# Charm meson resonances in $D$ semileptonic decays 

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#### Abstract

Motivated by recent experimental results we reconsider semileptonic $D \rightarrow P \ell v_{\ell}$ and $D \rightarrow V \ell v_{\ell}$ decays within a model which combines heavy quark symmetry and properties of the chiral Lagrangian. Using limits of soft collinear effective theory and heavy quark effective theory we parametrize the semileptonic form factors. We include excited charm meson states in our Lagrangians and determine their impact on the charm meson semileptonic form factors. Then we calculate branching ratios for all $D \rightarrow P \ell v_{\ell}$ and $D \rightarrow V l v_{l}$ decays.


The knowledge of the form factors which describe the weak heavy $\rightarrow$ light semileptonic transitions is very important for the accurate determination of the CKM parameters from the experimentally measured exclusive decay rates. Usually, the attention has been devoted to $B$ decays and the determination of the phase of the $V_{u b}$ CKM matrix element. At the same time in the charm sector, the most accurate determination of the size of $V_{c s}$ and $V_{c d}$ matrix elements is not from a direct measurement, mainly due to theoretical uncertainties in the calculations of the relevant form factors' shapes.

Recently, there have been new interesting results on $D$-meson semileptonic decays. The CLEO and FOCUS collaborations have studied semileptonic decays $D^{0} \rightarrow \pi^{-} \ell^{+} v$ and $D^{0} \rightarrow K^{-} \ell^{+} v$ [1, 2]. Their data provide new information on the $D^{0} \rightarrow \pi^{-} \ell^{+} v$ and $D^{0} \rightarrow K^{-} \ell^{+} v$ form factors. Usually in $D$ semileptonic decays a simple pole parametrization was used in the past. The results of Refs. [1, 2] for the single pole parameters required by the fit of their data, however, suggest pole masses, which are inconsistent with the physical masses of the lowest lying charm meson resonances. In their anlyses they also utilized a modified pole fit as suggested in [3] and their results indeed suggest the existence of contributions beyond the lowest lying charm meson resonances [1].

In addition to these results new experimental studies of charm meson resonances have provided a lot of new information on the charm sector [4, 5, 6, 7] which we can now apply to $D$ and $D_{s}$ semileptonic decays.

The purpose of our studies [8, 9] is to accommodate contributions of the newly discovered and theoretically predicted charm mesons in form factors which are parametrized using constraints coming from heavy quark effective theory (HQET) limit for the region of $q_{\max }^{2}$ and in the $q^{2} \simeq 0$ region using results of soft collinear effective theory (SCET). We restrain our discussion to the leading chiral and $1 / m_{H}$ terms in the expansion.

The standard decomposition of the current matrix elements relevant to semileptonic decays between a heavy pseudoscalar meson state $\left|H\left(p_{H}\right)\right\rangle$ with momentum $p_{H}^{v}$ and a light pseudoscalar meson state $\left|P\left(p_{P}\right)\right\rangle$ with momentum $p_{P}^{\mu}$ is in terms of two scalar
functions of the exchanged momentum squared $q^{2}=\left(p_{H}-p_{P}\right)^{2}-$ the form factors $F_{+}\left(q^{2}\right)$ and $F_{0}\left(q^{2}\right)$. Here $F_{+}$denotes the vector form factor and it is dominated by vector meson resonances, while $F_{0}$ denotes the scalar form factor and is expected to be dominated by scalar meson resonance exchange [10, 11]. In order that the matrix elements are finite at $q^{2}=0$, the form factors must also satisfy the relation $F_{+}(0)=$ $F_{0}(0)$.

The transition of $\left|H\left(p_{H}\right)\right\rangle$ to light vector meson $\left|V\left(p_{V}, \varepsilon_{V}\right)\right\rangle$ with momentum $p_{V}^{v}$ and polarization vector $\varepsilon_{V}^{V}$ is similarly parameterized in terms of four form factors $V, A_{0}, A_{1}$ and $A_{2}$, again functions of the exchanged momentum squared $q^{2}=\left(p_{H}-p_{V}\right)^{2}$. Here $V$ denotes the vector form factor and is expected to be dominated by vector meson resonance exchange, the axial $A_{1}$ and $A_{2}$ form factors are expected to be dominated by axial resonances, while $A_{0}$ denotes the pseudoscalar form factor and is expected to be dominated by pseudoscalar meson resonance exchange [11]. As in previous case in order that the matrix elements are finite at $q^{2}=0$, the form factors must also satisfy the well known relation $A_{0}(0)+A_{1}(0)\left(m_{H}+m_{V}\right) / 2 m_{V}-A_{2}(0)\left(m_{H}-m_{V}\right) / 2 m_{V}=0$.

