Halo Substructure and the Power Spectrum

Andrew R. Zentner[*] and James S. Bullock[†][**]

[*]*Department of Physics, The Ohio State University, 174 W. 18th Ave., Columbus, OH 43210-1173*
[†]*Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138*
[**]*Hubble Fellow*

Abstract.
   In this proceeding, we present the results of a semi-analytic study of CDM substructure as a function of the primordial power spectrum. We apply our method to several "tilted" models in the ΛCDM framework, with $n \simeq 1.1 - 0.8$, or $\sigma_8 \simeq 1.2 - 0.65$ when normalized to COBE. We also study a more extreme, warm dark matter-like spectrum that is sharply truncated below a scale of ~ 0.3 h^{-1} Mpc ($\sim 10^{10}$ h^{-1} $M_\odot$). Contrary to some expectations, we show that the mass fraction of halo substructure is not a strong function of spectral slope, so it likely will be difficult to constrain tilt using flux ratios of gravitationally lensed quasars. On the positive side, all of our CDM-type models yield projected mass fractions that are in good agreement with strong lensing estimates: $f \approx 1.5\%$ at $M \sim 10^8 M_\odot$. The truncated model produces a significantly smaller fraction, $f \approx 0.3\%$, suggesting that warm dark matter-like spectra are disfavored and potentially may be distinguished from CDM spectra using lensing. We also discuss the issue of dwarf satellite abundances, with emphasis on the cosmological dependence of the map between the observed central velocity dispersions of Milky Way satellites and the maximum circular velocities of their host halos. In agreement with earlier work, we find that standard ΛCDM over-predicts the estimated count of Milky Way satellites at fixed V_{max} by an order of magnitude, but tilted models do better because subhalos are less concentrated. Interestingly, under the assumption that dwarfs have isotropic velocity dispersion tensors, models with significantly tilted primordial power spectra (*e.g.*, $n \lesssim 0.85$, $\sigma_8 \lesssim 0.7$) *may underpredict* the number of large Milky Way satellites with $V_{\mathrm{max}} \gtrsim 40$ km s^{-1}.

1. INTRODUCTION

The ΛCDM model of a flat Universe dominated by cold, collisionless dark matter (CDM), and a cosmological constant (Λ) has emerged as the standard framework for the growth of cosmic structure. With $\Omega_{\mathrm{M}} \approx 0.3$, $h \approx 0.7$, and a nearly scale-invariant primordial spectrum of adiabatic density perturbations ($P(k) \propto k^n$, $n \approx 1$), ΛCDM is remarkably successful at reproducing large scale observations. In contrast, this paradigm faces several challenges on galactic and sub-galactic scales [1, 2]. In Zentner & Bullock [3] (ZB), we emphasized that inflation does not predict *exactly* scale-invariant (*i.e.*, $n = 1$) primordial spectra. Many models of inflation predict "tilted" spectra ($n \neq 1$), spectral index "running" ($dn/d\ln k \neq 0$), or other deviations from scale-invariance that have dramatic consequences on small scales. We showed that spectra with tilts of $n \sim 0.9$ and/or running and fixed by COBE on large scales can greatly reduce the predicted central densities of dark matter halos, alleviating the "central density problem" plaguing ΛCDM. Further, the neighborhood of $\sigma_8 \sim 0.75$ implied by these tilts is provocatively close to many recent estimates of "low" σ_8 values [4].

   In this proceeding, we report on results from follow-up work to ZB. We study the

TABLE 1.   Initial power spectra from the inflationary models discussed in ZB.

Model Description	Model Name	$n(k_{\mathrm{COBE}})$	$\mathbf{d}n(k_{\mathrm{COBE}})/\mathbf{d}\ln k$	σ_8
Scale-invariant	$n = 1$	$\equiv 1$	$\equiv 0$	$\simeq 0.95$
Inverted Power Law	IPL4	$\simeq 0.94$	$\simeq -0.001$	$\simeq 0.83$
Running-mass model I	RM I	$\simeq 0.84$	$\simeq -0.004$	$\simeq 0.65$
Running-mass model II	RM II	$\simeq 0.90$	$\simeq -0.001$	$\simeq 0.75$
Running-mass model III	RM III	$\simeq 1.1$	$\simeq -0.001$	$\simeq 1.21$
Broken scale-invariant	BSI	$= 1.0$	$= 0$	$\simeq 0.97$

