Individual strategies in complementarity games and population dynamics

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Abstract

We introduce and study an evolutionary complementarity game where in each round a player of population 1 is paired with a member of population 2. The game is symmetric, and each player tries to obtain an advantageous deal, but when one of them pushes too hard, no deal at all can be concluded, and they both loose. The game has many equilibria, and which of them is reached depends on the history of the interactions as the players evolve according to a fitness function that measures their gains across the interactions. We can then break the symmetry by assigning different strategy spaces to the populations, varying in particular with respect to the information available to the agents. The agents can, for example, adapt to the behavior of their opponents met in previous rounds, or they can try to copy the strategies of their successful friends. It turns out that, in general, the more restricted strategy spaces, that is, those that utilize less information, are more advantageous for a population as a whole as their adoption drives the equilibrium in a direction advantageous to that population. One reason is that a simpler strategy can be learned faster in an evolutionary setting, another is that it is good for a population to have some individuals that are unfit in the sense that they make offers that are individually unsuccessful, but have a systematic effect

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on the strategies of their opponents. All these effects are demonstrated through systematic simulations.

Keywords: Evolutionary complementarity game; Individual strategies; Population dynamics

1 Introduction

We introduce a simple new game between members of two populations that possesses some features hitherto not considered in game theory and therefore exhibits some new aspects and offers possibilities to investigate new issues.

The game is simple: We have two populations, called the "buyers" and the "sellers". For each interaction, a member of one of them is paired with a randomly drawn member of the other one. The buyer offers a certain amount x between 0 and K, and the seller asks an amount y within that same range. When $x \geq y$, a deal is concluded where the buyer pays x, or, equivalently, gains $K - x$, and the seller gains y. When $x < y$, they both gain nothing (or, in the first version, the buyer looses the amount K).

The rules are strict: In each interaction, the players do not know what the opponent is offering or asking, and the own bid cannot be adjusted. However, they can learn from experience. That is, the players are allowed to develop strategies that consist in choosing their bid in the present round for example on the basis of the bid of the opponent met in the previous interaction.

The outcomes of these interactions then are used to evaluate the fitness of the players in an evolutionary process. After each player has been chosen to play a given number of such interactions, the fitness values, that is, the accumulated gains of all the players in each population are compared, and a selection process is applied. That means that the chance for a player of the present generation to be represented in the next generation, or to be chosen as a parent in a recombination process from a standard genetic algorithm, is proportional to his relative fitness. In addition, we allow for mutations to occur with a certain probability when building the next generation.

Of course, there is ample room for changing the rules of the game, but let us analyze some important features of our game so that we can see what we can learn from it.

1. The competition takes place at two different levels. Each player plays against the members of the other population and in order to avoid the risk that the interaction becomes a complete failure, he has to make cautious bids. On the other hand, if a bolder bid is successful, he achieves a higher fitness gain than his more cautious fellows from his own population and thereby gains a selective advantage over them. Putting it the other way around, a higher gain in the case of a successful interaction is offset by a higher risk for the failure of that interaction.

2. Any value k between 0 and K can be an equilibrium in the following sense: When all the other members of the two populations always bid k, then

any player choosing a bid value different from k will be at a disadvantage: When he is a buyer and offers more than k, then he will pay more than the other buyers and become correspondingly less fit. When he offers less, then he cannot strike any deal and loses even more, namely the full amount K. For a seller, the situation is symmetric.

3. Whether an equilibrium is achieved, and what its value then is, depends entirely on the history. When such an equilibrium occurs at a value larger than $K/2$, the sellers are better off, else the buyers.

4. The situation between the populations is symmetric, but we can then break the symmetry in various ways to investigate the effect of certain parameters or strategic options. For example, we could make the population sizes or the mutation rates different. We could also allow one population to choose from one class of possible strategies, and the other one from another class. We can then see whether an equilibrium will be reached and what its value is.

5. Combining the insights from the previous points, in principle, if that were allowed in the game, one of the populations could coordinate its actions so as to drive the equilibrium in a direction favorable to it. For example, at an equilibrium at value k, the buyers could consistently offer an amount less than k. Eventually, the sellers would then react and also lower their bid correspondingly, to avoid the failure of all interactions. Now, we are not allowing such coordinated actions in our game, but also different mutation rates or different strategic options for the members of the two populations could drive the equilibrium value in a direction more favorable to one of the populations.