Next we follow the analysis of Ref. [3], where the $F_{+}$form factor in $H \rightarrow P$ transitions is given as a sum of two pole contributions, while the $F_{0}$ form factor is written as a single pole. This parametrization includes all known properties of form factors at large $m_{H}$. Using a relation which connects the form factors within large energy release approach [12] the authors in Ref. [3] propose the following form factor parametrization

$$
\begin{equation*}
F_{+}\left(q^{2}\right)=\frac{c_{H}(1-a)}{(1-x)(1-a x)}, \quad F_{0}\left(q^{2}\right)=\frac{c_{H}(1-a)}{1-b x}, \tag{1}
\end{equation*}
$$

where $x=q^{2} / m_{H^{*}}^{2}$.
Utilizing the same approach we propose a general parametrization of the heavy to light vector form factors, which also takes into account all the known scaling and resonance properties of the form factors. As already mentioned, there exist the well known HQET scaling laws in the limit of zero recoil [13] while in the SCET limit $q^{2} \rightarrow 0$ one obtains that all four $H \rightarrow V$ form factors can be related to only two universal SCET scaling functions [12].

The starting point is the vector form factor $V$, which is dominated by the pole at $t=m_{H^{*}}^{2}$ when considering the part of the phase space that is close to the zero recoil. For the heavy $\rightarrow$ light transitions this situation is expected to be realized near the zero recoil where also the HQET scaling applies. On the other hand, in the region of large recoils, SCET dictates the scaling described in [12]. In the full analogy with the discussion made in Refs. [3, 14], the vector form factor consequently receives contributions from two poles and can be written as

$$
\begin{equation*}
V\left(q^{2}\right)=\frac{c_{H}^{\prime}(1-a)}{(1-x)(1-a x)}, \tag{2}
\end{equation*}
$$

where $x=q^{2} / m_{H^{*}}^{2}$ ensures, that the form factor is dominated by the physical $H^{*}$ pole, while $a$ measures the contribution of higher states which are parametrized by another effective pole at $m_{\text {eff }}^{2}=m_{H^{*}}^{2} / a$.

An interesting and useful feature one gets from the SCET is the relation between $V$ and $A_{1}[12,15,16,17]$ at $q^{2} \approx 0$. When combined with our result (2), it imposes a single
pole structure on $A_{1}$. We can thus continue in the same line of argument and write

$$
\begin{equation*}
A_{1}\left(q^{2}\right)=\xi \frac{c_{H}^{\prime}(1-a)}{1-b^{\prime} x} \tag{3}
\end{equation*}
$$

Here $\xi=m_{H}^{2} /\left(m_{H}+m_{V}\right)^{2}$ is the proportionality factor between $A_{1}$ and $V$ from the SCET relation, while $b^{\prime}$ measures the contribution of resonant states with spin-parity assignment $1^{+}$which are parametrized by the effective pole at $m_{H_{\text {eff }}^{\prime *}}^{2}=m_{H^{*}}^{2} / b^{\prime}$. It can be readily checked that also $A_{1}$, when parametrized in this way, satisfies all the scaling constraints.

Next we parametrize the $A_{0}$ form factor, which is completely independent of all the others so far as it is dominated by the pseudoscalar pole and is proportional to a different universal function in SCET. To satisfy both HQET and SCET scaling laws we parametrize it as

$$
\begin{equation*}
A_{0}\left(q^{2}\right)=\frac{c_{H}^{\prime \prime}\left(1-a^{\prime}\right)}{(1-y)\left(1-a^{\prime} y\right)}, \tag{4}
\end{equation*}
$$

where $y=q^{2} / m_{H}^{2}$ ensures the physical $0^{-}$pole dominance at small recoils and $a^{\prime}$ again parametrizes the contribution of higher pseudoscalar states by an effective pole at $m_{H_{\text {eff }}^{\prime}}^{2}=m_{H}^{2} / a^{\prime}$. The resemblance to $V$ is obvious and due to the same kind of analysis [3] although the parameters appearing in the two form factors are completely unrelated.

