Two dimensional Quantum Chromodynamics as the limit of higher dimensional theories *

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Abstract

We define pure gauge QCD on an infinite strip of width L. Techniques similar to those used in finite TQCD allow us to relate 3D-observables to pure QCD_2 behaviors. The non triviality of the $L \to 0$ limit is proven and the generalization to four dimensions described. The spectrum of the theory in the small width limit is analyzed and compared to that of the two dimensional theory.

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1 Introduction

Since the pioneering work of 't Hooft [1] Quantum Chromodynamics in two dimensions (QCD_2) has been used as a model to understand some of the features of the true theory. It shares with its four-dimensional version many properties: confinement [2, 3], existence of massless pseudoscalars [4, 5] and asymptotic freedom [6]. The dimensionality of space-time allows the unveiling of the mechanism behind these in an exact manner. The drastic reduction in dimension, the only approximation when considering QCD_2 as a model for QCD, is however too extreme.

In here we develop a continuous procedure to lower the dimensionality of a gauge theory. We start by considering QCD in 2+1 dimension. One of the spatial dimensions is considered of finite length L (the strip) and we study how the three dimensional theory approaches the two dimensional one as the width of the strip is made to vanish. The choice of gauge is crucial for the process to proceed smoothly. We pay special attention to the spectrum and compare our findings with those of recent developments [7, 8].

The theory on the strip contains besides the conventional QCD_2 (2D) gluons an infinite set of covariant (non-gauge) interacting fields (ϕ , { V_{α}^n , $\alpha = 0, 1; n \neq 0$ }) belonging to the adjoint representation of the color group.

The ϕ field is a massless gauge-invariant degree of freedom. It is the remnant of the gauged-away *transversal* field A_2 . This scalar field couples to the 2D gluons through the covariant derivative to preserve the *longitudinal* gauge invariance. It interacts with the V fields and undergoes a mass renormalization process which provides it with mass. Unlike the 2D gluons, whose masslessness is protected by the 2D gauge invariance, there is no custodial symmetry for the scalar field. Moreover the non-abelian scalar field influences the 2D gluon dynamics by dressing the coupling constant g_2 .

The V fields of the non abelian strip theory self-couple with couplings which are not arbitrary, but determined by the 3D gauge invariance, lost after the implementation of the gauge fixing condition. The couplings are not gauge invariant under the 2D gauge symmetry because these modes are massive. The 2D gauge symmetry, however is instrumental in fixing the couplings of the 2D gluons with the V fields through covariant derivatives. All the couplings preserve the $U(1)^n$ topological conservation law, so that the global *n*-charge will be conserved in any vertex.

The theory on the strip carries a natural infrared cutoff, the strip width L. In the effective 2D action this scale parameter generates the perturbative masses of the V fields. These masses do not and should not survive the infinite volume limit, since they arise from the boundary conditions. However in QCD we expect mass scales to arise in the form of the renormalized 2D coupling constant g_2 by the effect of the scalar and V-particle loops ¹. This mass scale is fundamental in characterizing confinement in 3D [9].

¹Certainly the boson loops will provide the gluon correlator with an L dependence additional to that coming from the bare masses. If and how a mass parameter is generated depends on the resolution of the renormalization program.

2 Non-abelian gauge theory on the strip

A pure SU(N) gauge theory defined on an infinite strip of width L has the following action

$$S(L) = \int_{0}^{L} d\bar{x} \int dx \left\{ -\frac{1}{4} F_{\mu\nu}(x, \bar{x}) F_{\mu\nu}(x, \bar{x}) \right\}$$
(1)