dependence of CDM halo substructure on the primordial power spectrum (PPS). Our models of the PPS are the same as those in ZB. We COBE normalize all spectra and we assume a cosmological model with $\Omega_{\mathrm{M}} = 1 - \Omega_{\Lambda} = 0.3$, $\Omega_{\mathrm{B}}h^2 = 0.02$, and $h = 0.72$. The important characteristics of each input spectrum are summarized in Table 1. Numerical simulations cannot have both the resolution and the statistics needed to study substructure so we model substructure semi-analytically using host halo merger histories [5] and a scheme for approximating subhalo orbits and tidal mass loss. Our model expands on previous work by Bullock *et al.* [6] and Taylor and Babul [7]. We calibrated our model against available data from N-body simulations; nevertheless, our results *must* be regarded as preliminary estimates to be verified by extensive N-body work. We present results based on 100 merger tree realizations. We give a detailed description of our model and further results in a forthcoming paper [8].

2. SUBSTRUCTURE MASS FRACTIONS

Efforts have been made to use flux ratios in multiply-imaged quasars to detect substructure in galactic halos and to use these measurements to constrain cosmology. In particular, Dalal and Kochanek [9] (DK) considered bounds on the PPS. As such, it is important to understand the theoretical predictions for halo substructure as a function of the PPS and, more generally, substructure distributions and characteristics as a function of cosmology.

Our results on the substructure mass fraction and the PPS are summarized in Figure 1. DK took a typical lens mass of 3×10^{12} M$_\odot$ and the lenses in their sample have a median redshift of $z_\ell \approx 0.6$, so we present results for a 3×10^{12} M$_\odot$ halo at $z = 0.6$; however, our results do not change appreciably as a function of mass or redshift. Lensing measurements are sensitive to the mass fraction in substructure projected onto the plane of the lens at a halo-centric distance of order the Einstein radius, $R_{\mathrm{E}} \sim 5$ kpc. Consequently, we show in Fig. 1 the mass fraction in substructure for the entire halo *and* the mass fraction in substructure in a 2D projection of radius $R = 10$ kpc.

Notice that the substructure mass fraction is not a strong function of tilt and/or running. In tilted models, host halos are less concentrated and accrete their substructure later, and this compensates for the fact that the substructures are more fragile, and more easily destroyed by tides. It will be difficult to use substructure measurements to constrain these parameters. Only the BSI model, with a sharp drop in power at $\sim 10^{10}$
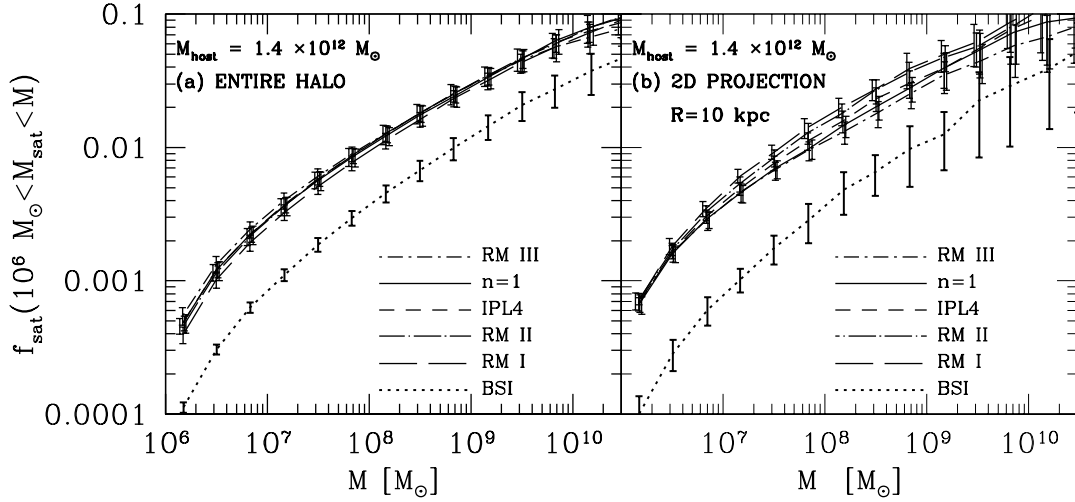
FIGURE 1. The fraction of the host halo mass bound up in substructures of mass between $10^6 M_\odot$ and M as a function of M. **(a)** The mass fraction in substructure for the entire halo. **(b)** The mass fraction in a 2D halo-centric cylindrical projection of radius $R = 10$ kpc. The lines represent the average mass fractions and the errorbars show the dispersion among the 100 realizations. The models are labeled in each panel.

