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Modeling and Economy's Dynamics and External Influences Through a System of Differential Equations

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MODELING AN ECONOMY'S DYNAMICS AND EXTERNAL INFLUENCES THROUGH A SYSTEM OF DIFFERETIAL EQUATIONS

THESIS

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Abstract

This research proposes a methodology to develop models for the greater understanding of the application of the economic instrument of national power through a selection of factors that define the economic condition of a country. The major components of an economy are identified as GDP per capita, a treasury bond yield, and a major stock market index. The components have interconnected dynamics along with external influences from the United Stated Federal Funds Rate and foreign direct investment. These connections are considered through a metamodel in the form of a system of differential equations which is solved as an inverse problem. The validity of the model is verified and the model is then used in making short term forecasts. What-if analysis of various policies is explored resulting in insight to policy changes. Through an increased understanding and awareness of the dynamics of the economic environment, a foundation for analysis is built to begin addressing the impacts of pecuniary warfare tactics.

Acknowledgments

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MODELING AN ECONOMY'S DYNAMICS AND EXTERNAL INFLUENCES THROUGH A SYSTEM OF DIFFERENTIAL EQUATIONS

I. Introduction

1.1 Background

In the 1990s, two Chinese colonels wrote *Unrestricted Warfare* (Liang 1999). Their book highlighted strategies a country could take to wage war against a militarily superior foe. One of the strategies purposed was pecuniary warfare. Pecuniary warfare consists of using economic tools as a method to degrade a country's ability to wage war. As the world economy develops, players enter and leave center stage; the yuan, China's currency, has recently been accepted by the International Monetary Fund as a reserve currency, while former shining stars, like the Brazilian real, recede behind the curtain ("Brazilian Waxing and Waning" 2015:1). In their fluid environment, new partnerships are formed, like the Association of Southeast Asian Nations (ASEAN), while old ones are threatened. The European Union has members on the brink of default, like Greece, and has members considering renouncing membership, like the United Kingdom. These events have cascading effects that impact the entire global economy.

1.2 Problem Statement

The world is a global community due to the evolution of technology that has created a level of interconnectedness unparalleled to anytime of the past. With this level of interconnectedness, a country's economic actions have impacts, not only on itself, but also on the other countries that comprise the world economy. At times, the characteristics of that impact are poorly defined, if known at all. The ability to investigate and evaluate those impacts would be a tool of great use for economic policy makers. Currently, the Department of Defense has a multitude of campaign level combat models that focus on the classic elements of a campaign. These models often use the attrition of physical assets as a metric to simulate the course of a campaign analyses. However, these models neglect the impacts of pecuniary warfare; highlighting the Department of Defense's need for a model that accounts for the pecuniary implications of warfare (Barone 2014:1). Having emphasized the need of a model, the following basic research questions still remain:

- Are historical methods useful for building an aggregate economic model?
- Can a baseline model be constructed for analyzing pecuniary warfare?
- Can we identify and model significant factors that influence countries engaged in pecuniary warfare?
- Does the base model behave differently for different regions or economies?

This research models the interactions of nations engaged in the international economy as a complex dynamical system. The system is then solved as an inverse problem. A solution methodology is presented and then an application using economic data from June 2006 to December 2013 demonstrates the dynamics captured from the actual data.

1.3 Research Objective and Scope

"Monetary policy cannot do much about long run growth, all we can try to do is to try to smooth out periods where the economy is depressed because of lack of demand"

–Ben Bernanke, Former Head of the Federal Reserve ("Highlights – Bernake Q&A Testimony" 2012:1)

"The Great Depression, like most other periods of severe unemployment, was produced by government mismanagement rather than by any inherent instability of the private economy."

–Milton Friedman, Nobel Prize winner in Economic Sciences (Friedman 2009:38)

These two quotes highlight the different views on how to describe an economy, what the key dynamic factors are which comprise an economy, and how to handle different economic conditions. The dynamic and unpredictable nature of an economy becomes increasingly evident when the second and third order effects are considered. Compounding the complexity is the fact that the world economy consists of many interrelated individual economies. This indicates a need for dynamic models and data that can help inform decision makers when national security objectives are in jeopardy.

In order to understand the importance and effects of United States pecuniary warfare actions on others and other's actions on the United States within an uncertain economic environment there must be a method to collect, analyze, and interpret data which provides insight into these actions.

The objective of this research is to provide a methodology that provides insight on the applications of the instruments of national power in an international economy. This research makes use of unclassified data so that it may be applied in multiple situations under different conditions. The methodology is generic enough to be expanded and applied to any nation and their measurable economic instruments of national power while remaining resilient to changes in the data structure that are required to conduct multiple assessments.

The methodology includes:

- Collecting and indexing aggregate economic data to capture the current economic environment of a country
- Conjecturing a functional form as a system of differential equations which accounts for interactions between the economic measures and the impacts of the economic instrument of national power
- Formulating a nonlinear program to solve for the parameters in the system of differential equations
- Using numerical methods with the results of the system of differential equations to gain insight on the system

1.4 Summary

The relevance of this research was provided in this chapter as well as a brief outline of the methodology underlying the model used in this research. This research uses data available to the public to help foster further development for understanding the dynamics of an international economic system.

The remainder of this document is organized as follows:

- Chapter II reviews the relevant literature that applies to this research
- Chapter III discusses the methodology of this research

- Chapter IV assesses the prediction quality of the model and an conducts a What-If analysis on the actions of the Federal Reserve after the 2008 United States financial crisis
- Chapter V presents a review of the significant insights and concludes with areas for future research

II. Literature Review

This literature review establishes a background of and justification for the methodology and model used in this study. This chapter examines the literature referenced in creating a methodology and model to solve the problem.

2.1 Macroeconomic Modeling

This section provides a description of prior research in the field of macroeconomics which is branch of economics that focuses on an economy as a whole, rather than individual entities. A brief, abbreviated description of the history of macroeconomic modeling is provided. This helps to identify the main branches of macroeconomic modeling and how they developed from one another over time. An example for each of the three main types is provided along with the strengths and weakness of that modeling type.

2.1.1 History of Macroeconomic Modeling

Macroeconomic modeling has been around since at least 1752 when David Hume defined a relationship between price level and output using the quantity theory of money (Hume 1752). Since then, the field has steadily grown by incorporating different techniques in an attempt to glean insight on different questions. These techniques include simple theoretical models, Empirical Forecasting Models, Dynamics Stochastic General Equilibrium models, Agent-Based Computational Economic models, and other mathematical models. There is a multitude of models built on the assumptions of different schools of economic thought such as Keynesian, new growth/synthesis, and business cycle theories. This section of the literature review focuses on models developed under newer developments of Keynes' principles because that is the framework which the model for this thesis follows. Broadly, Keynesian principles state that the output of an economy is influenced by the total spending in the economy and therefore a government can positively or negatively influence their economy using monetary or fiscal policy.

2.1.2 Simple Theoretical Models

Of the major categories, simple theoretical models were the first because they are the easiest to understand and do not delve deep into the advanced thought of macroeconomic implications. These models generally consist of diagrams and/or equations that depict the relationship between several aggregate macroeconomic indicators. Using these diagrams and equations, simple theoretical models attempt to describe, in extremely broad strokes, an entire economy or region of an economy. The Hicks-Hansen model, more commonly referred to as the IS-LM model as shown in Figure 1, is an example of a simple theoretical model. The model focuses on a short term static equilibrium that is represented by the intersection of the investment-savings (IS) curve and the liquidity preference-money supply (LM) curve (Hansen 1949:55-70). If there is an increase in consumption or a lack of desire to save, when the IS line shifts to the right, and the supply of money remains the same, the LM curve does not move, then the corresponding new equilibrium has an increased interest rate, i, and national income, Y.

Since it was one of the first mathematical formularizations of Keynesian's economics, the IS-LM model received a great deal of attention by macroeconomic thinkers from the 1940s to the 1970s. However, the weaknesses of the model, not accounting for the supply side, had come to light when it could not account for the simultaneous high inflation and unemployment rates during the late 1970s. Economist therefore began to explore more advanced models and methods such as empirical forecasting models.

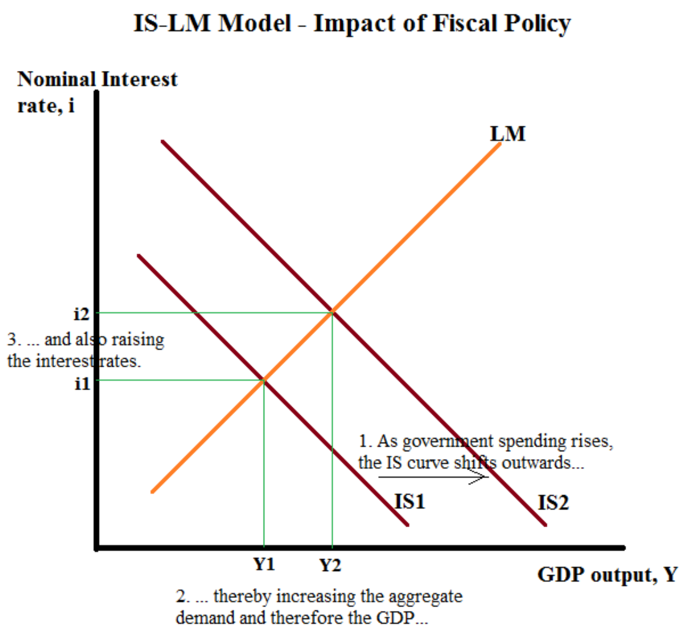


Figure 1: Hicks-Hansen model is an example of a simple theoretical model for macroeconomics (Natarajan 2012:1).

2.1.3 Empirical Forecasting Models

Webb suggest that the quantity and quality of macroeconomic data significantly increased after WWII. This lead economists to add more mathematical rigor and insight to their studies and simple theoretical models evolved into empirical forecasting models (Webb 1999:1-3). These models use statistical methods, such as regressions and correlations, to attempt to predict possible future states of an economy. Typically, these macroeconomic studies use empirical forecasting models to make prediction on an entire economy or a facet of an economy. More specifically, Lehmus modeled the Finnish economy using 71 endogenous variables and 70 exogenous variables in conjunction with 15 behavioral equations (Lehmus 2009:1). Those 141 variables are divided amongst equation sets that defined four facets of the economy: production function and factor demand equations, aggregate demand equations, price and wage equations, and public sector identities. The Lehmus model does an exceptional job at forecasting into the near future. It also highlights the aggregate nature of the macroeconomic environment through the equation sets. However, as shown in Hsieh's work on stock market returns, many of the aggregate macroeconomic variables are not independent and identically distributed (Hsieh 1991:1847-1858). This is a major weakness for empirical forecasting models as it is a fundamental assumption for the regression techniques used to build the models. If the assumptions are violated, such as no multicollinearity, the conclusions and insights drawn from those models should be subject to scrutiny.

2.1.4 Dynamic Stochastic General Equilibrium Models (DSGE)

In the 1980s, a crash happened that highlighted the need to attempt to model market shocks. This resulted in the development of dynamic stochastic general equilibrium models. Dynamic stochastic general equilibrium models looked to explain how shocks, such as an oil shortage, effect the equilibrium state of an economy as defined by more advanced versions of simple theoretical models, such as the optimizing IS-LM model (Christiano 2010:5, King 2000:49). Meese and Rogoff's work compared different structural and time series exchange rate models over a one month and one year time horizon. The compared models were a flexible price model (Frenkel-Bilson model), a sticky price model (Dornbusch-Frenkel model), and a sticky price model which incorporated current accounts (Hooper-Morton model). The study is a representation of a DSGE model using theoretical equations as a basis for modelling while highlighting the need to consider out of sample fits (Meese 1983:17). The lack of fit for out of sample data is one of the key weaknesses in DSGE models.

In his 1982 paper, Lucas conducted a theoretical study of the determination of interest rates. The study assumed a two country world modeled using difference equations and commonly used formulas for international trade, finance, and currency exchange rates. The analysis focused on monetary shocks and instability to the equilibrium state (Lucas 1982:342-348).

Dynamic general equilibrium models, based under the new Keynesian framework, are used to suggest to a government or central bank how to intervene after a shock to help stabilize an economic environment ("What are the different types of macroeconomic models?" 2015:1). Barndoff-Niessen used non-Gaussian processes, processes outside of the family of normal probability distributions, of the Ornstien-Uhlenbeck type, a modification of a random walk that tends to a mean in the long run, as the building blocks for a stochastic volatility model. Their work resulted in very simple expressions for a standard option pricing problem under stochastic volatility (Barndoff-Niessen 2001:31). Dynamic stochastic general equilibrium models are still referenced when policy makers need to create new policy.

2.1.5 Agent Based Computational Models (ACE)

Agent based computational models investigate macroeconomics from the reverse direction. Agent based computational models define rules for the agents, like household and firms, in a sector of an economy and simulate the interactions between these agents. Given these rules and the simulated interactions, agent based computational modeling define a particular sector of an economy by aggregating the interactions of all the individual agents. This can be repeated for each sector and then all the sectors can be aggregated to define an entire macroeconomic environment. The strength of ACE models comes from the ability of the modeler to specify how each agent is governed. This helps to reduce the number of assumptions made on the various agents in an economy. However, this is a double edged sword because the weakness of an ACE model is deciding how to model the minds of the computational agents that populate the model (Tesfatsion 2002:18-22). Additionally, the number of rules that govern the agents can make some problems computationally infeasible.

2.2 Defining a Country's Economic Environment

This section addresses prior research that highlights major factors that contribute to the economic environment and synthesizes how those factors affect one another. This section frames the problem that is addressed in this research, namely, the identification of three major endogenous factors used to define the economic environment in a country in this study; GDP per capita (GDP), a major stock market index (SM), and the 10-Year Treasury bond yield in a country (BY). Additionally, two exogenous factors that impact an economic environment are the Federal Funds Rate (FFR) issued by the Federal

Reserve and the Foreign Direct Investment (FDI) a country receives. The subsequent 5 sections further define each factor and highlight if a factor is related to another factor in the system as shown through previous studies.

2.2.1 Internal Factors

2.2.1.1 GDP per Capita

A country's Gross Domestic Product (GDP) is defined by Black as the annual value of goods sold and services paid for inside a country (Black 2012:170). By its definition, it would make sense that the GDP of a country is one of the most widely used measures of a country's economic output ("GDP" 2015:1). However, the total GDP of a nation maybe a poor measure for comparing countries because a country with a higher population tend to have a larger GDP than a less populated country. An example would be comparing India with a GDP of 1.8 trillion USD to Luxembourg with a GDP of 60 billion USD. To address this issue, the GDP per capita is used as it is a better indicator of relative performance when comparing two countries of significantly larger populations (i.e. China and Japan) ("Grossly Distorted Picture" 2015:1). Going back to the example, Luxembourg's GDP per capita is 110 thousand USD while India's GDP per capita is only 1.5 thousand USD. This makes sense when comparing the standard of living in India (low) versus Luxembourg (very high).

The GDP was shown to be forecasted by the treasury bond yield in Ang's 2004 study. The study uses bond yields as a measure for forecasting GDP in a Vector Auto-regression model (Ang 2004:371-373). This study concurs with the experience that if the government is willing to borrow money at a lower interest rate, then the output of the economy would increase because people take their money and use it to generate better returns through business rather than lending to the government. The relationship between GDP and stock markets has been shown to exist in many studies. Beck and Levine used three alternative panel specification tests to reject the hypothesis that stock markets do not have a significant impact on economic growth, as measured by GDP (Beck 2002:434-439). This is consistent with theories that stress an important role for financial development in the course of economic growth.

2.2.1.2 10 Year Treasury Bond Yield

A treasury bill is a debt that is issued by a country's government. The bills are traditionally issued for short, medium, and long term borrowing and mature at 3, 10, and 30 years respectively. Treasury bill rates represent what rate at which a country's government is willing and able to attempt to borrow money. Historically, when businesses are expanding a treasury bill's rate is high and when contracting the rate falls to lower levels. On the open market, investors trade treasury bills based on their yield, the expected payoff of a bill on its maturity date. Advanced methods for yield calculation take into account the inflation for a specific economy and other factors (Gurkaynak 2006:8-11). It could be said that a government's treasury bill rate is the government's expectation of their economy's health and that the yield is a measure of investors' expectation of how that economy's health should change in the future.

The treasury bill bond yield is not only related to GDP growth, as shown by Ang's research discussed in section 2.2.1.1, it has also been shown to be related to stock market returns. Using Ghana as a host country, Addo showed this relationship cointegration using a Vector Error Correcting model on monthly data for the Ghana Stock Exchange and Ghana treasury bill over the period from January 1995 to December 2011 (Addo 2013:18-20). The results of the coeintegration test showed a long run relationship between Ghana's treasury bill rate and the Ghana Stock Market All-Shares index. This evidence is consistent with the theory that if the government only borrows money at a low interest rate, people forego that investment and invest in the stock market instead for a higher return.

2.2.1.3 Stock Market Index

A financial market index is an aggregate value produced by combining several stocks or other investment vehicles together and expressing their total values against a base value from a specific date. They are intended to represent an entire stock market ("Market Index" 2015:1). A stock market index is used as a proxy to measure the state of the private sector of a country's economic environment. As highlighted by the Beck and Addo studies mentioned previously, the stock market has effects on both the GDP and bond yield factors when determining the economic environment. The next four sections describe four stock market indices.

2.2.1.3.1 Standard and Poor's 500 Index

Standard and Poor's 500 Index (SP500) is an index of 500 US companies chosen for market size, liquidity, and industry grouping. The SP500 is a reflection of the large capital companies in the United States ("S&P 500" 2015:1). The SP500 is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy in the United States through changes in the aggregate market value of 500 stocks representing all major industries. The index was developed with a base level of 10 for the 1941-43 base period ("SPX:IND" 2015:1).

2.2.1.3.2 Nikkei Stock Index

The Nikkei Stock Index is the leading and most respected index of Japanese stocks ("Nikkei" 2015:1). It is a price weighted index of Japan's top 225 companies and is comparable to the Dow Jones Industrial Average Index in the United States ("Nikkei" 2015:1). The Nikkei-225 Stock Average is a price-weighted average of 225 top-rated Japanese companies listed on the First Section of the Tokyo Stock Exchange. The Nikkei Stock Average was first published on May 16, 1949, where the average price was ¥176.21 ("NKY:IND" 2015:1).

2.2.1.3.3 Shanghai Shenzhen CSI 300 Index

The CSI 300 Index is a free-float weighted index that consists of 300 A-share, the highest quality type of share, stocks listed on the Shanghai or Shenzhen Stock Exchanges. The index had a base level of 1000 on 31 December 2004 ("SHSZ300:IND" 2015:1). Since the CSI 300 takes into account companies from both the Shanghai and Shenzhen Stock Exchanges, this index is a good measure for the private market in China as viewed by the Chinese.

2.2.1.3.4 Moscow Exchange Index

MICEX Index is cap-weighted composite index calculated based on prices of the 50 most liquid Russian stocks of the largest, dynamically developing Russian issuers presented on the Moscow Exchange. The MICEX Index was launched on September 22, 1997 at base value 100. The MICEX Index is calculated in real time in Russian rubles and denominated by the Moscow Exchange. (“INDEXCF:IND” 2015).

2.2.2 External Factors

2.2.2.1 Foreign Direct Investment

Foreign direct investment (FDI) is the net inflow of investment to acquire a lasting management interest (10 percent or more of voting stock) in an enterprise operating in an economy other than that of the investor (“Foreign Direct Investment” 2015:1). It is the sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments. This study uses the net inflows of foreign direct investments, new investment inflows less disinvestment, as reported by foreign investors in U.S. dollars.

Bengoa’s study showed that foreign direct investment is correlated with economic growth as measured by GDP per capita. The study focused on 18 Latin American countries and used data that spanned from 1970 to 2000. A positive relationship between GDP per capita and FDI was found at the .01 significance level using panel data analysis (Bengoa 2003:534-542). As Folster and Henrekson point out, there is fear that a panel data analysis neglects the long run effects from the business cycle if composed of annual data (Folster 2001:15). Bengoa followed the recommended solution to circumvent this problem which is to use 5 year periods instead of yearly observations. As shown in Claessens’ research, the level of FDI has an effect on the stock market of a country. This research used a sample of 77 countries with data values covering the period from 1975 to 2000 to provide regressions estimated through random effects models with robust standard errors (Claessens 2001:Ch 4). The regressions in Claessens' study indicate that FDI inflows have a positive correlation with stock market development at the .01 significance level. Both the Bengoa and Claessens studies show that FDI is an influential factor on a country's economic environment, specifically on the Stock Market and GDP per capita factors.

2.2.2.2 Federal Funds Rate

The Federal Funds Rate (FFR) is the rate at which banks lend each other money held at the Federal Reserve for an overnight loan. Targets for the FFR are set and maintained by the Federal Open Market Committee, the main monetary policymaking branch of the Federal Reserve. Even though changing the FFR is an action of monetary policy, it has effects on both monetary and financial conditions of an economy. For this reason, it is considered one of the most influential interest rates in the US economy ("Federal Funds Rate" 2015:1). The FFR determines how expensive it is to borrow money between extremely creditworthy institutions for a very short term loan. Therefore, the FFR is typically viewed as the base rate that determines the level of all other interest rates in the US economy ("Federal Funds Rate" 2015:1).

The FFR has documented effects on all three factors of the economic environment. In 2003, Sarno and Thornton found that there is a long run relationship between the FFR and treasury bill rates (Sarno 2003:8-10). Using daily data over the period of 1974 to 1999, Sarno and Thornton used co-integration in conjunction with an error correction model to provide empirical evidence of the long run equilibrium relationship. Ioannidis and Kontonikas used regression models to show that there is a significant relationship between monetary policy, i.e. the FFR, and stock market prices, and therefore a stock market index (Ioannidis 2008:42-49). Their study examined 13 OECD countries over the period from 1972-2002 and found that restrictive monetary policy changes resulted in decreased stock returns. Finally, John B Taylor proposed the Taylor rule for monetary policy in 1993. His monetary policy rule was defined by the equation

$$r = p + .5y + .5(p - 2) + 2 \tag{2.1}$$

where r is the federal funds rate, p is the rate of inflation over the previous four quarters, and y is the percent deviation of real GDP from a target. Subsequent studies have shown that the rule may exhibit better performance by including changes to a functional form (Hofman 2012:38). However, those researchers have yet to argue that there is no relationship between the FFR and GDP. Though not a law, the Taylor Rule shows that there is a relationship between GDP and Federal Funds Rate.

The FFR also has a major impact on international markets. At the time of writing this research (January 2016), the USD is approximately 70% of the world's reserve currency. Changing the interest rate on the dollar has major global effects (Morgan 2009:1,5). It is likely that these effects stem from the fact that US interest rates are not the only rates that track the FFR closely. London Internank Offered Rate (LIBOR), prime rate, and other international rates track the FFR extremely close, see Figure 2.

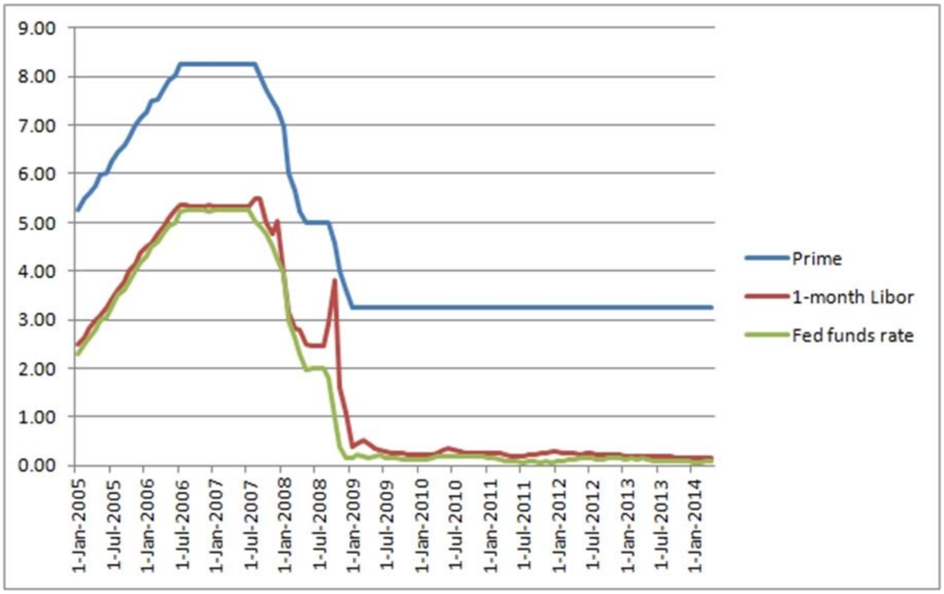


Figure 2: Prime and LIBOR rates closely track the FFR over time. ("Federal Funds Rate" 2015:1)

2.3 Modeling with Dynamic Systems

This section addresses relevant literature in the field of dynamic systems. Specific examples are highlighted in the following sections to show the application of using dynamic systems to model complex systems in different fields of research. The final section addresses the strengths and weaknesses of applying dynamic systems.

2.3.1 Dynamic System Applications

According to Boccara, a dynamical system is a set or system of equations whose solution describes the evolution or trajectory, as a function or parameter (time) along a set of states (phase space) of the system (Boccara 2010:11). Dynamic theory focuses on the asymptotic properties of the system as time approaches infinity. Researchers have been using dynamic systems to model real world phenomena since Poincare modeled the mechanics of celestial bodies (Poincare, Goroff 1992:17-23). Lotka-Volterra modeled the interactions between predator and prey populations using a dynamic system (Volterra

1928:5). This is one of the most commonly cited instances of dynamic system modeling being used to describe the interactions between two populations. Lanchester used a dynamic system to model the interactions between two forces engaged in aerial combat (Lanchester 1916). This system resulted in the Lanchester Equations and the subsequent inverse problem used to solve for the equations' parameters is one of the most commonly used examples of dynamic systems being used in military applications (Lucas, T.W. 2004:95-97). Heathcote used dynamic systems to model the dispersion of disease through not only a single population but up to four different populations (Heathcote 1989). Heathcote's work showed that the use of least squares minimization is acceptable when solving for parameter values that define the differential system. Saie used a dynamic system to model the interaction that US instruments of national power have on a counterinsurgency (Saie 2012). Saie's work highlighted the applicability of dynamic systems on aggregate indices and showed the usefulness of using the dynamic system with forcing functions for an analysis of alternatives. The Poincare, Lotka-Volterra, Lanchester, Heathcote, and Saie models illustrate the varied application of dynamic systems to many different fields.

2.3.2 Dynamic System Strengths and Weaknesses

Dynamic systems modeling is subject to both positive and negative critique. On a positive note, dynamic systems modeling allows for the analysis of a real world phenomena that is otherwise too complex to be analyzed with convention models. This analysis in turn can provide decision makers with clarity of action. However, dynamic systems are also a simplification of real world systems and therefore may not be able to predict unexpected events due to misspecification. Additionally, the analysis of a dynamic system does not specify the causes in a system, only the effects. For example in the predator/prey situation, dynamic systems can highlight that a species goes extinct when a certain state is reached, but it cannot specify what makes a species reach that state. These disadvantages can be mitigated with more research, both mathematical and nonmathematical, into the system that is being modeled.

2.4 Summary

This literature review presents four different types of macroeconomic modeling techniques. The four techniques discussed were simple theoretical models, empirical forecasting models, dynamic stochastic general equilibrium models, and agent based computational models. The strengths and weaknesses for each technique are also highlighted with an example from the literature. We then defined which factors contribute to or affect the economic environment of a country and how they interact with each other. This is important to provide a framework for this research. Finally, a discussion was provided on how modeling with dynamic systems has been used in the past on other complex systems and what some of the strengths and weaknesses are when using that methodology. Setting the stage to model economic systems as dynamic systems within this study.

Chapter III describes the application of these concepts when creating the model for this research.

III. Methodology

This research follows a solution methodology to solve the inverse problem with the following steps: Data collection, Index Formation, Model Generation, and Model Analysis as shown in Figure 3. A conjectured model utilizing a system of differential equations is proposed to model the dynamic system of the world economies. The determined coefficients of the conjectured model are the foundation for analysis of the international economic system and provide insight on the dynamics of the relationships between countries in an international economic system. Figure 3 is a graphical representation of the methodology and the subsequent sections describe the four steps this methodology followed in greater detail.

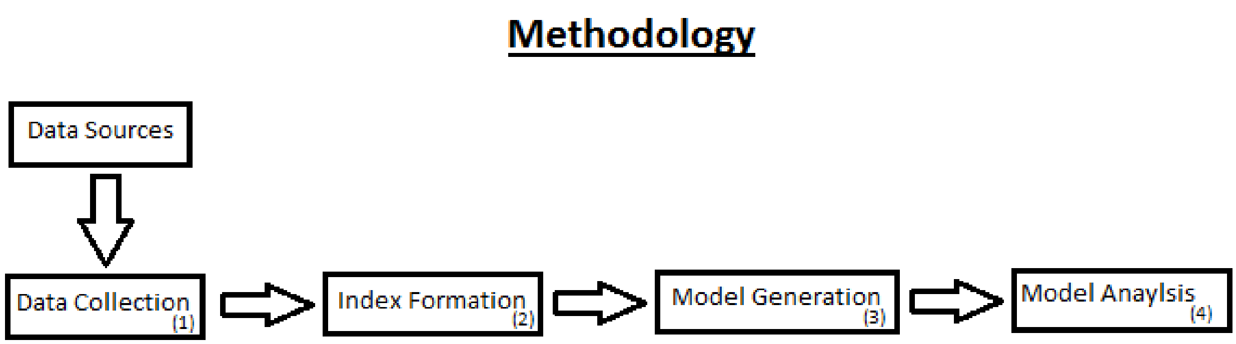


Figure 3: Graphical Depiction of the methodology steps followed in this research.