We are calling the infinitely extended coordinates (space and time) jointly by x and the bounded transversal one by \bar{x} . We compactify the transversal degrees of freedom by requiring periodic boundary conditions on the field strength $F^a_{\mu\nu}$. In order to describe the gauge dynamics on the strip we need the boundary conditions on the gauge field

$$A^{a}_{\mu}(x,\bar{x}+L) = A^{a}_{\mu}(x,\bar{x})$$
⁽²⁾

Periodicity implies that A^a_{μ} can be expanded in Fourier modes

$$A^{a}_{\mu}(x,\bar{x}) = \frac{1}{\sqrt{L}} \left[a^{a}_{\mu} + \sum_{n=1}^{\infty} \left(V^{na}_{\mu}(x) e^{i\frac{2\pi n}{L}\bar{x}} + V^{na*}_{\mu}(x) e^{-i\frac{2\pi n}{L}\bar{x}} \right) \right]$$
(3)

Our aim is to eliminate the transversal component of the gauge field by an appropriate selection of the gauge. This can be done for all transverse modes except for the zero mode. In order to avoid the appearance of ghosts we choose the gauge $\partial_2 A_2 = 0$ [11] and obtain as a gauge condition for the Fourier modes (3),

$$a_{2}^{a}(x) = \phi^{a}(x) = \frac{1}{L} \int_{0}^{L} d\bar{x} A_{2}^{a}(x, \bar{x})$$
$$V_{2}^{na} = 0 \; ; \; \forall n \neq 0$$
(4)

The gauge field zero mode is therefore a gauge invariant object ². Moreover the above equations allow us to relate a_2^a to a very familiar object in Quantum Field Theory at finite temperature, the so called Polyakov loop. On the strip we may define the *transverse* Polyakov loop as

$$Trg(x) = Tr \exp[(ig_3\sqrt{L})\phi^a T_a]$$
(5)

where g_3 is the coupling constant of the 3D theory. As the ordinary Polyakov loop Trg(x) is gauge invariant and it cannot be suppressed by a gauge transformation because it is a physical degree of freedom. Nevertheless the physical interpretation of the time and transverse Polyakov loops are very different. Therefore analogies between them beyond their formal properties must be avoided.

Once the gauge has been chosen it is possible to write the new action in terms of the Fourier modes of the gauge field and then perform the integration in the transversal coordinate obtaining in this way the 2D action

$$\mathcal{L}_2 = \frac{1}{2} f_{\alpha\beta}^2 + (D_\alpha \phi)^2 +$$

²Notice that the standard axial gauge $A_2 = 0$ is not allowed on the strip because ϕ cannot be removed by a gauge transformation.

$$\sum_{n \neq 0} \left\{ \frac{1}{2} (D_{\alpha} V_{\beta}^{n} - D_{\beta} V_{\alpha}^{n}) (D_{\alpha} V_{\beta}^{n*} - D_{\beta} V_{\alpha}^{n*}) + V_{\alpha}^{n} M_{n}^{2}(\phi) V_{\alpha}^{n*} - i \frac{g_{3}}{L^{\frac{1}{2}}} f_{\alpha\beta} [V_{\alpha}^{n}, V_{\beta}^{n*}] \right\} - i \frac{g_{3}}{L^{\frac{1}{2}}} \sum_{n,l \neq 0} (D_{\alpha} V_{\beta}^{n*} - D_{\beta} V_{\alpha}^{n*}) [V_{\alpha}^{l}, V_{\beta}^{n-l}] - \frac{g_{3}^{2}}{2L} \sum_{l,l' \neq 0} [V_{\alpha}^{l*}, V_{\beta}^{l}] [V_{\alpha}^{l'*}, V_{\beta}^{l'}] - \frac{g_{3}^{2}}{2L} \sum_{n,l,l' \neq 0} [V_{\alpha}^{l*}, V_{\beta}^{n+l}] [V_{\alpha}^{l'*}, V_{\beta}^{l'-n}]$$
(6)

The squared color matrix M_n^2 is given in terms of the scalar Field by

$$M_n^2(\phi)_{ab} = (m_n^2 + \frac{g_3^2}{L}\phi^2)\delta_{ab} - \frac{g_3^2}{L}\phi_a\phi_b - 2i\frac{g_3}{L^{1/2}}m_n\epsilon_{abc}\phi_c$$
(7)

