Non-Gaussian Fluctuations in Biased Resistor Networks: Size Effects versus Universal Behavior

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Abstract

We study the distribution of the resistance fluctuations of biased resistor networks in nonequilibrium steady states. The stationary conditions arise from the competition between two stochastic and biased processes of breaking and recovery of the elementary resistors. The fluctuations of the network resistance are calculated by Monte Carlo simulations which are performed for different values of the applied current, for networks of different size and shape and by considering different levels of intrinsic disorder. The distribution of the resistance fluctuations generally exhibits relevant deviations from Gaussianity, in particular when the current approaches the threshold of electrical breakdown. For two-dimensional systems we have shown that this non-Gaussianity is in general related to finite size effects, thus it vanishes in the thermodynamic limit, with the remarkable exception of highly disordered networks. For these systems, close to the critical point of the conductor-insulator transition, non-Gaussianity persists in the large size limit and it is well described by the universal Bramwell-Holdsworth-Pinton distribution. In particular, here we analyze the role of the shape of the network on the distribution of the resistance fluctuations. Precisely, we consider quasi-one-dimensional networks elongated along the direction of the applied current or trasversal to it. A significant anisotropy is found for the properties of the distribution. These results apply to conducting thin films or wires with granular structure stressed by high current densities.

1 Introduction and Model

Strongly correlated systems usually exhibit non-Gaussian distributions of the fluctuations of global quantities, as a consequence of the violation of the validity conditions of the central-limit theorem. Since correlations become important near the critical points of phase transitions, non-Gaussian fluctuations are usually observed near criticality [1, 2, 3, 4, 5, 6, 7, 8, 9]. In these conditions, the self-similarity of the system over all the scales, from a characteristic microscopic length up to the size of the system, has important implications on the fluctuation distribution [2, 3, 4, 5, 6, 7, 8, 9]. On the other hand, far from criticality, the correlations among different elements of the systems can also be important. This is particularly true for systems in non-equilibrium stationary states, where non-Gaussian fluctuations are frequently present [1, 10, 11, 12, 13, 14, 15]. Therefore the study of non-Gaussian fluctuations and of their link with other features of the system can provide new insights into basic properties of complex systems [2, 3, 4, 5, 6, 7, 8, 9, 11]. On this respect, the observation made few years ago by Bramwell, Holdsworth and Pinton (BHP) [2] of a common behavior of the distribution of the fluctuations in two very different systems, (the power-consumption fluctuations in confined turbulent-flow experiments and the magnetization fluctuations in the two-dimensional XY model in the spin-wave regime at low temperature [2, 3, 4]), has given rise to several intriguing questions about the origin of this common behavior, stimulating many other experimental, analytical and numerical studies. Successive findings have highlighted that many scale invariant systems display the same functional form for the distribution of the fluctuations [3, 7, 5, 13, 14, 15]. Very recently, new light on these puzzling observations has been given by Clusel et al. [16]. These authors, on the basis of a study of the fluctuation properties of the 2D XY model, have proposed a criterion for universality-class-independent critical fluctuations [16]. Actually, in the relatively simple case of the 2D XY model it is possible a complete understanding of fluctuation phenomena. This is not possible for nonequilibrium systems due to the lack of microscopic theories. Thus, for these systems, we can rely only on phenomenological observations and on analogies with better understood systems.

Here, we study the distribution of the resistance fluctuations of biased resistor networks in nonequilibrium stationary states [17]. Networks of different size and shape and with different levels of internal disorder are considered. The resistance fluctuations are calculated by Monte Carlo simulations for currents close to the threshold for electrical breakdown. This last phenomenon consists of an irreversible increase of the resistance, occurring in conducting materials stressed by high current densities and it is associated with a conductor-insulator transition [18, 19, 20, 21, 22, 23]. In our study we make use of the Stationary and Biased Resistor Network (SBRN) model [24, 25]. This model provides a good description of many features associated with the electrical instability of composite materials [20, 22, 24] and with the electromigration damage of metallic lines [17, 21], two important classes of breakdown phenomena.