As set up, our game is not a zero-sum game; it could be made into one by letting the buyer pay, and the seller receive, the amount $\frac{x+y}{2}$ in each successful interaction. Such a modification should not substantially affect the above points. The rule adopted by us could be interpreted as giving the difference $x-y$ to some middleman or trader. One could also look at the formally equivalent game where two partners with complementary ingredients give the amounts x and $z = K - y$ towards some common cause; that cause succeeds only when $x + z \geq K$; in that case, they each pay what they commit, that is x or z, while when it fails, they both suffer a loss of K. Again, they both wish to minimize their costs or losses. Thus, with respect to those individual interactions, our game is similar to, and in fact even simpler than, standard cooperation games like the prisoner's dilemma. One difference is that the number of options is much larger here; the players can choose any number k between 0 and K. Even though in our simulations, we restrict k to be an integer, this does not cause much of a difference as K can be chosen rather large. The more important difference, and what makes our set-up really interesting, is the division into two complementary populations from which the players are taken. This allows us to investigate how any symmetry breaking, for example when one of the populations starts exploring a new set of strategies, affects the resulting equilibrium,

or perhaps even prevents an equilibrium.
Thus, our game can serve to study the efficiency of different strategy sets available to, or adopted by, a population. The simplest meaningful type of strategy would be for each player to simply select one fixed value k and play that value all the time. The selection process should then assure that the two populations reach an equilibrium value. When random fluctuations like mutations occur, one should expect some safety margin, that is the buyers will settle on a slightly higher value than the sellers. If we also allow learning from experience, we naturally arrive at the following type of strategy: Each player develops a look-up table from which he selects his current bid as a function of the bid his opponent(s) in the previous round(s) chose. Of course, this can get unwieldy if the players can remember too many rounds from the past. Here, however, already the first non-trivial finding from our simulations becomes relevant. Namely, we observe that a population whose members remember only the value of the opponent's bid from the one most recent round achieves a favorable equilibrium value against a population that can look two rounds back. There are two simple and apparently general conceivalbe reasons for this that are partly in conflict with each other, but both of which seem to play a role here: Firstly, when the set of available options is smaller, an optimal strategy within that set can be found more quickly. In other words, a more complicated strategy may have so many entries to adapt that it takes such a long time to test them through exposure in the game and subsequent fitness evaluation that by that time, the other population with the simpler strategy space has already settled into an advantageous state from which it cannot be disposed anymore. Thus, a strategy that can be learned more quickly beats another one that takes longer to develop, because, according to the rules of our game, it can determine a favorable equilibrium from which it cannot be driven anymore once attained. Secondly, an explanation in the opposite direction is that with a richer strategy set, a population should also be able to react more accurately or quickly to fluctuations in the behavior of the other population, and then, for example, the sellers could adjust their own bids downwards more readily in response to some erratic behavior of some of the buyers. This is similar to the red king effect coined by Lachmann and Bergstrom[1] where it also is advantageous to respond more slowly to the opponent's actions. This finding may also have some interest in the context of the rationality issue in economic theory. There, the ideal is an actor that is in full command of all the available information when selecting his actions. Of course, it is questionable to what extent such an assumption is realistic, and this then becomes an empirical question at the intersection of economics and psychology. Our findings, however, shed a little light on this issue from a different perspective. Namely, we observe that a population whose individuals consistently adopt a less rational strategy, in the sense that they utilize a smaller amount of the available information from their own experience can do better in competition with one whose members make more use of that information, and so are more rational in the sense of economic theory. Of course, the issue here is not that the individual agent is doing worse when he tries to act in a cleverer manner, and in fact, such an agent could certainly gain an individual advantage

in comparison with his more stupid fellows. The point is rather that a population of individuals that individually act more stupidly can drive an equilibrium between populations to its advantage.

Another choice of strategies is the more indirect one of imitating successful fellows instead of directly responding to the bid of the previous opponent. Formally, we introduce a network structure into a population. In order to be able to compare different network types, we fix a value m for the average connectivity of the nodes in the network. Those nodes represent the individual members of the population, and an edge between two nodes stands for some form of acquaintance or information sharing which we call "friendship" for simplicity. The friendship strategy then simply consists in taking the average (or some weighted average in a more refined version) of the bids of those friends of the player that have had a successful interaction in the previous round. This could, but need not, include the own previous bid in case it was successful as well. If none of those bids from the previous round has been successful, the player makes a random bid. This is, of course, the standard option in all such situations where the adopted strategy does not apply to compute a bid.

We can then let different network topologies, like regular, random, small-world or scale-free, play against each other. So far, no network type has been consistently superior to all other ones. When we let a population with such a friendship network strategy play against one that chooses the one-round opponent bid strategy, the latter in most cases, but not always, gets an advantage, in the usual sense of an equilibrium value that is more favorable to it. Letting a friendship network play against a two-round opponent strategy did not show a decisive advantage on average for either of them.

Another issue that might play some role here is the homogeneity or heterogeneity of a population. Since we are not allowing direct coordination of the actions of the members of our population, except indirectly as for example through a friendship network, homogeneity of a population can indirectly enforce strong similarities between the individual behaviors. This might then be advantageous for driving the equilibrium towards a value that is favorable for the population in question. On the other hand, any such equilibrium is stable once it is reached and, in particular, predictability cannot be exploited here. Therefore, the only chance to modify the equilibrium value after the transient period consists in triggering a reaction of the opposite population through random or systematic deviations in the preferred direction. This could be achieved through mutations, or by other means for making the population heterogeneous enough to always maintain some deviating strategies by some of its members. Again, those deviating members will probably be less successful than the more conforming ones, and so, they will be eliminated by selection. The conclusion is that a population as a whole can gain when it can develop a mechanism for producing such individually less successful members. This leads to the issue of group selection, amply discussed in theoretical evolutionary biology, which, however, we do not intend to enter here more deeply.