3.1 Data collection

"If we have data, let's look at data. If all we have are opinions, let's go with mine." –Jim Barksdale, Former CEO Netscape Communications

Data from reputable sources is utilized in the entirety of this research. The data used to create the indices in this study are retrieved from the following open source data sets:

- World Bank and OECD National Accounts (Last accessed 01 November 2015) (http://data.worldbank.org/indicator/NY.GDP.PCAP.CD)
- International Monetary Fund (Last accessed 01 November 2015) (http://data.worldbank.org/indicator/BX.KLT.DINV.CD.WD)
- Investing.com (Last accessed 01 November 2015) (http://www.investing.com/rates-bonds/u.s.-10-year-bond-yield)
- CIA World Factbook (Last accessed 01 November 2015) (https://www.cia.gov/library/publications/resources/the-world-factbook/)
- United States Treasury (Last accessed 01 November 2015) (http://www.federalreserve.gov/releases/h15/data.htm#fn1)

The data is readily available and reported from credible sources and covers the large economies discussed here. Much of the reviewed literature pertaining to macroeconomic modeling makes use of some of the same data sources. With the hope of future development using this methodology, this research utilizes data that is and will continue to be collected for the foreseeable future and to be readily available.

3.2 Index Formation

A state S is defined by a set of indicators, $S_t = \{X_t^j\}$. Each indicator, X_t^j, is a set of observations, $x_t^{(j)}$, where j is the enumeration of the indicators 1, 2,…, n and span over the entire time period, t, given by:

$$X_t^{(1)} = \left\{x_1^{(1)}, x_2^{(1)}, \dots, x_t^{(1)}\right\}$$

$$X_t^{(2)} = \left\{x_1^{(2)}, x_2^{(2)}, \dots, x_t^{(2)}\right\}$$

$$\vdots$$

$$X_t^{(n)} = \left\{x_1^{(n)}, x_2^{(n)}, \dots, x_t^{(n)}\right\} \tag{3.1}$$

where,

$$n = the\ total\ number\ of\ indicators\ in\ S$$

$$t = total\ number\ of\ time\ periods$$

Each observation, $x_t^{(j)} \in S$ measures the same economic environment, for this research. However some indicators in S may not have the same units or frequency of official reporting. All indicators are measured using a common timeline. Indicators that have missing data or quarterly/yearly reported data have the missing data filled using linear interpolation. With a common frequency and time scale, the data is normalized from [0, 1] so that each observation has a common score. Each indicator uses the minimum and maximum bench marks for the time period and are normalized as follows:

$$Index\ Value = 100 * \frac{x_t^{(j)} - min\left(X_t^{(j)}\right)}{max\left(X_t^{(j)}\right) - min\left(X_t^{(j)}\right)} \tag{3.2}$$

Even though equal weighting was applied in this research, further researchers may want to weight the contribution of each indicator differently by using the following example equation:

$$X_t^{(1)} = \sum_{j=1}^{n} \left(w_j\right) \left(Norm\left(x_t^j\right)\right) \tag{3.3}$$

where each $X_t^{(1)}$ is the first composite index at time t in this example.

For this research, the state is the economic condition of a country and the indicators of that state are the three internal factors described in section 2.2.1- GDP per capita, Stock Market, and 10 Year Treasury Bond Yield - and the two external factors described in section 2.2.2 – the Federal Funds Rate and Foreign Direct Investment. The observations are monthly and span from June 2006 to December 2013. See Appendix C for calculated index tables.

3.3 General Form of Differential Equations

A system of differential equations captures the interrelatedness of the indicators, X_t^j, that define a state, S. The general form for the system of differential equations used in this research to define a state is as follows:

$$\frac{dX_t^1}{dt} = \sum_{a=1}^{n} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=n+1}^{m} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=m+1}^{l} \delta_{ja} x_t^a$$

$$\frac{dX_t^2}{dt} = \sum_{a=1}^{n} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=n+1}^{m} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=m+1}^{l} \delta_{ja} x_t^a$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\frac{dX_t^n}{dt} = \sum_{a=1}^{n} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=n+1}^{m} \alpha_{ja}\left(\frac{x_t^a}{\beta_{ja}} - 1\right) + \sum_{a=m+1}^{l} \delta_{ja} x_t^a \quad (3.4)$$

The α and β coefficients are introduced to define the effect of the current and/or previous state an internal indicator has.

$$General\ Form\ of\ an\ Internal\ Indictor = \alpha\left(\frac{x_t^j}{\beta} - 1\right) \quad (3.5)$$

The α coefficient represents the weight of the endogenous function. The β coefficient, similar to a bifurcation point, represents the point where a change in the parameter causes a change in the dynamical property of the system.

The α and β coefficients are similar to the proportional growth rate and carrying capacity coefficients, *r* and *K*, in the Logistics Differential Equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{3.6}$$

Note that for the Logistic Differential Equation the quantity in the parenthesis is being subtracted from 1, whereas in our model 1 is subtracted from the quantity (Aiello 1990:11-13). The difference is due to the fact that, in this model, the economic variables do not have a capacity on their values like population would in the population growth model.

The δ coefficients are introduced to define the effect of the current state that an external indictor has on the rate of change for an internal indicator.

$$General\ Form\ of\ an\ External\ Indicator = \delta x_t^j \tag{3.7}$$

The δ coefficients represent the weights that external factors have when computing the derivatives of the internal factors at time *t*.

To build a functional form of the model, the parameters must be defined over a range. The range for the α, β, and δ are

$$\alpha_{ja} \in \mathbb{R}\ for\ a = 1,2,\dots,n;\ for\ j = 1,2,\dots,n$$

$$\beta_{ja} \in \mathbb{R}\ for\ a = n+1,\dots,m;\ for\ j = 1,2,\dots,n$$

$$\delta_{ja} \in \mathbb{R}\ for\ a = m+1,\dots,l;\ for\ j = 1,2,\dots,n$$

The coefficients of the final system of differential equations are derived by using a nonlinear least-squares method. Each differential equation in the system of differential equations corresponds to an economic variable. In addition, each equation expresses each point as a derivative of itself, data from other economic variable, data from the US economic instrument of national power (i.e. the FFR for this study), and data from the exogenous foreign direct investment. Therefore, each equation in the system of differential equations describes the interrelatedness of the economic variables, US instrument of national power, and the exogenous variables. Functionally represented, the model defines the rate of change of each state variable at time *t* as a function of the state variables and the forcing functions. An example using the rate of change of the Stock Market is shown below using $Stock\ \dot{M}arket_t$.

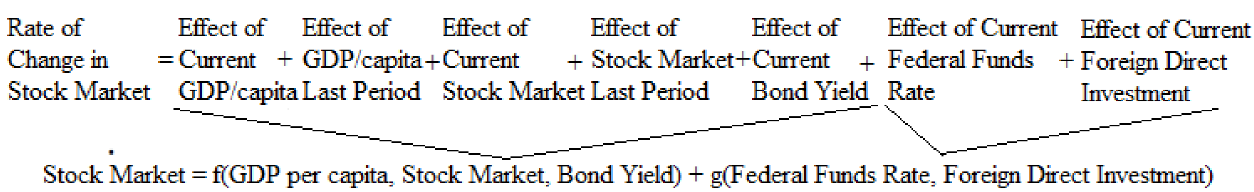


Figure 4: Depiction of the functional form

The differential equation form of $Stock\ \dot{M}arket_t$ is represented as

$$S\dot{M}_t = \alpha_{11}\left(\frac{GDP_t}{\beta_{11}} - 1\right) + \alpha_{12}\left(\frac{GDP_{t-1}}{\beta_{12}} - 1\right) + \alpha_{13}\left(\frac{SM_t}{\beta_{13}} - 1\right) + \alpha_{14}\left(\frac{SM_{t-1}}{\beta_{14}} - 1\right) +$$

$$\alpha_{16}\left(\frac{BY_t}{\beta_{16}} - 1\right) + \delta_{11}FFR_t + \delta_{12}FDI_t \qquad (3.8)$$

When applied to this research, the methodology thus far results in the following system of differential equations:

$$S\dot{M}_t = \alpha_{11}\left(\frac{GDP_t}{\beta_{11}} - 1\right) + \alpha_{12}\left(\frac{GDP_{t-1}}{\beta_{12}} - 1\right) + \alpha_{13}\left(\frac{SM_t}{\beta_{13}} - 1\right) + \alpha_{14}\left(\frac{SM_{t-1}}{\beta_{14}} - 1\right) +$$

$$\alpha_{16}\left(\frac{BY_t}{\beta_{16}} - 1\right) + \delta_{11}FFR_t + \delta_{12}FDI_t \qquad (3.8)$$

$$G\dot{D}P_t =$$

$$\alpha_{21}\left(\frac{GDP_t}{\beta_{21}} - 1\right) + \alpha_{22}\left(\frac{GDP_{t-1}}{\beta_{22}} - 1\right) + \alpha_{23}\left(\frac{SM_t}{\beta_{23}} - 1\right) +$$

$$\alpha_{26}\left(\frac{BY_t}{\beta_{26}} - 1\right) + \delta_{21}FFR_t + \delta_{22}FDI_t \qquad (3.9)$$

$$B\dot{Y}_t = \alpha_{31}\left(\frac{GDP_t}{\beta_{31}} - 1\right) + \alpha_{32}\left(\frac{GDP_{t-1}}{\beta_{32}} - 1\right) + \alpha_{33}\left(\frac{SM_t}{\beta_{33}} - 1\right) + \alpha_{34}\left(\frac{SM_{t-1}}{\beta_{34}} - 1\right) + \delta_{31}FFR_t$$

(3.10)

This system of differential equations can be defined in matrix notation with the following matrices. The $\left[\alpha_{ij}\right]_{5x5}$ matrix is made of the α coefficients. The $\left[\beta_{ij}\right]_{5x5}$ is a matrix of the inverse of the β coefficients.

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 0 & \alpha_{25} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{24} & 0 \end{bmatrix}, \quad \beta = \begin{bmatrix} \frac{1}{\beta_{11}} & \frac{1}{\beta_{12}} & \frac{1}{\beta_{13}} & \frac{1}{\beta_{14}} & \frac{1}{\beta_{15}} \\ \frac{1}{\beta_{21}} & \frac{1}{\beta_{22}} & \frac{1}{\beta_{23}} & 0 & \frac{1}{\beta_{25}} \\ \frac{1}{\beta_{31}} & \frac{1}{\beta_{32}} & \frac{1}{\beta_{33}} & \frac{1}{\beta_{33}} & 0 \end{bmatrix}$$

The economic variables compose the *X* matrix while the Federal Funds Rate and Foreign Direct Investment variable compose the F matrix.

$$X = \begin{bmatrix} GDP_t & GDP_{t-1} & SM_t & SM_{t-1} & BY_t \\ GDP_t & GDP_{t-1} & SM_t & 0 & BY_t \\ GDP_t & GDP_{t-1} & SM_t & SM_{t-1} & 0 \end{bmatrix}, \qquad F = \begin{bmatrix} FFR_t & FDI_t \\ FFR_t & FDI_t \\ FFR_t & 0 \end{bmatrix}$$

The δ coefficients compose the 3 x 2 matrix D as defined below:

$$D = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \\ \delta_{31} & 0 \end{bmatrix}$$

Using these matrices, two matrices of 1's, and the Hadamard product (o) operation, the system of differential equations is

$$\left[A \circ \left((X \circ \beta) - [1]_{5x5}\right)[1]_{5x1}\right] + [F \circ D] = \begin{bmatrix} S\dot{M}_t \\ G\dot{D}P_t \\ B\dot{Y}_t \end{bmatrix} \tag{3.11}$$

which also represents the estimated model, $\widehat{m}$.

3.3.1 Solving for α and β

To solve for the α and β coefficients, the following least-squares nonlinear minimization problem is formulated

$$\underset{\alpha,\beta,\delta,}{Minimize} \quad f(X) = \sum_t (\widehat{m} - m)^2 \tag{3.12}$$

$$Subject\ to: \alpha_{ij} \in \mathbb{R}$$

$$\beta_{ij} \in \mathbb{R}$$

$$\delta_{ik} \in \mathbb{R}$$

$$t \in \mathbb{Z}^+$$

The error as measured by the sum of squared error (SSE) of the system of differential equations is minimized by fitting the α, β, and δ coefficients. The generalized reduced gradient (GRG) method is used to solve the nonlinear program as implemented in the Excel Solver tool. Typical of many nonlinear problems (NLPs), the solution for α, β, and δ is not unique and that the solution is specific to the indicator during the time period being studied. Therefore, the parameters reported are not universal parameters and remain specific to this study and must be updated when introduced to new data.

The solution to the NLP is a mathematical expression of the economic environment in a country. Again, it is important to emphasize that the data used to create the indices cannot be altered without changing how the system is defined by the α, β, and δ coefficients. However, when testing the prediction accuracy, insight can be gained from testing changes in the internal or external environment variables. Specifically, modifications to the Federal Funds Rate and Bond Yield values can be implemented and then evaluated to see how these changes are reflected in the prediction.

3.4 Summary

This chapter described the model and solution methodology for the system of differential equations that model a country's economic condition. Within this chapter is a general overview of the model, a description of the model, and a rationale for the methodology. The methodology allows the user to vary exogenous and endogenous variables to evaluate the dynamics of different scenarios. The model in this research uses recent economic environment inputs to provide a decision maker with information to improve clarity of action when making policy decisions.

The model captures the interactions between the variables that compose an economic environment in a country. This is reflected in the functional form and the system of differential equations. In this model, each derivative calculated in the system of differential equations is composed of previously defined economic variables, as shown in section 2.2.1, and exogenous forcing functions, as shown in section 2.2.2.

The model is limited by what components are used to define the economic environment. When there are changes in the economic variables, the model provides insight as to the effects of those changes, but does not indicate a cause for the changes.

The methodology is implemented using a compiled data set that spans from June 2006 to March 2012. The prediction accuracy of the model is tested for two different prediction periods, 6 months and 21 months for four different countries. Chapter IV addresses construction of the model for this time frame and also alternate scenarios and their prediction capabilities.

IV. Implementation and Analysis

4.1 Implementation

> In 2008, the world economy faced its most dangerous crisis since the Great Depression of the 1930s. The contagion, which began in 2007 when sky-high home prices in the United States finally turned decisively downward, spread quickly, first to the entire U.S. financial sector and then to financial markets overseas (Havemann 2016:1).

The crisis highlighted how intertwined the economy of a country is with the rest of the economies in the global market. Even though the crisis had a "Made in America" label, its effects were felt throughout the world economy. This time period provides an excellent test bed to assess the validity of the methodology described in Chapter 3 and demonstrates how the model can be used in an analysis of alternative scenarios. To encompass the time leading up to and after the crisis, the time period that this study focuses on ranges from May 2006 to December 2013. The United States economic environment, shown in Figure 5, is the main example used throughout the remainder of the Implementation and Analysis chapter while the appendices contain the results, which are similar to the US results, for the other three countries. See Appendix A for a graphical depiction that includes Japan, Russia, and China.

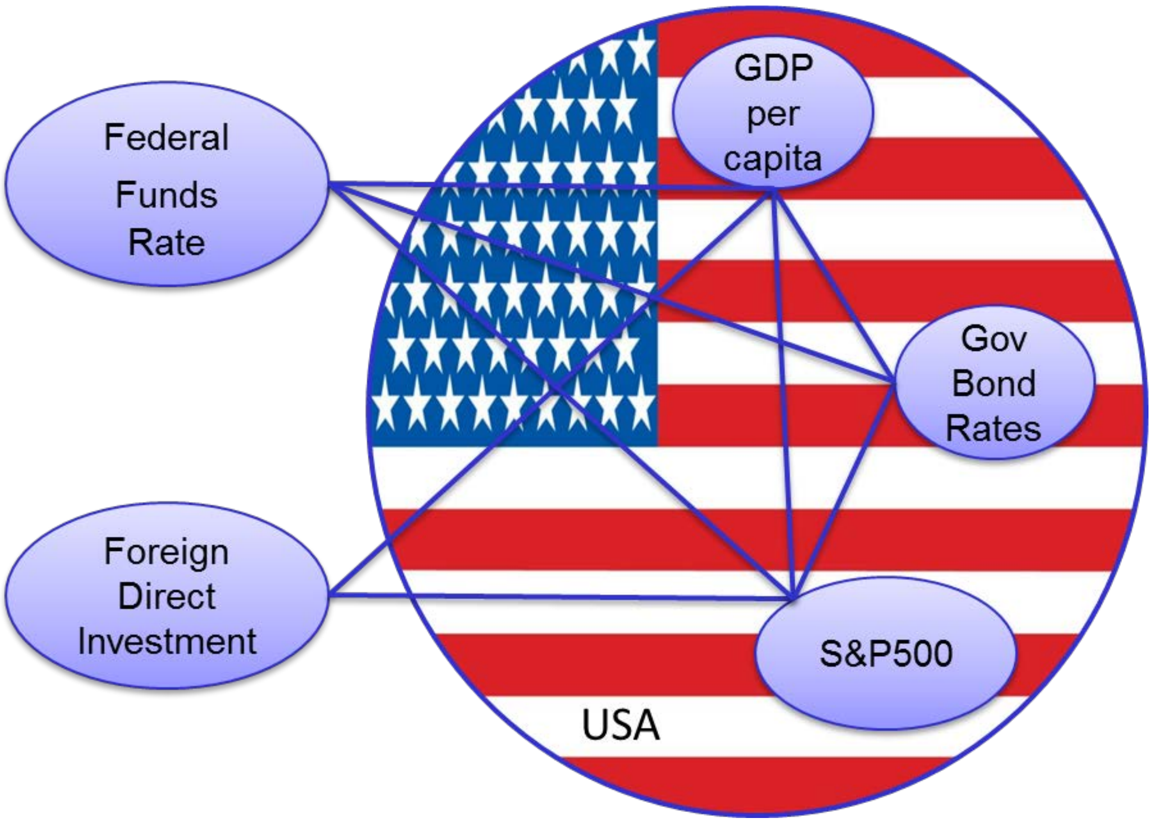


Figure 5: Graphical depiction of the US economy as defined by this research's methodology

4.1.1 Data Collection and Index Formation

Data collection includes data prior to the crisis (May 2006) through December 2013. Monthly data points were collected using the data sources listed in Section 3.1. The raw data is presented in Appendix B. Each data point is a monthly indicator or has been extrapolated to a monthly indicator. The extrapolated data, the GDP and FDI data, are assumed to have even effects throughout the year they are observed.

Having collected the data, the indices are formulated according to equation 3.2. There is an equal weighting for each factor in the calculation of the indices for a country's economic state. The calculated index values are presented in Appendix C.

Figure 6 shows the index values plotted over time for the US economy while Appendix D contains the plots for Russia, China, and Japan.

Figure 6: US Economic Condition Index Values plotted over time

The data from May 2006 to March 2012 was used to build the model while the data from April 2012 to December 2013 is used for testing the forecasting capability of the methodology.

4.1.2 Determining the Coefficients

The coefficients that define the system of differential equations are derived through the nonlinear least-squares method described in Section 3.3.1. To do this, the derivative at time t for an index as calculated by method 1 is defined as

$$\frac{dX_t^j}{dt} = \frac{x_t^j - x_{t-1}^j}{1} = m_t^i \tag{4.1}$$

Method 1's technique to compute the true value of the derivative at time t is similar to how the derivative is computed in the system of differential equations. An alternative method, method 2, for fitting the system of differential equations was also analyzed. Method 2 defined m_t as

$$\frac{dX_t^j}{dt} = \frac{x_{t+1}^j - x_{t-1}^j}{2} = m_t^i \tag{4.2}$$

This method to calculate the derivative is similar to the traditional secant method of approximating a derivative. The secant method captures whether there is an inflection point at time *t* by incorporating a future value into the calculation of the derivative. The difference between using method 1 and method 2 is impactful when minimizing the error between how well the model fits the derivative leading up to time *t*, equation 4.1, versus how well it fits the instantaneous derivative at time *t*, equation 4.2.

A nonlinear program according to equation 3.12 minimizes the SSE between m_t and $\widehat{m_t}$, as defined by equation 4.1 or 4.2, by changing the α, β, and δ coefficients. The nonlinear optimization is solved using the GRG Method implemented in Excel Solver. The resulting α, β, and δ coefficient values for both methods are shown in Table 1. Appendix E contains the coefficient values at full significant digit precision for all four countries modeled.

Table 1: Coefficient Values for US Economic Condition Model For Methods 1 and 2

	Method 1			Method 2		
	$\dot{SM}$	$\dot{GDP}$	$\dot{BY}$	$\dot{SM}$	$\dot{GDP}$	$\dot{BY}$
α_1	-0.033	0.081	1.3	22	2.7	0.015
α_2	0.040	1	-3.0	-32	1	-6.5
α_3	2.0	-0.12	0.81	14	-4.0	3.7
α_4	-2.1	1	-0.53	-0.091	1	-0.024
α_5	0.00060	0.045	0.98	-0.014	-0.035	0.98
β_1	2.7	1.3	0.53	23	63	0.014
β_2	3.2	1	1.1	40	1	5.5
β_3	2.0	3.4	2.6	28	140	9.6
β_4	2.0	1	2.6	0.16	1	0.069
β_5	3.2	4.2	1.0	0.27	76	1.0
δ_1	0.00013	0.022	-0.089	0.080	0.018	-0.042
δ_2	-0.00021	-0.011	0.99	0.041	-0.014	0.99

Having defined the coefficient values, the Sum of Squared Errors (SSE) and a R^2 value were calculated to statistically describe how well the system of differential equations fits the real data. The SSE and R^2 values were calculated by utilizing the following equations

$$SSE = \sum_t \left(x_t^j - x_t^{j'}\right)^2 \tag{4.3}$$

$$SST = \sum_t \left(x_t^j - \overline{x_t^{j'}}\right)^2 \tag{4.4}$$

$$R^2 = 1 - \frac{SSE}{SST} \tag{4.5}$$

Where x_t^j is the calculated index value for factor *j* at time *t* and $x_t^{j'}$ is the true index value for factor *j* at time *t*. A perfect fitting model would correspond to a R^2 value of 1 which would mean the SSE would be equal to 0. Table 2 summarizes the SSE and R^2 values for the indices for both methods.

Table 2: Sum of Squared Errors and $\boldsymbol{R^2}$ For Economic Condition Index Fittings

	Method 1		Method 2	
United States:	SSE	R^2	SSE	R^2
SM	0.004307	.99	530.2	.94
GDP	33.66	.95	28.75	.96
BY	3532	.69	1621	.77
Japan:				
SM	0.001531	.99	658.4	.96
GDP	42.80	.99	36.87	.99
BY	4991	.72	2268	.72
Russia:				
SM	0.06267	.99	999.1	.87
GDP	133.4	.92	129.1	.93
BY	6571	.28	3133	.49
China:				
SM	5.561	.98	744.8	.92
GDP	7.926	.99	3.115	.99
BY	7259	.60	4235	.41

Note that method 2 generally out performed method 1 in minimizing the SSE and its R^2 values were closer to 1, suggesting that using method 2 results in a better model fitting for the data set. Further analysis shows that method 2 outperformed method 1 in fitting the BY index and marginally outperformed method 1 when fitting the GDP index. However, method 1 outperformed method 2 in fitting the SM index. An analysis of the maximum errors is conducted for both methods to confirm or refute the results SSE analysis. To give a graphical depiction of the data, the calculated index and true index values are plotted for the United States' economic condition in Figures 7 and 8 while Appendix F contains the plots for Russia, China, and Japan.

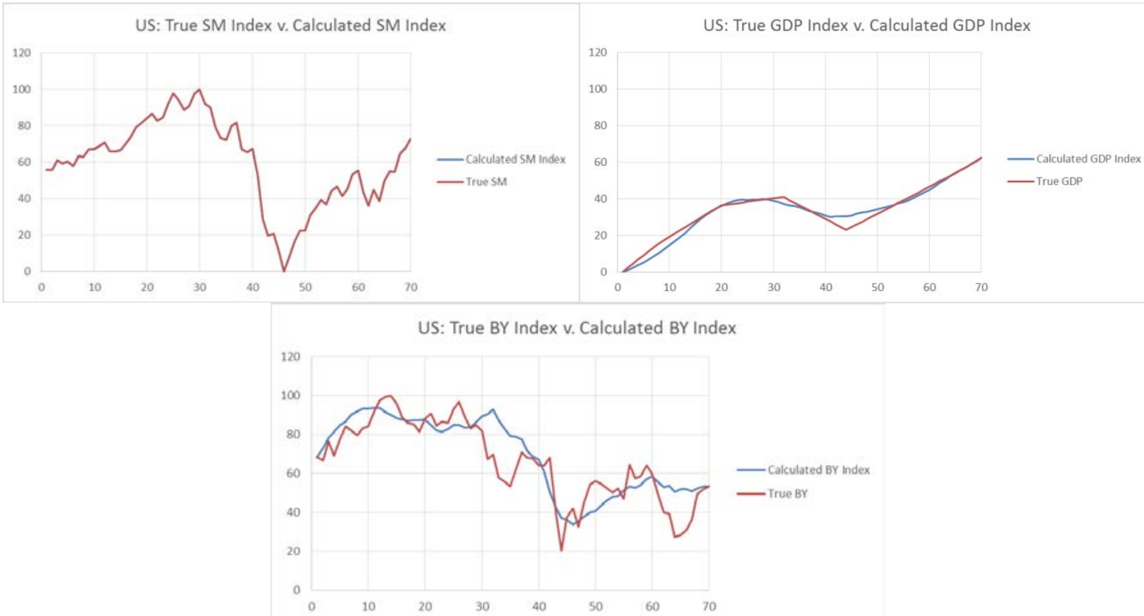


Figure 7: Plots of calculated US economic index and the true US economic index values over time using Method 1.

Method 1 resulted in a near perfect fitting, shown by the overlapping lines, of the US Stock Market index to the true US Stock Market index. The fittings for the US GDP and US BY indices were less accurate with the Bond Yield index having the worst fit for all countries. A possible explanation is that the fitting accuracy ranks are due to the fact that the Stock Market index has the most defining variables while the Bond Yield index has the least according to equations 3.8-3.10. Having more defining variables creates a larger space for solutions that could have a better optimal value when fitting an index. This highlights an area for further research which is discussed in 5.3.1.

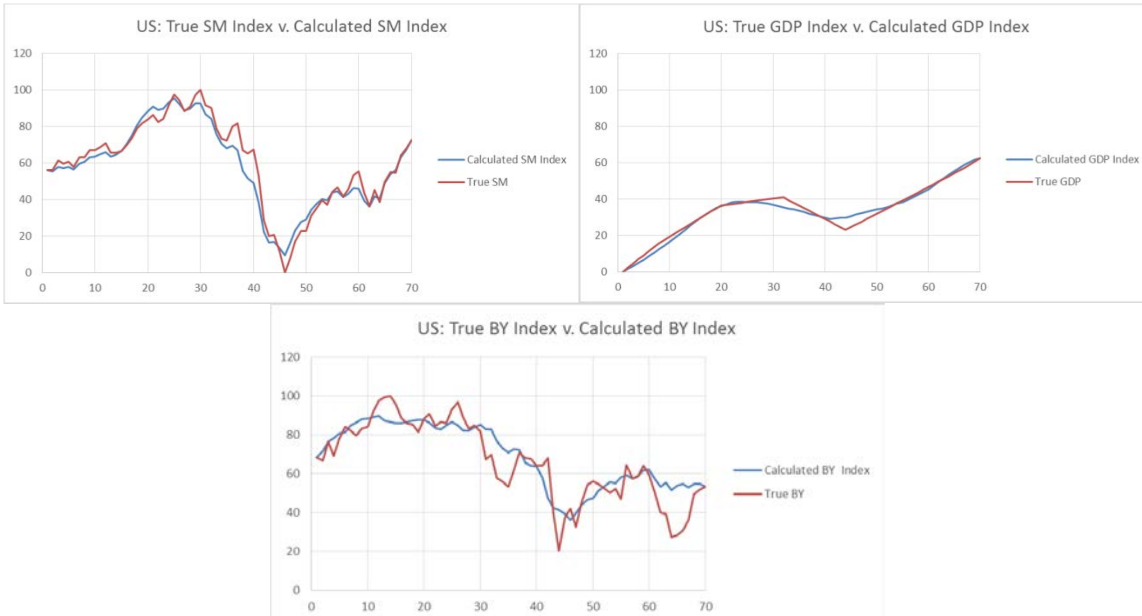


Figure 8: Plots of calculated US economic index and the true US economic index values over time using Method 2.

Method 2's fittings from best to worst were as follows: GDP per capita, Stock Market index, 10Year Treasury Bond Yield. In all cases, excluding China's BY index, method 2 had a better or equal fitting for the GDP and BY indices, method 1's worst fitting index and second best fitting index respectively.

When fitting a model, it is important to take into account the impact of outlier data points. Using the maximum error, an analyst may highlight potential outliers in the data that should be removed. The maximum error for an index is defined as the maximum difference between the model's calculated index value and the true index value. Table 3 highlights the maximum errors for each index using methods 1 and 2. The values for the maximum errors do not indicate that any individual data points may be outliers and therefore the data was not altered for the model fitting process.

Additionally, the maximum error can be used to assess which method had the most egregious worst case scenario. Method 2 slightly outperforms method 1 when fitting for the BY indices, whereas, method 1 significantly outperforms method 2 when fitting for the SM index. The marginal improvements in maximum error that method 2 has over method 1 for the BY indices do not justify the significant difference in the SM maximum error making method 1 the preferred method according to maximum error. This is contradictory to the SSE analysis.

Table 3: Maximum Error Values for model fittings

	Method 1 Maximum Error	Method 2 Maximum Error
United States:		
SM	.07 (*t*=42)	18.1 (*t*=38)
GDP	7.3 (*t*=42)	6.8 (*t*=42)
BY	29.6 (*t*=31)	25.8 (*t*=63)
Japan:		
SM	.05 (*t*=42)	17.9 (*t*=38)
GDP	7.6 (*t*=66)	9.5 (*t*=42)
BY	28.6 (*t*=13)	25.8 (*t*=63)
Russia:		
SM	.36 (*t*=10)	33.0 (*t*=36)
GDP	12.9 (*t*=30)	13.4 (*t*=30)
BY	67.9 (*t*=45)	55.5 (*t*=45)
China:		
SM	7.3 (*t*=50)	17.0 (*t*=28)
GDP	4.8 (*t*=35)	1.8 (*t*=66)
BY	41.8 (*t*=37)	44.3 (*t*=62)

Using the mean field theory approach, one may expect that the errors of the calculated index to the true index be normally distributed. A normal probability plot was constructed for each of the indices. A deviation from the diagonal indicates that the errors may not follow a normal distribution. The dotted lines that encompass the data points represent the 95% confidence interval for where the data could be to maintain the normally distributed hypothesis. A normal probability plot for the US economic condition indices using method 1 is depicted in Figure 9. The results for the normal probability plots for US economic condition when using method 2 are listed in Appendix G due to their similarity to the method 1 results. Additionally, Appendix G contains the normal probability plots for Russia, China, and Japan. The curves at the tails of the line are not a major concern unless both curves were to trend on the same side of the diagonal, i.e. both data curves plot below or above the diagonal. The curvature of the tails, symmetrical with heavy tails, indicates that the errors are symmetrically distributed but with fatter tails than a normal distribution (Neter 1996:107).

