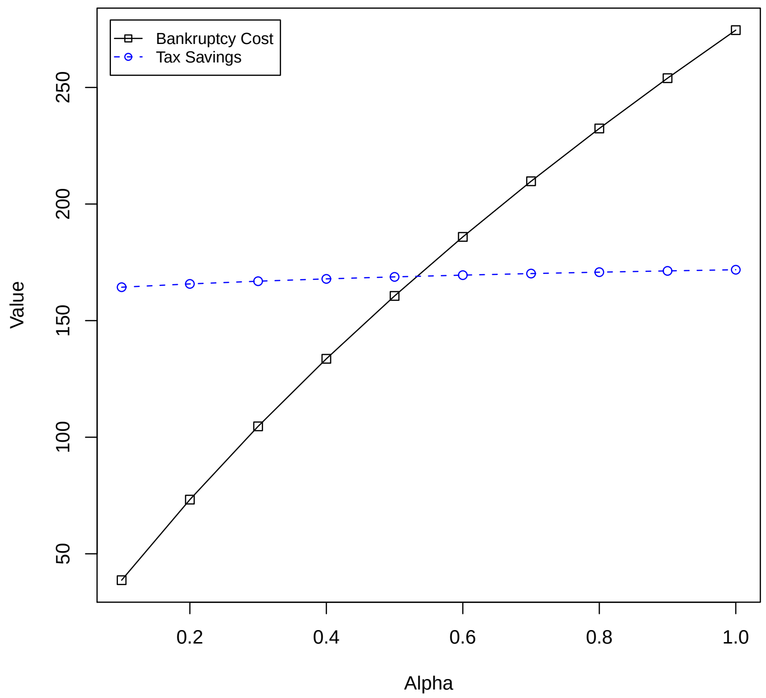


Figure B.13: Benefits of Merger (TS,BC) as a Function of Alpha

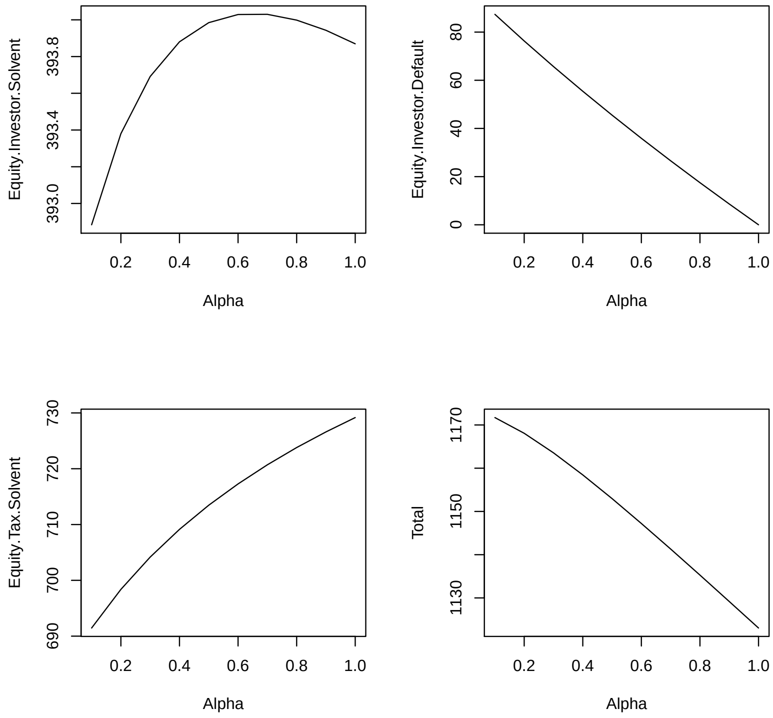


Figure B.14: Components of Equity as a Function of Alpha

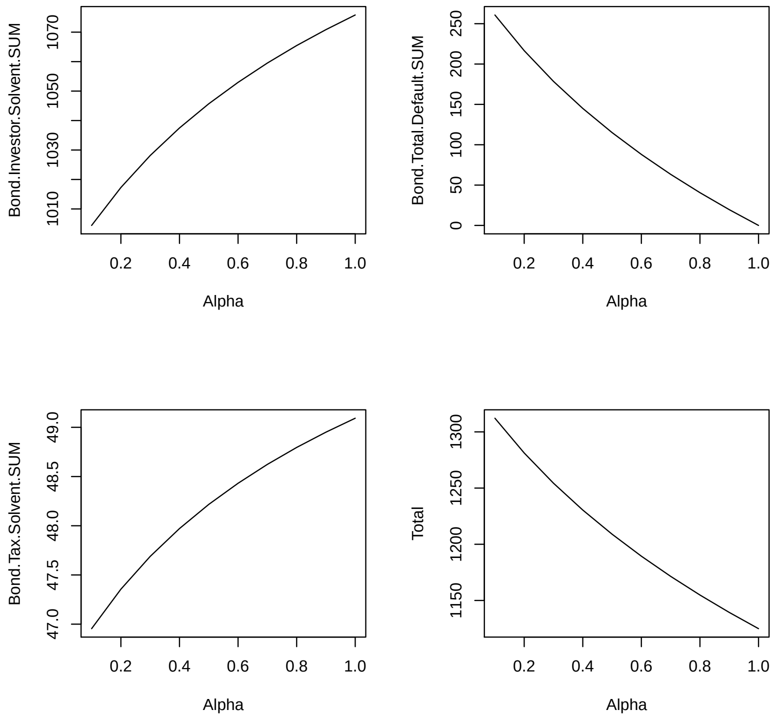


Figure B.15: Components of Debt as a Function of Alpha

B.4 Mu

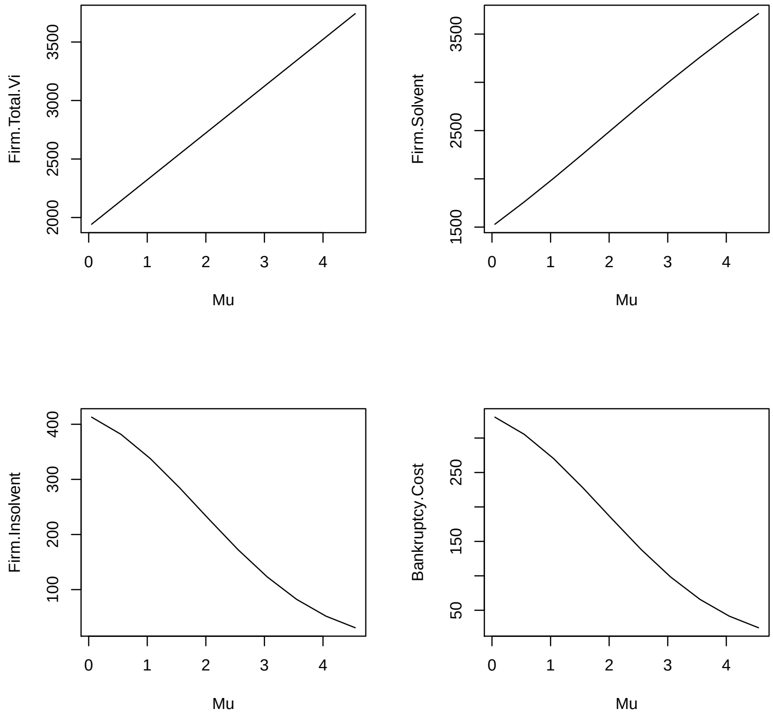


Figure B.16: Firm Value as a Function of Mu

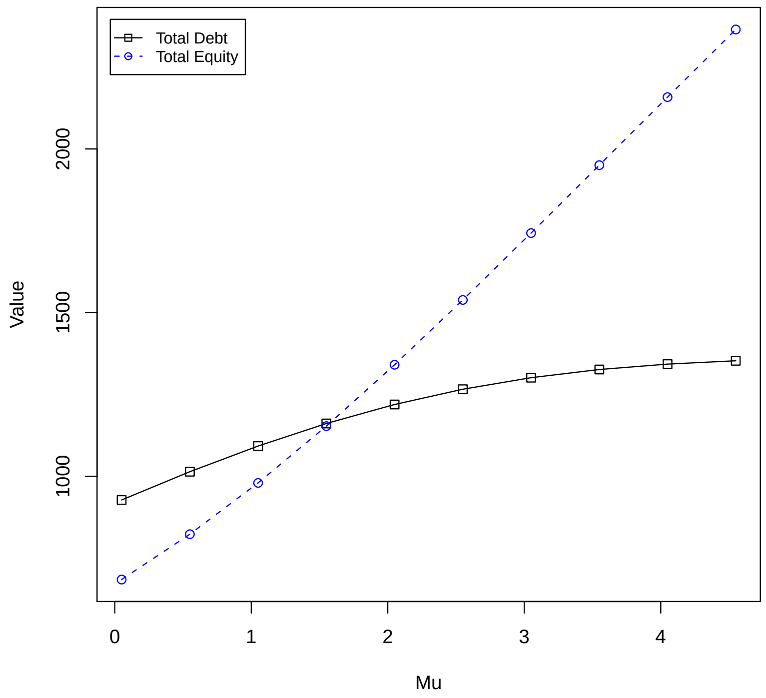


