Coupled pulsation and translation of a gas bubble and rigid particle

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Coupled pulsation and translation of a gas bubble and rigid particle

T. A. Hay, M. F. Hamilton, Yu. A. Ilinskii and E. A. Zabolotskaya

Applied Research Laboratories, The University of Texas at Austin, Austin, Texas 78713-8029, USA

Abstract. A nonlinear analytic model describing the interaction of a spherical gas bubble and spherical rigid particle is presented. Both the bubble and particle are free to translate. The model is accurate to fifth order in terms of a nondimensional expansion parameter R/d, where R is a characteristic radius and d is the distance separating the bubble and particle. Numerical simulation results are presented to demonstrate the effects of key particle parameters and an external acoustic source.

Keywords: bubble dynamics, particles
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INTRODUCTION

The theoretical framework for the model of bubble-bubble interaction developed by Ilinskii *et al*. [1] is applied to model the interaction between a spherical bubble and a spherical rigid particle. The investigation was motivated largely by the desire to account for the interaction of kidney stone fragments with cavitation clusters produced during shock wave lithotripsy. Extension of the model to clusters of bubbles and particles will be presented elsewhere. Here, we focus on the dynamics of a single bubble interacting with a single rigid particle.

MODEL EQUATIONS

The bubble and particle are positioned along the x axis in an incompressible liquid of density ρ as shown in Fig. 1. Variables with subscript 1 correspond to the gas bubble,

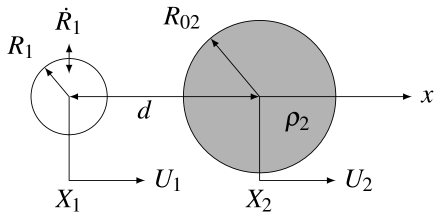

FIGURE 1. Notation and geometry for bubble and particle with collinear translational motion.

and variables with subscript 2 correspond to the particle. Their positions are defined by $X_{1,2}$ and the separation distance $d = |X_2 - X_1|$, while their instantaneous and equilibrium

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radii are given by R_1 and $R_{01,02}$, respectively. Their motions are described by the radial velocity of the bubble, $\dot{R}_1$, and translational velocities $U_{1,2} = \dot{X}_{1,2}$, where dots over quantities indicate time derivatives. The equations of motion are obtained as expansions in terms of R/d, where R is a characteristic radius, and they were derived following the formalism of Ilinskii *et al.* [1] The present model requires accuracy to order R^5/d^5, whereas the corresponding model for bubble-bubble interaction need only be accurate to order R^2/d^2. The radial equation of motion for the bubble is

$$\begin{aligned} R_1\ddot{R}_1 + \frac{3}{2}\dot{R}_1^2 = & \frac{P_1 - P_0}{\rho} + \frac{1}{4}U_1^2 + \frac{R_{02}^3}{2d^2}\dot{U}_2 - \frac{R_{02}^3}{2d^3}U_2\,(U_1 + 2U_2) \\ & - \frac{R_{02}^3}{4d^4}\left(2R_1^2\ddot{R}_1 + 4R_1\dot{R}_1^2\right) - \frac{R_{02}^3}{2d^5}\left[R_1^3\dot{U}_1 + 4R_1^2\dot{R}_1\,(U_1 - U_2)\right], \end{aligned} \tag{1}$$

where

$$P_1 = \left(P_0 + \frac{2\sigma}{R_{01}}\right)\left(\frac{R_{01}}{R_1}\right)^{3\gamma} - \frac{2\sigma}{R_1} \tag{2}$$

is the liquid pressure at the bubble wall, γ is the ratio of specific heats and σ is surface tension. The equations describing translation are $\dot{M}_1 = -F$ and $\dot{M}_2 = F$, where

$$M_1 = \frac{1}{2}\rho V_1 U_1 - \frac{3}{2}\rho V_{02}\left(\frac{R_1^3}{d^3}U_2 - \frac{R_1^5}{d^5}\dot{R}_1\right), \tag{3}$$