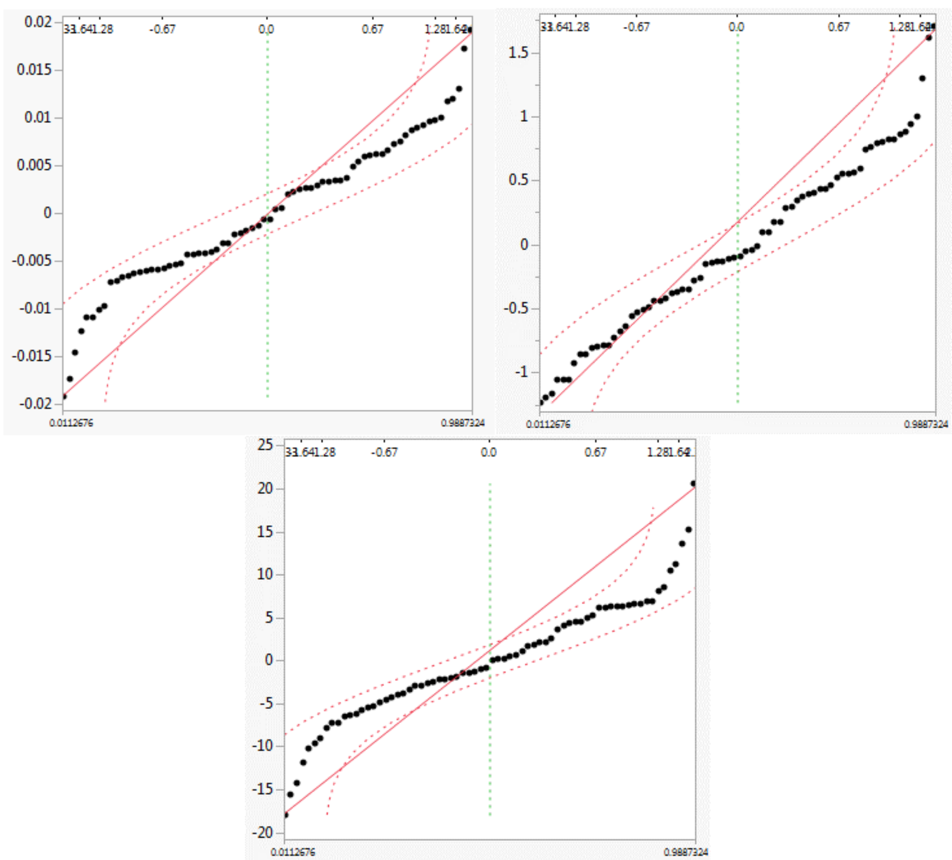


Figure 9: Normal Probability Plots for US Economic Condition Indices for Method 1. Clockwise: Stock Market, GDP per capita, Bond Yield

A Sharpiro-Wilk goodness of fit test is another method to test for normally distributed errors. A Sharpiro-Wilk test was conducted on the errors to supplement the Normal Probability Plots. For this test, H_0: population is from a normal distribution and a small p-value would reject the null hypothesis at the $\alpha = .01$ level. Table 4 contains the p-values from Sharpiro-Wilk goodness of fit tests on the errors and summarizes the results for each country's economic condition indices where a p-value less than the α level would be a failure.

Table 4: This table summarizes whether an index for a country's economic condition has normally distributed errors according to the Sharpiro-Wilk test. The α level considered is .01 and a p-value less than that corresponds to a failure of the index having normally distributed errors

	Method 1		Method 2	
United States:	*p*-value	Result	*p*-value	Result
SM	.8105	PASS	.3424	PASS
GDP	.2103	PASS	.0545	PASS
BY	.8739	PASS	.0269	PASS
Japan:				
SM	.0176	PASS	.4487	PASS
GDP	.1208	PASS	.0142	PASS
BY	.6954	PASS	.5805	PASS
Russia:				
SM	.1273	PASS	.0150	PASS
GDP	.0087	FAIL	.1492	PASS
BY	<.0001	FAIL	<.0001	FAIL
China:				
SM	.0207	PASS	.4407	PASS
GDP	.1021	PASS	.0558	PASS
BY	.2239	PASS	.0806	PASS

The results of the model fitting statistic, normal probability plots, and Sharpiro-Wilk test indicate that the model can be used for further analysis. Specifically, the fittings analysis resulted in method 1 being the better method to use when fitting the model with the subsequent analyses on the errors not providing a definitive argument for one method compared to another. Section 4.2 address the model's ability when making forecasts.

4.2 Model Forecast

The forecasting capability of how the economic condition would change was tested over the 21 month time period from April 2012 to December 2013, corresponding to *t*=70 to *t*=90. The Euler method is the numerical method used to make the forecasts based off an initial value. The Euler method is

$$u_{k+1} = u_k + ha_k \; for \; k = 0, 1, \dots, n \tag{4.6}$$

where $a_k = u'_k$ and h is the step size (Goldberg 1998:394-397). For the application in this research $h = 1$ and the initial index values are from the month of March 2012. This method allows the index values to be estimated based off an initial value and derivatives from the system of differential equations. Figures 10 and 11 shows the results of the Euler method for the US economic condition indices compared to the actual values using the coefficients from methods 1 and 2. The results for Russia, China, and Japan are in Appendix H. Note that the location of the initial value, at a peak or pit, has a significant impact on the accuracy of the prediction when using the Euler method. This helps to explain the prediction's major divergences from the true index values for the SM index when using method 2.

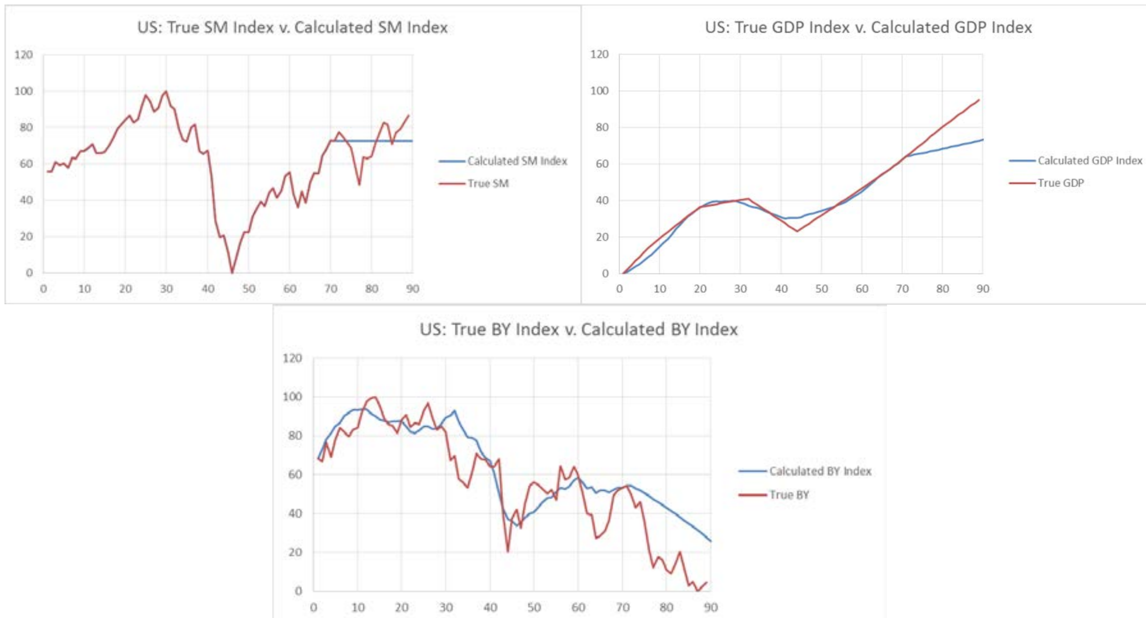


Figure 10: Plots for United State Economic Condition index predictions and true values over time using coefficients from method 1. The range for predictions is from t = 70-90.

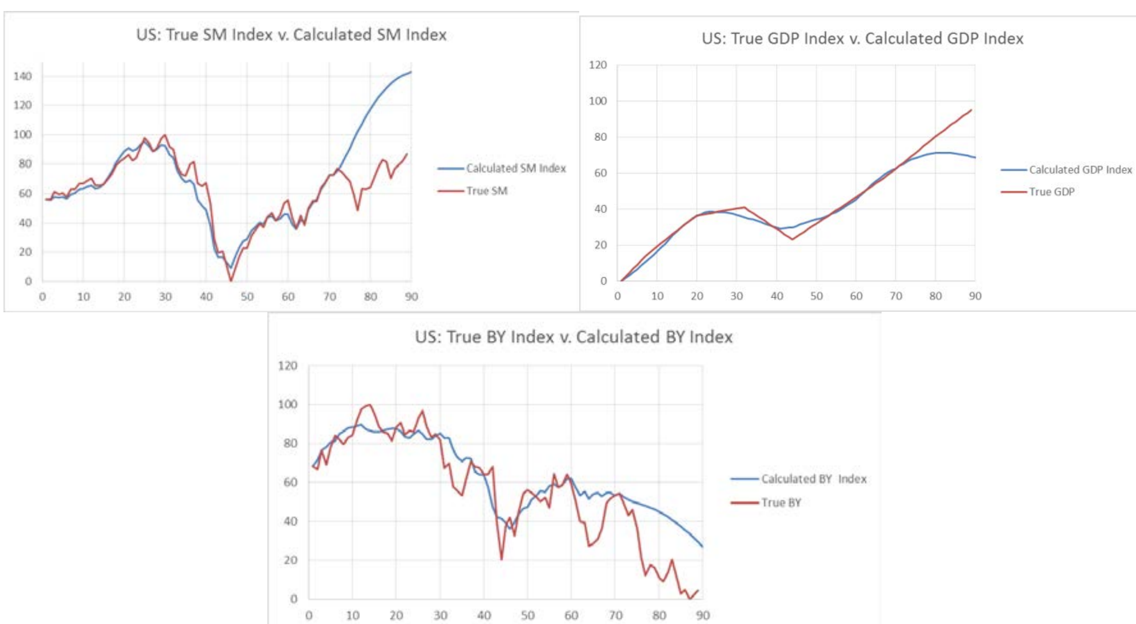


Figure 11: Plots for United State Economic Condition index predictions and true values over time using coefficients from method 2. The range for predictions is from t = 70-90.

When using forecasting techniques, an analyst typically calculates a weighted average rate of change for an index from the last few time periods and uses that rate to project forward in time. This generally produces a rather smooth result which is slow to react. For the sake of analysis and comparison, we define a naïve forecast as continuing the current rate of change in the data. The forecasts from this research's methodology consistently outperform the naïve forecast method for the Stock Market and Bond Yield index forecasts due to their interdependence on the overall state of the system (i.e. the rate of change being a function of the state variable in addition to the forcing function influences). Even though it captured the general trend, the GDP index typically did not result in a better forecast when compared to the naïve forecast. This results from smooth linear nature of recorded GDP that is reported in yearly figures and therefore a linear interpolation was conducted to gather monthly values to be able to fit the model. However, if a forecast began at or just prior to an inflection point this research's model forecast would most likely be the better forecast.

While it captures the general trend, the forecasts lose accuracy the farther out in time it is projected. Again, one would desire the model forecast to contain less error than the naïve forecast. To compute the fit of the model the Mean Squared Error (MSE) was calculated using equation 4.7.

$$MSE = \frac{1}{t}\sum_t \left(x_t^j - \overline{x_t^{j'}} \right)^2 \qquad (4.7)$$

where *t* is the number of time periods of the forecast, x_t^j is the calculated index value for factor j at time *t*, and $x_t^{j'}$is the true index value for factor j at time *t*. Unlike the R^2 statistic, the MSE does not include the mean of the true data. Therefore, the MSE estimator allows for the comparison of two different models for the same data. It is desirable to have as low of a MSE value as possible. Table 5 highlights the MSE for a 6 month forecast, from t=70 to t=75, compared with a forecast for the entire 21 month period, from t=70 to t=90.

Table 5: Mean Squared Error for 6mo and 21mo Predictions

	Method 1 MSE for:		Method 2 MSE for:	
United States:	6mo Prediction	21mo Prediction	6mo Prediction	21mo Prediction
SM	27.04703	249.7184	9.012533	290.0143
GDP	130.2342	94.71757	828.9539	2393.379
BY	398.8173	608.1712	388.2578	677.4021
Japan:				
SM	44.39105	2759.304	34.44212	2415.614
GDP	255.2526	241.918	43.28547	171.6481
BY	26.47125	366.8953	106.3751	575.9064
Russia:				
SM	89.3037	828.7402	137.1996	1505.97
GDP	750.8143	2110.255	1645.123	5673.04
BY	102.359	211.3094	163.5059	949.6596
China:				
SM	0.109499	24.17901	1.742902	118.7008
GDP	129.7824	689.2257	227.6116	7086.863
BY	259.8494	1526.822	48.42429	8536.404

The MSE for the 6 month forecasts are typically lower than the MSE for the 21 month forecasts. The exceptions were the US and Japan GDP indices. The cause of the exception is attributed to the fact that both indices had an unusually large error in the first six months that inflated the 6mo MSE value. This makes sense because the predicted values of the indices after the first prediction are based off of prior forecast values which estimated numbers. To gain more accuracy in a prediction, one would need to replace estimated values with the true values. Even without the additional data, the model captures the general trend of the indices very well for short term forecasts.

The results of the MSE over the fitted range and the prediction range show that defining the m_t^i according to method 1 equation 4.1 yield the results with the lowest MSE. When fitting the model the SSE and maximum error methods were inconclusive as to which method had a better fit. Therefore, the MSE results from the predictions are used as a tie breaker making method 1 the preferred method for the alternate scenario analyses. Next, Section 4.3 uses the method 1 fitted model to conduct a what-if analysis for immediately after the 2008 financial crisis.

4.3 What-If Scenarios

Beginning in July of 2007, the Federal Reserve began to lower the Federal Funds Rate and, by December of 2008, it was the lowest it had ever been in history. At the time of writing (January 2016), the interest rate is still currently held at a low level of ~0.38%. In this section, the methodology is applied to evaluate alternative scenarios which reflect possible modification to the Federal Funds Rate (FFR) starting in January of 2008. The following sections focus on two different scenarios. These are hypothetical scenarios that demonstrate the what-if analysis feature of the model.

4.3.1 Gradual Decrease to a Lower FFR Level

In the Gradual Decrease scenario, the Federal Reserve decides to moderate the rate at which the FFR is lowered and raise the minimum level that the rate reaches. The Gradual Decrease scenario assumes a more mundane policy is adopted; that the FFR does not change as rapidly and that it does not decrease to such a historic low. For the Gradual Decrease scenario, the inputs remain the same until the 21st time period, which corresponds to when historically the FFR index in the base case begins to rapidly reduce to 0, and then gradually decreases to an index level of 60. This differs from the historical data where at the 21st time period the FFR index drops to a near 0 level over the span of eight time periods. Appendix I contains the alterative FFR value sets used for the Gradual Decrease and No Adjustment scenarios.

Method 1 as described in Section 4.1.2 serves as the base case to which the impact of the alternate scenario changes is compared to. The coefficients that define the system of differential equations are not changed to allow for an evaluation of the alternate scenario under the conditions that took place. Put simply, if the only thing to change were the Federal Funds Rate, how would the indices have been affected by the new scenario's policy.

The observation here is how the indices change over time based upon a different Federal Funds Rate, while keeping the other variables constant. A change, in accordance with the Gradual Decrease scenario, to the FFR variable did have statistically significant impacts on all of the variables that define an economic condition. The change in the trajectories of the US economic condition indices is shown in Figure 12. Appendix J contains the results for the Japan, China, and Russia economy models.

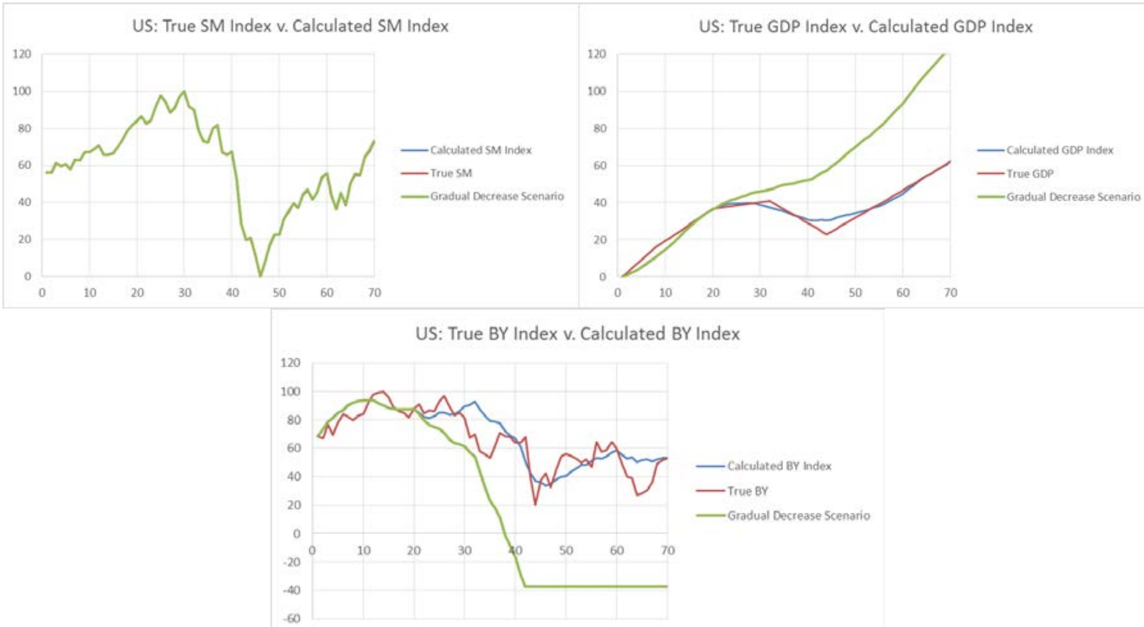


Figure 12: United States Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the Gradual Decrease policy begins.

Note that the plots in Figure 12 are identical up until month 21, the first month with a change in inputs. To insure stability of the dynamic system, each index was bounded to a historical low and 140% of the historical high which explains why the BY index flat lines at time period 42. If the BY did not flat line at a historical low index value of -37, the level of the GDP index would increase more rapidly. Figure 12 indicates that the change to the FFR policy did not have an impact on the SM index, had a positive impact on the GDP index, and had a negative impact on the BY index. The impacts on the GDP and BY indices are not consistent with what current theory states should happen and may indicate a misspecification of the functional form of the FFR variable in the system of differential equations.

A *t*-test on the difference of the means for each index would statistically test whether there was a difference between the base case scenario and the Gradual Decrease scenario. An assumption for the Student's t-test is that the variance of the two samples is equal. An F-test was conducted to test for equal variances for each country's index. It was found that this assumption of equal variances remained valid for the SM index for all four countries and the BY index for China and Russia. All other indices had non-equal variance and therefore a Welch t-test was conducted on the difference of the means for those indices. Both the Student and Welch t-tests are conducted with the following hypothesis:

$$H_o: \mu_{basecase} - \mu_{gradual\ change} = 0$$

$$H_o: \mu_{basecase} - \mu_{gradual\ change} \neq 0$$

$$\alpha = .05$$

This tests the hypothesis that there is no difference between the mean of alternate scenario, $\mu_{gradual\ change}$, and the mean actual scenario, $\mu_{basecase}$, where the μ value is calculated using a mean calculation of $\frac{1}{n}\sum_{t=1}^{n} x_t^i$ for scenario i. A p-value lower than the alpha level rejects the null hypothesis. The results from the t-test for all of the economic condition indices are reported in Table 6.

The results show that for the GDP and BY indices Ho is rejected because the p-value is less than the alpha level. The null hypothesis is not rejected for the SM index of all four countries and the BY index of Japan. This confirms that changing the FFR variable has a statistically significant effect on the economic condition at the α = .05 level for the scenario tested. Based on the results from the t-tests, it can be assumed that the Gradual Decrease to Federal Funds Rate scenario policy has statistically significant impacts on the economic condition GDP and BY indices.

Table 6: t-test results for Gradual Decrease Scenario using $\alpha = .05$. A low p-value indicates a significant difference between the base case and the scenario. An * indicates a Welch t-test was used.

	p-value	Result
United States:		
SM	.9733	Fail to Reject H_0
GDP*	2.268E-6	Reject H_0
BY*	1.947E-15	Reject H_0
Japan:		
SM	.9616	Fail to Reject H_0
GDP*	1.350E-8	Reject H_0
BY*	.6950	Fail to Reject H_0
Russia:		
SM	.4291	Fail to Reject H_0
GDP*	2.038E-4	Reject H_0
BY*	.001486	Reject H_0
China:		
SM	.6897	Fail to Reject H_0
GDP*	.005817	Reject H_0
BY	.02516	Reject H_0

4.3.2 No Adjustment to FFR Level

In the No Adjustment scenario, the Federal Reserve decides to maintain a constant FFR level. As shown in Appendix H, instead of rapidly decreasing the FFR index to 0 beginning at the 21st time period, the No Adjustment scenario value of the FFR index is kept steady at 96 from December 2007 through March 2012, the average level over the prior year and a half. This differs from the historical data where at the 21st time period the FFR index drops to a near 0 level over the span of eight time periods.

Method 1, as described in Section 4.1.2, serves as the base case to which the impact of the No Adjustment scenario policy is compared. The coefficients that define the system of differential equations are not changed to allow for an evaluation of the alternate scenario under the conditions that took place. Put simply, if the only thing to change were the Federal Funds Rate, how would the indices have been affected by the new scenario's policy.

The observation here is how the indices change over time based upon a different Federal Funds Rate, while keeping the other variables constant. A change, in accordance with the No Adjustment scenario, of the FFR variable did have impacts on some of the variables that define the economic condition of a county. The change in the trajectories of the US economic condition indices is shown in Figure 13. Appendix K contains the results for the Japan, China, and Russia economy models. Note that the plots are identical up until month 21, the first month with a change in the FFR input. As in the Gradual Decrease scenario, each index was bounded to a historical low and 140% of the historical high to insure stability of the dynamic system which explains the BY index flat lines at period 38. Had the BY index not been bounded at a historically low index value of -37, the level of the GDP index would increase at a higher rate. Once again, the GDP and BY index reactions to the scenario's change of the FFR differ from current theoretical expectations.

A *t*-test on the difference of the means for each index would statistically test whether there was a difference between the base case scenario and the Gradual Decrease scenario. Again, an assumption for the Student's *t*-test is that the variance of the two samples is equal. An F-test was conducted to test for equal variances for each country's index.

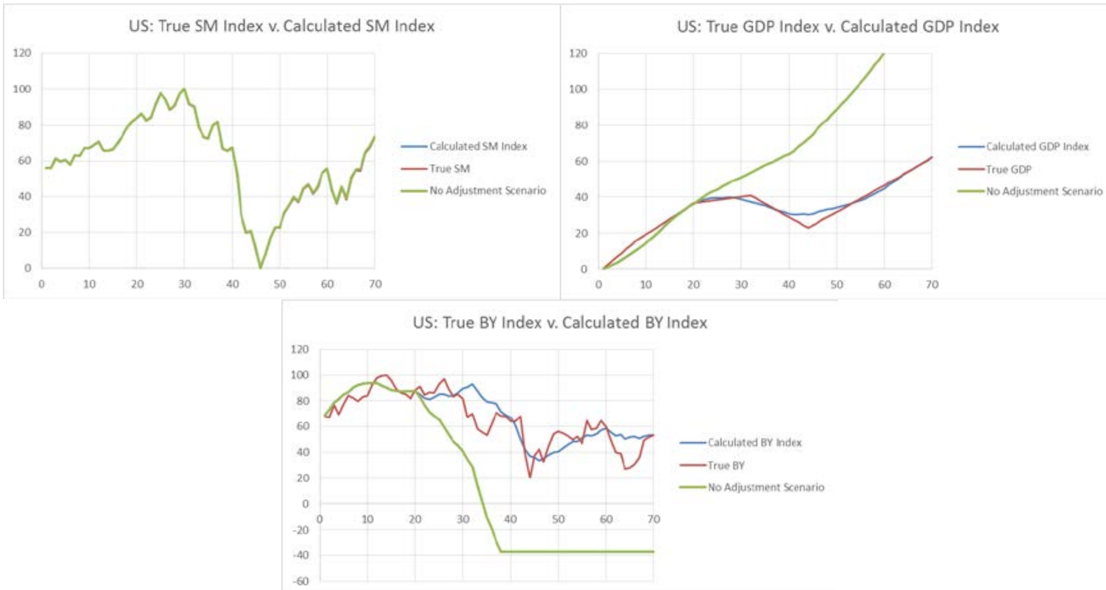


Figure 13: United States Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the No Adjustment policy begins.

It was found that this assumption of equal variances remained valid for the SM index for all four countries and the BY index for China and Russia. All other indices had non-equal variance and therefore a Welch *t*-test was again conducted on the difference of the means for those indices. The results of the F-test are identical to the F-test conducted in section 4.3.1. Both the Student and Welch *t*-tests are conducted with the following hypothesis:

$$H_o: \mu_{basecase} - \mu_{no\ adjustment} = 0$$

$$H_o: \mu_{basecase} - \mu_{no\ adjustment} \neq 0$$

$$\alpha = .05$$

This tests the hypothesis that there is no difference between the mean of alternate scenario and the mean actual scenario. The null hypothesis, H_o, is rejected when the *p*-values are less than the alpha level. Table 7 contains a summary of the results. The *t*-test results show that changing the FFR variable has a significant effect on the GDP index at the α = .05 level for all four countries. Changing the FFR did not result in significant effect on the SM index for all four countries at the α = .05 level. There were mixed results for the BY index across all four countries. There was a significant difference in the BY index for all countries except Japan at the α = .05 level. Based on the results from the model, it can be concluded that the No Adjustment scenario policy has statistically significant impacts on the economic condition of a country.

Table 7: *t*-test results for No Adjustment Scenario using α=.05. A low *p*-value indicates a significant difference between the base case and the scenario. An * indicates a Welch *t*-test was used.

	p-value	Result
United States:		
SM	.9582	Fail to Reject H_0
GDP*	3.419E-8	Reject H_0
BY*	4.701E-11	Reject H_0
Japan:		
SM	.9401	Fail to Reject H_0
GDP*	1.893E-15	Reject H_0
BY*	.5345	Fail to Reject H_0
Russia:		
SM	.2189	Fail to Reject H_0
GDP*	3.205E-6	Reject H_0
BY*	1.592E-5	Reject H_0
China:		
SM	.5330	Fail to Reject H_0
GDP*	.0002503	Reject H_0
BY	.0008999	Reject H_0

The model shows that the same indices are affected by a change in the FFR for all four countries. The magnitudes for the change are greater in the No Adjustment Scenario than in the Gradual Decrease scenario. The difference of magnitudes can be seen in the case for the US GDP and BY indices. For the Gradual Decrease scenario, shown in Figure 12, the GDP index reaches the 120 level at 68th time period whereas in the No Adjustment scenario, shown in Figure 13, the GDP index reaches the 120 level at the 60th time period. Similarly, the BY index reaches its minimum at the 41st time period for the Gradual Decrease scenario where as in the No Adjustment scenario it reaches the minimum value at the 38th time period.

4.4 Summary

This chapter illustrates the application of the model and solution methodology. Actual data from the 2008 financial crisis provided the framework for the application and what-if scenario analyses. Both the application and what-if scenario analysis demonstrate the usefulness of this research. The insights on the statistically significant impact that the FFR has on factors of the economic condition of a country may be used by policy makers to determine monetary policy and set a foundation for beginning pecuniary warfare campaign analysis.

This research provides a good step forward to evaluate the complexities of the world economic system. However, it remains imperfect and there are areas that need to be addressed in future research. Chapter V presents an overall conclusion and recommendations for future research.

V. Conclusions and Recommendations

This chapter reviews the significant insights of this research, identifies topics for future research, and provides a conclusion for this research.

5.1 Review

Due to the Department of Defense's need for models that pertain to pecuniary warfare, this research uses a methodology which evaluates the effects of using economic instruments of national power to affect the economic condition of countries in the world economy. The developed model attempts to capture the interrelatedness and complexities of the world economy. The model captures moments of a country's economic condition through an aggregation of internal and external variables.

As shown in *Unrestricted Warfare*, the fronts of warfare have evolved from an open battlefield pitting two armies against each other to more indistinct and poorly defined fronts such as cyberspace or the economy of a country (Liang 1999). These new fronts change how policy decisions impact the state of the deciding country and the economic state of an ally or enemy country. This model provides insight to the analyst on the application of an instrument of national power in terms of the economic condition being considered.

This research has proposed a baseline methodology that lays a foundation to satisfy the Department of Defense's need to model the effects of pecuniary warfare. This research shows that there are theories which relate the macroeconomic factors of a country and that there are models using the principle of dynamic systems that describe the complex interactions found in nature. However, there were no models found in the open literature which look at the tools available to conduct pecuniary warfare and evaluate their impact through the variable that describe a country's economic condition.

5.2 Insights

Through the available data and theoretical relationships of economic factors an estimated model was derived. Following the varied and applied works in the field of dynamics systems modeling, a system of differential equations was developed to capture the interrelatedness of the economic instrument of national power and the factors of a country's economic condition. The model is solved as an inverse problem. The model creates indices to approximate the factors that define a country's economic condition. The errors of the calculated value of an index and the true value of an index should be normally distributed to indicate a lack of bias in the model and, as shown in Table 3, this assumption is validated for the majority of the factor indices at the $\alpha = .01$ level. It is demonstrated that the models are country specific.

