Capital Accumulation of a Firm facing Environmental Constraints†

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SUMMARY

In this paper we consider the dynamic behaviour of a firm subject to environmental regulation. As a social planner the government wants to reduce the level of pollution. To reach that aim it can, among others, set an upper limit on polluting emissions of the firm. The paper determines how this policy instrument influences the firm's decisions concerning investments and abatement efforts.

Using standard control theory in determining the firm's optimal dynamic investment decisions it turns out that it is always optimal to approach a long run optimal level of capital. In some cases, this equilibrium is reached within finite time, but usually it will be approached asymptotically. Also a necessary condition is determined under which history dependent equilibria can occur. Some of them are saddle-point equilibria and others are unstable nodes or spirals (focuses). These history dependent equilibria can be identified as being good or bad for society. We show that it is possible to eliminate bad equilibria through government intervention.

Finally, we derive conditions under which equilibrium capital stock and equilibrium investment rate in the regulated case decrease compared to the unregulated case.

KEY WORDS

Optimal control, environmental problems, input substitution, history dependent equilibria, Skiba point

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1. Introduction

Currently, an important problem of a firm is how to react on environmental regulation. On the other hand, the government has several policy instruments at its disposal to give the firm an incentive to reduce pollution or to limit pollution by direct regulation. Examples of contributions in this research area that links the "dynamics of the firm" with environmental economics are Kort[1], Kort, Van Loon and Luptacik[2] and Xepapadeas[3].

In this paper we establish an optimal investment policy of the firm under the restriction that emissions cannot exceed a certain upper bound. Our model is different from the three contributions mentioned in the previous paragraph with respect to the specification of the pollution function. In the above references[1,2,3] a given abatement expenditure leads to a given pollution reduction, which is thus irrespective of the amount of pollution caused by the production process. This suggests that pollution can be driven to zero or even become negative, and, indeed, in these models a constraint appears that has to keep pollution non-negative. In our formulation we adopt the more realistic assumption that marginal costs of abatement increase sharply as the level of pollution shrinks, which implies that driving pollution to zero is very expensive, if not impossible.

In Section 2, the optimisation problem of the firm facing a pollution standard is specified as an optimal control problem, while Section 3 contains the mathematical analysis and the economic results. Section 4 concludes the paper.

2. Model Formulation

Consider a firm having a stock of capital goods K through which it produces output Q(K). Assume that

$$Q(0) = 0,\ Q'(K) > 0,\ Q''(K) < 0. \tag{1}$$

It is assumed that the production process generates emissions $E = \alpha Q(K)$, where α equals the emissions-to-output ratio.

The firm has the possibility to reduce the emissions-to-output ratio α by carrying out abatement expenditures A. Thus, let $\alpha = \alpha(A)$ and assume diminishing returns to such expenditures:

$$\alpha(A) > 0,\ \alpha'(A) < 0,\ \alpha''(A) > 0, \tag{2}$$

see, e.g., Dasgupta[4] or Van der Ploeg and Withagen[5].

As a practical example of such abatement expenditures consider the coal industry which, according to Jorgenson and Wilcoxen[6] (p. 328), belongs to the most heavily regulated industries in the USA. It is possible to reduce sulphur dioxide emissions through substitution of expensive low-sulphur for cheap high-sulphur fuel. The more the firm wants to reduce emissions the higher it can make the percentage of low-sulphur fuel. Now the extra cash outflow, that arises from switching from high-sulphur to low-sulphur coal, can be seen as abatement expenditures‡ (see Jorgenson and Wilcoxen[6], p. 317).

Note that A = 0 is associated with the production technology that would be chosen by a profit maximising firm in absence of any environmental regulations. Hence the emissions-to-output ratio associated with this technology is $\alpha(0)$.

In order to prevent a scenario that involves zero pollution we assume that

$$\lim_{A\to\infty} \alpha'(A) = 0, \qquad \lim_{A\to\infty} \alpha(A) = 0. \tag{3}$$

The second assumption in (3) says that every desired cleaning level can be achieved if only the abatement efforts are high enough.

Of course, abatement investments must always be non-negative:

$$A \geq 0. \tag{4}$$

As mentioned in the introduction, the government reduces pollution by imposing a fixed upper bound on emissions. Let Z be the maximum permitted volume of emissions at time t. The emission constraint for the firm at each instant of time is

$$\alpha(A)Q(K) \leq Z. \tag{5}$$

Clearly, by (3), this constraint can always be met by a sufficient amount of abatement, no matter how large K and therefore Q(K) is. The capital stock can be increased by productive investments I where investment cost equal C(I) for which we assume

$$C(0) = C'(0) = 0, \qquad C''(I) > 0. \tag{6}$$

On the other hand, the capital stock decreases by depreciation at rate a.

We assume that the firm is "small", in the sense that its product can be sold in the market at a fixed§ market price p, and that abatement investments face a horizontal supply curve so that their unit price is constant and equal to, say, v.

With the discount rate r (r > 0), we obtain the following dynamic model of the firm:

$$\max_{I,A} \int_0^\infty e^{-rt} \left\{ pQ(K) - vA - C(I) \right\} dt \tag{7}$$

‡ These expenditures refer to input substitution, i.e., measures to avoid pollution. An alternative approach would be to assume an „end of pipe“ technology, where pollution is created, a part of which is cleaned afterwards by abatement.