$$M_2 = \left(\frac{1}{2}\rho + \rho_2\right)V_{02}U_2 - \frac{3}{2}\rho V_{02}\left(\frac{R_1^2}{d^2}\dot{R}_1 + \frac{R_1^3}{d^3}U_1\right), \tag{4}$$

$$F = 3\rho V_{02}\left(\frac{R_1^2}{d^3}\dot{R}_1 U_2 + \frac{R_1^3}{d^4}U_1 U_2 - \frac{R_1^4}{d^5}\dot{R}_1^2\right), \tag{5}$$

are the generalized momenta (M_1, M_2), and translational force (F) that acts equally and oppositely on the bubble and particle. Quantities $V_1 = \frac{4}{3}\pi R_1^3$ and $V_{02} = \frac{4}{3}\pi R_{02}^3$ are the volumes of the bubble and particle. Assuming small linear bubble pulsation, $R_1(t) = R_{01} + \xi_0 \sin\omega t$ where $\xi_0 \ll R_{01}$, and averaging Eq. (5) over one acoustic cycle yields the time averaged force

$$\langle F \rangle = 3\rho V_{02}\left(\frac{\rho - \rho_{02}}{\rho + 2\rho_{02}}\right)\frac{R_{01}^4}{d^5}\omega^2\xi_0^2 + O(\xi_0^3). \tag{6}$$

Equation (6) is the expression derived by Coakley and Nyborg [2] by an entirely different approach. It indicates that for a particle having density greater than that of the liquid, $\rho_{02} > \rho$, the time-averaged interaction force is attractive while for a less dense particle, $\rho_{02} < \rho$, the bubble and particle repel. The result also demonstrates why it is essential to retain terms through order R^5/d^5.

FREE RESPONSE

The free response of the system is investigated by setting the initial bubble radius to a non-equilibrium value $R_1(0) > R_{01} = 100\,\mu$m and releasing it from rest. Parameters γ

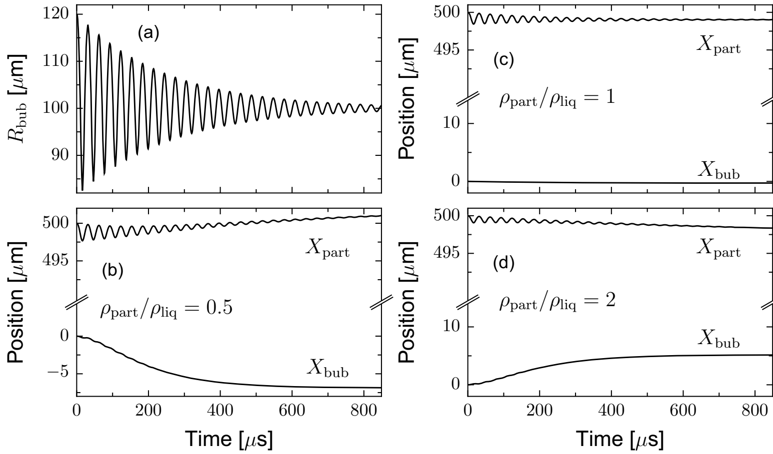

FIGURE 2. Transient response of bubble and rigid particle for different particle densities.

and σ were chosen to be 1.4 and 0.072 N/m, respectively. The particle is initially at rest in its equilibrium state and has radius $R_{02} = 2R_{01} = 200\,\mu$m. The initial positions of the bubble and particle are $X_1(0) = 0$ and $X_2(0) = 2.5R_{02} = 500\,\mu$m when the bubble is released from its initial radius $R_1(0) = 120\,\mu$m. Figure 2 shows results for three different particle densities: $\rho_2 = 500$, 1000, and $2000\,\mathrm{kg/m^3}$ ($\rho_2/\rho = 0.5\,, 1$, and 2). To aid the reader, the indices $i = 1$ and $i = 2$ on parameters shown in the figures are replaced with subscripts "bub" and "part", respectively, and the density of the liquid is denoted $\rho_{\rm liq}$.