A nonlinear least-squares minimization problem is solved to determine the coefficients of the system of differential equations. This problem was solved for two different fittings and it was concluded that method 1 as described in Section 4.1.2 resulted in a better fitting. The resultant model describes the effects of Federal Funds Rate on the economic condition factors. The methodology was done using Microsoft Excel and publicly available data. The results demonstrated that it is possible to construct a baseline model that incorporates influencing factors on an economy to lay a foundation for future pecuniary warfare modeling.

The applicability of this research in building a foundation for pecuniary warfare analysis was demonstrated through a prediction analysis and two what-if scenarios. The prediction analysis allows for an analyst to make judgements upon near term future state of a country's economic condition given historical data and an initial starting point. Though the models are specific to a country, they consistently produce predictions of the general trend that out perform a naïve forecast. Typically, the forecast trend of the BY index was most accurate with the trend predictions for the SM and GDP indices ranking second and third respectively. The prediction analysis demonstrates that the systems state is observable.

The what-if analysis allows for an analyst to detect statistical differences in the economic indices as a result of changing the Federal Funds Rate. The what-if analyses showed that a change to the Federal Funds Rate has an impact on the economic condition of a country in the scenario tested. The only index that did not have a significant change was the Stock Market index. For the indices that did change, the magnitude of change for the No Adjustment scenario was greater than the Gradual Decrease scenario. The what-if analysis demonstrates that the system's state is controllable through policy changes.

This methodology proposes a means for an analyst to collect, analyze, and interpret data which helps to understand the importance of the economic instruments of national power on the economic condition of a country. Understanding the dynamics that make up a country's economic condition is crucial to being able to model the effects of pecuniary warfare. This research's methodology allows analyst the ability to explore the application of monetary policy on economic condition factors.

The model may be used to provide relevant insight into the economic condition of a country in a specific environment. The provided information is limited by factors of the methodology. One of the major factors is that the methodology is data driven. Therefore, the availability of high quality data has significant impacts to the quality of the information produced by the model. It is recommended that an analyst be cautious to adhere to the weaknesses of this methodology and be vigilant to avoid the idea that the coefficients are universal. They are not and should be reevaluated when presented to a new scenario. As statistician George Box once said, "all models are wrong but some are useful" (Box 1976:1); this research shows promising results to the usefulness of this model and its methodology for describing the effects of actions that could be viewed as tools of pecuniary warfare.

5.3 Potential Future Research

Several ideas surfaced through the process of this research that fell outside the scope of the research but are now highlighted as potential avenues for future research. The topics for future research are discussed in this section.

5.3.1 Model Specification

The data for this baseline research effort specified the model using theoretical relationships defined under the Keynesian's macroeconomic school of thought. There are alternative schools of economic thought that can be considered when defining the economic condition of a country such as business cycle theory, new Keynesian theory, and post Keynesian stock flow consistent modeling theories. These alternative schools of thought would highlight different macroeconomic factors that would define the economic condition of a country. Additionally, the relationships of those factors would be different and potentially have a different differential form than the one used in this research.

Another method of determining the influential factors of a country's economic condition is to use multivariate techniques. Principle component analysis or factor would be used on publicly available data sets, such as the ones provide by the World Bank or individual country's treasury departments, to identify a subset of economic factors that explain the greatest amount of variance. This technique would have the added benefit of mathematical rigor when deciding to use a factor to define an economic state or relationship as opposed to just relying strictly on hypothesized influential factors or macroeconomic theory.

It is likely that these proposed model specification methods would result in new factors to define or impact the economic condition of a country. A likely factor that is not included in this research is the trade between countries. This could be modeled by adding another factor that may or may not have the effects describe by equations 3.5 or 3.7. in addition to trade, some potentially impactful factors that were not considered in this research are unemployment and consumer spending. Additionally, this research used an aggregate for the Foreign Direct Investment variable. For more fidelity, the FDI factors should be refined by defining which country invested with another. This would add an additional level of complexity to the model while providing more fidelity and insight to the model's results.

In reference to the fourth question of section 1.2, yes the model does behave differently for different economies but there is potential for further research. For example, there are typically four types of economic systems; traditional, command, market, and mixed. This research focused on the majority of economies which follow a mixed economic system where there is a balance between free market and government intervention. A potential area for further research would be investigating if there are fundamental characteristics shown in the model of economies that operate under one of the other three economic systems. Additionally, if there were to be a scaling as to how 'mixed' a specific mixed economic system, an inspection of the model and its results could generate clarity for pecuniary warfare modeling efforts. In general, categorizing the countries in the model (be it by economic system, geographic region, economic development, size or etc.) has the potential to provide an analyst significant insights to the understanding and impacts of pecuniary warfare actions.

There are shocks to economic systems that are also not considered in this research that should be addressed in future research. One shock is the impact of major corporation failures or a country reaching a limit on its accumulated debt. To model these shocks, factors should be added to the model. These factors could be constrained to represent the impacts of the shocks.

5.3.2 Retribution

The current model only accounts for the US instruments of national power. Dwight Eisenhower is quoted as saying "In preparing for battle I have always found that plans are useless, but planning is indispensable." What the former President of the United States was highlighting is that, once the plan is in effect, the environment changes and therefore the plan must be altered as well. The changes in the environment can be partially accounted for by considering the actions taken from countries other than the US when responding to their new economic condition. A detailed analysis would need to be conducted to identify potential actions that other countries are likely to take.

By adding forcing functions that account for the actions of the other countries, the model would be able to account for those environmental changes more effectively. A potential action to be added is the devaluation of currency taken by China. Modeling this effect could result in a better understanding of the effects that China's economic policy actions have on the US and other countries in the world economy. However, realized dynamics would be needed to calculate coefficients.

In line with this train of thought the implications that results from sanctions should be researched further. This prediction analysis can be used in conjunction with the what-if scenario of this research to model the effects of imposing or removing sanctions.

5.3.3 Mean Field Game Theory

Hoang defines Mean Field Game Theory as the study of strategic decision making in large populations that are composed of individuals that interact amongst each other (Hoang 2014:1). An example of an application of mean field game theory is in modeling a school of fish. Modeling the school of fish using classical game theory becomes computationally infeasible. Using mean field game theory, the problem becomes feasible by modeling how a fish interacts with the school of fish rather than modeling the interaction of a fish and all of the individual fish around it. Dynamic programming techniques can be used to describe how a fish reacts to the school of fish (Hoang 2014:1). The school of fish's actions is defined by the actions of the individual fish which can be defined using statistical mechanic principles (Hoang 2014:1). An important assumption of mean field game theory is time independency. Using the school of fish example, this means that the functions that define interactions between the fish and the school of fish and the actions of the individual fish do not change over time (Hoang 2014:1).

The application of mean field game theory on a complex multi-agent dynamic system such as the world economy could be better facilitated using this research. This research could be used to inform a modeler on how to specify the payoff functions for an individual country's action. Additionally, the functions that define the dynamics between the individual countries and the world economy would be better informed by the results of this research. The form of the system of differential equations used in this research is time independent making it a viable method for applying to mean field game theory.

5.4 Conclusion

This research provides a methodology to develop models for greater understanding of the application of the economic instrument of national power through the factors that define the economic condition of a country. The ability to perform what-if analysis and make predictions using this model is demonstrated. The understanding of the dynamics from the system can be implemented when considering the effects of economic policy as defined by FFR changes. Through an increased understanding and awareness of the dynamics of the economic environment, a baseline model for analysis is built to begin addressing the impacts of pecuniary warfare tactics.

Appendix A: Graphical Depiction of the Four Country Metamodel

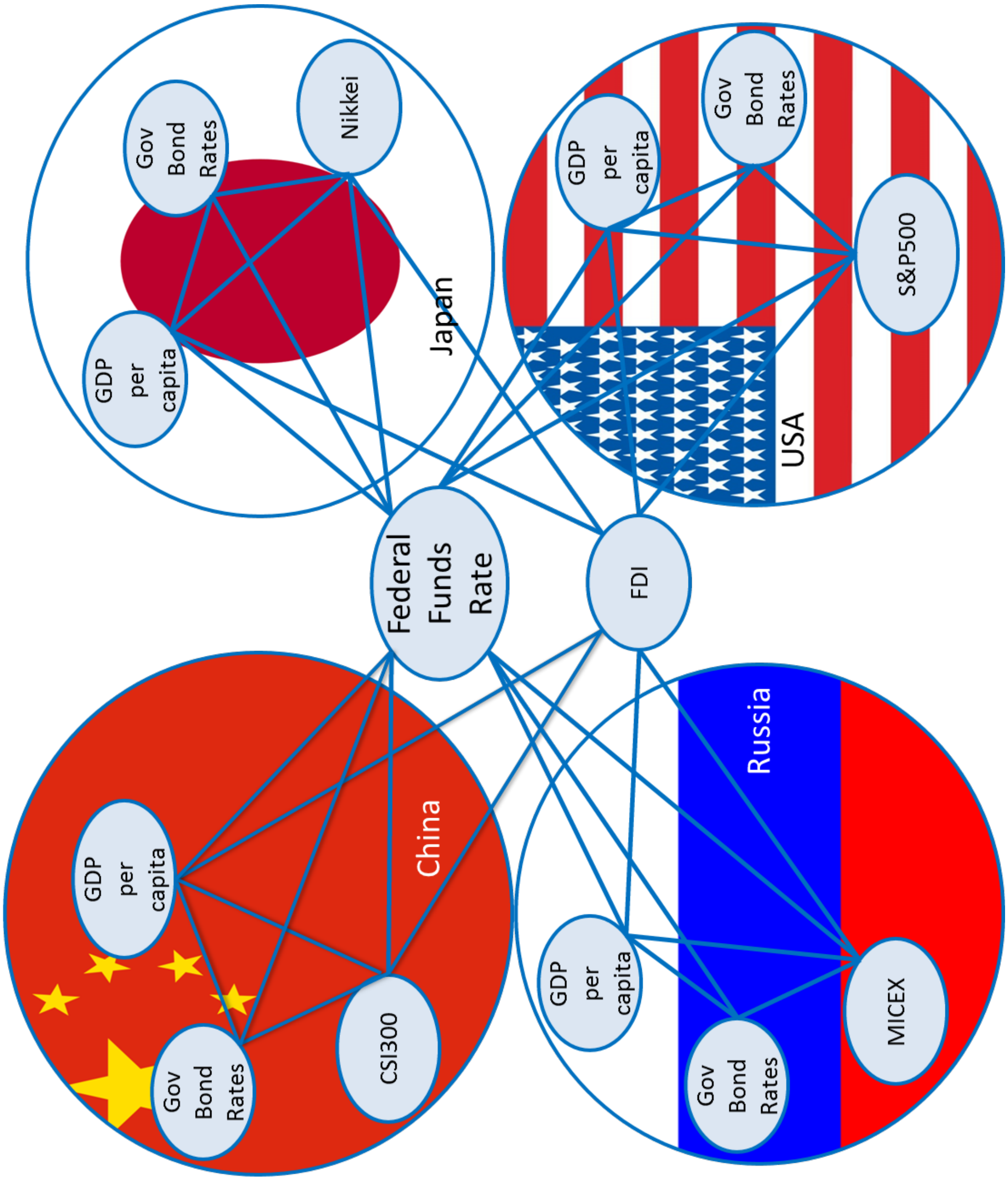


Figure 14: Metamodel of all four countries that were considered in this research. Note that FFR was used for every country.

Appendix B: Raw Data Values

All data was collected on 10 August 2015 from the online sources listed in section 3.1.

Table 8: US Economic Condition Raw Data Values

		United States			
Period	GDP	SM	BY	FDI	FFR
1	45195.03	1191.5	3.987	145966000000	4.94
2	45372.47	1191.33	3.921	145329416667	4.99
3	45549.91	1234.18	4.282	144692833333	5.24
4	45727.35	1220.33	4.014	144056250000	5.25
5	45904.79	1228.81	4.332	143419666667	5.25
6	46082.23	1207.01	4.557	142783083333	5.25
7	46259.67	1249.48	4.49	142146500000	5.25
8	46437.11	1248.29	4.395	141509916667	5.24
9	46572.47	1280.08	4.519	140873333333	5.25
10	46707.83	1280.66	4.557	140236750000	5.26
11	46843.19	1294.83	4.853	139600166667	5.26
12	46978.55	1310.61	5.057	138963583333	5.25
13	47113.9	1270.09	5.123	138327000000	5.25
14	47249.26	1270.2	5.145	151323750000	5.25
15	47384.62	1276.66	4.988	164320500000	5.26
16	47519.98	1303.82	4.732	177317250000	5.02
17	47655.34	1335.85	4.634	190314000000	4.94
18	47790.7	1377.94	4.604	203310750000	4.76
19	47926.06	1400.63	4.462	216307500000	4.49
20	48061.42	1418.3	4.7	229304250000	4.24
21	48089.76	1438.24	4.814	242301000000	3.94
22	48118.1	1406.82	4.577	255297750000	2.98
23	48146.44	1420.86	4.648	268294500000	2.61
24	48174.78	1482.37	4.628	281291250000	2.28
25	48203.12	1530.62	4.892	294288000000	1.98
26	48231.45	1503.35	5.027	298102750000	2
27	48259.79	1455.27	4.733	301917500000	2.01
28	48288.13	1473.99	4.527	305732250000	2
29	48316.47	1526.75	4.594	309547000000	1.81
30	48344.81	1549.38	4.473	313361750000	0.97
31	48373.15	1481.14	3.949	317176500000	0.39
32	48401.49	1468.36	4.035	320991250000	0.16
33	48284.82	1378.55	3.597	324806000000	0.15
34	48168.14	1330.63	3.519	328620750000	0.22
35	48051.47	1322.7	3.421	332435500000	0.18
36	47934.8	1385.59	3.734	336250250000	0.15
37	47818.13	1400.38	4.067	340065000000	0.18
38	47701.46	1280	3.975	339454083333	0.21
39	47584.79	1267.38	3.958	338843166667	0.16
40	47468.11	1282.83	3.825	338232250000	0.16
41	47351.44	1166.36	3.829	337621333333	0.15
42	47234.77	968.75	3.97	337010416667	0.12
43	47118.1	896.24	2.92	336399500000	0.12
44	47001.43	903.25	2.22	335788583333	0.12
45	47116.09	825.88	2.851	335177666667	0.11

Table 8 Continued…

46	47230.76	735.09	3.02	334566750000	0.13
47	47345.42	797.87	2.668	333955833333	0.16
48	47460.08	872.81	3.119	333344916667	0.2
49	47574.75	919.14	3.461	332734000000	0.2
50	47689.41	919.32	3.536	317821833333	0.18
51	47804.07	987.48	3.481	302909666667	0.18
52	47918.74	1020.62	3.401	287997500000	0.19
53	48033.4	1057.08	3.305	273085333333	0.19
54	48148.07	1036.19	3.388	258173166667	0.19
55	48262.73	1095.63	3.198	243261000000	0.19
56	48377.39	1115.1	3.837	228348833333	0.18
57	48496.24	1073.87	3.588	213436666667	0.17
58	48615.08	1104.49	3.619	198524500000	0.16
59	48733.92	1169.43	3.833	183612333333	0.14
60	48852.76	1186.69	3.659	168700166667	0.1
61	48971.6	1089.41	3.3	153788000000	0.09
62	49090.44	1030.71	2.935	162584333333	0.09
63	49209.28	1101.6	2.905	171380666667	0.07
64	49328.13	1049.33	2.47	180177000000	0.1
65	49446.97	1141.2	2.512	188973333333	0.08
66	49565.81	1183.26	2.603	197769666667	0.07
67	49684.65	1180.55	2.797	206566000000	0.08
68	49803.49	1257.64	3.288	215362333333	0.07
69	49944.52	1286.12	3.374	224158666667	0.08
70	50085.56	1327.22	3.422	232955000000	0.1
71	50226.59	1325.83	3.47	241751333333	0.13
72	50367.62	1363.61	3.29	250547666667	0.14
73	50508.65	1345.2	3.059	259344000000	0.16
74	50649.68	1320.64	3.16	259182833333	0.16
75	50790.72	1292.28	2.793	259021666667	0.16
76	50931.75	1218.89	2.234	258860500000	0.13
77	51072.78	1131.42	1.917	258699333333	0.14
78	51213.81	1253.3	2.116	258538166667	0.16
79	51354.84	1246.96	2.072	258377000000	0.16
80	51495.87	1257.6	1.876	258215833333	0.16
81	51624.72	1312.41	1.795	258054666667	0.14
82	51753.56	1365.68	1.974	257893500000	0.15
83	51882.4	1408.47	2.214	257732333333	0.14
84	52011.24	1397.91	1.919	257571166667	0.15
85	52140.09	1310.33	1.563	257410000000	0.11
86	52268.93	1362.16	1.643	254107333333	0.09
87	52397.77	1379.32	1.47	250804666667	0.09
88	52526.61	1406.58	1.548	247502000000	0.08
89	52655.45	1440.67	1.633	244199333333	0.08
90	52784.3	1412.16	1.694	240896666667	0.09
91	52913.14	1416.18	1.616	237594000000	0.08
92	53041.98	1426.19	1.757	234291333333	0.09

Table 9: Japan Economic Condition Raw Data Values

		Japan			
Period	GDP	SM	BY	FDI	FFR
1	35070.67	11276.59	1.251	7806977011	4.94
2	34928.57	11584.01	1.179	7611363789	4.99
3	34786.47	11899.6	1.308	7415750566	5.24
4	34644.37	12413.6	1.351	7220137344	5.25
5	34502.27	13574.3	1.485	7024524122	5.25
6	34360.17	13606.5	1.547	6828910899	5.25
7	34218.07	14872.15	1.444	6633297677	5.25
8	34075.98	16111.43	1.481	6437684455	5.24
9	34072.45	16649.82	1.564	6242071232	5.25
10	34068.93	16205.43	1.59	6046458010	5.26
11	34065.4	17059.66	1.766	5850844788	5.26
12	34061.88	16906.23	1.919	5655231565	5.25
13	34058.36	15467.33	1.831	5459618343	5.25
14	34054.83	15505.18	1.922	4804907670	5.25
15	34051.31	15456.81	1.939	4150196996	5.26
16	34047.79	16140.76	1.647	3495486323	5.02
17	34044.26	16127.58	1.677	2840775650	4.94
18	34040.74	16399.39	1.724	2186064977	4.76
19	34037.22	16274.33	1.665	1531354303	4.49
20	34033.69	17225.83	1.69	876643630	4.24
21	34353.02	17383.42	1.705	221932956.8	3.94
22	34672.35	17604.12	1.639	-432777716.5	2.98
23	34991.67	17287.65	1.662	-1087488390	2.61
24	35311	17400.41	1.634	-1742199063	2.28
25	35630.33	17875.75	1.746	-2396909736	1.98
26	35949.66	18138.36	1.86	-394566888.6	2
27	36268.98	17248.89	1.807	1607775959	2.01
28	36588.31	16569.09	1.623	3610118807	2
29	36907.64	16785.69	1.677	5612461654	1.81
30	37226.97	16737.63	1.611	7614804502	0.97
31	37546.29	15680.67	1.493	9617147350	0.39
32	37865.62	15307.78	1.509	11619490197	0.16
33	37987.04	13592.47	1.453	13621833045	0.15
34	38108.45	13603.02	1.367	15624175893	0.22
35	38229.87	12525.54	1.286	17626518740	0.18
36	38351.28	13849.99	1.625	19628861588	0.15
37	38472.7	14338.54	1.76	21631204436	0.18
38	38594.11	13481.38	1.595	21880674510	0.21
39	38715.53	13376.81	1.54	22130144585	0.16
40	38836.95	13072.87	1.42	22379614659	0.16
41	38958.36	11259.86	1.48	22629084734	0.15
42	39079.78	8576.98	1.49	22878554808	0.12
43	39201.19	8512.27	1.4	23128024883	0.12
44	39322.61	8859.56	1.175	23377494957	0.12
45	39621.5	7994.05	1.29	23626965032	0.11
46	39920.38	7568.42	1.265	23876435106	0.13
47	40219.27	8109.53	1.355	24125905181	0.16
48	40518.16	8828.26	1.43	24375375255	0.2
49	40817.04	9522.5	1.495	24624845330	0.2

Table 9 Continued…

50	41115.93	9958.44	1.35	23591647517	0.18
51	41414.81	10356.83	1.42	22558449704	0.18
52	41713.7	10492.53	1.305	21525251892	0.19
53	42012.59	10133.23	1.3	20492054079	0.19
54	42311.47	10034.74	1.415	19458856267	0.19
55	42610.36	9345.55	1.261	18425658454	0.19
56	42909.25	10546.44	1.291	17392460642	0.18
57	43183.78	10198.04	1.33	16359262829	0.17
58	43458.32	10126.03	1.309	15326065016	0.16
59	43732.86	11089.94	1.413	14292867204	0.14
60	44007.4	11057.4	1.29	13259669391	0.1
61	44281.93	9768.7	1.27	12226471579	0.09
62	44556.47	9382.64	1.09	11827680554	0.09
63	44831.01	9537.3	1.062	11428889530	0.07
64	45105.55	8824.06	0.985	11030098505	0.1
65	45380.09	9369.35	0.94	10631307481	0.08
66	45654.62	9202.45	0.935	10232516456	0.07
67	45929.16	9937.04	1.195	9833725431	0.08
68	46203.7	10228.92	1.116	9434934407	0.07
69	46243.33	10237.92	1.215	9036143382	0.08
70	46282.96	10624.09	1.259	8637352358	0.1
71	46322.59	9755.1	1.255	8238561333	0.13
72	46362.22	9849.74	1.206	7839770309	0.14
73	46401.85	9693.73	1.155	7440979284	0.16
74	46441.48	9816.09	1.135	6750004591	0.16
75	46481.11	9833.03	1.079	6059029898	0.16
76	46520.74	8955.2	1.032	5368055204	0.13
77	46560.37	8700.29	1.032	4677080511	0.14
78	46600	8988.39	1.05	3986105818	0.16
79	46639.63	8434.61	1.073	3295131125	0.16
80	46679.27	8455.35	0.988	2604156431	0.16
81	46008.8	8802.51	0.973	1913181738	0.14
82	45338.34	9723.24	0.968	1222207045	0.15
83	44667.88	10083.56	0.993	531232351.5	0.14
84	43997.41	9520.89	0.898	-159742341.8	0.15
85	43326.95	8542.73	0.829	-850717035.1	0.11
86	42656.49	9006.78	0.839	-709050605.1	0.09
87	41986.02	8695.06	0.798	-567384175.1	0.09
88	41315.56	8839.91	0.798	-425717745.2	0.08
89	40645.1	8870.16	0.773	-284051315.2	0.08
90	39974.63	8928.29	0.776	-142384885.2	0.09
91	39304.17	9446.01	0.712	-718455.2534	0.08
92	38633.71	10395.18	0.802	140947974.7	0.09

Table 10: China Economic Condition Raw Data Values

		China			
Period	GDP	SM	BY	FDI	FFR
1	1872.05	855.95	4.133	62108043001	4.94
2	1900.234	878.69	3.874	66199891563	4.99
3	1928.419	888.16	3.548	70291740126	5.24
4	1956.604	927.92	3.518	74383588688	5.25
5	1984.789	917.39	3.325	78475437250	5.25
6	2012.974	876.28	3.174	82567285813	5.25
7	2041.159	873.83	3.293	86659134375	5.25
8	2069.344	923.45	3.301	90750982937	5.24
9	2117.837	1009.6	3.134	94842831500	5.25
10	2166.33	1053.01	2.97	98934680062	5.26
11	2214.823	1061.09	2.948	103026528624	5.26
12	2263.316	1172.35	3.047	107118377187	5.25
13	2311.809	1365.45	3.047	111210225749	5.25
14	2360.302	1393.96	3.132	113048752643	5.25
15	2408.795	1294.33	3.39	114887279536	5.26
16	2457.288	1338.69	3.323	116725806430	5.02
17	2505.781	1403.27	3.302	118564333323	4.94
18	2554.274	1464.47	2.966	120402860217	4.76
19	2602.767	1714.36	3.01	122241387111	4.49
20	2651.26	2041.05	3.023	124079914004	4.24
21	2714.787	2385.34	3.008	125918440898	3.94
22	2778.315	2544.57	3.103	127756967791	2.98
23	2841.842	2781.78	3.435	129595494685	2.61
24	2905.37	3558.71	3.889	131434021578	2.28
25	2968.897	3927.95	4.3	133272548472	1.98
26	3032.424	3764.08	4.312	136282322996	2
27	3095.952	4460.56	4.399	139292097520	2.01
28	3159.479	5296.81	4.399	142301872043	2
29	3223.007	5580.81	4.459	145311646567	1.81
30	3286.534	5688.54	4.521	148321421091	0.97
31	3350.061	4737.41	4.613	151331195615	0.39
32	3413.589	5338.27	4.55	154340970139	0.16
33	3441.498	4620.4	4.3	157350744663	0.15
34	3469.408	4674.55	4.13	160360519186	0.22
35	3497.318	3790.53	4.15	163370293710	0.18
36	3525.227	3959.12	4.08	166380068234	0.15
37	3553.137	3611.33	4.176	169389842758	0.18
38	3581.046	2791.82	4.451	170840485073	0.21
39	3608.956	2805.21	4.587	172291127389	0.16
40	3636.866	2391.64	4.27	173741769705	0.16
41	3664.775	2243.66	3.704	175192412020	0.15
42	3692.685	1663.66	3.07	176643054336	0.12
43	3720.594	1829.92	3.3	178093696651	0.12
44	3748.504	1817.72	2.749	179544338967	0.12
45	3805.574	2032.68	3.099	180994981282	0.11
46	3862.644	2140.49	3.1	182445623598	0.13
47	3919.713	2507.79	3.16	183896265913	0.16
48	3976.783	2622.93	3.18	185346908229	0.2
49	4033.853	2759.71	3.06	186797550544	0.2

Table 10 Continued…

50	4090.922	3166.47	3.28	185153655391	0.18
51	4147.992	3734.62	3.268	183509760237	0.18
52	4205.062	2830.27	3.55	181865865083	0.19
53	4262.132	3004.8	3.54	180221969929	0.19
54	4319.201	3280.37	3.7	178578074776	0.19
55	4376.271	3511.67	3.56	176934179622	0.19
56	4433.341	3575.68	3.6	175290284468	0.18
57	4517.838	3204.16	3.6	173646389314	0.17
58	4602.336	3281.67	3.55	172002494161	0.16
59	4686.833	3345.61	3.58	170358599007	0.14
60	4771.33	3067.36	3.34	168714703853	0.1
61	4855.828	2773.26	3.31	167070808699	0.09
62	4940.325	2563.07	3.33	175897121497	0.09
63	5024.823	2868.85	3.31	184723434295	0.07
64	5109.32	2903.19	3.221	193549747093	0.1
65	5193.817	2868.85	3.321	202376059891	0.08
66	5278.315	2563.07	3.67	211202372688	0.07
67	5362.812	2773.26	3.9	220028685486	0.08
68	5447.309	3067.36	3.9	228854998284	0.07
69	5501.099	3345.61	3.89	237681311082	0.08
70	5554.888	3281.67	3.94	246507623880	0.1
71	5608.678	3204.16	3.9	255333936678	0.13
72	5662.467	3575.68	3.88	264160249476	0.14
73	5716.256	3511.67	3.84	272986562273	0.16
74	5770.046	3280.37	3.88	277870324646	0.16
75	5823.835	3004.8	4.12	282754087018	0.16
76	5877.624	2830.27	4	287637849391	0.13
77	5931.414	3734.62	3.87	292521611763	0.14
78	5985.203	3166.47	3.81	297405374135	0.16
79	6038.993	2759.71	3.62	302289136508	0.16
80	6092.782	2622.93	3.55	307172898880	0.16
81	6152.336	2507.79	3.4	312056661253	0.14
82	6211.89	2140.49	3.55	316940423625	0.15
83	6271.444	2032.68	3.55	321824185997	0.14
84	6330.998	1817.72	3.55	326707948370	0.15
85	6390.552	1829.92	3.5	331591710742	0.11
86	6450.106	1663.66	3.37	328594533773	0.09
87	6509.66	2243.66	3.33	325597356803	0.09
88	6569.215	2391.64	3.45	322600179834	0.08
89	6628.769	2805.21	3.47	319603002864	0.08
90	6688.323	2791.82	3.57	316605825895	0.09
91	6747.877	3611.33	3.57	313608648926	0.08
92	6807.431	3959.12	3.575	310611471956	0.09

Table 11: Russia Economic Condition Raw Data Values

		Russia			
Period	GDP	SM	BY	FDI	FFR
1	6008.866	603.89	8.392	15444370800	4.94
2	6142.957	639.98	8.339	15449678358	4.99
3	6277.048	700.65	8.311	15454985917	5.24
4	6411.139	784.28	7.507	15460293475	5.25
5	6545.23	892.5	6.68	15465601033	5.25
6	6679.32	842.52	7.068	15470908592	5.25
7	6813.411	944.55	6.938	15476216150	5.25
8	6947.502	1011	6.831	15481523708	5.24
9	7130.665	1171.44	6.676	15486831267	5.25
10	7313.827	1320.83	6.721	15492138825	5.26
11	7496.99	1299.19	6.729	15497446383	5.26
12	7680.153	1486.85	7.017	15502753942	5.25
13	7863.315	1281.5	6.777	15508061500	5.25
14	8046.478	1331.39	6.758	17348620042	5.25
15	8229.64	1380.24	6.721	19189178583	5.26
16	8412.803	1448.72	6.61	21029737125	5.02
17	8595.966	1367.24	6.53	22870295667	4.94
18	8779.128	1426.86	6.562	24710854208	4.76
19	8962.291	1550.58	6.56	26551412750	4.49
20	9145.454	1693.47	6.473	28391971292	4.24
21	9358.306	1656.94	6.57	30232529833	3.94
22	9571.158	1655.25	6.565	32073088375	2.98
23	9784.01	1698.08	6.496	33913646917	2.61
24	9996.862	1697.28	6.433	35754205458	2.28
25	10209.71	1570.34	6.317	37594764000	1.98
26	10422.57	1665.96	6.334	39118007025	2
27	10635.42	1734.42	6.341	40641250050	2.01
28	10848.27	1677.02	6.402	42164493075	2
29	11061.12	1759.44	6.513	43687736100	1.81
30	11273.97	1874.73	6.35	45210979125	0.97
31	11486.83	1850.64	6.338	46734222150	0.39
32	11699.68	1888.86	6.272	48257465175	0.16
33	11442.68	1574.33	6.281	49780708200	0.15
34	11185.68	1660.42	6.736	51303951225	0.22
35	10928.68	1628.43	6.432	52827194250	0.18
36	10671.68	1667.35	6.468	54350437275	0.15
37	10414.68	1925.24	6.709	55873680300	0.18
38	10157.68	1753.67	6.591	57449449192	0.21
39	9900.676	1495.33	7.211	59025218083	0.16
40	9643.675	1348.92	8.29	60600986975	0.16
41	9386.675	1027.66	7.459	62176755867	0.15
42	9129.674	731.96	8.595	63752524758	0.12
43	8872.674	611.32	8.134	65328293650	0.12
44	8615.673	619.53	9.487	66904062542	0.12
45	8790.181	624.9	12.575	68479831433	0.11
46	8964.689	666.05	12.693	70055600325	0.13
47	9139.197	772.93	12.796	71631369217	0.16
48	9313.705	920.35	10.677	73207138108	0.2
49	9488.213	1123.38	11.28	74782907000	0.2

Table 11 Continued…

50	9662.721	971.55	11.297	71599589617	0.18
51	9837.229	1053.3	11.316	68416272233	0.18
52	10011.74	1091.98	11.515	65232954850	0.19
53	10186.25	1197.2	10.878	62049637467	0.19
54	10360.75	1237.18	9.276	58866320083	0.19
55	10535.26	1284.95	9.087	55683002700	0.19
56	10709.77	1370.01	8.001	52499685317	0.18
57	10927.65	1419.42	7.751	49316367933	0.17
58	11145.52	1332.64	7.723	46133050550	0.16
59	11363.4	1450.15	6.933	42949733167	0.14
60	11581.28	1436.04	7.113	39766415783	0.1
61	11799.15	1332.62	7.55	36583098400	0.09
62	12017.03	1309.31	7.173	37131820958	0.09
63	12234.91	1397.12	7.077	37680543517	0.07
64	12452.78	1368.9	7.308	38229266075	0.1
65	12670.66	1440.3	7.264	38777988633	0.08
66	12888.53	1523.39	7.591	39326711192	0.07
67	13106.41	1565.52	7.67	39875433750	0.08
68	13324.29	1687.99	7.441	40424156308	0.07
69	13388.15	1723.42	8.25	40972878867	0.08
70	13452.01	1777.84	8.763	41521601425	0.1
71	13515.88	1813.59	7.817	42070323983	0.13
72	13579.74	1741.84	7.734	42619046542	0.14
73	13643.6	1666.3	8.179	43167769100	0.16
74	13707.47	1666.59	8.127	44160757717	0.16
75	13771.33	1705.18	7.71	45153746333	0.16
76	13835.2	1546.05	8.023	46146734950	0.13
77	13899.06	1366.54	8.711	47139723567	0.14
78	13962.92	1498.6	8.712	48132712183	0.16
79	14026.79	1499.62	8.306	49125700800	0.16
80	14090.65	1402.23	8.5	50118689417	0.16
81	14134.07	1514.03	8.29	51111678033	0.14
82	14177.49	1597.67	8.018	52104666650	0.15
83	14220.91	1517.34	7.825	53097655267	0.14
84	14264.33	1473.5	7.991	54090643883	0.15
85	14307.75	1306.42	8.706	55083632500	0.11
86	14351.17	1387.52	8.46	54708959350	0.09
87	14394.6	1407.02	7.976	54334286200	0.09
88	14438.02	1422.91	7.86	53959613050	0.08
89	14481.44	1458.26	7.761	53584939900	0.08
90	14524.86	1425.7	7.405	53210266750	0.09
91	14568.28	1405.97	6.93	52835593600	0.08
92	14611.7	1474.72	6.85	52460920450	0.09

Appendix C: Indexed Data Values

Index values were computed for each factor according to equation 3.2.