Finally for the $A_{2}$ form factor, due to the pole behavior of the $A_{1}$ form factor on one hand and different HQET scaling at $q_{\max }^{2}$ on the other hand, we have to go beyond a simple pole formulation. Thus we impose

$$
\begin{equation*}
A_{2}\left(q^{2}\right)=\frac{\left(m_{H}+m_{V}\right) \xi c_{H}^{\prime}(1-a)+2 m_{V} c_{H}^{\prime \prime}\left(1-a^{\prime}\right)}{\left(m_{H}-m_{V}\right)\left(1-b^{\prime} x\right)\left(1-b^{\prime \prime} x\right)} \tag{5}
\end{equation*}
$$

which again satisfies all constraints. Due to the relations between the form factors we only gain one parameter in this formulation, $b^{\prime \prime}$. This however causes the contribution of the $1^{+}$resonances to be shared between the two effective poles in this form factor.

At the end we have parametrized the four $H \rightarrow V$ vector form factors in terms of the six parameters $c_{H}^{\prime}, a, a^{\prime}, b^{\prime}, c_{H}^{\prime \prime}$ and $b^{\prime \prime}$.

In our heavy meson chiral theory ( $\mathrm{HM} \chi \mathrm{T}$ ) calculations we use the leading order heavy meson chiral Lagrangian in which we include additional charm meson resonances. The details of this framework are given in [8] and [9]. We first calculate values of the form factors in the small recoil region. The presence of charm meson resonances in our Lagrangian affects the values of the form factors at $q_{\text {max }}^{2}$ and induces saturation of the second poles in the parameterizations of the $F_{+}\left(q^{2}\right), V\left(q^{2}\right)$ and $A_{0}\left(q^{2}\right)$ form factors by the next radial excitations of $D_{(s)}^{*}$ and $D_{(s)}$ mesons respectively. Although the $D$ mesons mat not be considered heavy enough, we employ these parameterizations with model matching conditions at $q_{\max }^{2}$. Using HQET parameterization of the current matrix elements [8, 9], which is especially suitable for $\mathrm{HM} \chi \mathrm{T}$ calculations of the form factors near zero recoil, we are able to extract consistently the contributions of individual resonances from our Lagrangian to the various $D \rightarrow P$ and $D \rightarrow V$ form factors. We use physical pole masses of excited state charmed mesons in the extrapolation, giving for the
pole parameters $a=m_{H^{*}}^{2} / m_{H^{*}}^{2}, a^{\prime}=m_{H}^{2} / m_{H^{\prime}}^{2}, b^{\prime}=m_{H^{*}}^{2} / m_{H_{A}}^{2}$. Although in the general parameterization of the form factors the extra poles in $F_{+}, V$ and $A_{0,1,2}$ parametrized all the neglected higher resonances beyond the ground state heavy meson spin doublets $\left(0^{-}, 1^{-}\right)$, we are here saturating those by a single nearest resonance. The single pole $q^{2}$ behavior of the $A_{1}\left(q^{2}\right)$ form factor is explained by the presence of a single $1^{+}$state relevant to each decay, while in $A_{2}\left(q^{2}\right)$ in addition to these states one might also account for their next radial excitations. However, due to the lack of data on their presence we assume their masses being much higher than the first $1^{+}$states and we neglect their effects, setting effectively $b^{\prime \prime}=0$.

The values of the new model parameters appearing in $D \rightarrow P l v_{l}$ decay amplitudes [8] are determined by fitting the model predictions to known experimental values of branching ratios $\mathscr{B}\left(D^{0} \rightarrow K^{-} \ell^{+} v\right), \mathscr{B}\left(D^{+} \rightarrow \bar{K}^{0} \ell^{+} v\right), \mathscr{B}\left(D^{0} \rightarrow \pi^{-} \ell^{+} v\right), \mathscr{B}\left(D^{+} \rightarrow \pi^{0} \ell^{+} v\right)$, $\mathscr{B}\left(D_{s}^{+} \rightarrow \eta \ell^{+} v\right)$ and $\mathscr{B}\left(D_{s}^{+} \rightarrow \eta^{\prime} \ell^{+} v\right)$ [18]. In our calculations of decay widths we neglect the lepton mass, so the form factor $F_{0}$, which is proportional to $q^{\mu}$, does not contribute. For the decay width we then use the integral formula proposed in [19] with the flavor mixing parametrization of the weak current defined in [8].

