# Six More Mathematical Cuneiform Texts in the Schøyen Collection

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Almost all the mathematical cuneiform texts in the Schøyen Collection were published in Friberg's *A Remarkable Collection* = *MSCT* 1 (2007). However, six mathematical cuneiform problem texts in the collection were identified too late to be included. The present article will take care of that omission. It is a pleasure to be able place this additional material in a volume honouring Martin Schøyen, whose enthusiasm for seeing his objects published in academic editions is matched only by his benevolence towards those scholars who share this aim with him[1].

**Catalogue**

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|---|---|---|
| 1 | **MS 3299** 2 problems for rectangular prisms; 2 problems for squares<br>Tablet, portrait format, left corners missing, 25 + 1 + 11 lines, unruled | 94 x 62 x 22 mm |
| 2 | **MS 3976** A linear equation (finding an original amount of barleycorn)<br>Tablet, landscape format, complete, 11 + 1 lines, unruled | 62 x 75 x 25 mm |
| 3 | **MS 3895** A market rate problem (fine oil), leading to a quadratic equation<br>Tablet, landscape format, complete, obverse poor, 13 + 2 + 11 lines, unruled | 63 x 76 x 28 mm |
| 4 | **MS 3928** 2 quadratic equations (squares). No solution procedure or answer<br>Tablet, portrait format, complete, 10 + 10 lines, ruled | 65 x 46 x 27 mm |
| 5 | **MS 2833** 3 not very clear questions concerning a ditch. No solution procedure or answer<br>Tablet, portrait format, damage to right side, 13 lines, unruled, reverse uninscribed | 75 x 55 x 24 mm |
| 6 | **MS 4905** 2 not very clear problems, one for a rectangle, the other for a rectangular prism<br>Solution procedure but no answer. Tablet, landscape format, 5 + 6 lines, unruled | 40 x 47 x 23 mm |

## Introductory remarks

The scripts of these six tablets are conventional examples of Old Babylonian cursive. The orthographic style is mainly syllabic. The spelling conventions show that four of the tablets were certainly written in southern Babylonia: *e-pe-ši-ka* (**no. 1**: 3, 12, 20; **no. 2**: 4; **no. 3**: 8); *as-sú-uḫ* (**no. 1**: 9); *sà-*

[1] The Assyriological part of this article is the work of George, the mathematical part the work of Friberg, but each also contributed to the other's share. The cuneiform copies are by George, unless otherwise stated. – George records with pleasure his debt to Christopher Walker, Mark Ronan and other participants in the London Cuneiforum, where the tablets published here were read in the autumn of 2008.

*ri-iq* (**no. 2**: 1); *ta-na-as-sà-aḫ* (**no. 2**: 7); *sà-ni-iq* (**no. 6**: 5, 11). It is probable that texts nos. 1-5 stem from the same location. The majority of Old Babylonian archival documents in the Schøyen Collection came originally from Larsa, and Larsa is thus a plausible provenance for these mathematical tablets. A provenance elsewhere in the south is expected for text no. 6, for it entered the collection with a sixteenth-century archive dated to the first Sealand dynasty. Strictly speaking, this makes no. 6 a post-Old Babylonian tablet, but its text is clearly in the Old Babylonian tradition.

The texts' spelling, terminology and provenance are discussed in more detail in the mathematical commentary below. Noteworthy features of spelling and language are:

(a) *ip-se-e* = *ipsê* (no. 1: 7), the construct state of *ipsûm* "square root", clearly borrowed into Akkadian from ib.sá (ib.si$_8$), the Sumerian word of the same meaning, on which see now Attinger, *ZA* 98 (2008). Because it was only discovered recently, Akkadian *ipsûm* does not yet appear in any dictionary of Akkadian.

(b) GAM.DA (no. 1: 4, 18) for the vertical dimension of a rectangular pit, presumably = *šuplum* 'depth'.

(c) ì.GAM (no. 5: 2, 12) for *šapālum* 'to grow deep', probably also for *šuplum*[2].

(d) nam = *ana* (no. 2: 6, 9; no. 5: 12), as occasionally in Old Babylonian legal documents.

(e) ½ (*a*) duḫ = *mišil (a) paṭārum* (no. 3: 12) 'to calculate (lit. solve) half of *x*'.

(f) The question *kī maṣi* 'how much?' is abbreviated in no. 1: 11 to *kī maṣ (written ma-aṣ).*

(g) The unambiguous spelling [*tu-u*]*š-ta-ak-kal-ma* (no. 1: 23) with plene /kk/ resolves the disagreement over the derivation of this word in favour of *šutākulum* 'to consume oneself/each other' < *akālum*, against **šutakūlum* 'to make hold oneself/each other' < *kullum*, most recently advocated by Høyrup in *LWS* (2002), 23, and *ChV* (2001), 159, fn. 8. Unnoticed before, the same word, with the same spelling, appears also in the Larsa text YBC 4675 = *MCT* B, *obv*. 12, *rev*. 14. – The confirmation of the reading *tuštakkal* has consequences also for the proper understanding of the debated term *takīltum* < kullum, which occurs in several Old Babylonian solution procedures for quadratic equations. It can no longer be explained as derived from a preceding **tuštakāl* < **šutakūlum*, as in Høyrup, *op. cit.*, 199-200. The (presumably) correct meaning of the term is explained in Friberg, *MSCT* 1 (2007), 337, inspired by the discussion in Muroi, *HSc* 12 (2003), as 'that which holds (your mind)', in other words, something that you have made a mental note of. Indeed, normally in Old Babylonian metric algebra texts, the term stands for the coefficient in front of the linear term in a quadratic equation, multiplied by ½. In the solution of a quadratic equation by (geometric) "completion of the square", this halved coefficient is first squared *and made a mental note of*, to be used again in a later stage of the solution procedure. In one unique case, the term occurs in another context, but it is then explicitly stated to refer to a preceding *re-eš-ka li-ki-il* 'may it hold your head', in the following phrase: *a-na* 6 40 *ta-ki-il-ti ša re-eš-ka ú-ki-lu i-ší* 'raise(multiply) by 6 40, the *takīltum* that held your head'.

## 1. MS 3299. Two problems for rectangular prisms; two problems for squares

### 1.1. Presentation of the text

MS 3299 is an Old Babylonian tablet inscribed in a single column with 37 lines of cuneiform text. The tablet lacks the two left-hand corners, which were patched up with clay and small fragments of old tablets in modern times, presumably to improve the value of the tablet in the market place. These pieces, 20 in number, came adrift when the tablet was fired while in conservation at the Schøyen Collection. Upon close inspection, at least 7 of these small fragments turned out to hold text from the *Epic of Gilgamesh*. They were given the number MS 3263 and are published as text no. 6 in *Babylonian Literary Texts in the Schøyen Collection* (George, *MSCT* 4 (2009)).

[2] Note GAM for *šuplum* in Old Babylonian mathematical texts now in the Louvre, refs. *CAD* Š/3: 325. On the uses of the sign GAM in OB mathematical texts see Robson *MMTC* (1999), 132, who proposes that in the meaning 'depth' it can be read BÙR. BÙR, however, is a value of the sign U, not of GAM; nevertheless, the Sumerian 'deep' is /burud/, written bùr(u)-(d), so that GAM.DA should probably be read burud$_x$.da.

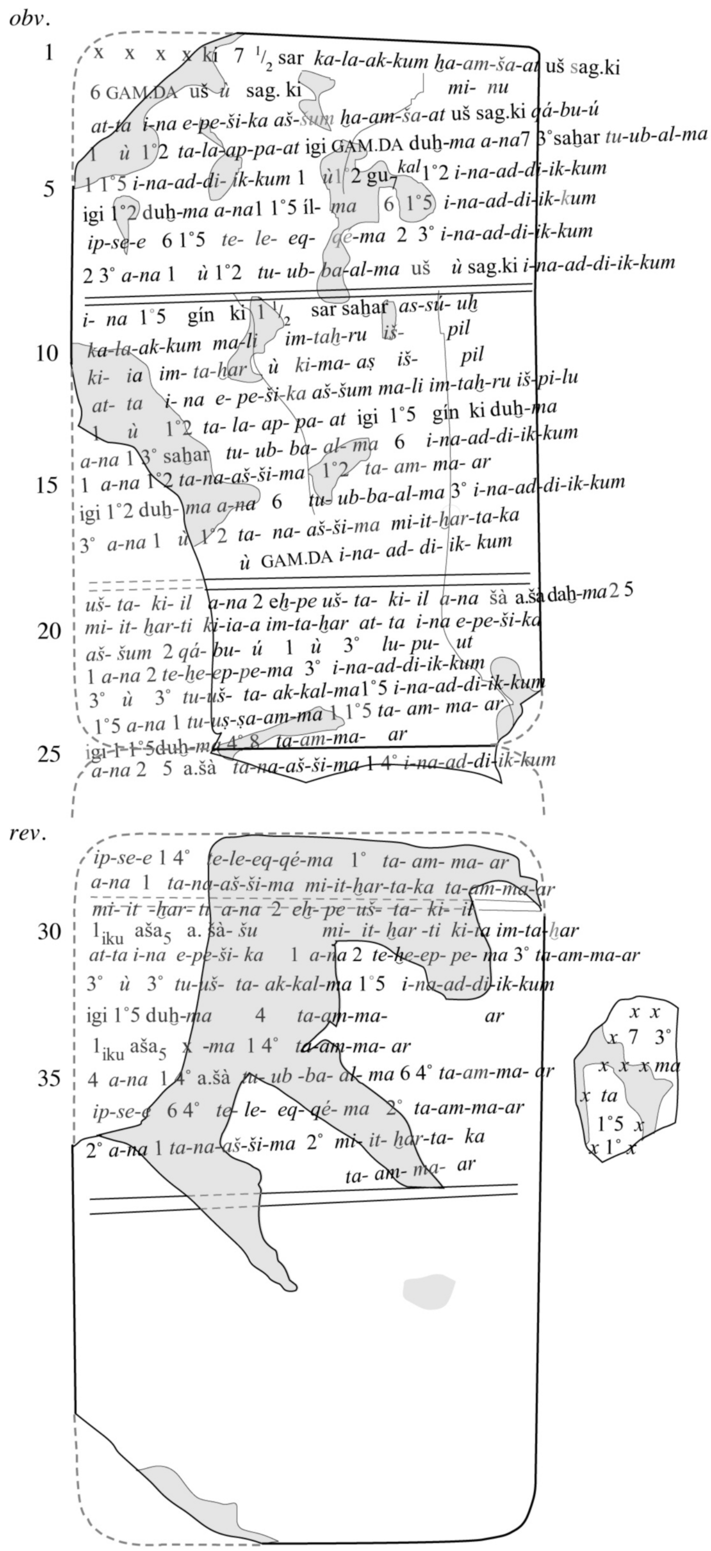


Fig. 1.1. MS 3299. Conform transliteration of the cuneiform text. Scale 1:1.

The photograph of MS 3299, published below as pl. 1, shows the baked tablet with two of the small pieces reattached to the reverse. The smaller is upside down and clearly does not belong where placed; nor could it be attached elsewhere. It is not reproduced in the hand copy of the reverse. After MS 3299 had been baked and the modern patching had been discovered, it was given the number MS 3263/15. The larger piece, a flake containing parts of six lines of text, was given the number MS 3263/9. It sits quite well as glued but does not make a tight join with the broken clay behind it. Because it is of mathematical content, it was assumed to belong to MS 3299, where it can only fit in the position it occupies on the photograph and in the hand copy. However, this assumption is not well-grounded, for the text of this part of the tablet is difficult to reconstruct if the flake is taken into account. In addition, the flake shows no sign of the tool mark that mars the adjacent surface of the reverse of MS 3299. Therefore, it is likely that the flake belongs to some other tablet entirely.

The tablet contains four mathematical problems. They are (# 1) to find the length and breadth of a pit excavated in the form of a rectangular prism, where volume and depth are known quantities and the breadth is one-fifth of the length (*ll*. 1-8); (# 2) to find the length, breadth and depth of a pit excavated in the form of cube, where surface area and volume are known quantities (*ll*. 9-18); (# 3) to find the sides of two square fields when one side is half the length of the other and when the combined area of the fields is a known quantity (*ll*. 19-28); (# 4) to find the side of a square field when a square field with half the side has a given area (*ll*. 29-38). All four problems are worked out, but only the last problem is concluded with an explicitly given answer.

### 1.2 a. MS 3299 # 1. A problem for a rectangular prism (*kalakkum* 'excavated pit')

The text of this exercise is fairly well preserved. Most of the missing text, in the upper and lower left corners of the obverse of the tablet, can be reconstructed without trouble.

**MS 3299 # 1. Transliteration**[3]

1 [x x x x k]i$^{?}$ 7 ½ *mušar*(sar) *ka-la-ak-kum ḫa-*⸢*am-ša*⸣*-at šiddim*(uš) *pūtum*([s]ag.ki)
2 [6 *šuplum*(GAM.DA)] *šiddum*([u]š) ⸢*ù*⸣ *pūtum*(sag.ki) *mi-*[*n*]*u*
3 [*at-ta*] *i-na e-pe-ši-ka aš-*[*šum*] *ḫa-am-*⸢*ša*⸣*-at šiddim*(uš) *pūtum*(sag.ki) *qá-bu-ú*
4 [1 *ù* 1]2 *ta-la-ap-pa-at igi*(igi) *šuplim*(GAM.DA) *tapaṭṭar*(duḫ)*-ma a-na* 7 30 *eperī*(saḫar) ⸢*tu*⸣*-ub-*<*ba*>*-al-ma*
5 ⸢1 15⸣ *i-na-ad-d*[*i-i*]*k-kum* 1 ⸢*ù*⸣ [1]2 *tuštakkal*(⸢gu$_7$⸣)$^{kal}$ 12 *i-na-ad-di-ik-kum*
6 *igi*(igi) 1[2] *tapaṭṭar*([d]uḫ)*-ma a-na* 1 15 *tanašši*(íl)-[*ma*] ⸢6 15⸣ *i-na-ad-di-ik-*[*k*]*um*
7 *ip-se-e* 6 15 ⸢*te-le*⸣*-eq-*⸢*qē*⸣*-ma* 2 30 *i-na-ad-di-ik-kum*
8 2 30 *a-na* 1 *ù* 12 *tu-ub-*⸢*ba*⸣*-al-ma šiddam*(⸢uš⸣) *ù pūtam*(sag.ki) *i-na-ad-di-ik-kum*

**Translation**

1 [x x x x x] (the volume) of an excavated pit was 7 ½ *mušar*, the front was a fifth of the length,
2 [the depth was 6 cubits]. What were the length and the front?
3 When [you] work it out, since "the front was a fifth of the length" was stated,
4 you shall note down [1 and 1]2, you shall solve (calculate) the reciprocal of the depth, and you shall carry (multiply) it by 7 30, the (volume of) earth:
5 it will give you 1 15. You shall let 1 and 12 eat each other (multiply them), it will give you 12.
6 You shall solve (calculate) the reciprocal of 12, and you shall raise (multiply) it by 1 15: it will give you 6 15.
7 You shall take the equalside (square side or square root) of 6 15: it will give you 2 30.

[3] According to the usual Assyriological convention, transliterations of Akkadian words are here written in Italic style, while transliterations of Sumerian words are in Roman style.

8 You shall carry (multiply) 2 30 by 1 and 12: it will give you the length and the front.

## MS 3299 # 1. Mathematical commentary

The statement of the problem (the "question") in this exercise originally began with a phrase that is now totally lost. It is likely that the phrase was purely descriptive. In any case, it cannot have specified any numerical parameters essential for what follows, since what seems to be a complete question is contained in the remainder of *ll.* 1-2. Thus, with the exception of the lost initial phrase, the text of the exercise is fairly well preserved, and the few missing parts are easily reconstructed.

The object considered in MS 3299 # 1, is a rectangular prism, called an "excavated pit" or "underground store" (*kalakkum* < Sum. ki.lá.(k)). The stated problem can be explained as follows (expressed for the readers' convenience in quasi-modern symbolic terms): Let $V$ be the volume of the prism, let $g$ be the depth (GAM.DA) of the prism, and let $u$ and $s$ be the length (uš) and front or width (sag), respectively, of the rectangular cross section of the prism. Then[4]

$u \times s \cdot g = V = 7\ ½$ sar, $s = u/5$, $g = 6$ (cubits) *ll.* 1-2

It is, of course, supposed to be known that the nindan (= c. 6 meters), the cubit (= $1/12$ nindan = c. 50 cm), and the sar or *mušar* (= 1 square nindan · 1 cubit) are the Old Babylonian basic units for horizontal length measure, vertical length measure, and volume, respectively.
The task set in the exercise is to find the values of the length $u$ and the front $s$:

$u = ?$, $s = ?$ *l.* 3

The solution procedure begins by recalling the condition that $s = u/5$ (*l.* 3, right). Then the numbers 1 and 12, for 1 and $1/5$, are noted down (*l.* 4, left), probably on a piece of clay serving as scratch pad.

Next, the area of the rectangular cross section of the prism (call it $A$) is computed as the volume divided by the height, that is, as follows:

$A = 1/g \cdot V = 1/6 \cdot 7;30 = 1;15$ (sq. nindan) *l.* 4, right and *l.* 5, left

Note the use here, *in the interpretation of the text*, of semicolons to specify the "absolute" values of the sexagesimal numbers, which in MS 3299 #1, as in all Old Babylonian mathematical texts, only have "relative" or "floating" values. Thus, the number 7 30 in the text is interpreted here as 7;30 = 7 ½, and the number 1 15 in the text is interpreted as 1;15 = 1 ¼.

The computed value for the area of the rectangular cross section of the prism is in the text compared with the area of a similar, but normalized, "reference rectangle". (The term is borrowed from Proust, *TMN* (2007), 230). In quasi-modern notations, if the sides of the reference rectangle are called $u'$ and $s'$, then saying that the reference rectangle is normalized means that $u' = 1$ (nindan), and saying that the reference rectangle is similar to the rectangular cross section of the prism means that $s' = u'/5 = 1/5 = ;12$ (nindan). Consequently, the area of the normalized reference rectangle can be computed as

[4] As first observed by Høyrup (see *LWS* (2002), 23), the *geometric multiplication* of two sides of a rectangle (conveniently in symbolic notation replaced by $\times$) is in Old Babylonian metric algebra texts often denoted by the verb *šutākulum*, the Št stem of *akālum* (= gu$_7$), while *arithmetic multiplication*, for instance of a geometric entity by a reciprocal or by a scaling factor (conveniently in symbolic notation replaced by · ) is denoted by use of the verbs *našûm* and *wabālum*. Also multiplication of a rectangular base by a height (or depth) is treated as an arithmetic multiplication!

$A' = u' \cdot s' = 1 \cdot ;12 = ;12$ (sq. nindan) *l.* 5, right

The ratio of the area of the rectangular cross section to the area of the reference rectangle is computed as

$1/A' \cdot A = 1/;12 \cdot 1;15 = 5 \cdot 1;15 = 6;15$ *l.* 6

Since the two rectangles are similar (that is, of the same form), this area ratio is the square of the linear "scaling factor" $f$ (a convenient name for the ratio of the lengths of the sides of the first rectangle to the lengths of the corresponding sides of the reference rectangle). The value of the scaling factor itself is computed as the 'equalside' (meaning the square root or, more correctly, in the geometric sense, the *square side*) of the area ratio, that is, as

$f =$ sqs. $6;15 = 2;30$ *l.* 7

The sides of the rectangular cross section of the prism can now, finally, be computed as

$u = f \cdot u' = 2;30 \cdot 1$ (nindan) and $s = f \cdot s' = 2;30 \cdot ;12$ (nindan) *l.* 8

The actual results of these easy computations are not given, but it is clear that

$u = 2;30$ (nindan) = 2 ½ nindan and $s = ;30$ (nindan) = ½ nindan.

It is easy to check the correctness of these computed values for the length $u$ and the front $s$, although there is no such check in the text of the exercise. Indeed, with the computed values,

$u \times s \cdot g = 2;30$ nindan $\cdot ;30$ nindan $\cdot$ 6 cubits = 7;30 sq. nindan $\cdot$ 1 cubit = 7 ½ volume-sar,

and

$u/5 = 2;30$ nindan$/5 = ;30$ nindan $= s$.

### 1.2 b. MS 3299 # 2. A problem for a cubical prism (another *kalakkum* 'excavated pit')

The text of exercise # 2 is more damaged than the text of exercise # 1. Luckily, at least the statement of the problem (the question) is largely intact, and therefore the text of the whole exercise could be reconstructed without any greater effort.

**MS 3299 # 2. Transliteration**

9 *i-na* 15 *šiqil*(gín) *qaqqarim*(ki) ⸢1⸣ ½ *mušar*(sar) *eperī*(saḫar) ⸢*as*⸣*-sú-uḫ*
10 [*k*]*a-*⸢*la*⸣*-ak-kum ma-l*[*i*] *im-*⸢*taḫ*⸣*-ru* ⸢*iš*⸣*-pil*
11 [*ki-i*]*a im-*⸢*ta*⸣*-*[*ḫar*] *ù* ⸢*ki*⸣ *ma-aṣ iš-pil*
12 [*at-ta*] *i-na e-pe-ši-*⸢*ka*⸣ *aš-šum*! *ma-li im-taḫ-ru*!(IM) *iš-pi-lu*
13 [1 *ù* 1]2 *ta-la-ap-pa-at igi*(igi) 15 *šiqil*(gín) *qaqqarim*(ki) *tapaṭṭar*(duḫ)*-ma*
14 [*a-na* 1 30 *eperī*(saḫa]r) *tu-ub-ba-*⸢*al-ma*⸣ 6 *i-na-ad-di-ik-kum*
15 [1 *a-na* 12 *ta-n*]*a-aš-ši-ma* ⸢12 *ta-am*⸣*-ma-ar*
16 [*igi*(igi) 12 *tapaṭṭar*(duḫ)*-ma a-n*]*a* 6 *tu-ub-ba-al-ma* 30 *i-na-ad-di-ik-kum*
17 [30 *a-na* 1 *ù*] ⸢12⸣ *ta-na-aš-ši-*⸢*ma*⸣ *mi-it-*⸢*ḫar*⸣*-ta-ka*
18 (vacat) *ù šuplam*(GAM.DA) *i-na-ad-di-ik-kum*

**Translation**

9 From 15 shekels(= ¼ *mušar*) of ground (area) I removed 1 ½ *mušar* (volume of) earth.
10 The excavated pit was as deep as it was equalsided (square).
11 How much was it equalsided (square) (each way), and how deep was it?
12 When [you] work it out, since it was as deep as it was equalsided (square),

13 you shall note down [1 and 12]. You shall solve (calculate) the reciprocal of 15 shekels of ground (area), and
14 you shall carry (multiply) it [by 1 30, the earth (volume)]: it will give you 6.
15 You shall raise (multiply) [1 by 12]; you will see 12.
16 [You shall solve (calculate) the reciprocal of 12, and] you shall carry (multiply) it by 6: it will give you 30.
17 You shall raise (multiply) [30 by 1 and] 12: your equalside (square side)
18 (empty space) and depth it will give you.

## MS 3299 # 2. Mathematical commentary

The object considered in the exercise MS 3299 # 2 is an excavated rectangular prism with its depth equal to the side of its square cross section. In modern (or more specifically, Greek) terms that is a *cube*. However, just like the object in exercise # 1, the excavated cube in exercise # 2 is called a *kalakkum*. The task of the exercise is to compute the square side $m$ (called *mitḫartu*, meaning something like 'equalside'), and the depth $g$ (called, unspecifically, 'as much as the pit is deep').

With the same use of quasi-modern symbolic notations as in the commentary above to exercise # 1, the question in exercise # 2 can be reformulated as follows:

$A$ = 15 area-shekels = ;15 area-sar, $V$ = 1 ½ volume-sar, *l.* 9
$m = g$ *l.* 10
$m$ = ?, $g$ = ? *l.* 11

Here, as in all similar Old Babylonian mathematical texts,

1 area-sar = 1 square nindan, and 1 volume-sar = 1 square nindan · 1 cubit.

The solution procedure starts by recalling (*l.* 12, right) that the depth of the excavated pit is supposed to be equal to the side of the square cross section, and by noting down (*l.* 13, left) the numbers 1 and 12, thus stating the different factors for calculating the sides (in nindan) and depth (in cubits), factors which are the key numbers in *l.* 17. (At least, this is what is suggested in the proposed reconstruction of the corresponding missing part of the text).

Then the depth ($g$) is computed as the volume divided by the area of the cross section, as follows:

$g = 1/A \cdot V$ = 1/;15 · 1;30 = (4 · 1;30 =) 6 (cubits) *l.* 13, right - *l.* 14

Next, the volume of a normalized "reference cube" is computed as

1 · 12 = 12 (sq. nindan · 1 cubit) *l.* 15

The explanation for this computation is that if a cube-shaped excavated pit is normalized in the sense that its square cross section has the side $m'$ = 1 (nindan), then its depth is $g'$ = 12 cubits. (Recall that 1 nindan = 12 cubits). The area of the square cross section of the cube is then 1 (sq. nindan), and the volume of the cube is 1 (sq. nindan) · 12 (cubits) = 12 (sq. nindan · 1 cubit).

The ratio of the depth of the given cube-shaped excavated pit to the depth of the normalized reference cube is, clearly,

$1/g' \cdot g$ = 1/12 · 6 = ;30 *l.* 16

This means that the "scaling factor" in this situation (the ratio between the lengths of the sides of the two cubes) is $f$ = ;30. Therefore, the sides of the original cube-shaped excaved pit can be computed as follows:

$m = f \cdot m'$ = ;30 · 1 (nindan) and $g = f \cdot g'$ = ;30 · 12 (cubits) *ll.* 17-18

Consequently, although this is not stated explicitly in the text, $m = g$ = ½ nindan = 6 cubits. Never mind that it should have been obvious, without any further computations, that if the depth of a cube is 6 cubits, as shown in *l.* 14, then all sides of the cube must have the common length 6 cubits. And never mind that if the square cross section of a cube is known to have the area ;15 sar = ¼ square nindan, as stated in the first line of the exercise, then the side of the cube must have the length sqs. (¼ square nindan) = ½ nindan = 6 cubits.

Apparently, the author of this exercise was so intent on giving another example of the method of using normalized reference objects and scaling factors that he did not bother to find the simplest solution to the stated problem! Or, maybe, the correct explanation is that this is an example of how instruction in Babylonian mathematics was based on learning procedure. Short cuts would risk the undermining of the rigid procedure, and the student who wrote the tablet was obliged to show that he knew the procedure.

### 1.2 c. MS 3299 # 3. A problem for two squares (‘equalsides’)

The text of exercise # 3 is extensively damaged. More than half the text is missing or unreadable. In addition, what remains of the question is awkwardly formulated and therefore quite obscure.

Fortunately, however, only the last few lines of the exercise are completely broken away, and their content, with the last steps of the solution algorithm, is easily reconstructed. Once the solution procedure had been reconstructed, in the way shown below, it was not too difficult to find also a reasonable reconstruction and interpretation of the question.

**MS 3299 # 3. Transliteration**

19 <*mi-it-ḫar-ti*> [*uš-ta-ki-il*] *a-na* 2 *eḫ*!(ḪAR)*-pe uš-ta-ki-il* ⸢*a*⸣*-na* ⸢*libbi*(šà)⸣ *eqlim*(a.šà) *uṣib*(daḫ)*-ma* ⸢2⸣ 05
20 [*mi-it-ḫar-ti k*]*i-ia-a im-ta-ḫar at-ta i-na e-pe-ši-ka*
21 [*aš-šum* 2 *qá*]*-bu-ú* 1 *ù* 30 *lu-pu-ut*
22 [1 *a-na* 2 *te-ḫe-e*]*p*-[*p*]*e-ma* 30 *i-na-ad-di-ik-kum*
23 [30 *ù* 30 *tu-u*]*š-ta-ak-kal-ma* 15 *i-na-ad-di-ik-kum*
24 [15 *a-na* 1 *tu-u*]*ṣ-ṣa-am-ma* ⸢1 15⸣ *ta-am-ma-ar*
25 [*igi*(igi) 1 15 *tapaṭṭar*(duḫ)*-ma*] ⸢48⸣ *ta-*⸢*am*⸣*-ma-ar*
26 [*a-na* 2 05 *ta-n*]*a-aš-ši-ma* 1 ⸢40 *i-na-ad*⸣*-di-i*[*k-kum*]
27 [*ip-se-e* 1 40 *te-le-eq-qé-ma* 10 *ta-am-ma-ar*]
28 [*a-na* 1 *ta-na-aš-ši-ma mi-it-ḫar-ta-ka ta*]*-*⸢*am-ma*⸣*-*[*ar*]

**Translation**

19 [I made <my equalside> eat itself (I squared it)], I broke it in 2 (and) I made it eat itself (I squared it). I added it (the area of the second square) onto the field (area) (of the original square): 2 05.
20 How much was [my equalside] equalsided (what was the square side)? When you work it out,
21 [since “2” was] stated, note down 1 and 30.
22 [You shall break 1 in 2]: it will give you 30.
23 [You] shall make [30 and 30] eat each other (you shall multiply them): it will give you 15.
24 [You] shall add [15 to 1]: you will see 1 15.
25 [You shall solve (calculate) the reciprocal of 1 15]: you will see 48.
26 [You shall] raise (multiply) it [by 2 05]: it will give [you] 1 40.
27 [You shall take the equalside (square root) of 1 40: it will give you 10.]
28 [You shall raise (multiply) it by 1: you] will see [your equalside (square side)].