Figure B.17: Debt and Equity Values as a Function of Mu

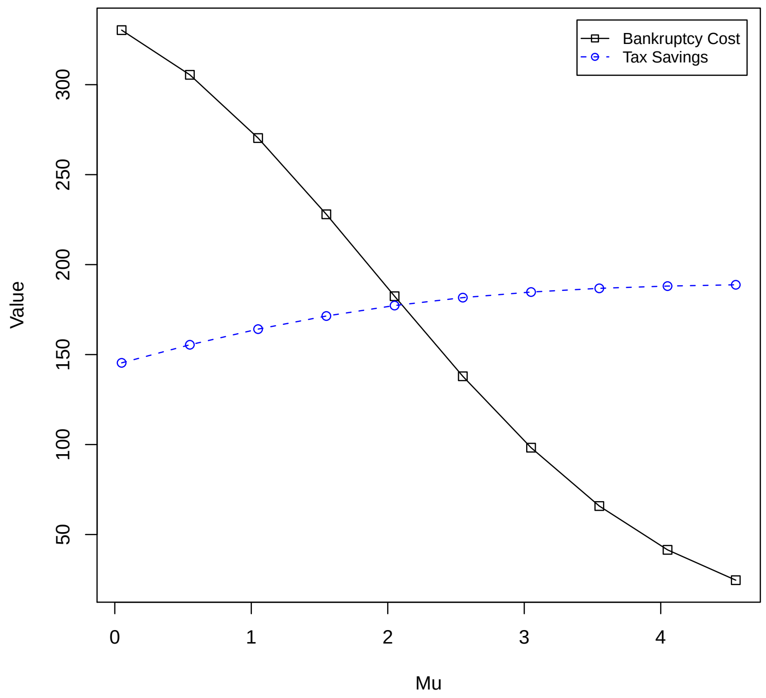


Figure B.18: Benefits of Merger (TS,BC) as a Function of Mu

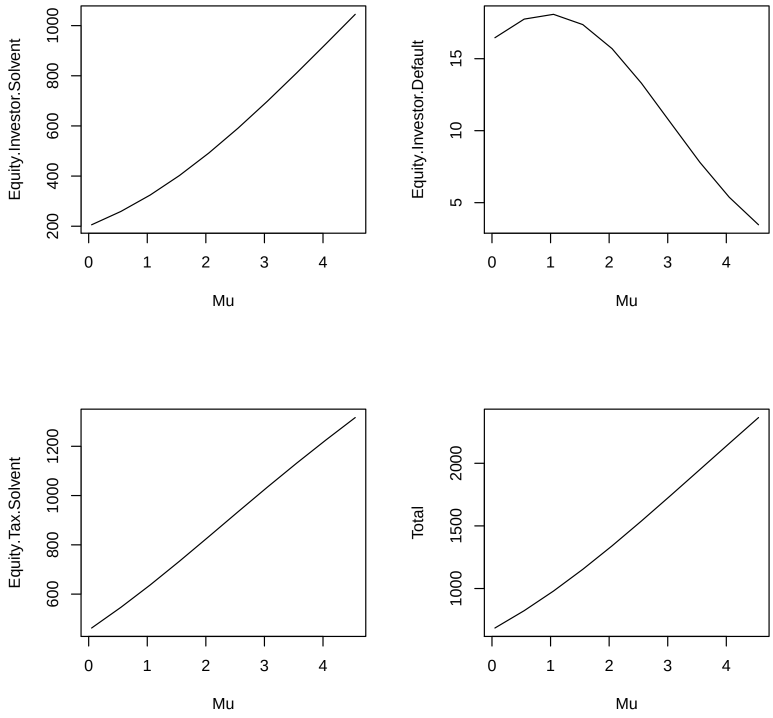


Figure B.19: Components of Equity as a Function of Mu

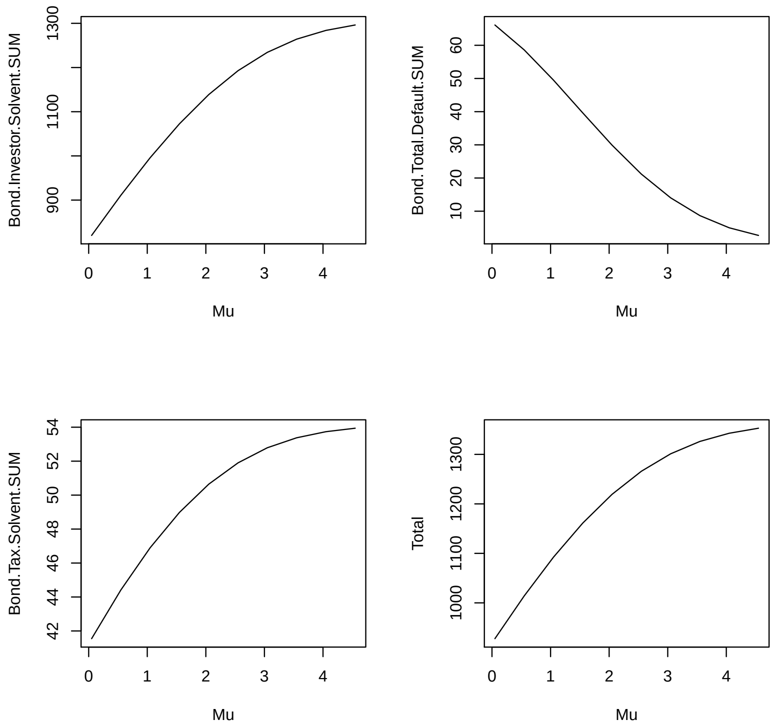


Figure B.20: Components of Debt as a Function of Mu

B.5 EBIT

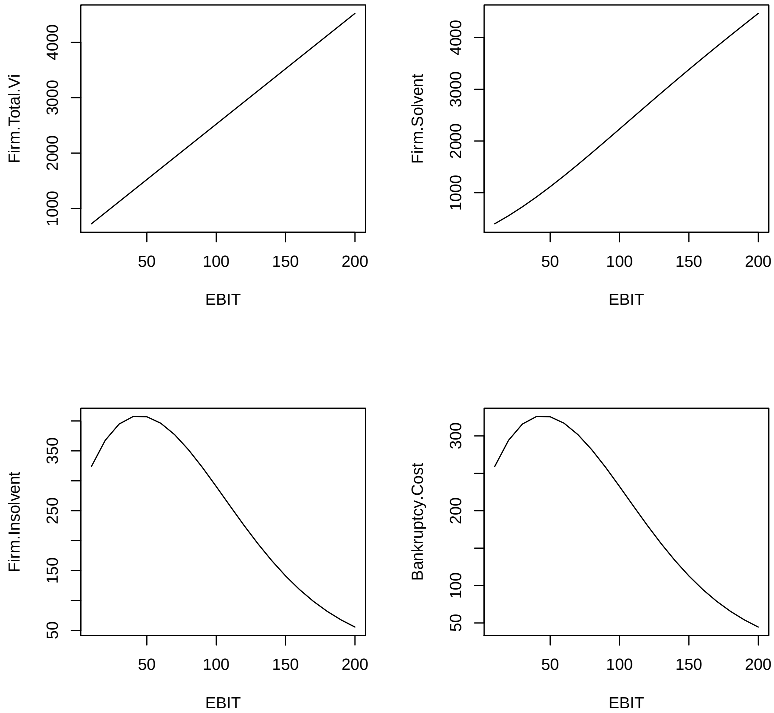


Figure B.21: Firm Value as a Function of EBIT

