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Non–semisimple gaugings of $D = 5$ $\mathcal{N} = 8$ Supergravity

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Abstract: The recent construction of the non–semisimple gaugings of maximal $D = 5$ supergravity is reported here. This construction is worked out in the so–called rheonomic approach, based on Free Differential Algebras and the solution of their Bianchi identities. In this approach the dualization mechanism between one–forms and two–forms is more transparent. The lagrangian is unnecessary since the field equations follow from closure of the supersymmetry algebra. These theories contain $12 - r$ self–dual two–forms and $15 + r$ gauge vectors, r of which are abelian and neutral. Such theories, whose existence has been proved and their supersymmetry algebra constructed, have potentially interesting properties in relation with domain wall solutions and the trapping of gravity.

1 Introduction

Gauged supergravity with a maximal compact group, $\mathcal{G} = SO(6)$ in $D = 5$ [1, 2], $\mathcal{G} = SO(8)$ in $D = 4$ [3] and $\mathcal{G} = USp(4)$ in $D = 7$ [4] has attracted much renewed attention in the last two years because of the AdS_{p+2}/CFT_{p+1} correspondence (for a general review see [5] and references therein). Indeed the maximally supersymmetric vacuum of these gauged supergravities is the AdS_D space and the compact gauge group $\mathcal{G}_{gauge}$ is the $\mathcal{R}$-symmetry of the corresponding maximally extended supersymmetry algebra. However the compact gaugings are not the only ones for extended supergravities. There exist also versions of these theories where the gauge group $\mathcal{G}_{gauge}$ is non–compact. Unitarity is preserved because in all possible extrema of the corresponding scalar potential the non–compact gauge symmetry is broken to some residual compact subgroup. Furthermore, there are models in which the gauge group is non–semisimple.

The non–semisimple gauged supergravities are relevant for a close relative of the *AdS/CFT* correspondence, namely the *Domain Wall/QFT correspondence* [6] between gauged supergravities and quantum field theories realized on certain domain wall solutions of either string theory or M–theory. Recently, it has been shown [7] that these domain wall solutions are related with localization of gravity. So, non–semisimple gauged supergravities could be good candidates to accomodate the Randall Sundrum scenario [8] into supergravity theories.

Here we report the recent construction [9] of the non–semisimple gaugings for $D = 5$ maximal supergravity. In the case of $D = 4$ maximal supergravity, these gaugings has been worked out in the eighties [10], but some difficulties prevented up to now their construction in the five dimensional case. These difficulties are related to the feature of one–form/two–form duality, typical of five–dimensions. As long as all vector fields are abelian we can consider them as one–form or two–form gauge potentials at our own will. Yet when we introduce non–abelian gauge symmetry matters become more complicated, since only 1–forms can gauge non–abelian groups while 2–forms cannot. On the other hand 1–forms that transform in a non–trivial representation of a non abelian gauge group which is not the adjoint representation are equally inconsistent. They have to be replaced by 2–forms and some other mechanism, different from gauge symmetry has to be found to half their degrees of freedom. This is self–duality between the 2–form and its field strength [11]. Hence gauged supergravity can only exist with an appropriate mixture of 1–forms and self–dual 2–forms. While this mixture was mastered in the case of compact and non–compact but semisimple gaugings, the case of non–semisimple algebras is more involved. Furthermore, it appears problematic to write a Lagrangian for the theories with non–semisimple gaugings. Again, the problems come from the mixture of 1–forms and self–dual 2–forms.

The catch of [9] is the use of the geometric approach (for a review of this topic see [12]) where the mechanism of one–form/two–form dualization receives a natural algebraic formulation and explanation. The result is that in the case of the non–semisimple gaugings there are $15+r$ gauge vectors and $12-r$ self–dual two–forms. 15 of the vectors gauge the non–semisimple algebra while r of them have an abelian gauge symmetry with respect to which no field in the theory is charged. At the same time these vectors are neutral with respect to the transformations of the gauge algebra.

In the case of $r > 0$ extra neutral vector fields, although field equations can be normally derived from closure of the supersymmetry algebra, a lagrangian of conventional type might not exist, just as it happens for type IIB supergravity in $D = 10$ (after all, this is not terribly surprising since $\mathcal{N} = 8$ supergravities in five dimensions should eventually be interpreted in terms of brane mechanisms and compactifications from type IIB superstring). This would make impossible the construction of the theory by means of lagrangian–based techniques. However in our construction, the existence of a Lagrangian is not fundamental, the existence of the theory following from the consistent closure of Bianchi identities.