### MS 3299 # 3. Mathematical commentary

The statement of the problem (the question) in this exercise is quite obscure in a number of ways (in addition to the fact that the very first word in the text of the question is missing). In particular, it is (probably) not said explicitly what the object originally considered should be. Nevertheless, it seems to be clear from by what remains of the question that the object originally considered in this exercise is a *mitḫartum* ‘equalside (square side)’, which is first squared, then cut in half, and then again squared.

After that, the sum of the areas of the two squares is given. In quasi-modern terms, if *m* is the original square side, the stated problem can be interpreted as the following equation:

sq. $m$ + sq. $(m/2) = 2\ 05$, $m = ?$ *ll.* 19-20

The somewhat curiously stated question in *l.* 20, "[My equalside, how] much is it equalsided?" obviously asks for the length of the square side *m*.

The solution procedure starts by recalling (in *l.* 21) the halving of the original square side, and by noting down '1' (for the whole square side) and '30' (for the halved square side).

Then the first step of the actual computation is to compute

$1 \cdot ½ = ;30$ and sq. $;30 = ;15$ *ll.* 22-23

In the next step of the solution procedure is computed

(sq. 1 + sq. ;30 =) 1 + ;15 = 1;15 *l.* 24

What is going on here is, of course, the computation of the combined area of a "normalized reference pair of squares", beginning with a square of side 1.

Next, the ratio of the combined area of the first pair of squares to the combined area of the normalized reference pair of squares is computed in the following way:

$^1/_{1;15} = ;48\ (^4/_5)$, $;48 \cdot 2\ 05 = 1\ 40$ *ll.* 25-26

Since this ratio, obviously, is the square of the scaling factor *f*, the value of *f* is computed as

sqs. 1 40 = 10 (nindan) *l.* 27

Consequently, the lengths of the original square side can be computed as follows:

10 (nindan) · 1 = 10 nindan *l.* 28

It is interesting to observe that this apparently complicated solution procedure in terms of a reference pair of squares and a scaling factor is mathematically equivalent to the modern way of solving the stated problem by use of symbolic notation and algebraic manipulation of equations. Indeed, the modern solution procedure would be as follows:

sq. $m$ + sq. $(m/2) = 125 \leftrightarrow (1 + ¼)$ sq. $m = 125 \leftrightarrow$ sq. $m = 125/1.25 = 100 \leftrightarrow m = \sqrt{100} = 10$.

## 1.2 d. MS 3299 # 4. A problem for a single square ('equalside')

The text on the reverse of MS 3299, including the entire text of exercise # 4, is partly lost, partly damaged. Nevertheless, with some effort it has been possible to reconstruct the entire text of exercise # 4 with relative confidence. The reconstruction is based on the reasonable assumption that this exercise is closely related to the preceding exercise.

**MS 3299 # 4. Transliteration**

29 [*mi-it-ḫar-ti a-na* 2 *eḫ-pe uš-ta-ki-i*]*l*
30 [1$_{iku}$ aša$_5$ a.šà-*šu mi-it-ḫar-ti ki-i*]*a im-ta-*[*ḫ*]*ar*
31 [*at-ta i-na e-pe-ši-ka* 1 *a-n*]*a* 2 *te-ḫ*[*e-ep-pe*]*-ma* 30 *ta-am-ma-ar*
32 [30 *ù* 30 *tu-uš-ta-ak-kal*]*-ma* 15 *i-*⌈*na-ad-di*⌉*-ik-kum*
33 [*igi*(igi) 15 *tapaṭṭar*(duḫ)*-ma* 4 *ta-a*]*m-ma-ar*
34 [1$_{iku}$ aša$_5$ x-*ma* 1 40 *t*]*a-am-ma-ar*
35 [4 *a-na* 1 40 *eqlim*(a.šà) *t*]*u-*⌈*ub*⌉ *-b*[*a-a*]*l-*⌈*ma*!⌉ 6 40 *ta-*⌈*am*⌉*-ma-ar*
36 [*ip-se-e* 6 40 *te*]*-le-eq-q*[*é-ma* 2]0 *ta-am-ma-ar*
37 20 *a-na* ⌈1⌉ [*ta-na-aš*]*-ši-ma* 20 *mi-i*[*t-ḫar*]*-ta-ka*
38 (vacat) *ta-am-*⌈*ma*⌉*-ar*

**Translation**

29 I broke [my equalside (square side) in 2, I made it eat itself (I squared it)].
30 [1 iku was its field (area). How much was my equalside (square side)] equalsided (what was its squareside)?
31 [When you work it out, you shall break [1] in 2: you will see 30.
32 [You] shall make [30 and 30 eat] each other (multiply them): it will [give] you 15.
33 [You shall solve(calculate) the reciprocal of 15: you] will see [4].
34 [You shall x (convert) 1 iku:] you will see 1 40.
35 You shall carry (multiply) [4 by 1 40, the field (area)]: you will see 6 40.
36 [You] shall take [the equalside (square root) of 6 40:] you will see [20].
37 [You] shall raise (multiply) 20 by 1: 20, your equalside(square side),
38 you will see.

## MS 3299 # 4. Mathematical commentary

According to our tentative reconstruction above of the question in *ll.* 29-30, of which hardly a trace remains, the object considered in this exercise is a single square side. The square side is halved, and then the halved square side is multiplied by itself. The result is a square with the given area 1 iku. In quasi-modern symbolic notations:

sq. ($m$/2) = 1 iku = 1 40 sar(sq. nindan), $m$ = ? *ll.* 29-30

This is a really simple equation with the obvious solution

$m$/2 = sqs. 1 40 = 10 (nindan), so that $m$ = 2 · 10 (nindan) = 20 nindan.

However, here again the author of MS 3299 sticks to his preferred method of using normalized reference objects and scaling factors. Therefore, he starts by computing the length of a halved normalized reference square side, which is, of course,

(½ · 1 =) ;30 · 1 = ;30 *l.* 31

The area of the square on this halved normalized square side is then, just as obviously,

sq. ;30 = ;15 *l.* 32

and its reciprocal is

1/;15 = 4 *l.* 33

The given area of the square on the halved original square side, on the other hand, is

1 iku = 1 40 sar(sq. nindan) *l.* 34

The ratio of this given area to the area of the square on the halved reference square side is

(1/;15 · 1 40 =) 4 · 1 40 = 6 40 *l.* 35

Since this ratio is the square of the scaling factor $f$, it follows that the value of $f$ itself is

sqs. 6 40 = 20 (nindan) *l.* 36

Consequently, the original square side must be, as stated in the text,

20 (nindan) · 1 = 20 nindan *ll.* 37-38

## MS 3299. Conclusion

MS 3299 is a quite brief “recombination text” with exercises ## 1-2 borrowed from one theme text (theme: excavated pits) and exercises ## 3-4 borrowed from another, unrelated theme text (theme: squares). MS 3299 is in its own right also a theme text, with the obvious theme “reference objects and scaling factors”.

### 1.3. IM 54478, *UET 5*, 859 # 2, and CBS 12648, parallel texts to MS 3299 ## 1-2

As mentioned above, it is likely that the exercises ## 1-2 in MS 3299 were borrowed from a theme text about various kinds of ‘excavated pits’ (rectangular prisms). Another such exercise is **IM 54478** (Baqir, *Sumer* 7 (1951), 30), where the excavated pit is cube-shaped, just as in MS 3299 # 2. The text is from Tell Harmal (the Eshnunna region), thus belonging to Goetze’s group 7.

**IM 54478**

| | |
|---|---|
| 1 | *šum-ma ki-a-am i-ša-al-ka um-ma šu-ú-ma* |
| 2 | *ma-la uš-ta-am-ḫi-ru ú-ša-pí-il-ma* |
| 3-4 | *mu-ša-ar ù zu-uz mu-ša-ri* / *e-pé-ri a-su-uḫ* |
| 5 | *ki-ia uš-tam-ḫi-ir* / *ki ma-ṣí ú-ša-pi-il* |
| 6 | *at-ta i-na e-pé-ši-ka* |
| 7-8 | [1 *ù*] 12 *lu-pu-ut-ma i-gi* 12 *pu-ṭú-ur-ma* / [5 *ta-mar a-na*1] 30 *e-pí-ri-ka* |
| 9-10 | *i-ši-ma* 7 30 *ta-mar* 7 30 / *mi-nam* íb.sá 30 íb.sá 30 *a-na* 1 |
| 11-12 | *i-ši-ma* 30 *ta-mar* 30 *a-na* 1 *ša-ni-im* / *i-ši-ma* 30 *ta-mar* 30 *a-na* 12 |
| 13-14 | *i-ši-ma* 6 *ta-mar* 30 *mi-it-ḫa-ar-ta-ka* / 6 *šu-pu-ul-ka* |

**Translation**

| | |
|---|---|
| 1 | If (somebody) so asks you, saying this: |
| 2 | I made deep as much as I made equal to itself (squared). |
| 3-4 | I dug out a *mušar* and a half *mušar* / of earth. |
| 5 | How much (each way) did I make equal to itself, / how much did I make deep? |
| 6 | You, in your doing it: |
| 7-8 | [1 and] 12 note down, then the reciprocal of 12 solve, then / 5 you will see. To 1 30, your earth, |
| 9-10 | raise it, then 7 30 you will see. 7 30, / how much does it make equalsided? 30 equalsided. 30 to 1 |
| 11-12 | raise, then 30 you will see. 30 to the second 1 / raise, then 30 you will see. 30 to 12 |
| 13-14 | raise, then 6 you will see. 30 your equalside, / 6 your depth. |

The text is written entirely in syllabic Akkadian, with the exception of the word íb.sá, twice. (For this reading of the word, see Attinger, *ZA* 98 (2008)). Note that all the verbs in the solution procedure (except *tammar*) are in the *imperative*.

In the question, it is given that the depth of the excavated pit is equal to the side of the square cross section, and that the volume is 1 ½ *mušar* (sar). Precisely as in MS 3299 ## 1-2, the solution procedure begins by noting down some needed values, in this case (probably) 1 and 12 for the lengths of the side and the depth of a normalized reference cube. By division, it is shown that the given volume of the excavated pit is ‘7 30’ times the volume of the reference cube. Since the cube side (íb.sá!) of ;07 30 (1/8) is ;30 (½), the scaling factor is ;30, so that the lengths of the two sides of the square cross section, and of the depth, are

;30 · 1 = ; 30 (= ½ nindan), ;30 · 1 = ; 30 (= ½ nindan), and ;30 · 12 = 6 (cubits).

This is the same solution as in the case of MS 3299 # 2.

Another parallel to MS 3299 is ***UET 5*, 859 ## 1-2** (Friberg, *RA* 94 (2000), 143, 184), an Old Babylonian mathematical text from Ur, written almost entirely in Sumerian. In exercise # 1, a rectangular prism with a square cross section has the given cross section area 10;40 16 (area-sar) and the given volume 1 04;01 36 (volume-sar). It is stated directly that the square side of the cross section must be 3;16 (nindan), and it is shown through division that the depth must be 6 (cubits).

In exercise # 2 of the same text, the volume and the cross section area of a cube are given, and asked for are the length, the front, and the depth. No answer is given. Note that here, just as in the case of the problem in MS 3299 # 2, it would have been enough to let just the volume (or the area) be given! Here is the Sumerian text of the exercise:

***UET 5*, 859 # 2** (Ur)

1-2 2 sar 15 gín a.šà / 40 ½ sar saḫar
uš sag *ù* bùru.bi ìb.sá
3 uš sag *ù* bùru.bi en.nam

1-2 2 sar 15 shekels is the field(area). / 40 ½ sar is the earth (volume).
Its length, front, and depth are equal.
3 Its length, front, and depth are what?

By use of the same method as in MS 3299 # 2, for instance, it is easily shown that

40;30/12 = 3;22 30 = cu. 1;30, $u = s$ = 1;30 · 1 = 1;30 = 1 ½ nindan, $g$ = 1;30 · 12 = 18 cubits.

More like MS 3299 # 1 is the best preserved exercise (# 2) in the Nippur text **CBS 12648** (Proust, *TMN* (2007), 228; Muroi, *Cent*. 31 (1989)). It, too, is written almost entirely in Sumerian. The destroyed end of exercise # 2 is reconstructed below in the same form as the preserved ends of exercises ## 1 and 3.

**CBS 12648 # 2** (Nippur)

1 2 še igi.12.gál še [a]$^?$ ⸢túl⸣$^?$ /
2-4 $^2/_3$.bi uš.a.kam sag / šu.ri.a sag.gá.kam / bùru.bi /
5-6 uš.bi sag.bi / *ù* bùru.bi [en.nam] /
7-10 uš [sag] / *ù* bùru.bi / ub.te.gu$_7$-*ma* / igi.bi e.duḫ-*ma*
11-14 saḫar.šè ba.e.ìl-*ma* / ib.sá / 15 37 30 / e$_{11}$.dè
15-[16] ib.sá 15 37 30 / [2 30]
*etc.* [uš.šè sag.šè *ù* bùru.še ba.e.íl-*ma*]
[uš.bi sag.bi *ù* bùru.bi ba.zu.zu.un]
[uš.bi ½ kùš sag.bi $^1/_3$$^1/_3$ kùš *ù* bùru.bi 5 šu.si]

1 2 barleycorns and a 12th-part of a barleycorn was (the volume of) a pit$^?$.
2-4 $^2/_3$ of the length was the front. / Half the front / was the depth.
5-6 Its length, its front, / and its depth were what?
7-10 The length, the front, / and the depth/ let eat each other, then / its reciprocal you solve.
11-14 To the earth you raise it, then / the equalside / of 15 37 30 / will come up.
15-[16] The equalside of 15 37 30 / [is 2 30].
*etc.* [To the length, to the front, and to the depth you raise it, then
its length, its front, and its depth you will know.
Its length is ½ cubit, its front is $^1/_3$ cubit, and its depth is 5 fingers].

In this exercise, a rectangular prism (actually in the size and form of a brick of the most common Old Babylonian format) has the given volume 2 $^1/_{12}$ barleycorns (= ;00 00 41 40 volume-sar). The front ($s$) is $^2/_3$ of the length ($u$), and the depth ($b$) ½ of the front. Consequently, the volume of a normalized reference prism is 1 · $^2/_3$ · (½ · $^2/_3$ · 12) = 1 · ;40 · 4 = 2;40 ($^8/_3$). The ratio of the given volume to the volume of the reference prism can then be computed as follows:

½;40 = ;22 30, and ;22 30 · ;00 00 41 40 = ;00 00 15 37 30.

Since the cube side of ;00 00 15 37 30 is ;02 30, it follows that

$u$ = ;02 30 nindan = ½ cubit, $s = \frac{2}{3} \cdot$ ½ cubit = $\frac{1}{3}$ cubit, and $g$ = ½ $\cdot \frac{1}{3}$ cubit = 5 fingers.

Note the suppression of most of the numerical details in the solution procedure above. This is a very unusual feature of an Old Babylonian mathematical exercise.

Note also that the computations were carried out without any use of modern notations, which required a clear understanding of orders of magnitude. Indeed, without the use of zeros and semi-colons, the Old Babylonian scribe could only see that the cube side of 15 37 30 is 2 30, where 2 30 must be interpreted, not as 2 ½ nindan, but as $\frac{1}{60}$ of 2 ½ nindan, because it was given that the volume of the prism should be as small as only 2 $\frac{1}{12}$ barleycorns.

## 1.4. An examination of the repertory of mathematical terms used in MS 3299

It was shown by Goetze (Neugebauer and Sachs, *MCT* (1945), Ch. 4) how unprovenanced Old Babylonian mathematical cuneiform texts can be divided into various internally connected groups by use of a detailed orthographic analysis of *the spelling of Akkadian words* in the texts. Goetze's analysis would have shown that MS 3299 is a "southern" text, in view of spellings such as

| | | |
|---|---|---|
| *as-sú-uḫ* | *sú* = ZU and *as* = AZ | *l.* 9; Goetze's criteria S 2-3 |
| *e-pe-ši-ka* | *pe* = PI | *l.* 3; Goetze's criterion S 4 |
| $\text{gu}_7^{kal}$ | phon. compl. $c + v + c$ | *l.* 5; Goetze's criterion S 7 |

Later, Goetze's analysis was extended and refined by Høyrup in several publications, most recently in *LWS* (2002), Ch. 9. At about the same time, it was shown by Friberg in *RA* 94 (2000), Sec. 7 b, how also an analysis of *the Sumerian terminology*, in the form of so called "Sumerograms", may provide important clues to the provenance of unprovenanced mathematical cuneiform texts. This new method presents an interesting alternative to the Goetze/Høyrup method of classification. Luckily, there turned out to be no conflict between the outcome of the two methods.

In the case of the new text MS 3299, which is of unknown provenance, and almost entirely written in Akkadian, the astonishingly rich repertory of Akkadian mathematical terms in the text will allow it to be shown most conclusively that MS 3299 belongs to the Goetze/Høyrup/Friberg group 1 a, which implies that the text is from Larsa.

Surprisingly, the analysis below demonstrates that in certain instances also *grammatical considerations* can be used as a third method for the classification of an unprovenanced Old Babylonian mathematical text. Indeed, it will be shown that when *verbs in the second person singular*, *durative tense*, are used consistently in the solution procedure of such a text, then that text is with very high probability from Larsa, belonging to one of the groups 1a, 1 b, or 1 c (defined in Friberg, *op. cit.*, 160-162).

Here follows a list of all mathematical terms appearing in the text of MS 3299, with explicit indication of the spelling of the Akkadian words in each case:

| | | | |
|---|---|---|---|
| *mi-it-ḫar-tum* | "equalside" | square (side); < *maḫārum* 5 'to be equal' | *ll.* 17, 37 |
| GAM.DA ($\text{burud}_\text{x}$.da)? | depth | depth; < bùru(d) *šuplum* 'depth' | *ll.* 4, 18 |
| *ma-li* (= *ma-la*) | as much as | as much as | *ll.* 10, 12 |
| *mi-nu* | what? | question | *l.* 2 |
| *im-ta-ḫar, im-taḫ-ru* | it was "equalsided" | it was a square side; < *maḫārum* 5 | *ll.* 10, 12, 20, 30 |
| *iš-pil, iš-pi-lu* | it was deep | its depth was; < *šapālum* 'to be deep' | *ll.* 10, 11, 12 |
| *ki-ia*(*-a*) *im-ta-ḫar* | how much each equalside? | question about more than one | *ll.* 11, 20, 29 |
| *ki ma-aṣ* (= *ki ma-ṣi*) | how much? | question; < *maṣûm* 'to be enough' | *l.* 11 |
| *i-na* (*a*) (*b*) *as-sú-uḫ* | from *a b* I tore out | I computed $a – b$; < *nasāḫum* 'to tear out' | *l.* 9 |
| (*a*) *ù* (*b*) *uš-ta-ki-il* | I let *a* and *b* eat each other | I computed $a \cdot b$; < *akālum* 'to eat' | *l.* 19 |

| | | | |
|---|---|---|---|
| (*a*) *a-na* 2 *eḫ-pe* | (*a*) in 2 I broke | I computed *a*/2; < *ḫepûm* 'to break' | *l.* 19 |
| *at-ta i-na e-pe-ši-ka* | when you work it out | solution procedure; < *epēšum* 'to do' | *ll.* 3, 12, 20 |
| *aš-šum* · · · *qa-bu-ú* | since · · · it was stated | recall an assumption; < *qabûm* 'to say' | *ll.* 12, 21 |
| *ta-la-ap-pa-at, lu-pu-ut* | (you shall) note down | recall given numbers; < *lapātum* 'to touch' | *ll.* 4, 13, 21 |
| (*a*) *a-na* (*b*) *tu-uṣ-ṣa-ab* | *a* to *b* you shall add | compute $a + b$; < *waṣābum* 'to add on' | *l.* 24 |
| (*a*) *a-na* šà (*b*) daḫ | *a* onto *b* (you shall) add | compute $a + b$; daḫ = *waṣābum* 'to add on' | *l.* 19 |
| (*a*) *a-na* (*b*) *ta-na-aš-ši* | *a* to *b* you shall raise | compute $a \cdot b$; < *našûm* 'to raise' | *ll.* 17, 28 |
| (*a*) *a-na* (*b*) íl | *a* to *b* (you shall) raise | compute $a \cdot b$; íl = *našûm* 'to raise' | *l.* 6 |
| (*a*) *a-na* (*b*) *tu-ub-ba-al* | *a* to *b* you carry | compute $a \cdot b$; < *wabālum* 'to carry' | *ll.* (5), 8, 16 |
| (*a*) *ù* (*b*) *tu-uš-ta-ak-kal* | *a* and *b* you shall let eat each other | compute $a \times b$; < *akālum* 'to eat' | *l.* 23 |
| (*a*) *ù* (*b*) $gu_7^{kal}$ | *a* and *b* you shall let eat each other | compute $a \times b$; $gu_7$ = *akālum* | *l.* 5 |
| (*a*) *a-na* 2 *te-ḫe-ep-pe* | (*a*) in 2 you shall break | compute *a*/2 (halve *a*); < *ḫepûm* 'to break' | *ll.* 22, 31 |
| igi (*n*) duḫ (or $du_8$) | solve the reciprocal of *n* | compute $^1/n$; duḫ = *paṭārum* 'to loosen' | *ll.* 4, 13, 16, 25 |
| *ip-se-e* (*a*) *te-le-eq-qé* | the equalside of *a* you shall take | compute $\sqrt{a}$; < *leqûm* 'to take' | *ll.* 7, [27, 36] |
| *i-na-ad-di-ik-kum* | it will give to you | the result is; < *nadānum* 'to give' | *ll.* 5, 7, 8, *etc.* |
| *ta-am-ma-ar* | you will see | the result is; < *amārum* 'to see' | *ll.* 24, 25, 28, 31, *etc.* |

This list of mathematical terms in MS 3299 can be compared with the following fairly complete list of occurrences of (some of) the same terms in other published Old Babylonian mathematical cuneiform texts. The texts in question were all initially unprovenanced, except, of course, the texts from Eshnunna, and the single text from Nippur.

| | | | | |
|---|---|---|---|---|
| *at-ta i-na e-pe-ši-i-ka* | AO 6770 (*MKT II*, 37) | *obv.* 2 | gr 1 a | Larsa |
| *i-na-ad-di-nam* | – | *obv.* 17; *rev.* 10 | – | – |
| *at-ta-na-aš-ši, a-na-aš-ši* | – | *rev.* 9, 18 | – | – |
| *tu-uš-ta-ka-al* | – | *obv.* 7 | – | – |
| *at-ta i-na e-pe-ši-i-ka* | AO 8862 (*MKT I*, 108) | *face* I, 8 | gr 1 a | Larsa |
| *ta-la-pa-at* | – | *face* II, 5, 22 | – | – |
| *tu-uṣ-ṣa-ab* | – | *face* II, 27, 30, *etc.* | – | – |
| *ta-na-sà-aḫ* | – | *face* I, 22 | – | – |
| *ub-ba-al* | – | *face* II, 9 | – | – |
| *uš-ta-kal* | – | *face* II, 13 | – | – |
| *i-na-di-ku*(*m*) | – | *face* II, 15, 20 | – | – |
| *te-ḫe-ep-pe-e, te-ḫe-pe-e* | – | *face* I, 12; II, 19, *etc.* | – | – |
| *uš-ta-kal* | – | *face* I, 1, 24, *etc.* | – | – |
| *i-na-an-di-kum* | YBC 4675 (*MCT* B, 45) | *obv.* 11; *rev.* 1 | gr 1 a | Larsa |
| *te-ḫe-pe-e* | – | *obv.* 8, 18; *rev.* 9 | – | – |
| *tu-uṣ-ṣa-ab* | – | *rev.* 4, 13 | – | – |
| *ta-na-aš-ši* | – | *obv.* 11, 20; *rev.* 2 | – | – |
| *tu-uš-ta-ak-ka-al* | – | *obv.* 12; *rev.* 15 | – | – |
| *te-le-qé-e* | – | *obv.* 15 | – | – |
| *ta-ta-na-aš-ši* | YBC 7997 (*MCT* Pa, 98) | *rev* 7 | gr 1 a | Larsa |
| *tu-ub-ba-al* | – | *rev.* 2 | – | – |
| *tu-uš-ta-kal* | – | *obv.* 4 | – | – |
| *ma-li* | YBC 9856 (*MCT* Q, 99) | *obv.* 2, 3, 6, 8 | gr 1 a | Larsa |
| *ta-at-ta-na-aš-ši-i* | YBC 9874 (*MCT* M, 90) | *rev.* 5, 7, 9 | gr 1 a | Larsa |
| *i-na-ad-di-ik-ku*(*m*) | – | *rev.* 8, 11 | – | – |
| *i-na e-pe-ši-i-ka* | YBC 6504 (*MKT III*, 22) | *obv.* 3, 12 | gr 1 b | Larsa |
| *ta-na-aš-ši* | – | *obv.* 16 | – | – |
| *te-ḫe-ep-pe* | – | *obv. 5,* 17, *etc.* | – | – |
| *tu-ṣa-ab* | BM 13901 (*MKT III*, 1) | *obv. i*: 3, 7, *etc.* | gr 1 c | Larsa |
| *ta-la-pa-at* | – | *obv. i,* 12, 19, *etc.* | – | – |
| *ta-na-ši* | – | *obv. i*: 11, 15, *etc.* | – | – |
| *tu-uš-ta-kal* | – | *obv. i*: 1, 2, 27, *etc.* | – | – |
| *te-ḫe-pe* | – | *obv. i*: 13 | – | – |
| *i-na-*(*ad-*)*di-ik-ku*(*m*) | YBC 4662 (*MCT*, 71) | *obv*. 5, 6, *etc.* | gr 2 a | southern (Ur???) |

| | | | | |
|---|---|---|---|---|
| *ta-mar* | – | *rev.* 22, 33, 34, 35 | – | – |
| *i-na-ad-di-ik-ku-um*, *etc.* | YBC 4663 (*MCT*, 69) | *obv*. 3, 11, *etc.* | gr 2 a | southern (Ur???) |
| *ta-na-aš-ši* | YBC 4608 (*MCT* D, 49) | *obv*. 19, 20 | gr 3 | Uruk |
| *ma-li* | —— | *rev.* 15 | – | – |
| *tu-uṣ-ṣa-ab* | Str 362 (*MKT I*, 239) | *obv*. 14 | gr 3 | Uruk |
| *ta-na-aš-ši* | – | *obv*. 8 | – | – |
| *te-le-qé-e* | Str 366 (*MKT I*, 257) | *obv*. 8 | gr 3 | Uruk |
| *ta-mar* | MLC 1950 (*MCT* Ca, 48) | *obv*. 5 | gr 3 | Uruk |
| *i-na-di-ku*(*m*) | YBC 8633 (*MCT*, 53) | *rev.* 7, 9 | gr 4 b | Uruk? |
| *i-na e-pe-ši-ka* | CBS 11681 (*TMN*, 224) | *obv*. 4; *rev*. 4 | no group | Nippur |
| *tu-ub-ba-al* | – | *obv*. 7 | – | – |
| *ta-la-ap-pa-at* | – | *rev*. 6 | – | – |
| *tu-uš-ta-ka-al* | – | *rev*. 7 | – | – |
| *at-ta i-na e-pé-ši-ka* | MLC 1842 (*MCT*, 106) | *obv*. 7 | gr 5 | northern? |
| *at-ta i-na e-pé-ši-ka* | IM 54478 (*Sumer* 7, 30) | *l*. 6 | gr 7 a | Eshnunna |
| *ki-ia* & *ki ma-ṣí* | – | *ll*. 4-5 | – | – |
| *ta-mar* | – | *ll*. 10-13 | – | – |
| *at-ta i-na e-pé-ši-ka* | IM 53953 (*Sumer* 7, 31) | *obv*. 5 | gr 7 a | Eshnunna |
| íb.si-*e* | – | *rev*. 3, 4 | – | – |
| *at-ta i-na e-pé-ši-ka* | IM 53965 (*Sumer* 7, 39) | *obv*. 6 | gr 7 a | Eshnunna |
| íb.si-*e* | – | *rev*. 6 | – | – |
| *ta-mar* | – | *rev*. 3, 4, 5 | – | – |
| *i-na-di-na-ku-um* | IM 54464 (*Sumer* 7, 43) | *rev*. 7 | gr 7 a | Eshnunna |
| *at-ta i-na e-pé-ši-ka* | $Db_2$-146 (*LWS*, 258) | *obv*. 3, 18 | gr 7 b | Eshnunna |
| *ta-mar* | IM 52301 (*Sumer 6*, 130) | *obv*. 21, 22, *etc.* | gr 7 b | Eshnunna |
| *tu-uš-ta-ka-al* | – | *edge* ii: 3 | – | – |

The list above of mathematical terms appearing in the text of MS 3299 makes the following structuration of the text immediately obvious:

1. *In the statement of the problem* (*the question*), the verbs are in the *first or third person singular, preterite.*
2. The solution procedure is preceded by the phrase *atta ina epēšika* 'you, in your doing it'.
3. *In the solution procedure* the verbs are in *the second person singular, durative.* The only exception is *lu-pu-ut* (imperative!) in *l*. 21.

