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A modified Corrado test for assessing abnormal security returns

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Event studies typically use the methodology developed by Fama et al. [1969. The adjustment of stock prices to new information. *International Economic Review* 10, no. 1: 1–21] to segregate a stock's return into expected and unexpected components. Moreover, conventional practice assumes that abnormal returns evolve in terms of a normal distribution. There is, however, an increasing tendency for event studies to employ non-parametric testing procedures due to the mounting empirical evidence which shows that stock returns are incompatible with the normal distribution. This paper focuses on the widely used non-parametric ranking procedure developed by Corrado [1989. A nonparametric test for abnormal security price performance in event studies. *Journal of Financial Economics* 23, no. 2: 385–95] for assessing the significance of abnormal security returns. In particular, we develop a consistent estimator for the variance of the sum of ranks of the abnormal returns, and show how this leads to a more efficient test statistic (as well as to less cumbersome computational procedures) than the test originally proposed by Corrado (1989). We also use the theorem of Berry [1941. The accuracy of the Gaussian approximation to the sum of independent variates. *Transactions of the American Mathematical Society* 49, no. 1: 122–36] and Esseen [1945. Fourier analysis of distribution functions: A mathematical study of the Laplace–Gaussian law. *Acta Mathematica* 77, no. 1: 1–125] to demonstrate how the distribution of the modified Corrado test statistic developed here asymptotically converges towards the normal distribution. This shows that describing the distributional properties of the sum of the ranks in terms of the normal distribution is highly problematic for small sample sizes and small event windows. In these circumstances, we show that a second-order Edgeworth expansion provides a good approximation to the actual probability distribution of the modified Corrado test statistic. The application of the modified Corrado test developed here is illustrated using data for the purchase and sale by UK directors of shares in their own companies.

Keywords: abnormal return; Corrado test; Edgeworth expansion; normal distribution; rank

JEL Classification: C14; G14

1. Introduction

Event studies use financial information to measure the impact that specific circumstances and events have on the market value of a firm's equity securities (MacKinlay 1997). The standard event study methodology was developed by Fama et al. (1969) and involves using a stock's sensitivity to variations in a well-diversified market index to segregate the stock's periodic return into expected and unexpected components. The latter component is normally referred to as the stock's abnormal return and is the principal focus of the typical event study. Event studies assess whether the average abnormal return over an event period and/or the accumulated average abnormal return around the event period are significantly different from zero across the sampled firms affected by the given event. Moreover, conventional practice assumes that abnormal returns evolve in terms of a normal distribution, in which case, one can assess the significance of the (accumulated)

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average abnormal returns by recourse to the ‘ t ’-test of Patell (1976) and others. A significant difficulty with this testing procedure, however, is that there is mounting empirical evidence which suggests that stock returns are incompatible with the normal distribution. In particular, stock return distributions appear to be leptokurtic and skewed and to exhibit fat tails (Theodossiou 1998; Harris and Küçüközmen 2001; Ashton and Tippett 2006). The skewness attribute is particularly troublesome, since it means that the commonly employed t -tests will have a tendency to reject (or accept) the null hypothesis of zero abnormal returns too often (or too little) depending on whether the stock’s returns are positively (or negatively) skewed.

Doubts about the exact distributional properties of stock returns have kindled a growing interest in non-parametric testing procedures for assessing the significance of the (accumulated) abnormal returns which arise out of the event study methodology. Non-parametric testing procedures make only minimal assumptions about the distributional properties of abnormal returns so much so that Conover (1971, 3) described non-parametric statistics as providing ‘approximate solutions to exact problems ... as opposed to the exact solutions to approximate problems furnished by parametric statistics’ such as the t -test developed by Patell (1976). Here, we would note that one of the best known and most commonly applied non-parametric tests in the event study literature was introduced by Corrado (1989). The Corrado (1989) test is valid when applied to skewed and/or leptokurtic distribution functions, and avoids many of the limitations implicit in alternative non-parametric tests of abnormal security-price performance (e.g. the symmetry assumptions on which the Wilcoxon signed-rank test is founded). Yet for all its virtues, the Corrado (1989) test is computationally cumbersome and little is known about its small sample properties. This note has a fourfold purpose. First, we determine a consistent estimator for the variance of the ranks of the abnormal security returns. We then use this consistent estimator for the variance to obtain an exact closed form, or modified expression for the Corrado (1989) test statistic. Second, we use the theorem of Berry (1941) and Esseen (1945) to determine the rate at which the distribution function of the modified Corrado (1989) test statistic converges towards the normal distribution function as the sample size grows in magnitude. Third, we determine the distributional properties of the sum of the ranks of the individual abnormal returns for a given firm over a given event window. This shows that describing the distributional properties of the sum of the ranks in terms of the normal distribution is highly problematic for small sample sizes and small event windows. Fortunately, in such circumstances, we also illustrate how a second-order Edgeworth expansion provides an accurate approximation to the distribution function for the sum of the ranks. Finally, we demonstrate how it is much ‘safer’ to employ the modified Corrado test developed here in preference to the conventional t -tests – such as that of Patell (1976) – as a means for assessing the likelihood of abnormal security-price performance. We illustrate the implementation of our results by using data for the purchase and sale by UK directors of shares in their own companies.