The scalar potential of these supergravities can be systematically derived, together with the complete field equations, from the closure of the supersymmetry algebra we have determined in [9]. This is completely algorithmic and straightforward, but it involves lengthy calculations that are postponed to a forthcoming publication [13].

2 $D = 5$ $\mathcal{N} = 8$ supergravity

In this section we recall the main features of $D = 5$ $\mathcal{N} = 8$ supergravity theory [1], [2]. The supersymmetry algebra for the ungauged theory is the superPoincaré superalgebra, whose external automorphism symmetry (the $\mathcal{R}$-symmetry) is $USp(8)$. The theory is invariant under local $ISO(4{,}1)\times USp(8)$ and global $E_{6(6)}$ transformations, and under local supersymmetry transformations, generated by 32 real supersymmetry charges, organized in 8 pseudo–Majorana spinors.

The theory contains: the graviton field, namely the fünfbein 1–form V^a, eight gravitinos $\psi^A \equiv \psi^A_\mu\, dx^\mu$ in the **8** representation of $USp(8)$, 27 vector fields $A^\Lambda \equiv A^\Lambda_\mu\, dx^\mu$ in the **27** of $E_{6(6)}$, 48 dilatinos χ^{ABC} in the **48** of $USp(8)$, and 42 scalars ϕ that parametrize the coset manifold $E_{(6)6}/USp(8)$, and appear in the theory through the coset representative $\mathbb{L}_\Lambda{}^{AB}(\phi)$, in the $(\mathbf{27},\overline{\mathbf{27}})$ of $USp(8)\times E_{6(6)}$. The local $USp(8)$ symmetry is gauged by a composite connection built out of the scalar fields.

Let us consider the gauged theories. In maximal supergravities, where no matter multiplets can be added, *gauging* corresponds to the addition of suitable interaction terms that turn a subgroup $\mathcal{G}$ of the global $E_{(6)6}$ duality group into a local symmetry. This is done by means of vectors chosen among the 27 A^Λ. The $E_{(6)6}$ symmetry is broken to the normalizer of $\mathcal{G}$ in $E_{(6)6}$, and after this operation the new theory has a local symmetry $USp(8)\times\mathcal{G}$ and a global symmetry $N(\mathcal{G}, E_{(6)6})$. From group theoretical considerations one can derive necessary conditions which strictly constrain the choice of $\mathcal{G}$. Such conditions are satisfied only by the semisimple groups $SO(p,q)$ with $p+q=6$ and their non–semisimple contractions $CSO(p,q,r)$, which will be discussed in section 5 (see [10, 14] for definitions). The possible gaugings are then restricted to these groups. The normalizer in $E_{(6)6}$ of all these groups is $SL(2,\mathbb{R})$. Therefore this latter is the residual global symmetry for all possible gaugings. The 27 vectors A^Λ are then decomposed into the vectors A_{IJ} in the $(\overline{\mathbf{15}},\mathbf{1})$, that gauge $\mathcal{G}$, and the vectors in the $(\mathbf{6},\mathbf{2})$, which do not gauge anything. In the $SO(p,q)$ gaugings, which have been built in [1, 2], the latter 12 vectors have to be dualized into two–forms $B^{I\alpha}$, as we will explain in the following. For all the admissible cases, the fifteen generators G^{IJ} of $\mathcal{G}$ have, in the fundamental **6**–dimensional representation, the form

$$(G^{IJ})^K{}_L = \delta^{[I}_L \eta^{J]K} \tag{1}$$

where η^{JK} is a diagonal matrix with p eigenvalues equal to 1, q eigenvalues equal to (-1) and, only in the case of contracted groups, r null eigenvalues. This signature completely characterizes the gauge groups and correspondingly the gauged theory. The covariant derivative with respect to $\mathcal{G}$ of a field V^I in the **6** of $SL(6,\mathbb{R})$ is defined as

$$DV^I \equiv \mathcal{D}V^I + g(G^{KL})^I{}_J A_{KL}\wedge V^J\,. \tag{2}$$

where $\mathcal{D}$ is the Lorentz–covariant exterior derivative.

