The Photon: A Virtual Reality

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ABSTRACT

It has been observed that every photon is, in a sense, virtual – being emitted and then sooner or later absorbed. As the motif of a quantum radiation state, the photon shares these characteristics of any virtual state: that it is not directly observable; and that it can signify only one of a number of indeterminable intermediates, between matter states that are directly measurable. Nonetheless, other traits of real and virtual behavior are usually quite clearly differentiable. How 'real', then, is the photon? To address this and related questions it is helpful to look in detail at the quantum description of light emission and absorption. A straightforward analysis of the dynamic electric field, based on quantum electrodynamics, reveals not only the entanglement of energy transfer mechanisms usually regarded as 'radiative' and 'radiationless'; it also gives significant physical insights into several other electromagnetic topics. These include: the propagating and non-propagating character in electromagnetic fields; near-zone and wave-zone effects; transverse and longitudinal character; the effects of retardation, manifestations of quantum uncertainty and issues of photon spin. As a result it is possible to gain a clearer perspective on when, or whether, the terms 'real' and 'virtual' are helpful descriptors of the photon.

Keywords: Virtual photon, photonics, quantum electrodynamics, resonance energy transfer, retardation, photon spin

1. INTRODUCTION

It is no longer so straightforward to explain what is meant by a 'photon'.[1] Although the term belongs to a concept first formulated a hundred years ago, the present conference eloquently bears witness to the present truth of this concise understatement. In recent literature, there is further disconcerting evidence in the number adjectival qualifiers that can be found attached to the term, as for example in 'superluminal',[2] 'electric',[3] 'magnetic',[4] 'ballistic',[5] 'transverse',[6] and 'longitudinal',[7] photons. 'Real' and 'virtual' photons are the subject of the present discourse. Based on the elementary definition that a virtual photon is one not directly observed, it has been correctly commented that every photon is, in a sense, virtual – being emitted and then sooner or later absorbed.[8] As the defining motif of a quantum radiation state, the photon exhibits the characteristic indeterminacy of any quantum virtual state, signifying its role as intermediary between states of matter that are directly measurable. Nonetheless, it is usually considered that traits of virtual behavior are distinctive and unambiguous. To address the question of what it means to categorize a photon as 'real' or 'virtual' in an optical context, this paper revisits the detailed quantum description of a photon history comprising creation and propagation. The photophysics exemplifies an interplay of quantum theory, electromagnetism and the principles of retardation; analysis based on quantum electrodynamics (QED) not only confronts key issues of photon character; it also elucidates a number of related matters such as the entanglement of 'radiative' and 'radiationless' mechanisms for energy transfer, two distinct senses of photon transversality, and photon spin issues.

2. QED FORMULATION

The photon has a character that, *inter alia*, reflects the electromagnetic gauge. In the Coulomb gauge the radiation field is ascribed an unequivocally transverse character,[9] in the sense that its electric and magnetic fields are orthogonally disposed with respect to the wave-vector. As will be shown, this transversality condition of electromagnetic fields is not necessarily transferable to a disposition with respect to the interpreted direction of electromagnetic energy transduction. To engage in a detailed study of these features it is appropriate to fully develop the theory of energy transfer within the framework of quantum electrodynamics, which treats both fields and matter on the same quantum basis. The system Hamiltonian comprises unperturbed operators for the radiation and for two material components, a source/donor A and a detector/acceptor B differentiated by a label ξ, and also two corresponding light-matter interaction terms;

$$H = H_{\mathrm{rad}} + \sum_{\xi=\mathrm{A,B}} H_{\mathrm{centre}}(\xi) + \sum_{\xi=\mathrm{A,B}} H_{\mathrm{int}}(\xi) \quad . \tag{1}$$