2. Fundamental results

We begin our analysis by noting that the detection of abnormal security performance requires a return-generating model. Suppose then that one follows the procedure laid down in MacKinlay (1997, 18–21) of assuming that the return on a given security bears a linear relationship to a well-diversified market index, the so-called ‘market model’ of empirical finance. One can then estimate the abnormal return, AR_{it} , for each of $i = 1, 2, 3, \dots, N$ securities across $t = 1, 2, 3, \dots, T$ time periods (days, weeks, months, etc.) by using the following procedure:

$$AR_{it} = R_{it} - (\hat{a}_i + \hat{b}_i R_{mt}), \quad (1)$$

where R_{it} is the return on the i th security during the t th time period, R_{mt} is the return on the market index during the t th time period and \hat{a}_i and \hat{b}_i are the estimated parameters of the market model for the i th security.¹ One can then let $1 \leq K(AR_{it}) \leq T$ be the rank for the i th firm of the abnormal return during the t th time period as summarised in the following matrix:

$$K = \begin{pmatrix} K(AR_{11}) & K(AR_{12}) & K(AR_{13}) & \dots & K(AR_{1T}) \\ K(AR_{21}) & K(AR_{22}) & K(AR_{23}) & \dots & K(AR_{2T}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K(AR_{N1}) & K(AR_{N2}) & K(AR_{N3}) & \dots & K(AR_{NT}) \end{pmatrix}.$$

We emphasise here that each row represents the ranks pertaining to the abnormal returns of a given firm. Thus, the first row contains the ranks from 1 to T of the abnormal returns of the first firm; that is, the lowest or most negative abnormal return is assigned a rank of 1 and the most positive abnormal return is assigned a rank of T . The second row contains the ranks from 1 to T of the abnormal returns of the second firm. The third row contains the ranks from 1 to T of the abnormal returns of the third firm and so on. Now, following Corrado (1989, 388), we assume that the ranks are randomly allocated across the T elements comprising each row of the above matrix. It then follows that the average of the ranks allocated to each of the $i = 1, 2, 3, \dots, N$ rows (i.e. the N firms) must be (Freund 1971, 421)

$$E[K(AR_{it})] = \frac{1}{T} \sum_{i=1}^T K(AR_{it}) = \frac{1}{T} \sum_{t=1}^T t = \frac{T+1}{2}, \tag{2}$$

where $E(\cdot)$ is the expectation operator.² Likewise, the variance of the ranks allocated to each row must be (Freund 1971, 421)

$$\text{Var}[K(AR_{it})] = \frac{1}{T} \sum_{i=1}^T \left[K(AR_{it}) - \frac{T+1}{2} \right]^2 = \frac{1}{T} \sum_{t=1}^T \left(t - \frac{T+1}{2} \right)^2 = \frac{T^2-1}{12}, \tag{3}$$

where $\text{Var}(\cdot)$ is the variance operator. Next, consider the sum of the ranks, $\sum_{i=1}^N K(AR_{it})$, allocated to each of the $t = 1, 2, 3, \dots, T$ columns of the above matrix; that is, the sum of the ranks across the N firms comprising the sample for a fixed or given time period (or column) (t). It then follows that the variance of the sum of the ranks for this sample of companies will be

$$\text{Var} \left[\sum_{i=1}^N K(AR_{it}) \right] = \sum_{i=1}^N \text{Var}[K(AR_{it})] + \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N \text{Cov}[K(AR_{it}), K(AR_{jt})]. \tag{4}$$

Here, $\text{Cov}[K(AR_{it}), K(AR_{jt})]$ is the covariance between the rank of the abnormal return contained in the i th row of column (t) and the rank of the abnormal return contained in the j th row of column (t). Since the rank allocated to the i th firm during the t th time period is independent of the rank allocated to the j th firm for the same time period, it necessarily follows that the covariance between the ranks allocated to the different elements of each column will be $\text{Cov}[K(AR_{it}), K(AR_{jt})] = 0$.³ One can then use Equations (3) and (4) to show that the variance

of the sum of ranks across the N firms will be

$$\text{Var} \left[\sum_{i=1}^N K(\text{AR}_{it}) \right] = \sum_{i=1}^N \text{Var}[K(\text{AR}_{it})] = \frac{N(T^2 - 1)}{12}. \quad (5)$$

Now, consider the Corrado (1989, 388) expression for the variance of the sum of excess ranks across these N firms:

$$S^2(K) = \frac{1}{T} \sum_{t=1}^T \left[\frac{1}{N} \sum_{i=1}^N \left\{ K(\text{AR}_{it}) - \frac{T+1}{2} \right\} \right]^2. \quad (6)$$

One can use this expression to compute the standardised variable:

$$z_c = \frac{1/N \sum_{i=1}^N [K(\text{AR}_{it}) - (T+1)/2]}{S(K)}. \quad (7)$$

However, the previously made assumption that the ranks are randomly distributed across the T elements of each row of the above matrix (Corrado 1989, 388) implies that a simpler expression exists for the standardised variable defined by Equation (7). This can be demonstrated by taking expectations across Equation (6), in which case, it follows that

$$E[S^2(K)] = \frac{1}{TN^2} \sum_{t=1}^T \sum_{i=1}^N \text{Var}[K(\text{AR}_{it})] = \frac{1}{TN^2} \sum_{t=1}^T \frac{N(T^2 - 1)}{12} = \frac{(T^2 - 1)}{12N} \quad (8)$$

provides a closed-form expression for the expected variance of the sum of the excess ranks across the N firms. Moreover, using this result, it follows that $S^2(K)$ is a consistent estimator of the population variance, or (Freeman 1963, 235–36)

$$\text{plim}_{N \rightarrow \infty} S^2(K) = \frac{(T^2 - 1)}{12N} = E[S^2(K)]. \quad (9)$$