(Verbs in the preterite are translated in this paper as verbs in the past tense, while verbs in the second person singular, durative, are translated as "you shall" or "you will" do so and so. Cf. the discussion of the grammatical structure of "rational practice texts" in Friberg (*AfO* 52, to appear), a paper about Babylonian tuning algorithms).

The use of the first or third person, preterite, in the question but the second person, durative, in the solution procedure is what is called the "standard format" for Old Babylonian mathematical problem texts in Høyrup, *LWS* (2002), 32.

In a few cases, the otherwise dominant use of Akkadian in MS 3299 is broken by the use of Sumerograms, as in the phrases

(*a*) *a-na* šà (*b*) daḫ, (*a*) *a-na* (*b*) íl, (*a*) *ù* (*b*) $gu_7$*kal*, and igi (*n*) duḫ.

The form of the phonetic complement in the third of these phrases shows, unambiguously, that also the Sumerograms were intended to be read as verbs in the durative tense.

Surprising is the appearance of the term (*a*) *a-na* (*b*) íl in *obv*. 5 of MS 3299, since it was stated in Friberg, *RA* 94 (2000), 166, on the evidence then available, that "the appearance of this phrase in a given text is sufficient to indicate that the text belongs to group 4 a!".

The comparative list above is intended to be a fairly complete enumeration of all the cases when mathematical terms parallel to the terms in MS 3299 appear in other published Old Babylonian mathematical problem texts. Unexpectedly, this comparative list shows that Høyrup's "standard format" is a standard format *only in mathematical problem texts belonging to group 1*, that is, in texts from Larsa. Conversely, it is easy to check that *all* known such texts use the format!

This observation, which ties together all the texts from group 1 in a simple way is all the more welcome in view of the following negative statement in Høyrup, *LWS* (2002), 338:

> "Beyond the shared orthographic characteristics noticed by Goetze, there is little that keeps the texts in question (that is, the texts in group 1) together as a coherent group".

The few instances when Akkadian verbs in the second person singular, durative tense, appear in texts belonging to the other "southern" groups, namely group 2 (Ur???) and group 3 (Uruk), must be considered as occasional deviations from the normal formats of those groups. Indeed, verbs are usually written as Sumerograms in texts from groups 2 and 3. See Friberg, *RA* (2000), 162-164.

Quite interesting, and quite curious, is the systematic appearance of verbs in the second person singular, durative tense, also in the Nippur text CBS 11681 (Proust, *TMN* (2007), 224). However, a look at the other two known mathematical problem texts from Nippur, namely CBS 19761 (*ibid.*, 234) and CBS 12648 (*ibid.*, 228) reveals that there is no common format shared by the three texts from Nippur. (See *ibid.*, 238-239). It is, for that reason, quite possible that CBS 11681 was, in some way, imported to Nippur from Larsa. (Either the clay tablet itself may have come from Larsa, or the person who wrote it).

Quite interesting, as well, is the considerable number of parallels to certain parts of the terminology of MS 3299 in several tablets belonging to the groups 7 a-b, that is in texts from Eshnunna. Note, in particular, that the phrase *atta ina epēšika* is characteristic, on one hand for texts belonging to group 1, on the other hand also for texts belonging to group 7. Does this mean that the two groups were in some way connected with each other? (See the discussion in Høyrup, *LWS* (2002), 358-361). Or does it mean that the phrase in both cases was a direct translation of a corresponding Sumerian phrase (such as za.e ak.da.zu.dè in some texts from groups 2 and 3) in as yet undiscovered Sumerian mathematical problem texts from the Ur III period, older than both the mathematical problem texts from Larsa and those from Eshnunna? This question is all the more interesting since it is known that the mathematical problem texts from Larsa and Eshnunna are older than all other Old Babylonian mathematical problem texts, except possibly those from Ur (Høyrup, *op. cit.*, 359)[5].
In MS 3299 # 2, the values for the sides of a cubic excavated pit are asked for with the phrase

[*ki-i*]*a im-*⸢*ta*⸣-[*ḫar*] *ù* ⸢*ki*⸣ *ma-aṣ iš-pil*
'How much (each way) was it equalsided (square), and how much was it deep?'.

Note the following parallel phrase in the Eshnunna text IM 54478, *obv*. 4-5 (above, sec. 1.3):

*ki-ia uš-tam-ḫir / ki ma-ṣí ú-ša-pí-il*
'How much did I make equal to itself (each way), / how much did I make deep?'.

A third, partial, parallel in CBS 43, *obv*. 2, 3, *etc*., (Robson, *SCIAMVS* 1 (2000), 39) is

*mitḫarti*(LAGAB$^{ti}$) *ki-ia im-ta-ḫar*
'My equalside, how much (each way) was it equalsided?'.

CBS 43 is an unprovenanced text (*contra* Høyrup, *op. cit.*, 253, fn. 284). It cannot be from Larsa, since

---

[5] Remember that it is now known that solutions to quadratic equations appear in at least one text firmly dated to the Sumerian Ur III period. This means that Old Babylonian mathematical texts with "metric algebra" problems must have had Sumerian predecessors, even if a documentation of this fact is otherwise non-existent. See Friberg, *MSCT* 1 (2007), 145-146, and Friberg, *CDLJ* 2009-3.

the verbs in the text are written as Sumerograms, except on one occasion when a verb is written in the imperative.

Perhaps the greatest surprise in the list above of mathematical terms appearing in MS 3299, which has now been shown to be a group 1 a text, is the appearance there of the term *ta-am-ma-ar* ‘you will see’, alternating with *i-na-ad-di-ik-kum* ‘it will give to you’, used to announce the results of computations. According to Høyrup, *op. cit.*, 360,

> “An easily perceived characteristic of most periphery groups (by which H. means groups 6 (Sippar), 7 (Eshnunna), and 8 (Susa)) is the use of “tammar” you see, when results are announced, borrowed from the lay traditions. The term is absent from all core groups (by which H. means the southern groups 1-4)”, *etc*.

It is now clear that Høyrup’s cited statement must be modified as follows: The term *ta-mar* is common in texts from groups 6, 7, and 8, but it also appears in one group 3 text (MLC 1950), in one group 2 text (YBC 4662), and in one group 1 a text (MS 3299), where it has the plene spelling *ta-am-ma-ar* (not before documented in mathematical texts, although it is common elsewhere).

## 2. MS 3976. A linear equation: Finding an original amount of barleycorn

### 2.1. Presentation of the text

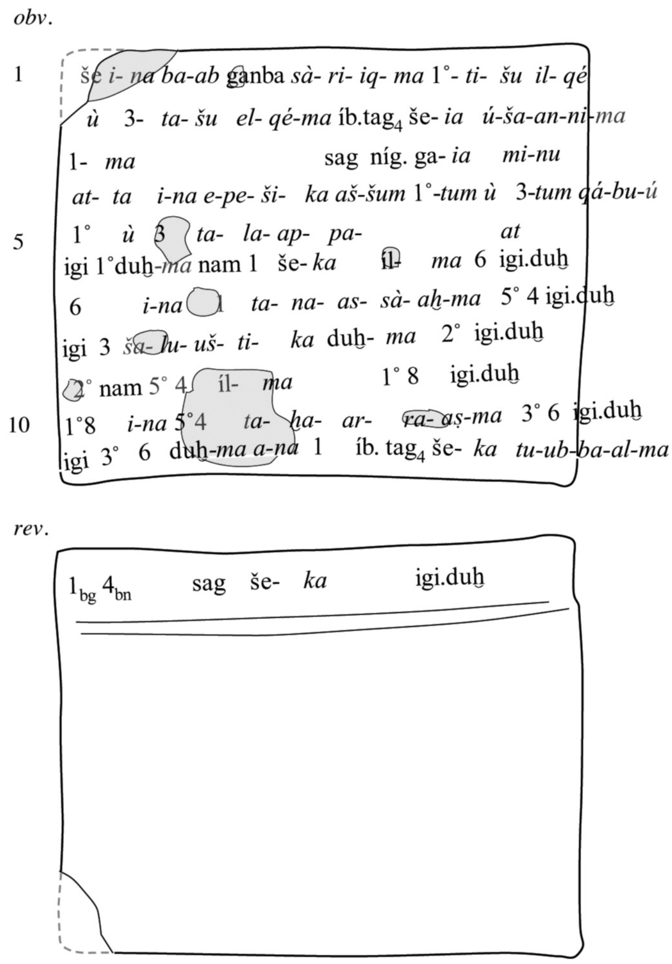


Fig. 2.1. MS 3976. Conform transliteration. Scale 1:1.

MS 3976 is a fairly well preserved clay tablet inscribed with 11 + 1 lines of text in one column. The text contains the statement of a single problem, a solution procedure, and the answer. The task is to find an original amount of barleycorn after a series of transactions, counting backwards from a given residual quantity.

**MS 3976. Transliteration**

1 [*še'*(še)-*i*] ⸢*i-na*⸣ *ba-ab maḫīrim*(ganba) *sà-ri-iq-ma ešrī*(10)-*ti-šu il-qé*
2 *ù šaluš*(3)-*ta-šu el-qé-ma rīḫti/šitti*(íb.tag$_4$) *še'i*(še)-*ia ú-ša-*⸢*an*⸣-*ni-*⸢*ma*⸣
3 1-*ma rēš*(sag) *makkūri*(níg.ga)-*ia mi-nu*
4 *at-ta i-na e-pe-ši-ka aš-šum ešrī*(10)-*tum ù šaluš*(3)-*tum qá-bu-*⸢*ú*⸣
5 10 *ù* ⸢3⸣ *ta-la-ap-pa-at*
6 *igi*(igi) 10 *tapaṭṭar*(duḫ)-⸢*ma*⸣ *ana*(nam) 1 *še'i*(še)-*ka tanašši*(íl)-*ma* 6 *tammar*(igi.duḫ)
7 6 *i-na* ⸢1⸣ *ta-na-as-sà-aḫ-ma* 54 *tammar*(igi.duḫ)
8 *igi*(igi) 3 ⸢*ša*⸣-*lu-uš-ti-ka tapaṭṭar*(duḫ)-*ma* 20 *tammar*(igi.duḫ)
9 ⸢20⸣ *ana*(nam) ⸢54 *tanašši*(íl)⸣-*ma* 18 *tammar*(igi.duḫ)
10 18 *i-*⸢*na*⸣ 5[4 *t*]*a-ḫa-ar-*⸢*ra-aṣ*⸣-*ma* 36 *tammar*(igi.duḫ)
11 *igi*(igi) 36 *tapaṭṭ*[*ar*(duḫ)-*ma*] ⸢*a-na*⸣ 1 *rīḫti/šitti*(íb.tag$_4$) *še'i*(še)-*ka tu-ub-ba-al-ma*
12 1 (bariga) 4 (bán) *rēš*(sag) *še'i*(še)-*ka tammar*(igi.duḫ)

**Translation**

1 [My barleycorn] was stored in the market gate, then (someone) took a 10th of it,
2 and I took a 3rd of it, then I remeasured the remainder of my barleycorn:
3 precisely 1 (bariga). What was the initial amount of my asset?
4 When you work it out, since "a 10th and a 3rd" were stated,
5 you shall note down 10 and 3.
6 You shall solve (calculate) the reciprocal of 10, then you shall raise (multiply) it by 1, your barleycorn: you will see 6.
7 You shall tear out (subtract) 6 from 1: you will see 54.
8 You shall solve (calculate) the reciprocal of 3, your 3rd: you will see 20.
9 You shall raise (multiply) 20 by 54: you will see 18.
10 You shall break off (subtract) 18 from 54: you will see 36.
11 You shall solve (calculate) the reciprocal of 36, then you shall carry (multiply) it by 1, the remainder of your barleycorn
12 you will see 1 (bariga) 4 (bán), the initial amount of your barleycorn.

## MS 3976. Mathematical commentary

The given remainder of barleycorn in this problem is 1 bariga = 1 00 (= 60) sìla[6], where the sìla is a Sumerian/Babylonian capacity unit equal to about 1 liter. In terms of modern symbolic notations, suppose that the original amount of barleycorn was $b$. Then the question in this exercise can be expressed in the following form:

$b - b \cdot 1/10 - (b - b \cdot 1/10) \cdot 1/3 = 1\ 00$ sìla, $b = ?$ *ll.* 1-3

Or, more compactly:

$b \cdot (1 - 1/10) \cdot (1 - 1/3) = 1\ 00$ sìla, $b = ?$

[6] The final double zero indicating multiplication by sixty is used here for the readers' convenience. It has no counterpart in the way in which sexagesimal numbers are expressed in Old Babylonian cuneiform texts.

This is a linear equation for the single unknown *b*. It is easy to solve the linear equation, for instance by computing the product $f = (1 - 1/10) \cdot (1 - 1/3)$ and dividing $r$ = 1 00 by *f*. However, this is not exactly the way the answer to the problem is found in the solution procedure of MS 3976. Instead, the solution proceeds in the following steps, making use of a normalized reference object (a "false assumption") and a scaling factor:

| | |
|---|---|
| It is (silently) assumed that the initial amount of barleycorn was 1 (00) (sìla). | |
| Then, in step 1, 1 (00) - 1 (00) · $1/10$ = 1 (00) - 6 = 54. | *ll.* 6-7 |
| Next, in step 2, 54 - 54 · $1/3$ = 54 - 18 = 36. | *ll.* 8-10 |
| However, the residual amount should be 1 00, not 36. To get the correct residual amount, the initial amount must be scaled up in the proportion 1 00 / 36 = 1;40. | |
| Consequently, the initial amount was 1 (00) sìla · 1 00 / 36 = 1 (00) sìla · 1;40 = 1 40 sìla = 1 bariga 4 bán | *ll.* 11-12 |

It is easy to check the correctness of this result. Indeed,

1 40 - 1 40 · $1/10$ = 1 40 - 10 = 1 30,   1 30 - 1 30 · $1/3$ = 1 30 - 30 = 1 00.

## 2.2. The mathematical terminology used in MS 3976

Goetze's analysis (*MCT* (1945), 147) would have shown that MS 3976 is a "southern" text, in view of spellings such as

| | | |
|---|---|---|
| *sà-ri-iq* and *ta-na-as-sà-aḫ* | *sà* = ZA | Goetze's criterion S 2 |
| *e-pe-ši-ka* | *pe* = PI | Goetze's criterion S 4 |

More information can be obtained from the mathematical terminology used in MS 3976. First there are a number of terms which also appear in MS 3299 (see above). These are:

| | | | |
|---|---|---|---|
| igi (*n*) duḫ (= $du_8$) | solve the reciprocal of *n* | compute $1/n$ | *ll.* 2, 11 |
| *mi-nu* | what? | = ? | *l.* 3 |
| *at-ta i-na e-pe-ši-ka* | when you work it out | solution procedure: | *l.* 4 |
| *aš-šum* · · · *qá-bu-ú* | since · · · it was stated | recall an assumption | *l.* 4 |
| (*a*) *a-na* (*b*) *tu-ub-ba-al* | *a* to *b* you shall carry | compute $a \cdot b$; < *wabālum* 'to carry' | *l.* 11 |
| *ta-la-ap-pa-at* | you shall note down | recall given numbers | *l.* 5 |

The last two terms are in the second person singular, durative, which is enough to show that MS 3976 belongs to group 1 a, just like MS 3299.

The list below of further mathematical terms appearing in MS 3976 includes two more verbs in the second person singular, durative, which confirms this conclusion.

| | | | |
|---|---|---|---|
| íb.$tag_4$ | remainder | the result of a subtraction | *ll.* 2, 1 |
| *ša-lu-uš-ti* | third | $1/3$ | *l.* 8 |
| *ú-ša-an-ni* | I remeasured | < *šanûm* II 'to repeat' | *l.* 2 |
| (*a*) nam (*b*) *íl* | *a* to *b* (shall) raise | compute $a \cdot b$; íl = *našûm* 'to raise' | *ll.* 6, 9 |
| igi.duḫ | you (will) see | the result is | *ll.* 7-10, 12 |
| (*a*) *i-na* (*b*) *ta-na-as-sà-aḫ* | *a* from *b* you shall tear out | compute $a - b$; < *nasāḫum* 'to tear out' | *l.* 7 |
| (*a*) *i-na* (*b*) *ta-ḫa-ar-ra-aṣ* | *a* from *b* you shall break off | compute $a - b$; < *ḫarāṣum* 'to break off' | *l.* 10 |

Of these new terms, (*a*) nam (*b*) íl and igi.duḫ are peculiar to MS 3976; they do not appear in any other known "southern" Old Babylonian mathematical cuneiform texts.

Here follows an account of occurrences of (some of) the terms listed above in other published Old Babylonian mathematical cuneiform texts.

| | | | | |
|---|---|---|---|---|
| *ta-na-as-sà-aḫ* | AO 6770 (*MKT II*, 37) | *obv*. 4 | gr 1 a | Larsa |
| *a-ḫa-ar-ra-aṣ* | – | *rev*. 8 | – | – |
| *ša-lu-uš-ti* | AO 8862 (*MKT I*, 109) | *face* I: 33 | gr 1 a | Larsa |
| *a-na-sà-aḫ, ta-na-sà-aḫ* | – | *face* I: 15, 22*, etc.* | – | – |
| *ta-ḫa-ar-ra-aṣ* | – | *face* I: 20 | – | – |
| *ta-ḫa-ar-ra-aṣ* | YBC 4675 (*MCT* B, 45) | *obv.* 14; *rev*. 5, 14 | gr 1 a | Larsa |
| *ša-lu-uš-ti* | YBC 9856 (*MCT* Q, 99) | *obv.* 7 | gr 1 a | Larsa |
| [*ta*]?*-na-as-sà-ḫu-ú* | Plimpton 322 (*MCT* A, 38) | *col. i*: 2 | no group | Larsa |
| *ša-lu-uš-ti* | BM 13901 (*MKT III*, 1) | *obv. i*: 9 | gr 1 c | Larsa |
| *ta-na-sà-aḫ* | – | *obv. ii*: 2, 7*, etc.* | – | – |
| igi.duḫ | YBC 4669 (*MKT III*, 26) | *rev. i*: 5, 7 | gr 2 b | Uruk |
| igi.duḫ | YBC 4673 (*MKT III*, 29) | *rev. iii*: 9, 13, 16, 20 | gr 2 b | Uruk |
| igi.dù | IM 55357(*LWS*, 232) | *obv*. 7, 8, *etc*. | gr 7 b | Eshnunna |
| (*a*) nam (*b*) íl | – | *obv*. 10, 13 | – | – |

This list of parallel occurrences looks just like the corresponding list in the case of the text MS 3299 (section 1.4 above). Thus it strengthens the conclusions drawn from that list, in particular the surprising observation that there are unexpected similarities between the terminology used in texts from group 1 a (Larsa) and in texts from group 7 (Eshnunna). As in that case, it is tempting to conjecture that the existence of some of the similarities between mathematical problem texts from Larsa and mathematical problem texts from Eshnunna may have been caused by the existence of common Sumerian predecessors to both groups of text. In particular, the Sumerian phrases igi.duḫ and (*a*) nam (*b*) íl can then be suspected of having been borrowed directly from older Sumerian mathematical problem texts.

Note: It is not at all clear why in MS 3976 (text no. 2) two different verbs, *nasāḫum* and *ḫarāṣum,* denote the action of subtraction. The verb *nasāḫum* is used in *l.* 7 when $^{1}/_{10}$ of 1 is subtracted from 1, and the verb *ḫarāṣum* is used in a perfectly similar situation in *l.* 10 when $^{1}/_{3}$ of 54 is subtracted from 54.

Similarly, it is not at all clear why in text no. 2 two different verbs, *našûm* (= íl) and *wabālum*, denote the action of multiplication.

The verb íl is used in lines 6 and 9, when 1 is multiplied by $^{1}/_{10}$, and when 54 is multiplied by $^{1}/_{3}$, respectively, while the verb *wabālum* is used in a fairly similar situation in line 11, when 1 is multiplied by $^{1}/_{36}$.

Also in MS 3299 (text no. 1) the two verbs *našûm* (= íl) and *wabālum* are used indifferently for the action of multiplication. The former is used, for instance, when a scaling factor multiplies 1 and 12 in *l.* 8, while the latter is used in a similar situation in *l.* 17 when the scaling factor 30 multiplies 1 and 12.

It is clear, on the other hand why *šutākulum*, the Št stem of *akālum* (= gu$_7$), is used to denote multiplication in *ll*. 5 and 23 of text no. 1. See fn. 4 above.

### 2.3. IM 53957, YBC 4669 B 4, YBC 4652, and AO 6770 # 3, parallel texts to MS 3976

Only four previously published cuneiform mathematical texts feature single linear equations like MS 3976, and the previous presentation and interpretation of all four texts is unsatisfactory in one or several respects. For that reason it seems to be well motivated to subject the four texts to a renewed discussion below.

A first example is **IM 53957** (***Sumer* 7 (1951), 37)**, No. 5 of the Old Babylonian mathematical texts from Tell Harmal published by T. Baqir in *Sumer* 7. This text is a parallel to MS 3976. Below is presented an amended transliteration, translation, and interpretation of the text.

**IM 53957** (group 7 a, Eshnunna)

1 *šum-ma* [*ki-a-am i-ša-al-ka um-ma šu-ú-ma*]
2-3 *a-na ši-ni-íp ši-ni-pí-ia* me sìla še / *ù ši-ni-pí ú- ṣí-im-ma*
4 [*še*]$^?$*-um i-ta-ag-ma-ar*
5 *re-ši-e-ia <<mi->> ki ma-ṣí*
6 *at-ta i-na e-pé-ši-ka*
7-8 *ša-na-pí ù ša-na-*[*pí*] / *šu-ta-ki-il-ma* 26 40 *ta-mar*
9-10 26 40 *i-na* [1] *ta-ba-al-ma* 33 [20] / *ši-ta-tum*
10-11 *i-gi* 33 20 *pu- ṭ*[*ú-ur-ma*] / [1] 48 *ta-mar*
12 1 48 *a-*[*na* 1 40] / [*i*]*-ši-ma* 3 *ta-mar*
3 *re-ši-e-im*

1 If (somebody) so asks you, saying this:
2-3 To two-thirds of my two-thirds a hundred sìla of barleycorn / and my two-thirds I added, then
4 the barleycorn completed itself.
5 How much was my initial barleycorn?
6 When you work it out:
7-8 Two thirds and two-thirds / let eat each other (multiply): you will see 26 40.
9-10 Carry away (subtract) 26 40 from 1: 33 20/ is the remainder.
10-11 Solve (calculate) the reciprocal of 33 20: / you will see 1 48.
12 Raise (multiply) 1 48 by 1 40: you will see 3.
3 is the initial barleycorn.

The stated problem can be expressed, in quasi-modern symbolic notations, as the following linear equation, where *b* stands for the initial amount of barleycorn:

$$b \cdot {}^2/_3 \cdot {}^2/_3 + 1\ 40 + b \cdot {}^2/_3 = b.$$

However, this must be a mistake, since an equation of this kind does not have a positive solution! Actually, the solution procedure gives the solution to the following, somewhat simpler, equation:

$$b \cdot {}^2/_3 \cdot {}^2/_3 + 1\ 40 = b.$$

(Therefore, the text must be a corrupt copy of another text with a correctly stated problem and a correct solution procedure). The solution proceeds, apparently, in the following steps:

It is clear that the initial amount of barleycorn minus two-thirds of its two-thirds should be 1 40 (sìla).
It is now (silently) assumed that the initial amount of barleycorn was 1 bariga = 1 (00) sìla.
Then, 1 (00) - 1(00) · $^2/_3$ · $^2/_3$ = 1(00) - ;40 · 40 = 1 (00) - 26;40 = 33;20.
However, the initial amount of barleycorn minus two-thirds of its two-thirds should be 1 40 (= 100), not 33;20.
To get the correct result, the initial amount of barleycorn must be scaled up in the proportion 1 40 / 33;20.
Consequently, the correct initial amount is
1 bariga · 1 40 / 33;20 = 1 bariga · 1 40 · ;01 48 = 1 bariga · 3 = 3 bariga.

Note that the answer is written as 3 (00), and not with the special cuneiform sign for 3 (bariga).

A second Old Babylonian text of a similar kind is **YBC 4669 B4** (***MKT III*** **(1937), 26 and pl. 3)**, one of the exercises in an Old Babylonian mathematical "recombination text". This example is a parallel to both MS 3976 and IM 53957. Below is presented an amended transliteration, translation, and interpretation of this text, too.

**YBC 4669 B4** (group 2 b, southern)

1-3 *a-na* $^2/_3$ $^2/_3$*-ia* / 1(bán) daḫ*-ma / še-e ma-ši-il*
4 sag še en.nam (erasure)

5 3 (bariga) še sag

1-3 To two-thirds of my two-thirds / I added 1 bán (= 10 sìla): / the barleycorn was halved.
4 What was the initial barleycorn?
5 3 bariga of barleycorn initially.

This is, again, in modern terminology, a linear equation for a single unknown *b* (the initial amount of barleycorn). In modern terms, the equation can be expressed as

$b \cdot 2/3 \cdot 2/3 + 10$ sìla $= b \cdot 1/2$.

Only the answer, but not the solution procedure is given in the text. Conceivably, the solution procedure was intended to proceed as follows:

Assume that the initial barleycorn *b* was 1 00 (sìla).
Then $b \cdot 1/2 - b \cdot 2/3 \cdot 2/3 = 30 - 40 \cdot ;40 = 30 - 26;40 = 3;20$.
However, the remainder should be 10 (sìla), not 3;20.
To get the correct result, the initial amount of barleycorn must be scaled up in the proportion 10 / 3;20.
Consequently, the initial amount was
1 00 sìla · 10 / 3;20 = 1 00 sìla · 10 · ;18 = 1 00 sìla · 3 = 3 00 sìla = 3 bariga.

A third Old Babylonian text parallel to MS 3976 is **YBC 4652** (Neugebauer and Sachs, *MCT* (1945), 100), a theme text with (initially) 22 closely related exercise. The first six exercises (those presumably simplest) are not preserved. The seventh is presented below:

**YBC 4652** (group 2 a, southern)

1 $na_4$ ì.pà ki.lá [nu.na.ta]g
2 igi.7.gál bí.daḫ igi.11.gá[l b]í.[da]ḫ / ì.lá 1 ma.na
sag $na_4$ en.nam sag $na_4$ 2/3 ma.na 8 gín 22 ½ še

1 I found a stone, the weight not marked.
2 I added a 7th-part to it, I added an 11th-part to it, / the weight was 1 mina.
What was the initial stone? The initial stone was 2/3 mina 8 shekels 22 ½ barleycorns.

This is, again, in modern terminology, a linear equation for a single unknown *w*, the initial weight of the (weight-)stone. In quasi-modern terms, the equation can be expressed as

$(w + w \cdot 1/7) + (w + w \cdot 1/7) \cdot 1/11 = 1$ mina.

Here again only the answer, but not the solution procedure is given in the text. Conceivably, the solution procedure was intended to proceed as follows:

Assume that the original weight was 7 · 11 = 1 17 (minas).
When a 7th-part of that is added to it, the result is 1 17 + 11 = 1 28 (minas).
When an 11th-part of that is added to it, the result is 1 28 + 8 = 1 36 (minas).
However, the result should be 1 mina (= 1 00 shekels, where 1 shekel = 3 00 barleycorns).
To get that result, the initial weight *w* must be 1 36 times smaller than the assumed 1 17 minas.
Since 1 17 / 1 36 = 12;50 / 16 = 3;12 30 / 4 = ;48 07 30, it follows that
*w* = ;48 07 30 mina = 2/3 mina 8;07 30 shekels = 2/3 mina 8 shekels 22 ½ barleycorns.