§ The results would be the same, qualitatively speaking, if the firm has some market power, i.e., p = p(Q), p'>0, p''<0 and if the revenue function R(Q) = Qp(Q) is concave. See, e.g., Kort[7] in which a related model is studied, but where an assumption is imposed that excludes history dependent equilibria.

$$\text{s.t.} \qquad \dot{K} = I - aK, \quad K(0) = K_0 \tag{8}$$

$$A \geq 0 \tag{9}$$

$$\alpha(A)Q(K) \leq Z. \tag{10}$$

Clearly, one must also have $K \geq 0$ which is however automatically satisfied if $I \geq 0$. We do not impose this constraint[7] since it can only be violated for extremely large values of K, where the emission tax forced the firm to reduce ist capital stock very quickly. Thus, the argument that $K \geq 0$ is always automatically satisfied remains true.

3. Mathematical Analysis and Economic Interpretations

3.1. Solution Using the Maximum Principle:

It is convenient to define $\overline{K} = \overline{K}(Z)$ as the largest capital stock, for which zero abatement is in accordance with the emission limit:

$$\alpha(0)Q(\overline{K}) = Z. \tag{11}$$

Since the control A does not enter the system dynamics (8), this model can be treated by applying a two step approach[8]:

Step 1: For every fixed K, solve the static problem of minimising the abatement expenditures s.t. the environmental standard:

$$\max_A \{-vA\} \tag{12}$$

$$\text{s.t.} \qquad A \geq 0 \tag{13}$$

$$\alpha(A)Q(K) \leq Z. \tag{14}$$

The solution is easily obtained as

$$A(K) = \begin{Bmatrix} 0 \\ A_Z(K) \end{Bmatrix} \quad \text{when} \quad \begin{Bmatrix} \alpha(0)Q(K) \leq Z \\ \alpha(0)Q(K) > Z \end{Bmatrix} \quad \text{i.e.} \quad K \begin{Bmatrix} \leq \\ > \end{Bmatrix} \overline{K} \tag{15}$$

where $A_Z(K)$ is an implicit function such that

$$\alpha(A_Z(K))Q(K) = Z. \tag{16}$$

From the implicit function theorem we compute the derivatives of $A_Z(K)$ for $K > \overline{K}$:

[7] Of course, this constraint could be imposed and would lead to boundary solutions for very large values of K. Due to space restrictions we refrain from doing the analysis with the corresponding Lagrange multiplier since this case of extremely large K is not very interesting.

$$A_Z'(K) = -\frac{Q'(K)\alpha(A)}{Q(K)\alpha'(A)} > 0 \tag{17}$$

$$A_Z''(K) = \frac{\alpha}{(\alpha')^3 Q^2}\left\{(\alpha')^2\left[-QQ'' + (Q')^2\right] + (Q')^2\left[(\alpha')^2 - \alpha\alpha''\right]\right\}. \tag{18}$$

Unfortunately, the sign of $A_Z''(K)$ is ambiguous because the sign of $(\alpha')^2 - \alpha\alpha''$ is, in general, unknown.

However, in the special case of an abatement effectiveness function which is exponential, i.e.

$$\alpha(A) = \alpha(0)e^{-bA}, \qquad b \text{ ... constant} \tag{19}$$

then $(\alpha')^2 - \alpha\alpha'' = 0$ and $A_Z''(K) < 0$.

Summing up the results of Step 1 we formulate

Proposition 1:
The optimal level of abatement only depends on the stock of capital. If $K < \overline{K}$, the firm will not carry out any abatement activities. For $K > \overline{K}$, abatement is positive and increases with K. In the special case of (19), abatement is a concave function of K.

Step 2: With A(K) computed in Step 1, solve the following control problem:

$$\max_I \int_0^\infty e^{-rt}\left\{pQ(K) - vA(K) - C(I)\right\}dt \tag{20}$$

$$\text{s.t.} \qquad \dot{K} = I - aK, \quad K(0) = K_0. \tag{21}$$

This leads to the Hamiltonian

$$H = pQ(K) - vA(K) - C(I) + \lambda(I - aK), \tag{22}$$

and the necessary optimality conditions are:

$$I = \arg\max_I H, \qquad \text{i.e.,} \qquad \lambda = C'(I) \tag{23}$$

$$\dot{\lambda} = r\lambda - H_K = (r+a)\lambda - pQ'(K) + vA'(K). \tag{24}$$

Strictly speaking, the adjoint equation (24) only holds for $K \neq \overline{K}$, where $A_Z(K)$ is differentiable in K. At the kink $K = \overline{K}$, the differential equation must be replaced by the differential inclusion

$$r\lambda - \dot{\lambda} \in \partial_K H .$$

where ∂_K denotes the generalised gradient of H w.r.t. K; see e.g. Clarke[9]. In our case, $\partial_K H$ evaluated at $K = \overline{K}$ is the interval between r.h.s. and l.h.s. derivative of H w.r.t. K. Thus, the differential inclusion becomes:

$$\dot{\lambda} \in \left[(r+a)\lambda - pQ'(\overline{K}), (r+a)\lambda - pQ'(\overline{K}) + vA'_Z(\overline{K}^+)\right]. \tag{25}$$

Usually in infinite horizon problems, it is difficult to verify, that the "limiting transversality condition"

$$\lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \tag{26}$$

is part of the necessary conditions. In this case however, it is possible to show that the additional assumption $Q(\infty) < \infty$ guarantees that $\partial\left[pQ(K) - vA(K) - C(I)\right]/\partial K$ is bounded so that the growth conditions imposed by Seierstad[10] are satisfied. His theorem then guarantees that the optimal solution satisfies (26). Hence, we will mainly look for those solutions of the canonical system for which K and λ remain bounded. See also the discussion in Section 3.2.1.