Substituting the latter result into Equation (7) leads to the following computationally more convenient modified Corrado test statistic:

$$z_1 = \frac{1/N \sum_{i=1}^N [K(\text{AR}_{it}) - (T+1)/2]}{\sqrt{(T^2 - 1)/12N}} = \sqrt{\frac{3}{N(T^2 - 1)}} \sum_{i=1}^N [2K(\text{AR}_{it}) - (T+1)]. \quad (10)$$

Note also that one can apply the central limit theorem to show that the distribution function, $F_N(z_1)$, of the random variable, z_1 , can be approximated by the standard normal distribution function, $\Phi(z_1) = 1/\sqrt{2\pi} \int_{-\infty}^{z_1} \exp(-x^2/2) dx$, as $N \rightarrow \infty$ (Fisz 1963, 197).⁴ Moreover, the theorem of Berry (1941) and Esseen (1945) shows that the absolute value of the error associated with the approximation of $F_N(z_1)$ by the standard normal distribution function, $\Phi(z_1)$, will be

$$|F_N(z_1) - \Phi(z_1)| \leq c \cdot \sqrt{\frac{27}{N} \left(1 - \frac{1}{T^2}\right)}, \quad (11)$$

where $0.4097 \leq c \leq 0.7056$ is known as the Berry–Esseen constant (Shevtsova 2007). Note how this result implies that the rate of convergence of $F_N(z_1)$ towards the standard normal distribution function is of the order of $1/\sqrt{N}$. The Berry–Esseen bound formalised through Equation (11) will

also enable those who use the modified Corrado (1989) test developed here to make assessments about how reliable the normal approximation is likely to be in their empirical work.

We have previously observed, however, that testing procedures in this area mainly focus on whether the sum (or average) of the abnormal returns for a particular sample of firms beyond a particular event period or date is significantly different from zero. We thus define the accumulated abnormal return for the i th firm, CAR_{itM} , for M periods beyond the event period (t) as

$$CAR_{itM} = \sum_{j=1}^M AR_{i(t+j)},$$

where, as previously, AR_{it} is the abnormal return for the i th firm during the t th time period. In the Corrado (1989) test, however, our concern is not so much with the abnormal return during any particular time period as it is with its rank relative to the other T abnormal returns for the particular firm and period under investigation. Given this, let

$$K(CAR_{itM}) = \sum_{j=1}^M K(AR_{i(t+j)}) \tag{12}$$

be the sum of the ranks of the individual abnormal returns over the M periods beyond the event period (t) for the i th of the N firms on which the analysis is based. Then, standard results show that the expected sum of the ranks, $K(CAR_{itM})$, for the abnormal returns arising beyond this event period must be (Freund 1971, 195)

$$E[K(CAR_{itM})] = \sum_{j=1}^M E[K(AR_{i(t+j)})] = M \cdot \frac{T+1}{2}. \tag{13}$$

Furthermore, the variance of the sum of the ranks, $\text{Var}[K(CAR_{itM})] = \text{Var}\left[\sum_{j=1}^M K(AR_{i(t+j)})\right]$, for the particular segment of the row containing the M abnormal returns beyond the event period turns out to be (Freund 1971, 44–5)

$$\begin{aligned} \text{Var}\left[\sum_{j=1}^M K(AR_{i(t+j)})\right] &= \sum_{j=1}^M \text{Var}[K(AR_{i(t+j)})] \\ &+ \sum_{\substack{j=1 \\ j \neq k}}^M \sum_{k=1}^M \text{Cov}[K(AR_{i(t+j)}), K(AR_{i(t+k)})]. \end{aligned} \tag{14}$$

Here, $\text{Cov}[K(AR_{i(t+j)}), K(AR_{i(t+k)})]$ is the covariance between the rank of the abnormal return contained in the $(t+j)$ th element of the i th row and the rank of the abnormal return contained in the $(t+k)$ th element of the i th row, where i denotes the i th of the N firms on which the analysis is based. However, from Equation (3), we know $\text{Var}[K(AR_{i(t+j)})] = (T^2 - 1)/12 = (T + 1)(T - 1)/12$. Moreover, Freeman (1963, 190) shows for $(j \neq k)$ that $\text{Cov}[K(AR_{i(t+j)}), K(AR_{i(t+k)})] = -(T + 1)/12$.⁵ Hence, substituting the latter two results into Equation (14) shows

$$\text{Var}[K(CAR_{itM})] = M \cdot \frac{(T+1)(T-1)}{12} - M(M-1) \frac{(T+1)}{12} = M \cdot \frac{(T+1)(T-M)}{12} \tag{15}$$

will be the variance of the sum of the ranks of the individual abnormal returns beyond the event period (t) for the i th of the N firms on which the analysis is based. It then follows that

$$z_2 = \frac{K(\text{CAR}_{itM}) - M \cdot (T + 1)/2}{\sqrt{M \cdot (T + 1)(T - M)/12}} = \sqrt{\frac{3}{M(T + 1)(T - M)}} [2K(\text{CAR}_{itM}) - M(T + 1)] \quad (16)$$

will be a standardised random variable with a mean of zero and unit variance.