Table 12: US Economic Condition Index Data Values

		United States			
Period	GDP	SM	BY	FDI	FFR
1	0	56.05006	68.4898	3.786594	93.8343
2	2.261255	56.02918	66.69388	3.471045	94.79769
3	4.52251	61.29143	76.51701	3.155495	99.61464
4	6.783765	59.59056	69.22449	2.839946	99.80732
5	9.045019	60.63196	77.87755	2.524396	99.80732
6	11.30627	57.95478	84	2.208847	99.80732
7	13.56753	63.17037	82.17687	1.893297	99.80732
8	15.82878	63.02423	79.59184	1.577748	99.61464
9	17.55378	66.92824	82.96599	1.262198	99.80732
10	19.27877	66.99947	84	0.946649	100
11	21.00377	68.73964	92.05442	0.631099	100
12	22.72876	70.67752	97.60544	0.31555	99.80732
13	24.45376	65.70141	99.40136	0	99.80732
14	26.17875	65.71492	100	6.442391	99.80732
15	27.90375	66.50825	95.72789	12.88478	100
16	29.62874	69.84367	88.7619	19.32717	95.37572
17	31.35374	73.77716	86.09524	25.76956	93.8343
18	33.07873	78.94608	85.27891	32.21195	90.36609
19	34.80373	81.73255	81.41497	38.65434	85.16378
20	36.52872	83.90254	87.89116	45.09673	80.34682
21	36.88987	86.3513	90.9932	51.53913	74.56647
22	37.25101	82.49272	84.54422	57.98152	56.06936
23	37.61215	84.21693	86.47619	64.42391	48.94027
24	37.9733	91.77075	85.93197	70.8663	42.58189
25	38.33444	97.69615	93.11565	77.30869	36.80154
26	38.69559	94.34722	96.78912	79.19963	37.1869
27	39.05673	88.44269	88.78912	81.09057	37.37958
28	39.41787	90.74163	83.18367	82.98152	37.1869
29	39.77902	97.22089	85.0068	84.87246	33.52601
30	40.14016	100	81.71429	86.7634	17.34104
31	40.5013	91.61969	67.45578	88.65434	6.165703
32	40.86245	90.05023	69.79592	90.54529	1.734104
33	39.37561	79.02099	57.87755	92.43623	1.541426
34	37.88877	73.13611	55.7551	94.32717	2.890173
35	36.40193	72.16225	53.08844	96.21811	2.119461
36	34.91509	79.88554	61.60544	98.10906	1.541426
37	33.42825	81.70185	70.66667	100	2.119461
38	31.94141	66.91842	68.16327	99.69717	2.697495
39	30.45457	65.3686	67.70068	99.39435	1.734104
40	28.96773	67.26596	64.08163	99.09152	1.734104
41	27.4809	52.9627	64.19048	98.78869	1.541426
42	25.99406	28.69494	68.02721	98.48587	0.963391
43	24.50722	19.79025	39.45578	98.18304	0.963391
44	23.02038	20.65112	20.40816	97.88021	0.963391
45	24.48163	11.14959	37.57823	97.57739	0.770713

Table 12 Continued…

46	25.94288	0	42.17687	97.27456	1.156069
47	27.40414	7.709784	32.59864	96.97173	1.734104
48	28.86539	16.91289	44.87075	96.66891	2.504817
49	30.32664	22.60251	54.17687	96.36608	2.504817
50	31.7879	22.62462	56.21769	88.97423	2.119461
51	33.24915	30.9951	54.72109	81.58238	2.119461
52	34.7104	35.0649	52.54422	74.19053	2.312139
53	36.17165	39.54242	49.93197	66.79869	2.312139
54	37.63291	36.977	52.19048	59.40684	2.312139
55	39.09416	44.27661	47.02041	52.01499	2.312139
56	40.55541	46.66765	64.40816	44.62314	2.119461
57	42.06991	41.60434	57.63265	37.23129	1.926782
58	43.5844	45.36467	58.47619	29.83945	1.734104
59	45.09889	53.33972	64.29932	22.4476	1.348748
60	46.61339	55.45936	59.56463	15.05575	0.578035
61	48.12788	43.51275	49.79592	7.663901	0.385356
62	49.64238	36.30402	39.86395	12.02418	0.385356
63	51.15687	45.00976	39.04762	16.38445	0
64	52.67136	38.59067	27.21088	20.74473	0.578035
65	54.18586	49.8729	28.35374	25.105	0.192678
66	55.70035	55.03813	30.82993	29.46528	0
67	57.21485	54.70533	36.10884	33.82556	0.192678
68	58.72934	64.17247	49.46939	38.18583	0
69	60.52662	67.66999	51.80952	42.54611	0.192678
70	62.3239	72.71734	53.11565	46.90638	0.578035
71	64.12118	72.54664	54.42177	51.26666	1.156069
72	65.91847	77.18626	49.52381	55.62694	1.348748
73	67.71575	74.9254	43.2381	59.98721	1.734104
74	69.51303	71.90927	45.98639	59.90732	1.734104
75	71.31031	68.42648	36	59.82743	1.734104
76	73.10759	59.41372	20.78912	59.74754	1.156069
77	74.90488	48.67185	12.16327	59.66765	1.348748
78	76.70216	63.63949	17.57823	59.58777	1.734104
79	78.49944	62.8609	16.38095	59.50788	1.734104
80	80.29672	64.16756	11.04762	59.42799	1.734104
81	81.93866	70.89857	8.843537	59.3481	1.348748
82	83.5806	77.44047	13.71429	59.26821	1.541426
83	85.22254	82.69535	20.2449	59.18832	1.348748
84	86.86448	81.39852	12.21769	59.10843	1.541426
85	88.50642	70.64314	2.530612	59.02854	0.770713
86	90.14836	77.00819	4.707483	57.39144	0.385356
87	91.7903	79.11555	0	55.75433	0.385356
88	93.43224	82.46325	2.122449	54.11722	0.192678
89	95.07418	86.64972	4.435374	52.48011	0.192678
90	96.71612	83.14851	6.095238	50.84301	0.385356
91	98.35806	83.64219	3.972789	49.2059	0.192678
92	100	84.87148	7.809524	47.56879	0.385356

Table 13: Japan Economic Condition Index Data Values

		Japan			
Period	GDP	SM	BY	FDI	FFR
1	8.200319	35.08222	43.92828	37.76175	93.8343
2	7.076612	37.99066	38.06031	37.03784	94.79769
3	5.952904	40.97639	48.57376	36.31393	99.61464
4	4.829197	45.83924	52.07824	35.59002	99.80732
5	3.70549	56.82038	62.99919	34.86611	99.80732
6	2.581782	57.12502	68.05216	34.1422	99.80732
7	1.458075	69.09907	59.6577	33.41829	99.80732
8	0.334368	80.82364	62.67319	32.69438	99.61464
9	0.306504	85.91723	69.43765	31.97047	99.80732
10	0.27864	81.71295	71.55664	31.24656	100
11	0.250776	89.79464	85.90057	30.52265	100
12	0.222912	88.34307	98.37001	29.79874	99.80732
13	0.195048	74.72994	91.19804	29.07483	99.80732
14	0.167184	75.08803	98.61451	26.65192	99.80732
15	0.13932	74.63041	100	24.22902	100
16	0.111456	81.10112	76.20212	21.80612	95.37572
17	0.083592	80.97643	78.64711	19.38322	93.8343
18	0.055728	83.54797	82.47759	16.96031	90.36609
19	0.027864	82.3648	77.66911	14.53741	85.16378
20	0	91.36674	79.7066	12.11451	80.34682
21	2.52521	92.85767	80.9291	9.691608	74.56647
22	5.05042	94.94567	75.55012	7.268706	56.06936
23	7.57563	91.95161	77.42461	4.845804	48.94027
24	10.10084	93.01841	75.14262	2.422902	42.58189
25	12.62605	97.5155	84.27058	0	36.80154
26	15.15126	100	93.56153	7.410114	37.1869
27	17.67647	91.58491	89.24205	14.82023	37.37958
28	20.20168	85.15346	74.24613	22.23034	37.1869
29	22.72689	87.20267	78.64711	29.64046	33.52601
30	25.2521	86.74799	73.26813	37.05057	17.34104
31	27.77731	76.74831	63.65118	44.46068	6.165703
32	30.30252	73.22047	64.95518	51.8708	1.734104
33	31.26267	56.99228	60.3912	59.28091	1.541426
34	32.22281	57.09209	53.38223	66.69103	2.890173
35	33.18296	46.89828	46.78077	74.10114	2.119461
36	34.1431	59.42862	74.40913	81.51125	1.541426
37	35.10324	64.05069	85.41157	88.92137	2.119461
38	36.06339	55.94128	71.96414	89.84459	2.697495
39	37.02353	54.95197	67.48166	90.76781	1.734104
40	37.98368	52.07645	57.70171	91.69103	1.734104
41	38.94382	34.92394	62.59169	92.61425	1.541426
42	39.90397	9.541776	63.40668	93.53746	0.963391
43	40.86411	8.929568	56.07172	94.46068	0.963391
44	41.82425	12.21521	37.73431	95.3839	0.963391
45	44.18782	4.026797	47.10676	96.30712	0.770713

Table 13 Continued…

46	46.55139	0	45.06927	97.23034	1.156069
47	48.91495	5.119329	52.40424	98.15356	1.734104
48	51.27852	11.91908	58.51671	99.07678	2.504817
49	53.64208	18.48714	63.81418	100	2.504817
50	56.00565	22.61148	51.99674	96.17642	2.119461
51	58.36921	26.38057	57.70171	92.35284	2.119461
52	60.73278	27.6644	48.32926	88.52927	2.312139
53	63.09635	24.26513	47.92176	84.70569	2.312139
54	65.45991	23.33334	57.29421	80.88211	2.312139
55	67.82348	16.81306	44.74328	77.05853	2.312139
56	70.18704	28.17443	47.18826	73.23496	2.119461
57	72.35806	24.87829	50.36675	69.41138	1.926782
58	74.52908	24.19702	48.65526	65.5878	1.734104
59	76.7001	33.31637	57.13121	61.76422	1.348748
60	78.87111	33.00851	47.10676	57.94064	0.578035
61	81.04213	20.81639	45.47677	54.11707	0.385356
62	83.21315	17.16396	30.80685	52.64125	0.385356
63	85.38417	18.62716	28.52486	51.16544	0
64	87.55519	11.87935	22.24939	49.68962	0.578035
65	89.7262	17.03822	18.58191	48.21381	0.192678
66	91.89722	15.45922	18.17441	46.73799	0
67	94.06824	22.40902	39.3643	45.26218	0.192678
68	96.23926	25.17044	32.92584	43.78636	0
69	96.55265	25.25558	40.9943	42.31055	0.192678
70	96.86605	28.90906	44.58028	40.83474	0.578035
71	97.17944	20.68772	44.25428	39.35892	1.156069
72	97.49284	21.58309	40.2608	37.88311	1.348748
73	97.80623	20.10712	36.10432	36.40729	1.734104
74	98.11963	21.26474	34.47433	33.85019	1.734104
75	98.43302	21.425	29.91035	31.29308	1.734104
76	98.74642	13.12004	26.07987	28.73598	1.156069
77	99.05981	10.70839	26.07987	26.17887	1.348748
78	99.37321	13.43404	27.54686	23.62177	1.734104
79	99.6866	8.194843	29.42135	21.06466	1.734104
80	100	8.39106	22.49389	18.50755	1.734104
81	94.69804	11.67547	21.27139	15.95045	1.348748
82	89.39608	20.3863	20.8639	13.39334	1.541426
83	84.09412	23.79522	22.90139	10.83624	1.348748
84	78.79216	18.47191	15.15892	8.279134	1.541426
85	73.4902	9.217744	9.535452	5.722029	0.770713
86	68.18824	13.60802	10.35045	6.246297	0.385356
87	62.88628	10.65891	7.008965	6.770565	0.385356
88	57.58433	12.0293	7.008965	7.294833	0.192678
89	52.28237	12.31549	4.971475	7.819101	0.192678
90	46.98041	12.86545	5.215974	8.343369	0.385356
91	41.67845	17.76349	0	8.867637	0.192678
92	36.37649	26.74339	7.334963	9.391906	0.385356

Table 14: China Economic Condition Index Data Values

		China			
Period	GDP	SM	BY	FDI	FFR
1	0	0	74.24893	0	93.8343
2	0.571078	0.470555	60.35408	1.518403173	94.79769
3	1.142156	0.666516	42.86481	3.036806346	99.61464
4	1.713233	1.489264	41.25536	4.55520952	99.80732
5	2.284311	1.271368	30.90129	6.073612693	99.80732
6	2.855389	0.420685	22.80043	7.592015866	99.80732
7	3.426467	0.369988	29.18455	9.110419039	99.80732
8	3.997545	1.396767	29.61373	10.62882221	99.61464
9	4.980104	3.179454	20.65451	12.14722539	99.80732
10	5.962663	4.077731	11.85622	13.66562856	100
11	6.945222	4.244929	10.67597	15.18403173	100
12	7.927781	6.547214	15.98712	16.70243491	99.80732
13	8.91034	10.543	15.98712	18.22083808	99.80732
14	9.8929	11.13295	20.54721	18.90307864	99.80732
15	10.87546	9.071326	34.38841	19.58531921	100
16	11.85802	9.98926	30.79399	20.26755977	95.37572
17	12.84058	11.3256	29.66738	20.94980033	93.8343
18	13.82314	12.59201	11.64163	21.6320409	90.36609
19	14.8057	17.76294	14.00215	22.31428146	85.16378
20	15.78825	24.52308	14.69957	22.99652202	80.34682
21	17.07544	31.64742	13.89485	23.67876259	74.56647
22	18.36262	34.94234	18.99142	24.36100315	56.06936
23	19.6498	39.85089	36.80258	25.04324371	48.94027
24	20.93699	55.92777	61.1588	25.72548428	42.58189
25	22.22417	63.5684	83.20815	26.40772484	36.80154
26	23.51135	60.17746	83.85193	27.52459198	37.1869
27	24.79853	74.58961	88.51931	28.64145912	37.37958
28	26.08572	91.89399	88.51931	29.75832625	37.1869
29	27.3729	97.77076	91.7382	30.87519339	33.52601
30	28.66008	100	95.06438	31.99206053	17.34104
31	29.94727	80.31842	100	33.10892766	6.165703
32	31.23445	92.75192	96.62017	34.2257948	1.734104
33	31.79995	77.89715	83.20815	35.34266194	1.541426
34	32.36545	79.01767	74.08798	36.45952907	2.890173
35	32.93095	60.72479	75.16094	37.57639621	2.119461
36	33.49645	64.21339	71.40558	38.69326335	1.541426
37	34.06195	57.01663	76.55579	39.81013048	2.119461
38	34.62745	40.05864	91.30901	40.34843484	2.697495
39	35.19295	40.33572	98.60515	40.88673919	1.734104
40	35.75845	31.77778	81.59871	41.42504354	1.734104
41	36.32395	28.71566	51.23391	41.9633479	1.541426
42	36.88946	16.71381	17.22103	42.50165225	0.963391
43	37.45496	20.1542	29.56009	43.0399566	0.963391
44	38.02046	19.90175	0	43.57826096	0.963391
45	39.1768	24.34988	18.77682	44.11656531	0.770713
46	40.33313	26.58078	18.83047	44.65486966	1.156069
47	41.48947	34.18126	22.04936	45.19317402	1.734104
48	42.64581	36.56383	23.12232	45.73147837	2.504817
49	43.80215	39.3942	16.68455	46.26978273	2.504817

Table 14 Continued…

50	44.95849	47.81122	28.48712	45.65976611	2.119461
51	46.11483	59.56785	27.84335	45.04974949	2.119461
52	47.27117	40.85428	42.9721	44.43973287	2.312139
53	48.42751	44.4658	42.43562	43.82971626	2.312139
54	49.58385	50.16813	51.01931	43.21969964	2.312139
55	50.74019	54.95438	43.50858	42.60968302	2.312139
56	51.89652	56.27893	45.65451	41.9996664	2.119461
57	53.6086	48.59113	45.65451	41.38964979	1.926782
58	55.32067	50.19503	42.9721	40.77963317	1.734104
59	57.03275	51.51813	44.58155	40.16961655	1.348748
60	58.74482	45.76035	31.70601	39.55959994	0.578035
61	60.45689	39.67458	30.09657	38.94958332	0.385356
62	62.16897	35.32516	31.16953	42.22485149	0.385356
63	63.88104	41.65261	30.09657	45.50011966	0
64	65.59312	42.3632	25.32189	48.77538783	0.578035
65	67.30519	41.65261	30.6867	52.05065601	0.192678
66	69.01726	35.32516	49.40987	55.32592418	0
67	70.72934	39.67458	61.74893	58.60119235	0.192678
68	72.44141	45.76035	61.74893	61.87646052	0
69	73.53128	51.51813	61.21245	65.15172869	0.192678
70	74.62116	50.19503	63.89485	68.42699687	0.578035
71	75.71103	48.59113	61.74893	71.70226504	1.156069
72	76.8009	56.27893	60.67597	74.97753321	1.348748
73	77.89077	54.95438	58.53004	78.25280138	1.734104
74	78.98065	50.16813	60.67597	80.06506793	1.734104
75	80.07052	44.4658	73.5515	81.87733448	1.734104
76	81.16039	40.85428	67.11373	83.68960104	1.156069
77	82.25027	59.56785	60.13948	85.50186759	1.348748
78	83.34014	47.81122	56.9206	87.31413414	1.734104
79	84.43001	39.3942	46.72747	89.12640069	1.734104
80	85.51988	36.56383	42.9721	90.93866724	1.734104
81	86.72656	34.18126	34.92489	92.75093379	1.348748
82	87.93324	26.58078	42.9721	94.56320035	1.541426
83	89.13991	24.34988	42.9721	96.3754669	1.348748
84	90.34659	19.90175	42.9721	98.18773345	1.541426
85	91.55327	20.1542	40.2897	100	0.770713
86	92.75994	16.71381	33.31545	98.88780756	0.385356
87	93.96662	28.71566	31.16953	97.77561513	0.385356
88	95.17329	31.77778	37.6073	96.66342269	0.192678
89	96.37997	40.33572	38.68026	95.55123026	0.192678
90	97.58665	40.05864	44.04506	94.43903782	0.385356
91	98.79332	57.01663	44.04506	93.32684538	0.192678
92	100	64.21339	44.3133	92.21465295	0.385356

Table 15: Russia Economic Condition Index Data Values

		Russia			
Period	GDP	SM	BY	FDI	FFR
1	0	0	32.4954	0	93.8343
2	1.558682	2.731298	31.68302	0.008945	94.79769
3	3.117364	7.322814	31.25383	0.017889	99.61464
4	4.676046	13.65195	18.9301	0.026834	99.80732
5	6.234728	21.84206	6.253832	0.035778	99.80732
6	7.79341	18.05956	12.2011	0.044723	99.80732
7	9.352092	25.78121	10.20846	0.053667	99.80732
8	10.91077	30.81016	8.568363	0.062612	99.61464
9	13.03987	42.95228	6.19252	0.071556	99.80732
10	15.16897	54.25815	6.882281	0.080501	100
11	17.29806	52.62043	7.004905	0.089445	100
12	19.42716	66.82257	11.41937	0.09839	99.80732
13	21.55625	51.28164	7.74065	0.107334	99.80732
14	23.68535	55.05733	7.449418	3.209127	99.80732
15	25.81445	58.7543	6.882281	6.31092	100
16	27.94354	63.93688	5.180871	9.412713	95.37572
17	30.07264	57.77046	3.954629	12.51451	93.8343
18	32.20174	62.28251	4.445126	15.6163	90.36609
19	34.33083	71.64567	4.41447	18.71809	85.16378
20	36.45993	82.45961	3.080932	21.81989	80.34682
21	38.93414	79.69501	4.56775	24.92168	74.56647
22	41.40835	79.56711	4.49111	28.02347	56.06936
23	43.88256	82.80849	3.433476	31.12526	48.94027
24	46.35676	82.74795	2.467811	34.22706	42.58189
25	48.83097	73.14111	0.689761	37.32885	36.80154
26	51.30518	80.37764	0.950337	39.89589	37.1869
27	53.77939	85.55871	1.057633	42.46293	37.37958
28	56.2536	81.21467	1.992643	45.02997	37.1869
29	58.72781	87.45223	3.694053	47.597	33.52601
30	61.20202	96.17739	1.195586	50.16404	17.34104
31	63.67623	94.35426	1.011649	52.73108	6.165703
32	66.15044	97.24676	0	55.29812	1.734104
33	63.16304	73.44307	0.137952	57.86516	1.541426
34	60.17565	79.95838	7.112201	60.4322	2.890173
35	57.18826	77.53737	2.452483	62.99923	2.119461
36	54.20087	80.48284	3.004292	65.56627	1.541426
37	51.21347	100	6.698345	68.13331	2.119461
38	48.22608	87.01555	4.889638	70.78887	2.697495
39	45.23869	67.46434	14.39301	73.44443	1.734104
40	42.25129	56.384	30.93194	76.09998	1.734104
41	39.2639	32.07099	18.19436	78.75554	1.541426
42	36.27651	9.69236	35.60699	81.4111	0.963391
43	33.28912	0.562304	28.54077	84.06666	0.963391
44	30.30172	1.183638	49.27958	86.72221	0.963391
45	32.33022	1.59004	96.61251	89.37777	0.770713
46	34.35871	4.70428	98.42121	92.03333	1.156069
47	36.38721	12.79298	100	94.68889	1.734104
48	38.4157	23.94975	67.51993	97.34444	2.504817
49	40.44419	39.31509	76.76272	100	2.504817

Table 15 Continued…

50	42.47269	27.82457	77.0233	94.63533	2.119461
51	44.50118	34.01143	77.31453	89.27066	2.119461
52	46.52968	36.93874	80.36481	83.90599	2.312139
53	48.55817	44.9018	70.60086	78.54132	2.312139
54	50.58667	47.9275	46.04537	73.17664	2.312139
55	52.61516	51.54274	43.14838	67.81197	2.312139
56	54.64365	57.9801	26.50215	62.4473	2.119461
57	57.17627	61.71945	22.67014	57.08263	1.926782
58	59.70888	55.15193	22.24096	51.71796	1.734104
59	62.24149	64.04511	10.13182	46.35329	1.348748
60	64.77411	62.97726	12.89086	40.98862	0.578035
61	67.30672	55.15041	19.58921	35.62395	0.385356
62	69.83933	53.38631	13.81055	36.54868	0.385356
63	72.37195	60.03179	12.33906	37.47341	0
64	74.90456	57.89609	15.87983	38.39814	0.578035
65	77.43717	63.29966	15.2054	39.32287	0.192678
66	79.96979	69.58792	20.21766	40.24761	0
67	82.5024	72.77633	21.42857	41.17234	0.192678
68	85.03501	82.04488	17.91845	42.09707	0
69	85.77737	84.72623	30.31882	43.0218	0.192678
70	86.51972	88.84474	38.1821	43.94654	0.578035
71	87.26207	91.55031	23.68179	44.87127	1.156069
72	88.00443	86.12026	22.40956	45.796	1.348748
73	88.74678	80.40338	29.23053	46.72073	1.734104
74	89.48913	80.42532	28.43348	48.39416	1.734104
75	90.23149	83.34582	22.04169	50.06759	1.734104
76	90.97384	71.30283	26.83936	51.74102	1.156069
77	91.71619	57.71749	37.38504	53.41445	1.348748
78	92.45855	67.71181	37.40037	55.08788	1.734104
79	93.2009	67.789	31.17719	56.76131	1.734104
80	93.94325	60.41851	34.15083	58.43474	1.734104
81	94.44798	68.87956	30.93194	60.10817	1.348748
82	94.95271	75.20944	26.76272	61.7816	1.541426
83	95.45744	69.13006	23.80441	63.45503	1.348748
84	95.96217	65.81224	26.34887	65.12846	1.541426
85	96.4669	53.16759	37.3084	66.80189	0.770713
86	96.97163	59.30526	33.53771	66.17047	0.385356
87	97.47636	60.78102	26.11895	65.53905	0.385356
88	97.98108	61.98358	24.3409	64.90764	0.192678
89	98.48581	64.65887	22.82342	64.27622	0.192678
90	98.99054	62.19473	17.36665	63.64481	0.385356
91	99.49527	60.70156	10.08584	63.01339	0.192678
92	100	65.90457	8.859595	62.38197	0.385356

Appendix D: Plots of Index Values v Time

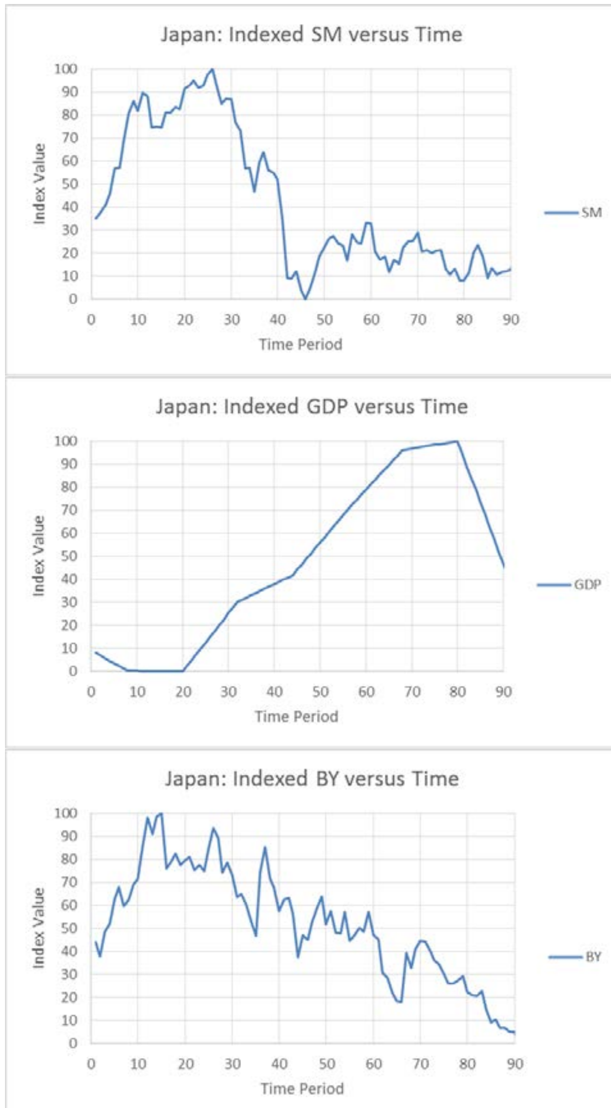


Figure 15: Japan Economic Condition Index Values plotted over time.