3.2. Qualitative Analysis in the Phase Plane:

To perform a phase plane analysis in the (K,I)-plane, first observe that the $\dot{K} = 0$ isocline is the straight line I = aK. Next, we have to derive a differential equation for I from the adjoint equation (24). This is done by differentiating (23) w.r.t. time t and subsequent elimination of the costate λ, using (24) for $K \neq \overline{K}$. This yields:

$$\dot{I} = \frac{1}{C''(I)}\left[(r+a)C'(I) - pQ'(K)\right] \quad \text{for} \quad K < Q^{-1}(Z/\alpha(0)) = \overline{K} \tag{27}$$

$$\dot{I} = \frac{1}{C''(I)}\left[(r+a)C'(I) - pQ'(K) + vA'_Z(K)\right] \quad \text{for} \quad K > Q^{-1}(Z/\alpha(0)) = \overline{K} . \tag{28}$$

With these, we can compute the slope of the $\dot{I} = 0$ isocline. It is convenient to treat the two cases separately. First, we consider the case of a small capital stock where the emission constraint is not binding, i.e., $K < \overline{K}$. In this case we obtain:

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0} = \frac{pQ''(K)}{(r+a)C''(I)} < 0 \tag{29}$$

On the other hand, when the capital stock is so large that the emission constraint is binding, i.e., $K > \overline{K}$, we obtain

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0} = \frac{pQ'' - vA''_Z}{(r+a)C''} = \frac{p(\alpha')^3 Q^2 Q'' + v\alpha(\alpha')^2\left[QQ'' - (Q')^2\right] + v(Q')^2\alpha\left[\alpha\alpha'' - (\alpha')^2\right]}{(r+a)C''(\alpha')^3 Q^2} \tag{30}$$

Unfortunately the sign of this expression is ambiguous. This means that the slope of the $\dot{I} = 0$ isocline is also ambiguous.

Let us now consider a steady state (K^*,I^*), which is defined by

$$I^* = aK^*, \qquad pQ'(K^*) = (r+a)C'(I^*) \tag{31}$$

if $K^* < \overline{K}$; see case (a) below. The second equation in (31) is standard (cf. Takayama[11], pp. 698-699), and says that in equilibrium marginal revenue equals marginal investment costs.

The determinant of the Jacobian of the dynamical system (21) and (27) evaluated at the equilibrium can easily be computed:

$$\det J = \frac{1}{C''}\left[-a(r+a)C'' + pQ''\right] < 0' \tag{32}$$

which means that:

Proposition 2:
If $K^* < \overline{K}$ then the equilibrium is a saddle point.

On the other hand, for $K^* > \overline{K}$, the steady state is defined by

$$I^* = aK^*, \qquad pQ'(K^*) = (r+a)C'(I^*) + vA'_Z(K^*); \tag{33}$$

see case (c) below. Here, opposite to the case $K^* < \overline{K}$, additional abatement investments are necessary to keep pollution equal to the standard level when capital stock increases marginally. Consequently, marginal abatement expenses, $vA'_Z(K^*)$, have to be added to the marginal investment costs, which results in a smaller equilibrium compared to the unregulated case. Now, the dynamical system consists of (21) and (28), implying that the determinant of the Jacobian, evaluated at the steady state, equals

$$\det J = \frac{1}{C''}\left[-a(r+a)C'' + pQ'' - vA''_Z\right] = (r+a)\left(-a + \left.\frac{dI}{dK}\right|_{\dot{I}=0}\right). \tag{34}$$

Since a is the slope of the $\dot{K} = 0$ isocline, this yields

Proposition 3:
If $K^* > \overline{K}$ then the equilibrium (K^*,I^*) is

- *a saddle point, if the* $\dot{I} = 0$ *isocline hits the* $\dot{K} = 0$ *isocline from above*
- *unstable, if the* $\dot{I} = 0$ *isocline hits the* $\dot{K} = 0$ *isocline from below; a necessary condition for this unstability is that* $A''(K) < 0$.

It is now convenient to consider two different cases: one where the $\dot{I} = 0$ isocline is always decreasing and the other where it is not.

3.2.1. The case of a globally decreasing investment isocline

Let us first consider the simplest case where the $\dot{I} = 0$ isocline is always decreasing. Since it is clear from (17), (27) and (28) that the $\dot{I} = 0$ isocline suffers a downward jump at the point $K = \overline{K}$, there are three possibilities where the two isoclines intersect; see also Figure 1:

(a) *intersection for* $K < \overline{K}$, *i.e. where constraint (10) is not binding*

(b) *intersection for* $K = \overline{K}$, *i.e. at the discontinuity of the* $\dot{I} = 0$ *isocline*

(c) *intersection for* $K > \overline{K}$, *i.e. where constraint (10) is binding*

Insert Figure 1 about here

The corresponding phase diagrams are illustrated in Figure 2. Subcases (a) and (c) refer to ordinary saddle points while the equilibrium in subcase (b) is approached within finite time; (see, e.g. Feichtinger and Hartl[12], pp. 397-402).

It is convenient to treat these subcases separately:

Insert Figure 2a about here

Scenario a. equilibrium for $K < \overline{K}$: By (21) and (27), this case is characterised by

$$(r + a)C'(a\overline{K}) > pQ'(\overline{K}) \tag{35}$$

Clearly, since C' is increasing and Q' is decreasing, this case corresponds to the situation of mild emission limits (i.e. large Z) for fixed parameters r, a, p, and v. Note that, by (11), there is a monotonically increasing relation between $\overline{K}$ and Z

Because marginal investment costs exceed marginal revenue for $K = \overline{K}$ it makes economic sense that the equilibrium occurs for K below $\overline{K}$. In this case constraint (10) is not binding in the equilibrium which is a saddle point. The long run behaviour is identical to the unregulated case. Only for initial capital stocks larger than $\overline{K}$, an initial time interval exists where abatement expenditures are necessary to satisfy the pollution constraint. On this interval, the investment rate is smaller than in the unregulated case.