$M_\odot$, shows deviation from the $n = 1$ model that is significant compared to the scatter. It may be possible to constrain models with such an abrupt break (*e.g.*, warm dark matter). DK found the halo mass fraction bound up in substructures of mass $M_{sat} \lesssim 10^9$ $M_\odot$ to be $0.006 \lesssim f_{sat} \lesssim 0.07$. All of our models are consistent with this bound, but the truncated model is just at the edge of the allowed region.

3. THE DWARF SATELLITES

The "dwarf satellite problem", namely that ΛCDM predicts roughly an order of magnitude more halos with $V_{max} \lesssim 40$ km s^{-1} than observed Milky Way (MW) satellites, is an often-discussed challenge to ΛCDM [1]. Stoehr *et al.* [10] (S02) and Hayashi *et al.* [11] (HO2) proposed that substructure halos may be significantly less concentrated than comparable field halos due to tidal effects. This implies that the values of V_{max} that correspond to the observed central velocity dispersions, $\sigma_\star$, of the MW satellites are larger than the values inferred by other authors. One must be cautious. Mass redistribution in subhalos is quite sensitive to the mass resolution of the simulation (S02) and the velocity function (VF) of satellite halos is sensitive to the initial concentrations and accretion times of substructure (H02). Our semi-analytic model represents one extreme; we do not allow for redistribution of mass within a subhalo's tidal limit. Using our model, we can also quantify the cosmology dependence of the mapping between $\sigma_\star$ and V_{max}. We have assumed that CDM halos can be well described by NFW [12] profiles with a particular V_{max} and R_{max} (the radius at which V_{max} is attained) and calculated all combinations of V_{max} and R_{max} that lead to the observed values of $\sigma_\star$ for each of the MW satellites. We have assumed that the stars have isotropic dispersion tensors and that the stellar distri-
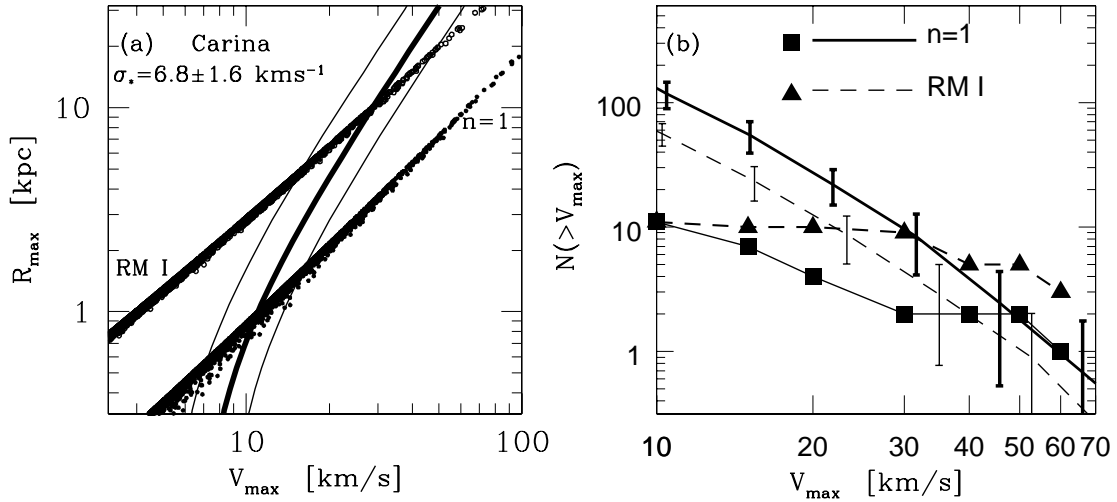
FIGURE 2. (a) The lower group of points represent a scatter plot of V_{max} vs. R_{max} for 10 realizations of the $n = 1$ model. The upper points correspond to RM I. The lines show the region that yields an observed value of $\sigma_\star = 6.8 \pm 1.6$ km s^{-1} for Carina. The thick solid line corresponds to the central value of $\sigma_\star$ while the thin solid lines correspond to the 1σ errors. Consistency demands that Carina resides in a halo that has structural properties that lie in the region of overlap between the thin solid lines and the scattered points for each cosmology. (b) The predicted VFs (lines) and scatter for the $n = 1$ and RM I models along with the "observed" VF (shapes) for each model inferred from the observed values of $\sigma_\star$. Squares represent satellite velocities that would be implied if their halo profiles reflect halos in the $n = 1$ model, triangles correspond to the RM I expectations.

butions are given by King profiles [13] with parameters given by Mateo [14].