The radial response of the bubble, which is independent of the density ratio $\rho_{\rm part}/\rho_{\rm liq}$, is shown in Fig. 2(a). Parts (b)–(d) of Fig. 2 display the positions of the bubble and particle for the three density ratios (note the split vertical axes). Consistent with the approximate analytical result for the time-averaged interaction force given by Eq. (6), the bubble and particle repel for $\rho_{\rm part}/\rho_{\rm liq} = 0.5$, they attract for $\rho_{\rm part}/\rho_{\rm liq} = 2$, and there is virtually no translation for $\rho_{\rm part}/\rho_{\rm liq} = 1$.

FORCED RESPONSE

The system may also be driven by an acoustic source as was included in the model of Ilinskii *et al*. [1] The acoustic pressure perturbation was defined to be a sinusoidal plane wave with amplitude 1 kPa. Figure 3 shows the positions and separation distances between the bubble and particle for the three density ratios ($\rho_{\rm part}/\rho_{\rm liq} = 0.5$, 1, 2) for a source located to the left [Fig. 3(a-c)] or right [Fig. 3(d-f)] of the bubble and particle.

At this small separation distance the direction of bubble and particle translation [Fig. 3(a,b,d,e)] is determined by the particle density. However, the source radiation pressure (primary Bjerknes force) on the bubble has a non-zero time average. Therefore the direction of bubble motion [Fig. 3(a,d)] is also influenced by the propagation direction of the acoustic excitation. This effect is most apparent for a neutrally buoyant particle, where the bubble-particle interaction force is negligible. In the case of a

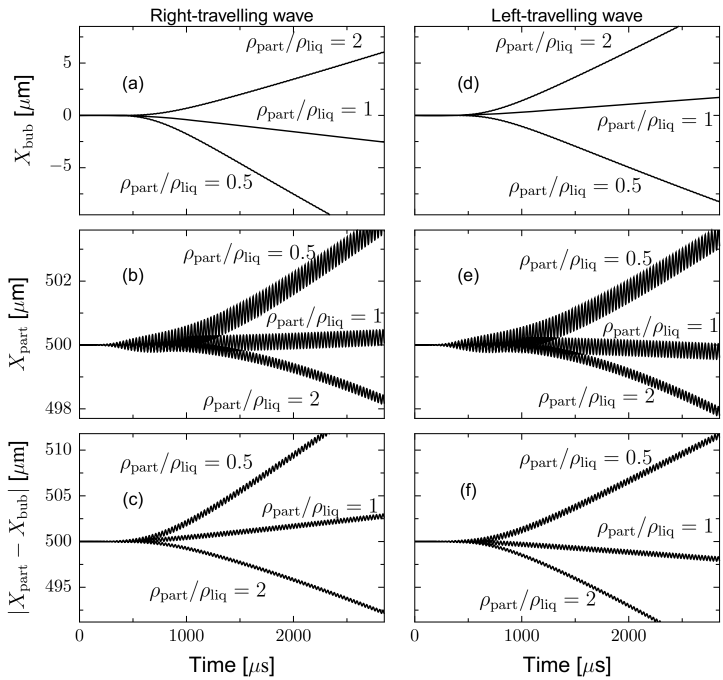

FIGURE 3. Bubble and particle translation for right- and left-traveling plane waves.

right-traveling wave the radiation pressure tends to push the bubble towards the particle, causing the separation distance to decrease [Fig. 3(c)]. In the case of a left-traveling wave, the radiation pressure tends to push the bubble away from the particle. If the initial separation distance were doubled the source radiation pressure would dominate the bubble-particle interaction force.

SUMMARY

Equations describing the interaction of a gas bubble and a rigid particle were presented. The equations are accurate to order R^5/d^5, the minimum required accuracy to account for the interaction force. The system dynamics show dependence on the density and size of the particle as well as acoustic source parameters.

REFERENCES

1. Yu. A. Ilinskii, M. F. Hamilton, and E. A. Zabolotskaya, "Bubble interaction dynamics in Lagrangian and Hamiltonian mechanics," *J. Acoust. Soc. Am.* **121**, 786 (2007).
2. W. T. Coakley, and W. L. Nyborg, "Cavitation Dynamics of Gas Bubbles," in *Applications in Ultrasound: Its Applications in Medicine and Biology*, edited by F. J. Fry, Elsevier, New York, 1978.