The other preserved exercises in the theme text YBC 4652 are all of the same general type, but increasingly complicated.

(Note that an alternative kind of solution procedure for YBC 4652 # 7 and all the following problems in the theme text is suggested by D. Melville in *Hist. Math.* 29 (2002)).

The exercises in the theme text YBC 4652 (above) are written entirely in Sumerian. Another exercise of the same type, but written (mainly) in Akkadian, is **AO 6770 # 3 (**Thureau-Dangin, *RA* 33 (1936), 78).

**AO 6770 # 3** (group 1 a, Larsa)

| | |
|---|---|
| 1-2 | $na_4$ šu.ba.an.ti-*ma* / *šu-qú-ul-ta-ša ú-ul i-de* |
| 3 | igi.7.gál 1/3 gín 15 še ba.zi |
| 4 | igi.11.gál *ša* ba.zi *ù* 5/6 gín *ù-te-er-ši-im-ma* |
| 5 | $na_4$ *a-na* ki.bi.$gi_4$-*ša it-tu-úr* |
| 6 | *re-iš ab-ni-ia mi-nu-um* |
| 7-8 | *na-al-pa-at-tum* 7 11 25 50 / *i-na* 7 1 ba.zi *a-na* 11 daḫ |
| 9 | *ša-pi-il-tam a-na* 50 *uš-ta-ka-al* |
| 10 | daḫ / 25 *i-na* šu.nigin-*ia a-ḫa-ar-ra-aṣ* |
| 11-12 | *ša-pi-il-tam a-na* 7 *at-ta-na-aš-ši* / *re-iš ab-ni-ia i-na-ad-di-nam* |

| | |
|---|---|
| 1-2 | I received a stone, / I did not know its weight. |
| 3 | I tore off a 7th-part (and) 1/3 shekel 15 barleycorns. |
| 4 | I gave back to it an 11th-part of what I tore off and 5/6 shekel. |
| 5 | The stone returned to its original state. |
| 6 | What was my initial stone? |
| 7-8 | Noted: 7, 11, 25, 50. / I shall tear off (subtract) 1 from 7, I will add (it) to 11. |
| 9 | I will let eat each other (multiply) the remainder by 50. |
| 10 | I will add. / I will break off 25 from the sum. |
| 11-12 | I will raise (multiply) the rest by 7. / It will give me my initial stone. |

Note that, since 1 shekel = 3 00 barleycorns, it clear that 1/3 shekel 15 barley corns = ;20 shekel + ;05 shekel = ;25 shekel. It is also clear that 5/6 shekel = ;50 shekel. Therefore, the stated problem in this exercise can be interpreted, in modern terms, as the following linear equation for the unknown initial weight $w$ (shekels).

$$w - (w/7 + ;25) + (w/7 + ;25)/11 + ;50 = w \quad \text{or simply} \quad (w/7 + ;25) - (w/7 + ;25)/11 = ;50.$$

An elegant way of solving this linear equation is to temporarily count with the new unknown

$$v = w/7 + ;25.$$

In terms of this new unknown, the simplified equation above (to the right) can be rewritten as

$$v - v/11 = ;50, \quad \text{or, after multiplication by 11,} \quad v \cdot (11 - 1) = ;50 \cdot 11.$$

Consequently,

$$v \cdot 10 = 9;10 \quad \text{so that} \quad v = 9;10 / 10 = ;55.$$

Hence, in terms of the original unknown $w$,

$$w/7 + ;25 = ;55, \quad \text{so that} \quad w/7 = ;30 \quad \text{and} \quad w = 7 \cdot ;30 = 3;30.$$

It is now obvious how the strange-looking solution procedure in AO 6770 # 3 can be explained. The text of the exercise was written by a scribe-school student who had listened somewhat inattentively to the teacher's instructions. He began his own version of the solution procedure correctly by writing down the numerical data in the stated problem, 7 and 11 for the two divisions, and 25 and 50 for the two added terms. Then he could not remember correctly the next step in the procedure, but he vaguely recalled that in similar situations it is sometimes a smart idea to make transformations of the type

$$(w - w/7) \cdot 7 = w \cdot (7 - 1) = w \cdot 6 \quad \text{and} \quad (w + w/11) \cdot 11 = w \cdot (11 + 1) = w \cdot 12.$$

That is why he wrote down, tentatively, that 7 should be diminished by 1 and that 11 should be increased by 1. Then he realized that he was stuck, and he bluffed, mentioning an unmotivated multiplication and an equally unmotivated addition. Luckily, he then remembered at least the final part of the teacher's explanation, namely that when at last the value of the temporary unknown $v = w/7 + 25$ had been computed, then the value of the original unknown $w$ could be obtained by first subtracting 25 and then multiplying by 7. This is what he wrote down in lines 10-12. Note that the name he quite appropriately gave to the temporary unknown $v = w/7 + 25$ was šu.nigin 'the sum'. On the other hand, he did not bother to give an exact numerical answer to the question.

Interestingly, there are other parallel texts in the Egyptian hieratic *Papyrus Rhind* (A. B. Chace, *et al.* (1929) pl. 59). One of the parallel texts is ***P. Rhind* # 37**. (Cf. the brief discussion of this parallel in Høyrup, *LWS* (2002), 321). The statement of problem # 37 is as follows:

I go down 3 times into the *hekat* (measure), 1/3 of me is added to me, 1/3 of 1/3 is added to me, 1/9 is added to me, I return, I (meaning the original *hekat*) am filled.

This incorrectly formulated problem can be correctly reformulated, in modern terms, as the following linear equation for an unknown capacity measure $c$:

$c \cdot (3 + 1/3 + 1/3 \cdot 1/3 + 1/9) = 1$ *hekat.*

The terms in the left hand side of this equation can be summed as follows, according to the entry for 2/9 in the $2/n$ table in *P. Rhind*:

$c \cdot (3 + 1/3 + 1/3 \cdot 1/3 + 1/9) = c \cdot (3 + 1/3 + 2 \cdot 1/9) = c \cdot (3 + 1/3 + 1/6 + 1/18) = c \cdot 3\ ½\ 1/18.$

The given sum, on the other hand, is 1 *hekat,* rather than 3 ½ 1/18 *hekat*. Consequently,

$c = 1$ *hekat* / 3 ½ 1/18.

Cf. the discussion of this problem in Friberg, *UL* (2005), 99, where it is pointed out that in *P. Rhind* all computations in the solution procedures are painstakingly accounted for in every detail. Accordingly, in the solution procedure for *P. Rhind* # 37, the division of 1 *hekat* by the "sum of parts" 3 ½ 1/18 is carried out in three different ways, with the answer first in the form of a sum of parts, then as a multiple of the *ro* (= 1/320 of a *hekat*), and finally in terms of the special *hekat* fractions (½, ¼, 1/8, 1/16, 1/32, and 1/64 of a *hekat*). Thus, the three forms of the answer are:

1) $c = 1$ *hekat* · ¼ 1/32, 2) $c = 90$ *ro*, 3) $c = ¼$ (*hekat*) 1/32 (*hekat*).

In addition to *P. Rhind* # 37, other texts in *P. Rhind* which can be interpreted as problems leading to single linear equations (essentially complicated "division problems") are discussed in some detail in Friberg, *UL* (2005), Sec. 2.1 b.
One such example is ***P. Rhind* # 28**, where the problem is stated as follows:

2/3 is what goes in, 1/3 is what goes out, 10 remains.

In modern terms, the problem is the following linear equation for an unknown $a$:

$a \cdot (1 + 1/3) \cdot (1 - 2/3) = 10.$

In Friberg, *UL* (2005), Sec. 2.1 b is also discussed the *Late Babylonian* mathematical cuneiform text **BM 34800**, which contains at least one problem of the same general type as the problem in MS 3976 and the problems in the parallel Old Babylonian and Egyptian hieratic texts mentioned above. BM 34800 is actually not a complete text; it is a small fragment of what appears to have been a large theme text, written in several columns and with several related exercises, all concerned with a $gur_7$ 'granary'. By a happy coincidence, the small preserved fragment contains a substantial part (probably somewhat less than half) of the text of one particular exercise, which was (with considerable effort) tentatively reconstructed in Friberg, *op. cit*. Since then, Kazuo Muroi has kindly informed the author in a letter about mistakes in the proposed reconstruction. The much improved reconstruction below, still very tentative, is based on Muroi's suggested corrections.

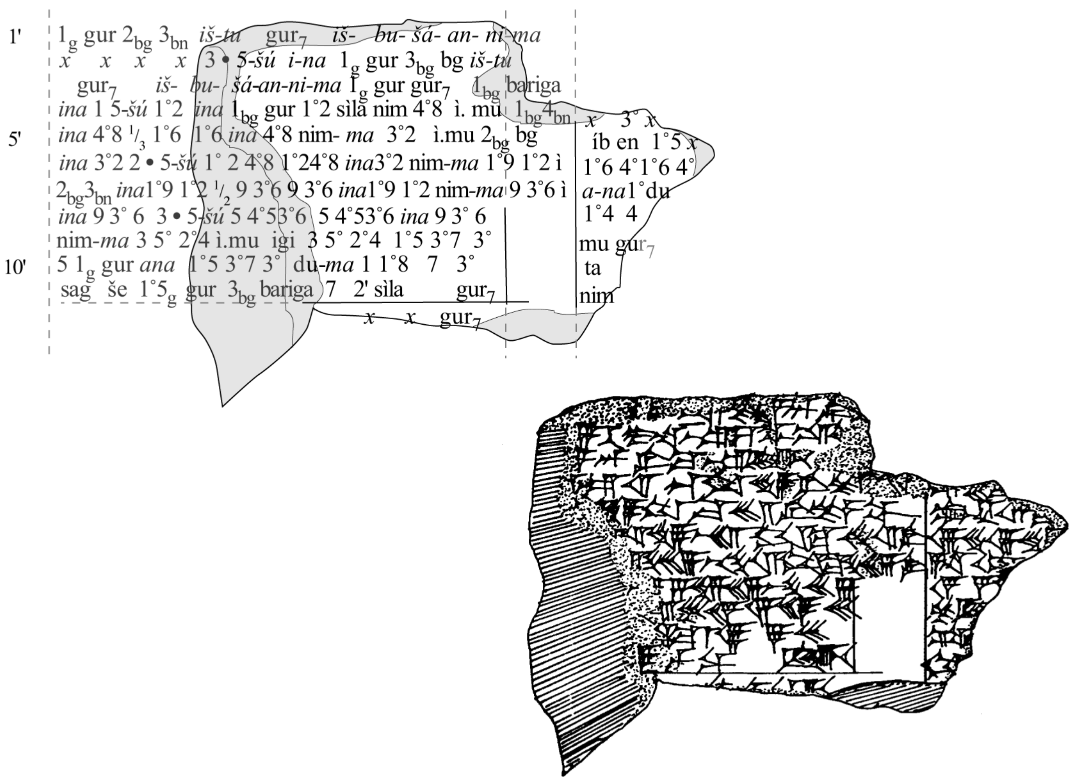

Fig. 2.2. BM 34800. Conform transliteration and cuneiform text (copy by Farouk Al-Rawi).

The reconstructed text of the exercise is divided below into its logical components. The intention has been to make it easy for the reader to discern the successive parts of the question, as well as the successive steps of the complicated but repetitive solution procedure.

**BM 34800** (Late Babylonian)

0'-1' [x x x *ina* / $1_g$ gur $2_{bg}$ $3_{bn}$ *iš-tu* gur$_7$ *iš-bu*]-*šá-an-ni*-[*ma*]
2' [x x x 3 •] 5-*šú*
3' *ina* $1_g$ gur $3_{bg}$ bariga *iš-t*[*u*] / [gur$_7$ *iš-bu-šá*]-*an-ni-ma* $1_g$ gur gur$_7$
4' [$1_{bg}$ bariga / *ina* 1 5-*šú* 12 *ina* $1_{bg}$] gur 12 sìla nim 48 ì.mu
5' [$1_{bg}$ $4_{bn}$ / *ina* 48 ¹/₃ 16 16 *ina*] 48 nim-*ma* 32 ì.mu
6' $2_{bg}$ bariga / [*ina* 32 2 • 5-*šú* 12 48] 12 48 *ina* 32 nim-*ma* 19 12 ì<.mu>
7' [$2_{bg}$ $3_{bn}$ *ina* 19 12 ½ 9 36] 9 36 *ina* 19 12 nim-*ma* 9 36 ì<.mu>
8' [*ina* 9 36 3 • 5-*šú* 5 45 36]
9' 5 45 36 *ina* 9 36 / [nim-*ma* 3 50 24 ì.mu igi] 3 50 24 15 37 30
10' [5 $1_g$ gur *ana* 15 37 30 d]u-*ma* 1 18 07 30
11' [sag še $15_g$ gur $3_{bg}$ bariga] 7 ½ sìla gur$_7$

0'-1' [x x x from / 1 gur 2 bariga 3 bán out of the granary he coll]ected (in taxes) from me:
2' [x x x x 3 ·] its 5th,
3' from 1 gur 3 bariga out of / [the granary he coll]ected from me, then (there was left) 1 gur (in) the granary.
4' [1 bariga. / From 1 its 5th, 12. From 1 bariga$^{!}$] 12 sìla lift (subtract), 48 it gives.
5' [1 bariga 4 bán. / From] 48 $1/3$, 16. 16 from 48 lift, 32 it gives,
6' 2 bariga. / [From 32, 2 · its 5th, 12;48.] 12;48 from 32 lift, then 19;12 it <gives>.
7' [2 bariga 3 bán. From 19;12 ½ 9;36.] 9;36 from 19;12 lift, then 9;36 it <gives>.
8' <3 bariga>. [From 9;36, 3 · its 5th, 5;45 36.]
9' 5;45 36 from 9;36 / [lift, then 3;50 24 it gives. The reciprocal] of 3;50 24 (is) 15;37 30.
10' [5 (00), 1 gur, by 15;37 30 g]o (multiply): 1 18 07;30.
11' [The initial barleycorn (was) 15 gur 3 bariga] 7 ½ sìla, (in) the granary.

In modern notations, the question in this exercise can be rephrased as the following single linear equation (or complicated division exercise):

$$b \cdot (1 - 1/5) \cdot (1 - 1/3) \cdot (1 - 2/5) \cdot (1 - 1/2) \cdot (1 - 3/5) = 1 \text{ gur} = 5\ 00 \text{ sìla}, \quad b = ?$$

It is interesting that the author of this text systematically expresses fractions in three different ways. While he counts with sexagesimal fractions, of course, he expresses certain of the fractions appearing in the coefficient in the linear equation both in terms of what seems to be a kind of "common fractions", namely 'its 5th', '2 · its 5th', '3 · its 5th' (unfortunately almost exclusively in the reconstructed part of the text), and in metrological terms, namely 'from 1 gur 1 bariga', 'from 1 gur 1 bariga 4 bán', *etc.* (unfortunately with one exception also only in the reconstructed part of the text). Here gur and bariga, *etc.* are well known Babylonian units of capacity measure with

1 gur = 5 bariga = 5 00 sìla, 1 bariga = 6 bán = 1 00 sìla,
1 bán = 10 sìla, 1 sìla = c. 1 liter.

The bán of 10 sìla in BM 34800 is the same as in Old Babylonian texts. Often, however, the bán contains only 6 sìla in cuneiform texts from the 1st millennium BC.

Note, by the way, that another example of a "common fraction" appearing in a Late Babylonian mathematical exercise is 4 *ḫa-an-za* '4 fifths' in VAT 7848 # 6 (Neugebauer and Sachs, *MCT* (1945), 141). Note also that five exercises in the demotic mathematical Papyrus BM 10520 (early Roman?) give the rules for calculating with common fractions (Friberg, *UL* (2005), Sec. 3.3 e).

It is interesting that in the linear equation in BM 34800 the fractions appearing in the coefficient $(1 - 1/5) \cdot (1 - 1/3) \cdot (1 - 2/5) \cdot (1 - 1/2) \cdot (1 - 3/5)$ were chosen with great care. Observe that

a) $1/5\ (= ;12) < 1/3\ (= ;20) < 2/5\ (= ;24) < 1/2\ (= ;30) < 3/5\ (= ;36)$.
b) $1/5,\ 1/3,\ 2/5,\ 1/2,\ 3/5$ are regular sexagesimal numbers.
c) $1 - 1/5 = 4/5,\ 1 - 1/3 = 2/3,\ 1 - 2/5 = 3/5,\ 1 - 1/2 = 1/2,\ 1 - 3/5 = 2/5$ are regular sexagesimal numbers, too.

(Here "regular sexagesimal number" is a modern term meaning that the number in question is a divisor of some power of 60, which ensures that the reciprocal of that number can be expressed as a terminating sexagesimal number. The symbol $<$ means "less than").

Just as in MS 3976, the solution procedure in BM 34800 makes use of a normalized reference quantity and a scaling factor, in (essentially) the following way:

| | | |
|---|---|---|
| Suppose that the original barleycorn is | 1 bariga = 1 00 (sìla), | end of *l.* 3' |
| then a fifth less is | 1 00 - 12 = 48 (sìla) (= 1 00 · $4/5$), | *l.* 4' |
| a third less of that is | 48 - 16 = 32 (= 1 00 · $4/5 \cdot 2/3$), | *l.* 5' |
| two fifths less of that is | 32 - 12;48 = 19;12 (= 1 00 · $4/5 \cdot 2/3 \cdot 3/5$), | *l.* 6' |
| a half less of that is | 19;12 - 9;36 = 9;36 (= 1 00 · $4/5 \cdot 2/3 \cdot 3/5 \cdot 1/2$), | *l.* 7' |
| three fifths less of that is | 9;36 - 5;45 36 = 3;50 24 (= 1 00 · $4/5 \cdot 2/3 \cdot 3/5 \cdot 1/2 \cdot 2/5$ ·), | *ll.* 8'-9' |
| 1 00 / 3;50 24 is | 15;37 30 (= $5/4 \cdot 3/2 \cdot 5/3 \cdot 2 \cdot 5/2 = 5^3 / 2^3$). | end of *l.* 9' |

Hence, if the original barleycorn is 1 00 sìla, the final barleycorn is 3;50 24 sìla,
and if the original barleycorn is 15;37 30 sìla (the scaling factor), the final barleycorn is 1 sìla.
However, the final barleycorn should be 1 gur = 5 00 sìla. beginning of *l.* 10'
Therefore, the original barleycorn in the granary was
5 00 · 15;37 30 sìla = 1 18 07;30 sìla = 15 gur 3 bariga 7 ½ sìla. *ll.* 10'-11'

## 3. MS 3895. A rectangular-linear system of equations: A market rate problem
### 3.1. Presentation of the text

MS 3895 holds the text of a single problem, in 24 lines on obverse and reverse. It is a market rate problem where a given quantity of oil is bought and then sold with a given difference between the market rates, and a given profit in silver. The problem is to determine the different market rates and the initial cost in silver. It can be reformulated as a rectangular-linear system of equations, and it is correctly solved[7].

The text on the obverse of this tablet is badly preserved. Luckily, the text on the reverse is in much better condition.

**MS 3895. Transliteration**

1 [*a-na šâ*]*m*(šám) 1 (bariga) 3 (bán) *šaman rūšti*$^{?}$(ì.sag) *i-na* ⸢*bīt*(é)⸣
*ummi'ānim*(um.⸢mi⸣.a) *kaspam*(kù$^{!}$.babbar)
1b *el-qé-ma*
2 *a-ša-*⸢*am-ma*⸣ *ma-ḫi-*⸢*ir*⸣ *a-*⸢*ša-mu ú-ul i-de*⸣
3 [*a*]*p-š*[*u-u*]*r-*⸢*ma ma-ḫi-ir*⸣ *ap-šu-ru* ⸢*ú-ul*⸣ *i-de*
4 [x x x x x] 5 [x x x x x]
5 [6 *šiqil*(gí]n) [*kaspam*(kù.babbar)] *né-me-lam* [*a*]*k-*⸢*mu-ur*⸣
6 [*ki-i*]*a-a* [*a-š*]*a-am-ma ki-ia-a ap-šu-ur*
7 [*ù*] *kaspum*(kù.babbar) *mi-nu-um*
8 [*at-t*]*a* [*i-n*]*a* ⸢*e*⸣*-pe-ši-ka*
*igi*(igi) 6 ⸢*nēmeli*(á.tuk)*-ka*⸣ [*tapa*]*ṭṭar*(duḫ)*-ma* 10 *tammar*(igi.duḫ)
9 [10 *a-n*]*a* 1 ⸢30⸣ *tuštakkal*(gu$_7$)$^{kal}$*-ma* 15
10 [*tammar*(igi.duḫ) 45 diri *ta-la-ap-pa-at*]$^{!?}$
11 [45 *a-na*] 15 *t*[*anašši*(íl)*-m*]*a* ⸢11 15⸣ *tammar*(igi.duḫ)
12 ½ 45 *ša x x x* [*ta-al*]*-pu-tu tapaṭṭar*(duḫ)*-ma*
13 ⸢22 30⸣ *tammar*(igi.⸢duḫ⸣)*-ma* ⸢*a-di*⸣ 2 *ta-la-ap-pa-at*
14 ⸢*ù*$^{?}$⸣ *tuštakkal*(gu$_7$)$^{kal}$ 8 26 15 *tammar*(igi.duḫ)
15 ⸢*a-na* 11⸣ 15 *tuṣṣab*(daḫ)*-ma* 11$^{!}$ 23 26 15 *tammar*(igi.duḫ)
16 *ipsâm*(íb.sá) *tu-še-el-le-ma* 3 22 30 *tammar*(igi.duḫ)
17 ⸢3 22 30 *a-di*⸣ 2 *ta-la-ap-pa-at*
18 22 30 ½ 45 ⸢*ša*⸣ *i-na x* 1*-ma*$^{?}$ *ta-al-pu-tu*
19 *a-na libbi*(šà) 3 22 30 *ša* 2 *ta-al-pu-tu*
20 *a-na* 1 *tuṣṣab*(daḫ)*-ma i-na* 2 *tanassaḫ*(ba.zi) 3 45 *ša ta-ša-mu*
20b *ù* 3 ⸢*ša ta-ap*⸣*-šu-ru tammar*(igi.duḫ)
21 *igi*(igi) 3 45 *ša ta-ša-m*[*u tapaṭṭar*(duḫ)*-m*]*a* 16 *tammar*(igi.duḫ)
22 *a-na* 1 30 ⸢*tuštakkal*(gu$_7$)⸣$^{k}$[$^{al}$*-ma* 2]4 *šiqil*(gín) *kaspam*(kù.babbar) *tammar*(igi.duḫ)

---

23 3 4[5 *ma-ḫi-ir a-š*]*a-mu*$^{!}$
24 ⸢3⸣ [*m*]*a-*[*ḫi-ir*] *ap-šu-ru*

---

7 The market rate of 3 sìla of ì.sag-quality oil for one shekel of silver was conventional in the Old Babylonian period, occurring in the laws of Eshnunna and elsewhere (see *CAD* R, 430).

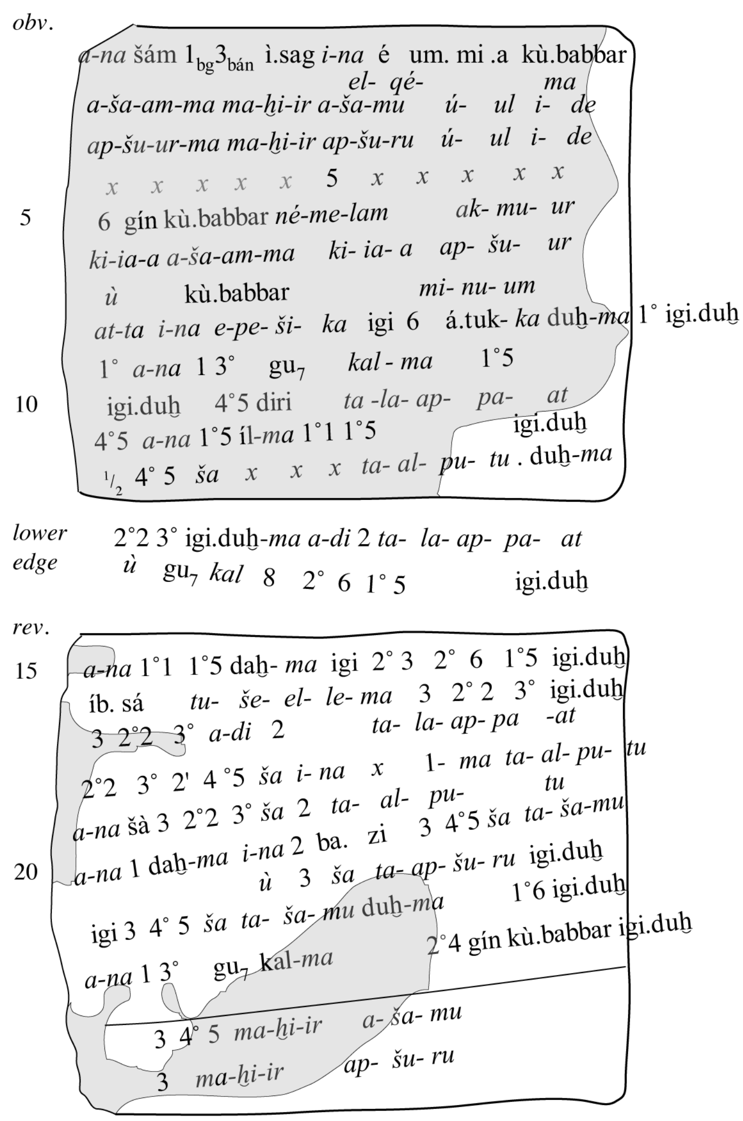


Fig. 3.1. MS 3895. Conform transliteration. Scale 1:1.

**Translation**

1-1b [To bu]y 1 bariga 3 (bán) of finest-quality oil, I borrowed silver from a lending-house, then
2 I bought, but I did no[t k]now the market rate at which I bought.
3 I so[ld, b]ut I did not know the market rate at which I sold.
4 [x x x x x] 5 [x x x x x].
5 I accumulated [6 shekels of silver] as profit.
6 [At what (price)] did I [b]uy, and at what (price) did I sell,
7 [and] what was the silver?
8 When [you] work it out:
(You shall) solve (compute) the reciprocal of 6, your profit: you (will) see 10.

9 You shall let eat each other (multiply) [10] by 1 30: 15
10 [you will see. You shall note down 45, the difference].
11 [You shall raise (multiply) 45 by] 15: you will see 11 15.
12 You shall solve (compute) ½ of 45 that x x x [you no]ted down,
13 you will see 22 30. You shall note it down twice,
14 and (you shall) let them eat each other (multiply), 8 26 15 you will see.
15 You shall add it on to 11 15: you will see 11 23 26 15.
16 You shall let go up (calculate) the equalside (square root): you will see 3 22 30.
17 You shall note down 3 22 30 twice.
18 22 30, ½ of 45, that x x x x x you noted down,
19 onto 3 22 30 that you noted down twice
20 you shall add on to the 1st, you shall tear out (subtract) from the 2nd, 3 45 at which you bought
20b and 3 at which you sold, you will see.
21 [You shall solve (compute)] the reciprocal of 3 45 at which you bought, then you will see 16.
22 You shall let eat each other (multiply) by 1 30: you will see [2]4 shekels of silver.

---

23 3 [45 was the rate at which I] bought,
24 3 [was the rate at which] I sold.

## MS 3895. Mathematical commentary

The data specified in the question in this exercise are not preserved. Luckily, however, so much is preserved of the solution procedure that it is has been possible to reconstruct the essential parts of the question, in particular, the lost specifications of the data.