2 Games between populations

To clarify our setting, we discuss here how it fits into the standard theory of games between two populations. A good reference for that topic is [4]. There, such games are treated as extensions of games within a single population. Players are randomly paired, but each player faces an opponent from the opposite population. It is instructive to subject the strategy evolution to replicator dynamics, as systematically developed in [3] and applied to 2-population games in [4]. To see how this applies to our example, we start with the simplified situation where each player has only two strategic options; to achieve as much symmetry as possible, we play the complementarity game described in the introduction as an equivalent version of our buyer-seller game.[1] Thus, each player has the options to offer either 1 or 2, and when the sum of the two offers is at least 3, each player receives 3 minus its own offer; else neither receives anything. Thus, the pay-off matrix is

$$\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Here, the ith row stands for the strategy i of a player, the jth column for the corresponding strategy of his opponent. When x^i denotes the proportion of players playing i in the first population, y^i the corresponding number in the second one, we can set up the corresponding replicator dynamics as

$$\dot{x}^1 = x^1(2y^2 - x^1(2y^2) - x^2(y^1 + y^2)) \quad (1)$$
$$\dot{x}^2 = 1 - \dot{x}^1 \quad (2)$$

since $x^1 + x^2 = 1$, and the symmetric formulae for the y^j. Putting $x := x^1$, $y := y^1$ for simplicity, we arrive at the system

$$\dot{x} = x(1-x)(1-2y) \quad (3)$$
$$\dot{y} = y(1-y)(1-2x) \quad (4)$$

This system has the two stable equilibria $(x, y) = (1, 0)$ and $(0, 1)$, with the line $x = y$ separating the two basins of attraction. Thus, whichever population has initially the larger number of players of strategy 1 will turn the game to its advantage.

The two basins of attraction will remain of equal size even when one of the population adapts more slowly than the other one. In the extreme case when the second one remains static, that is, $\dot{y} = 0$, the two basins of attraction[2] are

[1] After interchanging the rows in the pay-off matrix introduced below, the game becomes equivalent to the trading complementarities game analyzed in 10.4.3.1 of [4] who in turn refers to [2]. Thus, in particular, the analysis presented there applies to the problem at hand when played within a single population, and the extension to two populations is straightforward. The setting we adopt here, however, is better suited for our purpose of enlarging the strategy space.

[2] In this limiting case, the attractors are no longer points, but line segments, but that does not affect the discussion.

separated by the line $y = 1/2$; when y is below that value, x will tend to 1, else to 0.
When we extend the game to allow three values for the offers, 1,2, and 3, with a sum of 4 needed to give each player a pay-off of 4 minus its offer, the pay-off matrix is

$$\begin{pmatrix} 0 & 0 & 3 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

and the replicator dynamics for the strategies of the first population is

$$\begin{aligned} \dot{x}^1 &= x^1(3y^3 - x^1(3y^3) - x^2(2y^2 + 2y^3) - x^3) && (5) \\ \dot{x}^2 &= x^2(2y^2 + 2y^3 - x^1(3y^3) - x^2(2y^2 + 2y^3) - x^3) && (6) \\ \dot{x}^3 &= x^3(1 - x^1(3y^3) - x^2(2y^2 + 2y^3) - x^3) && (7) \end{aligned}$$

where we have used $y^1 + y^2 + y^3 = 1$ to simplify the coefficient of x^3, and again symmetric formulae for the strategies y^j of the second population. So, that population whose pay-off is largest will also grow fastest. In particular, when all three strategies are initially equally represented in the second population, that is $y^1 = y^2 = y^3$, then x^2 will grow at the fastest rate. Stable equilibria are at $x^1 = 1, y^3 = 1$, $x^2 = 1, y^2 = 1$ and $x^3 = 1, y^1 = 1$, among which the middle one has the largest basin of attraction. The reason is that it has the highest pay-off when averaged over the strategies of the opposite population. The pattern and its extension to more strategies is obvious.
The following heuristic consideration is useful for understanding some of the sequel better. As long as y^3 is not the dominant strategy in the other population, x^3 has a higher growth rate than x^1. Namely,

$$\left(\frac{x^3}{x^1}\right)^{\cdot} = \frac{x^3}{x^1}(y^1 + y^2 - 2y^3). \tag{8}$$