The covariant derivative is defined by

$$D_{\alpha} \equiv \partial_{\alpha} - i \frac{g_3}{L^{\frac{1}{2}}} [a_{\alpha}, .].$$
(8)

Finally the zero mode tensor $f_{\alpha\beta}$ is just the ordinary 2D gluon term

$$f_{\alpha\beta} = (\partial_{\alpha}a_{\beta} - \partial_{\beta}a_{\alpha}) - i\frac{g_3}{L^{\frac{1}{2}}}[a_{\alpha}, a_{\beta}]$$
(9)

Since our final goal is to connect the theory on the strip to two dimensional theories we must scale the coupling constant in the proper way. Our choice of normalization for the basis of the Fourier expansion leads to

$$[a_{\alpha}] = [V_{\alpha}^n] = [\phi] = 1 \tag{10}$$

the familiar dimensions of the two dimensional fields. Therefore only the coupling constant needs to be scaled in order to recover the canonical QCD_2 dimension, $[g_2] = M$. Its scaling becomes

$$g_2 = \frac{g_3}{L^{\frac{1}{2}}} \tag{11}$$

The effective 2D action Eq.(6) has a global $[U(1)]^n$ $(n \to \infty)$ symmetry whose associated conserved charges are the mode numbers. Its quantization is a necessary condition for the fields to fulfill the required periodic boundary conditions Eq.(2).

The effective 2D action is invariant under what we call *longitudinal* gauge transformations. This is the symmetry left after imposing the gauge fixing conditions, Eqs.(4). We observe that any gauge transformation behaving as

$$\partial_2 U(x, \bar{x}) = 0 \Leftrightarrow U = U(x) \tag{12}$$

preserves the axial gauge conditions and transforms the 2D Fourier fields as

$$\begin{aligned} a_{\alpha} &\to U a_{\alpha} U^{\dagger} - \frac{i}{g_2} (\partial_{\alpha} U) U^{\dagger} \\ \phi &\to U \phi U^{\dagger} \\ V_{\alpha}^n &\to U V_{\alpha}^n U^{\dagger} \end{aligned} \tag{13}$$

The non-covariant transformation law of the a_{α} field guarantees the covariance of the 2D field strength $f_{\alpha\beta}$ and thus the invariance of the first term of Eq.(6). For this reason these fields describe the two dimensional gluons. Besides the conventional gluons we find an infinite set of covariant (non-gauge) interacting fields (Φ , { V_{α}^{n} , n = 1, ...} belonging to the adjoint representation of the color group. Since they carry color charges they interact with the 2D gluons in a gauge invariant way. The derivatives terms of these latter fields are defined in terms of covariant derivatives, the non derivative terms depend on the covariant field V_{α}^{n} and therefore the invariance of the action (6) under *longitudinal* gauge transformations is explicit.

3 The small ϵ regime of QCD on the strip

In order to study the 2D effective action it is convenient to change the parameters defining the theory (g_3, L) into a more befitting set to work in 2D,

$$g_2 \equiv \frac{g_3}{L^{1/2}}; \quad [g_2] = M$$
 (14)

$$\epsilon \equiv g_3 L^{1/2}; \quad [\epsilon] = 1 \tag{15}$$

At a given width L and coupling constant g_3 we can characterize our theory by fixing the 2D gauge coupling constant g_2 and the dimensionless width ϵ . As far as the effective 2D theory is concerned ϵ acts as a dimensionless coupling constant. We proceed to find an expansion in this coupling.

In a naive analysis, and since the lagrangian masses of the V fields are

$$m_n = \frac{2\pi n}{L} = \frac{2\pi n g_2}{\epsilon} \tag{16}$$

we would expect the non-zero mode fields to disappear in the $\epsilon \to 0$ limit. Indeed, and just on classical grounds, the integration of the infinitely heavy particle sector has no consequence on the effective action for the remaining massless degrees of freedom, that is, the 2D gluons and the scalar field. At tree level, the sole contribution of the non-zero modes to the reduced action (6) would be through the substitution of the V field by its vev at $\epsilon = 0$

$$V^n_{\alpha} \to < V^n_{\alpha} >= 0 \tag{17}$$

So that,

$$\mathcal{L}_{cl}^{QCD_3} = \frac{1}{2} f_{\alpha\beta}^2 + (D_{\alpha}\phi)^2$$
(18)

However the classical approach Eq.(18) does not provide the correct answer to the problem of the real $\epsilon \to 0$ limit of QCD on the strip. The reason is that there are important quantum effects because the loop and ϵ expansions are *not* independent.