The field content of the (semisimple) gauged supergravity theories is the following:

#	Field	$(SU(2)\times SU(2))$–spin rep.	$USp(8)$ rep.	$\mathcal{G}$ rep.
1	V^a	$(1,1)$	**1**	**1**
8	ψ^A	$(1,1/2)\oplus(1/2,1)$	**8**	**1**
15	A_{IJ}	$(1/2,1/2)$	**1**	**15**
12	$B^{I\alpha}$	$(1,0)\oplus(0,1)$	**1**	$\mathbf{6}\oplus\overline{\mathbf{6}}$
48	χ^{ABC}	$(1/2,0)\oplus(0,1/2)$	**48**	**1**
42	$\mathbb{L}_\Lambda{}^{AB}(\phi)$	$(0,0)$	**27**	$\overline{\mathbf{27}}$

(3)

3 Gauged supergravities from F.D.A.'s and Rheonomy

Gauged maximal supergravities in $D = 5$ were originally constructed within the framework of Noëther coupling and component formalism [1],[2]. As we pointed out, the gaugings corresponding to the contracted groups $CSO(p, q, r)$ were left open in that approach.

In [9], these theories have been constructed by using the approach based on free differential algebras (F.D.A.'s) and rheonomy. To obtain this result, we started by reformulating the $D = 5$ $\mathcal{N} = 8$ supergravities [1] [2] with semisimple gauge groups in the rheonomic framework [12]. Then, the extension to non–semisimple gaugings has been worked out.

Here we do not describe the rheonomic formulation of supergravity. For a comprehensive review, see [12], while for a short summary, in the context of $D = 5$ $\mathcal{N} = 8$ supergravity, see [9]. We only recall that this approach is based on the closure of the Bianchi Identities of the superspace curvatures. In this context, the Bianchi Identities are not identically satisfied. Actually, they are the equations of the theory, determining its dynamics. Not only they give the parametrizations of the curvatures (from which one can read off the supersymmetry transformations), but they also fix the geometry of the scalar manifold and give the classical field equations satisfied by the spacetime fields. From this viewpoint, the explicit construction of the Lagrangian $\mathcal{L}$ is not really needed. Simply, when $\mathcal{L}$ exists, the determination of the field equations is more easily obtained by $\delta\mathcal{L}$ variations than through the analysis of the Bianchi Identities. When the Lagrangian exists, it can be obtained by means of a straightforward procedure starting from the curvature parametrizations [12].

This construction has been performed in [9], by solving the Bianchi Identities of $D = 5$ $\mathcal{N} = 8$ supergravity, both for the semisimple and non–semisimple cases. The parametrizations of the curvatures, and then the supersymmetry transformations, have been determined (modulo bilinear in the dilatinos). In the semisimple case, they are

$$\delta V^a_\mu = -\mathrm{i}\bar{\varepsilon}^A\gamma^a\psi_{\mu A} \tag{4}$$

$$\begin{aligned}\delta\psi_{A\mu} &= \mathcal{D}_\mu\varepsilon_A - g\frac{2}{45}T_{AB}\gamma_\mu\varepsilon^B + \frac{2}{3}\mathcal{H}_{AB|\nu\mu}\gamma^\nu\varepsilon^B - \frac{1}{12}\mathcal{H}^{\nu\rho}_{AB}\gamma^{\lambda\sigma}\varepsilon^B\epsilon_{\mu\nu\rho\lambda\sigma} \\ &\quad + \frac{3\mathrm{i}}{2\sqrt{2}}\chi_{ABC}\bar{\varepsilon}^B\psi^C_\mu - \frac{\mathrm{i}}{2\sqrt{2}}\gamma_\nu\chi_{ABC}\bar{\varepsilon}^B\gamma^\nu\psi^C_\mu + \mathcal{O}(\chi^\in)\end{aligned} \tag{5}$$

$$\begin{aligned}\delta\chi_{ABC} &= \frac{1}{\sqrt{2}}gA^D_{ABC}\varepsilon_D + \sqrt{2}\hat{P}_{ABCD|i}\partial_\nu\phi^i\gamma^\nu\varepsilon^D - \frac{3}{2\sqrt{2}}\mathcal{H}_{[AB|\mu\nu}\gamma^{\mu\nu}\varepsilon_{C]} \\ &\quad - \frac{1}{2\sqrt{2}}\Omega_{[AB}\mathcal{H}_{C]D|\mu\nu}\gamma^{\mu\nu}\varepsilon^D + \mathcal{O}(\chi^2)\end{aligned} \tag{6}$$