The first two components of (1) determine a basis in terms of which states of the system can be described, *i.e.* a direct product of eigenstates of the radiation field Hamiltonian and the Hamiltonian operators for the two components of matter. The third, radiation field-matter interaction, summation term can be expressed either in minimal coupling form (expressed in terms of coupling with the vector potential of the radiation field) or the generally more familiar multipolar formulation directly cast in terms of electric and magnetic fields. These two options lead to identical results for real processes, that is those subject to overall energy conservation;[10,11] for convenience the following theory is to be developed in multipolar form. [*Note*, in its complete form the multipolar interaction Hamiltonian can itself be partitioned as: (i) a linear coupling of the electric polarization field (accommodating all electric multipoles) with the transverse electric field of the radiation; (ii) a linear coupling of the magnetization field (all magnetic multipoles) with the magnetic radiation field; (iii) a quadratic coupling of the diamagnetization field with the magnetic radiation field. It may be observed that, although the following analysis focuses on electric polarization coupling, the same principles concerning the identity and transversality characteristics of real and virtual photons apply to each and every multipolar term.[12]] In equation (1), the absence of any terms with $\xi' \neq \xi$ signifies that the transduction of energy between A and B is not effected by direct instantaneous (longitudinal) interactions, but only through coupling with the quantum radiation field – a feature that is in marked contrast to most classical descriptions. In the lowest order, electric-dipole term in the multipole expansion, each $H_{\mathrm{int}}(\xi)$ operator is given by;

$$H_{\mathrm{int}}(\xi) = -\sum_{\xi} \boldsymbol{\mu}(\xi) \cdot \mathbf{e}^{\perp}(\mathbf{R}_{\xi}) \quad . \tag{2}$$

where the electric-dipole moment operator, $\boldsymbol{\mu}(\xi)$, operates on matter states and the transverse electric field operator, $\mathbf{e}^{\perp}(\mathbf{R}_{\xi})$ on radiation states. The latter operator is expressible in a plane-wave mode expansion summed over all wave-vectors, $\mathbf{p}$, and polarisations, λ;

$$\mathbf{e}^{\perp}(\mathbf{R}_{\zeta}) = \mathrm{i}\sum_{\mathbf{p},\lambda} \left(\frac{\hbar c p}{2\varepsilon_0 V} \right)^{1/2} \left[\mathbf{e}^{(\lambda)}(\mathbf{p}) a^{(\lambda)}(\mathbf{p}) e^{\mathrm{i}(\mathbf{p}\cdot\mathbf{R}_{\zeta})} - \overline{\mathbf{e}}^{(\lambda)}(\mathbf{p}) a^{\dagger(\lambda)}(\mathbf{p}) e^{-\mathrm{i}(\mathbf{p}\cdot\mathbf{R}_{\zeta})} \right] \quad . \tag{3}$$

Here $\mathbf{e}^{(\lambda)}(\mathbf{p})$ is the polarisation unit vector (plane or circular, but always orthogonal to $\mathbf{p}$) and $\overline{\mathbf{e}}^{(\lambda)}(\mathbf{p})$ is its complex conjugate; V is an arbitrary quantisation volume and $a^{\dagger(\lambda)}(\mathbf{p})$, $a^{(\lambda)}(\mathbf{p})$ respectively are photon creation and annihilation operators for the mode ($\mathbf{p}$, λ). Accordingly, each action of H_{int} signifies photon creation or annihilation.

Consider an energy transfer process for which the initial state $|i\rangle$ of the system may be written $|\mathrm{A}^{\alpha};\mathrm{B}^{0};0\rangle$ and the final state $|f\rangle$ as $|\mathrm{A}^{0};\mathrm{B}^{\beta};0\rangle$. Here the superscript 0 signifies the ground energy level, with α and β denoting the appropriate excited levels for the source and detector, respectively. Overall conservation of energy demands that $E_{\alpha 0}^{\mathrm{A}} = E_{\alpha}^{\mathrm{A}} - E_{0}^{\mathrm{A}} = E_{\beta 0}^{\mathrm{B}} = E_{\beta}^{\mathrm{B}} - E_{0}^{\mathrm{B}} \equiv \hbar ck$ where the last equality serves to introduce a convenient metric k. Energy transfer is mediated by coupling to the vacuum radiation field, invoking (a minimum of) one $a^{\dagger(\lambda)}(\mathbf{p})$ and also one $a^{(\lambda)}(\mathbf{p})$ operator, whose two distinct time-orderings correspond to: (*a*) the creation of a virtual photon at A and its subsequent annihilation at B; (*b*) *vice-versa*. Both pathways have to be considered, in order to take account of the non-energy conserving route allowed by the Uncertainty Principle at very short times; the virtual photon can be understood as 'borrowing' energy from the vacuum, consistent with an energy uncertainty $\hbar/t$, where t is the photon time-of-flight – here determined by the displacement of the detector from the source. This principle also indicates a temporary relaxation of exact energy conservation in the isolated photon creation and annihilation events. When the whole system enters its final state, i.e. after the virtual photon is annihilated, energy conservation is restored. With two virtual photon-matter interactions and $H_{\mathrm{int}}(\xi)$ acting as a perturbation, the quantum amplitude, $M_{fi}^{\text{e-e}}$, for energy transfer is calculated from the second term of an expansion in time-dependent perturbation theory;