The non-zero mode fields become infinitely heavy in the $\epsilon \to 0$ limit and a good description of the system can be given in terms of the lighter degrees of freedom. However the integration of the heavy modes must be performed in a more accurate way than previously

(Eq.(17)) presented. The effective potential for 2D gluons and scalar will include contributions arising from many heavy V loop diagrams. In all of them, 2D gluons and scalar particles will appear as external legs. The dependence on ϵ of these multi-loop diagrams will come through the V masses Eq.(16) exclusively. Thus we can find the order in ϵ of any diagram just by analyzing its m_n dependence.

Consider an arbitrary diagram of degree C in the coupling constant ($\sim g_2^C$) and B external legs at zero momentum, with a complicated structure in terms of the vertices and propagators defined by the Feynman rules of the action (6). Its behavior can be characterized as ³

diagram
$$\sim (g_2)^C m_n^D = (g_2)^{C+D} \epsilon^{-D} = (g_2)^2 \epsilon^{-D}$$
 (19)

where D can be expressed in terms of the superficial degree of divergence d, the number of internal transversal propagators $i_{(VV)_{\perp}}$, and the number of external ϕVV vertices of the diagram by

$$D = d - 2i_{(VV)\downarrow} + b_{\phi VV} \tag{20}$$

Using standard topological graph relations, we obtain

$$d = 4 - 2l - b' - 2b_{\phi VV} + 2i_{(VV)_{\perp}} \tag{21}$$

where l is the number of loops and b' stands for the number of external legs not of the ϕVV kind. That is

$$b = b' + b_{\phi VV} \tag{22}$$

After some elementary algebra we obtain an expression relating the D to the number of loops l,

$$D = 4 - 2l - b \tag{23}$$

It is now easy to understand why the classical action (18) cannot be the real $\epsilon \to 0$ limit of QCD_3 . This limit is defined by all the possible D = 0 operators we can construct out of the external 2D gluon and scalar fields. Since

$$\{b \ge 2, l \ge 0\}\tag{24}$$

the above condition can be fulfilled only if

$$\{b = 4, l = 0\}$$
 or $\{b = 2, l = 1\}$ (25)

The only operators which can appear in the effective action verifying the first condition *and* compatible with gauge invariance are

$$\begin{aligned}
f_{\alpha\beta}^2 & \stackrel{a_{\alpha}=u_{\alpha}=ctant}{\longrightarrow} & -g_2^2[u_{\alpha}, u_{\beta}]^2 = O(u^4) \\
D_{\alpha}\phi)^2 & \stackrel{u_{\alpha},\phi=ctant}{\longrightarrow} & -g_2^2[u_{\alpha}, \phi]^2 = O(\phi^2 u^2)
\end{aligned} \tag{26}$$

³Recall that all the operators contributing to the effective potential have to have mass dimension equals 2.

They give raise to the classical action (l = 0) Eq.(6). But we have also non-classical (l = 1) contributions to the effective potential. They correspond to operators arising from two external legs one-loop graphs $\{b = 2, l = 1\}$. The one-loop diagram with two external zero momentum gluon legs is zero, as local gauge invariance requires.⁴ On the contrary the scalar field is not a 2D gauge field. It can get a mass term through the one loop diagrams shown in Fig.1, which lead to

$$\mu_{\phi}^{2}(\epsilon) = \frac{g_{2}^{2}N}{\pi} + \frac{g_{2}^{2}N}{2\pi}\ln(\frac{1}{\epsilon^{2}}) + O(\epsilon^{2}\ln\epsilon)$$
(27)

Consequently, for small values of ϵ dynamics is not given by the classical action (18) but by the quantum corrected one,

$$\mathcal{S}^{QCD_3} = \int d^2x \left\{ \frac{1}{2} f_{\alpha\beta}^2 + (D_\alpha \phi)^2 + \mu_\phi^2(\epsilon) \phi^2 + O(\epsilon^2 \ln \epsilon) \right\}$$
(28)

which allows us to provide the exact *line limit* of QCD_3 .