It is not hard to show that the third moment of the standardised variable, z_2 , given here is zero. However, more complicated algebraic procedures also show that its excess fourth moment will be (Fix and Hodges 1955, 311)

$$E(z_2^4) - 3 = -\frac{6}{5} \left[\frac{T}{M(T - M)} - \frac{1}{T + 1} \right]. \quad (17)$$

Now, in most applications, the ‘test window’, M , surrounding the event date is relatively ‘small’. In contrast, the market model parameter estimation period, $T - M$, is normally relatively ‘large’. It is readily observed that the limiting value ($T \rightarrow \infty$) of Equation (17) in such circumstances is $-6/5M$. This shows that it is unlikely that the standardised variate, z_2 , can be normally distributed for small values of M . Fortunately, it is not hard to show that for small values of M , the approximation to the distribution function, $F_M(z_2)$, of the random variable, z_2 , can be improved considerably by employing the second-order Edgeworth expansion:

$$F_M(z_2) \approx \Phi(z_2) - \frac{1}{20} \left[\frac{T}{M(T - M)} - \frac{1}{T + 1} \right] \Phi^{(3)}(z_2) + \frac{[(T+1)^4 - 5(T+1)^2 + 5(T+1) + (M^4 + (T-M)^4) - 5M(T-M)(T+1) + 4]}{210[M(T-M)(T+1)]^2} \Phi^{(5)}(z_2), \quad (18)$$

where $\Phi^{(3)}(z_2) = 1/\sqrt{2\pi}(z_2^3 - 3z_2) \exp(-z_2^2/2)$ and $\Phi^{(5)}(z_2) = 1/\sqrt{2\pi}(z_2^5 - 10z_2^3 + 15z_2) \exp(-z_2^2/2)$ are the third and fifth derivatives (in terms of the ‘Hermite’ polynomials), respectively, of the standard normal distribution (Fix and Hodges 1955, 312).⁶

We conclude this section by recalling our earlier observation that the Patell (1976) t -test is typical of the parametric tests used for assessing the significance of abnormal returns in market-type models of the equity pricing process. These tests assume normally distributed returns in addition to the other assumptions on which the asset pricing models employed in the analysis are based. In contrast, the modified Corrado test is a distribution-free test which is almost as powerful as the t -test when the normality assumption turns out to be true. This is demonstrated by the fact that if the abnormal returns are generated by a normal distribution, then the (Pitman) asymptotic relative efficiency (ARE)⁷ of the modified Corrado test is $3/\pi \approx 0.9549$ when compared with that of the conventional t -tests applied in the literature (Hodges and Lehmann 1956).⁸ Moreover, the corresponding ARE of the modified Corrado test is at least unity in comparison with that of several other well-known probability distributions (Hodges and Lehmann 1956). In addition, an important safeguard provided by the modified Corrado test is that its ARE relative to that of the t -test can never fall below $108/125 = 0.864$. In contrast, the ARE of the t -test relative to that of the modified Corrado test may be as small as zero. These considerations mean that the modified Corrado test will always provide a satisfactory level of efficiency relative to conventional t -tests. In contrast, there is no guarantee that the t -test will always provide a satisfactory level of efficiency

relative to the modified Corrado test. Thus, based on this (Pitman) ARE criterion, it is always preferable to employ the modified Corrado test over parametric tests such as the Patell (1976) t -test.

3. Illustration of computational procedures

One can illustrate the application of the results developed in the previous section by determining the abnormal returns arising from an investment in the equity of Anglo American PLC as a consequence of the directors' share trading (purchase) activities on 1 October 2001 (the event date). The parameters of the market model as given by Equation (1) were estimated using the daily continuously compounded returns on Anglo American PLC ordinary shares and the FTSE All-Share Index over the period 13 September 2000–23 August 2001.⁹ Abnormal returns were then determined on a daily trading basis over the period 13 September 2000–9 October 2001. The event window encompasses eight trading days prior to the event date and 5 days subsequent to the event date and covers the period 18 September 2001–9 October 2001 – a total of 14 trading days. Moreover, there are $T = 227$ daily abnormal returns over the period 13 September 2000–9 October 2001, and these were ranked from lowest or most negative daily abnormal return (with a rank of 1) to highest or most positive daily abnormal return (with a rank of 227). A detailed summary of the rank test as it applies to the abnormal returns for Anglo American PLC can be found in Table 1.

Table 1. Rank of abnormal returns surrounding share purchase transactions by directors of Anglo American PLC on 1 October 2001 (the event date – time period zero).