$$\delta A_{IJ|\mu} = I\!\!L^{-1}_{ABIJ}\left[\frac{\mathrm{i}}{\sqrt{2}}\bar{\chi}^{ABC}\gamma_\mu\varepsilon_C + 2\mathrm{i}\bar{\varepsilon}^A\psi^B_\mu\right] \tag{7}$$

$$\begin{aligned}\delta B^{I\alpha}_{\mu\nu} &= I\!\!L^{I\alpha}{}_{AB}\left[-2\mathrm{i}g\bar{\varepsilon}^A\gamma_{[\mu}\psi^B_{\nu]} - \frac{\mathrm{i}}{2\sqrt{2}}g\bar{\chi}^{ABC}\gamma_{\mu\nu}\varepsilon_C\right] \\ &\quad + 2\mathcal{D}_{[\mu}\left[I\!\!L^{-1}_{AB}{}^{I\alpha}\left(2\mathrm{i}\bar{\varepsilon}^A\psi^B_{\nu]} + \frac{\mathrm{i}}{\sqrt{2}}\bar{\chi}^{ABC}\gamma_{\nu]}\varepsilon_C\right)\right]\end{aligned} \tag{8}$$

$$\hat{P}^{ABCD}_{,i}\delta\phi^i = 2\mathrm{i}\sqrt{2}\,\bar{\chi}^{[ABC}\varepsilon^{D]} + \frac{3\mathrm{i}}{\sqrt{2}}\,\Omega^{[CD}\chi^{AB]E}\varepsilon_E \tag{9}$$

as in [1]. In the semisimple case, also the Lagrangian and the equations of motion have been determined, and found to coincide with the results of [1]. We postpone to section 5 the analysis of the non-semisimple case.

4 The problem of the two–forms

It is a known fact [2], [1] that in order to consistently gauge the $\mathcal{N}=8$ theory, one has to dualize the vectors transforming in the $(\mathbf{6},\mathbf{2})$ of $SO(p,q)\times SL(2,I\!R)$ to massive two-forms obeying the self-duality constraint:

$$B^{I\alpha|\mu\nu} = m\epsilon^{\mu\nu\rho\sigma\lambda}\mathcal{D}_\rho B^{I\alpha}_{\rho\sigma\lambda} \tag{10}$$

with $m\sim g$. In the geometric formulation of the theory, this need for dualization emerges in a completely natural way. Indeed, let us start by considering the 12 vectors $A^{I\alpha}$. There is no way known to reconcile their abelian gauge invariance with their non-trivial transformation under the gauge group $\mathcal{G}$. Indeed, given the superspace curvatures

$$DA^{I\alpha} \equiv dA^{I\alpha} + g(G^{KL})^I{}_J A_{KL}\wedge A^{J\alpha} \tag{11}$$

it follows that the corresponding Bianchi Identities contain a term

$$DDA^{I\alpha} = g(G^{KL})^I{}_J \mathcal{F}_{KL}\wedge A^{J\alpha} \tag{12}$$

where the vectors $A^{J\alpha}$ appear naked. This makes impossible to write a parametrization of the curvatures covariant under the gauge group. Hence we have a clash between supersymmetry and the 12 abelian gauge invariances needed to keep the vectors $A^{J\alpha}$ massless. On the other hand, making them massive would destroy the equality of the Bose and Fermi degrees of freedom. So in the gauged case, where the 12 vectors $A^{J\alpha}$ acquire a non–trivial transformation under the non–abelian gauge symmetry, there is no way of fitting these fields into a consistent supersymmetric theory. The way out, as it was discussed in [1], is to interpret them as the duals of massive two-forms $B^{I\alpha}$, obeying a self–duality constraint which halves their degrees of freedom. This construction emerges naturally in the rheonomic framework [9]. In this context, one has to introduce superspace curvatures for the two–forms generalizing the Maurer–Cartan equations to a F.D.A. [12, 15]. At first sight it seems that we cannot escape from the problem described above, that affects the vectors $A^{I\alpha}$: indeed Bianchi identities do contain the naked fields $B^{I\alpha}$. Yet we can successfully handle this fact by considering the $B^{I\alpha}$ not as gauge potentials (that is, 2-forms defined modulo 1–form gauge transformations), but as physical fields, with their own explicit parametrization [1]. In this way, the two–forms loose their gauge freedom and become massive, as it can be found by solving the Bianchi identities. In fact, the Bianchi identities give directly the self–duality constraint (10) obeyed by the two–forms:

$$D_{[a}B^{I\alpha}_{bc]} = -\frac{1}{12} g I\!L^{I\alpha}{}_{AB}\mathcal{H}^{AB\ |de}\varepsilon_{abcde} + \text{fermion terms}\,. \tag{13}$$