The quantum fluctuations generated by the V-excitations are able to provide the classically massless scalar field with a very heavy mass. Thus the spectrum of QCD on the strip contains just one single massless field, the 2D gluon. The remaining particles, scalar and excited modes, are massive.

In QCD_3 the decoupling of the heavy sector is caused by quantum effects in the form of a divergent mass for the scalar particle in the $\epsilon \to 0$ limit,

$$\mu_{\phi}^{2}(\epsilon) = \frac{g_{2}^{2}N}{\pi} + \frac{g_{2}^{2}N}{2\pi}\ln(\frac{1}{\epsilon^{2}}) \stackrel{\epsilon \to 0}{\to} \infty$$
(29)

The 2D gauge field is the only massless degree of freedom of the theory and thus the long range 2D dynamics $(R \gg g_3^{-2})$ is completely determined by 2D gluon physics.

4 The spectrum of the effective small ϵ theory

The scalar field becomes, like its transversal partners V's, infinitely heavy in the completely reduced action ($\epsilon = 0$). In the small ϵ regime the scalar field ϕ , although still very heavy, is the lightest of all the transversal particles ($\mu_{\phi}^2 \sim \ln(1/\epsilon^2), m_n^2 \sim 1/\epsilon^2$). Consequently the appearance of transversal gluons effects takes place in a much softer way than in the classical reduced action Eq.(18). However QCD on the strip in the small ϵ regime (28) still contains highly non-trivial gluon physics.

We can take advantage of the fact that the scalar field is very heavy to give a nonrelativistic (NR) treatment to the action (28). Our interest lies in the calculation of (very massive $\sim \mu_{\phi}$) scalar bound states which do not show up in the $\epsilon \to 0$ limit (pure QCD_2).

We are now in an analogous situation to that found in 't Hooft's model. The heavy scalar field interacts with the 2D gluons as heavy fermions do. Although fermions and bosons verify different relativistic equations, they give raise to an identical NR formalism in

⁴In fact, 2D gauge invariance prevents the two gluon diagram at zero momentum to survive to all orders in the loop expansion. No gluon mass term is allowed by gauge symmetry.

2D when masses are large enough⁵. At lowest order in the inverse mass expansion $(1/\mu_{\phi})$, the suppressions in the equation for the bound-state amplitude are the same as those appearing in leading order in 1/N [1], namely, absence of sea quark effects, vertex corrections and non-planar diagram contributions.

The bound state mass equation is nothing but a one dimensional Schrödinger equation for a $\phi_{(1)}\phi_{(2)}$ system interacting through a color gauge field. Only the zero component of the gauge field contributes to the potential through the static term $g_2T^aa_0^a$. Other gauge field dependent contributions to the potential are suppressed by powers of $1/\mu_{\phi}$. In the $a_1 = 0$ axial gauge, the zero component of the gauge field is

$$a_0^a(x_{12}) = -\frac{g_2}{2}T_{(2)}^a|x_{12}| \quad \Rightarrow \quad \hat{V}_{12} = g_2 T_{(1)}^a a_0^a(x_{12}) = -\frac{g_2^2}{2}(T_{(1)}^a T_{(2)}^a)|x_{12}| \tag{30}$$

thus the eigenvalue equation for the bound state wave function $\Phi_{12}(x)$ becomes

$$-\frac{1}{\mu_R}\frac{d^2}{dx^2}\Phi_{12}(x) + \sigma|x|\Phi_{12}(x) = E\Phi_{12}(x)$$
(31)