We describe a thin conducting film with granular structure of length L, width W and thickness $t_h \ll W, L$ as a 2D resistor network of rectangular shape and square-lattice structure [17]. The network of resistance R is made by N_L and N_W resistors in the length and width directions respectively. Thus, the total number of resistors in the network (excluding the contacts) is: $N_{tot} = 2N_LN_W + N_L - N_W$. The external bias (here a constant current I), is applied to the network through electrical contacts realized by perfectly conducting bars at the left and right hand sides of the network. The network lies on an insulating substrate at temperature T_0, acting as a thermal bath. Each resistor has two allowed states [21, 26]: (i) regular, corresponding to a resistance $r_{reg,n}(T_n) = r_{ref}[1 + \alpha(T_n - T_{ref})]$ and (ii) broken, corresponding to a resistance $r_{OP} = 10^9 r_{reg,n}(T_0) \equiv 10^9 r_0$ (resistors in this state will be called defects). In the above expression α is the temperature coefficient of the resistance (TCR), r_{ref} and T_{ref} are the reference values for the TCR and T_n is the local temperature. The existence of temperature gradients due to current crowding and Joule heating effects is accounted for by taking the local temperature of the n-th resistor given by the following expression [21]:

$$T_n = T_0 + A[r_n i_n^2 + (3/4N_{neig}) \sum_{l=1}^{N_{neig}} (r_l i_l^2 - r_n i_n^2)] \tag{1}$$

where, i_n is the current flowing in the nth resistor and N_{neig} the number of its nearest neighbors over which the summation is performed. The parameter A represents the thermal resistance of each resistor and sets the importance of Joule heating effects. By taking the above expression for T_n we are assuming an instantaneous thermalization of each resistor at the value T_n [21, 26]. In the initial state of the network (no external bias) we take all the resistors identical (perfect network). We assume that two competing biased processes act to determine the evolution of the network [24, 25]. These two processes consist of stochastic transitions between the two possible states of each resistor and they occur with thermally activated probabilities [26]: $W_{Dn} = \exp[-E_D/k_BT_n]$ and $W_{Rn} = \exp[-E_R/k_BT_n]$, characterized by the two energies, E_D and E_R

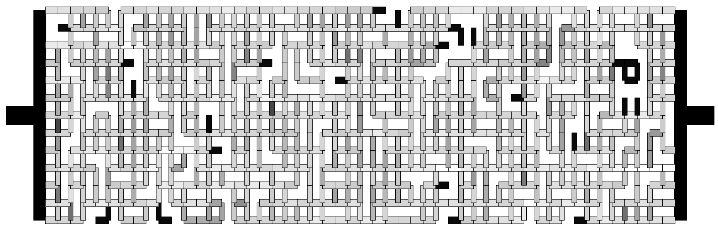

Figure 1: Pattern of a network 12×50 stressed by a current density $j = 0.32$ mA. The grey boxes show the backbone of the network, the black ones the "dangling bonds" (branches with zero-current, while the missing boxes correspond to the broken resistors. This pattern has been calculated at $t = 4 \times 10^4$ (time expressed in iteration steps).

(where k_B is the Boltzmann constant). The time evolution of the network is obtained by Monte Carlo simulations which update the network resistance after breaking and recovery processes, according to an iterative procedure described in detail in Ref. [24]. The sequence of successive configurations provides a resistance signal, $R(t)$, after an appropriate calibration of the time scale. Depending on the stress conditions (I and T_0) and on the network parameters (size, activation energies and other parameters dependent on the material, like r_{ref}, α and A), the network either reaches a stationary state or undergoes an irreversible electrical failure [17, 24, 25]. This latter possibility is associated with the achievement of the percolation threshold, p_c, for the fraction of broken resistors. Therefore, for a given network at a given temperature, a threshold current value, I_B, exists above which electrical breakdown occurs [24]. For values of the current below this threshold, the steady state of the network is characterized by fluctuations of the fraction of broken resistors, δp, and of the resistance, δR, around their respective average values $< p >$ and $< R >$. In particular, we underline that in the vanishing current limit (random percolation) [27], the ratio $\lambda \equiv (E_D - E_R)/k_B T_0$ determines the average fraction of defects and thus the level of intrinsic disorder inside the network [27]. In the following we analyze the results of simulations performed by considering networks of different size and shape stressed at room temperature, $T_0 = 300$ K, by a current density $j \equiv I/N_W = 0.32$ mA. We have taken: $\alpha = 3.6 \times 10^{-3}$ K^{-1}, $T_{ref} = 273$ K, $r_{ref} = 0.048\ \Omega$, $A = 2.7 \times 10^8$ K/W, $E_D = 0.41$ eV and $E_R = 0.35$ eV. This choice of the parameters is appropriate to describe the behavior under electromigration of metallic lines of Al-0.5%Cu studied in Ref. [17] and it corresponds to studying a network with an intermediate level of intrinsic disorder.