Time relative to announcement date (0)	Event window date (M)	Rank of abnormal return	z_2 Equation (16)	Accumulated probability z_2		
				Normal approximation	Edge approximation	Exact
-8	1	211	1.4803	0.9306	0.9266	0.9251
-7	2	13	-0.0433	0.4827	0.4847	0.4847
-6	3	196	0.6903	0.7550	0.7435	0.7438
-5	4	217	1.3903	0.9178	0.9150	0.9144
-4	5	150	1.4942	0.9324	0.9311	0.9301
-3	6	85	1.1844	0.8819	0.8786	0.8780
-2	7	138	1.2394	0.8924	0.8899	0.8893
-1	8	78	0.9646	0.8326	0.8293	0.8288
0	9	173	1.2171	0.8882	0.8862	0.8851
1	10	81	0.9948	0.8401	0.8375	0.8367
2	11	30	0.5554	0.7107	0.7082	0.7081
3	12	215	0.9891	0.8387	0.8366	0.8360
4	13	75	0.7829	0.7832	0.7809	0.7810
5	14	60	0.5293	0.7017	0.6998	0.6992

Notes: The first column represents the day relative to the event date (1 October 2001, which is time period zero). The second column gives the number of days, M , over which the ranks have been summed. The third column gives the rank of the abnormal return on the given day relative to the $T = 227$ abnormal returns covering the estimation and event windows. The fourth column summarises the standardised sum of the ranks, z_2 [Equation (16)], corresponding to the given day. The fifth column gives the accumulated probability on the assumption that the standardised sum of ranks in the fourth column is normally distributed. The sixth column gives the second-order Edgeworth approximation to the accumulated probability, $F_M(z_2)$ (Equation (18)), for the standardised sum of the ranks. Finally, the seventh column gives the exact accumulated probability for the standardised sum of the ranks, $F_M(z_2)$.

The first column in this table represents the day relative to the event date (1 October 2001, which is time period zero). The second column gives the number of days, M , over which the ranks have been summed. The third column gives the rank of the abnormal return on the given day relative to the $T = 227$ abnormal returns covering the estimation and event windows. The fourth column summarises the standardised sum of the ranks, z_2 [Equation (16)], corresponding to the given day. The fifth column gives the accumulated probability on the assumption that the standardised sum of ranks in the fourth column is normally distributed. Thus, on the event date (time period zero), the normal approximation shows that the probability of a standardised sum of ranks of 1.2171 or less is 0.8882. The sixth column gives the second-order Edgeworth approximation to the accumulated probability, $F_N(z_2)$ [Equation (18)], for the standardised sum of the ranks. Thus, on the event date (time period zero), the second-order Edgeworth approximation to the accumulated probability for a standardised sum of ranks of 1.2171 or less is 0.8862. Finally, the seventh column gives the exact accumulated probability for the standardised sum of the ranks, $F_N(z_2)$. Thus, for the event date, the exact accumulated probability corresponding to a standardised score for the sum of ranks of 1.2171 or less is 0.8851. Note how this table shows that for this example, the probability distribution of the standardised sum of ranks quickly converges towards the normal distribution. Indeed, by the fourth day (-5) of the test period ($M = 4$), there is virtually no difference between the normal approximation to the probability distribution for the standardised sum of ranks (0.9178) and the actual probability distribution for the standardised sum of the ranks (0.9144). Note also how this example shows that the Edgeworth approximation should be taken whenever there is a significant difference between the normal and Edgeworth approximations to the actual probability distribution for the standardised variable, z_2 .

4. Application to directors' share-trading activities

We now provide a large sample application of the results summarised in Section 2 by considering the large volume of empirical work which documents significant positive (negative) abnormal returns around directors' purchase (sale) of shares in their own firms. These abnormal returns are usually considered as the market's reaction to insiders' 'informational advantage' over outsiders about the operations of their firms (Fidrmuc, Goergen, and Renneboog 2006). Following this literature, we measured the abnormal returns around directors' trading for UK firms from January 1995 until December 2006. We calculated abnormal returns around insiders' purchase/sale transactions using the standard event study methodology based on the market model summarised by Equation (1).¹⁰ The market's return was approximated by the return on the FTSE All-Share Index. The estimation period for the parameters of the market model was from 221 trading days prior to the directors' sale or purchase transactions until 21 trading days prior to the directors' sale or purchase transactions. The event window encompasses 21 trading days prior to the directors' purchase/sale transactions and 5 days subsequent to the directors' purchase/sale transactions. Thus, our analysis is based on an estimation period of 200 trading days and an event window of 27 trading days.

We obtained directors' trading data for UK firms from Hemmington Scott. The original file contains information on 151,071 transactions by directors and other large shareholders, such as pension funds. In order to make our analysis comparable with the pre-existing work in the area (Lakonishok and Lee 2001; Fidrmuc, Goergen, and Renneboog 2006), we applied several filters to the source data file. For example, we based our analysis on open market sale and purchase transactions by (executive and non-executive) directors of non-financial firms only. Moreover, we deleted small transactions (less than 100 shares), transactions for firms with negative book values

and firms for which price data were not available from Datastream. We then determined the net purchase transactions and net sale transactions for each transaction date by aggregating multiple purchases and/or sales by directors for each transaction day for a given firm. For example, if on 1 January, the directors of a particular firm engage in five purchase transactions totalling 1000 shares and five sale transactions totalling 300 shares, then the net transaction for 1 January involves the purchase of $1000 - 300 = 700$ shares. Our final sample consisted of 16,043 net purchase and 5386 net sale transactions.