Suppose that the firm starts out with a capital stock larger than $\overline{K}$ and that this level is reached at t_2. Then the negative effect of abatement expenditures on the investment rate during the interval $[0,\ t_2)$ can be visualised by noting that for $t < t_2$ the investment level satisfies the following equation:

$$\int_t^{\infty} e^{-(r+a)(s-t)} pQ'(K(s))ds - \int_t^{t_2} e^{-(r+a)(s-t)} vA'(K(s))ds - C'(I(t)) = 0 \tag{36}$$

This expression is derived from (23), (24) and (31), and says that the net present value of marginal investment (NPVMI) equals zero. With other words, the additional investment expenditure C'(I), which is necessary to acquire an additional unit of capital stock, balances the future cash flow generated by this unit. This cash flow, that is corrected for depreciation and discounting, equals revenue from selling products made with the extra capital stock minus the additional abatement expenditures necessary to keep on meeting the pollution standard in the interval $[t,\ t_2)$.

Insert Figure 2b about here

A few words are in order concerning the role of saddle points in our model. We know from Section 3.1 that the optimal solution will satisfy (26). This condition is certainly satisfied by the solutions converging to the saddle point so that these are our main candidates, as usual. We must however make sure that all the other trajectories which might satisfy the necessary conditions do not make sense. We do this by economic arguments: Apparently diverging to ∞ is not optimal (because of the convex costs), and converging to 0 is also suboptimal because of the fact that marginal revenue, Q'(K), is large for K small. According to (1) and (6) it will hold that Q'(0) > (r+a) C'(0) = 0 so that it is always optimal to invest when you are in (0,0). Then the solutions that are left are trajectories converging to saddle point, so they must be optimal

Scenario b. equilibrium for $K = \overline{K}$: By (21), (27) and (28), this intermediate case is characterised by

$$(r+a)C'(a\overline{K}) \le pQ'(\overline{K}) \le (r+a)C'(a\overline{K}) + vA'_Z(\overline{K}) \tag{37}$$

which is the case of intermediate values of Z.

From the first inequality of (37) we infer that marginal revenue exceeds marginal investment costs; so it would be optimal to grow further if no abatement investments were necessary. But when the firm grows beyond $\overline{K}$, then abatement investments are needed to meet the standard; cf. (11). Hence, marginal costs increase with the abatement costs and the second inequality of (37) says that total marginal costs exceed marginal revenue. This means that it is optimal for the firm to keep the level of capital stock equal to $\overline{K}$.

In general, investment and capital stock are smaller than in the unregulated case. Mathematically, the equilibrium cannot be called a saddle point, since the r.h.s. of the differential equation for I is discontinuous there, see (27) and (28). More precisely, transformation of (25) from λ to I yields:

$$\dot{I} \in \left[\frac{1}{C''(I)}\left[(r+a)C'(I) - pQ'(\overline{K})\right],\quad \frac{1}{C''(I)}\left[(r+a)C'(I) - pQ'(\overline{K}) + vA'_Z(\overline{K})\right]\right] \tag{38}$$

It can be verified that following (27) or (28) for $K < \overline{K}$ or $K > \overline{K}$ and then, when the point $K = \overline{K}$ and $I = a\overline{K}$ is reached, remaining there, is in accordance with the state equation (8) and with (38).

Insert Figure 2c about here

Scenario c. equilibrium for $K > \overline{K}$: By (21) and (28), this case is characterised by

$$(r+a)C'(a\overline{K}) + vA'_Z(\overline{K}) \le pQ'(\overline{K}), \tag{39}$$

which is the case of very tight emission limits (i.e. small Z).

Here, marginal revenue exceeds marginal costs, so that it is actually optimal to grow beyond $\overline{K}$. In this case constraint (10) is binding in the equilibrium which is a saddle point.

Even when the constraint is not yet binding, investment is smaller than in the unregulated case. This is confirmed by the fact that also here the investment rate is determined such that the NPVMI equals zero. In case $K(0) \le \overline{K}$ and $t \le t'_2$ this leads to

$$\int_t^{\infty} e^{-(r+a)(s-t)} pQ'(K(s))\,ds - \int_{t'_2}^{\infty} e^{-(r+a)(s-t)} vA'(K(s))\,ds - C'(I(t)) = 0 \tag{40}$$

where t'_2 denotes the point of time at which the pollution limit becomes binding. From (40) we conclude that when it determines its current investment rate, the firm already takes into account future abatement expenditures, even when the pollution constraint is not yet binding.

In all three subcases the result, common to almost all capital accumulation models, is obtained: Investment is a monotonically decreasing function of capital stock

3.2.2. The case of an investment isocline with increasing parts

Now, we consider the more complicated case where the $\dot{I} = 0$ isocline is not globally decreasing. This is not a pathological case but can happen for large classes of functions. For instance, if we assume (19) for the abatement effectiveness α and that the production function is almost linear, i.e., Q" ≈ 0, then the isocline is upward sloping (However, we will not assume Q" = 0 in the sequel):

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0} \approx \frac{-(Q')^2 v\alpha}{(r+a)C''\alpha' Q^2} > 0 . \tag{41}$$

A general result is the following:

Lemma. Consider the region $K > \bar{K}$ in the (K,I)-plane. Then the $\dot{I} = 0$ isocline that corresponds to the regulated case is situated below the $\dot{I} = 0$ isocline of the unregulated case.

