Fermion mass and the pressure of dense matter

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Abstract. We consider a simple toy model to study the effects of finite fermion masses on the pressure of cold and dense matter, with possible applications in the physics of condensates in the core of neutron stars and color superconductivity.

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The role of finite quark masses in QCD thermodynamics has received increasing attention in the last few years. In the case of cold and dense QCD, it was generally believed that effects of nonzero quark masses on the equation of state were of the order of 5%, thereby yielding only minor corrections to the mass-radius diagram of compact stars [1]. In fact, mass, as well as color superconductivity gap, contributions to the pressure are supressed by two powers of the chemical potential as compared to zeromass interacting quark gas terms. Therefore, assuming a critical chemical potential for the chiral transition of the order of a few hundred MeV, naively those terms should not matter. However, recent results for the thermodynamic potential to two loops have shown that corrections are sizable, and may dramatically affect the structure of compact stars [2]. Moreover, the situation in which mass (as well as gap) effects are significant corresponds to the critical region for chiral symmetry breakdown in the phase diagram of QCD. Hence, not only the value of the critical chemical potential will be affected, but also the nature of the chiral transition. In particular, if the latter is strongly first-order there might be a new class of compact stars, smaller and denser, with a deconfined quark matter core [3]. Of course, contributions due to color superconductivity [4] as well as chiral condensation [5] will also affect this picture.

In what follows, we study a simple toy model – cold and dense Yukawa theory – to investigate the influence of fermion masses on the pressure. Here, we present a two-loop calculation of the pressure with massive fermions in the modified minimal subtraction (\overline{MS}) renormalization scheme [6], and briefly comment on possible implications to the physics of condensates in the core of neutron stars and effective models for color superconductivity. Higher-order corrections and a thorough analysis of renormalization group effects will be presented elsewhere [7].

We consider a gas of massive fermions whose interaction is mediated by a real scalar field, ϕ , with an interaction Lagrangian of the Yukawa form, $\mathcal{L}_I = g \overline{\psi} \psi \phi$, where g is the coupling constant. In the zero-temperature limit, the perturbative pressure results in

a power series of $\alpha_Y \equiv g^2/4\pi$.¹ Up to $O(\alpha_Y)$, the first non-trivial contributions to the pressure are given by the free massive gas term, P_0 , and the "exchange diagram", P_1 . Using standard methods of field theory at finite temperature and density [8], one can derive the free gas pressure for fermions of mass *m*, obtaining in the zero-temperature limit the following form:

$$\lim_{T \to 0} P_0 = \frac{1}{12\pi^2} \left[\mu p_f \left(\mu^2 - \frac{5}{2}m^2 \right) + \frac{3}{2}m^4 \ln \left(\frac{\mu + p_f}{m} \right) \right],\tag{1}$$

where μ is the chemical potential and $p_f = \sqrt{\mu^2 - m^2}$ denotes the Fermi momentum. The $O(\alpha_Y)$ renormalized correction reads [6]:

$$\lim_{T \to 0} P_1 = -\frac{\alpha_Y}{4\pi^3} \left[\frac{3}{4} u^2 - p_f^4 + m^2 \left(3 + 2\ln\frac{\Lambda^2}{m^2} \right) u \right],$$
(2)

where $u = \mu p_f - m^2 \ln[(\mu + p_f)/m]$ and Λ is the renormalization scale in the \overline{MS} scheme.

Fig. 1 illustrates the effect of modifying the mass on the total pressure to $O(\alpha_Y)$, $P = P_0 + P_1$. The choice of range for μ , and accordingly for the masses, are inspired by the scales found in the case of QCD [2]. In the same vein, the coupling is fixed to $\alpha_Y = 0.3$. It is clear from the figure that mass corrections bring significant changes to the pressure, even in the absence of renormalization group (RG) running for the coupling and the mass. The figure also shows the dependence on the renormalization scale Λ . The values chosen are motivated by the ones which appear in QCD, as before. Although the effects of varying Λ appear to be relatively small, it would be premature to conclude that this feature will remain after implementing the RG flow. In fact, the results presented in Fig. 2 most probably underestimate the scale dependence of the full correction, since not only the coupling but also the mass will run with Λ . In the Yukawa theory, in contrast to QCD, the effect will become larger as we increase the chemical potential. For fixed coupling, larger values of Λ yield larger modifications in the pressure. However, after the inclusion of RG running, this behavior can be mantained, as should be the case here, or become the opposite, as is the case in QCD, depending on the sign of the beta function. Since the Λ -dependence comes from the term $\sim m^2 \alpha_Y \ln(\Lambda/m)$ in (2), there will be a competition between the behavior of the renormalization scale Λ and that of m and α_Y as functions of μ .

Even at two loop order mass effects bring into play logarithmic corrections originated in the \overline{MS} subtraction scheme. As usual, they bring about a non-physical dependence on the renormalization scale Λ , since one has to cut the perturbative series at some order. Higher-order computations in this framework are in progress [7], and will give a better handle on the choice of this scale, which in our case should be a function of μ and m. On the other hand, one can also choose the scale in a phenomenological way in a given model, imposing physical constraints to the equation of state, as was done in Ref. [3] to model the non-ideality of QCD at finite density with massless quarks.

¹ Since we are concerned only with the zero-temperature limit, there are no odd powers of g coming from resummed contributions of the zero Matsubara mode for bosons in the perturbative series.



FIGURE 1. Pressure normalized by the free fermion gas pressure as a function of the fermion chemical potential. Left: $\Lambda = 2\mu$ and different values of the fermion mass. Right: m = 100 MeV and different values of the renormalization scale Λ .

The points discussed above might be relevant in the study of effective models for the cold and dense matter found in the interior of compact stars, especially because the effects seem to be significant near the critical region. In the context of the NJL model, e.g., it was shown that a self-consistent treatment of quark masses strongly affects the competition between different phases [5]. And the mechanism of pairing in color superconductivity will certainly be influenced [9] by the running of nonzero quark masses. The investigation of these issues, as well as the effect of nonzero fermion masses in the formation of other condensates in neutron star matter, is under way [7].

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