We determined the excess average ranks at the event period date (the day on which the directors' purchase/sale transactions occurred) and the variance of those ranks based on the $N = 16,043$ firms involved in directors' net purchase transactions across the $T = 227$ days constituting the sum of our event window ($M = 27$ days) and our estimation window ($T - M = 200$ days). Substituting the resulting calculations into Equation (7) returns a Corrado (1989) test statistic of $z_c = 6.8828$. The 'equivalent' figure for the modified Corrado test statistic, as defined by Equation (10), is much higher at $z_1 = 21.0683$. Finally, the conventionally applied Patell (1976, 257) t -test returned a test statistic of $z_p = 27.7197$. The corresponding figures for the directors' sales transactions are $z_c = -3.3722$ for the Corrado (1989) test, $z_1 = -7.4019$ for the modified Corrado test and $z_p = -7.6292$ for the Patell (1976, 257) t -test. All test statistics are asymptotically distributed as standard normal variates. Moreover, the Berry (1941) and Esseen (1945) theorem may be used to make assessments about the reliability of the normal approximation. For example, substituting $T = 227$ and $N = 16,043$ into Equation (11) for directors' purchase transactions shows the absolute value of the error associated with the approximation of the distribution function for the modified Corrado test statistic, $F_N(z_1)$, by the standard normal distribution function, $\Phi(z_1)$, will be $|F_N(z_1) - \Phi(z_1)| \leq 0.7056 \times 0.0410 = 0.0289$. This means that the difference between the actual distribution function for the modified Corrado test statistic and its normal approximation will be 2.9% at worst.

The important point here, however, is that the Corrado (1989) test provides significantly weaker results than either the modified Corrado test or the Patell (1976) t -test. Moreover, while the modified Corrado test returns slightly less compelling results when compared with the Patell (1976) t -test, it makes no assumptions about the nature of the underlying return distribution – in particular, the modified Corrado test does not assume that the return process is normally distributed as is the case with the Patell (1976) t -test. Indeed, as we have shown in Section 2, the modified Corrado test will always provide a satisfactory level of efficiency relative to the t -test, but there is no guarantee that the t -test will always provide a satisfactory level of efficiency when compared with the modified Corrado test. This, in turn, will mean it is always 'safer' to employ the modified Corrado test over the Patell (1976) t -test.

Now, suppose one has computed the z_2 statistics defined by Equation (16) for all $i = 1, 2, 3, \dots, N$ firms comprising the sample on which the analysis is based. It then follows that the sum of these statistics will possess a mean of zero and a standard deviation of \sqrt{N} . One can then use the central limit theorem to show that for a fixed event window, M , the distribution function, $F_N(z_3)$, of the standardised random variable

$$\begin{aligned} z_3 &= \frac{\sqrt{\frac{3}{M(T+1)(T-M)}} \sum_{i=1}^N [2K(CAR_{itM}) - M(T+1)]}{\sqrt{N}} \\ &= \sqrt{\frac{3}{MN(T+1)(T-M)}} \sum_{i=1}^N [2K(CAR_{itM}) - M(T+1)] \end{aligned} \quad (19)$$

can be approximated by the standard normal distribution function, $\Phi(z_3) = 1/\sqrt{2\pi} \int_{-\infty}^{z_3} \exp(-x^2/2)dx$, as $N \rightarrow \infty$ (Fisz 1963, 197).¹¹ Again, one can apply the theorem of Berry (1941) and Esseen (1945) to show that the rate of convergence to the standard normal distribution is of the order of $1/\sqrt{N}$. However, since previous analysis shows that for individual firms, the standardised random variable defined by Equation (16) converges quickly towards the normal distribution for quite modest values of M , it necessarily follows that the sum (or average) of the standardised random variables across many firms will converge even more quickly towards the normal distribution.

We demonstrate the implementation of the above result by again considering the abnormal returns arising around the time of UK directors' purchase and/or sale of shares in their own firms. We begin by determining the excess ranks for the abnormal returns for each of the $N = 16,043$ firms involved in directors' net purchase transactions across the $T = 227$ days which constitute the sum of our estimation and event period windows. We then determined the sum of the excess ranks for the abnormal returns from the first ($M = 1$) to the fifth ($M = 5$) days after the directors' purchase transactions occurred. Substituting the resulting calculations into Equation (19) returns the modified Corrado (1989) test statistics for directors' purchase transactions summarised in Table 2 (Net share purchases). Thus, for an $M = 2$ day event window, the modified Corrado (1989) test statistic is $z_3 = 23.7080$. For an $M = 3$ day event window, the modified Corrado (1989) test statistic is $z_3 = 22.4510$ and so on. The 'equivalent' modified Corrado test statistic for the $N = 5386$ directors' sales transactions is summarised in Table 2 (Net share sales). Table 2 also summarises the Patell (1976, 257) t -test statistics for the directors' purchase and sale transactions. This shows that for an $M = 4$ day window, the Patell (1976) statistic for directors' purchase transactions is $z_p = 30.5517$. Likewise, for an $M = 5$ day window, the Patell (1976) statistic for directors' sales transactions is $z_p = -12.4500$. Note again that while the modified Corrado test returns slightly less compelling results when compared with the Patell (1976) t -test, it does not assume that abnormal returns are normally distributed. Finally, Campbell and Wasley (1993, 85) determined the distributional properties of the Corrado (1989, 388) test statistic for multi-period

Table 2. Market's reaction to directors' trades in shares of their own firms.