Proof: For the $\dot{I} = 0$ isocline of the unregulated case we have from (27) that

$$pQ'(K) = (r+a)C'(I). \tag{42}$$

For the $\dot{I} = 0$ isocline of the regulated case we have from (28) that

$$pQ'(K) = (r+a)C'(I) + vA'(K). \tag{43}$$

The result of the Lemma follows from (42), (43) and the fact that $A'_Z > 0$; see (17). ❑

The lemma has the implication that in the regulated case an equilibrium with $K > \bar{K}$ can occur if and only if the saddle point equilibrium in the unregulated case occurs for $K > \bar{K}$. By combining the observation of the Lemma with the fact that the $\dot{K} = 0$ isocline is upward sloping we conclude that:

Proposition 4:
In any "regulated equilibrium" we have a lower level of capital stock K compared to the "unregulated equilibrium".

In the appendix it is shown that for certain classes of model functions, additional properties of the $\dot{I} = 0$ isocline can be derived. In particular, an increasing part is also concave and whenever it is non-increasing for some capital stock $\tilde{K}$ ☢ $\bar{K}$ then it is decreasing for all $K > \tilde{K}$. Furthermore, the $\dot{I} = 0$ isocline converges to the K-axis as K→∞. Figure 3 shows a typical case in which multiple equilibria occur.

Insert Figure 3 about here

In this case the abatement effectiveness function is given by (19) so that $A''_Z < 0$. Compared to the solutions depicted in Figure 2, this causes the $\dot{I} = 0$ isocline to increase on a certain K-interval and this, in turn, results in a second saddle-point equilibrium denoted by G. Furthermore, since the other equilibrium for $K > \overline{K}$, i.e. the one along the increasing part of the $\dot{I} = 0$ isocline is unstable, there exists a Skiba-point[13, 14] K_S such that for "large" initial capital stocks, $K > K_S$, it is optimal to approach the larger long run optimal equilibrium K_G.

On the other hand, for "small" initial capital stocks, $K < K_S$, the firm would have to carry out many additional abatement investments in order to reach K_G. This is due to the fact that $A'' < 0$ on a K-interval that is bounded from below by $\overline{K}$ and that contains at least all K-values for which the $\dot{I} = 0$ isocline increases. Therefore, it is more profitable to approach the other equilibrium B for $K = \overline{K}$ which is of the same type as the one in Figure 2b. Here, revenue is lower but no abatement expenditures are needed to meet the standard.

3.2.3. A model with an investment grant

Comparing the two saddle point equilibria of subsection 3.2.2 we conclude that production in G ("good" equilibrium) exceeds production in B ("bad" equilibrium), while in both G and B the pollution standard in binding so that pollution is equal. Hence, from a social point of view it is clear that equilibrium G is more preferable than equilibrium B. Therefore it would be worthwhile for the government to develop a policy that eliminates the "bad" equilibrium B (cf. Matsuyama[15], pp. 639 - 642).

This can be done by, e.g., distributing an investment grant. Receiving such a grant as a reward for investing makes it more profitable for the firm to expand and equilibrium G becomes more attractive compared to equilibrium B. If the grant rate is denoted by g, the objective of the firm becomes:

$$\max_{I,A} \int_0^\infty e^{-rt} \left\{ pQ(K) - vA - C(I) + gI \right\} dt \tag{44}$$

Performing the same calculations as before leads to the following expressions for $\dot{I}$:

$$\dot{I} = \frac{1}{C''(I)} \left\{ (r+a)\left[C'(I) - g\right] - pQ'(K) \right\} \quad \text{for} \quad K < \overline{K} \tag{45}$$

$$\dot{I} = \frac{1}{C''(I)} \left\{ (r+a)\left[C'(I) - g\right] - pQ'(K) + vA'_Z(K) \right\} \quad \text{for} \quad K > \overline{K}. \tag{46}$$

Comparing (45) - (46) with (27) - (28) we conclude that the $\dot{I} = 0$ isocline shifts upwards in the (K, I) - plane while the slope does not change. From Figure 3 it can be easily inferred that equilibrium B is eliminated if this upward shift is large enough. From (46) we conclude that:

Proposition 5:
To eliminate "bad" equilibrium B, the investment grant must satisfy the following inequality:

$$g > C'(a\overline{K}) + \left\{ vA'(\overline{K}) - pQ'(\overline{K}) \right\} \Big/ (r + a) \tag{47}$$

Due to this elimination it is clear that all trajectories will end at the "good" equilibrium G. Moreover, this equilibrium is shifted to the right so that production is increased in G.

Of course, on the basis of this analysis we cannot conclude whether it pays for the government to distribute such a grant to the firm. To answer this question requires another model with the government as decision maker. Here, we just focus on the optimal response of a firm to government policy.