$$M_{fi}^{\text{e-e}} = \frac{\langle f|H_{\text{int}}|r_a\rangle\langle r_a|H_{\text{int}}|i\rangle}{\left(E_i - E_{r_a}\right)} + \frac{\langle f|H_{\text{int}}|r_b\rangle\langle r_b|H_{\text{int}}|i\rangle}{\left(E_i - E_{r_b}\right)} \quad . \tag{4}$$

The ensuing calculation leads into some relatively straightforward vector analysis and contour integration; the major didactic issues and also some of the mathematical intricacies have both been the subject of recent reviews.[13,14] Using the convention of summation over repeated Cartesian indices, the result for the transfer quantum amplitude emerges as follows:

$$M_{fi}^{\text{e-e}} = \mu_i^{0\alpha(\text{A})} V_{ij}(k,\mathbf{R}) \mu_j^{\beta 0(\text{B})} \quad , \tag{5}$$

Here $\mathbf{R} = \mathbf{R}_{\text{B}} - \mathbf{R}_{\text{A}}$ is the source-detector displacement vector, the source transition dipole moment is $\boldsymbol{\mu}^{0\alpha(\text{A})} \equiv \langle \text{A}^0 | \boldsymbol{\mu}^{(\text{A})} | \text{A}^\alpha \rangle$, and for the detector $\boldsymbol{\mu}^{\beta 0(\text{B})} \equiv \langle \text{B}^\beta | \boldsymbol{\mu}^{(\text{B})} | \text{B}^0 \rangle$; also $V_{ij}(k,\mathbf{R})$ is the retarded resonance electric dipole – electric dipole coupling tensor, expressible as;

$$V_{ij}(k,\mathbf{R}) = \frac{e^{\text{i}kR}}{4\pi\varepsilon_0 R^3}\left\{\left(\delta_{ij} - 3\hat{R}_i\hat{R}_j\right) - (\text{i}kR)\left(\delta_{ij} - 3\hat{R}_i\hat{R}_j\right) - (kR)^2\left(\delta_{ij} - \hat{R}_i\hat{R}_j\right)\right\} \quad . \tag{6}$$

3. RETARDED ELECTRIC FIELDS AND PHOTON TRANSVERSALITY

The quantum amplitude (5) can legitimately be interpreted as the dynamic dipolar interaction of the detector with a retarded electric field $\mathbf{e}_{\text{R}}(\text{B})$, generated by the source. From equation (5) it follows that this field has Cartesian components given by;

$$e_{\text{R}\,j}(\text{B}) = -\mu_i^{0\alpha(\text{A})} V_{ij}(k,\mathbf{R}) \, . \tag{7}$$

Notwithstanding its quantum electrodynamical derivation outlined above, the result has an identical form[15,16] to that which, when cast in SI units, emerges from classical retarded electrodynamics;[17]

$$\mathbf{e}_{\text{R}} = k^2\left(\hat{\mathbf{R}}\times\boldsymbol{\mu}^{0\alpha}\right)\times\hat{\mathbf{R}}\frac{e^{\text{i}kR}}{4\pi\varepsilon_0 R} + \left[3\hat{\mathbf{R}}\left(\hat{\mathbf{R}}\cdot\boldsymbol{\mu}^{0\alpha}\right) - \boldsymbol{\mu}^{0\alpha}\right]\left(\frac{1}{4\pi\varepsilon_0 R^3} - \frac{\text{i}k}{4\pi\varepsilon_0 R^2}\right)e^{\text{i}kR} \quad . \tag{8}$$