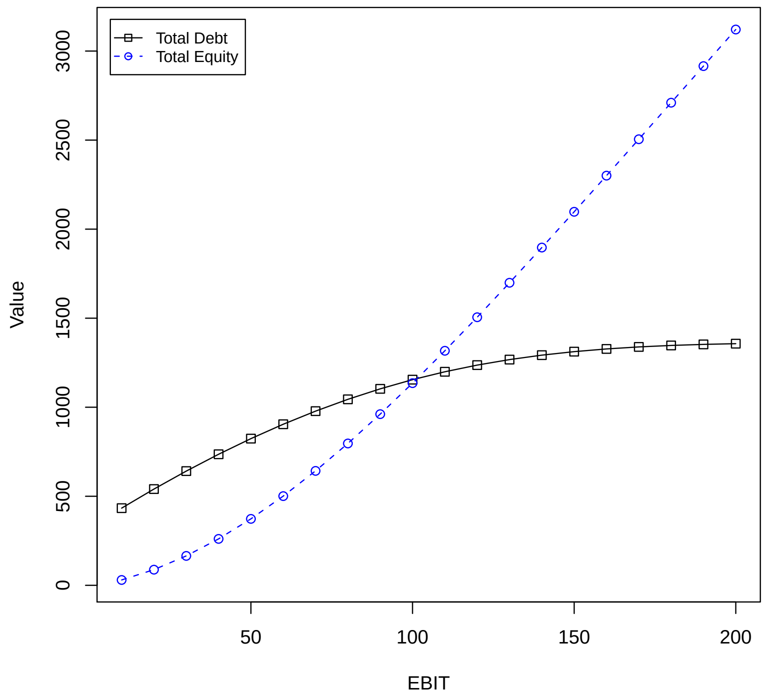


Figure B.22: Debt and Equity Values as a Function of EBIT

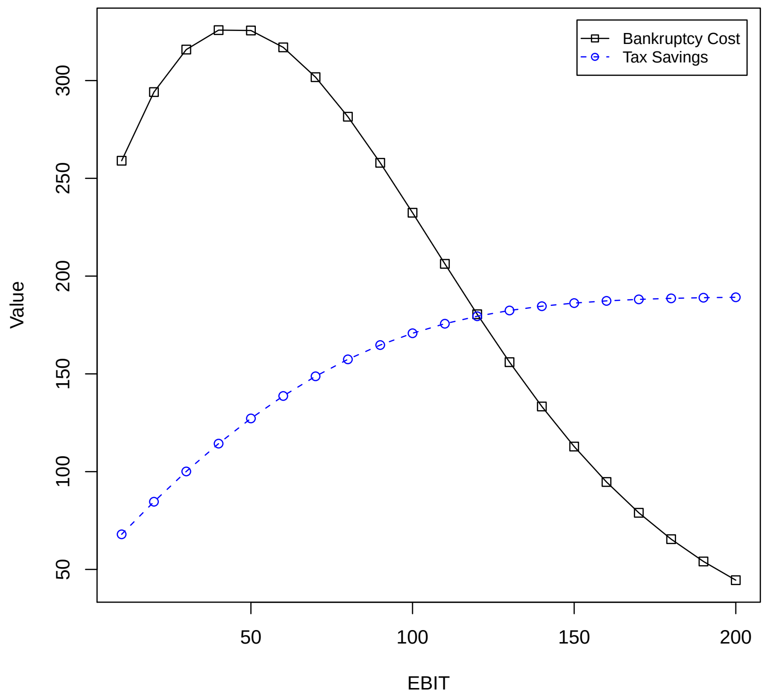


Figure B.23: Benefits of Merger (TS,BC) as a Function of EBIT

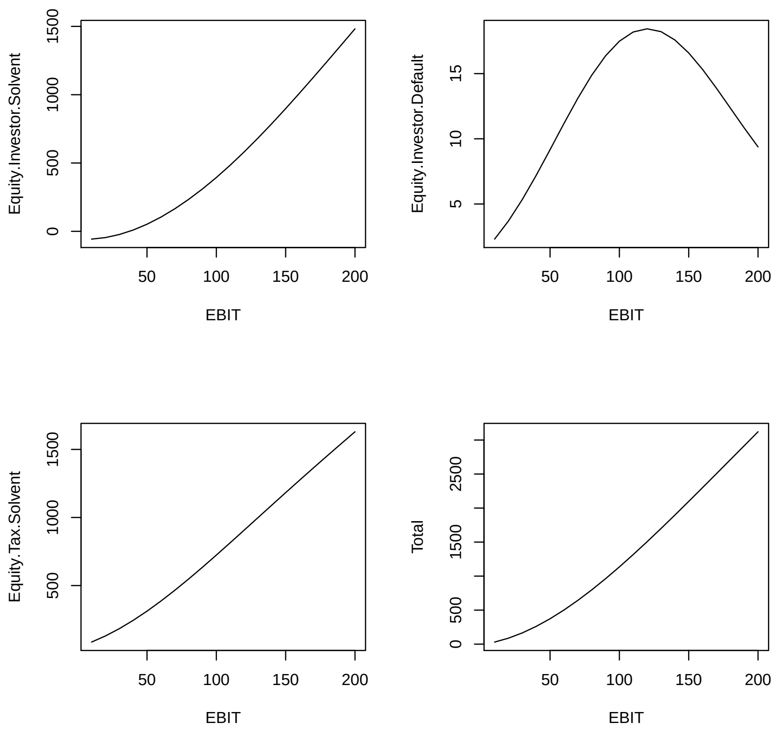


Figure B.24: Components of Equity as a Function of EBIT

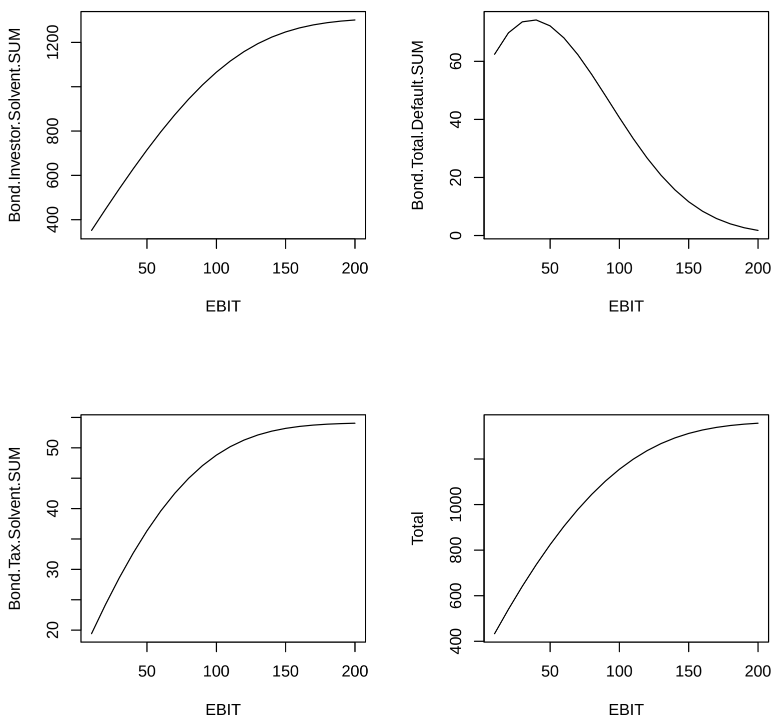


Figure B.25: Components of Debt as a Function of EBIT

B.6 Theta

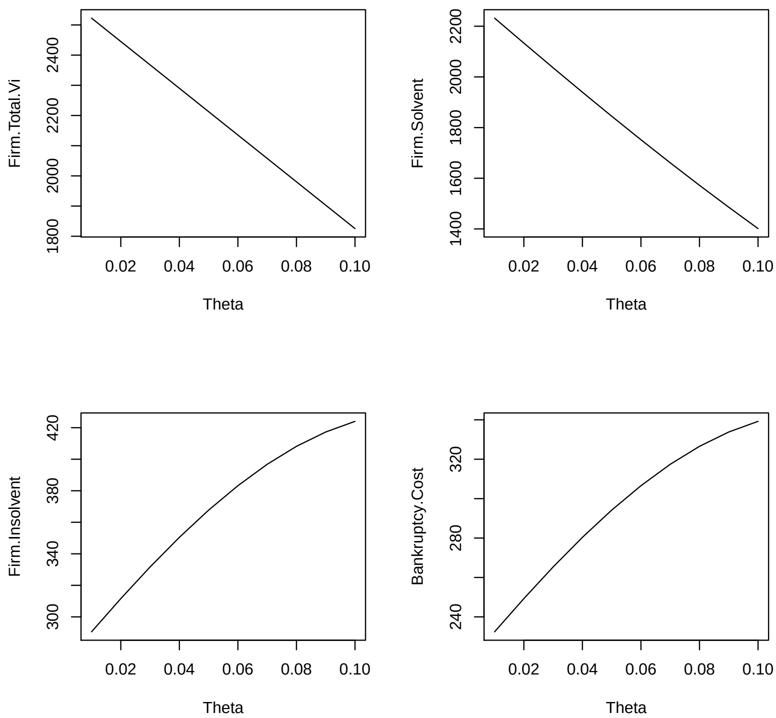