Thus, for example when we are near the equilibrium point $x^2 = 1, y^2 = 1$ and the second population is subjected to some random perturbation while the first one then is allowed to adapt, we expect a preference for strategy 3 over 1 to develop. Thus, the population that can adapt more quickly will tend to prefer the more cautious strategy 3 over the bolder strategy 1. The slower population will then see an increase of 3 in the other population and can thus increase the rate for its own strategy 1. Thus, the situation will develop in its favor. This is similar to the red king effect of [1], with the difference that here not the faster responding player is put at an individual disadvantage, but rather the faster and individually beneficial responses of the members of a population ultimately are disadvantageous to the population as a whole and thus also for its individual members.
Another reason why a faster evolving population could prefer strategy 3 over 1 is a finite size effect. Namely, the players of 1 undergo a higher risk of being

eliminated when their performance over finitely many interactions is counted as their fitness.
However, the slight preference for 3 over 1 should not be seen as a result of stochastic instability because the stable and dominant strategy is 2.
In any case, we should point out that neither of the preceding is the correct explanation for the phenomena we see below where a population with simpler strategies beats those with more complex ones. Essentially, a more restricted set of strategic options will focus the agents better in the relevant subset of the strategy space.
In any case, if one population would have a central coordination mechanism and sufficient foresight, it could clearly drive the other, uncoordinated one into an equilibrium that is advantageous for itself. However, we shall not allow any such thing in our simulations.

3 Metastrategies

Thus, the standard replicator analysis of our game is rather trivial. It depends on the initial relative frequencies of the strategies in the two populations which of the many possible equilibria is reached. As explained in the introduction, we wish to apply here a different evolutionary setting than the one encoded in the replicator dynamics. Namely, we offer the players richer strategic options; for example, they could take their past experiences into account, or follow their successful friends. (Those particular options are similar to, but not identical with those considered in [4], called "best response" and "imitation".) The question addressed in our simulations then is which kind of metastrategy when applied by all players in a population will bring that population into the basin of a favorable attractor.
One emergent feature of our simulation results is that typically a population with a more restricted set of strategic options does better in competition with one with a larger set. A reason for this is that when there are fewer response options available each single response has a higher frequency of being chosen and thereby subjected to an evolutionary test. It thus has more opportunities to exhibit its fitness and to adapt.
Let us consider the more concrete case where the members of one population, labelled 1, can only use the simplest possible strategy, namely to choose one fixed value for their offer whereas the members of the other population, called 2, are able to determine their current bid on the basis of the bid of the opponent from the previous encounter. That is, they have a look-up vector with K entries where the kth element gives their response, that is, their current bid, when the previous opponent had been asking k. Of course, in principle, such an agent could just choose a constant vector, that is, select its current bid independently of its previous experience, and so, the richer strategy set includes the simpler one. Nevertheless, it turns out that the resulting equilibrium is in favor of the population with the simpler strategies. This can be understood as follows. The

members of 2 have only one value which then is subjected to many evolutionary test, and so, the population 2 might rather quickly settle their values within some restricted range. Population 1 will then only adapt those values of the look-up vectors that correspond to those values exhibited by 2. A mutant from 2 with a more risky strategy then, while having a lower expectation value for its fitness, still has a positive probability to gain a higher fitness because he can explore a region in which the members of 1 have not yet adapted and behave more or less randomly. Some more members of population 2 can then follow when we renew the population evolutionarily according to their fitness. This then forces the members of 1 to adapt accordingly, because those that do not bow to the more aggressive behavior of their opponents risk being eliminated by our evolution algorithms since many failed trades lower their fitness. The point is that the mutant from 2 were essentially facing a random behavior from 1 whereas the members of 2 now are confronted with a more systematic behavior from 2 which forces them to yield.

4 Simulation Results

In our simulations, we choose two populations of $N = 400$ individuals each, the buyers and the sellers. In each round, each buyer is randomly paired with a seller, and they both make offers between 0 and $K = 49$ or $K = 99$ (only in one simulation K is chosen to be 19). After 1000 rounds, the population for the next generation is determined, by letting the present population reproduce differentially according to the fitness accumulated during those rounds. When the strategy consists of a look-up vector or matrix where the ith entry, or the ij matrix element tells the agent what to bid when his opponent from the previous round bid i, or the two previous opponents bid i and j, resp., we also adopt genetic recombination as in standard genetic algorithms to construct the next generation. We use a cross-over probability of 0.70.

Before presenting the simulations of our game, we would like to define some variables that will be used later. The *success rate* of deals taking place at time step t is defined as $m(t)/N$, where $m(t)$ represents the number of successful deals, that is, those where the buyer offered at least as much as the seller asked, at time t and N is the number of buyers or sellers. We define the $median\ fee$ as

$$\frac{\sum_{i=1}^{m(t)} Off_{buy}(i,t)}{m(t)} - \frac{\sum_{i=1}^{m(t)} Off_{sel}(i,t)}{m(t)}, \tag{9}$$

where $Off_{buy}(i,t)(Off_{sel}(i,t))$ is the offer of the ith successful buyer (seller) at t. In addition, we use $evolutionary\ difference$ to monitor only the decreasing values of $median\ fee$, that is, its value at t is the minimum of all values of $median\ fee$ between time steps 0 and t.