The string tension σ is given in terms of the 2D coupling constant and the color invariant C_N as $\sigma = \frac{1}{2}g_2^2 C_N^{-6}$. The mass μ_R is the renormalized scalar mass, obtained by dressing the ϕ mass Eq.(29) by the action of 2D gluons. The calculation of the renormalized scalar mass has been carried out in the context of finite temperature QCD_3 under the name of *electric* mass, or inverse Debye screening length [11], which properly applied to the strip becomes

$$\mu_R^2(\epsilon) = \frac{g_2^2 N}{4\pi} \ln(\frac{1}{\epsilon^2}) \tag{32}$$

The solution of the one dimensional differential equation (31) is known. It is given in terms of the Airy function [11, 12]

$$\Phi_r(x) = \operatorname{Ai}[(\mu_R \sigma)^{1/3} x - \varepsilon_r]$$
(33)

where $-\varepsilon_r$ is the *r*th zero of Ai or Ai' for odd or even states, respectively. The energy of the bound state and thus the mass of the *r*th glueball is given by

$$E_{r}(\epsilon) = 2\mu_{R} + \varepsilon_{r} \left(\frac{\sigma^{2}}{\mu_{R}}\right)^{1/3}$$

$$= g_{2} \sqrt{\frac{2N}{\pi}} \left[\ln\left(\frac{1}{\epsilon^{2}}\right)\right]^{1/2} + \varepsilon_{r} g_{2} \left(\frac{C_{N}^{2}}{2} \sqrt{\frac{\pi}{2N}}\right)^{1/3} \left[\ln\left(\frac{1}{\epsilon^{2}}\right)\right]^{-1/6}$$
(34)

The mass of these glueball states grows as ϵ approaches zero. They decouple eventually leaving ordinary 2D gluons as the only remnants of color interaction.

⁵In higher dimensions this limit is also the same with the exception of the term proportional to the spin S.B. This terms is not present in 2D because of the absence of spin.

 $^{{}^{6}}C_{N}$ is the color expectation value $-\langle (12)|T^{a}_{(1)}T^{a}_{(2)}|(12)\rangle$, where the color state $|(12)\rangle$ is a $|T_{tot} = 0, \{N^{2}-1\} \otimes \{N^{2}-1\}\rangle$ state. For $N = 2, C_{N} = 3/4$.

In the small ϵ regime the lowest glueball spectrum corresponds to light glueball $\phi\phi$ bound states. But they are certainly not the only glueball states one can generate from the strip action (6). The non-zero mode fields V_n are also present. They can likewise yield glueball particles as massive $V_n V_n^*$ bound states. Since V states are heavier than the scalar particles we expect the lightest $V_n V_n^*$ spectrum to be above the lowest mass scalar glueballs Eq.(34).

The way to proceed with $V_n V_n^*$ bound states is identical to that we followed for scalar glueballs. First of all, NR approach is even better in this case than for scalar particles since $\mu_{\phi} < m_n$ for small ϵ 's. The only difference lies on the presence of V selfcouplings and of an additional $V\phi$ interaction in the V action Eq.(6).

As far as the V self-interaction is concerned we can simply neglect it to lowest order in ϵ . As we saw in Eq.(17), in order to calculate the lowest ϵ contributions we must perturbate V around its classical value at $\epsilon = 0$. That is, we must expand the non-zero mode field V_n around zero

$$\hat{V}^n_{\alpha} = <\hat{V}^n_{\alpha} > +\delta\hat{V}^n_{\alpha} = \delta\hat{V}^n_{\alpha} \tag{35}$$

At lowest order in the ϵ expansion only quadratic terms in V are relevant in the strip action. Higher power V terms are suppressed as ϵ^2 .