5 Gauging the non–semisimple $CSO(p,q,r)$ groups

Let us consider the gauging of the $CSO(p,q,r)$ $p+q+r=6$ groups in $D=5$ $\mathcal{N}=8$ supergravity. We begin with a short description of the $CSO(p,q,r)$ algebras (see also

[1]The same happens to matter two–form fields coupled with $\mathcal{N}=2$ supergravity [16].

[10, 14]). The generators of $SO(p,q)$ (with $p+q=n$) satisfy

$$[G^{IJ},G^{KL}] = f^{IJ,KL}_{MN} G^{MN} \tag{14}$$

where

$$f^{IJ,KL}_{MN} = -2\delta^{[I}_{[M}\eta^{J][K}\delta^{L]}_{N]} \tag{15}$$

and

$$\eta^{IJ} \equiv \operatorname{diag}(\overbrace{1,\ldots,1}^{p},\overbrace{-1,\ldots,-1}^{q})\,. \tag{16}$$

Their generalization, studied by Hull in the context of supergravity [10] are the algebras $CSO(p,q,r)$ with $p+q+r=n$, defined by the structure constants (15) with

$$\eta^{IJ} \equiv \operatorname{diag}(\overbrace{1,\ldots,1}^{p},\overbrace{-1,\ldots,-1}^{q},\overbrace{0,\ldots,0}^{r})\,. \tag{17}$$

Decomposing the indices as $I=(\bar I,\hat I)$ $\bar I = 1,\ldots,p+q$, $\hat I = p+q+1,\ldots,n$, we have that $G^{\bar I\bar J}$ are the generators of $SO(p,q)\subset CSO(p,q,r)$, while the $r(r-1)/2$ $G^{\hat I\hat J}$ are central charges

$$[G^{\bar I\hat J},G^{\bar K\hat L}] = \frac{1}{2}\eta^{\bar I\bar K}G^{\hat J\hat L}\,. \tag{18}$$

They form an abelian subalgebra, and

$$SO(p,q)\times U(1)^{\frac{r(r-1)}{2}} \subset CSO(p,q,r)\,. \tag{19}$$

As in the case of $SO(p,q)$ (see eq. (1)), the

$$(G^{IJ})^K{}_L = \delta^{[K}_J\eta^{L]I} \qquad I,J,K,L = 1,\ldots,n \tag{20}$$

are generators of a representation of $CSO(p,q,r)$. This representation is not faithful, because the generators of the central charges are identically null

$$(G^{\hat I\hat J})^K_L = 0\,. \tag{21}$$

The gauged versions of $\mathcal{N}=8$, $D=5$ supergravity constructed in [1], [2] and based on a semisimple choice of the gauge group $\mathcal{G}=SO(p,q)$ ($p+q=6$) can be generalized (see [9]) to the non–semisimple gauge groups $\mathcal{G}=CSO(p,q,r)$ ($p+q+r=6$).

The new gaugings can be obtained by taking for the matrix η^{IJ} the definition (17), with some null entries on the diagonal. Let us discuss the consequences of this in the theory, in order to see if any pathology occurs. One has

$$(G^{KL})^{\hat I}{}_J = \delta^{[K}_J\eta^{L]\hat I} = 0\,, \tag{22}$$

so the covariant derivative of a *contravariant field* (2), along the contracted directions, reduces to the ordinary Lorentz–covariant derivative:

$$DV^{\hat I} = \mathcal{D}V^{\hat I} + g(G^{KL})^{\hat I}{}_J A_{KL}\wedge V^J = \mathcal{D}V^{\hat I}\,. \tag{23}$$

This, however, does not happen for the covariant derivative of a *covariant field*:

$$DV_{\hat I} \equiv \nabla V_{\hat I} - g(G^{KL})^J{}_{\hat I}A_{KL}\wedge V^J = \nabla V_{\hat I} - g\eta^{\bar L\bar J}A_{\hat I\bar L}\wedge V_{\bar J}\,. \tag{24}$$