Results for Carina are shown in Figure 2, along with a scatter plot of V_{max} vs. R_{max} for the surviving satellites in 10 realizations of a $M_{host} = 1.4 \times 10^{12}$ M$_\odot$ host halo at $z = 0$ for the $n = 1$ and RM I models. This plot shows how allowing for less concentrated halos helps to alleviate the dwarf satellite problem. Less concentrated halos (larger R_{max}) require a larger V_{max} because the outer stellar radius of Carina is much smaller than the radius at which the halo's rotation curve peaks. Larger halos are intrinsically scarcer objects, helping to explain the paucity of dwarf satellites. Feedback mechanisms (*e.g.*, [6]) then explain the dearth of smaller halos. Figure 2 also demonstrates that the mapping between $\sigma_\star$ and V_{max} is dependent upon cosmology, in particular the PPS, and so the same observational data imply a *cosmology dependent* "observed" VF. Our estimates for dwarf velocities in the $n = 1$ case compare well to the estimates made by Klypin and collaborators [1].

In the right panel of Fig. 2 we present the predicted VFs along with separate "observed" velocity functions for the $n = 1$ and RM I models. The RM I VF is a factor of ~ 2 lower than the $n = 1$ VF mainly because typical halos are less concentrated in this model, so that V_{max} is lower at a given mass. Also notice that the "observed" VF is shifted significantly higher at high V_{max}. This suggests that significantly tilted power spectra $n \lesssim 0.85$ may *underpredict* the number of MW satellites at high V_{max}. Moreover, subhalos are likely less centrally concentrated than field halos (as suggested by S02 and H02), and this serves to make the underprediction *more* pronounced. However,

we recommend circumspection. Our results concerning VFs are sensitive to several assumptions such as the isotropy of the dispersion tensor.

REFERENCES

1. Moore, B. *et al.*, ApJ, **524**, L19-L22 (1999); Klypin, A. A. *et al.*, ApJ, **522**, 82 (1999).
2. Debattista, V. P. and Sellwood, J. A., ApJ, **543**, 704 (2000); Keeton, C. R., ApJ, **561**, 46 (2001); Alam, S. M. K. *et al.*, ApJ, **572**, 34 (2002); McGaugh, S. *et al.*, ApJ, **584**, in press, (2003).
3. Zentner, A. R. and Bullock, J. S., PRD, **66**, 043003 (2002), (ZB).
4. Viana, P. T. *et al.*, ApJ, **569**, 75; Bahcall, N. *et al.*, ApJ, **585**, in press (2002); Schuecker, P. *et al.*, A&A, in press (2002); Allen, S. W. *et al.*, astro-ph/0208394, (2002); Brown, M. L. *et al.*, astro-ph/0210213, (2002); Hamana, I. *et al.*, astro-ph/0210450 (2002); Peirpaoli, W. J. *et al.*, astro-ph/0210567, (2002).
5. Somerville, R. S. and Kolatt, T. S., MNRAS, **305**, 1, (1999).
6. Bullock, J. S., Kravtsov, A. V., and Weinberg, D. H., ApJ, **539**, 517, (2000).
7. Taylor, J. E., and Babul, A., ApJ, **559**, 716, (2001).
8. Zentner, A. R. and Bullock, J. S., in preparation.
9. Dalal, N. and Kochanek, C., ApJ, **572**, 25, (2001); Dalal, N. and Kochanek, C., PRL, submitted, (2002).
10. Stoehr, F. *et al.*, MNRAS, **335**, L84, (2002).
11. Hayashi, E. *et al.*, ApJ, in press, astro-ph/0203004, (2002).
12. Navarro, J. F., Frenk, C. S., and White, S. D. M., ApJ, **490**, 493, (1997).
13. King, I., ApJ, **67**, 471, (1962).
14. Mateo, M., Ann. Rev. Astron. Astrophys., **36**, 435, (1998).