Figure B.26: Firm Value as a Function of Theta

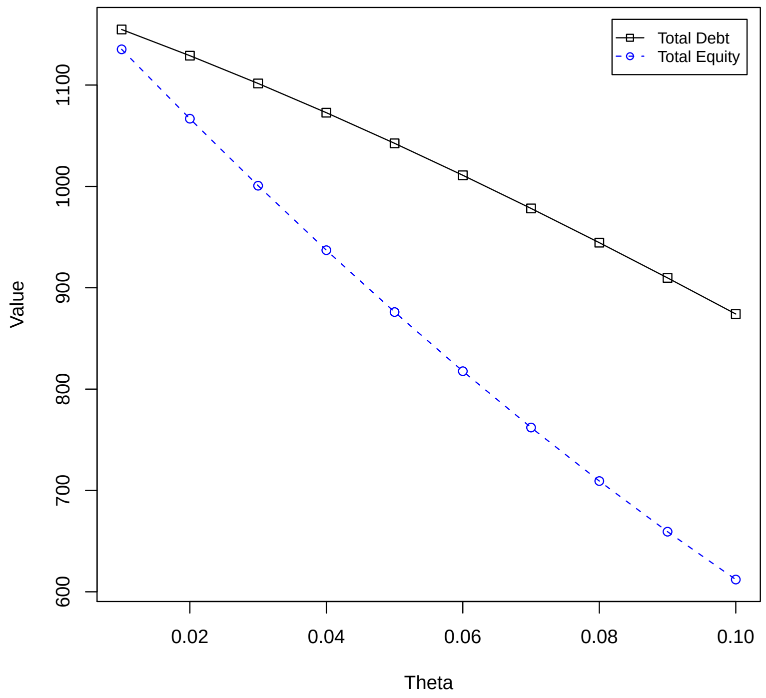


Figure B.27: Debt and Equity Values as a Function of Theta

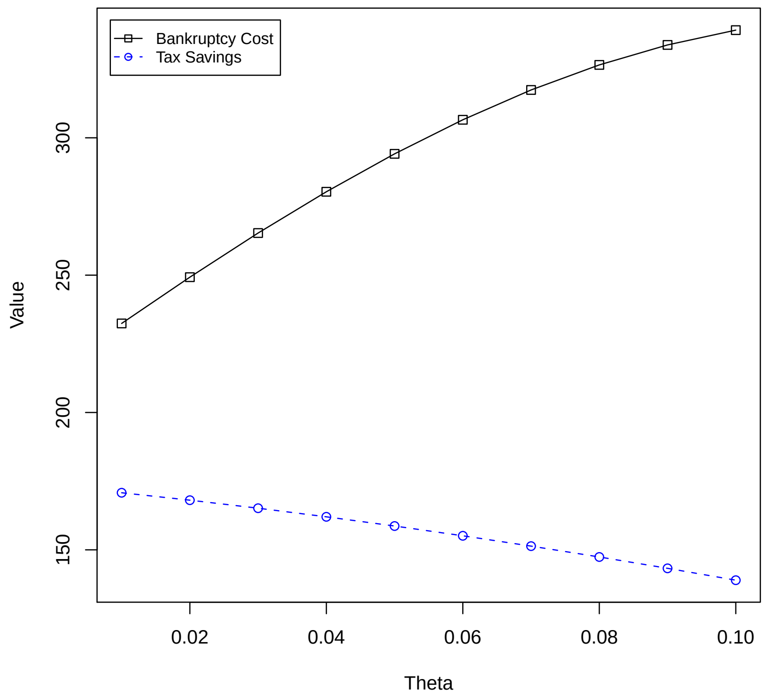


Figure B.28: Benefits of Merger (TS,BC) as a Function of Theta

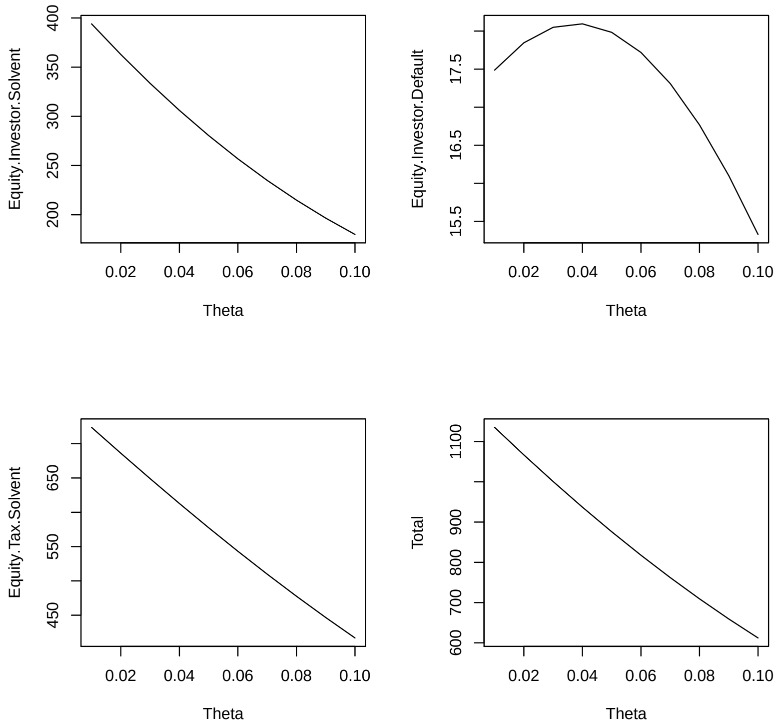


Figure B.29: Components of Equity as a Function of Theta

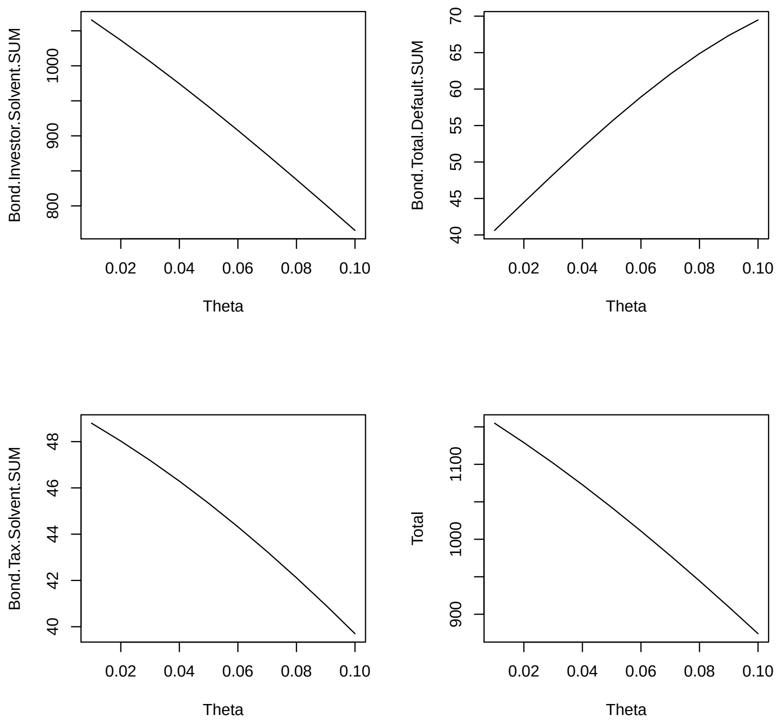


Figure B.30: Components of Debt as a Function of Theta