2 RESULTS AND CONCLUSIONS

Figure 1 displays the pattern of a network 12×50 calculated at a given time, $t = 4 \times 10^4$, (expressed in iteration steps) in the stationary regime of the network, i.e. for $t > \tau_{rel}$, where $\tau_{rel} \approx 8 \times 10^3$ is the relaxation time for the achievement of the nonequilibrium stationary state. The network in this figure is stressed by a current density ($j = 0.32$ mA) close to the breakdown value, $j_B = I_B/N_W$. The resistance evolution for the same network is reported in Fig. 2. In this figure the grey line shows the average value of the resistance, $< R >$. We note that both the average resistance and the relative variance of the resistance fluctuations, $< (\delta R)^2 > / < R >^2$, depend on j. A detailed analysis of the behavior of these two quantities as a function of the current can be found in Refs. [24, 25]. In previous works [13, 14, 15] we have analyzed the effects on the distribution of the resistance fluctuations of the biasing current [13], of the intrinsic

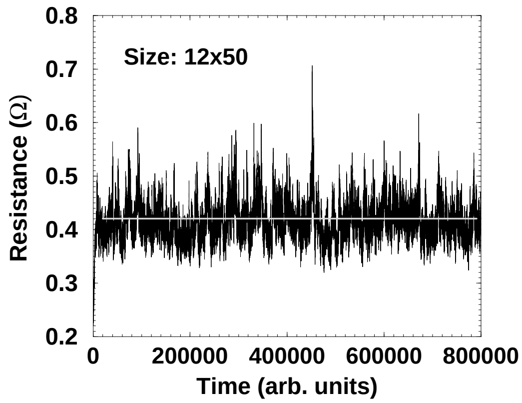

Figure 2: Resistance evolution of the network in Fig. 1. The time is expressed in arbitrary units (iteration steps), the resistance in Ohm. The grey line shows the average value of the resistance.

disorder and of the size of the network [14, 15], by limiting ourself to discuss square networks. Here, we focus our discussion on shape effects: precisely we analyze the effect on the distribution of the fluctuations of scaling the size of the network separately in the two directions, i.e. of scaling separately the width, N_W and the length, N_L, of the network. Figure 3(a) shows the distributions of the resistance fluctuations obtained for two networks of size 12×50 (big circles) and 50×12 (triangles) stressed by the same current density. In this figure (and in the followings) we denote with Φ the probability density function (PDF) of the distribution and with σ the root mean square deviation from the average value. This normalized representation, by making the distribution independent of its first and second moments, is particularly convenient to explore the functional form of a distribution [3]. A lin-log scale is adopted for convenience to plot the product $\sigma\Phi$ as a function of $(< R > -R)/\sigma$. The PDFs in Fig. 3 and all the others in this paper have been calculated by considering time series containing about 10^6 resistance values. For comparison, in Fig. 3 we also report the Gaussian distribution (dashed curve) and the BHP distribution (continuous curve) [2, 3]. The PDF obtained for the network 12×50 (corresponding to the signal in Fig. 2) exhibits a strong non-Gaussianity, well described by the BHP curve. By contrast, the PDF obtained for the network 50×12 is nearly Gaussian. At a first insight, this result can seem surprising: in fact the two networks are composed by nearly the same number of resistors, moreover the dissipated electric power per unit volume, $RI^2/(LW) \propto j^2$, is the same in both cases. As a consequence, the average fraction of defects $p \approx 0.19$ is also the same. However, the percolation threshold p_C is different for the two networks [17]. Therefore, the nearly Gaussian distribution of the 50×12 network is due to the higher value of p_C (and thus to the higher stability) of this network [17]. For comparison, we report in Fig. 3(b) the PDFs calculated for two square networks 12×12 and 50×50 biased by the same current density. Again, the dissipated power density and the average fraction of defects are the same for both networks. However, for square $N \times N$ networks the percolation threshold is roughly independent of the size, even for biased percolation [17]. Thus, for these networks the higher instability and the stronger non-Gaussianity for decreasing N is mainly related with the increase in magnitude of the fluctuations associated with the smaller size [13, 14].

The normalized PDFs of the resistance fluctuations calculated for several networks of different size are reported in Fig. 4. Figure 4(a) displays PDFs obtained for networks elongated along the direction of the applied current (precisely networks of a given width, $N_W = 12$, and with