We start with the simplest conceivable strategy, namely the one where each agent can only choose one fixed value for all his bids. This value then is subjected to the standard evolutionary scheme, that is, after a fixed number of

rounds has been played, agents reproduce differentially according to their fitness.
So far, the situation is symmetric between the two populations, but we can easily break the symmetry by introducing mutations between generations and to assign different mutation rates to the two populations. Thereby, we can understand the role of the random mutation rate in our game and find out whether an optimized value of this mutation rate exists and what its value then is. In most simulations, the population with a small random mutation rate, like 0.01, is doing slightly worse than the opponent side with a much higher mutation rate like 0.04 or 0.05 (see Fig. 1 (a)). We also observed that the side being subject to random mutations with a very high rate, like 0.1, can do slightly worse than the other side who is subject to random mutations with relatively lower rates, like 0.05, as in Fig. 1 (b). However, 0.04 or 0.05 is not the best value in all simulations, and there is no exception-free conclusion possible here. Nevertheless, the values from 0.01 to 0.05 will be good parameters, given the population size of our models.
After these preparations, we can discuss simulations with richer strategic options for the players. First we come to the simulations of our game where all the players, both the buyers and the sellers, choose their bids on the basis of their own past experiences. More specifically, players' offers at time $t+1$ are functions of offers of their opponents met at time t. Say, if a buyer's last-round opponent asks k, then his bid at the current round will be the kth entry in the vector of his own look-up table. In these simulations, the random mutation rate is 0.01. Fig. 2 (a) shows in one simulation the variation of success rate with respect to the time t. As we can see from the figure, the success rate is improved very quickly. After only 15 generations, the success rate has been improved from 0.5 to around 0.8. The success rate is lowered then a little bit but reaches almost 0.9 after 20 generations and gets even higher after 25 generations. At the same time, the median fee has been lowered down to a very small value, around 1.0, which has been plotted in Fig. 3 (a). The above two figures indicate that the learning process of the players is ideal. We also plotted from the same simulation evolutionary difference versus time in Fig. 4, which shows first rapid, then slow and smooth decrease.
We next discuss the simulation results of our game where both populations choose their bids on the basis of their friends' past successful experiences in the same population. The friendship strategy is implemented by constructing networks where players are located. The four types of standard networks adopted in our simulations are regular, small-world (as constructed by the Strogatz-Watts method [5]), random (after Erdös-Renyi [6]) and scale-free (as constructed by the Barabasi-Albert algorithm [7]). In order to make the results of different network topologies comparable, we set the average connectivity of all networks to be fixed, for instance, 4. Here the offer of a player at the present round is a function of the average of his friends' last-round successful offers. In case there is no successful friend in the previous round, he will simply select a random value between 0 and K. One minor thing is that the successful experiences of a player himself can also be considered. But according to our simulations includ-

ing it or not does not cause too much difference. Fig. 3 (b) shows the success rate versus the time t when both populations are located on regular friendship networks. Surprisingly, the improvement of the success of deals here is even more remarkable than what has been shown in Fig. 2 (a). We see that after only around 25 generations the success rate approaches 1.0 and then remains quite stable. Fig. 3 (b) gives the median fee versus the time t. Again, this difference can be significantly decreased. We have also run extensive simulations of our game when the network topologies are small-world, random and scale-free. These simulations display similar behaviors as what we have seen in Fig. 2 (b) and Fig. 3 (b). Another finding is that there is no network type among the four ones considered that is systematically superior to the other ones. Also, adding another layer of complexity, namely an evolutionary optimization of networks based on the combined fitnesses of their members does not lead to a particular network paradigm. Fig. 5 shows part of our simulations when buyers and sellers have different friendship network strategies. The outcome depends more on the (random) initial conditions than on the network types involved.

We next investigate whether more available information will be definitely advantageous to the players who have them. Buyers and sellers will now be allowed to utilize different amounts of information. More concretely, we say that they have a 1-round opponent strategy when the members of the population can only look one round back (they recall only their opponents' bids of the most recent round). If they can look two rounds back, then they use a 2-round opponent strategy. Obviously, the players who have a 2-round opponent strategy have access to more information than those who have the 1-round opponent strategy. We then compare the intrinsic dynamic of a game with both populations using a 1-round opponent strategy with the one where both use 2 rounds. The comparison is plotted in Fig. 6. We note that when the members of both populations can use only the most recent round information the learning process is much quicker and more efficient than when they have a 2-round opponent strategy. In the 1-round opponent strategy simulation, the success rate becomes optimized and stable after only 10 generations. While in the 2-round one, it takes almost 100 generations to reach the same position as the former one. To make the comparison more convincing we can play 1-round opponent strategy against 2-round opponent strategy. This means we let, for example, buyers take a 1-round opponent strategy and sellers adopt a 2-round opponent strategy, or reversely. The simulation of such a game will allow us to check whether the symmetry will be broken, whether an equilibrium will be reached, and most importantly, for which side the equilibrium value will be more favorable. Our extensive simulations show that the equilibrium will favor the side holding less information, i.e., with 1-round opponent strategy. Fig. 7 (a) shows one simulation where buyers can recall one round back and the sellers can recall two. As we can see, the equilibrium value of buyers is 20, smaller than 25. Fig . 7 (b) shows the reverse case of Fig. 7 (a) and the situation then is more favorable to the sellers who have less information.