Conclusions

In our model, we have obtained the following results:

- ☞ In most situations, if the capital stock is low, the investment rate should be high and vice versa. This is a result common to many investment models. However, in the case of history dependent equilibria (see below) this need not be true for all levels of capital stock.
- ☞ If there is no economic incentive for voluntary abatement activities, the firm will not start cleaning before the emission constraint is hit. This is an argument for the introduction of an emission tax which, in general, affects the firms' behaviour for all levels of emission.
- ☞ During the whole planning period the firm's investment level is determined such that marginal investment expenditures balance the future cash flow generated by this extra unit of investment. This means that the net present value of marginal investment equals zero. From this feature it could be concluded that (future) abatement expenditures have a negative effect on the growth of the firm.
- ☞ It is always optimal to approach a long run optimal level of capital. In some cases, this equilibrium is reached within finite time, but usually it will be approached asymptotically.
- ☞ History dependent equilibria may occur, i.e., the long run optimal level of capital can depend on the initial stock of capital. This means that for small initial capital stocks it can be too expensive for the firm to carry out additional abatement investments in

order to reach a high equilibrium capital stock and that it is more profitable to approach another equilibrium for which the standard is met without requiring abatement expenditures. A necessary condition for history dependent equilibria to occur is that there exists an interval of K-values on which marginal abatement efforts, necessary to satisfy the pollution restriction when K increases marginally, decrease with increasing capital stock.

☞ These history dependent equilibria can be identified as being good or bad for society. It is possible to eliminate bad equilibria through government intervention such as investment grants.

☞ Consider the situation where equilibrium capital stock is that large that, without abatement investments, the pollution standard would be violated. Then, in the regulated case, equilibrium capital stock and equilibrium investment rate are lower compared to the unregulated case.

Clearly, this model is only a first step in modelling and analysing more complicated models of the firm with abatement decisions[7] and subject to environmental regulations. Possible extensions and generalisations are:

☞ The consideration of an environmental tax rather than constraint. This will be considered in a forthcoming paper.

☞ To study a scenario where the pollution standard becomes tighter over time in order to give the firm some time to adjust to stronger environmental regulation, i.e., $Z = Z(t)$ with $\dot{Z} < 0$.

☞ The assumption that the firm has some market power or even a monopoly. As mentioned in the footnote in Section 2, all the results of this paper remain valid, if price is not fixed but follows an inverse demand function $p = p(Q)$. All we need to assume in this case is that the revenue $R(Q) = Qp(Q)$ is a concave function of production Q, which is not unusual. An even weaker assumption, which would also suffice is that revenue $p(Q(K))Q(K)$ is a concave function of capital K.

☞ The consideration of oligopoly rather than a polypoly. Modelling this situation will lead to a differential game which will not be easy to solve.

☞ An important extension would be to consider that abatement expenditures not only have a cleaning effect on current emissions but are accumulated in a stock of abatement capital (filters and other abatement devices can usually be used for some period of time after they are purchased). The resulting model will then have two state variables and it will be more difficult to obtain the same kind of analytical results that we derived by solving our one state variable model.

Appendix:

In order to obtain additional results for the shape of the $\dot{I}=0$ isocline we assume that the model functions belong to the following general classes:

$$Q(K)=c\left(1-e^{-dK}\right), \quad c, d \text{ ... constants} \tag{48}$$

$$C(I)=\beta I^2/2, \quad \beta \text{ ... constant} \tag{49}$$

and that $\alpha(A)=\alpha(0)e^{-bA}$ as assumed in (19). Then for $K<\overline{K}$ the curvature of the $\dot{I}=0$ isocline can be derived by applying the implicit function theorem to (29):

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0}=-\frac{pcd^2e^{-dK}}{(r+a)\beta}<0 \tag{50}$$

$$\left.\frac{d^2I}{dK^2}\right|_{\dot{I}=0}=\frac{d}{dK}\left[\left.\frac{dI}{dK}\right|_{\dot{I}=0}\right]=\frac{pcd^3e^{-dK}}{(r+a)\beta}>0 \tag{51}$$

For $K>\overline{K}$ we can apply the implicit function theorem to (30):

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0}=\frac{d^2e^{-dK}}{(r+a)\beta}\left[-pc+\frac{v}{b\left(1-e^{-dK}\right)^2}\right] \tag{52}$$

$$\left.\frac{d^2I}{dK^2}\right|_{\dot{I}=0}=\frac{d}{dK}\left[\left.\frac{dI}{dK}\right|_{\dot{I}=0}\right]=-d\left.\frac{dI}{dK}\right|_{\dot{I}=0}-\frac{2vd^3e^{-2dK}}{(r+a)\beta b\left(1-e^{-dK}\right)^3} \tag{53}$$

From this, we see that $\left.\frac{d^2I}{dK^2}\right|_{\dot{I}=0}<0$ when $\left.\frac{dI}{dK}\right|_{\dot{I}=0}\geq 0$. This implies that, whenever $\left.\frac{dI}{dK}\right|_{\dot{I}=0}\leq 0$ for some $K=\tilde{K}$ then the $\dot{I}=0$ isocline decreases for all $K>\tilde{K}$.

Finally the limiting behaviour of the $\dot{I}=0$ isocline for $K\to\infty$ can be obtained. From (28) we see that $\dot{I}=0$ leads to pQ'(K) = (r+a)C'(I) + vA'(K), which now reads:

$$pcde^{-dK}=(r+a)\beta I+\frac{vde^{-dK}}{b\left(1-e^{-dK}\right)} \tag{54}$$

Thus, if $K\to\infty$, then $I\to 0$.