	$M = 1$	$M = 2$	$M = 3$	$M = 4$	$M = 5$
Net share purchases					
Corrado	6.8828	9.8062	9.6579	9.8918	9.7949
Modified Corrado	21.0683	23.0780	22.4510	22.8258	22.4038
Patell	27.7197	29.8720	29.9381	30.5517	29.9718
Net share sales					
Corrado	-3.3722	-6.0231	-6.5379	-7.6127	-7.6540
Modified Corrado	-7.4019	-9.8928	-10.8730	-12.6794	-12.9040
Patell	-7.6292	-9.7394	-10.3603	-11.9730	-12.4500

Notes: 'Net share purchase' is based on the abnormal returns for each of the $N = 16,043$ firms involved in directors' net purchase transactions across the $T = 227$ days which constitute the sum of our estimation and event period windows. We then determined the sum of the excess ranks for the abnormal returns from the first ($M = 1$) to the fifth ($M = 5$) days after the directors' purchase transactions occurred. Substituting the resulting calculations into Equation (19) returns the modified Corrado test statistics for directors' purchase transactions summarised in Panel A. Likewise, 'Net share sale' is based on the abnormal returns for each of the $N = 5386$ firms involved in directors' net sales transactions across the $T = 227$ days which constitute the sum of our estimation and event period windows. The above table also summarises the Patell (1976, 257) ' t '-test statistics for the directors' purchase and sale transactions. Campbell and Wasley (1993, 85) determine the distributional properties of the Corrado (1989, 388) test statistic for multi-period event windows. This statistic is also summarised in the above table.

event windows. Hence, Table 2 also summarises the multi-period Corrado (1989) test statistic for both directors' purchase and sale transactions. The important point here is that the Corrado (1989) test provides much less compelling results when compared with both the modified Corrado and Patell (1976) t -tests. Of course, the Patell (1976) test assumes that abnormal returns are normally distributed. In contrast, the modified Corrado test is a distribution-free test which returns results that are significantly better than those of the original Corrado test and almost as compelling as those of the Patell (1976) test.

5. Conclusions

We employ a consistent estimator for the variance of the ranks of abnormal security returns and then use it to obtain an exact closed-form expression for the Corrado (1989) test statistic. This simplifies the computational procedures behind the Corrado (1989) test considerably – to the point where they can be implemented using only a hand-held calculator. Moreover, we also extend the original Corrado (1989) analysis by determining the distributional properties of the sum of the ranks of the individual abnormal returns over a given event window. This shows that describing the distributional properties of the sum of the ranks in terms of a normal distribution is highly problematic for small sample sizes and small event windows. In such circumstances, however, we also demonstrate that a second-order Edgeworth expansion provides a good approximation to the actual probability distribution of the Corrado (1989) test statistic. Our analysis also shows that the original Corrado (1989) test holds considerably less power when compared with the closed-form (i.e. modified) version of the Corrado test developed here. Moreover, since the modified Corrado test makes very few distributional assumptions and has relatively high efficiency when compared with the conventional parametric tests, it represents an attractive alternative to the existing tests for assessing the significance of abnormal security-price performance. Finally, we demonstrate how the theorem of Berry (1941) and Esseen (1945) may be used to make assessments about the reliability of approximating the distribution function of a given test statistic by the standard normal distribution function.

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Notes

1. See MacKinlay (1997) for a more detailed exposition of the event study methodology than can be provided here.
2. The important point here is that the elements of the matrix, K , are *not* based on a global ranking across all NT abnormal returns arising on the N firms across the T available periods. Rather, each row ranks the abnormal returns from 1 to T for a given firm. Since there are N firms, the total of the ranks will be $NT(T+1)/2$. The average of these ranks is $1/NT \cdot NT(T+1)/2 = (T+1)/2$ – as captured by Equation (2) given in the text. Against this, using a global ranking across all NT abnormal returns shows that the total of the ranks will be $NT(NT+1)/2$. The average of the ranks based on this global ranking procedure will then be $1/NT \cdot NT(NT+1)/2 = (NT+1)/2$. We emphasise again that our analysis is not based on this global ranking approach.
3. Since by assumption, the first T integers (ranks) are randomly allocated to each row of the matrix, summing the columns is equivalent to a random drawing of N of these T integers, but with replacement after each drawing is made; that is, after an integer is drawn (for a particular element of a given column), it is replaced before the next random drawing occurs (for the immediately ensuing element of the given column). Freeman (1963, 187–91) shows that the act of replacement means $\text{Cov}[K(AR_{it}), K(AR_{jt})] = 0$ for all $i \neq j$.

4. If, however, there are different sample sizes for each of the $i = 1, 2, 3, \dots, N$ firms, then the above result takes the following 'equivalent' form:

$$z_1 = \sqrt{\frac{3}{N}} \sum_{i=1}^N \frac{[2K(AR_{it}) - (T_i + 1)]}{\sqrt{T_i^2 - 1}},$$

where T_i is the number of abnormal returns computed for the i th firm (Fisz 1963, 203).