Similar to the US economy, the global financial crisis can be seen in the drop of the SM index starting after t=20. This indicates the health of the SM index of Japan could be dependent upon the health of the United States SM index.

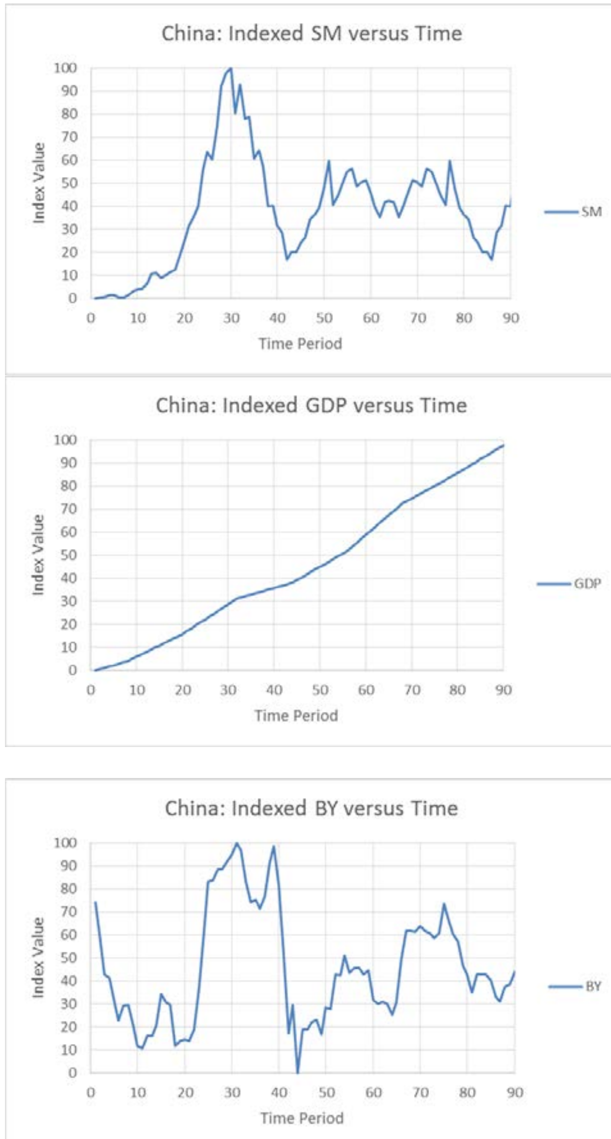


Figure 16: China Economic Condition Index Values plotted over time.

As with the US economy, the global financial crisis can be seen in the drop of the SM index starting after t=20. Note the linear trend of the Chinese GDP that displays their increasing economic output and highlights the fact that they are becoming a larger player in the world economy.

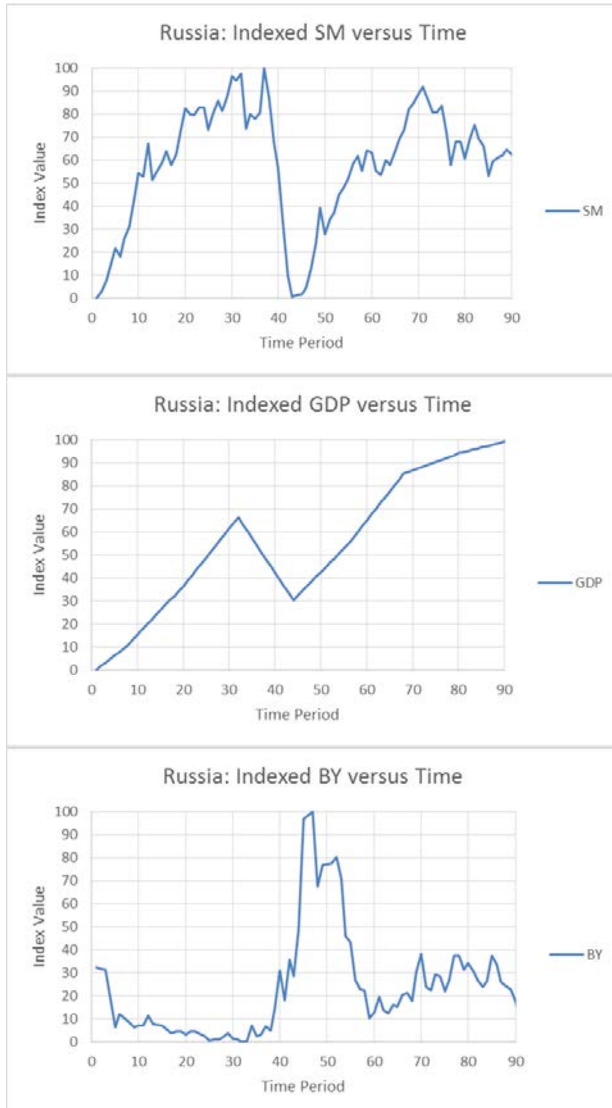


Figure 17: Russia Economic Condition Index Values plotted over time.

The global financial crisis can be seen in the drop of the SM index starting after *t*=30, while *t*=20 for the US economy. This could indicate that the Russian stock market is less dependent upon the US stock market than the other two countries. The major spike in the BY index could be a source of error when fitting the model. This spike may or may not be the result of the Russia government attempting to counteract the financial crisis.

Appendix E: Coefficient Values for Each Country

These coefficient values define the system of differential equations for each country modeled. The sign and magnitude of the coefficient has a significant impact when conducting what-if analysis.

Table 16: US α, β, and δ coefficient values with 14 digits of precision after the decimal point

	Method 1			Method 2		
	SM_t	GDP_t	BY_t	SM_t	GDP_t	BY_t
α_1	-0.03333511714007	0.08066856522335	1.34872344599628	22.56166101334500	2.77406825872357	0.01551685514811
α_2	0.04090206358247	1.00000000000000	-3.00933085305630	-32.56991547307270	1.00000000000000	-6.59297614293303
α_3	2.04744305981442	-0.11697634215290	0.81327475671854	14.80528191488310	-4.00556487184182	3.78042851732247
α_4	-2.06984227090032	1.00000000000000	-0.53516470103875	-0.09093448320865	1.00000000000000	-0.02486582000373
α_5	0.00060262650020	0.04501875991672	0.98419693608174	-0.01452932943961	-0.03508676284861	0.98419693608174
β_1	2.71639570354657	1.32813408298641	0.53059014762421	23.49700677432070	63.68705729915320	0.01474745842430
β_2	3.21892972828790	1.00000000000000	1.09019846173835	40.09798793004630	1.00000000000000	5.51042710247800
β_3	2.04692544743897	3.45768095870912	2.61476571086859	28.98691122297380	144.48255864301100	9.63877056946508
β_4	2.06850546508281	1.00000000000000	2.60408566303586	0.16556019697330	1.00000000000000	0.06982407678714
β_5	3.29180793381251	4.29126065207461	1.01395499094116	0.27042537366103	76.30569720801030	1.01395499094116
δ_1	0.00013895290737	0.02255020161932	-0.08927736229519	0.08005653594769	0.01874600093210	-0.04265823358094
δ_2	-0.00020643520575	-0.01135426105670	0.99599964337692	0.04161985521546	-0.01484206588792	0.99599964337692

Table 17: Japan α, β, and δ coefficient values with 14 digits of precision after the decimal point

	Method 1			Method 2		
	SM_t	GDP_t	BY_t	SM_t	GDP_t	BY_t
α_1	-0.02136677090853	0.04753138835717	-0.07745187439498	1.21769866336384	-1.58682616321501	-0.03450285513410
α_2	0.02163997236461	1.00000000000000	0.12543673842996	-12.52784325995360	1.00000000000000	0.19985667098523
α_3	1.10165777244476	0.05212151278543	0.90909685189513	0.10349220897746	-1.97883374147890	-1.19594902887278
α_4	-1.12492572556745	1.00000000000000	-0.37234866043354	-0.81715941993961	1.00000000000000	-0.02765086114152
α_5	0.00030435596401	0.01434816922513	0.98419693608174	-0.00037621104904	-0.02824547409605	0.98419693608174
β_1	3.24198112707851	3.33043112101949	0.14388522034927	75.37174778352060	143.86799873027400	0.07278416597828
β_2	3.31672694314407	1.00000000000000	0.22671429506394	152.80426598946500	1.00000000000000	0.42305432464262
β_3	1.10160937819954	4.80607354449845	2.34472437117090	0.22006222230540	276.72693660187400	-5.03924876328278
β_4	1.12479912256761	1.00000000000000	0.99994068584853	1.37872197987651	1.00000000000000	0.10968003663861
β_5	2.33504296367745	0.65653355343903	1.01395499094116	1.42318861484766	-2.12127361368424	1.01395499094116
δ_1	-0.00025970127666	-0.02476622312318	0.00099027306552	0.00206698607220	-0.03756842098306	0.01252281089089
δ_2	-0.00015004340712	-0.00447404810001	0.99599964337692	-0.07022360387744	-0.02117662710346	0.99599964337692

Table 18: China α, β, and δ coefficient values with 14 digits of precision after the decimal point

	Method 1			Method 2		
	SM_t	GDP_t	BY_t	SM_t	GDP_t	BY_t
α_1	0.52940198654221	0.12853236791750	0.25854581523735	0.24286562704637	0.05678677615250	15.30044742178620
α_2	-0.31375553357311	1.00000000000000	-0.49646336224604	-0.31817138697877	1.00000000000000	-15.13767227235470
α_3	24.15594877964920	0.57368506413955	10.64976378332240	7.15237320449931	0.02263154569921	9.49462266458029
α_4	-24.39677385623800	1.00000000000000	-0.07175058684017	-1.58162301680769	1.00000000000000	-1.32411780752177
α_5	0.07315340917544	-0.13133835707209	0.98419693608174	0.21788196147197	0.00353199585621	1.00000000000000
β_1	3.52054298593069	3.55647868763555	0.02493727555069	0.03697015093163	1.78797783428471	1.69841603489383
β_2	2.63644411414284	1.00000000000000	0.04743786578432	0.04669339575213	1.00000000000000	1.66384762687073
β_3	24.30524493318520	26.01601974775400	42.04891743289230	17.99953653876700	1.93915843954137	47.99193770058190
β_4	24.33233748880310	1.00000000000000	0.37366751864718	3.40334133729883	1.00000000000000	8.65289345829390
β_5	18.82288222258590	52.68935175483080	1.01395499094116	11.77077718897320	-0.66214796999515	1.00000000000000
δ_1	0.00177718400525	0.01410331999282	-0.00864879773809	-0.00618856698208	0.01017622032476	-0.00924048030835
δ_2	-0.03122247215168	-0.02324745295611	0.99599964337692	0.26460989035990	-0.01344988008676	1.00000000000000

Table 19: Russia α, β, and δ coefficient values with 14 digits of precision after the decimal point

	Method 1			Method 2		
	SM_t	GDP_t	BY_t	SM_t	GDP_t	BY_t
α_1	-0.02389031077591	0.25358532772723	-0.33843388914567	16.34386249294980	4.40533191663794	-0.00321841086971
α_2	0.01307283563172	1.00000000000000	0.35759644883871	-11.76716649534540	1.00000000000000	1.88516712621399
α_3	0.29106352457659	0.05830498701572	-1.17810048433669	8.68642708378173	-2.14635106542973	-1.46282611603519
α_4	-0.69077759974694	1.00000000000000	0.11628863337573	-0.06606015208972	1.00000000000000	0.00626634791497
α_5	0.00178063226431	1.36163834719677	0.98419693608174	-0.01216575745160	3.30339859968052	0.98419693608174
β_1	3.53801571268901	5.15027599740817	0.44920651179403	32.69104480766550	41.11898913715090	0.00251030577023
β_2	6.28683469161090	1.00000000000000	0.44173240447983	50.23373172019090	1.00000000000000	1.37219663874877
β_3	0.29066918916838	3.43685012413528	8.95773517891909	19.91602465356330	151.75785563934300	11.81354471357030
β_4	0.69076114345093	1.00000000000000	1.46515589164827	0.11570948179270	1.00000000000000	0.09089325339117
β_5	1.89303963444703	19.89104540867800	1.01395499094116	0.55886029912347	70.54730773581230	1.01395499094116
δ_1	-0.00382605846173	0.01858463032766	0.01004324401904	0.16173141205537	0.06279900534765	0.03147570784983
δ_2	-0.00364597543197	-0.05434167402968	0.99599964337692	0.09190507892658	-0.00896389390767	0.99599964337692

Appendix F: Calculated Index Values and True Index Values over Time

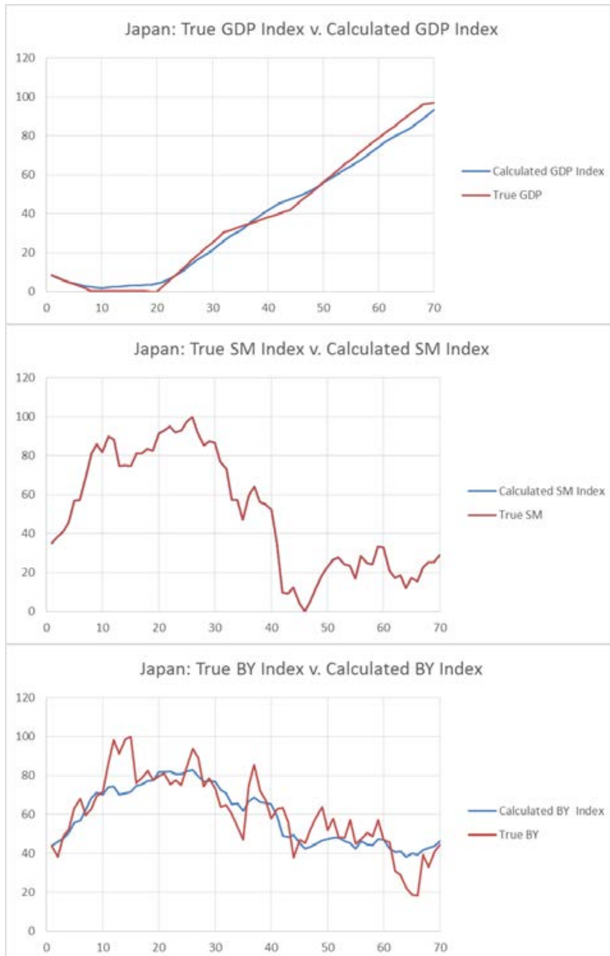


Figure 18: Japan True Index and Calculated Index plotted over time using method 1.

The SM index had a near perfect fit and resulted in the lines overlapping. The trend of the index is accurately captured for the GDP and BY indices. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

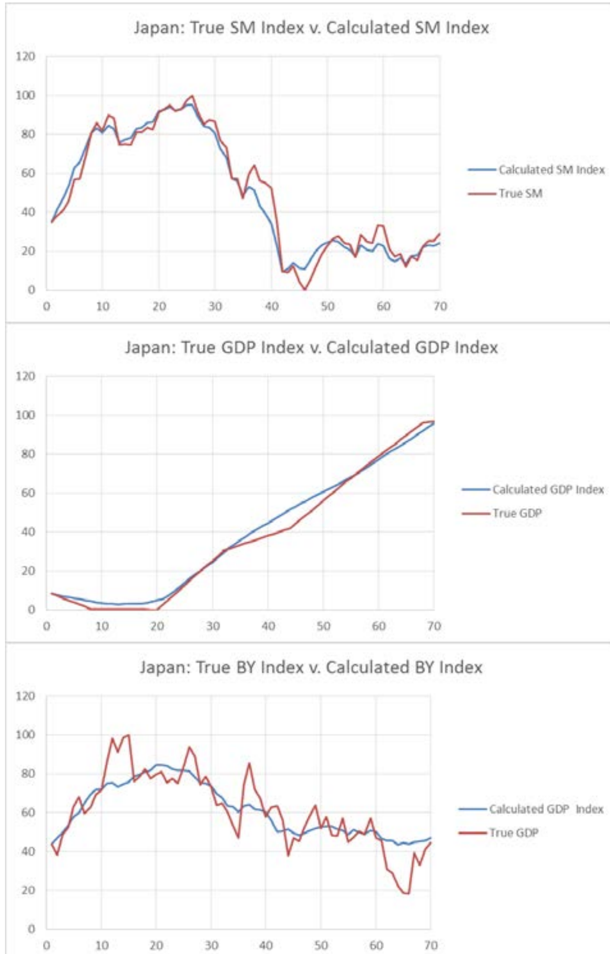


Figure 19: Japan True Index and Calculated Index plotted over time using method 2

The fitting for the SM index was not nearly as close as method 1's fitting yet the general trend of all the indices is sufficiently captured. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

Figure 20: China True Index and Calculated Index plotted over time using method 1

The SM index seems to consistently fit under the true values. The GDP index was a near perfect fit for the time interval. The BY once again contained the most error for the fitting but still capture the general trend of the data. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

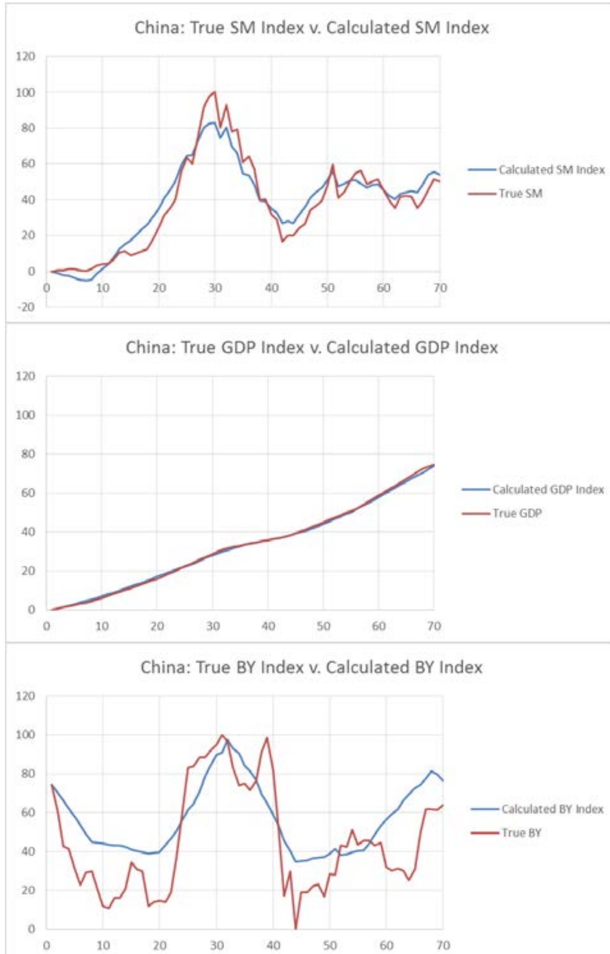


Figure 21: China True Index and Calculated Index plotted over time using method 2

Using method 2, the SM index fitting seemed to fit above the true values which is opposite of what happened using method 1. Again, there is a near perfect fit for the GDP index. The calculated BY index using method 2 seems to fit even more above the data than the method 1 results. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

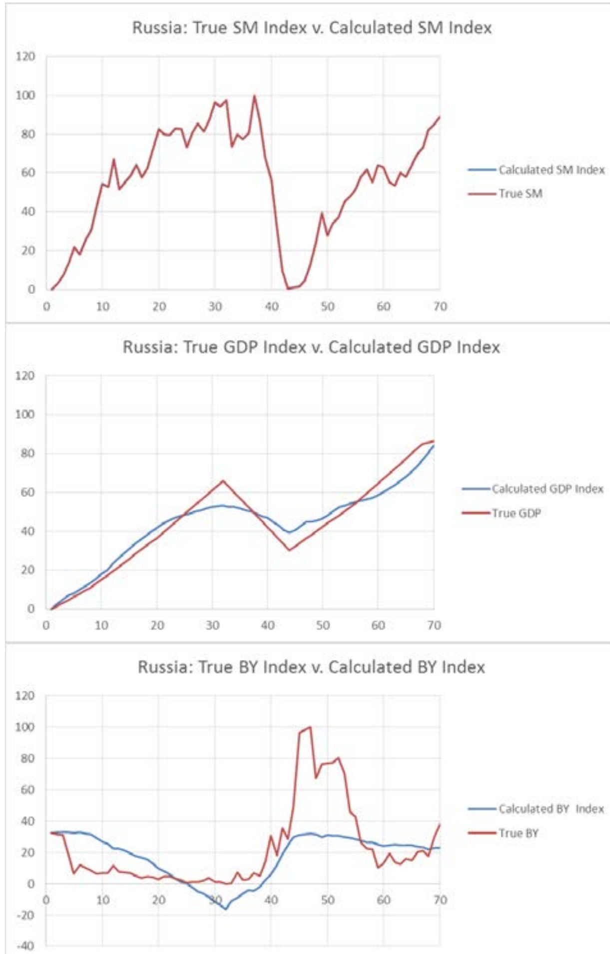


Figure 22: Russia True Index and Calculated Index plotted over time using method 1

Method 1 resulted in a near perfect fit of the SM index. The calculated GDP index captured the inflection points at t=32 and t=43. Similarly the calculated BY index captures the general trend of the true data. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

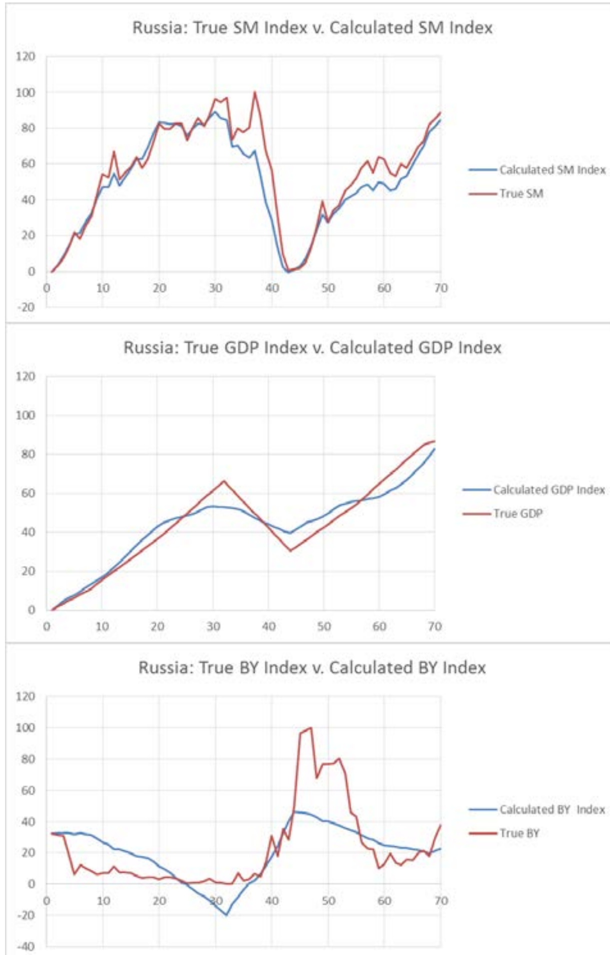


Figure 23: Russia True Index and Calculated Index plotted over time using method 2

When using method 2, the calculated SM index seems to consistently plot below the true SM index. Once again the inflection points in the GDP index were captured by the calculated GDP index. While initial inspection shows that method 2 may have resulted in a better BY fitting than method 1. See Table 2 and Table 3 for information on the index's SSE and Maximum Error values.

Appendix G: Normal Probability Plots for Country Indices

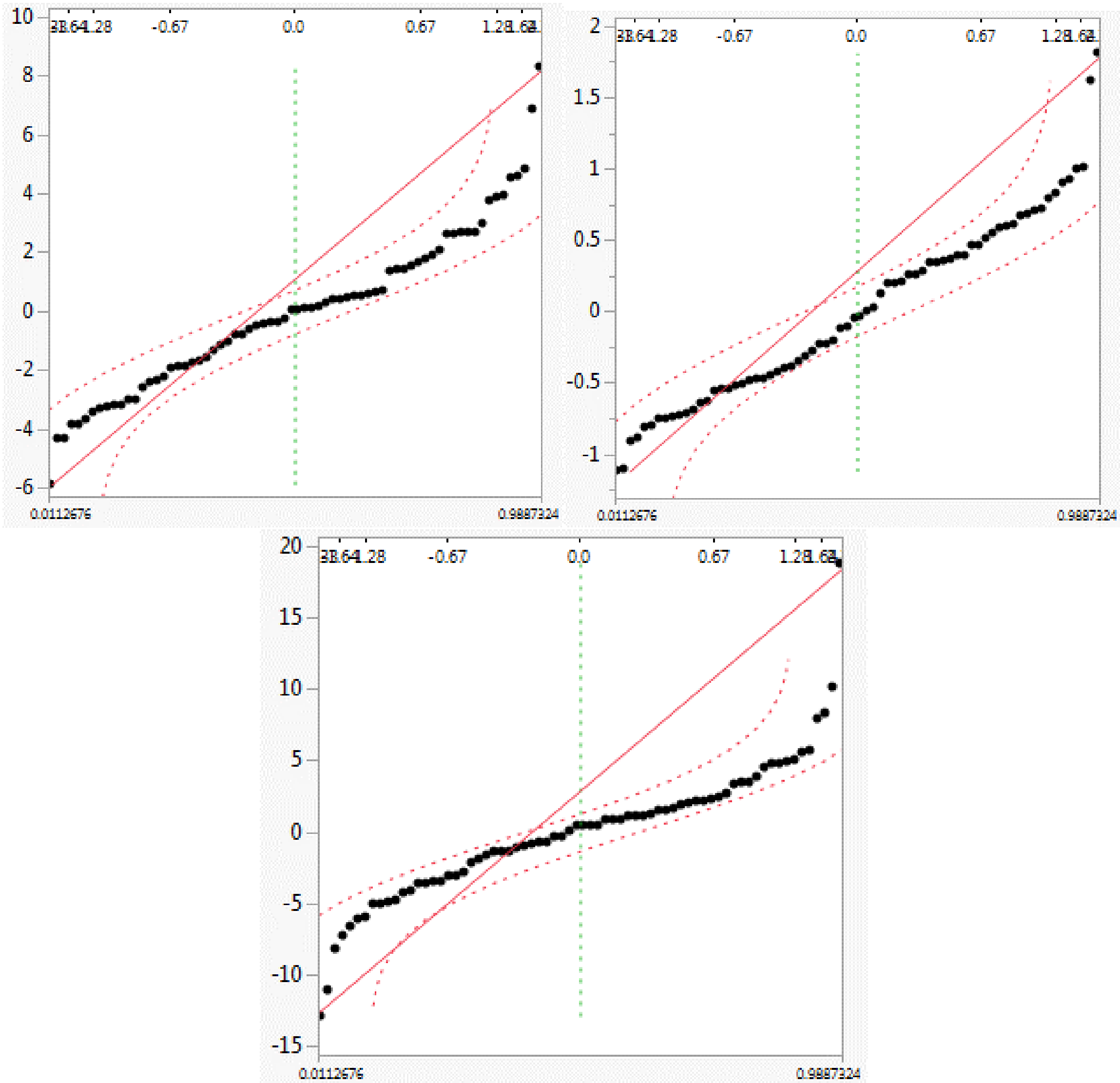


Figure 24: US Normal Probability Plot for errors using method 2. Clockwise: Stock Market, GDP per capita, Bond Yield.

These normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation.

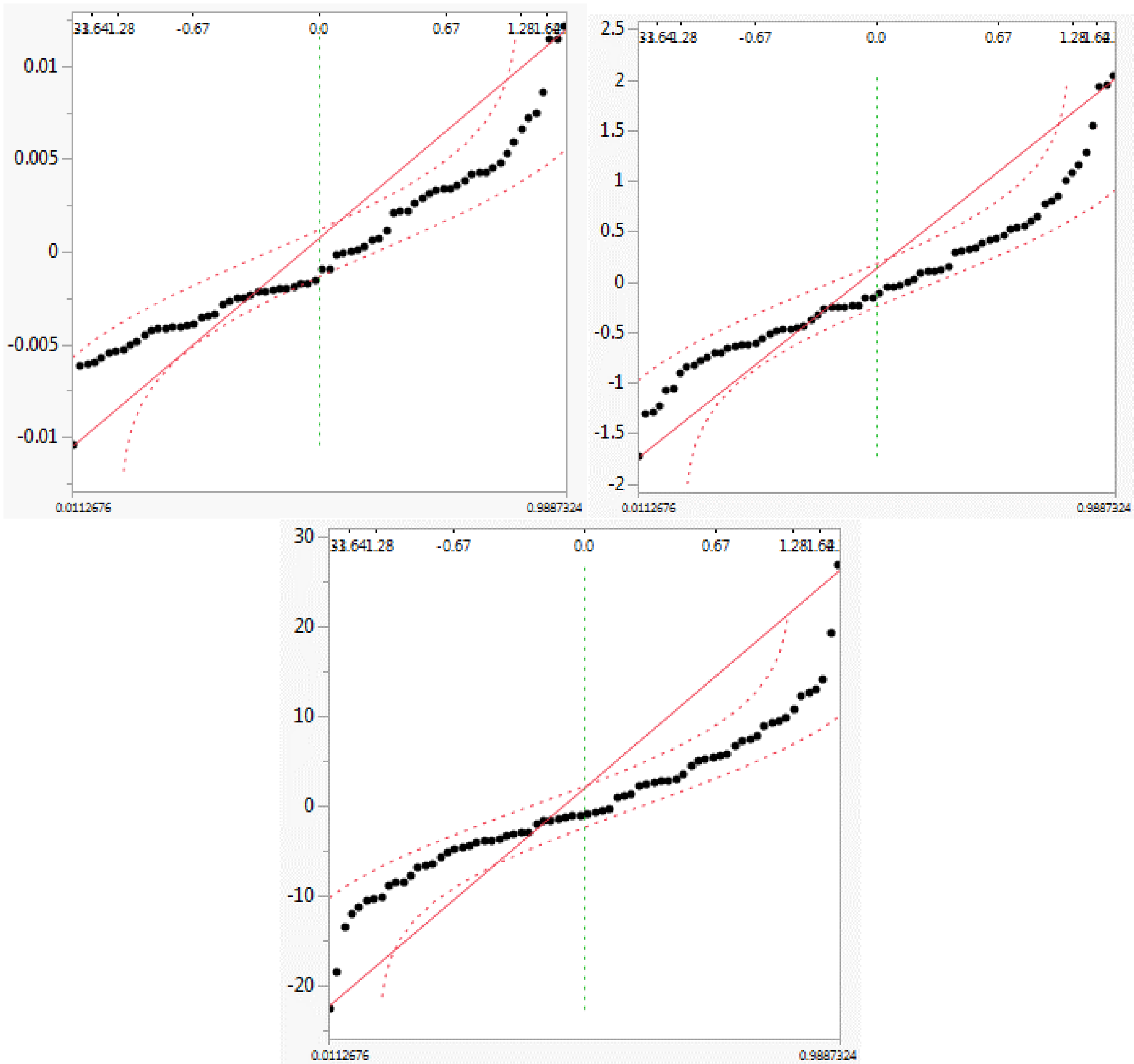


Figure 25: Japan Normal Probability Plot for errors using method 1. Clockwise: Stock Market, GDP per capita, Bond Yield.