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List of Figure Captions:

Fig. 1: The three cases

Fig. 2a: Case (a) with a saddle point for $K < \overline{K}$

Fig. 2b: Case (b) with an equilibrium for $K = \overline{K}$ which is reached in finite time

Fig. 2c: Case (c) with a saddle point for $K > \overline{K}$

Fig. 3: The case with multiple equilibria and a Skiba-point

(Kort-f)

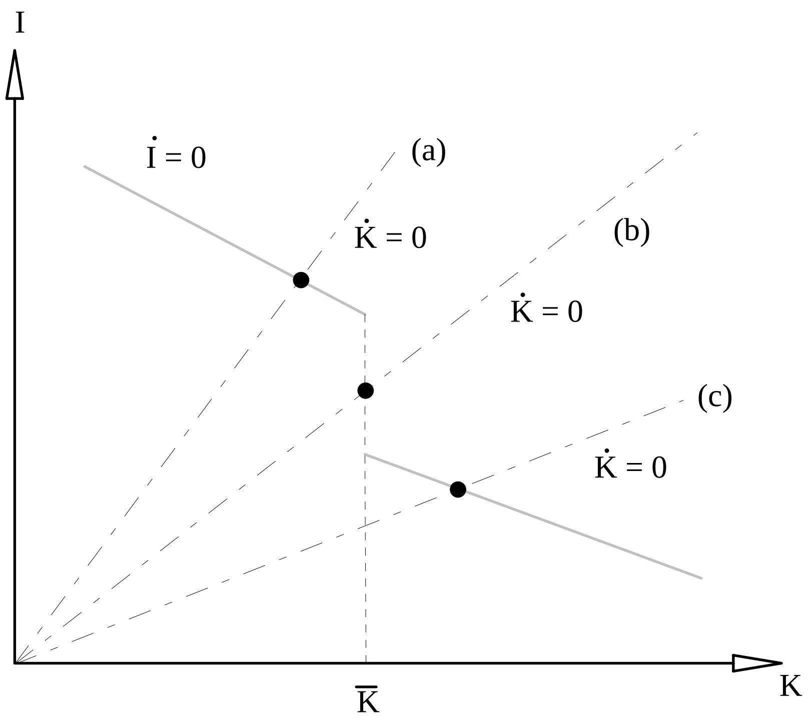


Fig. 1: The three cases

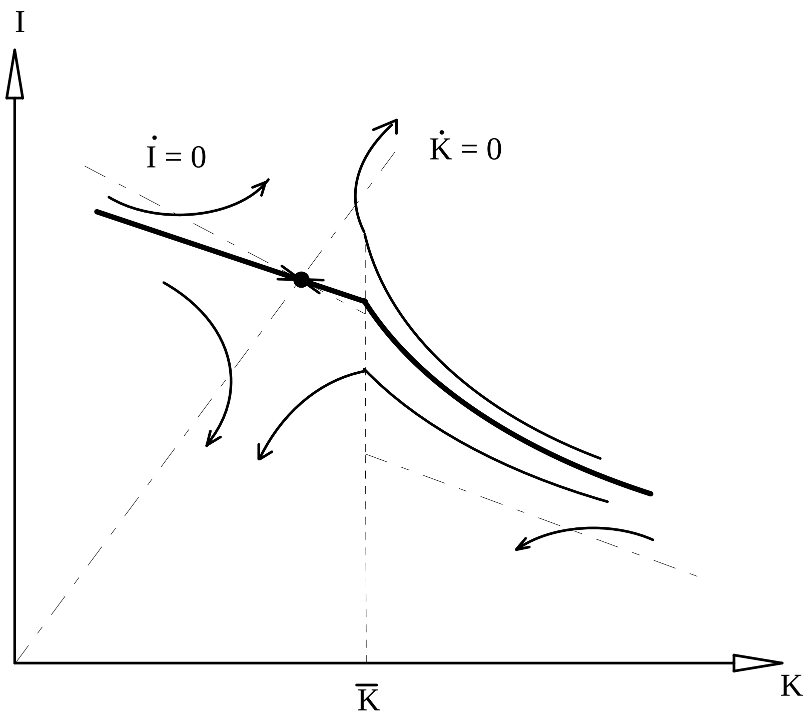


Fig. 2a: Case (a) with a saddle point for $K < \overline{K}$

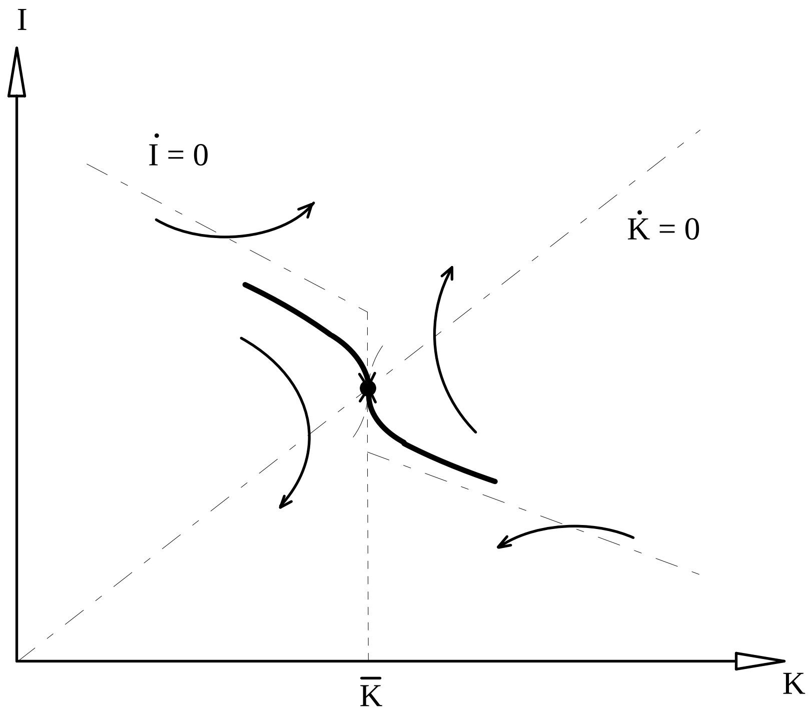


Fig. 2b: Case (b) with an equilibrium for $K = \overline{K}$ which is reached in finite time