The NR limit of the equation for a $V_n V_n^*$ bound state is then exactly the same as Eq.(31) except for an extra interaction term in the potential. This is due to the existence of an additional $VV\phi$ vertex supplying an interaction which survives to the NR limit. However for this piece of the potential is a 2D OSEP (One Scalar Exchange Potential), it is strongly short range decaying exponentially with the scalar mass ($\sim e^{-\mu_{\phi}|x|}$). As a first approximation (recall μ_{ϕ} is very large for small ϵ) we do not consider it and we evaluate the spectrum taking into account the confining 2D OGEP (One Gluon Exchange Potential) exclusively. Therefore we can calculate with a rather good accuracy the ϵ dependence of the V glueball spectrum just by copying the result we obtained for scalar glueballs and making the substitution

$$\mu_R \to m_n \tag{36}$$

That is, the mass of the *r*th glueball state made out of two massive gluons with U(1) charge $n (V_n V_n^* \text{ state})$ will be

$$E_r^{(n)}(\epsilon) = 2m_n + \varepsilon_r \left(\frac{\sigma^2}{m_n}\right)^{1/3}$$
$$= g_2 \frac{4\pi n}{\epsilon} + g_2 \varepsilon_r \left(\frac{C_N^2 \epsilon}{8\pi n}\right)^{1/3}$$
(37)

The equations (34) and (37) are valid for the low energy spectrum. High energy modes (binding energy $\sim \mu_{\phi}$) have to be treated in a relativistic framework. The highest modes (ultrarelativistic ones) can be obtained using the results of Demerterfi et al. [7]⁷

⁷Notice they calculate with the *massless* adjoint action (18) which is not the reduction QCD_3 into the line limit.

5 Conclusions

In recent times there have been a growing interest in the study of QCD_2 coupled to massless adjoint matter[7, 8]. That is in the study of the classical reduced action (18). This theory possesses some appealing properties. It is a supersymmetric theory, it is super-renormalizable, it owns a non-trivial glueball spectrum and besides it shows confinement and asymptotic freedom. It is hoped that, as in 't Hooft's model, one can keep track of some interesting features of the real theory by studying the action (18) as a 2D model of QCD. This hope would be mainly based on the dimensional reduction process. In this way the role of the scalar field would be to give a *transversality* component to the theory ⁸ absent in pure gauge QCD_2 .

However, as we have just proved, Eq.(18) does not provide the right dimensional reduction of QCD in 2 + 1 dimensions. The massless character of the scalar field is a classical feature. Vacuum fluctuations corresponding to the massive transversal modes renormalize the scalar mass. This renormalization process is perfectly defined and gives rise to an ϵ -dependent *non-zero* scalar mass Eq.(29). Consequently the scalar mass is not an arbitrary parameter of the theory but a well defined function of the dimensionless width ϵ .

Another important fact indicated by the small ϵ expansion action Eq.(28) is that the line limit $(\epsilon \to 0)$ of gluon QCD_3 is gluon QCD_2 . In the framework of an interpolating model of QCD_3 like that given by Eq.(6), this means that QCD_2 and the successive effective theories obtained by increasing the value of ϵ are those which can be connected to real QCD_3 in the $\epsilon \gg 1$ regime. For instance we expect that if there exists some property of the line limit $(\epsilon \to 0)$ theory surviving in the infinite volume limit this will correspond to QCD_2 and it will be respected by all the effective actions interpolating between $(\epsilon \to 0)$ and $(\epsilon \to \infty)$. The flux generated by the flow of the effective actions with ϵ is perfectly defined. For the complexity of the effective 2D theories representing QCD_3 increases rapidly with the value of ϵ Eq.(23), we come to the conclusion that pure QCD_2 and the one loop action (28) are the simplest 2D models one can relate to 3D QCD^9 . Certainly there is a limit in the validity of the perturbative approach. The ϵ expansion will break down for large values of ϵ . Nevertheless this flow can be extrapolated to the non-perturbative region by means of lattice calculations [13]. Finite temperature lattice results can be likewise interpreted on the strip after a suitable euclidean rotation. The interesting finite temperature operators in the strip framework are spatial-like, which become time-like ones on the strip. This is the case of the finite temperature *spatial* Wilson loop corresponding to the ordinary strip Wilson loop. The analysis of lattice results points out the existence of a continuous behavior of the spatial-like Wilson loop over the phase transition point. Moreover it seems that the spatiallike Wilson loop gets stabilized already at this point reaching its zero temperature value. When properly interpreted in the strip language this fact means that physical properties, as the glueball spectrum or the string tension, can be continuously extrapolated over the non-perturbative region to the three dimensional ϵ regime [9].