B.7 Simulation with Dynamic Default Barrier

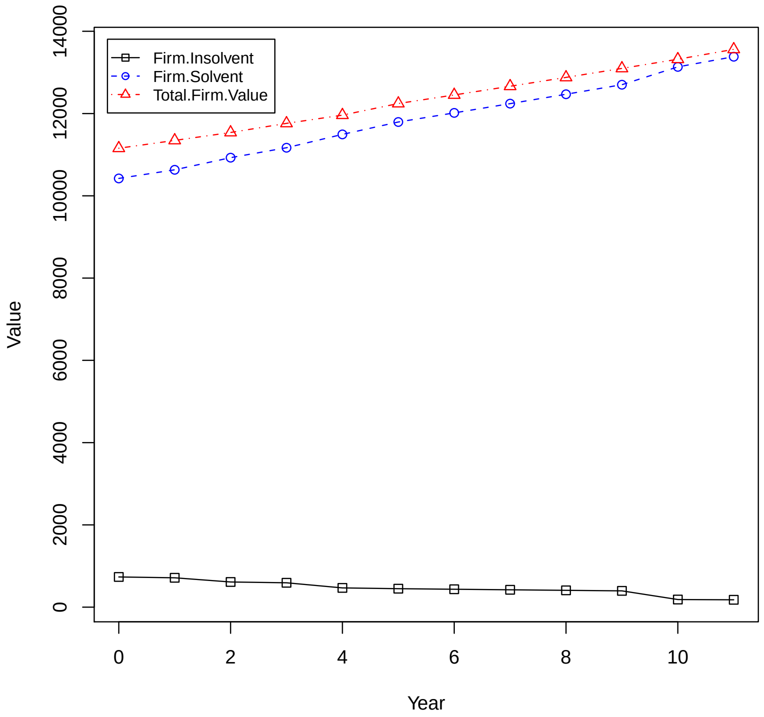


Figure B.31: Firm Value with dynamic default barrier, V_B

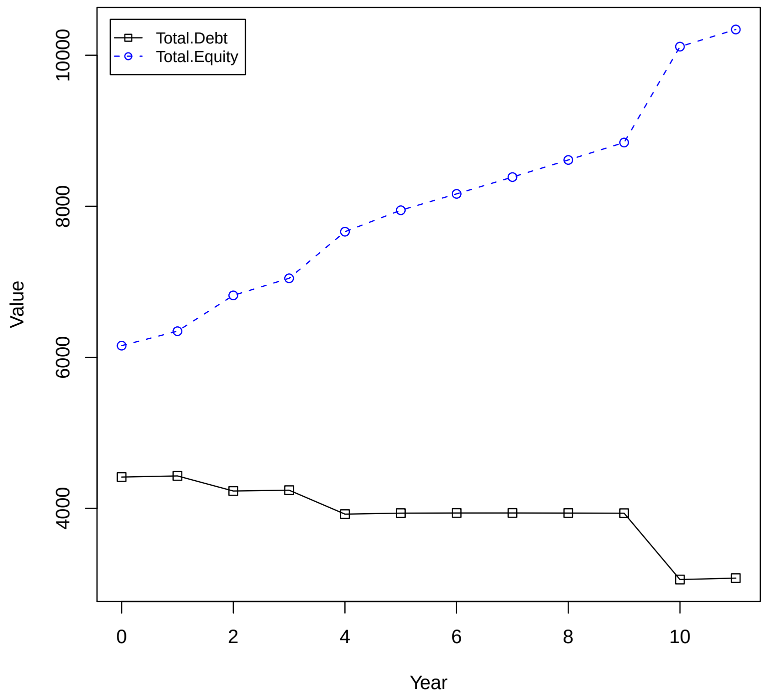


Figure B.32: Debt and Equity with dynamic default barrier, V_B

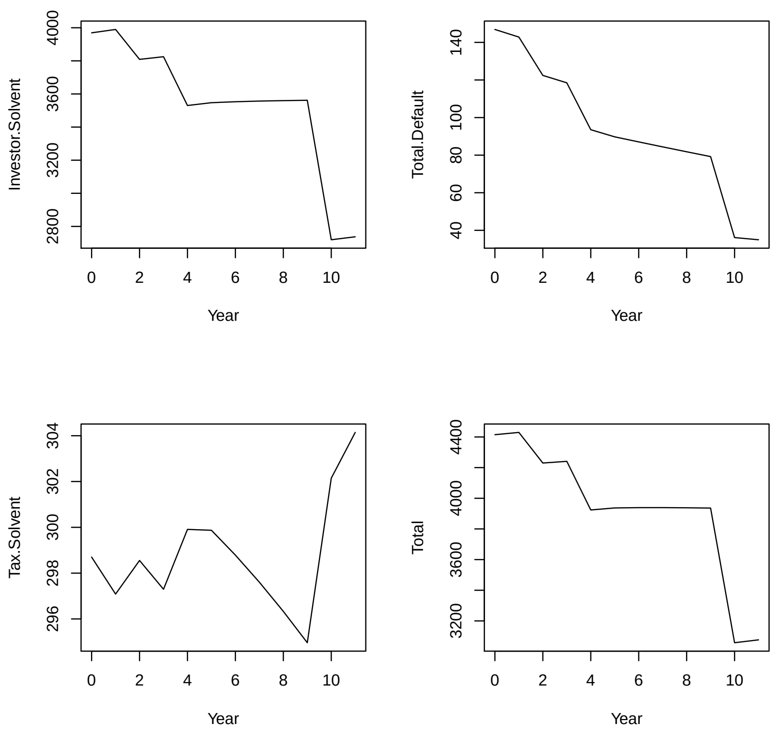


Figure B.33: Components of Bond with dynamic default barrier, V_B

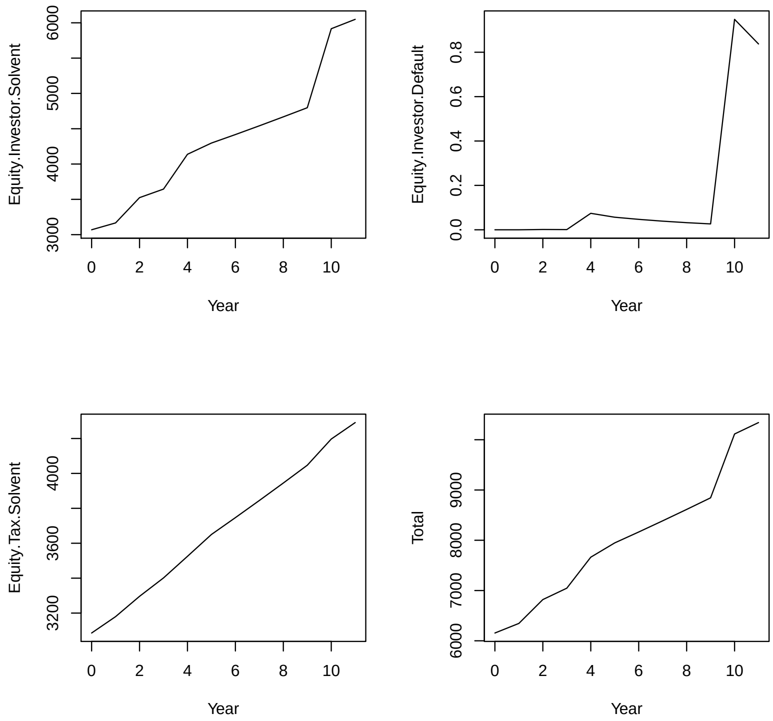


Figure B.34: Components of Equity with dynamic default barrier, V_B