A commodity trader has bought a certain amount of finest-quality oil (ì.sag) for a certain amount of silver (kù.babbar = *kaspum*) at one "market rate" (*maḫīrum*, expressed in sìla of oil per shekel of silver), then sold it at another market rate. Not known are the invested amount of silver (call it $S$), the market rate at which the oil was bought (call it $m$), and the market rate at which it was sold (call it $m'$). Known are, instead, the capacity measure of the oil (call it $C$), the difference between the two market rates ($m - m'$), and the profit ($P$) earned by the trader through the transaction. The (reconstructed) given data are:

$C$ = 1 bariga 3 bán = 1 30 sìla (where 1 sìla = c. 1 liter) *l.* 1
$m - m'$ = ⅔ sìla 5 gín / shekel = ;45 sìla / shekel *l.* 4
$P$ = 6 shekels *l.* 5

The unknowns are asked for as follows:

$m = ?,\quad m' = ?,\quad S = ?$ *ll.* 6-7

The inverted market rate $1/m$, expressed in shekels of silver per sìla of oil, is the "unit price" at which the oil was originally bought, while $1/m'$ is the unit price at which it was sold. Therefore, $S = C \cdot 1/m$ is the invested amount of silver, while $S' = C \cdot 1/m'$ is the amount of silver the trader ended up with after the transaction. Knowing this, it is easy to see that

$$P = S' - S = C \cdot 1/m' - C \cdot 1/m = C \cdot (1/m' - 1/m).$$

If this equation is multiplied by the product of the market rates, one gets the new equation

$$P \cdot m \cdot m' = C \cdot (1/m' - 1/m) \cdot m \cdot m' = C \cdot (m - m').$$

Consequently, the values of the two unknown market rates can be calculated as the solutions to the following "rectangular-linear" system of equations:

$m \cdot m' = 1/P \cdot C \cdot (m - m') = 1/(6 \text{ shekels}) \cdot 1\ 30 \text{ sìla} \cdot ;45 \text{ (sìla / shekel)} = 11;15 \text{ sq. (sìla / shekel)}$ *ll.* 8-11
$m - m' = 45$ sìla / shekel *l.* 4

The way in which Old Babylonian mathematicians would solve a rectangular-linear system of equations of this kind is well known. See, for instance, Friberg, *MSCT* 1 (2007), 327. The solution procedure can be explained as follows. If, for given values of $A$ and $q$,

$m \cdot m' = A$ and $m - m' = q$

then

sq. $p/2$ = sq. $q/2 + A$, where $p = m + m'$ (*)

This is, of course, a modern-type equation in symbolic notation. The Old Babylonian counterpart was a piece of "metric algebra", where $m$ and $m'$ were interpreted as the sides of a rectangle and where the product $m \cdot m'$ was interpreted as the area of that rectangle. If four such rectangles are put together as in Friberg, *op. cit.*, Fig. 11.2.10, left, to form a ring of rectangles, then $p = m + m'$ is the side of a square bounding the ring from the outside, while $q = m - m'$ is the side of a square bounding the ring from the inside. Therefore, as in Fig. 11.2.10, right, the Babylonian geometric counterpart of the modern equation (*) above is the straightforward observation that the area of the square with the side $p/2 = (m + m')/2$ is equal to the area of a square with the side $q/2 = (m - m')/2$ plus the area of the rectangle with the sides $m$ and $m'$.

In the metric algebra solution procedure in MS 3895, which started with the computation of the product $m \cdot m'$ in *ll.* 8-11, the rectangular-linear system of equations for the unknowns $m$ and $m'$ is solved in the following series of computations (here, for the readers' convenience, expressed in terms of quasi-modern symbolic notations and sexagesimal numbers with semi-colons):

| | |
|---|---|
| $(m - m')/2$ = ;45/2 = ;22 30, two copies of ;22 30 are noted down | *ll.* 12-13 |
| sq. $(m - m')/2$ = sq. ;22 30 = ;08 26 15 | *l.* 14 |
| sq. $(m - m')/2 + m \cdot m'$ = 11;15 +;08 26 15 = 11;23 26 15 | *l.* 15 |
| $(m + m')/2$ = sqs. 11;23 26 15 = 3;22 30, two copies of 3;22 30 are noted down | *ll.* 16-17 |
| recall that $(m - m')/2$ = ;45/2 = ;22 30 was noted down twice | *l.* 18 |
| $m = (m + m')/2 + (m - m')/2$ = 3;22 30 + ;22 30 = 3;45 (sìla/shekel), and | |
| $m' = (m + m')/2 - (m - m')/2$ = 3;22 30 - ;22 30 = 3 (sìla/shekel) | *ll.* 19-20b |
| $1/m$ = $^1/_{3;45}$ = ;16, $S = 1/m \cdot C$ = ;16 · 1 30 = 24 (shekels) | *ll.* 21-22 |

The main result of the computations, giving the values of the two market rates, is repeated below a horizontally drawn line in *ll*. 22-23.

The details of the computation of the square side (square root) in *l*. 16 are not given. The square side was probably computed more or less in the following way by use of the Babylonian "additive square side rule" (Friberg, *BaM* 28 (1997), 317):

sqs. 11;23 26 = appr. 3 + 2;23 26 / 2 · 3 = appr. 3 + ;24 = 3;24,
sq. 3;20 = 11;06 40,
sqs. 11;23 26 15 = appr. 3;20 + ;16 40 / 2 · 3;20 = 3;20 + ;02 30 = 3;22 30.

### 3.2. The mathematical terminology used in MS 3895

Goetze's analysis (*MCT* (1945), 147) would have shown that MS 3976 is a "southern" text, in view of the spelling

| | | |
|---|---|---|
| *e-pe-ši-ka* | *pe* = PI | Goetze's criterion S 4 |

More information can be obtained from the mathematical terminology used in MS 3895. A number of terms are identical with, or at least close to, those which appear in MS 3299 or MS 3976 (see above). These are:

| | | | |
|---|---|---|---|
| *mi-nu-um* | what? | = ? | *l.* 7 |
| *ki-ia-a* | how much (each)? | = ? (of several) | *l.* 6 |

| | | | |
|---|---|---|---|
| *at-ta i-na e-pe-ši-ka* | when you work it out | solution procedure: | *l.* 8 |
| (*a*) *a-na* šà (*b*) daḫ | *a* onto *b* add | compute *a* + *b*; daḫ = *waṣābum* 'to add on' | *l.* 19 |
| igi (*n*) duḫ (= $du_8$) | solve the reciprocal of *n* | compute 1/*n* | *ll.* 8, 21 |
| *ta-la-ap-pa-at* | you shall note down | recall given numbers; < *lapātum* 'to touch' | *ll.* 13, 17 |
| (*a*) *a-na* (*b*) $gu_7^{kal}$ | *a* to *b* you shall let eat each other | compute *a* × *b*; $gu_7$ = *akālum* 'to eat' | *ll.* 9, 14 |
| igi.duḫ | you (will) see | the result is | *ll.* 8, 11, etc. |

MS 3895 makes more use of Sumerograms for verbs than either one of MS 3299 and MS 3976, for instance in the term (*a*) *i-na* (*b*) ba.zi '*a* from *b* tear out (subtract)' in *l.* 20. Still, it is clear that the verbs in the solution procedure are thought of as being in the second person singular, durative. This is shown by two of the terms listed above, namely *ta-la-ap-pa-at* and $gu_7^{kal}$, but also by the new term

| | | | |
|---|---|---|---|
| íb.sá *tu-še-el-le* | you shall let the equalside go up | compute the square side; < *elûm* III 'to go up' | *l.* 16 |

Obviously, therefore, MS 3895 belongs to group 1 a (Larsa), just like MS 3299 and MS 3976.

A term without any known parallel in other mathematical cuneiform texts is

| | | | |
|---|---|---|---|
| ½ (*a*) duḫ | ½ of *a* solve | compute *a*/2 | *l.* 12 |

On the other hand, the term

| | | | |
|---|---|---|---|
| *ú-ul i-de* | I did not know | = ? | *l.* 3 |

has parallels in other "southern" texts, namely

| | | | | |
|---|---|---|---|---|
| *ú-ul i-de* | AO 6770 (*MKT II*, 37) | *rev*. 11 | group 1 a | Larsa |
| *ú-ul i-de* | Str 362 (*MKT I*, 239) | *obv*. 3 | group 3 | Uruk |
| *ú-ul i-de* | VAT 8391 (*MKT I*, 322) | *rev i*: 27 | group 4 a | Uruk? |
| *ša la ti-du-ú* | Str 368 (*MKT I*, 311) | *obv*. 7 | group 3 | Uruk |
| *ša la ti-du-ú* | VAT 7532 (*MKT I*, 294) | *obv.* 8, 12 | – | – |
| *ša la ti-du-ú* | VAT 7535 (*MKT I*, 303) | *obv.* 9, 12 | – | – |

Note that the term $gu_7^{kal}$ = *tuštakkal* 'you shall let them eat each other' is used as a term for multiplication in lines 9, 14, and 22 of the present text. In line 14, it is used, essentially, in the same sense as in line 5 of text no. 1 above (see also footnote 4), namely as a term for the multiplication of two sides of a rectangle or square. In lines 9 and 22, on the other hand, the term is used non-geometrically. Other examples of a non-geometrical use of the term can be found, for instance, in IM 53967 and AO 6770 # 3 (sec. 2.3 above).

### 3.3. YBC 4698, MLC 1842, *TMS* 13, and IM 54464, parallel texts to MS 3895

Parallel texts to the market rate problem in MS 3895 were discussed already in Sec. 4.6 of Friberg, *UL* (2005). Here follows an abridged version of that discussion.

**YBC 4698** (Friberg, *op. cit.*, 61, 215) is a small Old Babylonian theme text on the theme "commercial mathematics", belonging to the "southern" group 2 b. It is written almost entirely in Sumerian, and it contains 2 interest exercises and 15 market rate exercises. Exercise # 9 is particularly close to MS 3895, although it consists of only question and answer.

**YBC 4698 # 9** (group 2 b, southern)

1-4 1 (gur) gur ì.giš / *i-na* šàm 1 gín / 2 sìla sì / 7 ½ gín kù.babbar diri
5-6 en.nam šám.ma / en.nam búr.ra
7-8 1 (bán) šàm.ma / 8 sìla búr.ra

1-4 1 gur sesame oil. From the buying (rate) for 1 shekel, 2 sìla were given.
7 ½ shekels of silver was the excess.
5-6 What did I buy at, then what did I sell at?
7-8 I bought at 1 bán, then I sold at 8 sìla.

In quasi-modern symbolic notations, as above, the problem can be expressed as follows:

$C$ = 1 gur = 5 00 sìla, $m - m'$ = 2 sìla / shekel, $C \cdot (1/m' - 1/m) = P$ = 7 ½ shekels, $m, m'$ = ?

An equivalent rectangular-linear system of equations for $m$ and $m'$ then is, of course,

$m \cdot m' = 1/P \cdot C \cdot (m - m')$ = 1 / 7;30 · 5 00 · 2 = 1 20 sq. (sìla / shekel), $m - m'$ = 2 (sìla / shekel).

The solution is, as indicated in the text,

$m$ = 10 sìla / shekel = 1 bán / shekel, $m'$ = 8 sìla / shekel.

Another close parallel to the market rate problem in MS 3895 is a market rate problem in Bruins and Rutten, ***TMS*, text 13** (1961). Amended transliterations and translations of that text have been published in Gundlach/von Soden, *AMSUH* 26 (1963), and Høyrup, *LWS* (2002), 206.

***TMS* 13** (group 8 a, Susa)

1 2 (gur) 2 (bariga) 5 (bán) ì.giš šám
2 *i-na* šám 1 gín kù.babbar / 4 sìla.ta.am *ak-ší-iṭ-ma*
3 ²/₃ ma.na <<20 še>>? kù.babbar *ne-me-la a-mu-úr*
4 *ki ma-ṣí / a-šà-am ù ki ma-ṣí ap-šu-úr*
5 za.e 4 sìla ì.giš gar *ù* 40 ma.na *ne-me-la* gar
*etc.*

1 I bought 2 gur 2 bariga 5 bán of oil.
2 I cut off each time 4 sìla from the market rate (for) 1 shekel of silver, then
3 ²/₃ mina <<20 barleycorns>>? was the profit that I saw.
4 How much / did I buy at and how much did I sell at?
5 You: set 4 sìla of oil, and set 40 mina as profit.
*etc.*

This is precisely the same problem as in the case of MS3895, only the data are different:

| | | |
|---|---|---|
| $C$ = | 2 gur 2 bariga 5 bán = 12 50 sìla | *l.* 1 |
| $m - m'$ = | 4 sìla / shekel | *ll.* 2, 5 |
| $P = C \cdot (1/m - 1/m')$ = | ²/₃ mina = 40 shekels | *ll.* 3, 5 |

Thus, in this case, the rectangular-linear system of equations for the two market rates is

| | |
|---|---|
| $m \cdot m' = 1/P \cdot C \cdot (m - m')$ = 1/(40 shekels) · 12 50 sìla · 4 (sìla / shekel) = 1 17 sq. (sìla / shekel) | *ll.* 6-7 |
| $m - m'$ = 4 sìla / shekel | *l.* 2 |

Therefore, by the same reasoning as in the case of MS 3985

| | |
|---|---|
| sq. $(m - m')/2 + m \cdot m'$ = 4 + 1 17 = 1 21 | *ll.* 8-9, left |
| $(m + m')/2$ = sqs. 1 21 = 9, two copies of 9 are noted down | *ll.* 9, right-10, left |
| $m = (m + m')/2 + (m - m')/2$ = 9 + 2 = 11 (sìla/shekel) | *ll.* 10, right-11, left |
| $m' = (m + m')/2 - (m - m')/2$ = 9 - 2 = 7 (sìla/shekel) | *ll.* 11, right-12 |
| $S = 1/m \cdot C$ = 12 50 / 11 = 1 10 (shekels) = 1 mina 10 shekels | *ll.* 13-14 |

The last computation in TMS 13 is without counterpart in MS 3895. Asked for is how much oil can be bought at the second, lower market rate for the profit in silver that resulted from the transaction. The answer is:

| | |
|---|---|
| 7 sìla/shekel · 40 shekels = 4 40 sìla of oil | *ll.* 15-17 |

A somewhat less close parallel to MS 3895 is MLC 1842 (*MCT* Sb, 106), a text belonging to group 5 (“northern”). The question in this text is phrased in the following way:

**MLC 1842** (group 5, northern)

1 ganba.e *i-li-i-ma* 30 še gur *a-ša-am*
2 ganba *iš-pí-il-ma* 30 še gur *a-ša-am*
3 *ma-ḫi-ri-ia ak-mu-ur-ma* 9 /
4-5 kù.babbar *ma-ḫi-ri-ia ak-mu-ur-ma* / 1 ma.na 7 ½ gín
6 ganba *a-ša-am ù ki-ia ap-šu-ur*

1 The market rate increased, I bought 30 gur of barleycorn,
2 the market rate decreased, I bought 30 gur of barleycorn.
3 I added together my market rates, 9.
4-5 I added together the silver of my market rates: 1 mina 7 ½ shekels.
6 (At which) market rate did I buy, and at what each (sic!) did I sell?

This is a variant of the problem type in MS 3895. The problem can be rephrased as follows:

| | | |
|---|---|---|
| $C =$ | 30 gur = 2 30 00 sìla | *l.* 1 |
| $m + m' =$ | 9 (00) sìla / shekel | *ll.* 2, 5 |
| $Q = C \cdot (1/m + 1/m') =$ | 1 mina 7 ½ shekels = 1 07;30 shekels. | *ll.* 3, 5 |
| $m, m' =$ ? | (this question is incorrectly formulated)! | *l.* 6 |

The corresponding rectangular-linear system of equations for the two market rates is:

$m \cdot m' = 1/Q \cdot C \cdot (m + m')$ = 1/(1 07;30 shekels) · 2 30 00 sìla · 9 00 (sìla / shekel) = 20 00 00 sq. (sìla / shekel)
$m + m'$ = 9 00 sìla / shekel

The answer is that

$m$ = 5 00 sìla / shekel, and $m'$ = 4 00 sìla / shekel.

Note that the market rate for barleycorn was much greater than the market rate for oil. The zeros indicating the correct sizes of the sexagesimal numbers were inserted above for the readers’ convenience only.

It is somewhat strange that in the question in this text the sum of the market rates is given simply as 9 (sixties) and not more precisely, in terms of capacity measures, as 1 gur 4 bariga. The text on the obverse of MLC 1842 is badly preserved, so it is not known how the answer was formulated. Note, however, that 5 00 sìla = 1 gur and 4 00 sìla = 4 bariga. Indeed, the usual Old Babylonian market rate for barleycorn was a round 1 gur (per shekel).

Another, very interesting, Old Babylonian market rate exercise can be found in the Eshnunna text **IM 54464** (Baqir, *Sumer* 7 (1951); von Soden, *Sumer* 8 (1952)). The question in this exercise is corrupt (an important part of it is missing), and also damaged in several crucial places. In addition, the vague and shifting meaning of the word *maḫīrum* = ganba (KI.LAM) (‘market’, ‘market rate’, ‘exchange rate’, ‘market price’, market value, *etc*.) makes the translation of certain parts of the text quite difficult. For these reasons, since its publication in 1951, the text has never been adequately explained. Fortunately, however, it is possible to work backwards from the correct answer and the correct solution procedure and in this way reconstruct the damaged question.

**IM 54464** (group 7 a, Eshnunna)

1 *šum-ma ki-a-am i-ša-al-ka um-ma šu-ú-ma*
2 *i-na ma-ḫi-ir* 1 (bán) 5 sìla ì.šaḫ 1 (bán) ì.giš
3 *ši-ni-ip ma-ḫi-ir na-ḫi-im ù* <*ul-lu-um wa-at-ru-um ma-ḫi-ir ul-li-im* 6 še kù.babbar>? *ul-li-*⌜*im*⌝?

4 *wa-at-ri-im* ⸢1? gín⸣ kù.babbar *na-ši-a-ku*
5 ì.giš *ù* ì.šaḫ *ša-*[*ma-am*] ⸢*at-ta i-na e-pé-ši-ka*⸣
6 *i-gi* 15 *pu-ṭú-ur-ma* [4 *i-li*]
7 4 *a-na* 1 *i-ši-i-ma* 4 [*i-li-a-k*]*um*
8 *na-ás-ḫi-ir-ma i-gi* 10 ⸢*pu-ṭú-ur-ma*⸣
9 6 *i-li* 6 *a-na* 40 [*i-š*]*i-i-ma* 4 *i-l*[*i*]
10 *na-ás-ḫi-ir-ma* ⸢ganba⸣? *ul-*[*li-i*]*m*? *wa-at-r*[*i*]-
10a *im*
11 *i-na* ⸢1 gín kù.babbar⸣ 6 še kù.babbar
12 *ḫu-ru-uṣ-ma* 58 *ša-pí-il-tum*
13 *na-ás-ḫi-ir-ma* 4 *ù* 4 *ku-mu-ur-ma*
14 8 *i-li* 8 *ša i-li-kum i-gi-šu pu-ṭ*[*ú-ur-ma*]
15 7 30 *i-*[*li*] 7 30 *ša i-*[*li-kum*]
16 *a-na* 58 kù.babbar ⸢*i-ši*⸣-[*ma*] ⸢*x*⸣ 7 15 [*i-li*]
17 7 15 *ša i-li-kum a-na* 4 *i-ši-i-ma*
18 29 *i-na-di-na-ku-um na-ás-ḫi-*[*ir*]*-ma*
19 7 15 *a-na* 4 *i-ši-i-ma* 2⸢9⸣ *i-li*
20 6 še kù.babbar *ša i-na* kù.babbar *ta-su-ḫu*
21 *a-na iš-te-en ṣí-ma*
22 *iš-te-en* 29 *iš-te-en* 31
23 *ša-am iš-te-en* 7 15
24 *iš -te-en* 5 10

1 If (somebody) so asks you, saying this:
2 At a market rate of 1 bán 5 sìla of lard, 1 bán of oil,
3 two-thirds the traded amount of lard plus <the oil surplus was the traded amount of oil. 6 barleycorns was the silver>? of the oil
4 surplus. I was carrying (could spend) ⸢1? shekel⸣ of silver.
5 Buy [me] lard and oil! When you work it out:
6 The reciprocal of 15 solve (compute), then [4 will come up].
7 Raise 4 to 1, then 4 [will come up] for you.
8 Come again: solve the reciprocal of 10:
9 6 will come up. [Li]ft 6 to 40, then 4 will come up.
10 Come again: the market value of the oil surp-
10a lus,
11 from 1 shekel of silver 6 barleycorns of silver
12 break off, 58 is the remainder.
13 Come again: 4 and 4 heap, then
14 8 will come up. 8 that came up for you, solve (compute) its reciprocal:
15 7 30 will com[e up]. 7 30 that came up for you
16 raise to 58, the silver[:] ⸢*x*⸣ 7 15 will come up.
17 7 15 that came up for you raise to 4, then
18 29 it will give you. Come again:
19 raise 7 15 to 4, then 29 will come up.
20 6 barleycorns of silver that you tore out from the silver
21 double (add!) to one.
22 One is 29, one is 31.
23 Buy of one 715,
24 of one 5 10.

In this exercise, the market rates for ì.šaḫ ‘pig-fat’ or ‘lard’ ($m$ = 15 sìla/shekel) and ì.giš ‘(sesame) oil’ ($m'$ = 10 sìla/shekel) are explicitly given at the beginning of the exercise. The initially unknown quantities of lard and oil, respectively, and the likewise initially unknown amounts of silver paid for them, are calculated in the exercise and explicitly given in the answer. In quasi-modern symbolic notations, let $C$ and $C'$ denote the calculated quantities of lard and oil, respectively (in sìla measures), and let $S$ and $S'$ denote the calculated amounts of silver (in shekels) paid for the lard and oil respectively. Then

$S$ = '29' (clearly meaning ;29 shekels), and $S'$ = '31' (clearly meaning ;31 shekels) *l.* 22
$C$ = '7 15' (clearly meaning 7;15 sìla), and $C'$ = '5 10' (clearly meaning 5;10 sìla) *ll.* 23-24

It follows that

$S + S'$ = ;29 shekels + ;31 shekels = 1 shekel, and

$2/3\ C + 1/3$ sìla = 4;50 sìla + ;20 sìla = 5;10 sìla = $C'$.

It is only in view of these observed relations that it has been possible to find out what the correct form should be of the corrupt and damaged question in the exercise.

Note that what is called ganba *ul-li-im wa-at-ri-im* 'the market value of the oil surplus', according to the text in lines 11 and 20 has the value '6 barleycorns of silver', and recall that 1 shekel = 3 00 (180) barleycorns. At the given market rate of oil (10 sìla/shekel), 6 barleycorns of silver = ;02 shekel of silver corresponds to ;20 (1/3) sìla of oil. This, then, must be the amount of the curiously named 'oil surplus'.

In view of these considerations, it turns out that the corrupt and damaged formulation of the question in IM 54464 can be amended as follows: In *obv*. 4-5, the phrase

'[1] shekel of silver is brought to you. Lard and oil buy to me'

corresponds to the modern equation

$S + S'$ = 1 shekel.

More importantly, in *l.* 3, the incomplete and damaged phrase

'Two-thirds of the traded amount of lard plus [x] of the oil surplus'

should be corrected to

'Two-thirds of the traded amount of lard plus <the oil surplus is the traded amount of oil.
6 barleycorns of silver is the market value of> the oil surplus'.

These phrases correspond to the modern equation

$2/3\ C + m' \cdot s' = C'$, where $m'$ = 10 sìla / shekel and $s'$ = 6 barleycorns = ;02 shekel.

In the transliteration and translation of IM 54464 above, the text within angular brackets in *l.* 3 is a suggested reconstitution of the missing part of the text. (Apparently, the one who produced this clay tablet copied the text from another clay tablet, and when he should have copied the first instance of 'the oil surplus' he jumped ahead to the second instance of the same phrase and continued copying from there). Anyway, it should now be clear that the mysterious phrase 'oil surplus' is a quite appropriate name for how much the given quantity of oil surpasses 2/3 of the given quantity of lard.

Wrapping it all up, it seems to be clear that the question in IM 54464 can be reconstructed as

$2/3\ C + m' \cdot s' = C'$, where $m'$ = 1 bán (10 sìla) per shekel, and $s'$ = 6 barleycorns = ;02 shekel *ll.* 3-4

$S + S' = C/m + C'/m'$ = 1 shekel of silver, where $m$ = 1 bán 5 sìla (15 sìla) per shekel. *ll.* 4-5

This is *a system of two linear equations for the two unknowns C and C'.* Very few examples of such systems of linear equations are known to appear in Old Babylonian mathematical problem texts, and none with the same kind of solution procedure as the one in IM 54464. (See the discussion in Friberg, *MSCT* 1 (2007), 333, Sec. 11.2 m). Actually, the solution method used in IM 54464 is identical with a modern way of solving systems of two linear equations for two unknowns.

Indeed, here is a way in which the problem in this exercise can be solved by use of modern notations and modern algebraic operations. Given are the equations

$;40\ C + m' \cdot s' = C'$, where $m' = 10$ (sìla/shekel) and $s' = 6$ barleycorns = ;02 (shekel) *ll.* 3-4
$S + S' = 1$ (shekel) *ll.* 4-5

These equations can easily be transformed into the equivalent equations

$;40 \cdot C \cdot 1/m' + s' = C' \cdot 1/m'$
$C \cdot 1/m + C' \cdot 1/m' = 1$ shekel, where $m' = 15$ (sìla/shekel)

If the expression for the product $C' \cdot 1/m'$ in the first of these equations is inserted into the second equation, the result is the new equation

$C \cdot 1/m + ;40 \cdot C \cdot 1/m' + s' = 1$ (shekel), $s' = ;02$ (shekel).

This new equation, in its turn, can be simplified to

$C \cdot (1/m + ;40 \cdot 1/m') = 1$ shekel - 6 barleycorns = 1 shekel - ;02 shekel = ;58 (shekel). (*)

The solution to this single linear equation for the single unknown $C$ is, of course,

$C = ;58$ (shekel) $\cdot 1/(1/m + ;40 \cdot 1/m')$. (**)

Now, compare this explicit solution by use of modern symbolic notations and modern algebraic operations with the following steps of the solution procedure in the text of IM 54464:

| | |
|---|---|
| $1/m = 1/15 = ;04$ (shekel/sìla), $1 \cdot 1/m = 1 \cdot ;04 = ;04$ (shekel/sìla) | *ll.* 6-7 |
| $1/m' = 1/10 = ;06$ (shekel/sìla), $;40 \cdot 1/m' = ;40 \cdot ;06 = ;04$ (shekel/sìla) | *ll.* 8-9 |
| 1 shekel - 6 barleycorns = 1 shekel - ;02 shekel = ;58 shekel | *ll.* 10-12 |
| $1 \cdot 1/m + ;40 \cdot 1/m' = ;04$ (shekel/sìla) + ;04 (shekel/sìla) = ;08 (shekel/sìla) | *ll.* 13-14 |
| $1/(1 \cdot 1/m + ;40 \cdot 1/m') = 1/;08$ (sìla/shekel) = 7;30 (sìla/shekel) | *ll.* 14-15 |
| $C = ;58$ shekel $\cdot 1/(1/m + ;40 \cdot 1/m') = ;58 \cdot 7;30$ (sìla) = 7;15 (sìla) | *ll.* 15-16 |
| $S = C \cdot 1/m = 7;15$ (sìla) $\cdot$ ;04 (shekel/sìla) = ;29 shekel | *ll.* 17-18 |
| $S = C \cdot 1/m = 7;15$ (sìla) $\cdot$ ;04 (shekel/sìla) = ;29 shekel (note the repetition!) | *l.* 19 |
| $S' = S + 6$ barleycorns = ;29 shekel + ;02 shekel = ;31 shekel | *ll.* 20-22 |
| $C = 7;15$ (sìla), $C' =$ (;31 shekel $\cdot$ 10 sìla/shekel =) 5 10 (sìla) | *ll.* 23-24 |

It is clear that the steps from *l.* 6 to *l.* 16 in the solution procedure in IM 54464 almost exactly match the modern solution formula in (**) above. The further steps from *l.* 17 to *l.* 24 are more or less straightforward computations of the values of $S'$, $C$, and $C'$, once the value of $S$ is known.

Note that the fact that in *ll.* 6-7 and 13-14 the product $1 \cdot 1/m$ is explicitly computed (instead of just $1/m$) and then added to $;40 \cdot 1/m'$, *etc.* is a clear indication that the equation (*) above was solved by use of a normalized “reference object” (1 sìla of lard) and a scaling factor. (Cf. sec. 1.2 above).

Note also that there is a quite surprising step in the solution procedure of IM 54464 in *ll.* 20-22. There the silver paid for the lard is computed as a *second* amount of ;29 barleycorns, redundantly computed in *l.* 19, plus 6 barleycorns = ;02 shekel, the silver paid for the ‘oil surplus’. (A more obvious procedure would have been to compute, with the same result, the silver paid for the oil as *1 shekel minus the silver paid for the lard*). This surprising step in the solution procedure can be explained as follows:
A crucial step in the solution of the stated problem was the transformation of the given equation

$;40\ C + m' \cdot s' = C'$

into the equivalent equation

$;40 \cdot C \cdot 1/m' + s' = C' \cdot 1/m'$, with $m' = 15$ (sìla/shekel) and $s' = 6$ barleycorns = ;02 (shekel).