It is also interesting to compare the value of the information obtained from the successful experiences of friends and that from one's own past experiences

based on the opponents' behaviors. So we can let one population choose the friendship network strategy and another side have a 1-round opponent strategy or a 2-round opponent strategy. Again, there are many possibilities here. Our simulations indicate that in most cases, but not always, the 1-round opponent strategy gains an advantage over friendship network strategies. As shown in both Fig. 8 (a) and Fig. 8 (b), the equilibrium values here are more favorable to the side taking 1-round opponent strategy. Simulations were also done for other friendship network topologies against the 1-round opponent strategy and showed similar results. Another finding of our simulations was that the play between the friendship network strategies and the 2-round opponent strategy does not lead to a decisive conclusion. In some runs, the population with the network strategies can achieve a better position. But when the initial conditions are changed, the situation may favor the population with the 2-round opponent strategy. Two of our simulations of friendship network strategy against 2-round opponent strategy were plotted in Fig. 9. One interesting byproduct of our simulations is that a population with simpler friendship network strategies, like averaging over all friends without distinguishing them by success or not, have better chances to win over the population with 1-round opponent strategy than that with more refined friendship network strategies, like averaging (or even weighted, or evolved weighted) only over successful friends. This important finding suggests again, among other things, that simpler and more flexible strategies may be good choices in the competition.
The following simulations support our conclusion that the achieved equilibrium may favor the population with more restricted strategic options. In such a case, each member of one population takes the simplest strategy discussed above, of selecting a fixed value as his offer. The simplest strategy is then subject to a selection process with only random mutations occurring at a very small probability. The members of the other side, however, can have richer and more complex strategies. They could have a 1-round opponent strategy or a network friendship strategy. We played the two populations against each other and found that in most cases the simple strategy can get a slight advantage over the 1-round opponent strategy, shown in Fig. 10. The play of the simple strategy against the friendship network strategies displays, however, a more interesting behavior. As shown in Fig. 11 (a), initially the population with the simple strategy has a prominent advantage over the opponent side with the friendship network strategies. We notice that the evolution of the population with the friendship network strategies is almost suppressed and appears rather slow. As the time goes on and the generations continue, however, the population with the network strategies may have a good chance to win over its opponent side, see Fig. 11 (b) and (c). But such a process takes much longer time than ever required in other simulations. For example, to acquire a better position, the population may need 1000 or more generations.

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Figure Captions:

Fig. 1: The average successful offer versus time when both populations take simple strategies but with different random mutation rates. Average successful offer includes the average offer of successful buyers and that of successful sellers. (a) Mutation rate in the buyer population is 0.01 and for the sellers, 0.05; (b) Mutation rate for buyer population is 0.05 and for sellers, 0.1. Simulations were done with $N = 400$ and $k = 99$.

Fig. 2: Success rate versus time for the game where both populations make offers according to (a) their own past experiences (based on the behaviors of their opponents met in the most recent round) and (b) their friends' successful experiences (here players are interacted through regular networks). The simulations were done with $N = 400$ and $K = 49$.

Fig. 3: Median fee versus time for the game where both populations make offers according to (a) their own past experiences (based on the behaviors of their opponents met in the most recent round) and (b) their friends' successful experiences (here players are interacting through regular networks). The simulations were done with $N = 400$ and $K = 49$.

Fig. 4: Evolutionary difference versus time for the game where both populations make offers according to their own past experiences. The simulation was done with $N = 400$ and $K = 49$.

Fig. 5: The average successful offer versus time for friendship network strategies playing against each other. (a) Buyers adopt a small-world network strategy and sellers use a regular network strategy; (b) Buyers take a random network

strategy and sellers take a regular network strategy; (c) Buyers take a scale-free network strategy and sellers take a random network strategy; (d) Buyers take a scale-free network strategy and sellers take a small-world network strategy. Simulations were done with $N = 400$ and $k = 49$.

Fig. 6: Comparison of success rate versus time for two simulations where both populations use the 1-round opponent strategy and the 2-round opponent strategy, respectively. The top curve corresponds to the game with 1-round opponent strategy and the bottom to two-round opponent strategy. The simulations were done with $N = 400$ and $K = 19$.

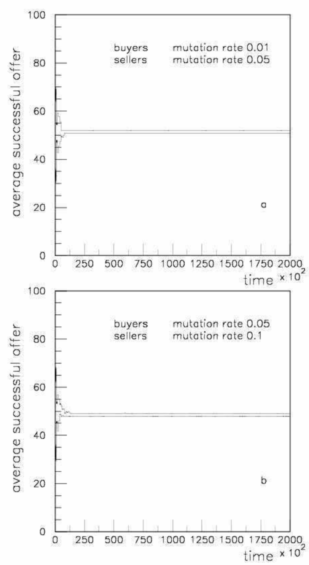
Fig. 7: The average successful offer versus time for 1-round opponent strategy playing with 2-round opponent strategy. (a) Buyers use the 1-round opponent strategy and sellers the 2-round opponent strategy; (b) Buyers have the 2-round opponent strategy and sellers have the 1-round opponent strategy. Simulations were done with $N = 400$ and $k = 49$.