Previous analyses have mostly focused on the striking variation in range-dependence exhibited within the results. Both in equations (6) and (8) the first term, proportional to R^{-3}, is dominant in the short-range or near-zone region ($kR \ll 1$), whereas the third term, proportional to R^{-1}, dominates in the long-range or wave-zone ($kR \gg 1$). Consequently short-range energy transfer is characterized by a (Fermi Rule) *rate* that runs with R^{-6}, familiarly known as 'radiationless' (*Förster*) resonance energy transfer,[18] whereas the long-range transfer rate carries the R^{-2} dependence that is best known as the *inverse square law*. These two cases are asymptotic limits of a completely general rate law illustrated in Fig. 1. The Uncertainty Principle again affords a simple way of understanding the exhibited behavior. In terms of a transit time, *t*, for the energy transfer we have; $\hbar^{-1}\Delta E\Delta t \equiv c\Delta k\Delta t \equiv \Delta k\Delta R \sim 1$. It is because energy is transferred that the propagating electric field does not display the same inverse power dependence on the separation *R* for all times. For energy transfer over very short times, associated with short-range transfer distances $kR \ll 1$, the energy cannot be localized in either A or B and the result essentially reflects the R^{-3} form of a static dipolar field. However at distances

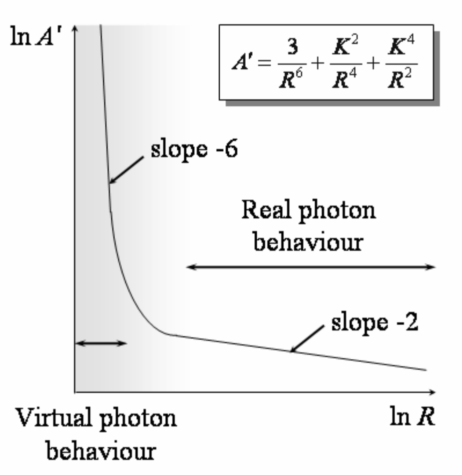

Figure 1. Logarithmic plot of the rate of dipole-dipole energy transduction against distance, with short- and long-range asymptotes. The formula for the dimensionless function A' (insert) determines the rate for an isotropically oriented system; for details see ref. 19.

where $kR \gg 1$, corresponding to relatively large times, the propagating character of the energy becomes more evident, and leads to the characteristic radiative R^{-1} behavior.

Despite the fact that the virtual photon formulation leading to (6) is cast in terms of electromagnetic fields that are purely transverse with respect to the photon propagation direction $\hat{\mathbf{p}}$, the field (8) contains elements that are manifestly non-transverse against $\hat{\mathbf{R}}$. To exhibit this explicitly, the given expression can be decomposed into terms that are transverse (⊥) and longitudinal (‖) with respect to $\hat{\mathbf{R}}$;

$$\mathbf{e}_{\perp} = \frac{e^{\mathrm{i}kR}}{4\pi\varepsilon_0 R^3}\left[\hat{\mathbf{R}}\left(\hat{\mathbf{R}}\cdot\mu^{0\alpha}\right)-\mu^{0\alpha}\right]\left(1-\mathrm{i}kR-k^2R^2\right); \tag{9}$$

$$\mathbf{e}_{\parallel} = \frac{e^{\mathrm{i}kR}}{2\pi\varepsilon_0 R^3}\hat{\mathbf{R}}\left(\hat{\mathbf{R}}\cdot\mu^{0\alpha}\right)\left(1-\mathrm{i}kR\right); \tag{10}$$

One immediate conclusion to be drawn from the prominence of the longitudinal component in the short-range region is the fact that photons with $\mathbf{p}$ not parallel to $\hat{\mathbf{R}}$ are involved in the energy transfer – which is consistent with the position-momentum Uncertainty Principle. By contrast the absence of an overall R^{-1} term in equation (10), compared to (9), signifies that the component of the field that is longitudinal with respect to $\hat{\mathbf{R}}$ is not sustained in the wave-zone $kR \gg 1$ (equivalently $R \gg ƛ$, where $ƛ = 2\pi/k$ designates the wavelength regime of the energy being transferred). Physically, this relates to the fact that with increasing distance the propagating field loses its near-field character and is increasingly dominated by its transverse component, conforming ever more closely to what is expected of 'real' photon transmission.