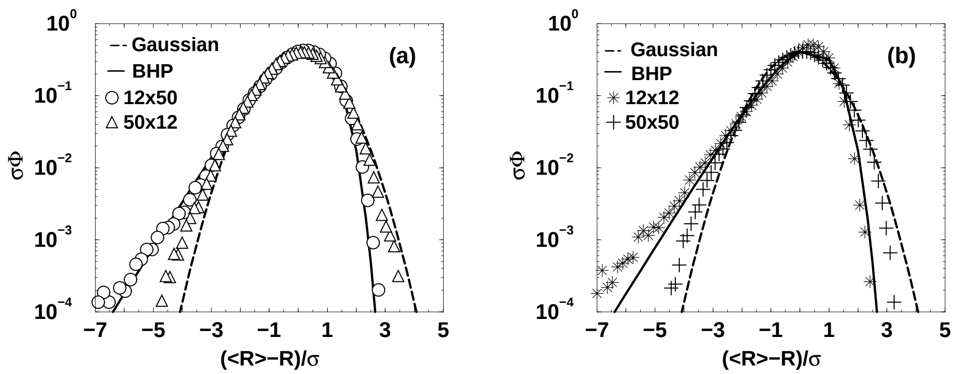

Figure 3: Normalized PDF of the resistance fluctuations of networks of different size and stressed by the same current density $j = 0.32$ mA. Precisely, in (a) the size is: 12×50 (big circles) and 50×12 (triangles); in (b): 12×12 (stars), 50×50 (plus). The solid and dashed curves refer to the BHP and Gaussian distributions, respectively.

increasing length, $N_L = 50 \div 400$), while Fig. 4(b) shows PDFs obtained for networks elongated in a direction trasversal to the current applied (precisely networks of a given length, $N_L = 12$, and with increasing width, $N_W = 50 \div 400$. We can see that for trasversal networks the PDF is rather insensitive to the width and the small non-Gaussianity for small widths completely vanishes already for networks with $N_W = 200$. By contrast, for longitudinal networks, the PDF is sensitive to the length. However, it should be noted that the PDF obtained for $N_L = 200$ practically overlaps with that obtained for $N_L = 400$ and both exhibit non-Gaussian tails. Since the correlation length, ξ, for these networks is estimated to be $\xi < 5$, networks with $N_L = 400$ can be considered as infinitely long. Thus, Fig. 4(a) suggests a persistent non-Gaussianity for longitudinal networks in the limit $N_L \to \infty$, associated with the finite size of the network in the transversal direction. Furthermore, the magnitude of this non-Gaussianity is expeceted to be controlled by the level of intrinsic disorder.

In conclusions, we have studied the distribution of the resistance fluctuations of biased resistor networks in nonequilibrium stationary states. We have considered networks biased by currents close to the threshold of electrical breakdown. As a general trend, the distribution of the fluctuations is found to exhibit relevant deviations from Gaussianity, which are in general related to finite size effects [13, 14]. However, for systems close to the critical point of the conductor-insulator transition, the non-Gaussianity persists in the large size limit [13, 14] and it is well described by the universal Bramwell-Holdsworth-Pinton distribution. Furthermore, we have analyzed the role of the shape of the network on the distribution of the resistance fluctuations, by considering quasi-one-dimensional networks elongated along the direction of the applied current or trasversal to it. A significant anisotropy is found for the properties of the distribution. These results apply to conducting thin films or wires with granular structure stressed by high currents.

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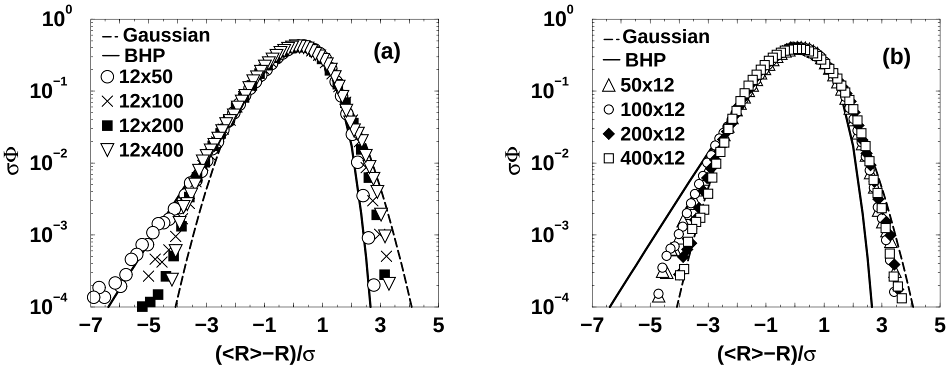

Figure 4: Normalized PDF of the resistance fluctuations of networks of different size and stressed by the same current density $j = 0.32$ mA. Precisely, in (a) the size is: 12×50 (big circles), 12×100 (crosses), 12×200 (full squares) and 12×400 (down triangles); in (b) the size is: 50×12 (triangles), 100×12 (small circles), 200×12 (full diamonds) and 400×12 (squares). The solid and dashed curves have the same meaning of Fig. 3.

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