However

;40 · *C* · 1/*m*′ = *C* · 1/*m*, where *m* = 10 (sìla/shekel).

Therefore

*C* · 1/*m* + *s*′ = *C*′ · 1/*m*′,

which is the same as saying that

*S* + *s*′ = *S*′ with *s*′ = 6 barleycorns = ;02 (shekel).

Evidently, the one who constructed the original version of this market rate problem in IM 54464 knew perfectly well what he was doing!

Connected to the market rate problems discussed above are the following entries in Goetze's "compendium", text 2 = IM 52685 (Goetze, *Sumer* 7 (1951), 152, 47′-50′):

| | |
|---|---|
| [*m*]*a-ḫ*[*i-ra-tim na*]*-sa-ḫa-am ša-ma-am ù k*[*a-ma-ra*]*-am* | *l*. 47′ |
| *ne-mé-l*[*a-am ka-ma-r*]*a-am kaspam*(kù.babbar) *ša-qa-la-am* | *l*. 48′ |
| *šamnam*(ì.giš) *ù nāḫam*(ì.[šaḫ]) *ša-ma-am* | *l*. 49′ |
| *kasap*(kù.babbar) *šamnim*(ì.giš) *ù nāḫim*(ì.[šaḫ]) *ka-ma-r*[*a-am*] | |
| *ma-ḫi-ir šamnim*(ì.[giš]) *ù nāḫim*(ì.[šaḫ]) *ka-ma-ra-a*[*m*] | *l*. 50′ |

| | |
|---|---|
| to subtract, buy at, add together market rates | *l*. 47′ |
| [to collect] a profit, to weigh up silver | *l*. 48′ |
| to buy oil and lard | *l*. 49′ |
| to add together the silver for oil and lard | |
| to add together the market rates for oil and lard | *l*. 50′ |

## 4. MS 3928. Two quadratic equations (Squares). No solution procedure or answer
## 4.1. Presentation of the text

MS 3928 holds twenty lines of text, inscribed between irregularly drawn lines in a single column. It contains the statements of two quadratic equations, but no solution procedures and no answers. The tablet is the work of an inept scribe, who writes *ú* (*l*. 2) instead of *ù* 'and', *mitḫatum* instead of *mithartum* 'equalside' (*ll*. 2, 8, 11, 13, 16, 20), and who erroneously writes *mīnum* 'what?' twice in *ll*. 19-20. Some signs are poorly formed or incomplete and there are several erasures. In addition, in a strange way, the words of sentences are not held together, but are distributed over two or more consecutive lines of text. Therefore, this is a most unusual kind of Old Babylonian mathematical school text. Normally, only advanced and relatively skilled students wrote clay tablets with mathematical problem texts.

**MS 3928. Transliteration**

| | |
|---|---|
| 1-4 | *eqlī*(aša$_5^{li}$) / *ú* (sic!) *mi-it-ḫa-<ar>-ti* / *ak-mu-ur-ma* / 20 |
| 5-8 | *mi-nu-um* {*na*} / *eqlī*(aša$_5^{li}$) / *mi-nu-um* / *mi-it-ḫa-<ar>-ti* |
| 9-12 | *eqel*(aša$_5^{qé-el}$) / *ša-la-aš* / *mi-it-ḫa-<ra-ti>-*⸢*ia*$^{!}$⸣ / 42 21$^{!}$ 40$^{!}$ |
| 13-16 | 1 *mi-it-ḫa-<ar>-tum* / 40 *ši-ni-pi-it* / 30 *mi-ši-el* / *mi-it-ḫa-<ar>-tim*$^{?}$ |
| 17-20 | *mi-nu-um* / *eqlī*(aša$_5^{li}$) / *mi-nu-um* / *mi-nu-um mi-it-ḫa-<ar>-ti* |

**Translation**

| | |
|---|---|
| 1-4 | My field / and my equalside / I heaped (added together) / 20. |
| 5-8 | What / was my field? / What / was my equalside? |
| 9-12 | The field of / my three / equalsides / 42 21$^{!}$ 40$^{!}$. |

13-16 1 the equalside / 40 two-thirds, / 30 half / of the equalside.
17-20 What / was my field? / What / What was my equalside?

Note that in a mathematical text without solution procedure, like this one, and also no. 5 below, it cannot be concluded through inspection of the tense of the verbs in the solution procedure, as in texts nos. 1-3 above, that it is likely that the text is from Larsa.

## MS 3928. Mathematical commentary

It is important to understand the difference between the modern and the Babylonian way of thinking about squares. (Cf. Høyrup, *HM* 17 (1990)). In the modern view, a square is a given equal-sided and equal-angled two-dimensional quadrilateral, and the square sides are the line segments bounding that figure. In the Babylonian view, apparently, the *mitḫartum*, approximately meaning

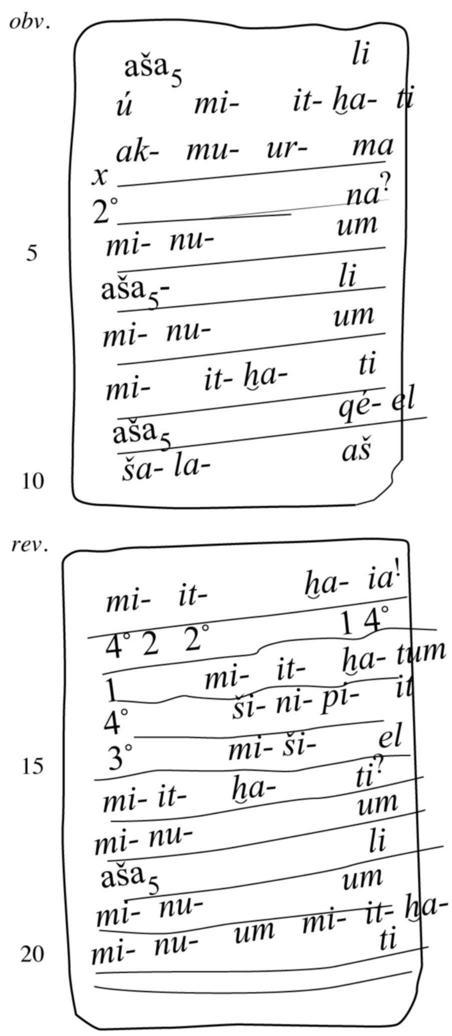


Fig. 4.1. MS 3928. Conform transliteration. Scale 1:1.

'that which is mutually equal', here loosely translated as 'equalside', is a given line segment, and the square, called the corresponding 'field', is constructed with departure from this line segment and another one like it.

Therefore, in MS 3299 # 1, *l.* 11 (Sec. 1.1 a above), the phrase *kiyā imtaḫar* means something like 'How much was it equalsided (square) (each way)?', rather than the modern 'What are the sides of the square?'. Similarly, in MS 3299 # 3, *ll.* 19-20 (Sec. 1.1 c above), the difficult phrase [*uš-ta-ki-il*] *a-na* 2 *eḫ-pe uš-ta-ki-il a-na* šà a.šà daḫ-*ma* 2 05. [*mi-it-ḫar-ti k*]*i-ia-a im-ta-ḫar* '[I made it eat itself], I broke it in 2, I made it eat it self, I added it onto the field: 2 05. How much was [my equalside] equalsided?' presumably means that on an 'equalside' of unknown length a square field is constructed, then the equalside is halved, and a new square field is constructed on the second equalside. Finally, the areas of the two square fields are added together, with the given result. Then the task is to find the length of the original 'equalside'.

In **MS 3928 # 1**, *ll.* 1-4, the 'field' is the field of an 'equalside', and the curious phrase

'My field and my equalside I added together, 20. What is my field? What is my equalside?'

means that the area of the square field constructed on an 'equalside' of unknown length plus the 'equalside' itself is a given number. The task is to find the area of the square field and the length of the 'equalside'. In modern symbolic notations, with *m* as the length of the 'equalside':

sq. $m + m = 20$ *ll.* 1-4
sq. $m = ?$, $m = ?$ *ll.* 5-8

Geometrically, this is clearly an impossible equation. How can a line segment be added to a square figure? A couple of ways to circumvent this difficulty have been suggested in a number of similar situations. The simplest explanation seems to be to reinterpret the question (in quasi-modern symbolic notations) in the following way (cf. Friberg, *MSCT* 1 (2007), 310):

sq. $m + 1 \cdot m = 20$

This means that the sum of the areas of the square constructed on the 'equalside' plus a rectangle of length 1 constructed on the 'equalside' is a given (area) number.

The way in which an Old Babylonian student of mathematics would solve an equation of this kind is well known. (See Friberg, *op. cit.*, and Høyrup, *LWS* (2002), 51). See also the brief discussion in Sec. 3.1 above of the way in which an Old Babylonian student of mathematics would solve a rectangular-linear system of equations). In quasi-modern notations, the solution method can be explained as follows:

sq. $(m + ;30)$ = sq. $m + 1 \cdot m$ + sq. $;30 = 20$ + sq. $;30 = 20;15$ ($^{81}/_4$)
$m + ;30$ = sqs. $20;15 = 4;30$
$m = 4;30 - ;30 = 4$, sq. $m = 16$

This result is obviously correct.

In **MS 3928 # 2**, *ll.* 9-20, the question is strangely formulated as follows:

The field of / three / equalsides / 42 21 40.
1 the equalside, 40 two-thirds, 30 half of the equalside.
What was my field? What was my equalside?

It can be interpreted, in quasi-modern symbolic notations, as the equation

sq. $m$ + sq. $m_2$ + sq. $m_3$ = 42 21 40, where $m_2 = {}^2/_3\, m$, and $m_3 = {}^1/_2\, m$ *ll.* 9-16
sq. $m = ?$, $m = ?$ *ll.* 17-20

This is a quadratic equation similar to, but more complicated, than the equation in MS 3299 # 4,

sq. (*m*/2) = 1 iku = 1 40 sar(sq. nindan),

discussed in Sec. 1.2 d above. Although there is no solution procedure provided by the text, the problem can, of course, be solved by an adaptation of the solution method used in MS 3299 # 4.
Thus, consider the "normalized reference equalside" 1. Then

sq. 1 + sq. ⅔+ sq. ½ = 1 +;26 40 + ;15 = 1;41 40.

This is not a regular sexagesimal number. Therefore its reciprocal cannot be computed. However,

42 21 40 · 6 = 4 14 10, so that 42 21 40 · 36 = 25 25,
1 41 40 · 6 = 10 10, so that 1 41 40 · 36 = 1 01,
42 21 40 / 1 41 40 = 25 25 / 1 01 = 25.

Note that in *l*. 12 of MS 3928 the given number 42 21 40 is written as 42 20 (+) 1 40. This was almost certainly a clever device invented by the teacher to make the student attacking the problem aware of the fact that the best way to divide 42 21 40 by 1 41 40 was to start by eliminating the common factor 1 40 in the two numbers!

An obvious difficulty is that the text does not specify what the "absolute" value should be of the given number 42 31 40. Only its "relative" value is given. However, in Old Babylonian mathematical problem texts, the sides of rectangles and squares are most of the time a moderately big number of nindan units. Therefore, it is reasonable to assume that the intended value of the area number 42 21 40 should be 42;21 40. Therefore, the "scaling factor" *f* in this case can be computed as follows:

sq. *f* = 42;21 40 / 1;41 40 = 25, so that *f* = 5.

Consequently, the answer to the exercise MS 3928 # 2 is that (in this order!)

sq. *m* = 25, *m* = 5.

Although it is not asked for, it is easy to see that then the sides of the three squares are

1 *m* = 5, ⅔ *m* = 3;20, and ½ *m* = 2;30.

It is also easy to check that then, indeed,

sq. (1 *m*) + sq. (⅔ *m*) + sq. (½ *m*) = 25 + 11;06 40 + 6;15 = 42 21 40.

### 4.2. The mathematical terminology used in MS 3928

MS 3928 is a brief text with much repetitiveness. Therefore not much can be said about its mathematical terminology. For what it is worth, however, here it is:

| | | | |
|---|---|---|---|
| *eqlī*(aša$_5^{li}$), *eqel*(aša$_5^{qé\text{-}el}$) | | < *eqlum* = aša$_5$(gán) or a.šà 'field, area' | *ll*. 1, 9 |
| *mi-it-ḫa-<ar>-tum* | equalside | square side | *ll*. 2, 8, *etc*. |
| (*a*) *ú* (*b*) *ak-mu-ur* | *a* and *b* I heaped | I computed *a* + *b*; < *kamārum* 'to heap' | *l*. 3 |
| *mi-nu-um* | what? | = ? | *ll*. 5, 7, *etc*. |
| *ša-la-aš* | three | 3 | *l*. 10 |
| *ši-ni-pi-it* | two-thirds | ⅔ | *l*. 14 |
| *mi-ši-el* | half | ½ | *l*. 15 |

Note the spurious NA between *ll*. 4 and 5. It can, possibly, be explained as the second syllable of a *mi-na* that was changed to *mi-nu-um*.

Already the brief partial review below of other texts that have a common vocabulary with MS 3298 is enough to show quite clearly that MS 3298, too, is a text from Larsa (group 1), just like the three texts MS 3299, 3976, and 3895 discussed in the sections above.

| | | | | |
|---|---|---|---|---|
| a.šà$^{lam}$, a.šà$^{lim}$ | AO 8862 (*MKT I*, 108) | *face* I:2, 5; II:35 | gr 1 a | Larsa |
| (*a*) *ú* (*b*) *ak-mu-ur* | – | *face* I:37; III:23 | – | |
| *mi-nu-um* | – | *face* I:7; II:1; III:26 | – | – |
| *ša-la-š(a)* | – | *face* III:27; IV:17 | – | – |
| *ši-ni-ip-pa-a-at* | – | *face* IV:20 | – | – |
| a.šà$^{lam}$, a.šà$^{lim}$ | BM 13901 (*MKT III*, 37) | *obv. i*:1, 10; *rev. ii*:11 | gr 1 c | Larsa |
| (*a*) *ù* (*b*) *ak-mu-ur* | – | *obv. i*:1; *ii*:3, *etc.* | – | |
| *mi-it-ḫar-tum* | – | *obv. i*:1, 5, *etc.* | – | – |
| *ša-la-aš* | – | *rev. i*:39; *ii*:17 | – | – |
| *ši-ni-pa-(a)-at* | – | *obv. i*:3; *ii*:45, *etc.* | – | – |
| *mi-ši-el* | – | *rev. i*:13, *ii*:19 | – | – |

### 4.3. BM 13901 # 1 and MS 5112 # 1, parallel texts to MS 3289 # 1

The quadratic equation in MS 3289 # 1 is of an unusually simple kind. Normally, quadratic equations in Old Babylonian "metric algebra" texts are more complicated.There are, however, at least two earlier published exercises quite similar to MS 3289 # 1. They will be displayed below. The first parallel exercise, **BM 13901 # 1** (Høyrup, *ChV* (2001), 165, and *LWS* (2002), 50) is from a text belonging to group 1 c (Larsa), while the other one, MS 5112 is from a text belonging to no identified group, possibly a Kassite (post-Old Babylonian) text.

**BM 13901 # 1** (group 1 c, Uruk)

1 *a.šà*$^{l[am]}$ *ù mi-it-ḫar-ti ak-m*[*ur-m*]*a* 45.e
2 1 *wa-ṣi-tam* / *ta-ša-ka-an ba-ma-at* 1 *te-ḫe-pe*
3 [3]0 *ù* 30 *tu-uš-ta-kal* / 15 *a-na* 45 *tu-ṣa-ab-ma*
1.[e] 1 íb.sá
4 30 *ša tu-uš-ta-ki-lu* / *lib-ba* 1 *ta-na-sa-aḫ-ma* 30 *mi-it-ḫar-tum*

1 The field and my equalside I heaped (added together), it is 45.
2 1, the projection / you shall set, half of 1 you shall break (= 30).
3 30 and 30 you shall let eat each other (= 15), 15 to 45 you shall add on (= 1).
1 makes 1 equalsided.
4 30 that you made eat itself, out of 1 you shall tear out, then 30, the equalside.

In quasi-modern symbolic notations, the equation solved in this exercise is

sq. $m + 1 \cdot m = 45$.

In *l*. 2, the coefficient '1' in this equation is called *wāṣītum* 'that which goes out, projection', which is a suitable name for one of the sides of the rectangle with the other side *m* which may be thought of as projecting out from the square with the side *m*. A geometric interpretation of the solution procedure is illustrated in Høyrup, *op. cit*., 51, Fig. 2. The idea is to divide the projecting rectangle in two equal halves and to let one half project out horizontally, the other vertically. This is the first step in a solution of the quadratic equation through a *geometric* "completion of the square". The explicitly given solution is that the square side is *m* = '30'. However, what this means in "absolute" numbers is open to debate. One possibility is that the projection '1' is interpreted as 1, and that '45' is interpreted as ;45, and then it follows that *m* = ;30. Indeed, it is easy to check that

sq. ;30 + 1 · ;30 = ;15 + ;30 = ;45.

Another possibility is that '1' is interpreted as 1 00, and that '45' is interpreted as 45 00. In that case, it follows that $m$ = 30. Indeed, it is easy to check that

sq. 30 + 1 00 · 30 = 15 00 + 30 00 = 45 00.

A third possibility is that the one who constructed this exercise made a simple mistake. He may have intended the solution to be $m$ = 30, and the projection to be 1 (both in absolute numbers), but then computed the third term in the equation incorrectly as

sq. 30 + 1 · 30 = 15 + 30 = 45 (instead of 15 (00) + 30 = 15 30).

A second exercise parallel to MS 3928 # 1 is **MS 5112 # 1** (Friberg, *MSCT* 1 (2007), 309). In the geometric interpretation of that exercise there is already from the beginning two rectangles projecting out from the square, one horizontally, the other vertically. The solution method is again a geometric application of the method of "completion of the square". The explicitly given solution is that the square side is $m$ = 10 nindan. The writer of the text forgot to mention that then the field (area) of the square is sq. $m$ = 1 40 sq. nindan. Note that therefore, as required in the question,

sq. $m$ + 2 · $m$ = 1 40 sq. nindan + 2 nindan · 10 nindan = (1 40 + 2 · 10) sq. nindan = 2 (00) sq. nindan.

**MS 5112 # 1** (Kassite?)

| | |
|---|---|
| *obv. i*: 1 | [a.šà ù 2 téš.a.s]i gar.gar-*ma* 2<br>a.šà *ù* téš.a.si en.nam |
| 2 | [za.e ak.da.zu.d]è |
| 3 | *a-na* 2 ša gar.gar 1 *wa-ṣi-tam* daḫ-*ma* / [2 01 íb.s]á-*šu* duḫ-*ma* 11<br>*i-na* 11 *wa-ṣi-tam* 1 zi-*ma* |
| 4 | [10] nindan.[t]a.àm téš.a.si |

| | |
|---|---|
| *obv. i*: 1 | I heaped (added together) the field and two samesides (square sides): 2.<br>What are the field and the sameside? |
| 2 | When you work it out: |
| 3 | Add on to 2 of the heap (sum) 1 the projection: 2 01. Solve (compute) its equalside (square root): 11.<br>Tear off (subtract) 1, the projection, from 11. |
| 4 | 10 nindan each way is the sameside. |

Note that in this text square sides are denoted not by the usual term *mitḫartum* = íb.sá but by the otherwise never observed term téš.a.si, possibly meaning 'given together'. To emphasize the use of this unique term, téš.a.si is translated as 'sameside' instead of as 'equalside'.

### 4.4. BM 13901 ## 14, 24 and Str. 363, texts related to MS 3928 # 2

As was remarked above, MS 3928 appears to be written by a scribe school student at the beginning of the more advanced stage of his mathematical education. After finishing his study of arithmetical and metrological lists and tables, and elementary arithmetical exercises (see Friberg, *MSCT* 1 (2007), Chs. 1-3; Proust, *TMN* (2007), Chs. 5-6), he was, perhaps for the very first time, listening to his teacher's dictation and writing down the questions of a couple of unusually simple mathematical problem texts. Note that the format of the clay tablet is the same as that normally used for brief excerpts from arithmetical or metrological table texts.

MS 3928 # 1 is a quadratic equation of the simplest possible kind, which looks like the first exercise of a theme text with the theme 'quadratic equations'. Similarly, MS 3928 # 2 looks like one of the first exercises of a theme text with the theme 'equations for two or more squares'. An Old Babylonian mathematical "recombination text", containing two interesting exercises apparently excerpted from a certain (lost) theme text of precisely this kind, is the Larsa text **BM 13901** (Neugebauer, *MKT*

*III*, 1-5), in which exercise # 24 is a more complicated variant of MS 3892 # 2. Below is reproduced only the question of that exercise.

**BM 13901 # 24** (group 1 c, Uruk)

*rev. ii*: 17 a.šà *ša-la-aš mi-it-ḫ*[*a-r*]*a-ti-i*[*a*] *ak-mur-ma* 29 10

18 *mi-it-ḫar-tum ši-ni-pat*[*mi-i*]*t-ḫar-tim ù 5 nindan*
19 [*mi-š*]*i-e*[*l m*]*i-it-ḫar-tim ù* 2 [30] *nindan*

rev. ii: 17 I heaped (added together) the fields (areas) of my three equalsides (squaresides): 29 10.

18 The equalside, two-thirds of the equalside and 5 nindan,
19 half of the equalside and 2;30 nindan.

In quasi-modern symbolic notations, this vaguely formulated question can be understood as

sq. $m$ + sq. $m_2$ + sq. $m_3$ = 29 10, where $m_2 = \tfrac{2}{3}\, m$ + 5 nindan, and $m_3 = \tfrac{1}{2}\, m_2$ + 2;30 nindan

In the exercise, the answer is given as

$m$ = 30 (nindan), $m_2$ = 25 (nindan), $m_3$ = 15 (nindan).

The solution procedure, contains several errors which, however, (probably not by accident) manage to cancel each other out so that the answer becomes correct. Indeed,

sq. 30 + sq. 25 + sq. 15 = 15 (00) + 10 25 + 3 45 = 29 10.

A simpler, but similar, exercise from the same recombination text has the following question:

**BM 13901 # 14** (group 1 c, Larsa)

*obv. ii*: 44 a.šà *ši-ta mi-it-ḫa-ra-ti-ia ak-mur-ma* [25] 25
45 *mi-it-ḫar-tum ši-ni-pa-at mi-it-ḫar-tim* [*ù* 5 nin]da

*obv. ii*: 44 I heaped (added together) the fields (areas) of my two equalsides (square sides), then [25] 25.
45 The equalside, two-thirds of the equalside [and 5 nin]da.

In quasi-modern symbolic notations, this question can be reformulated as

sq. $m$ + sq. $m_2$ = 25 25, where $m_2 = \tfrac{2}{3}\, m$ + 5 nindan.

The solution procedure, which is correct, essentially proceeds by reducing the given problem to the following quadratic equation (again in quasi-modern symbolic notations):

1;26 40 sq. $m$ + 2 · 3;20 $m$ = 25 (00).

The answer states, correctly, that

$m$ = 30 (nindan), $m_2$ = 25 (nindan).

A related exercise, published in Neugebauer, *MKT I* (1935), 243, has the following question:

**Str. 363 # 1** (group 3, Uruk)

*obv.* 1 a.šà 2 íb.sá gar.gar-*ma* 16 40
2 íb.sá $^2/_3$ íb.sá / 10 *i-na* íb.sá tur ba.zi
íb.sá en.nam /

*obv.* 1 I heaped (added together) the fields (areas) of 2 equalsides (squaresides): 16 40.
2 Equalside ⅔ of equalside, I tore off 10 from the small equalside.

Note that, in contrast to BM 13901, this text is written predominantly by use of Sumerograms. The question can be reformulated as follows:

sq. $m$ + sq. $m_2$ = 16 40, where $m_2$ = ⅔ $m$ - 10.

A solution procedure of exactly the same kind as the one in BM 13901 # 14, essentially proceeds by reducing the given problem to the following quadratic equation:

1;26 40 sq. $m$ - 2 · 6;40 $m$ = 15 (00).

The answer states, correctly, that

íb.sá gu.la 'the big equalside' = $m$ = 30 (nindan),
íb.sá tur.ra 'the small equalside' = ⅔ $m$ - 10 (nindan) = 10 (nindan).

Related to MS 3928 # 2 are also the following entries in Goetze's "compendium", text 2 = IM 52685 (Goetze, *Sumer* 7 (1951), 147), 13´-15´:

| | |
|---|---|
| a.šà *ši-ta mi-it-ḫa-ra-tim ka-ma-ra-am* | to add the areas of two equalsides |
| a.šà *ša-la-aš mi-it-ḫa-ra-tim ka-ma-ra-am* | to add the areas of three equalsides |
| a.šà *er-bé-e mi-it-ḫa-ra-tim ka-ma-ra-am* | to add the areas of four equalsides |

## 5. MS 2833. Three questions concerning ditches. No solution procedure or answer
## 5.1. Presentation of the text

MS 2833 is inscribed only on the obverse, with a text of thirteen lines, set in a single column and divided into three sections by rulings. The content is a series of questions about ditches (*ikum*, strictly speaking a ditch and dike in combination) of particular dimensions. Damage prevents a full decipherment and impedes complete understanding. There is no solution procedure or answer.

**MS 2833. Transliteration**

1 [*ik*]*um*(e) *i-na* ½ nindan 3 ⅓ *šiqil*(gín) *eperū-šu*(saḫar.[bi])
2 ⌜2⌝ *ammat*(kùš) *išpil*(ì.GAM) *i-na* ⌜*išdim*(suḫuš)⌝ *ki m*[*a-ṣi*]
3 *a-ḫa-ar-ra-ar e-le-nu m*[*īnam*(⌜e⌝[n.ta.àm])?]
4 *a-ḫa-ar-*[*r*]*a-a*[*r*]

---

5 *i-na* ½ nindan 3 ⌜⅓⌝ g[ín saḫar.bi (. . .)]
6 *ù e-le-nu* 4 k[ùš . . . ]
7 *ù* ⌜*ikum*(e)? *šū* (bi)⌝ *mīnum*(⌜en⌝.t[a.àm])

---

8 *ikum*(e) 5 nindan *šiddum*(ús) 1 nindan *i-na išdim*(suḫuš) *ah-ru-u*[*r*]
9 ½ nindan *e-le-nu aḫ-ru-ur* ½ nindan *e-x x*
10 *i-na-an-na ki-ma iku*/*im*(e) *kalakkam*(ki.lá!?=diš.aš) *iškunu*/*aškun*(in.gar)
11 *i-qá-ab-bi-a mu-ú im-ta-aḫ-ru-ni-in-ni*
12 2 ½ *ammat*(kùš) *išpil*(ì.GAM) *ki ma-ṣi ana*(nam) *šuplim*(ì.GAM)
13 *lu ikam*(e) *lu šiddam*(uš)*-ma ikam*(e) *lu-uk-ṣú-ur*
rev. uninscribed

**Translation**

1 [A] ditch. In ½ nindan (of length) [its] earth(volume) was 3 ⅓ shekels.
2 It was 2 cubits deep. At the base how [much]

3 shall I dig, at the top [what?]
4 shall I dig?

---

5 In ½ nindan (of length) [its earth (volume) was] $3\ ^{1}/_{3}$ [shekels ( . . . )],
6 and at the top 4 cu[bits . . . ]
7 and that ditch?, what (were they each)?

---

8 A ditch. 5 nindan was the length, 1 nindan at the base I dug,
9 ½ nindan at the top I dug, ½ nindan I . . .
10, 11 Now, (someone) tells me that the ditch formed an excavated pit,
(so that) the waters were equal-sided around me,
12 (and) it was 2 ½ cubits deep, how much to the depth
13 should I construct either ditch or side and ditch?

## MS 2833. Mathematical commentary

Regrettably, the text of exercise # 1 is badly preserved, and even less remains of the text of exercise # 2. Just as troubling is that the text in its original form of all three exercises ## 1-3 was too brief, with several omissions and badly formulated sentences. For these reasons, and since there is no solution procedure and no answer, the mathematical commentary below will be both incomplete and tentative.

In **exercise # 1**, apparently an excavated ditch has a trapezoidal cross section, although this assumption is not mentioned explicitly. Any section of the ditch of length ½ nindan (= 1 'reed') has the given volume $3\ ^{1}/_{3}$ volume-shekels = ;03 20 volume-sar. The given depth of the ditch is 2 cubits (c. 1 meter). Since 1 volume-sar = 1 sq. nindan · 1 cubit, it follows that the *average width* of the trapezoidal cross section of the ditch is

*obv*.