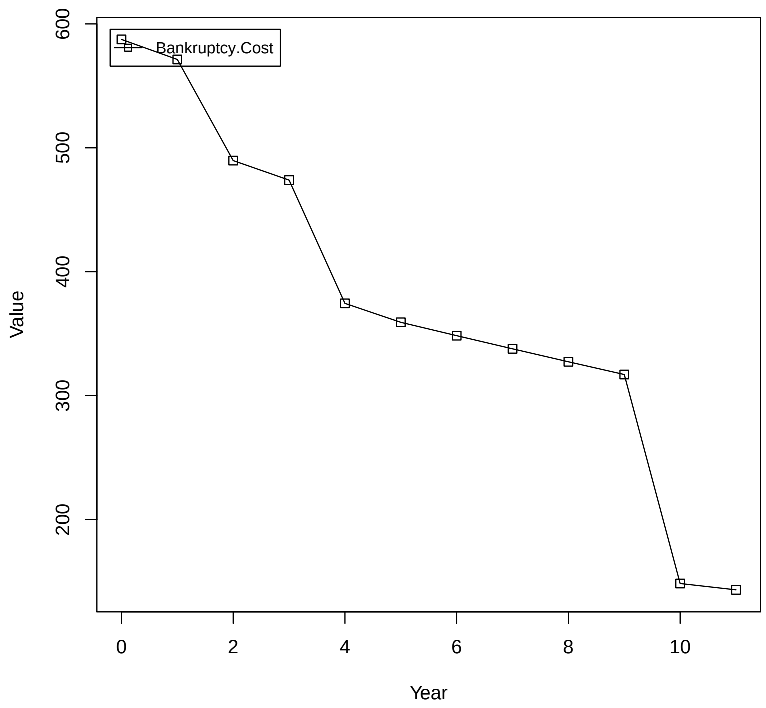


Figure B.35: Bankruptcy Cost with dynamic default barrier, V_B

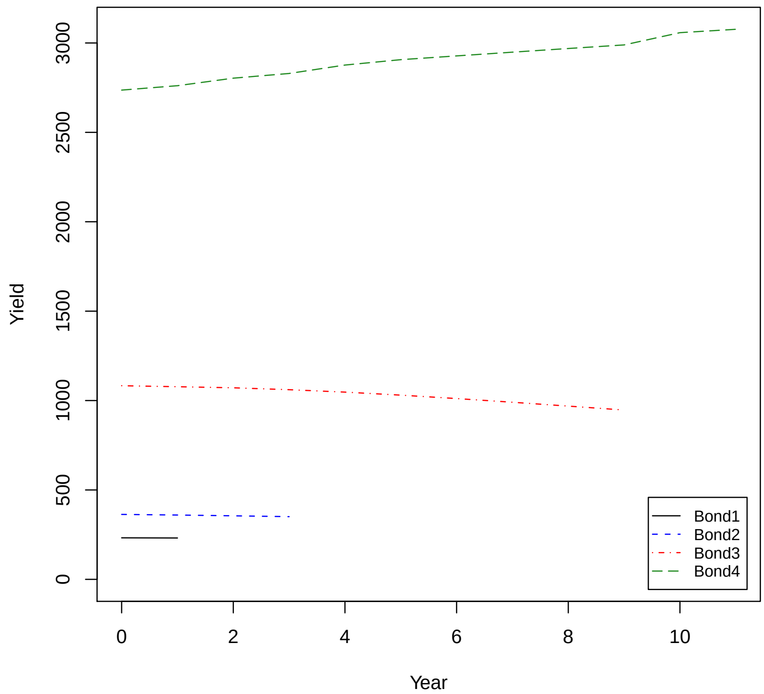


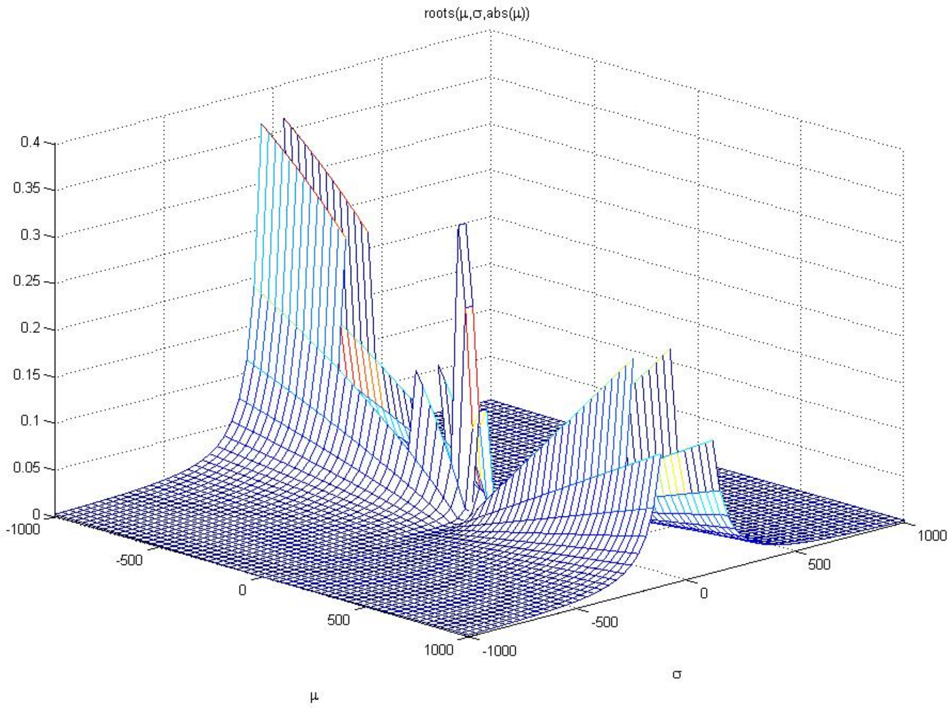
Figure B.36: Bond Prices in Time with dynamic default barrier, V_B

Appendix C

Computer Codes

C.1 Roots

```
function [x] = roots( mu,sigma,r)
%Returns the value of x or y in Goldstein et al 2001 (eq.10)
s2= sigma^2;
ms2=mu-s2/2;
x= (1/s2) * (ms2+sqrt(ms2^2+2*r*s2) );
% ezmesh(@(mu,sigma)roots(mu,sigma,abs(mu)),[-1000,1000,-1000,1000])
end
```



C.2 Trinomial Probabilities

Following pseudo code calculates all the paths reaching nodes specified by N and x.

```
i = 2
```

```
N = 150 'Node number

    For X = 0 To N

        For a = 0 To Int((N - X) / 2)

               c = X + a

               b = N - c - a    'a,b,c,N and X

               i = i + 1

        Next a

        i = i + 1

    Next X
```