Similarly in the case of $D \rightarrow V l v_{l}$ transitions we have to fix additional model parameters [9] and we again use known experimental values of branching ratios $\mathscr{B}\left(D_{0} \rightarrow\right.$ $\left.K^{*-} \ell^{+} v\right), \mathscr{B}\left(D_{s}^{+} \rightarrow \Phi \ell^{+} v\right), \mathscr{B}\left(D^{+} \rightarrow \rho^{0} \ell^{+} v\right), \mathscr{B}\left(D^{+} \rightarrow K^{* 0} \ell^{+} v\right)$, as well as partial decay width ratios $\Gamma_{L} / \Gamma_{T}\left(D^{+} \rightarrow K^{* 0} \ell^{+} v\right)$ and $\Gamma_{+} / \Gamma_{-}\left(D^{+} \rightarrow K^{* 0} \ell^{+} v\right)$ [18]. We calculate the decay rates for polarized final light vector mesons using helicity amplitudes $H_{+,-, 0}$ as in for example [20]. By neglecting the lepton masses we again arrive at the integral expressions from [19] with the flavor mixing parametrization of the weak current defined in [9].

We first draw the $q^{2}$ dependence of the $F_{+}$and $F_{0}$ form factors for the $D^{0} \rightarrow K^{-}$, $D^{0} \rightarrow \pi^{-}$and $D_{s} \rightarrow K^{0}$ transitions. The results are depicted in Fig. Uur model results, when extrapolated with the double pole parameterization, agree well with previous theoretical [21, 22] and experimental [1, 2] studies whereas the single pole extrapolation does not give satisfactory results. Note that without the scalar resonance, one only gets a soft pion contribution to the $F_{0}$ form factor. This gives for the $q^{2}$ dependence of $F_{0}$ a constant value for all transitions, which largely disagrees with lattice QCD results [22] as well as heavily violates known form factor relations.

We also calculate the branching ratios for all the relevant $D \rightarrow P$ semileptonic decays and compare the predictions of our model with experimental data from PDG. The results are summarized in Table 11 For comparison we also include the results for the rates obtained with our approach for $F_{+}\left(q_{\text {max }}^{2}\right)$ but using a single pole fit. It is very interesting that our model extrapolated with a double pole gives branching ratios for $D \rightarrow P \ell v_{\ell}$ in rather good agreement with experimental results for the already measured decay rates. It is also obvious that the single pole fit gives the rates up to a factor of two larger than the experimental results. Only for decays to $\eta$ and $\eta^{\prime}$ as given in Table 1 an agreement with experiment of the double pole version of the model is not better but worse than for the single pole case.

We next draw the $q^{2}$ dependence of all the form factors for the $D^{0} \rightarrow K^{-*}, D^{0} \rightarrow \rho^{-}$ and $D_{s} \rightarrow \phi$ transitions. The results are depicted in Fig. 2, Our extrapolated results for the shapes of the $D \rightarrow V$ semileptonic form factors agree well with existing theoretical studies [20, 21, 23, 24], while currently no experimental determination of the form

TABLE 1. The branching ratios for the $D \rightarrow P$ semileptonic decays. Comparison of our model fit with experiment as explained in the text.

| Decay | $\mathscr{B}($ model, double pole $[\%]$ | $\mathscr{B}($ model, single pole) [\%] | $\mathscr{B}($ Exp. [18]) [\%] |
| :---: | :---: | :---: | :---: |
| $D^{0} \rightarrow K^{-}$ | 3.4 | 4.9 | $3.43 \pm 0.14$ |
| $D^{0} \rightarrow \pi^{-}$ | 0.27 | 0.56 | $0.36 \pm 0.06$ |
| $D_{s}^{+} \rightarrow \eta$ | 1.7 | 2.5 | $2.5 \pm 0.7$ |
| $D_{s}^{+} \rightarrow \eta^{\prime}$ | 0.61 | 0.74 | $0.89 \pm 0.33$ |
| $D^{+} \rightarrow \bar{K}^{0}$ | 9.4 | 12.4 | $6.8 \pm 0.8$ |
| $D^{+} \rightarrow \pi^{0}$ | 0.33 | 0.70 | $0.31 \pm 0.15$ |
| $D^{+} \rightarrow \eta$ | 0.10 | 0.15 | $<0.5$ |
| $D^{+} \rightarrow \eta^{\prime}$ | 0.016 | 0.019 | $<1.1$ |
| $D_{s}^{+} \rightarrow K^{0}$ | 0.20 | 0.32 |  |