5. We are here summing the ranks beyond the announcement period for a given firm; that is, we are summing the ranks across a given row. Recall, however, that a given rank can only appear once in each row. Hence, summing the rows is equivalent to a random drawing of M of the T integers (ranks), but without replacement; that is, after an integer is drawn, it is not replaced before the next random drawing occurs. Freeman (1963, 187–91) shows that non-replacement induces the negative serial correlation in the sum of ranks across the given row reported here.
6. See Freeman (1963, 156–61) for a more detailed exposition of the computational procedures that lie behind the approximation of a particular probability distribution using an Edgeworth expansion based on the standard normal distribution.
7. Suppose n_1 and n_2 are the sample sizes necessary for two tests, Q_1 and Q_2 to have equivalent power under the same level of significance, α . If the level of significance, α , and the probability of a type II error, β , remain fixed, then the limit n_1/n_2 , as n_1 approaches infinity, is called the (Pitman) ARE of the first test relative to that of the second test, if that test is independent of α and β (Nikitin 1995, 15). There are, however, alternative measures of efficiency. Bahadur (1967), for example, computed the ARE for two tests, Q_1 and Q_2 , by fixing all parameters, except for α which is allowed to approach a limiting value of zero. In contrast, Hodges and Lehman (1956) fixed all parameters, except for β which is allowed to approach a limiting value of unity. The Pitman efficiency is generally easier to calculate, and it is this which probably explains why it is the most commonly used measure of ARE in the literature (Nikitin 1995, 15). Moreover, Wieand (1976) shows that under certain mild regularity conditions, the Pitman and Bahadur measures of ARE will coincide. We should emphasise that these regularity conditions are satisfied by our analysis (Nikitin 1995, 18).
8. Both this result and those which follow assume that there is 'slippage' in the location parameter on which the two distributions are based (Hodges and Lehmann 1956, 325–26).
9. There were no significant differences between the results obtained using ranks based on the ordinary least squares procedure and ranks based on the Dimson (1979) technique.
10. The parameters of the market model, as summarised by Equation (1), were estimated using the daily continuously compounded returns on the ordinary shares of the affected firms, and the FTSE All-Share Index was used as the market index. The parameters of the market model were estimated using both ordinary least squares and the Dimson (1979) technique. There were no significant differences between the results obtained using ranks based on the ordinary least squares procedure and ranks based on the Dimson (1979) technique.
11. If, however, there are different sample sizes for each of the firms, then the above result takes the following 'equivalent' form:

$$z_3 = \sqrt{\frac{3}{MN}} \sum_{i=1}^N \frac{[2K(CAR_{itM}) - M(T_i + 1)]}{\sqrt{(T_i + 1)(T_i - M)}},$$

where T_i is the number of abnormal returns computed for the i th firm (Fisz 1963, 203).

References

- Ashton, D. and M. Tippett. 2006. Mean reversion and the distribution of United Kingdom Stock Index Returns. *Journal of Business Finance and Accounting* 33, no. 9&10: 1586–609.
- Bahadur, R. 1967. Rates of convergence of estimates and test statistics. *Annals of Mathematical Statistics* 38, no. 2: 303–24.
- Berry, A. 1941. The accuracy of the Gaussian approximation to the sum of independent variates. *Transactions of the American Mathematical Society* 49, no. 1: 122–36.
- Campbell, C. and C. Wasley. 1993. Measuring security price performance using daily NASDAQ returns. *Journal of Financial Economics* 33, no. 1: 73–92.
- Conover, W. 1971. *Practical nonparametric statistics*. New York: Wiley.
- Corrado, C. 1989. A nonparametric test for abnormal security price performance in event studies. *Journal of Financial Economics* 23, no. 2: 385–95.
- Dimson, E. 1979. Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics* 7, no. 2: 197–226.

- Esseen, C. 1945. Fourier analysis of distribution functions: A mathematical study of the Laplace–Gaussian law. *Acta Mathematica* 77, no. 1: 1–125.
- Fama, E., L. Fisher, M. Jensen, and R. Roll. 1969. The adjustment of stock prices to new information. *International Economic Review* 10, no. 1: 1–21.
- Fidrmuc, J., M. Goergen, and L. Renneboog. 2006. Insider trading, news releases, and ownership concentration. *Journal of Finance* 61, no. 6: 2931–73.
- Fisz, M. 1963. *Probability theory and mathematical statistics*. New York: Wiley.
- Fix, E. and J. Hodges. 1955. Significance probabilities of the Wilcoxon test. *Annals of Mathematical Statistics* 26, no. 2: 301–12.
- Freeman, H. 1963. *Introduction to statistical inference*. Reading, MA: Addison-Wesley Publishing Company.
- Freund, J. 1971. *Mathematical statistics*. Englewood Cliffs, NJ: Prentice-Hall, Inc.
- Harris, R. and C. Küçüközmen. 2001. The empirical distribution of U.K. and U. S. stock returns. *Journal of Business Finance and Accounting* 28, no. 5&6: 715–40.
- Hodges J. and E. Lehmann. 1956. The efficiency of some nonparametric competitors of the ‘*t*’ test’. *Annals of Mathematical Statistics* 27, no. 2: 324–35.
- Lakonishok, J. and I. Lee. 2001. ‘Are insiders’ trades informative?’ *Review of Financial Studies* 14, no. 1: 79–111.
- MacKinlay, A. 1997. Event studies in economics and finance. *Journal of Economic Literature* 35, no. 1: 13–39.
- Nikitin, Y. 1995. *Asymptotic efficiency of nonparametric tests*. Cambridge: Cambridge University Press.
- Patell, J. 1976. Corporate forecasts of earnings per share and stock price behavior: Empirical test. *Journal of Accounting Research* 14, no. 2: 246–76.
- Shevtsova, I. 2007. Sharpening of the upper bound of the absolute constant in the Berry–Esseen inequality. *Theory of Probability and Its Applications* 51, no. 3: 549–53.
- Theodossiou, P. 1998. Financial data and the skewed generalized *T* distribution. *Management Science* 44, no. 12(1): 1650–61.
- Wieand, H. 1976. On a condition under which the Pitman and Bahadur approaches to efficiency coincide. *Annals of Statistics* 4, no. 5: 1003–11.