Fig. 8: The average successful offer versus time when friendship network strategy plays against 1-round opponent strategy. (a) Buyers have regular network strategy and sellers have the 1-round opponent strategy. (b) Buyers have the 1-round opponent strategy and sellers have the regular network strategy. Simulations were done with $N = 400$ and $k = 49$.

Fig. 9: The average successful offer versus time when friendship network strategy plays against 2-round opponent strategy. (a) Buyers take the random network strategy and sellers have the 2-round opponent strategy. (b) Buyers use the 2-round opponent strategy and sellers the small-world network strategy. Simulations were done with $N = 400$ and $k = 49$.

Fig. 10: The average successful offer versus time for simple strategy playing against 1-round opponent strategy. (a) Buyers adopt the 1-round opponent strategy and sellers the simple strategy; (b) Buyers take the simple strategy and sellers have the 1-round opponent strategy. Simulations were done with $N = 400$ and $k = 99$.

Fig. 11: Buyers use the simple strategy and sellers have the scale-free network strategy. Here we show the average successful offer versus time (a) from 0 to 400,000 time steps; (b) from 400,001 time steps to 800,000 time steps and (c) from 800,001 time steps to 1000,000 time steps. Simulations were done with $N = 400$ and $k = 99$.



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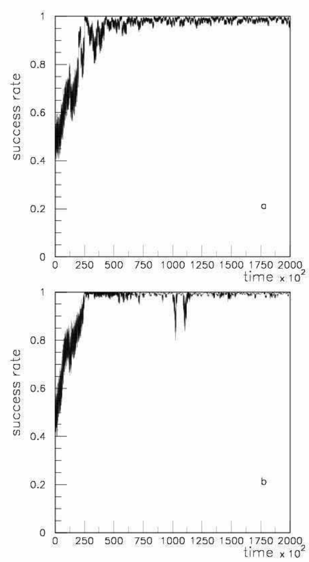

Fig. 2

Figure 2: Success rate versus time for the game where both populations make offers according to (a) their own past experiences (based on the behaviors of their opponents met in the most recent round) and (b) their friends' successful experiences (here players are interacted through regular networks). The simulations were done with $N = 400$ and $K = 49$.

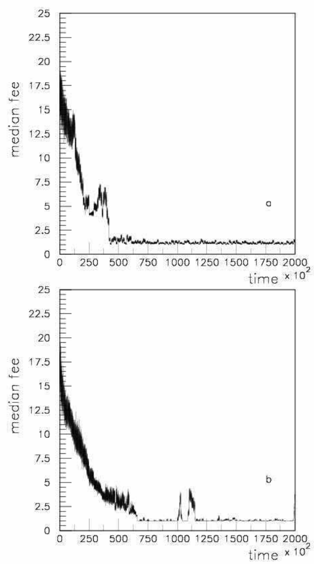

Fig. 3

Figure 3: Median fee versus time for the game where both populations make offers according to (a) their own past experiences (based on the behaviors of their opponents met in the most recent round) and (b) their friends' successful experiences (here players are interacting through regular networks). The simulations were done with $N = 400$ and $K = 49$.

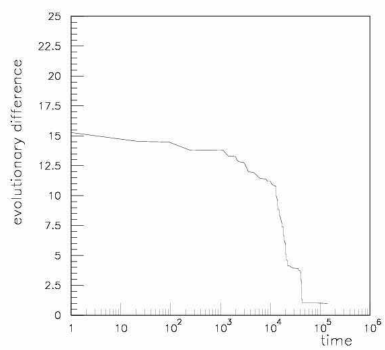

Fig. 4

Figure 4: Evolutionary difference versus time for the game where both populations make offers according to their own past experiences. The simulation was done with $N = 400$ and $K = 49$.

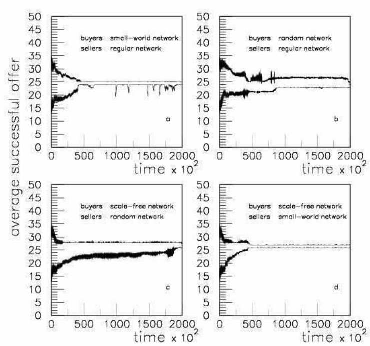

Fig. 5

Figure 5: The average successful offer versus time for friendship network strategies playing against each other. (a) Buyers adopt a small-world network strategy and sellers use a regular network strategy; (b) Buyers take a random network strategy and sellers take a regular network strategy; (c) Buyers take a scale-free network strategy and sellers take a random network strategy; (d) Buyers take a scale-free network strategy and sellers take a small-world network strategy. Simulations were done with $N = 400$ and $k = 49$.