4. QUANTUM PATHWAYS

It is of passing interest to note the results of a recent analysis which, for the first time, allowed the identification of contributions to the propagating field (8) separated on another basis, reflecting terms arising through either one of the two alternative quantum pathways discussed in Sect. 2. These signify (*a*) the physically intuitive propagation of a virtual photon from A to B; (*b*) the counterintuitive case of virtual photon propagation from B to A. In the short-range,

$kR \ll 1$, both such contributions to the field unequivocally exhibit R^{-3} dependence; both play a significant role in the mechanism for energy transfer, as is once again consistent with quantum mechanical uncertainty. However in the long-range (which features only terms transverse to $\hat{\mathbf{R}}$), contributions of type (*a*) carry an R^{-1} radiative dependence, whereas those arising from type (*b*) unexpectedly fall off as R^{-4}. Although it was anticipated that the 'reverse propagation' terms would dwindle in importance compared to type (*a*), as distance increases and the photon acquires an increasingly real character, it was not previously recognized that the rate of diminution actually increases with distance.[20]

5. SPIN AND PHOTON ANGULAR MOMENTUM

While a number of issues associated with the interplay of transversality and angular momentum have been explored in the general context of spontaneous emission,[21] the developing technology of *spintronics*[22] invites a consideration of energy transduction between quantum dots. In determining the transverse field produced by an electric dipole spin transition, it transpires that noteworthy features arise in the case of a source whose transition moment is spin-aligned with respect to $\hat{\mathbf{R}}$, i.e.; whose complex transition moments lie in a plane orthogonal to the transfer direction and therefore expressible as:

$$\boldsymbol{\mu}^{0\alpha}_{(\pm)} = \frac{\mu}{\sqrt{2}}[\hat{\mathbf{i}} \pm \mathrm{i}\hat{\mathbf{j}}]. \tag{11}$$

Here, the corresponding result for the electric field, from equation (9), is:

$$\mathbf{e}_{\perp(\pm)} = -\frac{\mu e^{\mathrm{i}kR}}{4\pi\varepsilon_0 R^3}\left[\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} \pm \mathrm{i}\hat{\mathbf{j}})\right]\left(1 - \mathrm{i}kR - k^2R^2\right). \tag{12}$$

As is readily shown, the complex vector in equation (12) that is designated by the terms in square brackets corresponds to a circularly polarized photon of left/right helicity, signifying retention of ±1 units of *spin* angular momentum.[23] This feature has the potential for considerable importance in connection with energy migration down a column of quantum dots, oriented in a common direction.[24] Even though, in the technically most significant near-zone region, the coupling cannot be ascribed to real photon propagation – and the power law on distance also changes between near-zone and far-zone displacements – the fundamental symmetry properties are the same in each regime and angular momentum is therefore conserved.

Finally, it is of interest to make an observation prompted by the rise to prominence of the technology of twisted laser beams – beams with a helical wavefront that convey what has become termed *orbital* angular momentum.[25] The connotations of the term 'photon' in such a context have been the subject of much recent work, particularly in connection with Laguerre-Gaussian modes, and it has been shown that the photons in such beams convey multiples of the usual spin, the integer multiplier corresponding to the topological charge. Intriguingly, there have also been recent cases of non-integer vortex production.[26] Here, there is an obvious issue to be addressed concerning a rapprochement with the bosonic character of quantized radiation states; the validity of the photon concept in the case of such beams therefore remains to be established. In processes where photon emission and absorption are together encapsulated within a theory of energy transduction, it is legitimate to use any complete basis set for the photon of *de facto* virtual character and there is nothing to be gained (or lost) by employing vortex modes.

6. CONCLUSION

Based on a consideration of the 'life' of a photon as it propagates from its source of creation towards the site of its annihilation at a detector, a case can be made that every such photon in principle exhibits both virtual and 'real' traits. In the short-range limit significant retardation is absent and the virtual nature of the photon in a sense justifies the widely adopted term ‘radiationless’ as a descriptor of the energy transfer. The effect of increasing transfer distance is to diminish the virtual character of the coupling; the energy transfer exhibits an increasingly ‘radiative’, propagating behaviour – though a partly virtual character always remains; the coupling photons are never *fully* real. Thus the radiative and radiationless mechanisms for energy transduction, traditionally viewed as separate, are accommodated within a single theoretical construct, and it is significant that they never compete. Further analysis reveals hitherto unsuspected features in the asymptotic behaviour of the quantum pathways for resonance energy transfer. The results formally vindicate the accommodation of both source-creator and detector-creator pathways in the near-zone, and the domination of the source-creator pathway in the wave-zone. Physically, this behaviour is consistent with a rapid diminution in significance of the pathway in which the virtual photon propagates from the detector ‘back’ to the source, consistent with a diminishing virtual character for the coupling photon. Finally, a consideration of the angular momentum aspects of the photon field shows that the possibility for retention of angular momentum, associated with circular photon polarizations, can apply even in the near-zone. The result offers new possibilities for implementation in spintronic devices.

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