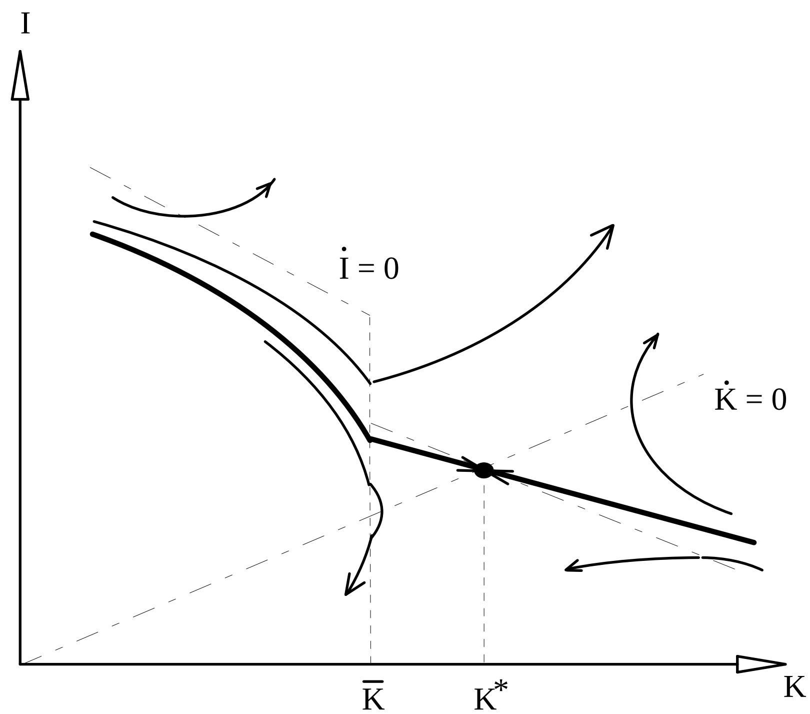
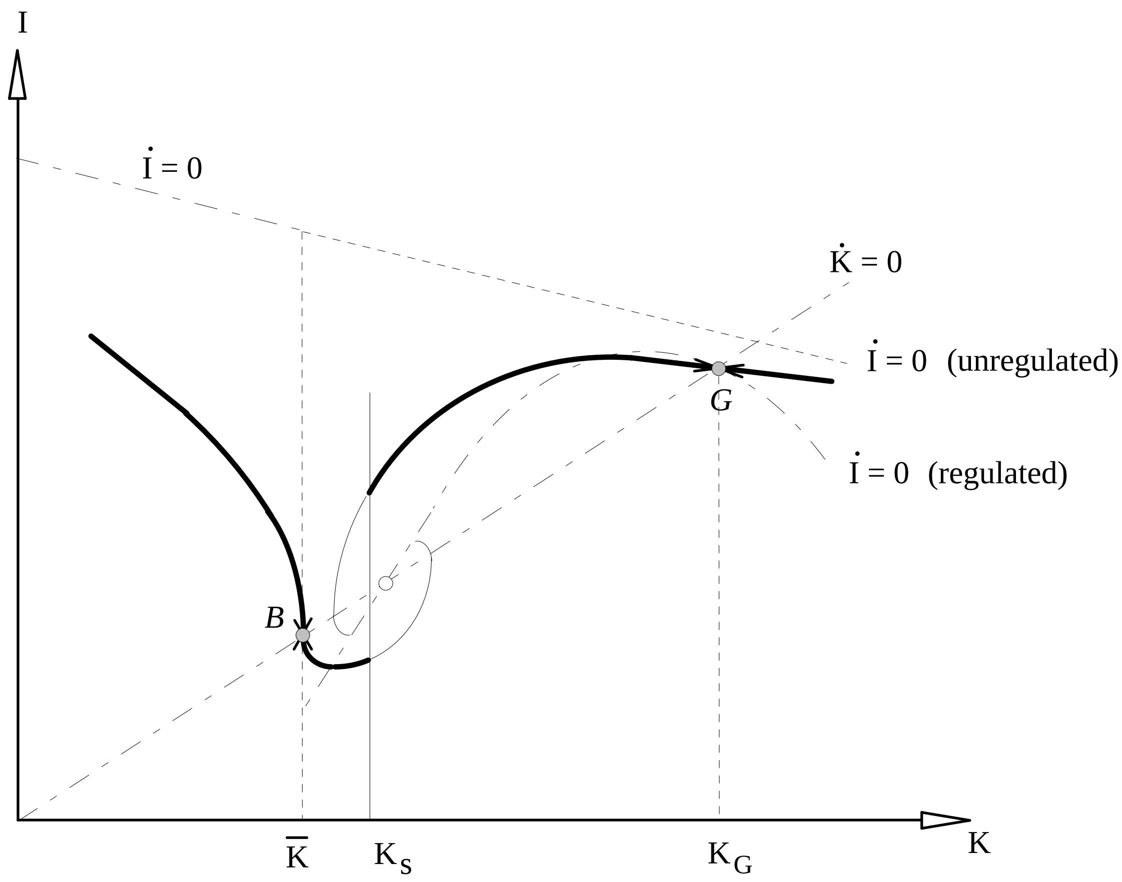


Fig. 2c: Case (c) with a saddle point for $K > \overline{K}$



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