Let us consider now the most subtle part of the theory: the two–forms. First of all we notice that, because of (23), the two–forms along the contracted directions $B^{\hat I\alpha}$ are discharged under the gauge group. Furthermore, one finds that the Bianchi identities of the two–forms corresponding to the contracted directions (the $B^{\hat{I}\alpha}$) are cohomologically trivial, so that these fields are actually field strengths of one–form fields

$$B^{\hat{I}\alpha} \equiv dA^{\hat{I}\alpha} + \text{ fermions} \tag{25}$$

having a $U(1)$ gauge invariance, as argued in [1]. Let us stress that the calculation of [9] shows that there are no consistency conflicts between the two types of gauge invariances, and therefore no need arises to introduce massive vectors as proposed in [1]. Indeed, in the F.D.A. rheonomic approach we see in a transparent way where the consistency conflicts arise and how they are solved. Summarizing it goes as follows. When a vector field is charged with respect to the gauge group, but does not gauge any generator of the gauge algebra it appears naked in its own Bianchi identities. This requires dualization to a two–form, so the correct number of degrees of freedom is got through self–duality instead of gauge invariance. On the other hand when a contraction is performed on some direction $\hat{I}$, in the Bianchi identities of the fields $A^{\hat{I}\alpha}$ (25) the naked gauge fields disappear. Therefore, the two gauge invariances are not inconsistent, and the corresponding vectors can stay massless. Note that in this case the Bianchi identities look very different from those along the non-contracted directions. Now the self–duality constraint disappears and the halving of degrees of freedom is due to the recovered $U(1)$ gauge symmetry.

In this way new gauged $D = 5$ $\mathcal{N} = 8$ supergravities arise, with $(12 - r)$ two–forms, $(15 + r)$ one–forms, and gauge group $CSO(p, q, r)$. It is worth noting that the r vectors $A^{\hat{I}\alpha}$ are coupled with the other fields, even if they don't gauge anything, and so are the abelian vectors $A_{\hat{I}\hat{J}}$. Indeed,

$$\mathcal{H}^{AB}_{ab} = I\!\!L^{IJAB} F_{ab|IJ} + I\!\!L_{\bar{I}\alpha}{}^{AB} B^{\bar{I}\alpha}_{ab} + I\!\!L_{\hat{I}\alpha}{}^{AB} B^{\hat{I}\alpha}_{ab} \tag{26}$$

and $\mathcal{H}^{AB}_{ab}$ does appear in the equations of motion of the two–forms (13) along the non–contracted directions, which we have derived from the Bianchi identities and don't change in the contracted gaugings.

The supersymmetry transformation rules for the new theories are obtained by substituting, for the contracted directions $\hat{I}$, the supersymmetry transformation rule (8) for $B^{\hat{I}\alpha}$ with the supersymmetry transformation for $A^{\hat{I}\alpha}$

$$\delta A^{\hat{I}\alpha}_{\mu} = I\!\!L^{-1}_{AB}{}^{\hat{I}\alpha} \left[\frac{\mathrm{i}}{\sqrt{2}} \bar{\chi}^{ABC} \gamma_\mu \varepsilon_C + 2\mathrm{i} \bar{\varepsilon}^A \psi^B_\mu \right] \tag{27}$$

all other transformation laws remaining unchanged. The new theories are completely sensible and well defined, however apparently there is not a lagrangian formulation of them. In fact, it seems impossible to write the terms giving the equations of motion of the r one–form fields $A^{\hat{I}\alpha}$ in a covariant and gauge–invariant way. A possible argument to motivate this situation is the following. The existence of r extra neutral vectors besides the 15 charged ones implies a sort of Hodge dualization for the corresponding two–forms. Specifically, what happens here is that the field strength $H^{[3]}$ for r of the $B^{[2]}$ fields is identically zero, so that we have to interpret the $B^{[2]}$ themselves as field strengths of new gauge vectors $A^{[1]}$. In other words, we have traded r "electric" two–form fields $B^{[2]}$ for just as many "magnetic" one–forms $A^{[1]}$. In view of this, it is not too surprising if the $15 + r$ vectors are not mutually local, which would be necessary to admit a common lagrangian description.

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