To end we extend our results to four dimensions in a descriptive fashion. Consider pure gauge QCD defined on $\mathcal{R}^2 \otimes [0, L] \otimes [0, L]$. We perform a double compactification in the

⁸Recall ϕ is nothing but the A_2 zero mode.

⁹In terms of 2D gauge degrees of freedom. It is not excluded an alternative formulation using strings, for example [10]. In any case this approach should be equivalent to that presented here.

transverse coordinates. The gauge fixing procedure introduces now two 2D scalar fields instead of one (one per each compactified dimension). They are massless classically. The existence of heavy non-zero $U(1) \otimes U(1)$ modes allows the reduction of dynamics to the scalar sector in the small L regime. By means of the same compactification techniques used for QCD on the strip one can construct the effective action [9]. The lowest order in the ϵ expansion of the 2D effective action for QCD_4 becomes

$$S^{QCD_4} = \int d^2x \left\{ \frac{1}{2} f_{\alpha\beta}^2 + (D_\alpha \phi)^2 + \mu^2(\epsilon) \phi^2 + (D_\alpha \phi')^2 + \mu^2(\epsilon) \phi'^2 + a(\epsilon) \phi \phi' - g_2^2 [\phi, \phi']^2 + O(\epsilon^2 \ln \epsilon) \right\}$$
(38)

The integration of the $U(1) \otimes U(1)$ non-zero mode fields is performed analogously as on the strip. The non-zero mode fields loops give a very heavy mass $(\mu^2(\epsilon) \sim \ln(1/\epsilon))$ to the scalar sector. They also give raise to an additional $\phi \phi'$ term. This crossed term has the effect of breaking the mass degeneracy of the scalar doublet. This can be manifestly seen by introducing the field combinations

$$\Phi_{+} \equiv \frac{1}{\sqrt{2}}(\phi + \phi')$$

$$\Phi_{-} \equiv \frac{1}{\sqrt{2}}(\phi - \phi')$$
(39)

which yields to the following effective action in the small ϵ regime

$$S^{QCD_4} = \int d^2 x \left\{ \frac{1}{2} f_{\alpha\beta}^2 + (D_\alpha \Phi_+)^2 + \mu_+^2(\epsilon) \Phi_+^2 + (D_\alpha \Phi_-)^2 + \mu_-^2(\epsilon) \Phi_-^2 \right\}$$
(40)

The masses are given by

$$\mu_{+}^{2}(\epsilon) = \mu^{2}(\epsilon) + \frac{a(\epsilon)}{2} \stackrel{\epsilon \to 0}{\to} \infty$$

$$\mu_{-}^{2}(\epsilon) = \mu^{2}(\epsilon) - \frac{a(\epsilon)}{2} \stackrel{\epsilon \to 0}{\to} \infty$$
 (41)

We see that, like in the strip, scalar particles become extremely heavy in the small ϵ regime. They decouple eventually from the 2D gluons in the $\epsilon \to 0$ limit. The parallelism between the previous action and the 2D effective action we found for the strip Eq.(28) is evident. Thus we expect to reproduce *qualitatively* the main features of QCD on the strip at small ϵ 's. The main ingredients are identical in both theories. Namely, the existence of a very heavy constituent matter and the presence of a linearly confining interaction supplied by the exchange of 2D gluons.

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Figure Captions

Fig.1 : Diagrams contributing to the scalar mass in lowest order in ϵ . Solid lines represent adjoint scalar fields ϕ . Double wavy lines represent heavy vector modes V_{α}^{n} .

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