As with the US's, Japan's normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation.

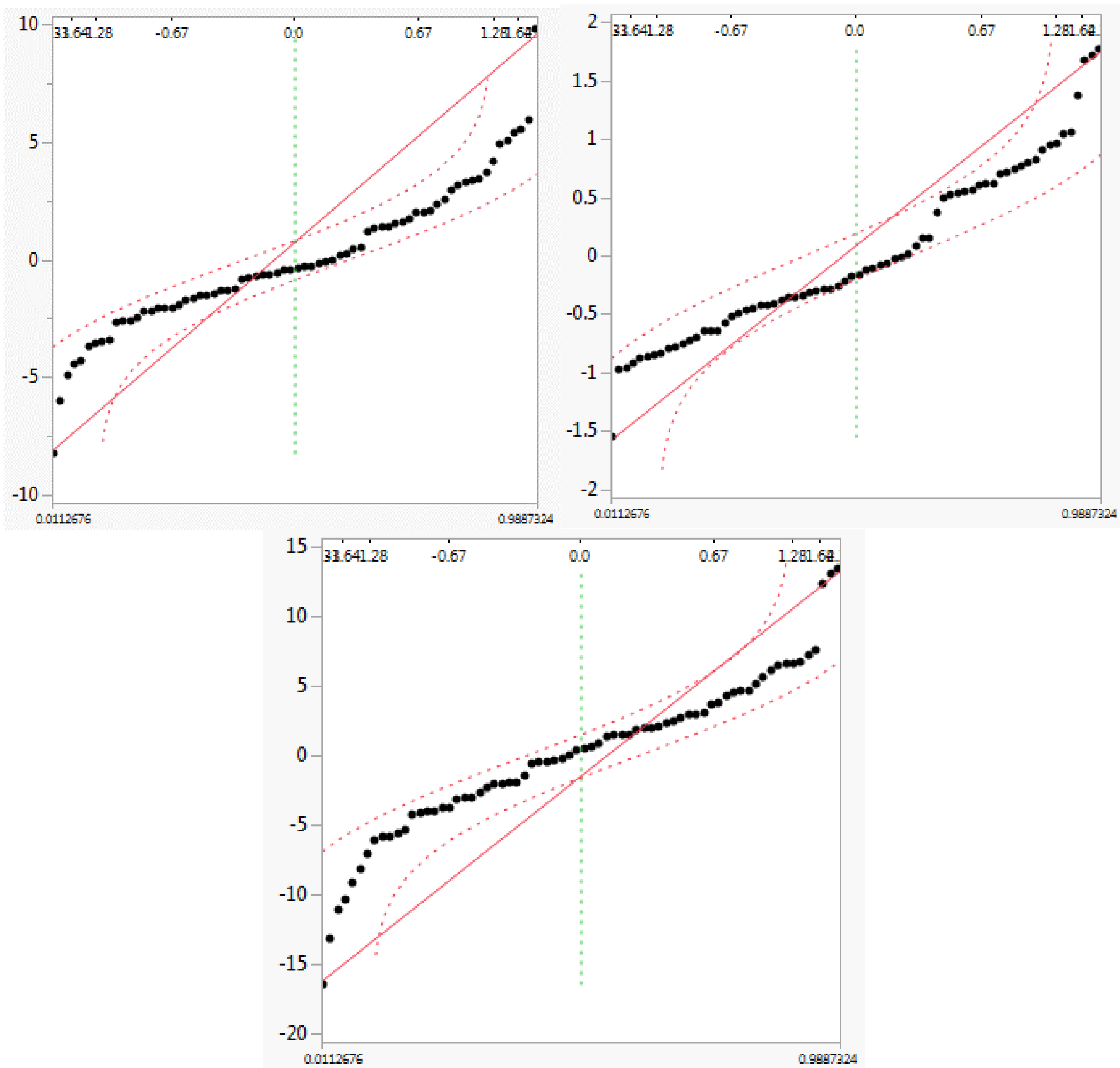


Figure 26: Japan Normal Probability Plot for errors using method 2. Clockwise: Stock Market, GDP per capita, Bond Yield.

These normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation.

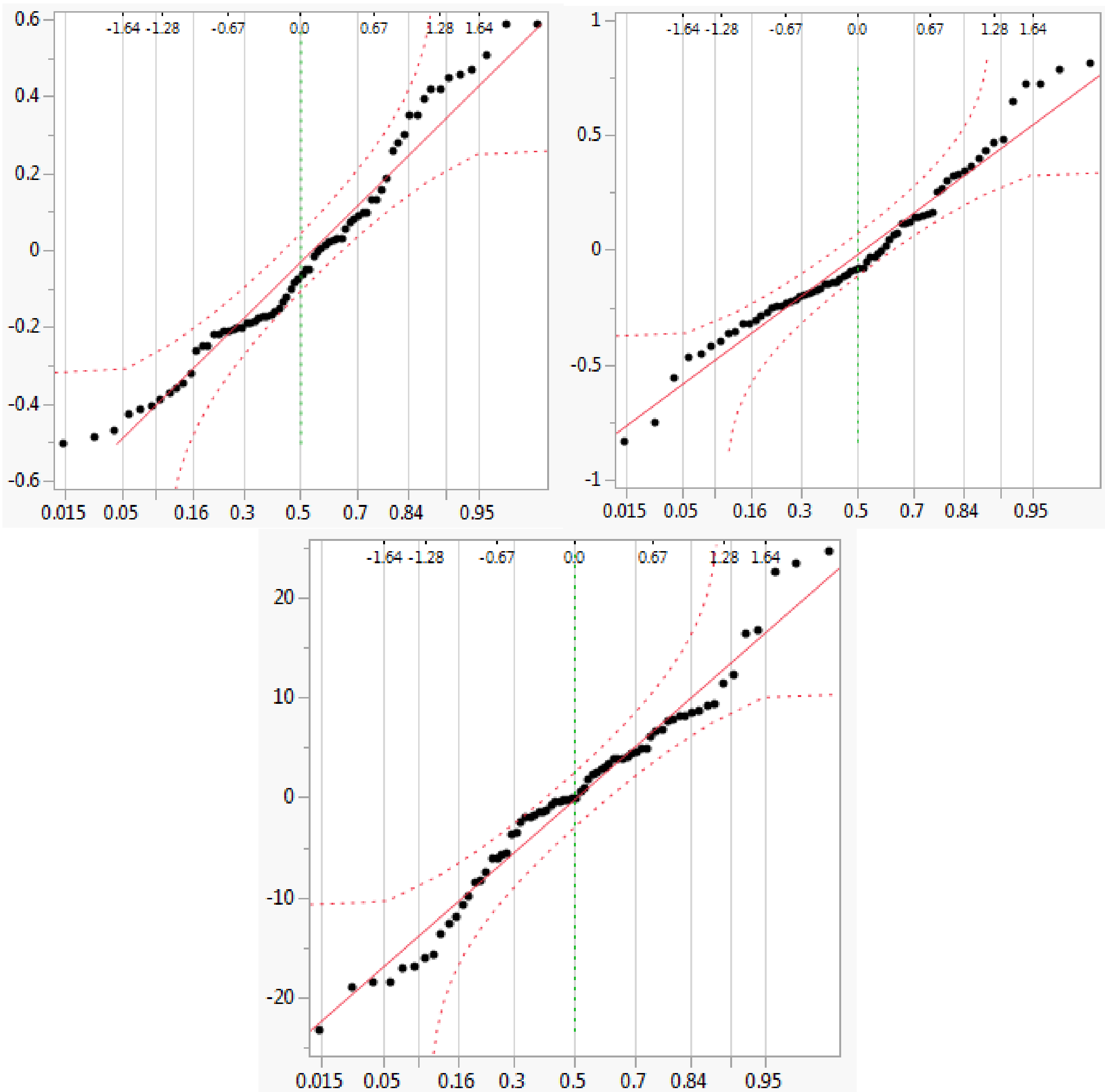


Figure 27: China Normal Probability Plot for errors using method 1. Clockwise: Stock Market, GDP per capita, Bond Yield

Similar to the US's, China's normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation.

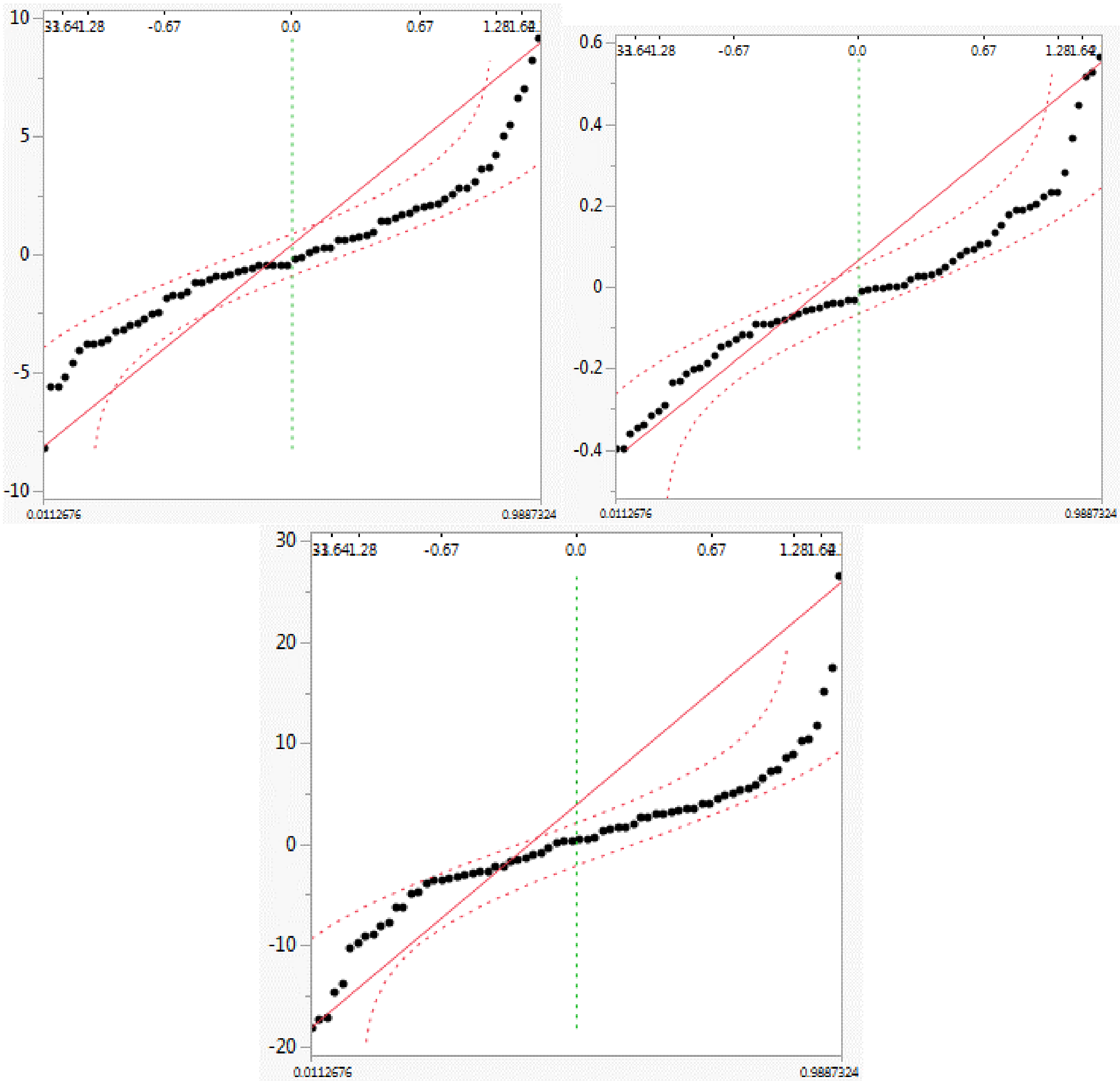


Figure 28: China Normal Probability Plot for errors using method 2. Clockwise: Stock Market, GDP per capita, Bond Yield

These normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation.

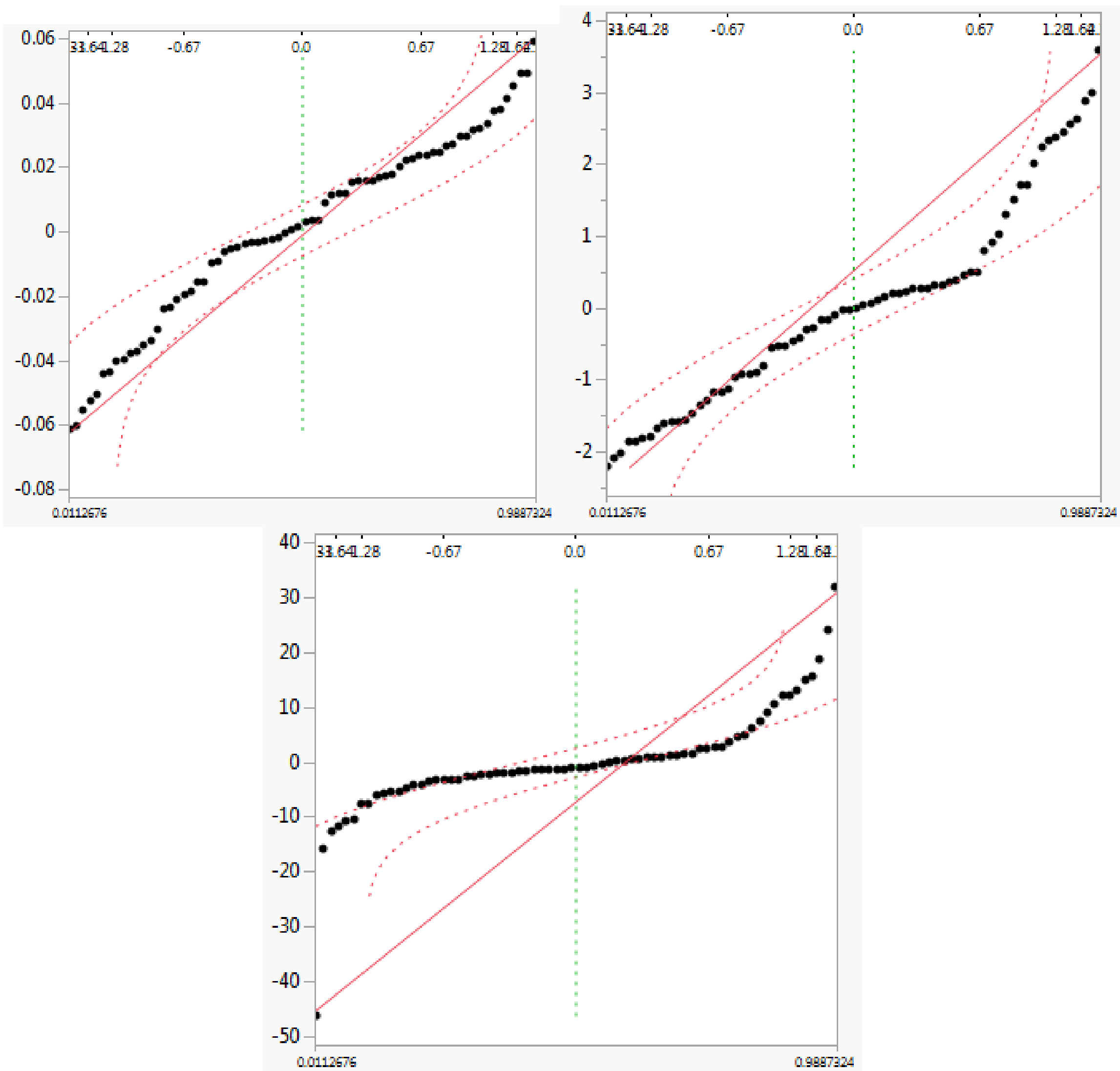


Figure 29: Russia Normal Probability Plot for errors using method 1. Clockwise: Stock Market, GDP per capita, Bond Yield.

These normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation. The GDP index errors and the BY index errors did not pass the Shapiro-Wilk test and therefore are not normally distributed. Further research should address this deficiency.

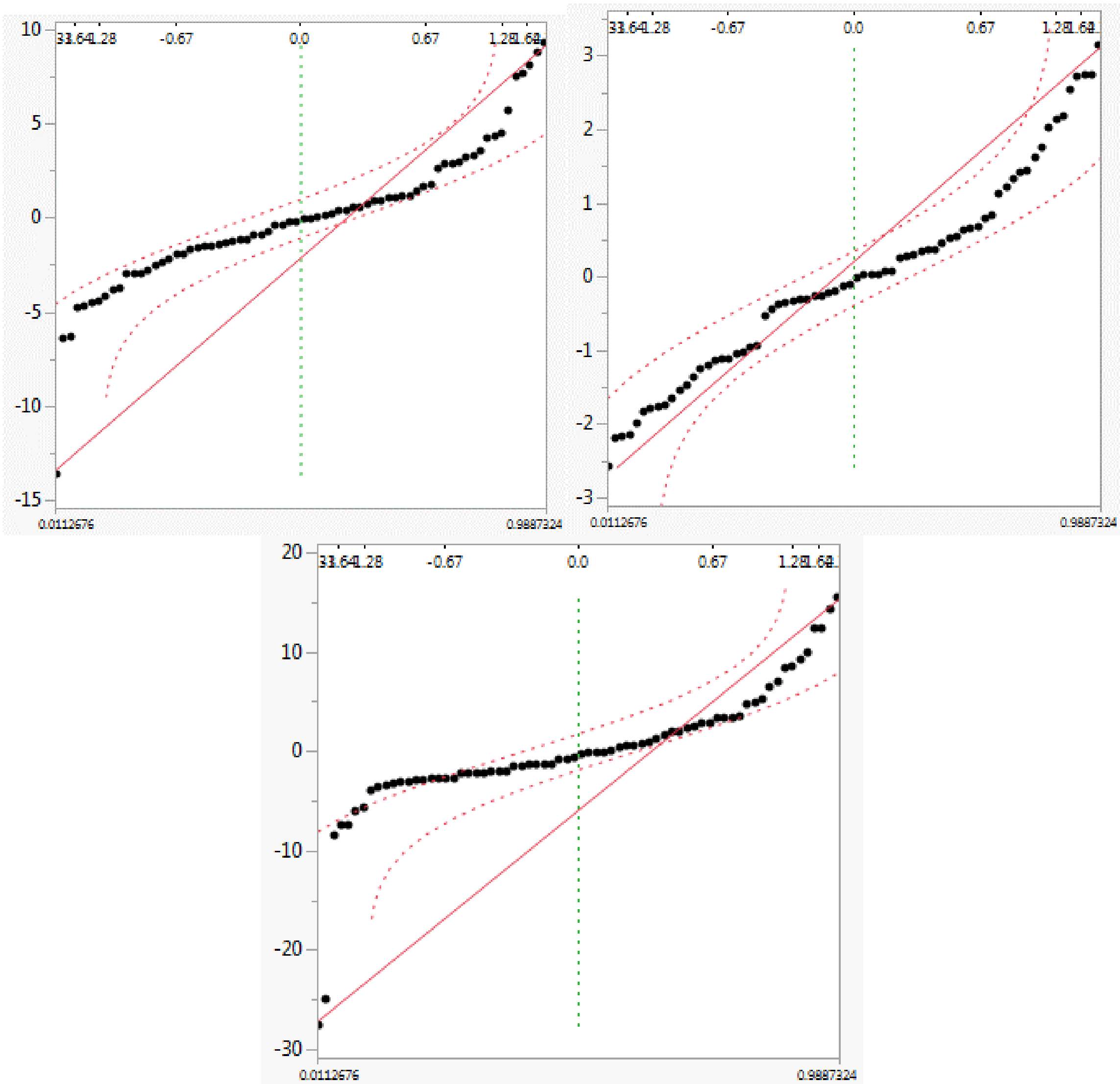


Figure 30: Russia Normal Probability Plot for errors using method 2. Clockwise: Stock Market, GDP per capita, Bond Yield.

These normal probability plots are heavy tailed symmetrical meaning that the errors associated with the fitting of the respective index are symmetrical like a normal distribution but have fatter tails. A Shapiro-Wilk test was conducted for confirmation. The BY index errors did not pass the Shapiro-Wilk test and therefore are not normally distributed. Further research should address this deficiency.

Appendix H: Prediction Plots for Each Index

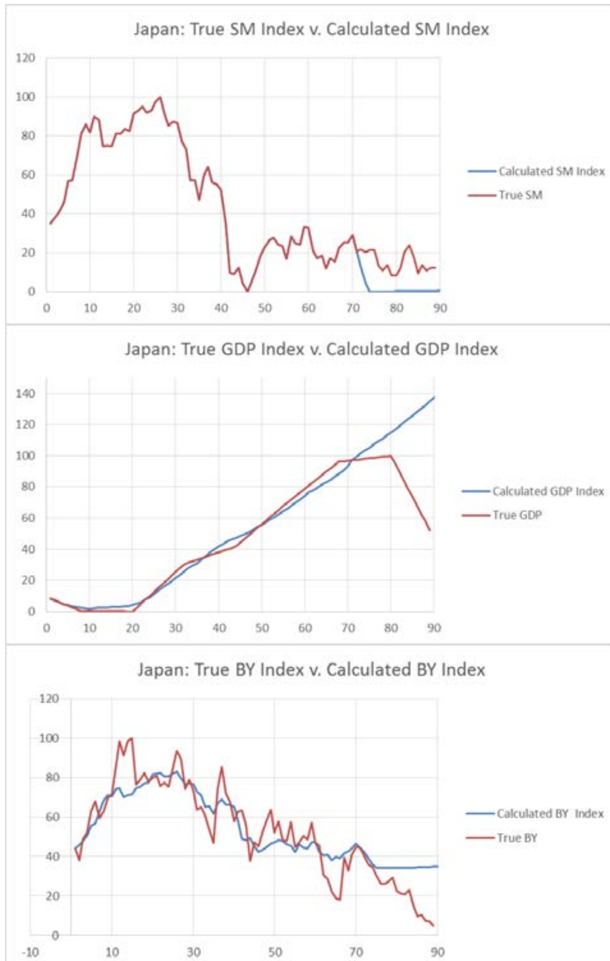


Figure 31: Japan Predicted Index and True Index plotted over time using method 1, prediction begins at t=71.

The prediction for the SM index using method 1 decreases and then maintains a flat rate. This behavior is similar to the true values but more exaggerated in the decrease. The GDP index prediction captured the initial increase but failed to capture the subsequent decrease in the index at *t*=80. The BY index prediction was accurate for the short term but failed to capture the continuing decrease of BY index in the long term.

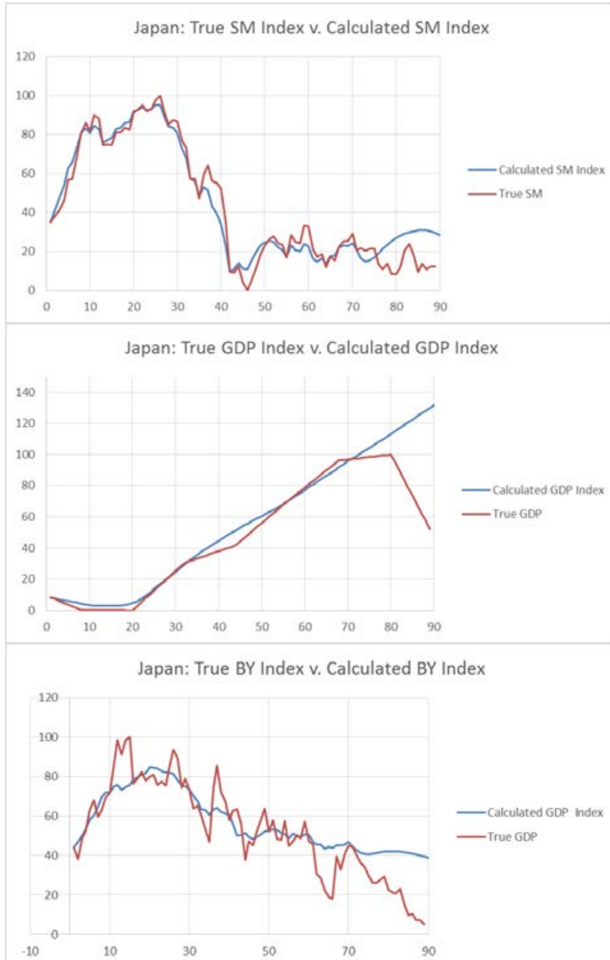


Figure 32: Japan Predicted Index and True Index plotted over time using method 2, the prediction begins at t=71.

The prediction for the SM index captured the general trend of the true index value, highlighting the capture of the inflection point at *t*=83. Again, the GDP index prediction captured the initial increase but failed to capture the subsequent decrease in the index at *t*=80. While the BY index prediction captured the negative trend just understated its magnitude.

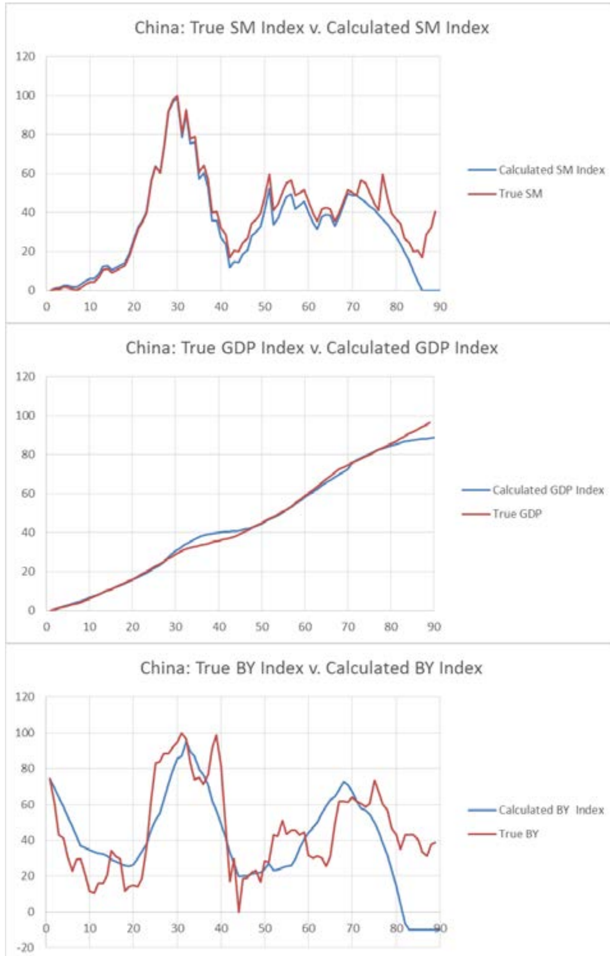


Figure 33: China Predicted Index and True Index plotted over time using method 1, prediction begins at t=71.

The SM index prediction captured the general trend of the true SM index well for the first 15 months and then falls off in accuracy for the remaining 6 months. The GDP index prediction was extremely accurate but understated the true level towards the end of the prediction range. The BY index prediction captured the general trend of the true BY index during the initial decrease and then the leveling out of the index after t=80.

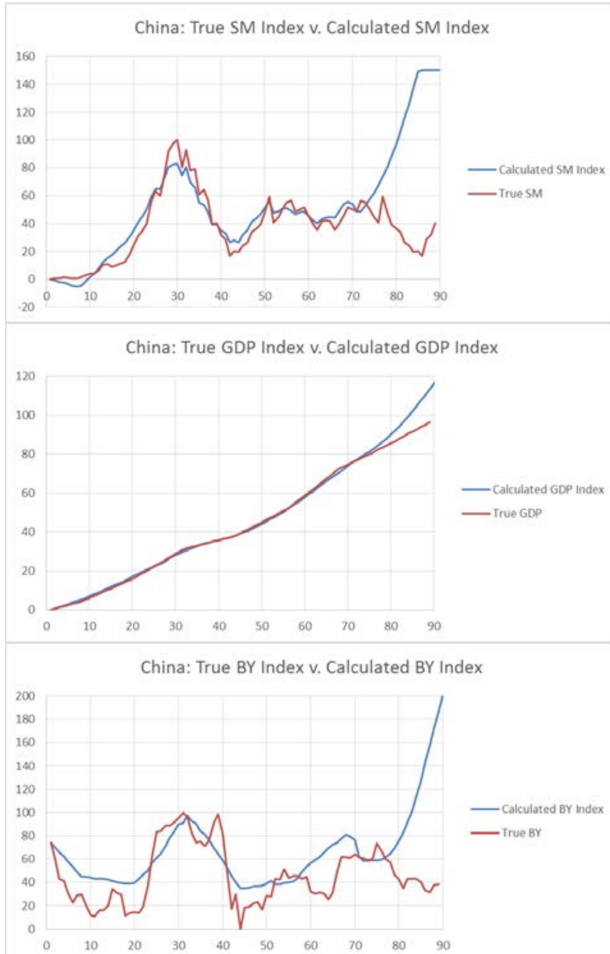


Figure 34: China Predicted Index and True Index plotted over time using method 2, prediction begins at t=71.