Appendix D

Formulas and Derivations

D.1 Multi Period Model

D.1.1 Bankruptcy

$$\tau = infs \geq t_0 : \eta_s = \eta_B(s).$$

$$V_B(\tau) = E_\tau^Q \int_\tau^T \eta_s e^{-r(s-\tau)} ds$$

$$\Phi(t_0, T, \eta_{t_0}, \eta_B(t)) = 1 - \int_{\eta_B(T)}^{\infty} \phi(t_0, T, \eta_{t_0}, \eta_B(t)) d\eta_s$$

The density of first passage time, ψ is found by taking the partial derivative of Φ with respect to time.

$$\psi(t_0, s, \eta_{t_0}, \eta_B(t)) = \frac{\partial \Phi(t_0, T, \eta_{t_0}, \eta_B(t))}{\partial T}.$$

The price of Arrow-Debreu security is found by integrating the density of first passage time and taking the discounted value by

$$p_B(t_0, T, \eta_{t_0}, \eta_B(t)) = \int_{t_0}^T e^{-r(s-t_0)} \psi(t_0, s, \eta_{t_0}, \eta_B(t)) ds.$$

D.1.2 Arrow-Debreu Prices for Future Intervals

$$t_0 \leq T' < T$$

$$\Phi(T', T, \eta_{t_0}, \eta_B(t)) = \Phi(t_0, T, \eta_{t_0}, \eta_B(t)) - \Phi(t_0, T', \eta_{t_0}, \eta_B(t))$$

therefore, p_B for a future time interval is found by

$$p_B(T', T, \eta_{t_0}, \eta_B(t)) = p_B(t_0, T, \eta_{t_0}, \eta_B(t)) - p_B(t_0, T', \eta_{t_0}, \eta_B(t)).$$

D.1.3 Splitting Firm Value

Assuming $\eta_B(s) = \eta_B$ is constant, the firm value can be split by bankruptcy time $\tau(w)$.

$$V = E^Q_{t_0} \int_{t_0}^{\tau(w)} \eta_s e^{-r(s-t_0)} ds + E^Q_{t_0} \left[e^{-r(\tau(w)-t_0)} \int_{\tau(w)}^{\infty} \eta_s e^{-r(s-\tau(w))} ds \right]$$

Using law of iterated expectations

$$V = V^+ + E^Q_{t_0} \left[e^{-r(\tau(w)-t_0)} E^Q_{\tau(w)} \left[\int_{\tau(w)}^{\infty} \eta_s e^{-r(s-\tau(w))} ds \right] \right],$$

$$V = V^+ + \int_{t_0}^{\infty} e^{-r(u-t_0)} E^Q_u \left[\int_u^{\infty} \eta_s e^{-r(s-u)} ds \right] P(\tau(w) \in du),$$

$$V = V^+ + V_B \int_{t_0}^{\infty} e^{-r(u-t_0)} \psi(t_0, u, \eta_{t_0}, \eta_B(t)) du,$$

$$V = V^+ + p_B(t_0, \infty, \eta_{t_0}, \eta_B) V_B,$$

$$V = V^+ + V^-.$$

D.1.4 GBM Process

The firm EBIT follows a process given by

$$\frac{d\overline{\eta}}{\overline{\eta}} = \overline{\mu}_\eta dt + \overline{\sigma}_\eta dz^P$$

$$\frac{d\overline{\eta}}{\overline{\eta}} = (\overline{\mu}_\eta - \overline{\theta} \cdot \overline{\sigma}_\eta)dt + \overline{\sigma}_\eta dz^Q$$

$$\overline{\mu} = \overline{\mu}_\eta - \overline{\theta} \cdot \overline{\sigma}_\eta$$

$$\overline{V}_t = \frac{\overline{\eta}_t}{r - \overline{\mu}}$$

$$\frac{d\overline{V}}{\overline{V}} = \overline{\mu} dt + \overline{\sigma}_\eta dz^Q$$

$$d\ln(\eta) = d\ln(\bar{V}) = (\overline{\mu} - \frac{\overline{\sigma}_\eta^2}{2})dt + \overline{\sigma}_\eta dz^Q$$

D.1.5 ABM Process

$$d\eta = \mu dt + \sigma_\eta dz^Q$$

$$dV = \frac{\partial V}{\partial \eta} d\eta + \frac{1}{2}\frac{\partial^2 V}{\partial \eta^2} d\eta^2$$

$$\eta dt + dV = rVdt$$

$$V = A + B\eta$$

$$V = \frac{\mu}{r^2} + \frac{\eta_{t_0}}{r}$$

$$dV = \frac{1}{r}\left[\mu dt + \sigma_\eta dz^Q\right]$$

$$d\eta = \mu dt + \sigma_\eta dz^P$$

$$\mu = \mu_\eta - \theta\sigma_\eta$$

$$\theta = \frac{\mu_\eta - r}{\sigma_\eta} + \frac{r + r\eta_{t_0} - r^2 V}{\sigma_\eta}$$

$$dV + \eta dt = rVdt + \frac{\sigma_\eta}{r}dz^Q$$

$$dV = rVdt + \frac{\sigma_\eta}{r}dz^Q - \eta dt$$

Appendix E

Definitions and Some Background

E.1 Feynman-Kac Theorem

One method to obtain general valuation formulas for securities and derivatives is to use martingale methods which requires solving conditional expectations. Another method is to use PDE approach which requires solving partial differential equations under boundary conditions. These two methods are linked to each other with Feynman-Kac (FK) formula (Sondermann, 2006, pp.76).

FK formula allows switching between PDE approach and martingale approaches. For example, time-dependent expectation of a function of a Markovian stochastic process can be found by solving a partial differential equation, subject to boundary and end conditions (Tavella, 2002, pp.31).

The Feynman-Kac theorem states that given a SDE, $dX(t) = a(X,t)dt + b(X,t)dW(t)$ the expectation of a function of $X(T)$, for $0 \leq t \leq T$, given by

$$g(t,x) = E_{(t,X(t)=x)}[f(X(T))]$$

satisfies the partial differential equation

$$\frac{\partial g}{\partial t} + a(x,t)\frac{\partial g}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 g}{\partial x^2} = 0 \tag{E.1}$$

subject to the end condition $g(t = T, x) = f(x)$.

Opposite is also true. Given a PDE the solution can be written as a conditional expectation.

E.2 Law of Iterated Expectations

$$E[Y] = E_X[E[Y|X]]$$

Proof:

$$E_X[E_{Y|X}[Y|X]] = \int_{-\infty}^{\infty} (\int_{-\infty}^{\infty} y f_{Y|X}(Y|X)dy) f_X(x)dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f_{Y|X}(Y|X) f_X(x)dydx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f_{YX}(Y, X)dydx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f_{X|Y}(X|Y) f_Y(y)dydx$$

$$= \int_{-\infty}^{\infty} y(\int_{-\infty}^{\infty} f_{X|Y}(X|Y)dx) f_Y(y)dy$$

$$= E_Y[Y]$$

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