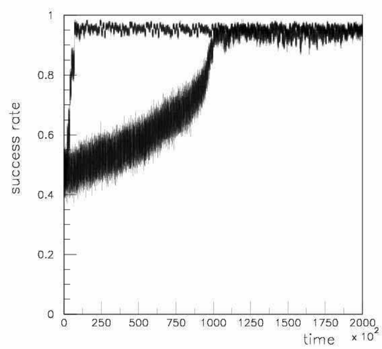

Fig. 6

Figure 6: Comparison of success rate versus time for two simulations where both populations use the 1-round opponent strategy and the 2-round opponent strategy, respectively. The top curve corresponds to the game with 1-round opponent strategy and the bottom to two-round opponent strategy. The simulations were done with $N = 400$ and $K = 19$.

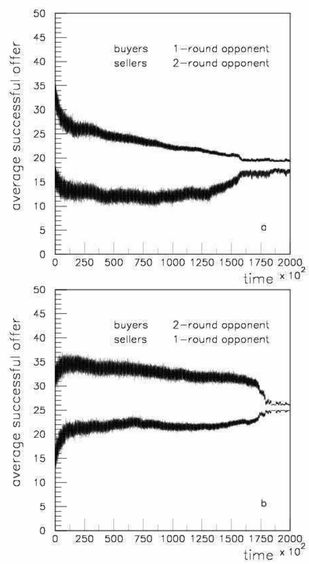

Figure 7: The average successful offer versus time for 1-round opponent strategy playing with 2-round opponent strategy. (a) Buyers use the 1-round opponent strategy and sellers the 2-round opponent strategy; (b) Buyers have the 2-round opponent strategy and sellers have the 1-round opponent strategy. Simulations were done with $N = 400$ and $k = 49$.

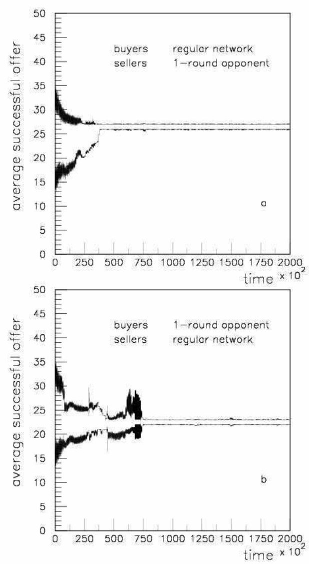

Fig. 8

Figure 8: The average successful offer versus time when friendship network strategy plays against 1-round opponent strategy. (a) Buyers have regular network strategy and sellers have the 1-round opponent strategy. (b) Buyers have the 1-round opponent strategy and sellers have the regular network strategy. Simulations were done with $N = 400$ and $k = 49$.

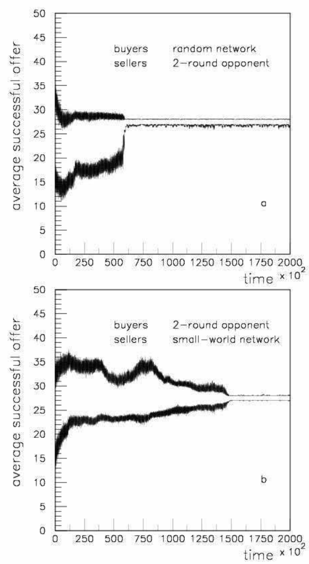

Figure 9: The average successful offer versus time when friendship network strategy plays against 2-round opponent strategy. (a) Buyers take the random network strategy and sellers have the 2-round opponent strategy. (b) Buyers use the 2-round opponent strategy and sellers the small-world network strategy. Simulations were done with $N = 400$ and $k = 49$.

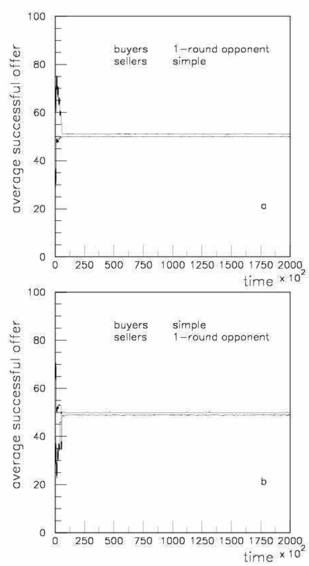

Figure 10: The average successful offer versus time for simple strategy playing against 1-round opponent strategy. (a) Buyers adopt the 1-round opponent strategy and sellers the simple strategy; (b) Buyers take the simple strategy and sellers have the 1-round opponent strategy. Simulations were done with $N = 400$ and $k = 99$.

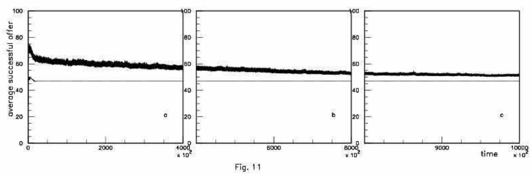

Figure 11: Buyers use the simple strategy and sellers have the scale-free network strategy. Here we show the average successful offer versus time (a) from 0 to 400,000 time steps; (b) from 400,001 time steps to 800,000 time steps and (c) from 800,001 time steps to 1000,000 time steps. Simulations were done with $N = 400$ and $k = 99$.