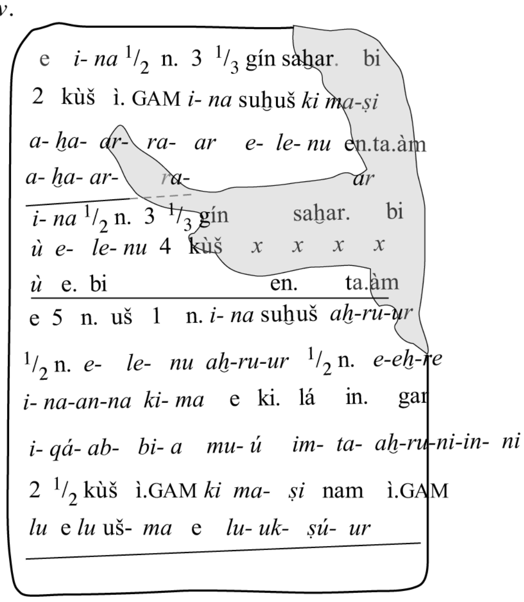


Fig. 5.1. MS 2833, *obv*. Conform transliteration. Scale 1:1.

;03 20 sq. nindan · cubit / ;30 nindan · 2 cubits = ;03 20 nindan = ⅔ cubit.

Already this result is strange, because a ditch of this depth and width would be very narrow.

In addition, the question is how wide the ditch shall be at the top and at the bottom, but no information is given about how great the vertical decrease (the 'feed') of the sides of the ditch was supposed to be. If, for instance, the intended answer was that the upper width should be 1 cubit and the lower width ⅓ cubit, then the corresponding 'feed' would have been

(1 cubit - ⅓ cubit) / 2 cubits = ⅙ cubit/cubit = 5 fingers/cubit = ;00 50 nindan/cubit.

In **exercise # 2**, any section of the ditch of length ½ nindan (= 1 'reed') has again the given volume 3 ⅓ volume-shekels = ;03 20 volume-sar. In addition, the upper width is given, as 4 cubits (which means that the ditch in this case is no longer unreasonably narrow, at least if the depth is the same as before). Because of the extensive damage to the text of this exercise, it is not clear what else is given, or what should be computed.

In **exercise # 3**, a ditch of length 1 nindan has the lower width$^{?}$ 1 nindan, and the upper width$^{?}$ ½ nindan. Something, possibly the depth is ½ nindan. However, in *l.* 12, something else (the 'waters'?) is 2 ½ cubits deep, and possibly square.

It is very strange that the lower width is greater than the upper width, unless in this exercise the focus has shifted from the excavated ditch to the dike formed by the excavated earth. (What is the meaning of the phrase 'The ditch formed an excavated pit'?).

Mathematically, these exercises are exceedingly simple, but because of the mentioned difficulties it remains unclear what they are about. Unfortunately no parallel is known to exercise # 3, potentially the most interesting of the three exercises.

## 6. MS 4905. Two problems concerning brick piles. Solution procedure but no answer
## 6.1. Presentation of the text

This small tablet entered the Schøyen collection with archival tablets dated to the first Sealand dynasty (mostly MS 2200), but was discovered by Stephanie Dalley, in her work on the archive (Dalley, *MSCT* 3 (2009), to be of mathematical not administrative content. It nevertheless bears a physical resemblance to some of the Sealand tablets and probably has the same provenance. Its text thus becomes important as a witness to the continuity of the southern Old Babylonian mathematical tradition in the period immediately after the end of Ḫammurapi's dynasty. The contents appear to be two similar mathematical procedures, perhaps involving piles of bricks.

**MS 4905. Transliteration**

**# 1**
1 [x (x x) x *la i*]-*de-ma*$^{?!}$ x x x
2 *šiddam u pūtam*([u]š sag.bi) *uš-ta-ka-al-ma*
3 *i-gi-a-šu a-pa-ṭa-ar-ma*
4 *a-na* 25 (sup. ras.) *mu-ta-ši-i-tum* (= *tim*$^{!?}$)
5 *sà-ni-iq*
**# 2**
6 *ši-id*$^{?}$*-di*$^{?}$ *amarim*(sig$_4$$^{!}$.⸢anše⸣)*-ma la i-*⸢*de-*{*x*}⸣*-ma*
7 *šiddam u pūtam*(uš.sag.bi) *uš-ta-ka-al-ma*
8 *a-na me-li-im at-ta-na-ši-ma*
9 *i-gi-a-am a-pa-ṭa-ar-ma*
10 *a-na* 4 *a-na-ši-i-ma*
11 [x ]x x *sà-ni-iq*

**Translation**

**# 1**

1 [ . . . I do not] know and . . . ,
2 I shall let eat each other (multiply) length and width and
3 I shall solve (calculate) its reciprocal, and
4 (multiply) by 25, the one that always raises (multiplies).
5 It will be in order (?).

**# 2**

6 The length(?) of a brick-pile I do not know, and
7 I shall let eat each other (multiply) length and breadth and
8 I shall always raise (multiply) it by the height and
9 I shall solve (calculate) the reciprocal and
10 I shall raise (multiply) by 4:
11 . . . will be in order (?).

This small clay tablet contains two mathematical exercises. The question of exercise # 1 (*l.* 1) is too damaged to be readable. The question of exercise # 2 (*l.* 6) is not quite as badly damaged but is still only tentatively readable. Presumably, both exercises are about the dimensions of brick piles ($sig_4$.anše).

Just like text no. 4 above, the present text appears to have been written by a student in a scribe-school, inattentively listening to his teacher's exposition of the two problems and their solutions, and making notes lacking many crucial bits of information, in particular the given numerical data.

Nevertheless, the solution procedure of exercise # 1 is clear enough. It is an obvious parallel to the solution procedure of exercise # 1 in text no. 1 above (MS 3299). Similarly, the solution proce-

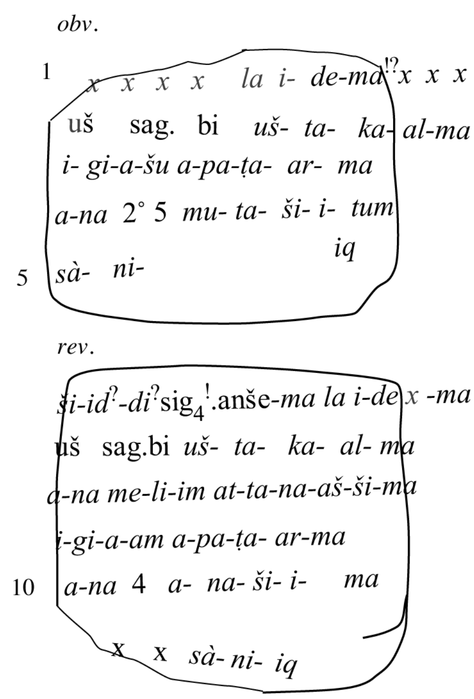


Fig. 6.1. MS 4905. Conform transliteration.

dure in exercise # 2 seems to be parallel to the solution procedure of CBS 12648 # 2 (sec. 1.3 above), although the lack of numerical details doesn't allow anything more to be said.

### 6.2. The mathematical terminology used in MS 4905

Goetze's analysis (*MCT* (1945), 147) would have shown that MS 4905 is a "southern" text, in view of the spelling

| | | | |
|---|---|---|---|
| *sà-ni-iq* | *sà* = ZA | Goetze's criterion | S 2 |

More information can be obtained from the mathematical terminology used in the solution procedures of MS 4905 (*ll.* 2-4 and 7-10). The verb forms appearing there are:

| | | | |
|---|---|---|---|
| (*a*) (*b*) *uš-ta-ka-al* | *a* and *b* I shall let eat each other | compute $a \times b$; < *akālum* 'to eat' | *ll.* 2, 7 |
| *i-gi-a-am a-pa-ṭa-ar* | I shall solve the reciprocal | compute $1/n$; < *paṭārum* 'to loosen' | *ll.* 3, 9 |
| (*a*) *a-na* (*b*) *at-ta-na-ši* | I shall always raise *a* to *b* | compute $a \cdot b$; < *našûm* 'to raise' | *l.* 8 |
| (*a*) *a-na* (*b*) *a-na-ši-i* | I shall raise *a* to *b* | compute $a \cdot b$; < *našûm* 'to raise' | *l.* 10 |

These verbs are in the durative tense, just as the verbs in the solution procedures of texts nos. 1-3 above, and as the verbs in the solution procedures of all other Old Babylonian mathematical problem texts belonging to group 1 (Larsa). However, all the verbs are consistently in the 1st(!) person singular, which almost never happens in solution procedures in Old Babylonian mathematical texts. An exception is the solution procedures of exercises ## 3 and 5 in the mathematical "recombination text" AO 6770 (group 1 a; Neugebauer, *MKT II* (1935), 37), where the following verb forms appear:

| | | | |
|---|---|---|---|
| *uš-ta-ka-al* | I shall let eat each other | < *akālum* 'to eat' | *rev.* 7 |
| *a-ḫa-ar-ra-aṣ* | I shall break off (subtract) | < *ḫarāṣum* 'to break off' | *rev.* 8 |
| *at-ta-na-aš-ši* | I shall always raise | < *našûm* 'to raise' | *rev.* 9 |
| *a-na-aš-ši* | I shall raise | < *našûm* 'to raise' | *rev.* 18 |
| *a-pa-ṭa-ar* | I shall solve | < *paṭārum* 'to loosen' | *rev.* 18 |
| *e-ṣí-ip* | I shall double | < *eṣēpum* 'to double' | *rev.* 19 |
| *e-ḫe-pe* | I shall break (in two) | < *ḫepûm* 'to break' | *rev.* 20 |

In contrast to this, the solution procedures in exercises ## 1 and 2 in AO 6770 both begin with the phrase *at-ta i-na e-pe-ši-i-ka* 'when *you* work it out', and correspondingly have the verbs in the 2nd person singular[8]!

The indicated strong similarity between MS 4905 and AO 6770 ## 3 and 5 (group 1 a, from Larsa; see above, sec. 2.3 for a discussion of # 3) suggests that, after the Old Babylonian period, the Larsa tradition of mathematics continued elsewhere in the south, under the Sealand dynasty.

In exercise # 1 as well as exercise # 2 of MS 4905, the last word is *saniq*, stative of *sanāqum* 'to check (that something is in order)'. It is likely that it plays the same role in this mathematical text as the phrase *ki-a-am né-pe-šu* 'such is the procedure' in YBC 4663 (group 2 a) and *ki-a-am ne-pé-šum* or simply *ne-pé-šum* in BM 85194 and other texts from Sippar (group 6), namely to state that the solution procedure is completed. The word was previously known from only one Old Babylonian math-

[8] The solution procedure in the early Old Babylonian exercise AO 8862 # 2 (group 1 a; Neugebauer, *MKT I*, 109-110), which also begins with the phrase 'you, in your procedure', alternates in an erratic way between verbs and pronominal suffixes in the 1st and 2nd person singular!

ematical text, namely Neugebauer, *MCT* (1945), text B = YBC 4675 (group 1 a, Larsa). It appears there *twice in the question*, in the phrase

*ki ma-ṣi lu-uš-ku-un-ma* 1 (bùr) aša$_5$ *lu-ú sà-ni-iq* *obv.* 5, 7
How much shall I set so that 1 bùr (area measure) will be in order (?)?

(The translation is amended here). It also appears *twice in the answers*, in the phrase

1 (bùr) aša$_5$ *sà-ni-iq* *rev.* 6, 16
1 bùr (area measure) will be in order (?).

Note the absence in exercises ## 1 and 2 of MS 4905 of a proper question and of detailed computations, as well as the use of non-standard terminology, such as *mu-ta-ši-i-tum* and *at-ta-na-aš-ši*, both words derived from the Gtn stem of *našûm* 'to carry, multiply' indicating *something that should always be done in a certain way*, and the stative *sà-ni-iq* 'it is in order (?)', vaguely indicating that *if the problem is solved in this way, the correct answer will be obtained*. All these unusual features suggest that MS 4905 is not a direct copy of an older text provided as a model by the teacher, but rather what a rather young student wrote, listening to the teacher's *oral presentation* of the problem and the proper way to solve it. Maybe also the unusual use of verbs in the 1st person can be explained in this way: When the inexperienced student heard the teacher say 'you shall do this', he did not copy this *verbatim* but instead obediently wrote 'I shall do this'.

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# APPENDIX

**Unpublished Mathematical and Related Texts in the Schøyen Collection**

The list presented hereunder is extracted from the database of the Schøyen Collection. It comprises all tablets not published by Jöran Friberg in *MSCT* 1 (2007) but identified in the collection's database as mathematical, excluding some pieces that were discovered to be falsely described. It has not been possible to verify every description but the list gives a fair impression of the extent and character of the residue of the collection's mathematical cuneiform tablets.

| | |
|---|---|
| 2318/01 | Scratch pad: numbers 5, 30 then 2, 40, 9, 40. Old Babylonian, 73 x 48 x 20 mm, 1 column, 2 lines. |
| 2769 | Conversion table, capacity measure (**bariga**, **bán**) into sexagesimal numbers. Old Babylonian, 75 x 44 x 25 mm, 3 cols., 16 lines. |
| 2833 | Problems for ditches. Old Babylonian on clay, 75 x 55 x 24 mm, 1 column, 13 lines. Published above, text no. **5**. |
| 2854 | Multiplication table for 3 20, from 1 to 20, then 30, 40 and 50 in the sexagesimal system. Old Babylonian, 89 x 44 x 22 mm, 3 columns, 23 lines. |
| 3263/9 | Problem text? Old Babylonian, 13 x 21 x 3 mm, flake, 6 lines. Cuneiform published above, falsely joined to text no. 1. |
| 3299 | 2 problems about rectangular prisms, 2 problems about squares. Old Babylonian, 94 x 62 x 22 mm, 1 column, 25+2+13 lines. Published above, text no. **1**. |
| 3846 | Multiplication table for 24. Early Old Babylonian? 80 x 47 x 29 mm, 1 column, 10+6 lines. |
| 3847 | Scratch pad. Old Babylonian? 56 x 51 x 27 mm, 1 column, 1 line. |
| 3860 | Scratch pad. Old Babylonian? 56 x 59 x 32 mm, 1 column, 4 lines. |
| 3862 | Mathematical table. Old Babylonian, 84 x 49 x 28 mm, 1 column, 15+7 lines. |
| 3869/02 | Conversion table, linear measure (**kùš**, **nindan**) into sexagesimal numbers. Old Babylonian, 72 x 48 x 22 mm, 1 column, 4+5 lines. |
| 3869/03 | Conversion table, linear measure (**šu.si**, **kùš**, **nindan**) into sexagesimal numbers, with colophon. Old Babylonian, 101 x 44 x 22 mm, 1 column, 20+11+4 lines. |
| 3869/04 | Conversion table, capacity measure (**bariga**, **bán**) into sexagesimal numbers. Old Babylonian, 70 x 44 x 23 mm, 1 column, 15+2+13 lines. |
| 3869/09 | Conversion table, capacity measure (**bán**, **sìla**) into sexagesimal numbers. Old Babylonian. Upper 1/3 of a tablet, 45 x 50 x 23 mm, 1 column, 11+0 lines. |
| 3869/10 | Conversion table, capacity measures (**gur**, **bariga**) into sexagesimal numbes. Old Babylonian, 71 x 55 x 21 mm, 1 column, 9+2 lines. |
| 3889 | Scratch pad, numbers in a grid. Old Babylonian, 84 x 86 x 29 mm, 1 column, 3 lines. |
| 3892 | Scratch pad. Old Babylonian?, 33 x 28 x 16 mm, 1 column, 1 l., other face erased. |
| 3893 | Scratch pad, grid. Old Babylonian?, 58 x 76 x 22 mm, 1 column, 2 lines. |
| 3894/01 | Multiplication table, with colophon (**im.gíd.da**). Early Old Babylonian? 63 x 46 x 23 mm, 1 column, 13+6 lines. |
| 3894/02 | Multiplication table for 50, with colophon (**im.gíd.da**). Early Old Babylonian? 67 x 46 x 20 mm, 1 column, 13+11 lines. |
| 3894/03 | Multiplication table for 50. Early Old Babylonian? 63 x 38 x 18 mm, 1 column, 12+12 lines. |
| 3895 | Problem text. Old Babylonian, 63 x 76 x 28 mm, 1 column, 13+2+11 lines. Published above, text no. **3**. |
| 3896 | Scratch pad, grid. Old Babylonian? 78 x 62 x 28 mm, 2 columns, 3+2 lines. |
| 3897 | School text, including mathematical calculation; Old Babylonian, 79 x 82 x 26 mm, 2 columns, 4+1+1 lines. |
| 3901 | Area calculation and diagram. Old Babylonian? 57 x 48 x 22 mm, 1 column, 5+6 lines. |

3903 Multiplication table. Old Babylonian, 63 x 47 x 22 mm, 1 column, 1+9 lines.
3904 Scratch pad. Old Babylonian? 39 x 31 x 15 mm, 1 column, 4+3 lines.
3907 Mathematical diagram and text. Old Babylonian? Lenticular tablet, 67 x 32 mm, 1 column, 12+1 lines.
3910 Mathematical diagram and calculations. Old Babylonian, lenticular tablet, 52 x 19 mm, 2+2 columns, 2+2+3+3 lines.
3911 School text: administrative text practice; mathematical calculation. Old Babylonian, 60 x 44 x 20 mm, 1 column, 4+4 lines.
3914 Multiplication table. Old Babylonian, 58 x 39 x 22 mm, 1 column, 21 lines.
3918 Scratch pad. Old Babylonian, ½ of a tablet, 52 x 63 x 27 mm, 1 column, 2 lines.
3921 Numerical content. Old Babylonian + modern pastiche, 52 x 52 x 22 mm, 1 column, 8+8 lines.
3924 Scratch pad with calculations, partly erased. Old Babylonian, 73 x 78 x 27 mm, 2 columns, obverse: 2 erased lines., reverse: 4+1 lines with ruling between lines.
3928 Problems for squares. Old Babylonian, 65 x 46 x 27 mm, 1 column, 10+10 lines. Published above, text no. **4**.
3935 Mathematical content? Old Babylonian, 69 x 50 x 26 mm, 1 column, 5 lines.
3943 List of numbers, non-mathematical? Old Babylonian, 97 x 58 x 23 mm, 1 column, 3+2 lines.
3953 Multiplication exercises? Old Babylonian, lower part of tablet, 35 x 41 x 18 mm, 1 column, 3 lines.
3954 Multiplication table for 7. Old Babylonian, upper ½ of a tablet, 39 x 44 x 24 mm, 1 column, 7 lines.
3958 Scratch pad, grids with numbers. Old Babylonian, 97 x 68 x 22 mm, 3+4 columns, 8+2 lines.
3959 Scratch pad with calculations. Old Babylonian, lenticular tablet, diam. 56 x 29 mm, 1 column, 5 lines.
3960 Mathematical table. Old Babylonian, 58 x 51 x 22 mm, 1 column, 10+10 lines., cf. MS 3970.
3975 Scratch pad, many-place sexagesimal numbers, drawing. Old Babylonian, 60 x 62 x 27 mm, 1 column, 2 lines., drawing on reverse.
3976 Problem about barleycorn. Old Babylonian, 62 x 75 x 25 mm, 1 column, 11+1 lines. Published above, text no. **2**.
3980 Mathematical table. Old Babylonian?, 59 x 51 x 17 mm, 1 column, 12 lines.
3982 Mathematical text. Old Babylonian, detailed description lacking.
3983 Mathematical text. Old Babylonian, detailed description lacking.
3984 Combined multiplication table for 4 and 3. Old Babylonian?, 108 x 52 x 27 mm, 1 column, 29+14 lines.
4905 2 problems for a rectangle and a rectangular prism. Sealand period. 40 x 47 x 23 mm, 1 column. 5 + 6 lines. Published above, text no. **6**.
5016 Numbers. Old Babylonian, 123 x 65 x 34 mm, 1 column, 73 lines.

FIGURES

Photographs courtesy of the Schøyen Collection, drawings by Andrew George.