Only the GDP prediction resembled the true index value in the long term when using method 2 coefficient values for China's economic condition. The explosive increase in the SM and BY indices could be due to the large alpha coefficient the SM index has in the BY derivative calculation. This large value would spiral the two indices to infinity over time.

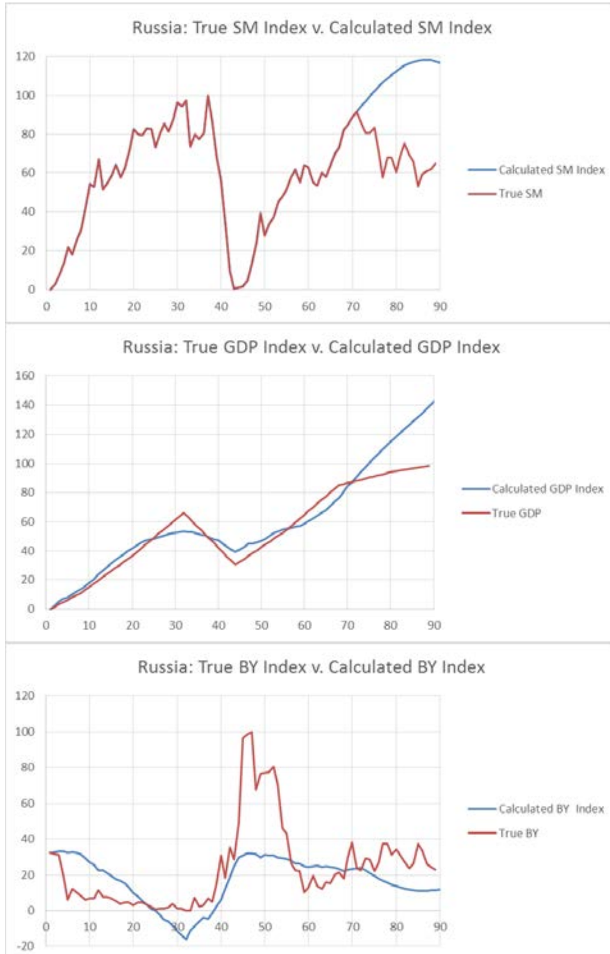


Figure 35: Russia Predicted Index and True Index plotted over time using method 1, prediction begins at t=71.

The SM prediction does not capture the trend of the true index value. This could be have been a results of the initial point used in the Euler method being an inflection point. Note that it does then back down toward the true value the farther the prediction is. The true GDP index trend was sufficiently captured in the GDP index prediction. The trend of the BY index falling slightly but maintaining a relative mean was captured by the BY index prediction.

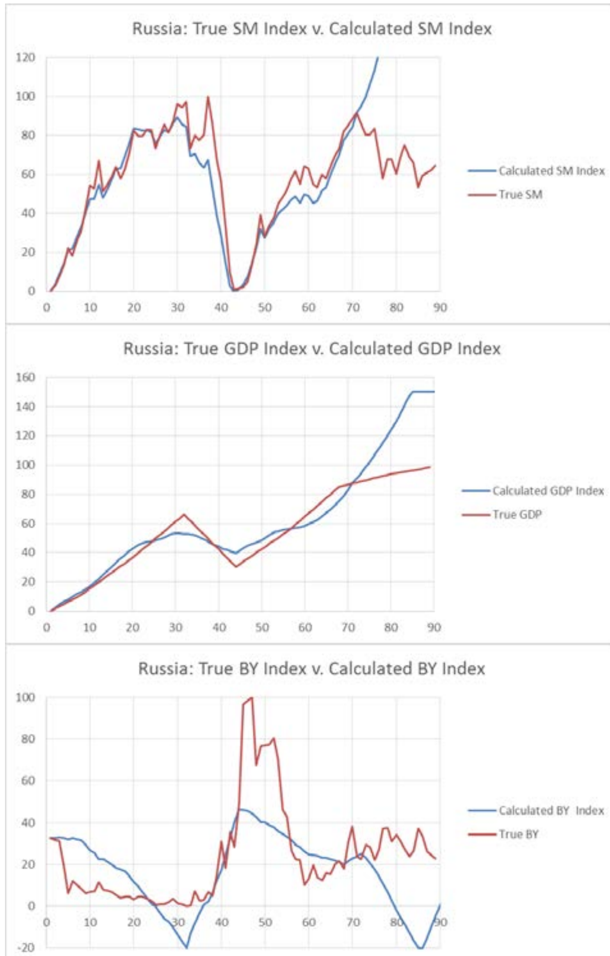


Figure 36: Russia Predicted Index and True Index plotted over time using method 2, prediction begins at t=71.

Again, the SM index prediction did not capture the trend of the true SM index. The explosive growth inflated the GDP index prediction due to the large positive alpha value for the SM index that was used when defining the GDP derivative. Subsequently, the BY index was severely depressed because of the larger negative alpha values for the GDP and SM indices when defining the BY derivative.

Appendix I: FFR Values for Basecase, Gradual Decrease, and No Adjustment Scenarios

Table 20: Alternate FFR_t values used for base case, Gradual Decrease, and No Adjustment scenarios for the what-if analysis section.

Period	FFR_t Base case	FFR_t No Change	FFR_t Gradual Change
1	94.797687861271700	94.7976878612717	94.7976878612717
2	99.614643545279400	99.6146435452794	99.6146435452794
3	99.807321772639700	99.8073217726397	99.8073217726397
4	99.807321772639700	99.8073217726397	99.8073217726397
5	99.807321772639700	99.8073217726397	99.8073217726397
6	99.807321772639700	99.8073217726397	99.8073217726397
7	99.614643545279400	99.6146435452794	99.6146435452794
8	99.807321772639700	99.8073217726397	99.8073217726397
9	100	100	100
10	100	100	100
11	99.807321772639700	99.8073217726397	99.8073217726397
12	99.807321772639700	99.8073217726397	99.8073217726397
13	99.807321772639700	99.8073217726397	99.8073217726397
14	100	100	100
15	95.375722543352600	95.3757225433526	95.3757225433526
16	93.834296724470100	93.8342967244701	93.8342967244701
17	90.366088631984600	90.3660886319846	90.3660886319846
18	85.163776493256300	85.1637764932563	85.1637764932563
19	80.346820809248600	80.3468208092486	80.3468208092486
20	74.566473988439300	96.0	80.0
21	56.069364161849700	96.0	80.0
22	48.940269749518300	96.0	75.0
23	42.581888246628100	96.0	75.0
24	36.801541425818900	96.0	75.0
25	37.186897880539500	96.0	70.0
26	37.379576107899800	96.0	70.0
27	37.186897880539500	96.0	70.0
28	33.526011560693600	96.0	70.0
29	17.341040462427700	96.0	70.0
30	6.165703275529870	96.0	65.0
31	1.734104046242770	96.0	65.0
32	1.541425818882470	96.0	65.0
33	2.890173410404620	96.0	65.0
34	2.119460500963390	96.0	65.0
35	1.541425818882470	96.0	65.0
36	2.119460500963390	96.0	65.0
37	2.697495183044320	96.0	65.0
38	1.734104046242770	96.0	65.0
39	1.734104046242770	96.0	65.0
40	1.541425818882470	96.0	65.0
41	0.963391136801541	96.0	65.0
42	0.963391136801541	96.0	65.0

Table 20 Continued…

43	0.963391136801541	96.0	65.0
44	0.770712909441233	96.0	65.0
45	1.156069364161850	96.0	65.0
46	1.734104046242770	96.0	65.0
47	2.504816955684010	96.0	65.0
48	2.504816955684010	96.0	65.0
49	2.119460500963390	96.0	65.0
50	2.119460500963390	96.0	65.0
51	2.312138728323700	96.0	60.0
52	2.312138728323700	96.0	60.0
53	2.312138728323700	96.0	60.0
54	2.312138728323700	96.0	60.0
55	2.119460500963390	96.0	60.0
56	1.926782273603080	96.0	60.0
57	1.734104046242770	96.0	60.0
58	1.348747591522160	96.0	60.0
59	0.578034682080925	96.0	60.0
60	0.385356454720616	96.0	60.0
61	0.385356454720616	96.0	60.0
62	0.0	96.0	60.0
63	0.578034682080925	96.0	60.0
64	0.192678227360308	96.0	60.0
65	0.0	96.0	60.0
66	0.192678227360308	96.0	60.0
67	0.0	96.0	60.0
68	0.192678227360308	96.0	60.0
69	0.578034682080925	96.0	60.0
70	1.156069364161850	96.0	60.0

Appendix J: Gradual Decrease Scenario Plots

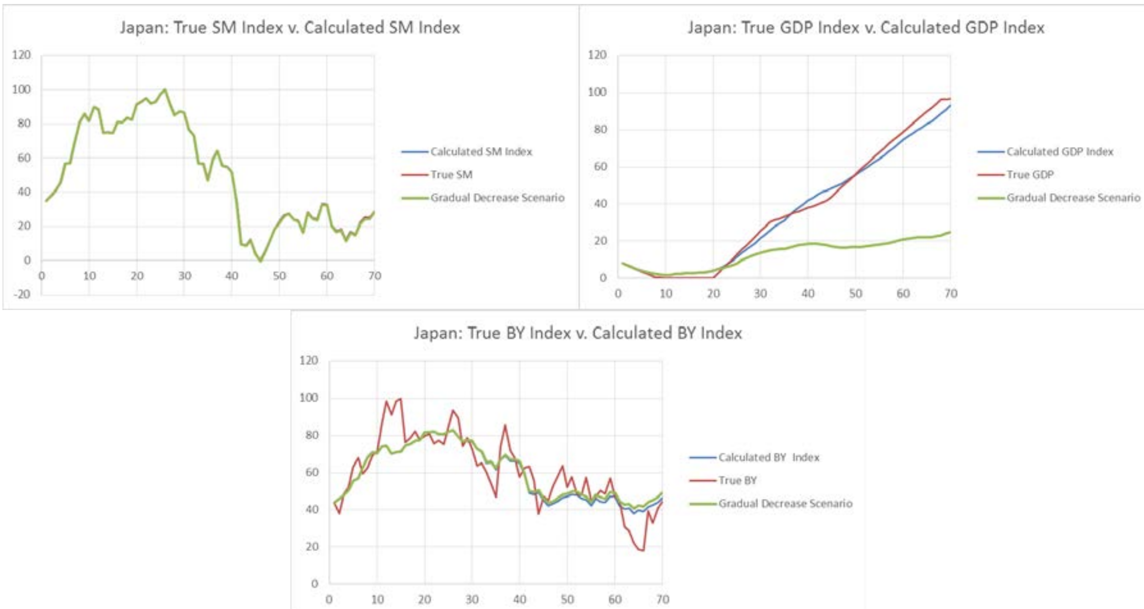


Figure 37: Japan Economic Condition Index, Calculated Index, and True Index Values over time. The 21^{st} period is when the Gradual Decrease policy begins.

The Gradual Decrease policy did not have a statistically significant impact on Japan's SM index nor Japan's BY index at the α=.05 level. However, it did have a statistically significant negative impact to the GDP index of Japan at the α=.05 level.

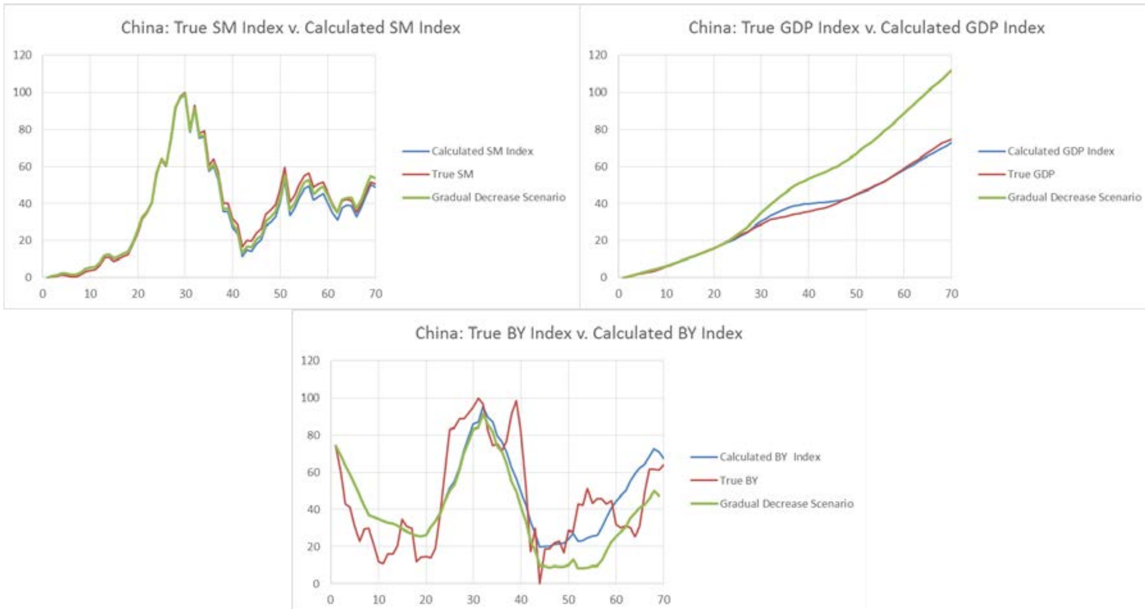


Figure 38: China Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the Gradual Decrease policy begins.

On the GDP and BY indices of China, the Gradual Decrease policy to the FFR has a statistically significant positive and negative impacts, respectively, at the α=.05 level. Again, the SM index was not significantly impacted at the α=.05 level according to a *t*-test of the difference of means.

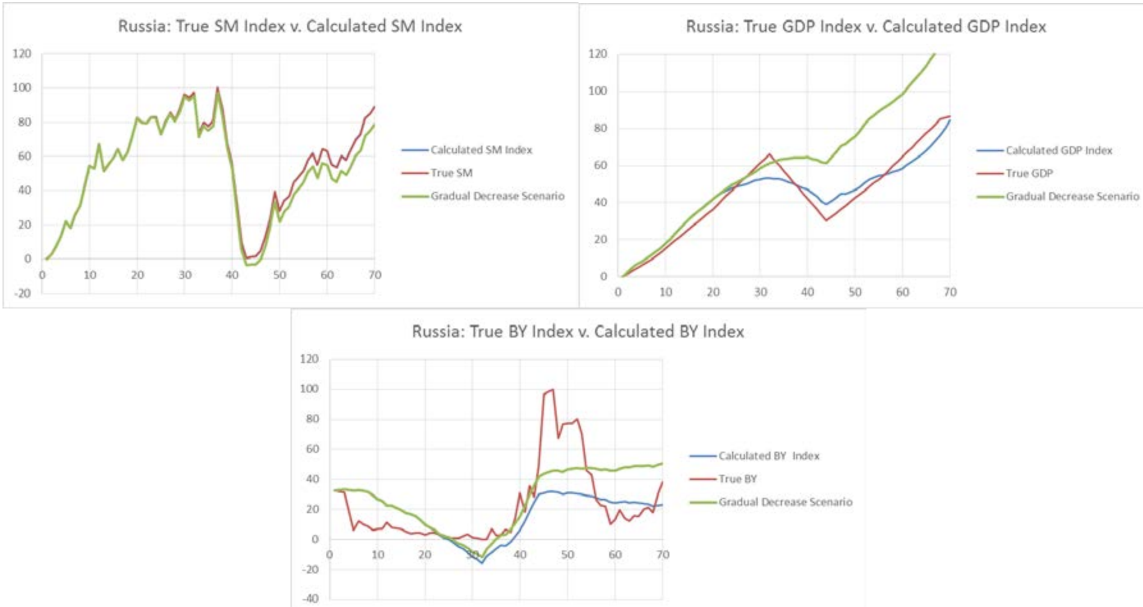


Figure 39: Russia Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the Gradual Decrease policy begins.

The Russian SM index was not statistically significantly changed by the Gradual Decrease policy at the α=.05 level. The GDP index change was positive and statistically significant at the α=.05 level. The negative change to the BY index for Russia was statistically significant at the α=.05 level.

Appendix K: No Adjustment Scenario Plots

The statistical results for the No Adjustment scenario are identical to the results from the Gradual Decrease scenario for all four countries and therefore an individual analysis is not provided for each country's indices. Note that the magnitude of the change in the indices was greater for the No Adjustment scenario. This makes sense given that the No Adjustment scenario altered the same variable as in the Gradual Decrease scenario but to a greater degree.

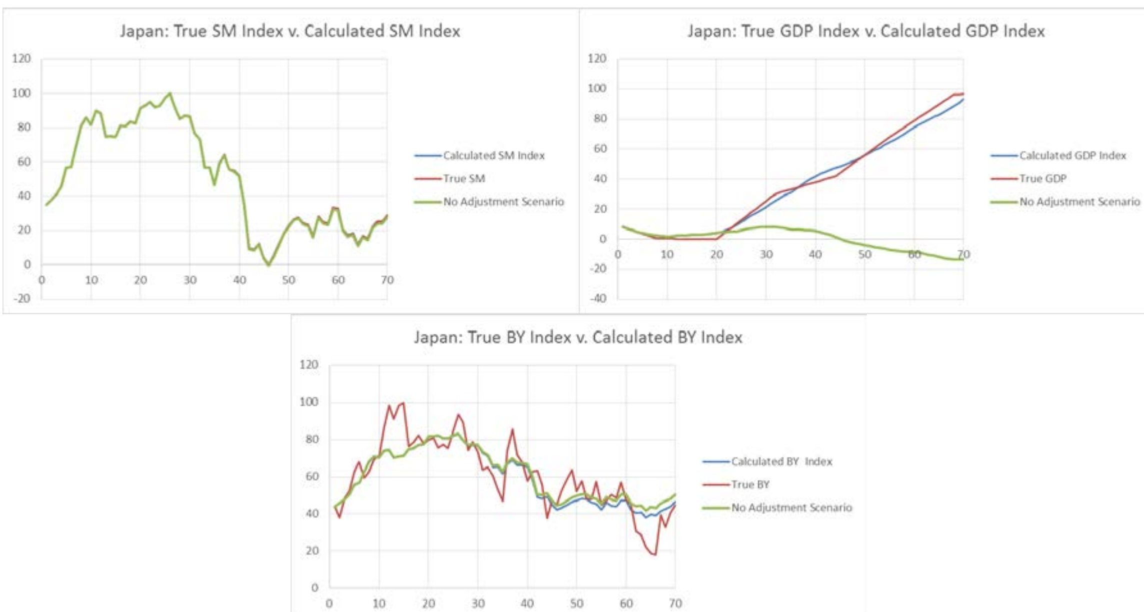


Figure 40: Japan Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the No Adjustment policy begins.

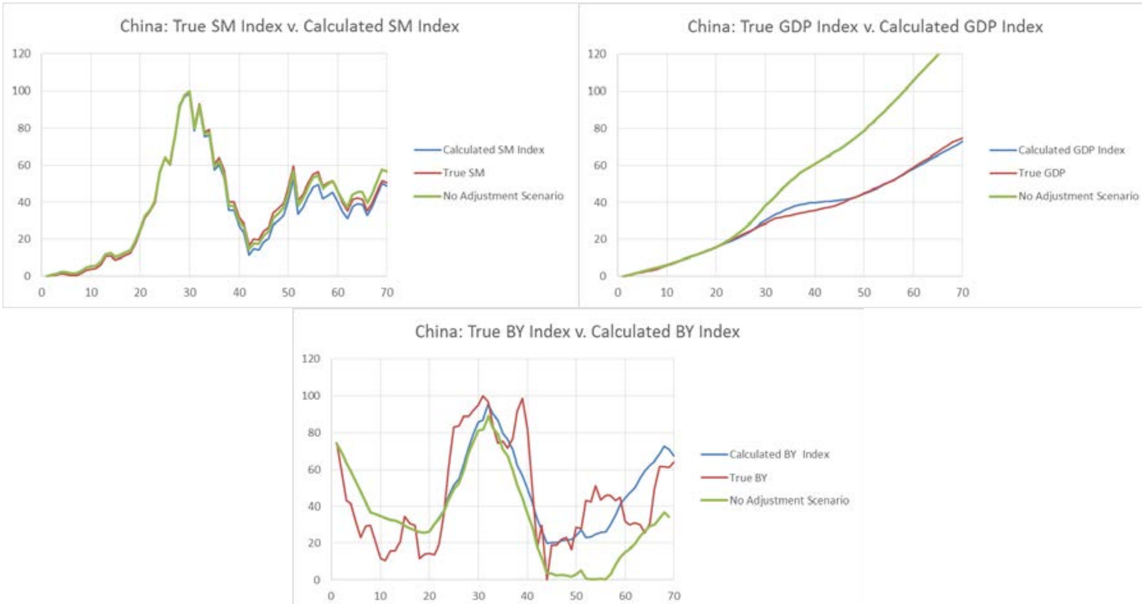


Figure 41: China Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the No Adjustment policy begins.

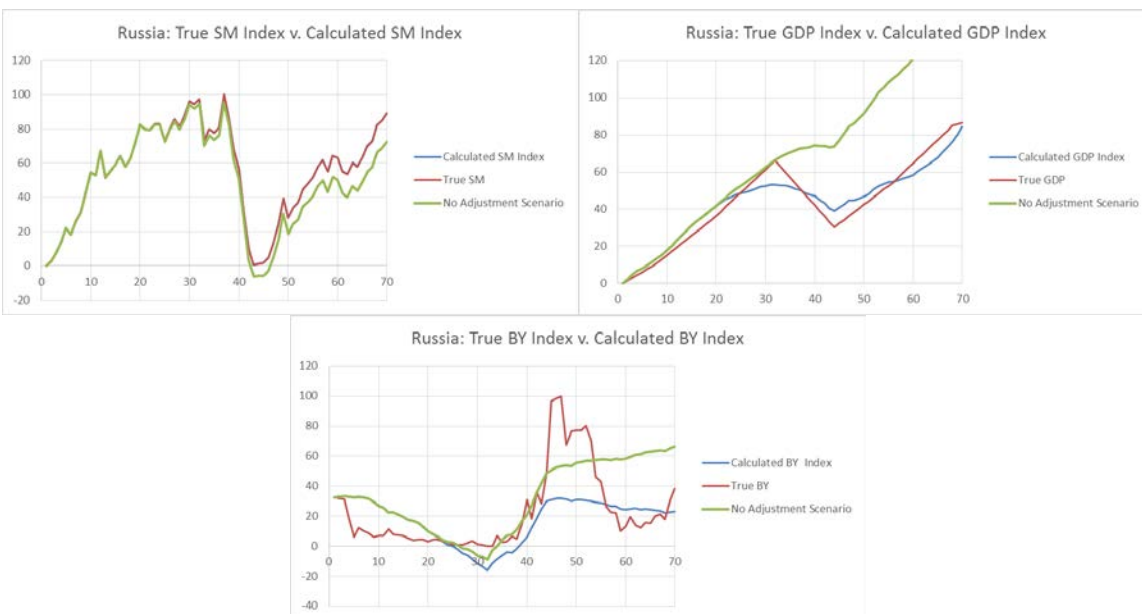


Figure 42: Russia Economic Condition Index, Calculated Index, and True Index Values over time. The 21st period is when the No Adjustment policy begins.

Appendix L: Storyboard

Modeling an Economy's Dynamics and External Influences Through a System of Differential Equations

Problem Statement

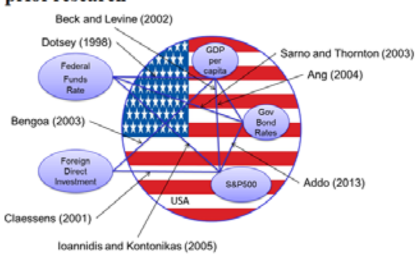
- From the open literature, the DoD does not have any models that address pecuniary warfare.
- Historical macroeconomic models do not sufficient capture the complex and dynamic nature of pecuniary warfare tactics.
- Dynamic systems modeling has been used to model other complex systems in nature

Objective

To build a foundation for analysis to begin addressing the impacts of pecuniary warfare.

1. Conjectured Single Country Model

- Defined the internal factors of an economy and the external factors that affect the economy
- Established relationships between the factors using prior research



2. Data Collection & Processing

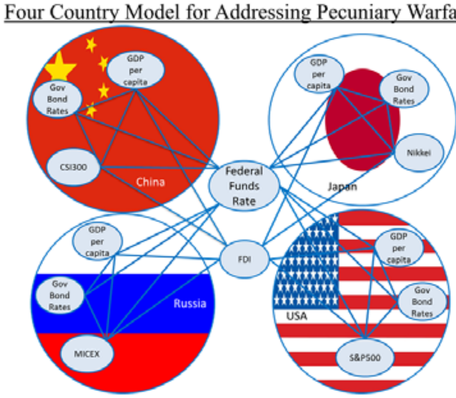
Sources
- World Bank and OECD National Accounts
- International Monetary Fund
- Investing.com
- United States Treasury

Format
- Monthly data to capture the short to medium term impacts
- Ranges from May 2006 to December 2013 (90 months)

Note: Data capture the 2008 financial crisis and contemporary economic condition

2d Lt Thomas Dickey
Advisor: Darryl K. Ahner, PhD, P.E.
Reader: Richard Deckro, DBA
Department of Operational Sciences (ENS)
Air Force Institute of Technology

Four Country Model for Addressing Pecuniary Warfare



3. Create the Functional Form of the System

$$\dot{SM}_t = \alpha_{11}\left(\frac{GDP_t}{\beta_{11}} - 1\right) + \alpha_{12}\left(\frac{GDP_{t-1}}{\beta_{12}} - 1\right) + \alpha_{13}\left(\frac{SM_t}{\beta_{13}} - 1\right) + \alpha_{14}\left(\frac{SM_{t-1}}{\beta_{14}} - 1\right) + \alpha_{16}\left(\frac{BY_t}{\beta_{16}} - 1\right) + \delta_{11}FFR_t + \delta_{12}FDI_t$$

$$\dot{GDP}_t = \alpha_{21}\left(\frac{GDP_t}{\beta_{21}} - 1\right) + \alpha_{22}\left(\frac{GDP_{t-1}}{\beta_{22}} - 1\right) + \alpha_{23}\left(\frac{SM_t}{\beta_{23}} - 1\right) + \alpha_{26}\left(\frac{BY_t}{\beta_{26}} - 1\right) + \delta_{21}FFR_t + \delta_{22}FDI_t$$

$$\dot{BY}_t = \alpha_{31}\left(\frac{GDP_t}{\beta_{31}} - 1\right) + \alpha_{32}\left(\frac{GDP_{t-1}}{\beta_{32}} - 1\right) + \alpha_{33}\left(\frac{SM_t}{\beta_{33}} - 1\right) + \alpha_{34}\left(\frac{SM_{t-1}}{\beta_{34}} - 1\right) + \delta_{31}FFR_t$$

- Effects on the current state from internal factors are represented using a modified form of the Logistic Differential Equation

Effect on internal factor i from internal factor j: $\alpha_{ij}\left(\frac{internal\ factor\ j}{\beta_{ij}} - 1\right)$

- Effects on the current state from external factors are represented using a single weight value

Effect on internal factor i from external factor k: $\delta_{ik}(external\ factor\ k)$

4. Solve for α, β, and δ Coefficients

$$\underset{\alpha,\beta,\delta,}{Minimize} \sum_{t=1}^{70}\left(\widehat{\dot{GDP}_t} - \dot{GDP}_t\right)^2 + \left(\widehat{\dot{SM}_t} - \dot{SM}_t\right)^2 + \left(\widehat{\dot{BY}_t} - \dot{BY}_t\right)^2$$

$$\alpha_{ij} \in \mathbb{R}\ for\ i = 1,2,3; j = 1,2,\ldots,5$$

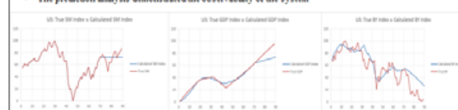
$$\beta_{ij} \in \mathbb{R}\ for\ i = 1,2,3; j = 1,2,\ldots,5$$

$$\delta_{ik} \in \mathbb{R}\ for\ i = 1,2,3;\ k = 1,2$$

Model was fit by two approaches: slope between one lag and the current time and the slope calculate by the secant method.

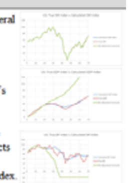
5. Prediction Analysis

- 6 month and 1.5 year predictions were calculated using Euler Method and the derivatives defined by the dynamic system
- The dynamic systems modeling predicted the factors state in the short term extremely well
- The model generally captured the long term trends of the factors
- The prediction analysis demonstrated the observability of the system



6. What-If Analysis

- Gradual Decrease Scenario: What if the Federal Reserve didn't drop the FFR to historic lows after the 2008 crisis?
- No Change Scenario: What if the Federal Reserve maintained the FFR at the past year's average?
- What-If Analysis showed that changes to the FFR policy had statistically significant impacts on the GDP per capita and BY indices of a country at the α=.05 level, but not the SM index.



Findings

- Repeatable methodology that uses open source data and common software
- Captures the interactions of an country's economy and the influence of external factors
- Demonstrates the capability to conduct near term forecasts and what-if analysis
- Developing models of pecuniary warfare may help to inform decision maker on the impacts of future policy

Impact

- Fills a gap, the first methodology to address pecuniary warfare
- Gives an analyst the ability to "turn the dial" on the economic instrument of national power and see the impact to the economic condition of a country

Future Research

- Enhance data collection to find the "best" data set.
- Incorporate additional factors, both internal and external, like trade or a debt limit
- Account for other country's' economic instruments of national power in the model
- Application of the model to inform the cost, payoff, or behavioral functions in mean field game theory

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