factors' shapes in these decays exists.
We complete our study by calculating branching ratios and partial decay width ratios also for all relevant $D \rightarrow V l v_{l}$ decays. They are listed in Table 2 together with known experimentally measured values.

Finally, we summarize our results: We have investigated semileptonic form factors for $D \rightarrow P$ and $D \rightarrow V$ decays within an approach which combines heavy meson and chiral symmetry. The form factors are parametrized to satisfy all constraints coming from HQET and SCET. The contributions of excited charm meson states are included


FIGURE 1. $q^{2}$ dependence of the $D^{0} \rightarrow K^{-}$(upper left), $D^{0} \rightarrow \pi^{-}$(upper right) and $D_{s} \rightarrow K^{0}$ (lower) transition form factors.

TABLE 2. The branching ratios and partial decay width ratios for the $D \rightarrow V$ semileptonic decays. Comparison of our model fit with experiment as explained in the text.

| Decay | $\mathscr{B}$ (Mod.) [\%] | $\mathscr{B}$ (Exp.) [\%] | $\Gamma_{L} / \Gamma_{T}$ (Mod.) | $\Gamma_{+} / \Gamma_{-}$(Mod.) |
| :--- | :---: | :---: | :---: | :---: |
| $D_{0} \rightarrow K^{*}$ | 2.2 | $2.15 \pm 0.35[18]$ | 1.14 | 0.22 |
| $D_{0} \rightarrow \rho$ | 0.20 | $0.194 \pm 0.039 \pm 0.013[25]$ | 1.11 | 0.14 |
| $D^{+} \rightarrow K_{0}^{*}$ | 5.6 | $5.73 \pm 0.35[18]$ | $1.13^{*}$ | $0.22^{\dagger}$ |
| $D^{+} \rightarrow \rho_{0}$ | 0.25 | $0.25 \pm 0.08[18]$ | 1.11 | 0.14 |
| $D^{+} \rightarrow \omega$ | 0.25 | $0.17 \pm 0.06 \pm 0.01[25]$ | 1.10 | 0.14 |
| $D_{s} \rightarrow \Phi$ | 2.4 | $2.0 \pm 0.5[18]$ | 1.08 | 0.21 |
| $D_{s} \rightarrow K_{0}^{*}$ | 0.22 |  | 1.03 | 0.13 |

* Exp. $1.13 \pm 0.08$ [18]
${ }^{\dagger}$ Exp. $0.22 \pm 0.06$ [18]
into analysis. The values of form factors at $q_{\max }^{2}$ are calculated using heavy meson chiral Lagrangian. The second poles of the $F_{+}\left(q^{2}\right), V\left(q^{2}\right)$ and $A_{0}\left(q^{2}\right)$ form factors are saturated by the presence of the next radial excitations of $D_{(s)}^{*}$ and $D_{(s)}$. The single pole $q^{2}$ behavior of the $A_{1}\left(q^{2}\right)$ form factor is explained by the presence of a single $1^{+}$state relevant to each decay, while in $A_{2}\left(q^{2}\right)$ in addition to $1^{+}$states one might include their next radial excitations.

The obtained $q^{2}$ dependence of the form factors is in good agreement with recent experimental results and existing theoretical studies. The calculated branching ratios are


FIGURE 2. $\quad q^{2}$ dependence of the $D^{0} \rightarrow K^{*-}$ (upper left), $D^{0} \rightarrow \rho^{-}$(upper right) and $D_{s} \rightarrow \phi$ (lower) transition form factors.
close to the experimental ones. We hope that the ongoing experimental studies will help to shed more light on the shapes of the $D \rightarrow P, V$ form factors.

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