Plate 1. Text no. 1. MS 3299, with MS 3263/9 and 15 falsely attached

MS 3299

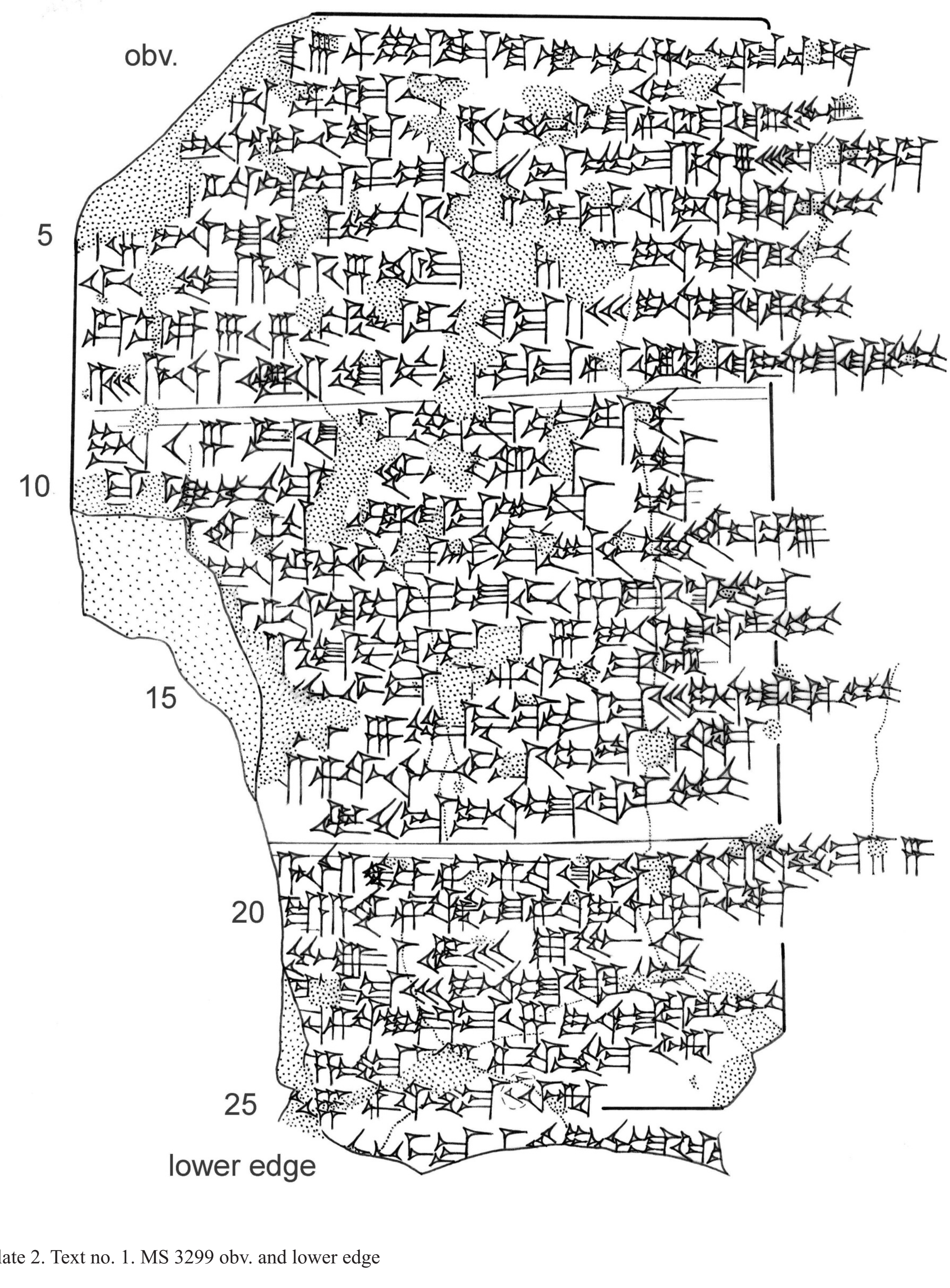



Plate 2. Text no. 1. MS 3299 obv. and lower edge

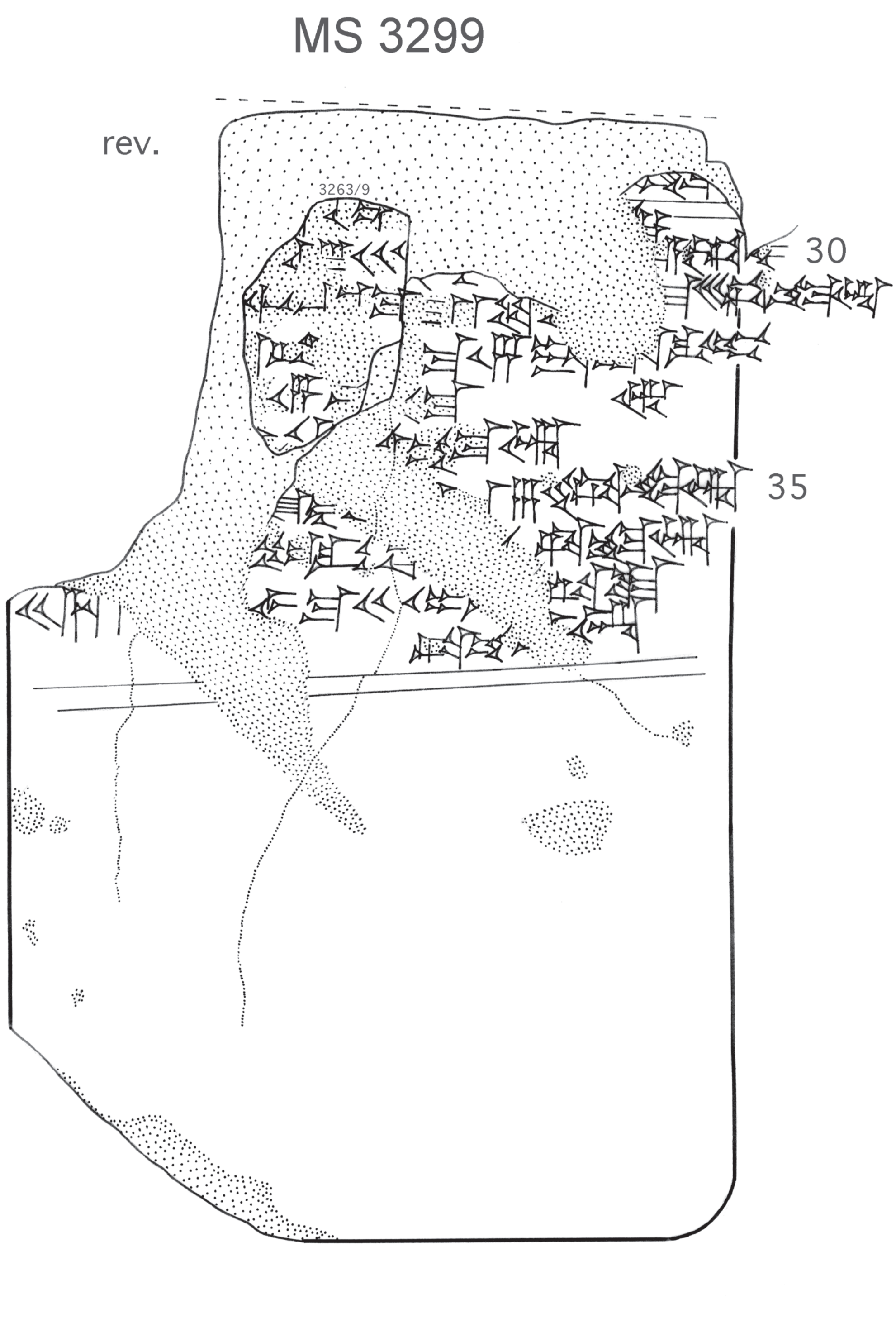



Plate 3. Text no. 1. MS 3299 rev., with MS 3263/9 falsely attached


Plate 4. Text no. 2. MS 3976

# MS 3976

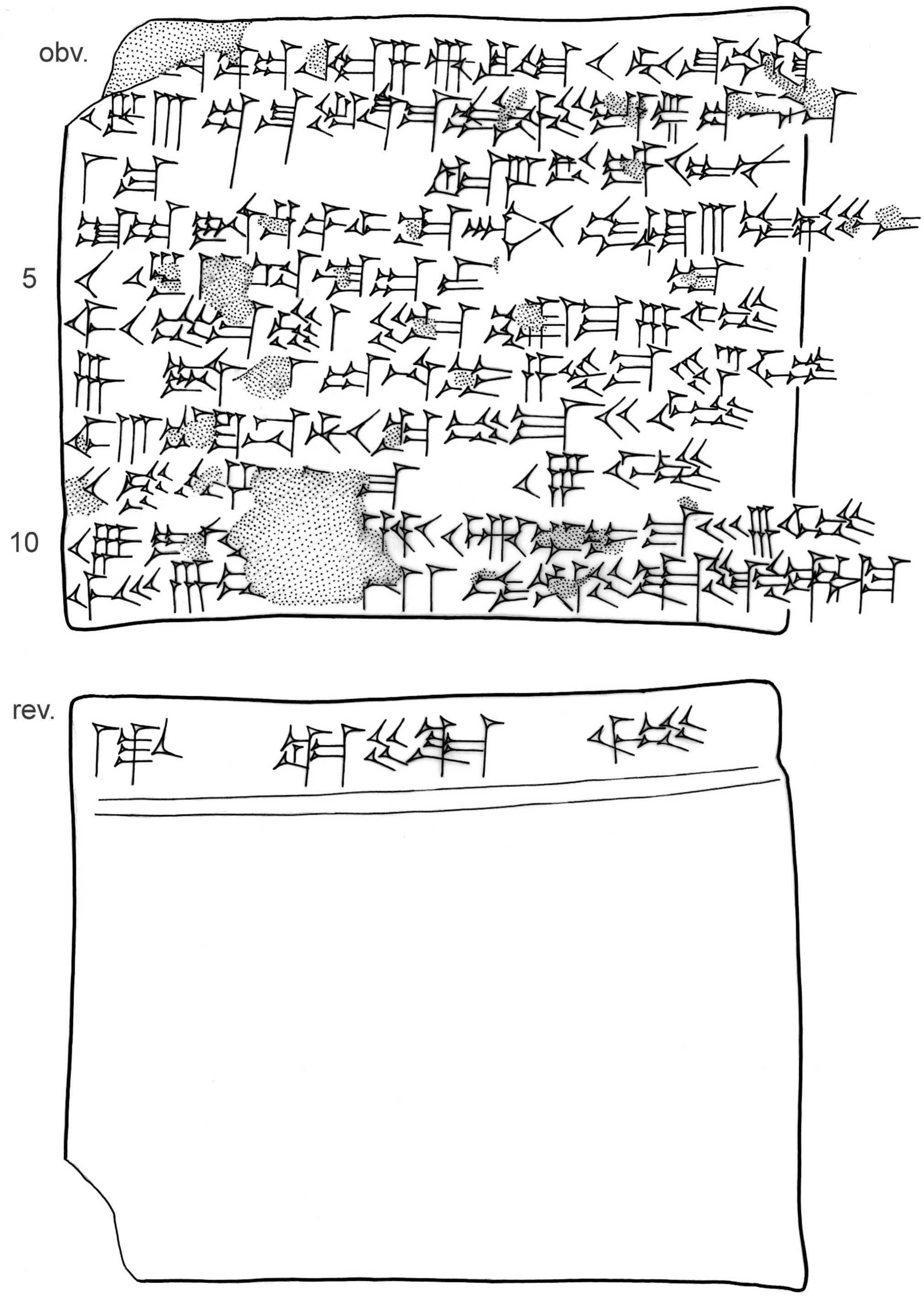



Plate 5. Text no. 2. MS 3976

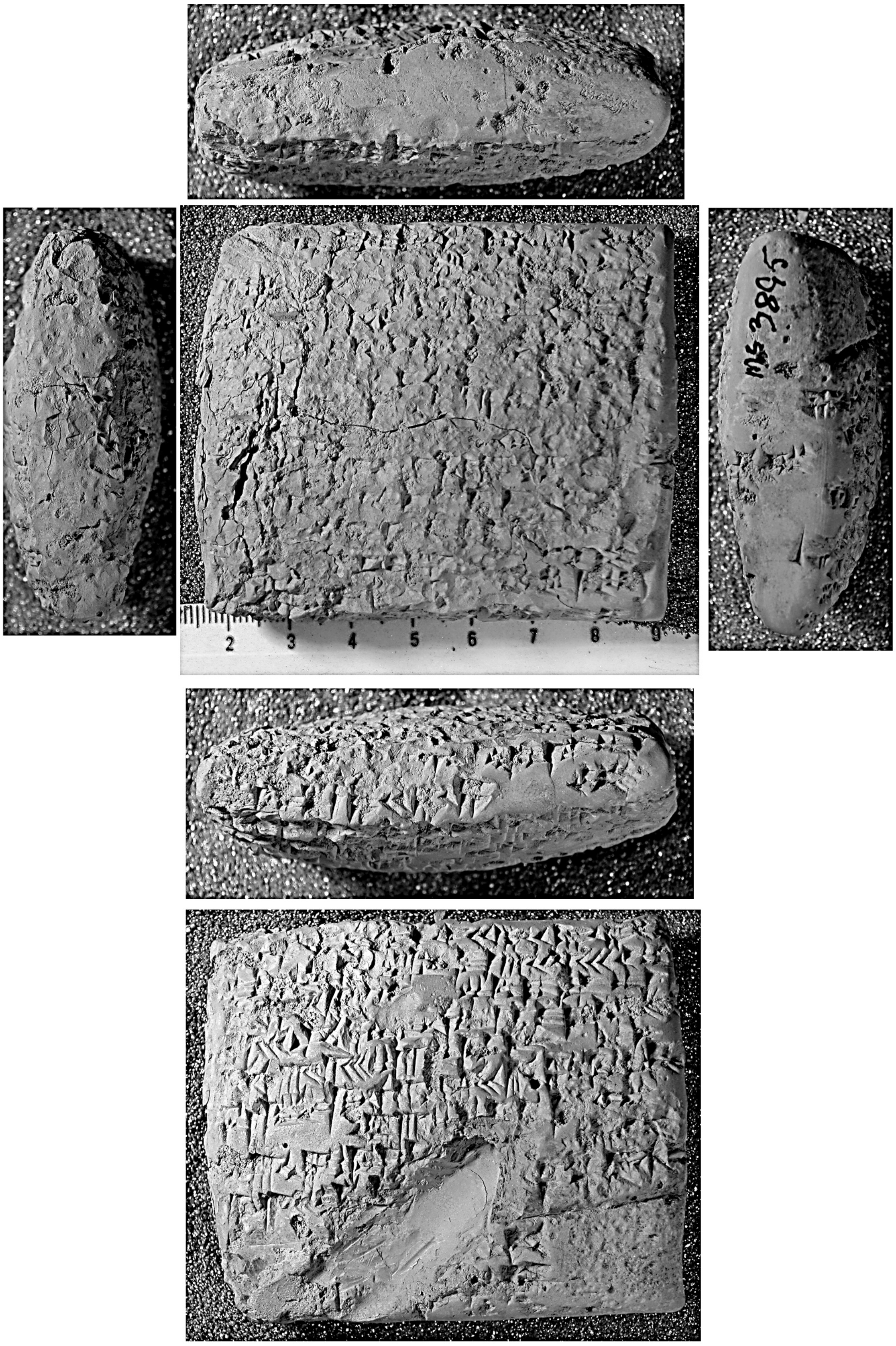



Plate 6. Text no. 3. MS 3895

# MS 3895

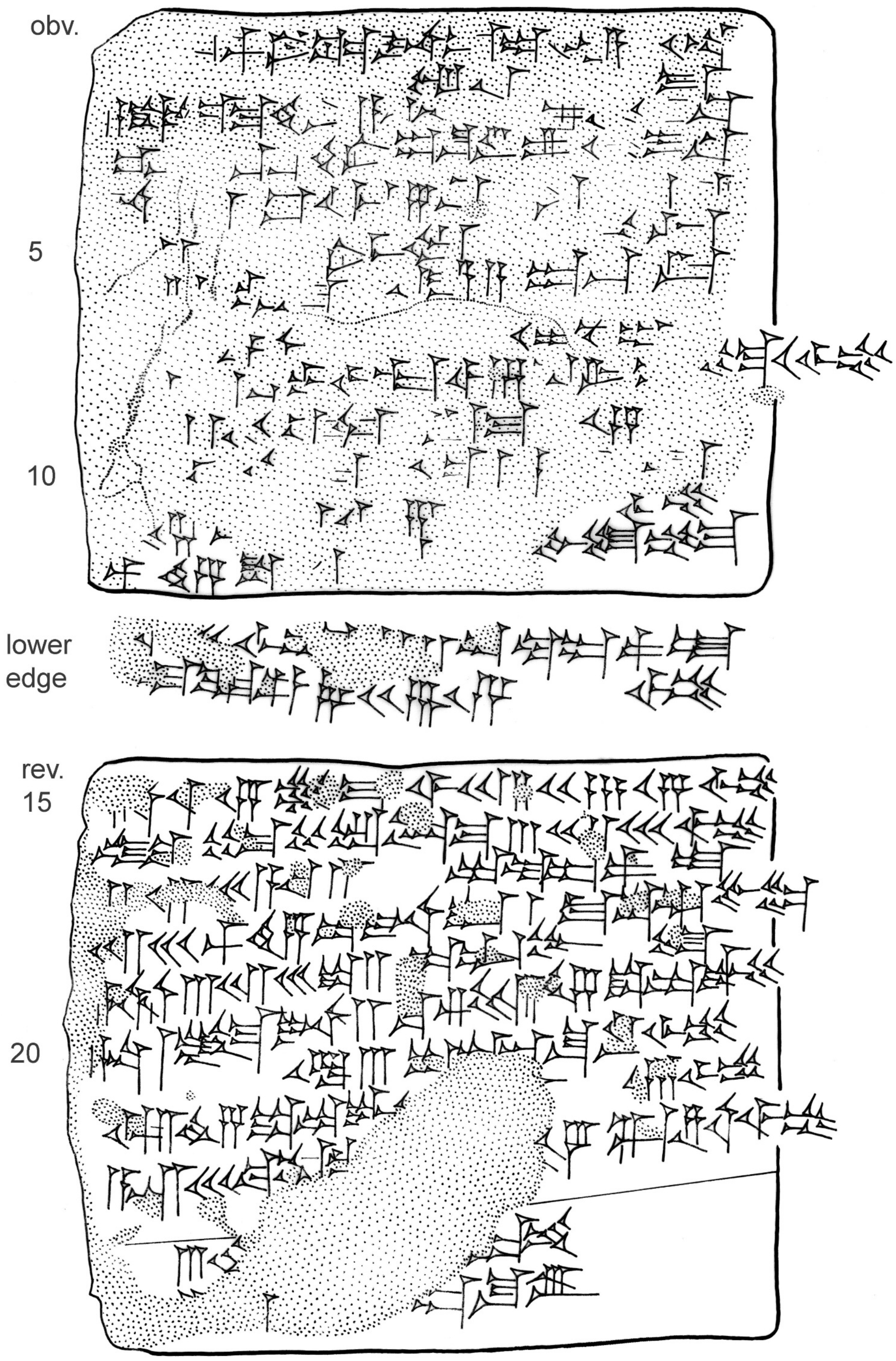



Plate 7. Text no. 3. MS 3895

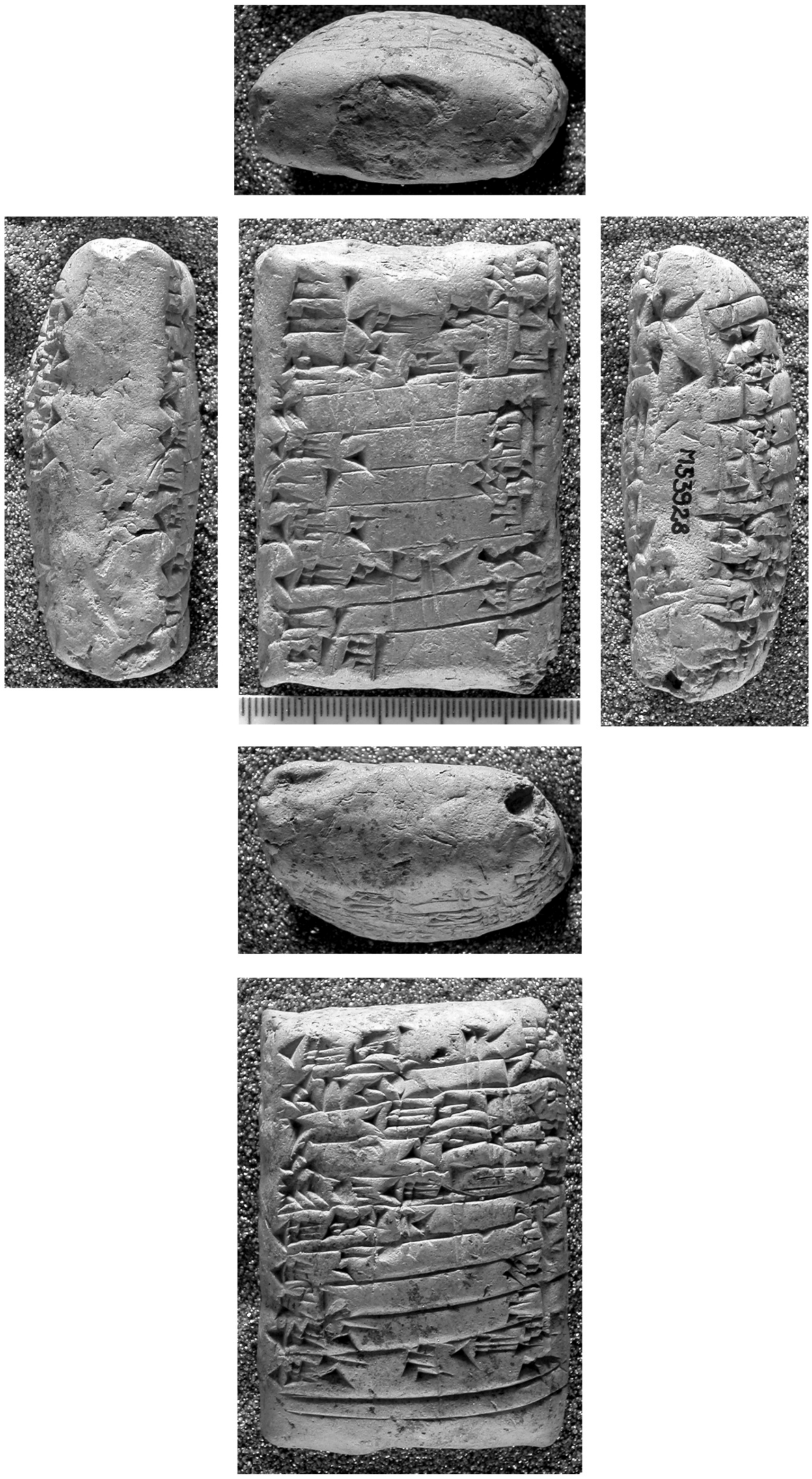



Plate 8. Text no. 4. MS 3928

MS 3928

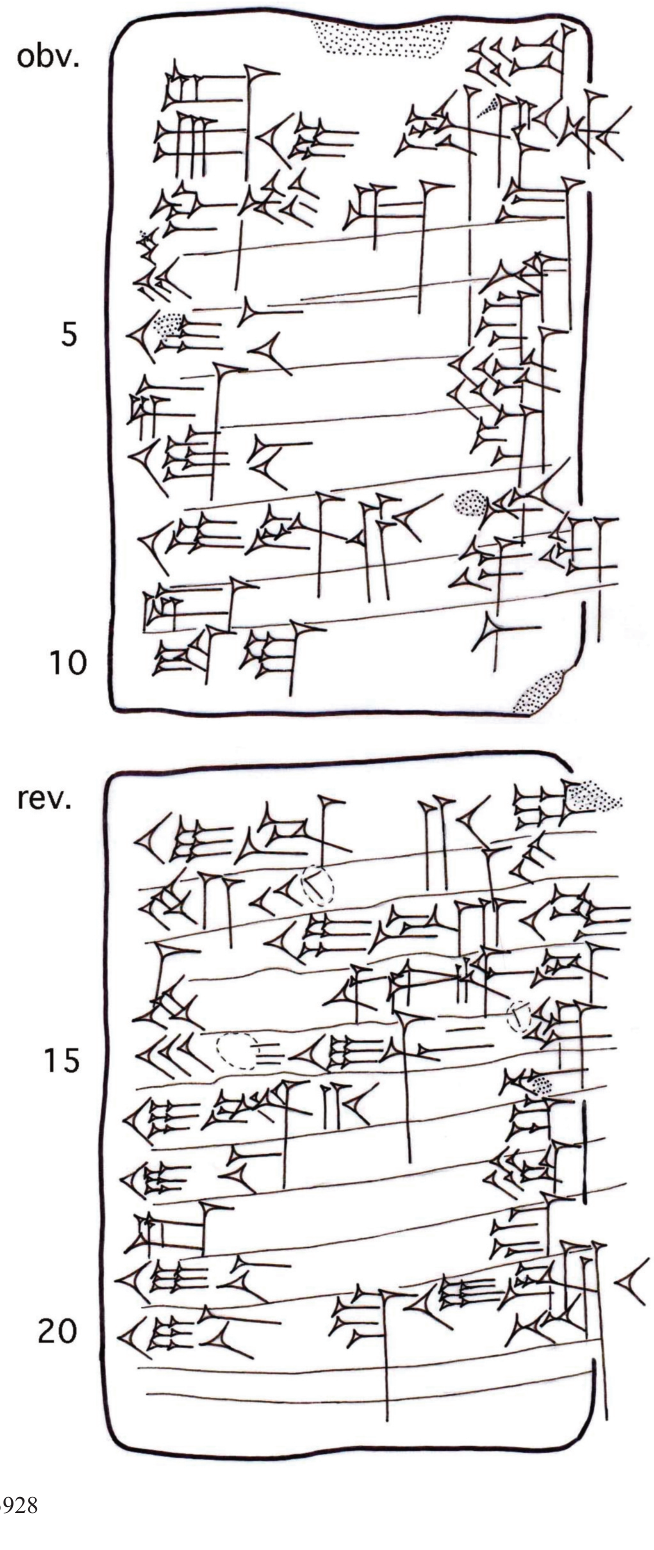



Plate 9. Text no. 4. MS 3928

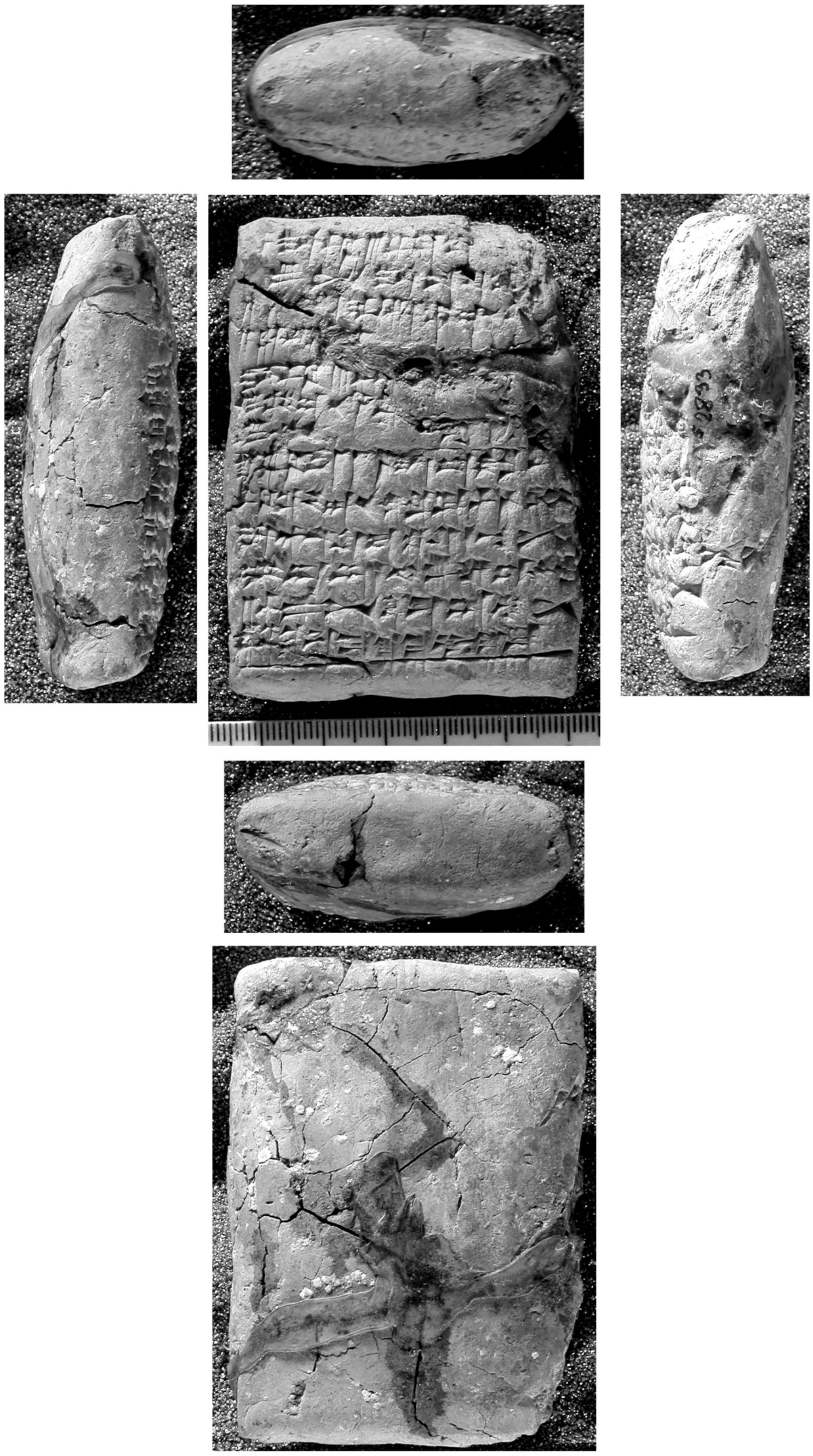
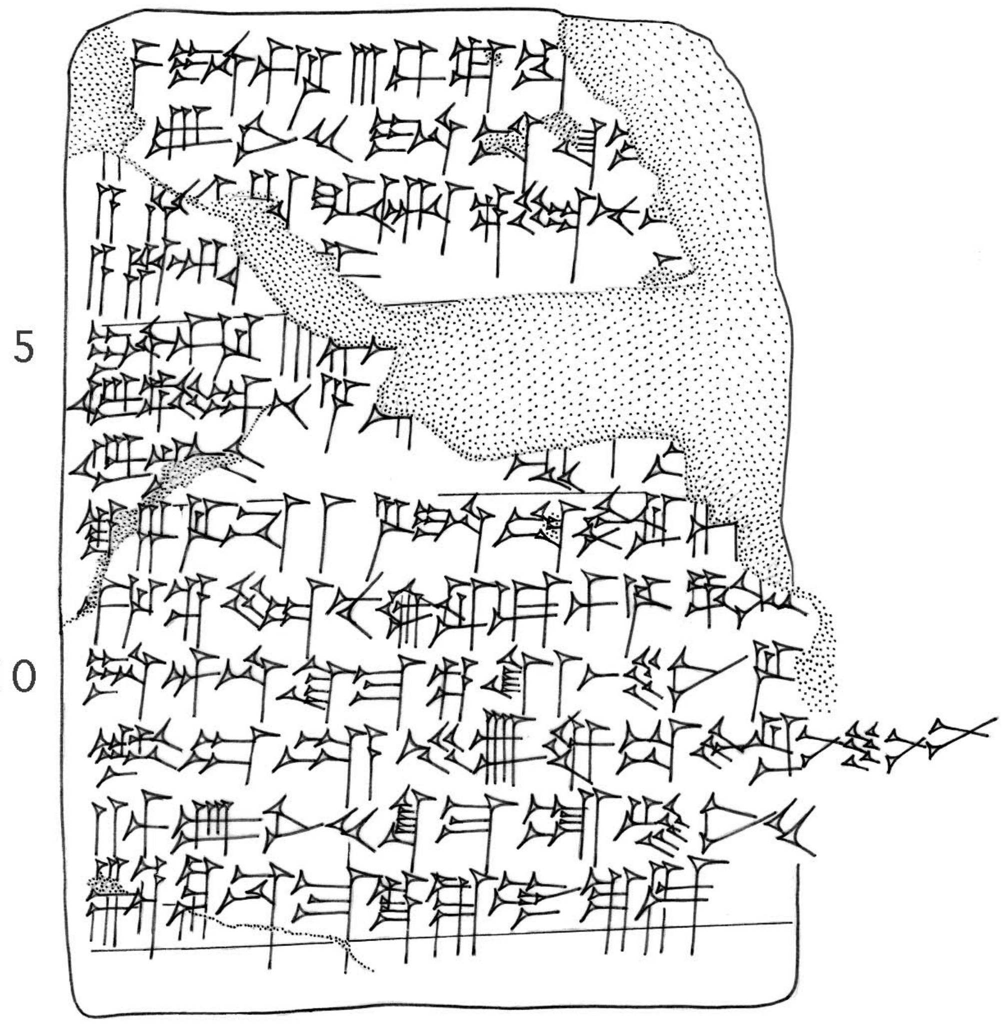


Plate 10. Text no. 5. MS 2833

MS 2833

obv.




rev. uninscribed

Plate 11. Text no. 5. MS 2833

Plate 12. Text no. 6. MS 4905

# MS 4905

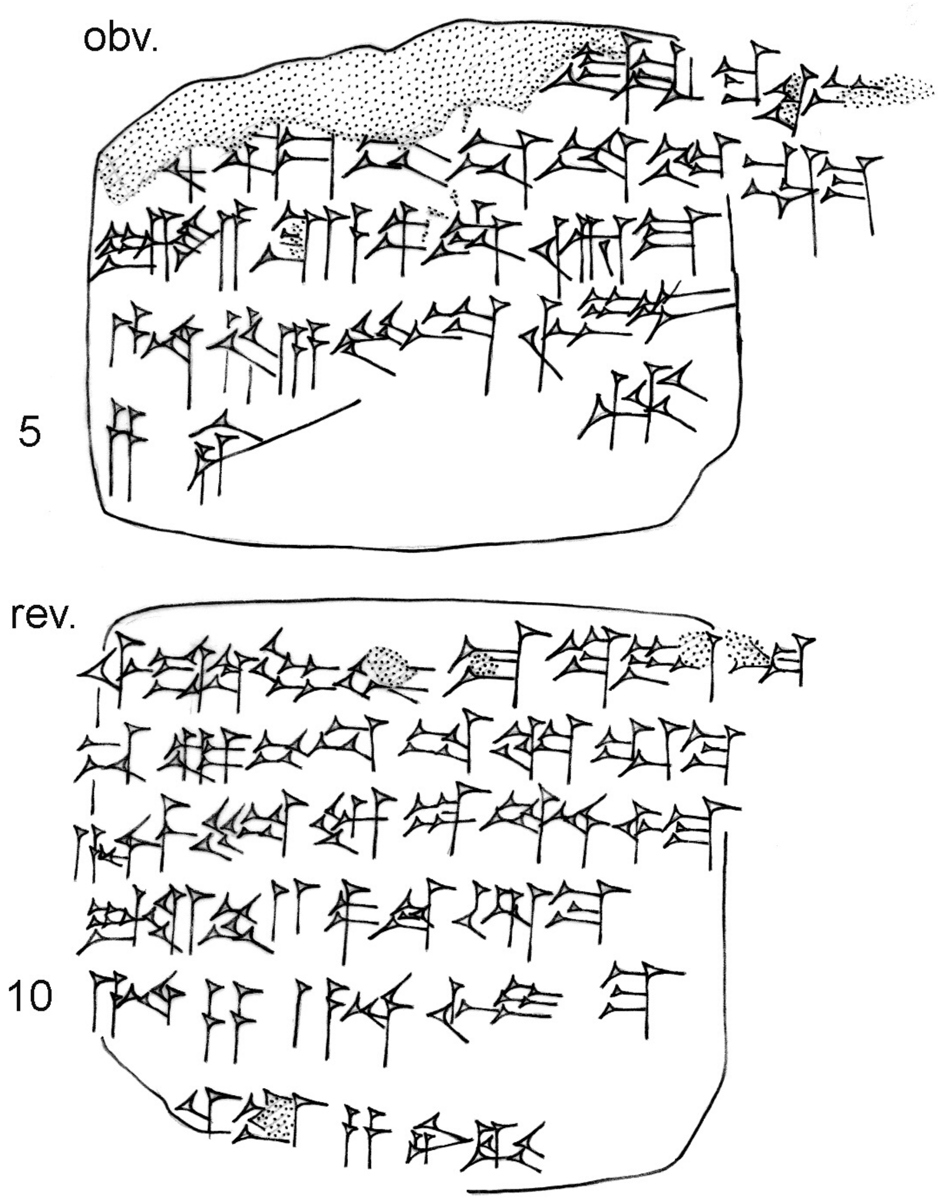



Plate 13. Text no. 6. MS 4905